OECD Economics Department Working Papers No. 143

GREEN - The Reference Manual

Dominique van der Mensbrugghe

https://dx.doi.org/10.1787/054442860076

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Economics Department
Working Papers
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Dominique van der Mensbrugghe

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT

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GREEN: THE REFERENCE MANUAL

This document provides a full description of the GREEN model. It is intended to accompany the GREEN code, i.e. the implementation of the model, and to enable the user to understand the links between the theoretical framework of the model and its practical implementation.

The document lists all the model equations, provides a data dictionary to link the equation variables with the variables in the code, explains details which are traditionally bypassed in technical papers, and provides an explanation of the data base and the data management part of the code.

The document is organised as follows. Following a non-technical overview of the model in Part I, Part II presents the structure of the model with a complete description of the equations, the variables, and parameters which are part of the GREEN model. Part III explains the data management in GREEN.

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GREEN : LE MANUEL DE RÉFÉRENCE

Ce document contient une description complète du modèle GREEN. Il est destiné à accompagner sa version informatique et à permettre ainsi à l'utilisateur de faire le lien entre la version théorique du modèle et sa mise en oeuvre pratique.

Ce document décrit toutes les équations du modèle, contient un dictionnaire qui relie les variables des équations aux variables qui se trouvent dans le programme informatique, et détaille certains aspects techniques qui sont souvent omis des documentations de référence.

Ce document est organisé comme suit. La partie I donne, en termes non-techniques, un résumé du modèle. La partie II décrit la structure du modèle et de chacune de ses équations, ainsi que les variables et les paramètres de GREEN. La partie III explique la gestion des données dans le modèle.

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GREEN: THE REFERENCE MANUAL

by

Dominique van der Mensbrugghe[1]

Introduction

The OECD Secretariat has developed a multi-country, multi-sector, dynamic applied general equilibrium (AGE) model to quantify the economy-wide and global costs of policies to curb emissions of carbon dioxide (CO_2). It is called the **G**ene**R**al **E**quilibrium **EN**vironmental model, hereafter referred to as GREEN.

The GREEN project was initiated at the OECD in 1990 at the request of Working Party No. 1 of the Economic Policy Committee. The first phase of the project, which was directed by John P. Martin, consisted of specifying the model, developing its computer software and simulating it for a wide range of policy-relevant scenarios. Drawing from the experience of the Secretariat in modeling agricultural policies (the WALRAS model), the base specification and structure of GREEN were designed by Jean-Marc Burniaux who also carried out the early stages of the model building. He was subsequently joined by Giuseppe Nicoletti and Joaquim Oliveira-Martins who contributed to enrich the model structure and to elaborate its methodological documentation. For an earlier description of the GREEN specification, see Burniaux et. al. (1992), which will be referred to as WP116. WP116 complements this reference manual. It provides more information on the parameterization and data of the GREEN model, as well as more detailed theoretical justification for the model specification. This reference manual is meant to accompany the source code of the model and to document the latest version of GREEN Many of the results of simulations using GREEN to quantify the economic costs of curbing CO_2 emissions have been published in a special issue of *OECD Economic Studies*, No. 19, Winter 1992.

The second phase of the GREEN project, underway since Autumn 1992, comprises various modifications and extensions, as well as the preparation of a *user-friendly* version to be made available to users outside the OECD Secretariat. This latter task, including the preparation of a User Manual and a Reference Manual for GREEN, has been carried out by Dominique van der Mensbrugghe.

This document presents the structure and functioning of the GREEN model. It lists all the model equations, provides a data dictionary to link the equation variables with the variables in the code, explains details which are traditionally bypassed in technical papers, and provides an explanation of the data base and the data management part of the code.

The new version of GREEN has been coded in C (rather than FORTRAN). Although FORTRAN has the advantage of being better known by the current generation of economists (and scientists), C is the preferred language for most new programming projects on work stations and microcomputers. It has also become the language of choice for teaching computer programming in many computer science departments, so that a growing number of economists are familiar with it. While C has a rich set of instructions, particularly with respect to data management within the program, the basic elements of C are similar to FORTRAN structures. The new GREEN code does, however, use many of the advanced features of C which are intended to enhance its readability and ease of use; hence, for those readers who are only familiar with FORTRAN there will be some startup costs to understand these features.

The document is organised as follows: following a non-technical overview of the model in Part I, Part II presents the structure of the model with a complete description of the equations, the variables, and parameters which are part of the GREEN model. Part III explains the data management in GREEN.

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[1] The author would like to express his gratitude to John Martin and Peter Sturm for constructive advice and comments on various drafts of the GREEN Reference Manual, as well as to participants of GREEN workshops. The author owes a great debt to Joaquim Oliveira-Martins who provided extensive explanations of the more intricate parts of the GREEN model, and was a patient and critical beta-tester of the new version of GREEN and its accompanying documentation.

I. Brief Overview of Green

The key dimensions of GREEN are set out in Annex 1. It runs over a 65-year time horizon from 1985 to 2050, using time intervals of five years between 1985 and 2010 and twenty years thereafter. It consists of twelve detailed regional sub-models: four OECD regions – United States (USA), Japan (JPN), EC (EEC) and Other OECD (OOE) – and eight non-OECD regions – the former USSR (FSU), the Central and Eastern European Countries (EET), China (CHN), India (IND), the Energy-exporting LDCs (EEX), the Dynamic Asian Economies (DAE), Brazil (BRA), and the Rest of the World (ROW).

The remainder of this section outlines briefly the main characteristics of supply, demand, the dynamics and the policy instruments of the model. Part II describes in full detail each block of the model.

1. Supply, Demand and Foreign Trade

(a) Production

There are fifteen producing sectors in GREEN. Twelve sectors concern the supply and distribution of energy: coal mining, crude oil, natural gas, refined oil, electricity, gas and water distribution[2] and seven back-stop technologies. The latter are defined as known technologies which are assumed to become commercially available in the future. The seven back-stop technologies – a carbon-based back-stop for coal, crude oil and gas, a carbon-free back-stop for the same fuels, and a carbon-free electric back-stop – are assumed to come on stream in all regions only by 2010. The remaining sectors are broad aggregates of the rest of the economy: agriculture, energy-intensive industries and other industries and services.

All sectors are assumed to operate under constant returns to scale and cost optimisation. Production technology is modeled mainly by a nesting of constant-elasticity-of-substitution (CES) functions, which is depicted in Figures 1a-b. There are a few exceptions to the CES nesting: all inputs are assumed to be used in fixed proportions (Leontief technology) in the production of fossil fuels (coal, crude oil, natural gas), petroleum products and the back-stop technologies.

In each period, the supply of **primary** factors – capital and labour – is usually predetermined.[3] However, supplies of agricultural land, the carbon-free electric resource (nuclear, hydro, and geothermal), oil, natural gas and coal are all assumed to be sensitive to their contemporaneous prices.

The model includes adjustment rigidities. An important feature is the distinction between *old* and *new* capital goods. In addition, capital is assumed to be partially mobile, reflecting differences in the marketability of capital goods across sectors.[4]

Once the optimal combination of inputs is determined, sectoral output prices are calculated assuming competitive supply (zero-profit) conditions in all markets.

(b) Energy Prices

When demand does not exceed potential supply (whose determination is described below), coal and gas prices are determined by the supply elasticity of their respective fixed factor (i.e. their resource base). This elasticity is assumed to be asymmetric with respect to output changes. Generally, it is higher in response to downward than to upward variations.

The real world price of crude oil is endogenous in GREEN.[5] This is implemented by introducing a supply equation for oil in the Energy-exporting LCD's. This region (mainly OPEC) is assumed to have a finite supply elasticity. Nonetheless, there is an upward bound to oil production in the Energy-exporting LCD's which is set by the level of their available reserves. All other producers are price-takers, their supply of oil being strictly determined by their resource constraint.

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2 Because of data constraints it was not possible to isolate the electricity sector from the gas and water distribution sector.

3 Capital supply is to some extent dependent on the current period's investment level.

4 For simplicity, it is assumed that old capital goods supplied in second-hand markets and new capital goods are homogeneous. This formulation makes it possible to introduce downward rigities in the adjustment of capital without increasing excessively the number of equilibrium prices to be determined by the model (see Fullerton, 1983).

5 The real world price of oil is computed with respect to a weighted average of real exchange rates in the OECD regions. In each country, the real exchange rate is defined as the ratio of a weighted average of domestic primary factor prices to the numéraire of the model.

The prices of back-stop technologies are exogenous; they were taken from the Stanford-based Energy Modeling Forum Study no. 12 (EMF 12) entitled "Global Climate Change: Energy Sector Impacts of Greenhouse Gas Emission Control Strategies". By definition, back-stop technologies, once they come on stream, are available in all regions in unlimited quantities at constant marginal costs (see Manne and Richels, 1991). As a consequence, this rules out any incentive to trade in "new" energy sources between regions.

(c) Consumption and the closure rule

All income generated by economic activity is assumed to be distributed to consumers. A single representative consumer allocates optimally his/her disposable income among four broad consumer goods (food and beverages, fuel and power, transport and communication and other goods and services) and saving. The structure of household demand is depicted in Figure 2. The consumption/saving decision is completely static: saving is treated as a "good" and its amount is determined simultaneously with the demands for the other four goods, the price of saving being set arbitrarily equal to the average price of consumer goods.[6] Given energy prices, consumers choose an optimal mix of fuels, except in the transport and communication sector where the sole energy input is refined petroleum products.

The government collects carbon or energy taxes, income taxes and indirect taxes on intermediate inputs, outputs and consumer expenditures. Revenues of carbon taxation or from trade in emission rights are recycled by assuming revenue-neutrality: the changes in the government budget are automatically compensated by changes in marginal income tax rates. This assumption is considered the appropriate closure to apply to the government sector for long-term simulations. Government expenditures are exogenous in real terms, growing at the same rate as GDP.

Each region runs a current-account surplus (deficit) which is fixed in nominal terms. The counterpart of these imbalances is a net outflow (inflow) of capital, which is subtracted (added to) the domestic flow of saving. In each period, the model equates gross investment to net saving (equal to the sum of saving by households, the net budget position of the government and foreign capital inflows). This particular closure rule implies that investment is driven by saving.

(d) Foreign Trade

The world trade block is based on a set of regional bilateral flows. The basic assumption in GREEN is that imports originating in different regions are imperfect substitutes. Therefore, in each region, total import demand for each good is allocated across trading partners according to the relationship between their export prices. This specification of imports – commonly referred to as the Armington specification – implies that each region faces downward-sloping demand curves for its exports. This is implemented for all goods except crude oil, which is assumed to be a homogeneous commodity across regions, implying a unique world oil price. Natural gas and coal are assumed to be heterogeneous goods across regions because their transportation costs are much higher than for oil.

2. Dynamic Features and Calibration

The current version of GREEN has a simple recursive dynamic structure as agents are assumed to be myopic and to base their decisions on static expectations about prices and quantities. Dynamics in GREEN originate from three sources: i) depletion of fossil-fuel resources; ii) accumulation of productive capital; and iii) the putty/semi-putty specification of technology.

(a) Resource Depletion

While coal reserves are assumed to be infinite over the current time horizon in GREEN, the supplies of crude oil and natural gas are derived from a resource depletion sub-model. The sub-model makes potential supply dependent on the initial levels of proven and unproven (so-called "yet-to-find") reserves, the rate of reserve discovery and the rate of extraction.[7] Whether potential output increases or decreases over time depends on whether extracted resources are balanced by newly discovered resources. In the version of GREEN used in this

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[6] The demand system used in GREEN is a version of the Extended Linear Expenditure System (ELES) which was first developed by Lluch (1973). The formulation of the ELES used in GREEN is based on atemporal maximisation — see Howe (1975). In this formulation, the marginal propensity to save out of supernumerary income is constant and independent of the rate of reproduction of capital.

[7] The rate of reserve discovery is the rate at which unproven reserves are converted into proven reserves, while the rate of extraction is the rate at which proven reserves are converted into output.

paper, the levels of yet-to-find reserves are fixed and correspond to the highest estimates of resources taken from the guidelines laid down by EMF 12. Nonetheless, the depletion mechanism embodies some price sensitivity via the rate of reserve discovery which is assumed to be a function of the world oil and gas prices.

(b) Capital Accumulation

In the aggregate, the basic capital accumulation function equates the current capital stock to the depreciated stock inherited from the previous period plus gross investment. However, at the sectoral level, the specific accumulation functions may differ because the demand for (old and new) capital can be less than the depreciated stock of old capital. In this case, the sector contracts over time by releasing old capital goods. Consequently, in each period, the new capital vintage available to expanding industries is equal to the sum of disinvested capital in contracting industries plus total saving generated by the economy, consistent with the closure rule of the model.

(c) The putty/semi-putty specification

The substitution possibilities among production factors are assumed to be higher with the *new* than with the *old* capital vintages – technology has a putty/semi-putty specification. Hence, when a shock to relative prices occurs (e.g. the imposition of a carbon tax), the demands for production factors adjust gradually to the long-run optimum because the substitution effects are delayed over time. The adjustment path depends on the values of the short-run elasticities of substitution and the replacement rate of capital. As the latter determines the pace at which the new vintages are installed, the larger is the volume of new investment, the greater the possibility to achieve the long-run total amount of substitution among production factors.

(d) Dynamic calibration

The model is calibrated on exogenous growth rates of population, GDP per capita and an autonomous energy efficiency improvement in energy use (known as the AEEI factor). In the so-called Business-as-Usual (BaU) scenario, the dynamics are calibrated in each region by imposing the assumption of a **balanced growth** path. This implies that the ratio between labour and the capital/fixed-factor bundle (in efficiency units) is held constant over time.[8] When alternative scenarios around the baseline are simulated, the growth of capital is endogenously determined by the saving/investment relation.

3. Policy Instruments

GREEN has several policy instruments to achieve CO_2 emission reductions: i) carbon, energy or mixed taxes (computed either as equilibrium shadow prices of a carbon constraint or set exogenously); and ii) tradable permits.

The *carbon tax* is an excise tax, which is expressed as a **fixed** absolute amount of US$ per ton of carbon emitted by each fuel. The tax is applied at the level of consumers of **primary** fuels[9], thereby avoiding distortions between domestic and imported fuels; it is applied prior to any indirect taxation included in the model.

GREEN also incorporates the possibility that any international agreement to curb CO_2 emissions could include a provision allowing countries to *trade emission rights*. In the present version of the model, countries are endowed with initial quotas of emission rights set equal to the upper bounds on emissions imposed in the no-trade case and this is fixed in all time periods. In this case, a unique constraint on carbon emissions is imposed at the world level and a single world price of permits – the shadow price of carbon – is determined with free trade in permits. Regions with a lower carbon tax in the no-trade case will want to sell rights, while those in the opposite situation will want to buy them. Trade in emission rights gives rise to flows of income between regions which modify the current account constraint. It is assumed that these income flows increase or decrease government revenues, depending upon whether the region in question is a seller or a buyer of emission permits.

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[8] This involves computing in each period a measure of Harrod-neutral technical progress in the capital/fixed-factor bundle as a residual, given that the growth of the labour force (in efficiency units) is equal to the exogenous growth in GDP. This is a standard calibration procedure in dynamic AGE modelling — see Ballard et al. (1985).

[9] Alternative implementations of the carbon tax in the model e.g. levying it at the point of production, would probably produce different results (see Whalley and Wigle, 1991).

II. Structure of the GREEN Model

Overall Structure

The following list describes the overall structure of the GREEN program:

1. Initialise Model

   Allocate memory for variables and coefficients and setup data base ;
   Read the control file ;
   Read the base data ;
   Read the base parameters (mainly elasticities) ;
   Calibrate all other parameters and initialise all variables not read in from base data set ;

2. Solve Model

   The model is solved one period at a time, progressing from 1985 to 2050 ;
   Within each time period, the model is solved as a sequence of equations; The algorithm stops iterating through the model when all factor markets clear (within a given tolerance level).

3. Print the results

   Results can be printed in several formats. The program will generate a series of tables detailing the key results for the current simulation. The results will also be dumped in data files which can be read by downstream applications for further analysis and comparisons with other simulations.

The next sections will describe in more detail each part of the program.

Notation conventions.

File names and user input data will be printed using a fixed space font such as `Courier`. Subroutines will appear in bold and italics, e.g. ***solve***. Variable names will appear in italics, e.g. *gap*.

Time and region subscripts will be dropped from most equations unless they are strictly necessary. For example, time subscripts need to appear in all transition equations, and regional subscripts are necessary for describing the bilateral trade flows. A superscript *d* refers to demand, and *s* refers to supply. The meaning of the other subscripts/superscripts is described in Annex 1.

Demand and demand prices are agent-specific. For notational purposes, agents are identified by a single letter, appended to the name of the corresponding variable, rather than by an index. For example, *XM*, refers to the aggregate volume of imports. *XMc*, refers to household volume of imports. There are five types of agents in the economy: producer, designated by the letter *p*, consumer (or household), designated by the letter *c*, government, *g*, investment, *i*, and stock building, *st*.

All commodity prices start with the letter *P*. Most sector specific volumes start with the letter *X* (exports are an exception). This is followed by another letter(s) describing the type of commodity. *D* refers to a commodity produced and consumed domestically, e.g. *PD*. *M* refers to an imported commodity, e.g. *PM*. *A* refers to the Armington composite commodity. *E* refers to the export price.

Share parameters always start with the letter *a*. Subsequent letters and numbers help to identify the nature of the parameter. For example *al*, refers to the labour share parameter, and *aex*, designates a share parameter used in the determination of energy demand, where *x* is replaced by an appropriate agent abbreviation (*p*, *c*, *g*, *i*, or *st*). A number refers to the sequencing of the share in a nested structure (only needed in energy). For example, in energy demand by households, *aec1* is the share parameter for the splitting of the energy bundle into fuel components, *aec2* is the share parameter for splitting the fossil fuel components into conventional fuels and the back-stops, and *aec3* is the share parameter for splitting the electricity fuel component into conventional electricity and the back-stop electric option.

A. Model Initialisation

Model initialisation occurs in five steps.

1. Allocate memory for variables and coefficients and setup the variable and coefficient data bases.
2. Read the parameters in the control file. The control file contains file names and simulation options (it is exhaustively described in the GREEN User Manual).
3. Read the base data. The base data is contained in a separate data file (which is in GAMS compatible format). It is described in the GREEN User Manual.
4. Read the base parameters. The base parameters are contained in a separate data file. The file is described in the GREEN User Manual.
5. Calibrate the model parameters.

[Note: This section will be elaborated in a future version of the manual. For the moment, it mainly indicates the name of the subroutines and the files where the pertinent code is located.]

1. Data base initialisation

Memory for all regional variables and coefficients is allocated at the beginning of the program. For variables, this occurs in the file `vars.c`. The main subroutine controlling memory allocation for variables is called ***AllocVars***. Not only does this subroutine allocate memory, it also sets up the variable data base. A similar subroutine exists for the coefficients, it is called ***AllocCofs***, and is in the file `cofs.c`.

2. Reading of control file

The subroutine ***ReadControl*** in the file `readctl.c` handles the input of the parameters and simulation options in the control file. This subroutine is called from the ***InitProg*** subroutine. The ***ReadControl*** subroutine also reads the carbon tax specification file.

3. Reading of base data file

The subroutine ***ReadData*** in the file `readdata.c` handles the reading of the base data file (and the parameter files). All data and parameter files are in GAMS compatible format. All GAMS compatible files are read using a subroutine called ***ReadGams***. This is a flexible routine which will read any GREEN variable or parameter assuming the user uses the corresponding name of a GREEN variable or parameter. (These are detailed in the GREEN User Manual.) If modifications are made to the GREEN code which include the introduction of new variables (or parameters), the programmer does not need to do anything about input, the ***ReadGams*** routine will automatically assign the data in the file to the variable defined in the code.

4. Reading of parameter file(s)

Parameter files are also read in by the ***ReadData*** subroutine. There are three parameter files: the base parameters, used to calibrate the model in the base year; the dynamic parameters (or the scenario definition), parameters which are time subscripted; and the back-stop parameters, which determine the cost of the back-stop substitutes and demand structure.

5. Parameter calibration

Typical to most CGE models, GREEN is a calibrated model. Using the base data and a set of exogenous parameters (most often elasticities), all share and shift parameters in the model are calibrated in such a way to re-produce the base year data set using the model equations. Calibration can be done on a block by block basis, e.g. the production block, followed by the consumption block, etc. The following example shows how to calibrate a general CES function.[10] Suppose we have a two-good CES, the two goods being D and M, with respectively, price PD and PM. Let the substitution elasticity be given by σ. Normally D, M, PD, and PM are given in the base data set, and σ is given (or sometimes independently estimated). The CES first order

[10] See Annex 2 for a description of the CES function.

conditions are inverted in order to determine the share parameters. (Assumptions have to be made on the aggregate (CES) price, or the aggregate volume. Often times, the aggregate price will be set to 1 (and the goods prices are also often set at 1). If the aggregate price is set to 1, the aggregate volume is given by:

$$V = PD\,D + PM\,M$$

The share parameters are calibrated using the following formulas:[11]

$$\alpha_d = \frac{D}{V}\left(\frac{PD}{P}\right)^{\sigma}$$

$$\alpha_m = \frac{M}{V}\left(\frac{PM}{P}\right)^{\sigma}$$

The entire calibration process occurs in the subroutine ***Calibrate*** contained in the file `calibrat.c`. The calibration process is split by block, with a separate calibration subroutine for each block.

There are some degrees of freedom in calibrating CGE models, particularly with respect to prices (or more accurately with respect to disaggregating the value data into a price and a quantity). Typically, in CGE models, all prices are set to 1, hence volumes are the same as their values, and the share parameters in the CES function are true value share parameters. In this version of GREEN, almost all prices are set to 1, including factor prices. The only exceptions are for sectoral variables subject to price wedges (i.e. distortions). The rule which has been adopted is that the producer (or output) price, inclusive of producer taxes/subsidies, is equal to 1. This means that the unit cost of production will be different from 1 in sectors with production taxes/subsidies. The sales price of domestic goods also differs from 1 if there are sales taxes. World import prices are set to 1, but domestic import prices may differ from 1 in the presence of tariffs. No export distortions are included in the model, hence the export price is also set to 1. No other distortions exist in the base year data, hence all other prices are initially set to 1.

B. Model Equations

The main module (contained in the file `green.c`) passes control over to ***solve*** after the initialisation process. The subroutine ***solve*** controls the iterations, both over time, and within each time period. The following provides the pseudo-code for ***solve***:

[11] The CES share parameters always incorporate the elasticity exponent in the code. In other words, all CES share parameters in the code are not the real share parameters, but the share parameters raised to the power of the subsitution elasticity.

```
Loop from 1985 to 2050
    Initialise current time period ;
        Initialise the values of endogenous variables by copying their values from t-1 to t [12] ;
        Update the "old" capital stocks ;
        Update other exogenous variables ;
        Update exogenous growth factors ;
        Re-calibrate the "old" vintage share parameters ;
        Calibrate the back-stop parameters (if back-stops are implemented) ;
    Loop until convergence is achieved ;
        Save the values of the previous iteration ;
        Loop over all regions ;
            Calculate the model equations for each region ;
            Update the factor prices ;
        Update equations for global module ;
        Check for convergence ;
```

1. Initialisation at the beginning of each new period

At the beginning of each period, the following steps are taken:

All endogenous variables are initialised to the values of the previous period (X[t] = X[t-1]) ;[13]
All pre-determined variables are updated ;
The "old" vintage share parameters are re-calibrated ;
The back-stop parameters are initialised ;

Discussion in this section relates to the subroutine ***InitPeriod*** which is contained in the file `solve.c`. ***InitPeriod*** is called once at the beginning of each period from the ***solve*** subroutine.

a) In order to make the model "gap" independent, the period gap is calculated and is stored in the global variable *gap*.[14] For example, in 1990, the variable *gap* has the value 5 (1990-1985). In the period 2030, the variable *gap* has the value 20 (2030-2010). This means that the transition are not hard coded depending on the size of the gap. (Note that the capital accumulation equation is independent of the size of the gap, which in general will tend to overstate the share of new capital if the gap is high, say 20 years. In other words, the share of new capital in the year 2050 will be different depending on whether the model jumps from 2030 to 2050, or whether it solves for 2035, 2040, and 2045 as well.)

b) There are two possibilities for initialising the endogenous variables at the beginning of a new time period. The default is to initialise the endogenous variables with the solution of the previous period, i.e. X[t] = X[t-1]. However, a user can choose to initialise the endogenous variables with the solution of a previous simulation (which will have been stored in a data file), i.e. X[t] = X'[t], where X'[t] is the solution from a previous simulation. The latter option is useful when simulations are run with policy changes occurring in later years. In this case, the program can quickly jump to the year in which policy changes occur, and avoid wasting computer time solving periods where there are no changes.

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12 The time subscript of variables has a range of 0-9. 0 corresponds to the base data which happens to be 1985. The range 1-8 corresponds to each of the solution periods of the model, i.e. 1 corresponds to 1985, 2 to 1990, etc. Therefore, there are two copies of the 1985 data. The index 9, is used as temporary storage. In between each iteration (within the current period), the values of the previous iteration are stored in this temporary location. This makes it possible to compare variable levels between two iterations.

13 There can be some exceptions. Only endogenous variables which are in the regional database and the global database are initialised this way. The carbon taxes are defined at a zone level (which is defined below), i.e. they are neither part of a region, nor in the global database. The carbon tax (for the zone) is initialised at a value of $50 per ton of carbon. This may lead to convergence difficulties.

14 The model is only gap independent in the sense that the specification of each equation does not need to know the step-size. This has implications for the share of new capital in total capital. If the step-size is large, the quantity of old capital will be relatively smaller than with a short step size because of the way in which depreciation is accounted for (see Equation 20.1.1). In other words, the quantity of old capital in the year 2050 will differ depending on whether the step size between 2030 and 2050 is 20 years, or 5 years. While the specification of the model does not depend on the time framework, the input data files do. The GREEN User Manual describes the time structure of the input files.

c) Many (but not all) transition equations are predetermined, i.e. they can be determined prior to starting the iteration for the next period and do not rely on any current variables. One exception is the total capital stock. It will respond to changes in current investment since the transition equation uses the change in investment (between periods) to calculate a growth rate for new capital. The following two tables present the pre-determined transition equations:

Table 1.1: **Initial Supply of Old Capital**

(1.1.1) $$KO^{s}_{i,t} = (1-\delta)^{n} K^{d}_{i,t-n}$$

(1.1.2) $$KO^{s}_{g,t} = (1-\delta)^{n} K^{d}_{g,t-n}$$

In Table 1.1 KO^{s}_{i} represents the installed old capital at the beginning of each period by sector.[15] It is simply equal to the sector's previous period's total (depreciated) capital stock. The end of period stock of old capital (in a given sector) may be less than the initial stock. If the sector is declining, old capital will be disinvested and the actual stock of old capital will be less than the initial stock. KO^{s}_{g}, in Equation (1.1.2), represents the relevant variable for the government sector.

Table 1.2: **Other Pre-Determined Exogenous Variables**

(1.2.1) $$L^{s}_{t} = (1+\gamma^{y}_{t})^{n} L^{s}_{t-n}$$

(1.2.2) $$Pop_{t} = (1+\gamma^{p}_{t})^{n} Pop_{t-n}$$

(1.2.3) $$TG_{t} = (1+\gamma^{y}_{t})^{n} TG_{t-n}$$

(1.2.4) $$TRG_{t} = (1+\gamma^{y}_{t})^{n} TRG_{t-n}$$

In Table 1.2 L^{S} represents labour supply (in efficiency units). Labour supply grows at an exogenous rate which is the sum of the population growth rate and the growth rate of labour efficiency. This aggregate growth is designated by γ^{y} and is equal to $\gamma^{p} + \gamma^{l}$ where γ^{p} is the population growth rate, and γ^{l} is the growth rate of labour efficiency. Pop_{t} is the population at time t. *TG* is the level of total real government non-transfer expenditures (including goods, services, labour, and capital). *TRG* is the real level of transfers from government to households. Both *TG* and *TRG* are assumed to grow at the same rate as the economy.

The energy efficiency factors are also exogenous and pre-determined leading to the following set of transition equations:

15 The base year data does not provide capital stock data by sector, only capital remuneration by sector. Since the price of capital in each sector is set to 1 in the base year, the sectoral capital stock data should be seen as an index volume and not a volume which has been observed. Since it is further assumed that the aggregate capital stock is the sum of the sectoral capital stock, this implicitly implies that the price of capital is the same across sectors in the base year.

Table 1.3: **Energy Efficiency Factors**

(1.3.1) $$\lambda^{e,p}_{i,v,t} = (1+\gamma^{e,p}_{i,v,t})^n \lambda^{e,p}_{i,v,t-n}$$

(1.3.2) $$\lambda^{e,c}_{k,t} = (1+\gamma^{e,c}_{k,t})^n \lambda^{e,c}_{k,t-n}$$

(1.3.3) $$\lambda^{e,g}_{i} = (1+\gamma^{e,g}_{t})^n \lambda^{e,g}_{i-n}$$

(1.3.4) $$\lambda^{e,i}_{i} = (1+\gamma^{e,i}_{t})^n \lambda^{e,i}_{i-n}$$

(1.3.5) $$\lambda^{e,st}_{i} = (1+\gamma^{e,st}_{t})^n \lambda^{e,st}_{i-n}$$

The annual autonomous energy efficiency factors are given by $\gamma^{e,p}$, $\gamma^{e,c}$, $\gamma^{e,g}$, $\gamma^{e,i}$, and $\gamma^{e,st}$ representing respectively the growth in energy efficiency in production, household consumption, government demand, investment demand, and stock-building demand. The energy efficiency factors in production are specific to both sector and vintage. The energy efficiency factors in consumption are commodity specific.[16] The cumulative factors are given by the λ variables.

d) GREEN has a vintage structure of capital which is based on an assumption of a putty/semi-putty structure of production. It is further assumed that the substitutability of capital differs across vintage, with old capital typically less substitutable than new capital. There are only two vintages, old and new. New capital is generated by investment in the previous period. Old capital is the installed capital in the previous period. Over time, the structure of old capital changes as the previously new capital gets merged into the old capital. Rather than keep track of each vintage over time, we modify the structural parameters of the old capital to reflect its changing composition. The key rule that has been adopted is that the share parameters associated with old capital should be able to produce all of the previous period's production (with the substitution elasticities of the old capital). For example, assume we have a CES production function in capital (K), labour (L), and energy (E). Production then has the form:

$$X_v = [a_{k,v}K_v^{\rho_v} + a_{l,v}L_v^{\rho_v} + a_{e,v}E_v^{\rho_v}]^{1/\rho_v}$$

where

$$\sigma_v = \frac{1}{1-\rho_v}$$

and X is output (by vintage), K_V is capital by vintage, L_V is labour, and E_V is energy. The share parameters are vintage specific as is the substitution elasticity. The first order conditions for cost minimisation lead to:

$$K_v = \alpha_{k,v} X_v \left(\frac{P_v}{r_v}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{k,v} = a_{k,v}^{\sigma_v}$$

$$L_v = \alpha_{l,v} X_v \left(\frac{P_v}{w}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{l,v} = a_{l,v}^{\sigma_v}$$

$$E_v = \alpha_{e,v} X_v \left(\frac{P_v}{e_v}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{e,v} = a_{e,v}^{\sigma_v}$$

where r, w, and e are respectively the price of capital, labour, and energy. (Note that the model uses the modified share parameters, i.e. the share parameters of the CES function raised to the power of the substitution elasticity. The former share parameters are never formally employed in the model since only

[16] Currently the reference scenario assumes a uniform 1 per cent per annum increase in all regions for all economic sectors, i.e. $\gamma = 0.01$, for all agents, sectors, and vintages, except in the production of fossil fuels and refined petroleum products.

the first order conditions and the CES price function are used.) P is the CES dual price which is given by the following equation:

$$P_v = \left[\alpha_{k,v} r_v^{1-\sigma_v} + \alpha_{l,v} w^{1-\sigma_v} + \alpha_{e,v} e_v^{1-\sigma_v}\right]^{1/(1-\sigma_v)}$$

We assume that the production structure of output associated with new capital is constant over time, i.e. the share parameters and substitution elasticities are not time dependent (except for the efficiency factors). However, old capital changes over time as in each time period previously new capital is added to the old capital stock. In order to account for the change in old capital the share parameters for the production structure associated with old capital are modified in such a way that the total of the factors in the previous period could produce all of the previous period's output assuming the old substitution elasticities. To continue with the above notation, we re-calibrate the share parameters according to the following formula:

$$\bar{\alpha}_{k,o} = \frac{K_{t-1}}{X_{t-1}}\left(\frac{r_{t-1}}{P_{t-1}}\right)^{\sigma_o}$$

$$\bar{\alpha}_{l,o} = \frac{L_{t-1}}{X_{t-1}}\left(\frac{w_{t-1}}{P_{t-1}}\right)^{\sigma_o}$$

$$\bar{\alpha}_{e,o} = \frac{E_{t-1}}{X_{t-1}}\left(\frac{e_{t-1}}{P_{t-1}}\right)^{\sigma_o}$$

where the re-calibrated share parameters, $\bar{\alpha}$, are calibrated at the beginning of each period, and all the volumes are the sum of the old and new vintages from the previous period, and the prices are the average prices (N.B. the subscript o is used for old capital, and n for new capital):

$$X_{t-1} = X_{o,t-1} + X_{n,t-1}$$

$$K_{t-1} = K_{o,t-1} + K_{n,t-1}$$

$$E_{t-1} = E_{o,t-1} + E_{n,t-1}$$

$$P_{t-1} = \left[P_{o,t-1} X_{o,t-1} + P_{n,t-1} X_{n,t-1}\right] / X_{t-1}$$

$$r_{t-1} = \left[r_{o,t-1} K_{o,t-1} + r_{n,t-1} K_{n,t-1}\right] / K_{t-1}$$

$$e_{t-1} = \left[e_{o,t-1} E_{o,t-1} + e_{n,t-1} E_{n,t-1}\right] / E_{t-1}$$

With the above definitions of the aggregate factors and average factor prices, the production function associated with the $\bar{\alpha}$ parameters is consistent with the aggregate output X_{t-1}.

Rather than repeat the above formulas for all the nested CES production functions, users are referred to the code which should be self-explanatory. As described in more detail below, the production structure can be represented by a nested tree structure of CES and Leontief functions. Within this structure, there are five CES aggregation functions whose old-vintage share parameters are re-calibrated in the manner described above.

The Regional Models

Until convergence is achieved within each time period, the ***solve*** subroutine loops over all the regional (and global) equations of the model. The regional equations are contained in a set of subroutines which will be described below. The subroutine driving the regional equations is named ***RegionalModel*** (in the file `model.c`). The sequence of the subroutines is listed in the following table:

Name	*Description*
KapStock ;	Update capital stocks (and calibrate capital efficiency coefficient)
Depletion ;	Update depletion modules for natural gas and crude oil
BSPrice;	Update prices in back-stop sectors
ProdPrice ;	Update producer prices
TradePrice ;	Update top level Armington prices
ConsPrice ;	Update consumer prices
InvStPrice ;	Update prices in investment and stock building final demand
GovPrice ;	Update government expenditure prices
ydist ;	Update income distribution
Consumption ;	Update household consumption
InvSt ;	Update final demand by investment and stock building
GovExp ;	Update government expenditures (including labour and capital)
Armington ;	Update top level Armington structure (split A into D and M)
prod ;	Update production (for both vintages)
KapSupply ;	Update disinvestment (if any)
FFactSupply ;	Update supply of fixed factors
Equil ;	Use tâtonnement to update factor prices
Taxes ;	Update tax revenues

The global module performs the following calculations (see ***World*** in `world.c`):

Update world trade prices (i.e. the 2nd level Armington nest)
Update the world trade matrix ;
Update export demand ;
Update world price indices ;
Calculate Walras' Law ;
Update the oil market variables ;
Update energy consumption and carbon emissions
Update carbon/energy taxes ;

The sequence of the equations can be critical and should only be modified if the model's convergence properties will be improved by a re-ordering (user beware!). Optimally, the order of the equations should be as close as possible to the recursive nature of the model. If the model were entirely linear, one would try to order the equations such that the matrix describing the model were lower triangular. In practice, with these types of non-linear applied general equilbrium models, the typical sequence starts with an update of prices, then income, final demand, production, and then factor prices. However, this may not be the most natural way to describe the model. Therefore, the description of the model will proceed in a sequence different from the order of the model equations yet maintaining the structure of the subroutines.

2. Production

Production is modeled as a nested structure of constant elasticity of substitution (CES) functions. We use the term CES somewhat loosely since it will often be the case that the substitution elasticity will be zero, which is more often referred to as a Leontief function.[17]

Figure 1a provides a graphical depiction of the nested CES production structure. The top level structure is a Leontief structure in non-energy intermediate inputs and an aggregate bundle designated by *KLEF*. The *KLEF* bundle is an aggregate of capital, labour, an energy bundle, and the fixed factor. Due to the existence of vintage capital, each producing sector is modeled as comprising two distinct technologies, producing an homogeneous good, but with different production parameters. Thus, each intermediate and factor demand is indexed by vintage (using the index v). Table 2.1 provides the top level demands for the non-energy intermediate inputs and

[17] Annex 2 provides a brief description of the CES function.

the *KLEF* bundle. The relevant code is contained in the file `prod.c`. The production structure is calculated in the subroutine ***Production*** which is called from the ***prod*** subroutine.

Table 2.1: **Top Level Production Nest**

$$XAp_{nf,j} = \sum_{v} a^{v}_{nf,j} XPv^{v}_{j} \tag{2.1.1}$$

$$KLEF^{v}_{j} = aklef^{v}_{j} XPv^{v}_{j} \tag{2.1.2}$$

XAp is intermediate demand (at the Armington level, i.e. before dis-aggregation into import demand and demand for domestically produced commodities). The index *nf* identifies elements pertaining to the set of non-energy commodities (in the current version of the model this set is {Agric, EnerInt, OtherInd}). In Equation (2.1.1) aggregate intermediate demand is determined directly (i.e. summing over vintage), since non-energy intermediate demand is not dependent on the vintage. (The Armington structure could of course be modeled as dependent on the vintage; however, this is not the case in GREEN.) The matrix *a*, is the matrix of input-output coefficients for non-energy intermediate inputs, and *XPv* represents aggregate sectoral output for each vintage. *KLEF* represents the capital, labour, energy, fixed factor bundle, and it is determined using a fixed coefficient function, with *aklef* being the share parameter.

At the next stage in production, the *KLEF* bundle is split into labour on the one hand, and the *KEF* bundle on the other. This is done using a CES function with the substitution elasticity given by $\rho^{k,v}_{1,j}$. Table 2.2 gives the reduced form first order conditions for this level of the nest.

Table 2.2: **Demand for Labour and the KEF Bundle**

$$L^{d}_{j} = \sum_{v} al^{v}_{j} \lambda^{l\,(\rho^{k,v}_{1,j}-1)}_{j} KLEF^{v}_{j} \left(\frac{Pklef^{v}_{j}}{w} \right)^{\rho^{k,v}_{1,j}} \tag{2.2.1}$$

$$KEF^{v}_{j} = akef^{v}_{j} KLEF^{v}_{j} \left(\frac{Pklef^{v}_{j}}{Pkef^{v}_{j}} \right)^{\rho^{k,v}_{1,j}} \tag{2.2.2}$$

Labour demand is homogeneous across vintages and therefore summed directly in Equation (2.2.1) where L^{d} represents sectoral labour demand.[18] *Pklef* is the aggregate (or CES dual) price of the *KLEF* bundle, *w* is the uniform economy-wide wage rate, and *Pkef* is the price of the *KEF* bundle. The share parameters are *al* for labour, and *akef* for the *KEF* bundle. The cumulative labour productivity factor is given by λ^{l}.

The next level of the CES nesting disaggregates the *KEF* bundle into the energy bundle on one side, and the *KF* bundle on the other. The equations in Table 2.3 provide the reduced form first order conditions for demand for *Ep* and *KF*.

[18] Labour supply (and demand) is defined in efficiency units. Labour supply grows at the same rate as the population growth rate **plus** the rate of growth of labour efficiency. Because labour demand is also defined in efficiency units, the λ coefficient for labour is always equal to 1

Table 2.3: **Demand for the Energy and KF Bundles**

$$(2.3.1) \qquad Ep_j^v = ae_j^v KEF_j^v \left(\frac{Pkef_j^v}{PEp_j^v} \right)^{\rho_{2,j}^{k,v}}$$

$$(2.3.2) \qquad KF_j^v = akf_j^v KEF_j^v \left(\frac{Pkef_j^v}{Pkf_j^v} \right)^{\rho_{2,j}^{k,v}}$$

Ep is demand for the energy bundle (by vintage), *PEp* is the price of the energy bundle, *KF* represents the capital-fixed factor bundle, and *Pkf* is the price of the *KF* bundle. The share parameters are *ae* for the energy bundle, and *akf* for the *KF* bundle. Both sides of this nest are further disaggregated. The *KF* bundle is split into the capital and fixed factor components, the energy bundle is split into the fuel composites (including the back-stops). Table 2.4 presents the demand for capital and the fixed factor. These demands are for homogenous factors and therefore there is no further nesting at this end of the tree.

Table 2.4: **Demand for Capital and the Fixed Factor**

$$(2.4.1) \qquad Kv_j^{d,v} = ak_j^v KF_j^v \; / \; \lambda_j^k$$

$$(2.4.2) \qquad FF_j^d = \sum_v af_j^v KF_j^v \; / \; \lambda_j^f$$

Kv^d is the demand for capital by vintage, FF^d is the aggregate demand for the fixed factor. The Leontief share parameters are *ak* for capital, and *af* for the fixed factor. There are cumulative productivity factors for both capital and the fixed factor. The cumulative productivity factors are the same for both capital and the fixed factor, i.e. the productivity factor is of the Hicks-neutral type of disembodied technical progress in the KF bundle. This factor is further discussed in the section on capital accumulation.

Each decision unit in the economy (producers, consumers, and units determining other final demand) has a demand for an energy bundle. In the case of production, energy demand is also vintage specific, and the substitution possibilities across fuels is generally lower for old capital than for new capital. The CES structure of energy demand is the same for all agents, however, the share and substitution parameters are allowed to differ.

The structure of energy demand is also a nested CES structure (see Figure 1b). In the case of production, the top level energy demand is given by *Ep*. The first stage of the nesting decomposes the energy bundle into five fuel **composites**, designated by the variable *FUx*, with price *PFUx*. With the introduction of back-stop technologies, these fuel composites are further decomposed into a conventional fuel component, and the relevant back-stop substitute components.

The decomposition of the energy bundle into the fuel composites is modeled using three levels of nesting.[19] The energy bundle is first decomposed into an electric fuel composite component and a non-electric bundle. The non-electric bundle is split into a coal fuel composite, and a bundle composed of crude oil, natural gas, and refined oil, which will be designated by Oil+Gas bundle. The Oil+Gas bundle is then decomposed into its three fuel composite components. The CES substitution elasticities in production are vintage and sector specific.

Once the fuel composites have been determined – coal, crude oil, natural gas, refined oil products, and electricity – they are decomposed into their respective conventional and back-stop components. Prior to the introduction of the back-stop technologies (in 2010, cf. below), the fuel composites are the same as the counterpart conventional fuels, i.e. there is no further nesting. After the introduction of the back-stops, each of the fuel composites (except for refined oil), are bundles of the conventional fuel and one or more of the back-

[19] This is a significant change from the previous version of the model which had only one level of nesting. This implied that inter-fuel substitutions were the same for all combinations of fuels. The current version of the model assumes the same substitution elasticities, therefore, the model results are identical.

stops, where the substitution between the conventional fuel and the back-stops is modeled using a CES function. Table 2.5 provides the demand for the fuel composites, i.e. the disaggregation of the energy bundle.

Table 2.5: **Demand for the Fuel Composites**

$$FUp_{Elec,j} = \sum_{v} aep^{v}_{E,j} \lambda^{e,p(\rho^{p,v}_{j}-1)}_{j,v} \left(\frac{PEp^{v}_{j}}{PFUp_{Elec,j}} \right)^{\rho^{p,v}_{j}} Ep^{v}_{j} \quad (2.5.1)$$

$$NEp_{v,j} = aep^{v}_{NE,j} \lambda^{e,p(\rho^{p,v}_{j}-1)}_{j,v} \left(\frac{PEp^{v}_{j}}{PNEp_{v,j}} \right)^{\rho^{p,v}_{j}} Ep^{v}_{j} \quad (2.5.2)$$

$$FUp_{Coal,j} = \sum_{v} aeNEp^{v}_{Coal,j} \left(\frac{PNEp^{v}_{j}}{PFUp_{Coal,j}} \right)^{\rho^{NEp,v}_{j}} NEp^{v}_{j} \quad (2.5.3)$$

$$OGp_{v,j} = aeNEp^{v}_{OG,j} \left(\frac{PNEp^{v}_{j}}{POGp_{v,j}} \right)^{\rho^{NEp,v}_{j}} NEp^{v}_{j} \quad (2.5.4)$$

$$FUp_{Oil,j} = \sum_{v} aeOGp^{v}_{Oil,j} \left(\frac{POGp^{v}_{j}}{PFUp_{Oil,j}} \right)^{\rho^{OGp,v}_{j}} OGp^{v}_{j} \quad (2.5.5)$$

$$FUp_{Gas,j} = \sum_{v} aeOGp^{v}_{Gas,j} \left(\frac{POGp^{v}_{j}}{PFUp_{Gas,j}} \right)^{\rho^{OGp,v}_{j}} OGp^{v}_{j} \quad (2.5.6)$$

$$FUp_{RefOil,j} = \sum_{v} aeOGp^{v}_{RefOil,j} \left(\frac{POGp^{v}_{j}}{PFUp_{RefOil,j}} \right)^{\rho^{OGp,v}_{j}} OGp^{v}_{j} \quad (2.5.7)$$

Equations (2.5.1) and (2.5.2) reflect the first level of the energy nesting, i.e. the split of the energy bundle into demand for the electric composite fuel, and demand for the non-electric bundle. In Equation (2.5.1), the demand for the composite can be directly summed over vintages since the split into the conventional and the back-stop components is not vintage dependent. The split of the non-electricity bundle, *NEp*, is assumed to be vintage specific. The price of the non-electric bundle is given by *PNEp*. The autonomous energy efficiency improvement factor enters at this level of the energy nesting. It is specific to the production sector and the vintage (but, currently, not to the type of fuel demand). The CES share parameters are given by aep (in the code they are indexed as '0' and '1', not 'E' and 'NE'). The CES substitution elasticity is $\rho^{p,v}_{j}$

The next level in the energy nest splits the *NEp* bundle into demand for the coal fuel composite on the one hand, FUp_{Coal}, and the oil+gas bundle on the other, *OGp*. Equation (2.5.3) determines the demand for the coal composite fuel. Equation (2.5.4) determines the demand for the oil+gas bundle. The CES substitution elasticity is given by $\rho^{NEp,v}_{j}$, and the share parameters by *aeNEp*. The price of the fuel composites is represented is given by *PFUp*, and the price of the oil+gas bundle is given by *POGp*.

The final level in the energy nest (prior to the back-stop determination), is the split of the oil+gas bundle into demand for the composite fuels of crude oil, natural gas, and refined oil products. Equations (2.5.5)-(2.5.7) determine these demands. The CES substitution elasticity is given by $\rho^{OGp,v}_{j}$, and the share parameters by *aeOGp*.

The next stage of the energy decomposition is to split the fuel composite demands, *FUp*, into their conventional and back-stop components. Coal, crude oil, and natural gas have the same CES substitution structure with the back-stops, albeit with potentially different parameters. In these three sectors, the conventional fuel is

substitutable with a carbon-based back-stop and a carbon-free back-stop. Let *bf* index the set B = {Coal, Oil, Gas}, and let *CBS* and *CFBS* represent the carbon-based back-stop and the carbon-free back-stop, respectively. Table 2.6 presents the demand for the conventional fuels and the back-stops for the fuels in set B.

Table 2.6: **Dis-Aggregation of Fuel Demand for Coal, Oil, and Gas**

(2.6.1) $$XAp_{bf,j} = aep1^{0}_{bf,j} FUp_{bf,j} \left(\frac{PFUp_{bf,j}}{PAp_{bf,j}} \right)^{\rho^{p,1}_{bf,j}}$$

(2.6.2) $$XAp_{bfCBS,j} = aep1^{1}_{bf,j} FUp_{bf,j} \left(\frac{PFUp_{bf,j}}{PAp_{bfCBS,j}} \right)^{\rho^{p,1}_{bf,j}}$$

(2.6.3) $$XAp_{bfCFBS,j} = aep1^{2}_{bf,j} FUp_{bf,j} \left(\frac{PFUp_{bf,j}}{PAp_{bfCFBS,j}} \right)^{\rho^{p,1}_{bf,j}}$$

As above, *XAp* represents the Armington demand and will be further disaggregated into domestic and import demand. *PAp* is the corresponding Armington price. If the back-stops are not active, the equations are simply reduced to:

$$XAP_{bf,j} = FUp_{bf,j}$$

and demand for the back-stops is zero. Table 2.7 lists the equation for electricity demand, including the carbon free electricity back-stop, denoted by *ElecBS*. There are no back-stop substitutes for refined oil products.

Table 2.7: **Dis-Aggregation of Fuel Demand for Refined Oil and Electricity**

(2.7.1) $$XAp_{RefOil,j} = FUp_{RefOil,j}$$

(2.7.2) $$XAp_{Elec,j} = aep2^{0}_{j} FUp_{Elec,j} \left(\frac{PFUp_{Elec,j}}{PAp_{Elec,j}} \right)^{\rho^{p,2}_{j}}$$

(2.7.3) $$XAp_{ElecBS,j} = aep2^{1}_{j} FUp_{Elec,j} \left(\frac{PFUp_{Elec,j}}{PAp_{ElecBS,j}} \right)^{\rho^{p,2}_{j}}$$

This completes the description of the production structure. Starting from output, *XPv*, the nested CES tree structure of production unfolds until at the end of each branch a basic commodity (at the Armington level) or factor of production is specified. The next section will describe the formulation of prices in the production sector. The description of prices proceeds in the opposite direction. It starts at the bottom of the tree, using the fundamental (or the economy's equilibrium prices), and moves up the tree to define the price of the different CES aggregate bundles.

Table 2.8 provides the data dictionary for the variables calculated in the subroutine ***Production***, i.e. it provides the link between the variables described in the equations above and the variables as they are called in the program (with the appropriate dimensions).

Table 2.8: **Data Dictionary for Subroutine *Production***
`prod.c`

$XAp_{i,j}$	`XAp[r][i][j][t]`	Intermediate (Armington) demand
$KLEF_j^v$	`klef[r][j][v][t]`	Demand for the KLEF bundle
L_j^d	`LabD[r][j][t]`	Demand for labour
KEF_j^v	`kef[r][j][v][t]`	Demand for the KEF bundle
Ep_j^v	`EnerP[r][j][v][t]`	Demand for the energy bundle
KF_j^v	`kf[r][j][v][t]`	Demand for the KF bundle
$Kv_j^{d,v}$	`KapDv[r][j][v][t]`	Demand for capital by vintage
FF_j^d	`FFactD[r][j][t]`	Demand for the fixed factor
NEp_j^v	`N_Elecp[r][j][v][t]`	Demand for the non-electric energy bundle
OGp_j^v	`OilGasp[r][j][v][t]`	Demand for the oil+gas energy bundle
$FUp_{e,j}$	`FUp[r][e][j][t]`	Demand for fuel composites

3. Production Prices

In this section (see the subroutine ***ProducerPrice*** in the file `pprice.c`), it is assumed that all equilibrium prices are given. The equilibrium prices include the Armington prices and the factor prices. The aggregate prices are all determined going from the bottom up. Table 3.1 describes the (CES) price of the fuel composites which are determined from the Armington prices.

Table 3.1: **Price of the Fuel Composites in Production**

(3.1.1) $$PFUp_{bf,j} = \left[aep1^0_{bf,j}\, PAp_{bf,j}^{(1-\rho^{p,1}_{bf,j})} + aep1^1_{bf,j}\, PAp_{bfCBS,j}^{(1-\rho^{p,1}_{bf,j})} + aep1^2_{bf,j}\, PAp_{bfCFBS,j}^{(1-\rho^{p,1}_{bf,j})}\right]^{\frac{1}{1-\rho^{p,1}_{bf,j}}}$$

(3.1.2) $$PFUp_{Refoil,j} = PAp_{Refoil,j}$$

(3.1.3) $$PFUp_{Elec,j} = \left[aep2^0_j\, PAp_{Elec,j}^{(1-\rho^{p,2}_j)} + aep2^1_j\, PAp_{ElecBS,j}^{(1-\rho^{p,2}_j)}\right]^{\frac{1}{1-\rho^{p,2}_j}}$$

Equation (3.1.1) in Table 3.1 defines the CES dual price, *PFUp*, for the fuel composite aggregates represented by the index *bf*, i.e. coal, crude oil, and natural gas. Equation (3.1.2) defines the composite price of refined oil which is simply the Armington price since there are no back-stop substitutes for refined oil. Equation (3.1.3) defines the price of the electricity composite for which there is one back-stop substitute.

Table 3.2 provides the aggregate prices for the various energy related bundles – oil+gas, non-electric, and the energy aggregate – and the *KF* bundle.

Table 3.2: **Price of the Energy and the KF Bundles in Production**

(3.2.1) $$POGp_j^v = \left[\sum_{e \in \{Oil,Gas,Refoil\}} aeOGp_{e,j}^v \, PFUp_{e,j}^{(1-\rho_j^{OGp,v})} \right]^{\frac{1}{1-\rho_j^{OGp,v}}}$$

(3.2.2) $$PNEp_j^v = \left[aeNEp_{Coal,j}^v \, PFUp_{Coal,j}^{(1-\rho_j^{NEp,v})} + aeNEp_{OG,j}^v \, POGp_{v,j}^{(1-\rho_j^{NEp,v})} \right]^{\frac{1}{1-\rho_j^{NEp,v}}}$$

(3.2.3) $$PEp_j^v = \left[aep_{E,j}^v \lambda_{j,v}^{e,p(\rho_j^{p,v}-1)} \, PFUp_{Elec,j}^{(1-\rho_j^{p,v})} + aep_{NE,j}^v \lambda_{j,v}^{e,p(\rho_j^{p,v}-1)} \, PNEp_{v,j}^{(1-\rho_j^{p,v})} \right]^{\frac{1}{1-\rho_j^{p,v}}}$$

(3.2.4) $$Pkf_j^v = ak_j^v \frac{r_j^v}{\lambda_j^k} + af_j^v \frac{Pff_j}{\lambda_j^f}$$

Equation (3.2.1) in Table 3.2 determines the CES dual price for the oil+gas bundle, *POGp*, by vintage. Equation (3.2.2) determines the CES dual price for the non-electric bundle, *PNEp*, which is composed of the coal fuel composite and the oil+gas bundle. Equation (3.2.3) determines the price of the aggregate energy bundle, *PEp*. The energy bundle is composed of the electric composite fuel and the non-electric bundle. The CES price incorporates the energy efficiency factor. Finally, Equation (3.2,4) determines the price of the *KF* bundle, adjusted for the efficiency factor of capital and the fixed factor. The rental rate is given by *r* and is sector and vintage specific. The rental rate is specified this way to simplify notation. In fact there is only one rental rate for new capital, and the rental rate on old capital is only sector specific if a sector is declining. Discussion of the capital market will be postponed until later. The price of the fixed factor is given by *Pff* and it is sector specific. Table 3.3 provides the equations describing the remaining prices in production.

Table 3.3: **Price of the KEF and KLEF Bundles, and Unit Production Cost**

(3.3.1) $$Pkef_j^v = \left[ae_j^v \, PEp_j^{v(1-\rho_{2,j}^{k,v})} + akf_j^v \, Pkf_j^{v(1-\rho_{2,j}^{k,v})} \right]^{\frac{1}{1-\rho_{2,j}^{k,v}}}$$

(3.3.2) $$Pklef_j^v = \left[al_j^v \left(\frac{w}{\lambda_j^l} \right)^{(1-\rho_{1,j}^{k,v})} + akef_j^v \, Pkef_j^{v(1-\rho_{1,j}^{k,v})} \right]^{\frac{1}{1-\rho_{1,j}^{k,v}}}$$

(3.3.3) $$PX_j = \frac{\sum_v \left[\sum_{nf} a_{nf,j}^v \, PAp_{nf,j} + aklef_j^v \, Pklef_j^v \right] XPv_j^v}{XP_j}$$

(3.3.4) $$PP_j = PX_j(1+\tau_j^p)$$

Equation (3.3.1) defines the CES dual price, *Pkef* for the *KEF* bundle. The price of the *KLEF* bundle, *Pklef*, is given by equation (3.3.2). The unit cost of aggregate production in sector *j*, *PX*, is given by Equation (3.3.3). The top level of the nested CES structure is a Leontief function in non-energy intermediate goods and the *KLEF* bundle. The unit cost by vintage is the sum of the Leontief coefficients weighted by the respective prices. The aggregate unit cost is the average unit cost by vintage weighted by the shares of production of each vintage over total production. (In other words $PX\,XP = PXv^o XPv^o + PXv^n XPv^n$, where the unit cost per vintage PXv^v is given by the term in the brackets in Equation (3.3.3).) Equation (3.3.4) determines the output price (or the producer price), *PP*. The output price is equal to the unit production cost plus a producer tax/subsidy designated by τ_j^p.

This finishes the section on prices in production. The disaggregation of the Armington demand will be discussed below. Table 3.4 provides the data dictionary for the variables determined in the subroutine ***ProducerPrice***.

Table 3.4: **Data Dictionary for Subroutine *ProducerPrice***
`pprice.c`

$PFUp_{e,j}$	`PFUp[r][e][j][t]`	Price of fuel composites (incorporating back-stops)
$POGp_j^v$	`POilGasp[r][j][v][t]`	Price of the oil+gas bundle
$PNEp_j^v$	`PN_Elecp[r][j][v][t]`	Price of the non-electric bundle
PEp_j^v	`PEnerP[r][j][v][t]`	Price of the energy bundle
Pkf_j^v	`pkf[r][j][v][t]`	Price of the KF bundle
$Pkef_j^v$	`pkef[r][j][v][t]`	Price of the KEF bundle
$Pklef_j^v$	`pklef[r][j][v][t]`	Price of the KLEF bundle
PX_j	`PX[r][j][t]`	Producer price (i.e. unit cost) exclusive of producer taxes/subsidies
PP_j	`PP[r][j][t]`	Producer price (i.e. unit cost) inclusive of producer taxes/subsidies

4. Income Distribution

GREEN has only one household which receives most of its income from value added. Other sources of income include depreciation allowance and rents generated by price wedges in the production of back-stops and crude oil, and transfers from the government. Table 4.1 lists the equations determining household income (the equations in this section are in the subroutine ***ydist*** in the file `ydist.c`).

Table 4.1: **Household Income and GDP Statistics**

(4.1.1) $$DeprY_t = \delta \; R_t \; K_t$$

(4.1.2) $$BSRents = \sum_{bs} XP_{bs}\left(PD_{bs} - PP_{bs}\right)$$

(4.1.3) $$\begin{aligned} Y \;=\; & w\left(L_g^d + \sum_i L_i^d\right) + \sum_v \left[r_g^v K_g^{d,v} + \sum_i r_i^v K_i^{d,v}\right] + \sum_i Pff_i FF_i^d \\ & - \; DeprY + OilRents + BSRents \end{aligned}$$

(4.1.4) $$Yd = Y - TaxH + cpi\; TRG$$

(4.1.5) $$RGDP = L_g^d + K_g^d + \sum_i \left(L_i^d + \lambda_t^k K_i^d + \lambda_t^f FF_i^d\right)$$

(4.1.6) $$PGDP_t = \frac{Y_t + DeprY_t}{L_t^s + K_t^s + \sum_i FF_{i,t}^s + OilRents_0 + BSRents_0}$$

Equation (4.1.1) defines the depreciation allowance on the total capital stock of the previous period.[20] *DeprY* is the value of the depreciation allowance, δ is the depreciation rate (defined on the aggregate capital stock, but region specific), and R is the economy-wide rental rate. Equation (4.1.2) defines aggregate rents accruing from production of back-stops, *BSRents*. These rents are generated by the price wedge between the sales price of back-stops, *PD*, and the producer price, *PP*. The sales price are (essentially) fixed, therefore there is no equilibrating mechanism which forces the sales price to be equal to the producer price. These rents can be thought of as a payment to an undetermined fixed factor in the production of the back-stops. Equation (4.1.3) defines total household income. It is the sum of payments to production factors (including payments by the government), less depreciation, plus rents generated by price wedges in the production of crude oil and the back-stop goods. *Y* is total household income, *OilRents* is the value of income generated by the price wedges in the production of crude oil, and *BSRents* is the value of rents in the production of the back-stop commodities. Equation (4.1.4) determines household disposable income, *Yd*. *Yd* is equal to aggregate income, less direct taxes, *TaxH*, plus transfers from government to households, *TRG*. *TRG* needs to be multiplied by a price in order to preserve the homogeneity of the model. The consumer price index, *cpi*, was chosen as the appropriate deflator.

Equation (4.1.5) defines real GDP, *RGDP*.[21] It is the sum of factor demand in efficiency units. Equation (4.1.6) defines the GDP deflator, *PGDP*.[22] The GDP deflator is defined as the value of factor payments, including oil and back-stop rents, divided by the sum of factor volumes.

The convergence properties of the model are enhanced if total income is calculated using a weighted average of value added calculated using factor *demands* and factor *supplies*. Factor supplies are more stable than factor demands and this leads to a more stable convergence path. Value added income using factor supplies is given by:

$$Y = wL^S + \sum_{i \in Decl} r_i^v KOld_i^s + \phi r_g^v KOld_g^s + R\left(K^s - \sum_{i \in Decl} KOld_i^s - \phi KOld_g^s\right) + \sum_i Pff_i FF_i^s$$

L^S is aggregate labour supply, the sum of supply of old capital is over declining sectors (*Decl* represents the set of declining sectors), the flag ϕ is equal to 1 if the government sector is declining, else it is equal to 0, R is the uniform economy-wide rent on new capital, and FF^S is the supply of the fixed factor. The code calculates value added income using both demand and supply and uses the average of the two, using a weight of 0.5. At equilibrium, of course, the two incomes are identical.

Table 4.2 presents the government and investment closure rules.

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20 There are two variables representing the aggregate capital stock, K and K^S, each representing a different unit of account. K is in millions of 1985 USD and in the base year, represents the total value of the capital stock. K^S is also in millions of 1985 USD, but in the base year, it represents aggregate capital remuneration. The reason for using K^S is to be able to normalise the capital rental rate to the value 1 in the base year. A numerical example may clarify the difference. In the US, aggregate capital remuneration in the base year, as determined from the US national accounts, is \$1,216 billion. The aggregate value of the capital stock is estimated to be \$10,399 billion. Given the rental rate normalisation rule, the first number is used in the model as the base year value of the capital stock. If instead, the second number were used (or the true value were used), the imputed rental rate would be 0.117. In fact, it makes no difference, since it is only an indexing issue. However, it does make a difference when imputing the depreciation income, hence the use of K. (K is discussed again in the section on capital accumulation.)

21 It is assumed that there is no change in the efficiency of capital in the government sector, though there is efficiency improvement in the use of labour (since labour is implicitly expressed in efficiency units).

22 All base year factor prices are equal to one, therefore, this implies that the denominator is evaluated in base year prices.

Table 4.2: **Government Saving and Investment**

(4.2.1) $$SectTax = \sum_i \left[PTxY_i + STxY_i + TarY_i + FuTxY_i \right] + \sum_i \tau_i^{ff} \, Pff_i \, FF_i^d$$

(4.2.2) $$GovExp = wL_g^d + \sum_v r_g^v K_g^v + \sum_i PAg_i \, XAg_i + cpi \; TRG$$

(4.2.3) $$\begin{cases} TaxH = cpi \; \zeta + \tau_h Y & \text{if} \quad S_g \text{ is endogenous} \\ TaxH = GovExp - SectTax + S_g & \text{if} \quad S_g \text{ is exogenous} \end{cases}$$

(4.2.4) $$\begin{cases} S_g = TaxH + SectTax - GovExp & \text{if} \quad S_g \text{ is endogenous} \\ S_{g,t} = PGDP_t \; S_{g,0} & \text{if} \quad S_g \text{ is exogenous} \end{cases}$$

(4.2.5) $$TI \; PI = S_h + S_g + P \, S_f + DeprY - \sum_i PAst_i \, XAst_i$$

Equation (4.2.1) defines government revenues from sectoral taxes, *SectTax*. Sectoral taxes include: production taxes, *PTxY*, sales tax on domestic production sold domestically, *STxY*, import taxes, *TarY*, the fuel taxes, *FuTxY*, and taxes on fixed factors (only used to implement an energy tax on the fixed factor in the electricity sector). Equation (4.2.2) defines government expenditures, *GovExp*. Expenditures include wages, payments to capital, purchases of goods and services, and transfers to households. *XAg* represents government demand for the Armington goods, and *PAg* are the relevant Armington prices.

The government closure rule is user determined. There are two options. Government saving is either fixed (in real terms) at the base year level, or government saving is endogenous. Equations (4.2.3) and (4.2.4) determine household taxes, *TaxH*, and government saving, S_g, under each closure rule. In the case of endogenous government saving, the household tax schedule is constant and is given in the first part of Equation (4.2.3). It is a linear schedule, with an intercept given by ζ (deflated by the cpi), and a slope (or marginal tax rate) given by τ^h. Government saving is simply the difference between government revenue and government expenditure and is determined by the first part of Equation (4.2.4). If government saving is exogenous (which is the case for the reference scenario), the household tax schedule is endogenous. Household taxes are then determined by the second part of Equation (4.2.3), i.e. *TaxH* is the equilibrating variable to achieve the fixed government balance. In the case of exogenous government saving, it is held constant in real terms at its base year level. This implies that as a share of GDP, government saving (or the deficit), will decline.

Equation (4.2.5) determines the level of real investment, *TI*. Aggregate fixed investment (in value) is the sum of saving, plus depreciation, less expenditures on stock building. Saving includes household saving, S_h, government saving, S_g, and foreign saving, S_f. Foreign saving is exogenous in each time period and is deflated by a world price index to be defined below. The sum of foreign saving across all regions must be zero. Closure will be discussed below. Changes in stocks are represented by *XAst* (at the Armington level), and the associated prices are *PAst*.[23] The investment deflator, *PI*, will be defined below. *TI* represents the volume of real investment. Note that because the sum of foreign saving (across regions) must be equal to zero, the investment-saving closure rule equation for one of the regions is redundant.

Table 4.3 describes the variables calculated in the subroutine ***ydist***, and the correspondence between the variables above and those in the code.

23 In the reference simulation stocks are set to zero starting in 1990.

Table 4.3: **Data Dictionary for Subroutine ydist**
ydist.c

DeprY	`Deprec[r][t]`	Depreciation allowance on total capital stock
BSRents	`BSRents[r][t]`	Rents generated by back-stop production
Y	`Y[r][t]`	Total household income
Yd	`yd[r][t]`	Household disposable income
PGDP	`pgdp[r][t]`	GDP deflator
RGDP	`rgdp[r][t]`	Volume of real GDP
TaxH	`taxh[r][t]`	Household direct taxes
S_g	`savg[r][t]`	Government saving/deficit
TI	`ti[r][t]`	Aggregate "Real" investment

5. Household Consumption

Total household consumption is determined by a single consumer maximising utility, subject to the usual budget constraint. The GREEN model uses the utility function associated with the extended linear expenditure system (ELES). The ELES is similar to the familiar LES, but incorporates household saving into the utility function. The basket of consumer goods is different from the sectoral definition of producer commodities. Demand for consumer goods are mapped into demand for produced goods using a nested Leontief-CES structure. There are four consumer commodities: Food and Beverages, Energy, Transport and Communication, and Other Goods and Services.

Consumers under the ELES are assumed to maximise the following utility function:

$$\max U = \sum_k \mu_k \ln(C_k - \theta_k) + \mu_s \ln\left(\frac{S}{P}\right)$$

subject to the budget constraint:

$$\sum_k PC_k C_k + S = Yd$$

C is consumer spending, S is saving (in value), Yd is disposable income, PC are consumer prices, and μ and θ are the ELES parameters.[24] Consistency requires the following constraints on the parameters μ:

$$\sum_k \mu_k + \mu_s = 1$$

The following demand functions can be derived:

$$C_k = \theta_k + \frac{\mu_k}{PC_k}\left(Yd - \sum_{k'} PC_{k'}\theta_{k'}\right)$$

The usual interpretation of this demand function is that consumption is composed of two parts. The first part has been referred to as the subsistence minima, θ. The term in parenthesis, represents residual income, or supernumerary income, i.e. it is the residual income after subtracting expenditures on the subsistence minima.

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[24] In the utility function, S needs to be deflated by an appropriate price, which would represent the consumer spot price of future consumption. This price does not need to be specified for the model since household saving can be derived as a residual from the budget constraint. For welfare calculations, the consumer price index, *cpi*, has been chosen as the saving deflator since there is no forward looking behavior in GREEN.

Therefore the second part of consumption is a share of supernumerary income. Note that there is no minimal consumption of savings, i.e. θ_S is 0. Saving can be determined via the budget constraint:

$$S = Yd - \sum_k PC_k C_k$$

The income and price elasticities are given by the following formula:

$$\eta_k = \frac{\mu_k Yd}{PC_k C_k} = \frac{\mu_k}{\chi_k}$$

$$\varepsilon_k = \frac{\theta_k (1-\mu_k)}{C_k} - 1$$

The income elasticity is equal to the ratio of the marginal propensity to consume good *k* out of supernumerary income, μ, over the average propensity to consume good *k* out of income.

The relevant equations in GREEN are presented in Table 5.1 (see the subroutine ***Consumption*** in the file `cons.c`).

Table 5.1: **Household Consumption**

(5.1.1) $$C_k = \theta_k Pop + \frac{\mu_k}{PC_k}\left[Yd - Pop \sum_{k'} PC_k \theta_{k'} \right]$$

(5.1.2) $$S_h = Yd - \sum_k PC_k C_k$$

Equation (5.1.1) defines consumer demand, C_k. The subsistence minima are calibrated in the base year on a per capita basis, therefore they are multiplied each period by the total population in order to grow with population. The index *k* identifies the four consumer goods. Equation (5.1.2) defines household saving, S_h.

The next stage maps household demand in terms of consumer commodities into demand for produced commodities. The top level assumes a fixed coefficient Leontief structure in non-energy commodities and an aggregate energy bundle yielding the equations in Table 5.2

Table 5.2: **Transformation of Consumption into Produced Goods**

(5.2.1) $$XAc_{nf,k} = ac_{nf,k} C_k$$

(5.2.2) $$Ec_k = aec_k C_k$$

Equation (5.2.1) defines the demand for non-energy goods, where *XAc* is (Armington) demand, and *ac* is the fixed transformation coefficient. Equation (5.2.2) defines demand for the energy bundle for each consumer good, with *Ec* representing demand for the energy bundle, and *aec* is the fixed coefficient of energy content in the consumption good.

The next level of transformation splits energy into demand for the fuel composites. The nested structure of energy demand in consumption is identical to the structure in production. The equations are listed in Table 5.3

Table 5.3: **Demand for the Fuel Composites in Consumption**

(5.3.1) $$FUc_{Elec,k} = aec1_{0,k}\lambda_k^{e,c(\rho_k^{c,1}-1)}\left(\frac{PEc_k}{PFUc_{Elec,k}}\right)^{\rho_k^{c,1}} Ec_k$$

(5.3.2) $$NEc_k = aec1_{1,k}\lambda_k^{e,c(\rho_k^{c,1}-1)}\left(\frac{PEc_k}{PNEp_k}\right)^{\rho_k^{c,1}} Ec_k$$

(5.3.3) $$FUc_{Coal,k} = aeNEc_{Coal,k}\left(\frac{PNEc_k}{PFUc_{Coal,k}}\right)^{\rho_k^{NEc}} NEc_k$$

(5.3.4) $$OGc_k = aeNEc_{OG,k}\left(\frac{PNEc_k}{POGc_k}\right)^{\rho_k^{NEc}} NEc_k$$

(5.3.5) $$FUc_{Oil,k} = aeOGc_{Oil,k}\left(\frac{POGc_k}{PFUc_{Oil,k}}\right)^{\rho_k^{OGc}} OGc_k$$

(5.3.6) $$FUc_{Gas,k} = aeOGc_{Gas,k}\left(\frac{POGc_k}{PFUc_{Gas,k}}\right)^{\rho_k^{OGc}} OGc_k$$

(5.3.7) $$FUc_{RefOil,k} = aeOGc_{RefOil,k}\left(\frac{POGc_k}{PFUc_{RefOil,k}}\right)^{\rho_k^{OGc}} OGc_k$$

The energy nesting determines demand for the five fuel composites, which are further disaggregated into conventional and back-stop components. The top level splits aggregate energy demand (by consumer sector), into the electric composite fuel, and a non-electric bundle (see Figure 1b for a graphical depiction of the nesting). Equations (5.3.1) and (5.3.2) represent this level of the nesting. Equation (5.3.1) determines demand for the electric fuel composite, FUc_{Elec}, and Equation (5.3.2) determines demand for the non-electric bundle, *NEc*. The CES share parameters are given by *aec1*, and the substitution elasticity is $\rho_k^{c,1}$. Note, the energy efficiency factor enters at this level of the nesting. At the next level, the non-electric bundle is split into two components – the coal fuel composite and the oil+gas bundle. Equation (5.3.3) determines demand for the coal fuel composite, and Equation (5.3.4) determines demand for the oil+gas bundle, *OGc*. The CES share parameters are given by *aeNEc*, and the substitution elasticity is ρ_k^{NEc}. The third stage of the energy nesting involves the decomposition of the oil+gas bundle into three fuel composites – crude oil, natural gas, and refined oil. Equations (5.3.5)-(5.3.7) determine, respectively, demand for these fuel composites. The share CES share parameters are given by *aeOGc*, and the substitution elasticity is ρ_k^{OGc}.

The next level requires dis-aggregating the fuel composites into conventional fuels and their associated back-stop substitutes.

Table 5.4: **Dis-Aggregation of the Fuel Composites**

$$XAc_{bf,k} = aec2^{0}_{bf,k} FUc_{bf,k} \left(\frac{PFUc_{bf,k}}{PAp_{bf,k}} \right)^{\rho^{c,2}_{bf,k}} \tag{5.4.1}$$

$$XAc_{bfCBS,k} = aec2^{1}_{bf,k} FUc_{bf,k} \left(\frac{PFUc_{bf,k}}{PAc_{bfCBS,k}} \right)^{\rho^{c,2}_{bf,k}} \tag{5.4.2}$$

$$XAc_{bfCFBS,k} = aec2^{2}_{bf,k} FUc_{bf,k} \left(\frac{PFUc_{bf,k}}{PAc_{bfCFBS,k}} \right)^{\rho^{c,2}_{bf,k}} \tag{5.4.3}$$

$$XAc_{RefOil,k} = FUc_{RefOil,k} \tag{5.4.4}$$

$$XAc_{Elec,k} = aec3^{0}_{k} FUc_{Elec,k} \left(\frac{PFUc_{Elec,k}}{PAc_{Elec,k}} \right)^{\rho^{c,3}_{k}} \tag{5.4.5}$$

$$XAc_{ElecBS,k} = aec3^{1}_{k} FUc_{Elec,k} \left(\frac{PFUc_{Elec,k}}{PAc_{ElecBS,k}} \right)^{\rho^{c,3}_{k}} \tag{5.4.6}$$

Equations (5.4.1)-(5.4.3) define the demand for the conventional fuel and the two back-stop substitutes for the basic fuels, i.e. coal, crude oil, and natural gas, with CES share parameters *aec2* and substitution elasticity $\rho^{c,2}$. Equations (5.4.5) and (5.4.6) compute derived demand for conventional and back-stop electricity, respectively, with share parameters *aec3* and substitution elasticity $\rho^{c,3}$. *PAc* are the relevant (Armington) prices in consumption.

Table 5.5 provides the data dictionary for the consumer demand system.

Table 5.5: **Data Dictionary for the Subroutine *Consumption***
`cons.c`

C_k	`cons[r][k][t]`	Household consumption of "consumer" commodities
S_h	`savh[r][t]`	Household saving
$XAc_{i,k}$	`XAc[r][i][k][t]`	Household (Armington) consumption of "production" commodities
Ec_k	`EnerC[r][k][t]`	Household consumption of the energy bundle
NEc_k	`N_Elecc[r][k][t]`	Household consumption of the non-electric bundle
OGc_k	`OilGasc[r][k][t]`	Household consumption of the oil+gas bundle
$FUc_{e,k}$	`FUc[r][e][k][t]`	Household consumption of the fuel composites

6. Consumer Prices

In this section (see the subroutine ***ConsPrice*** in the file `cprice.c`) consumer prices are determined from the bottom up. Table 6.1 describes the (CES) price of the fuel composites which are determined from the Armington prices.

Table 6.1: **Price of the Fuel Composites in Consumption**

(6.1.1) $$PFUc_{bf,k} = \left[aec2^{0}_{bf,k}\, PAc_{bf,k}^{(1-\rho^{c,2}_{bf,k})} + aec2^{1}_{bf,k}\, PAc_{bfCBS,k}^{(1-\rho^{c,2}_{bf,k})} + aec2^{2}_{bf,k}\, PAc_{bfCFBS,k}^{(1-\rho^{c,2}_{bf,k})}\right]^{\frac{1}{1-\rho^{c,2}_{bf,k}}}$$

(6.1.2) $$PFUc_{Refoil,k} = PAc_{Refoil,k}$$

(6.1.3) $$PFUc_{Elec,k} = \left[aec3^{0}_{k}\, PAc_{Elec,k}^{(1-\rho^{c,3}_{k})} + aec3^{1}_{k}\, PAc_{ElecBS,k}^{(1-\rho^{c,3}_{k})}\right]^{\frac{1}{1-\rho^{c,3}_{k}}}$$

Equation (6.1.1) in Table 6.1 defines the CES dual price, *PFUc*, for the fuel composite aggregates represented by the index bf, i.e. coal, crude oil, and natural gas. Equation (6.1.2) defines the composite price of refined oil which is simply the Armington price since there are no back-stop substitutes for refined oil. Equation (6.1.3) defines the price of the electricity composite for which there is one back-stop substitute.

Table 6.2 provides the aggregate prices for the energy bundle and the consumer commodities.

Table 6.2: **Price of Energy and Aggregate Consumer Prices**

(6.2.1) $$POGc_{k} = \left[\sum_{e\in\{Oil,Gas,Refoil\}} aeOGc_{e,k}\, PFUc_{e,k}^{(1-\rho^{OGc}_{k})}\right]^{\frac{1}{1-\rho^{OGc}_{k}}}$$

(6.2.2) $$PNEc_{k} = \left[aeNEc_{Coal,k}\, PFUc_{Coal,k}^{(1-\rho^{NEc}_{k})} + aeNEc_{OG,k}\, POGc_{k}^{(1-\rho^{NEc}_{k})}\right]^{\frac{1}{1-\rho^{NEc}_{k}}}$$

(6.2.3) $$PEc_{k} = \left[aec1_{0,k}\, \lambda_{k}^{e,c(\rho^{c,1}_{k}-1)}\, PFUc_{Elec,k}^{(1-\rho^{c,1}_{k})} + aec1_{1,k}\, \lambda_{k}^{e,c(\rho^{c,1}_{k}-1)}\, PNEc_{k}^{(1-\rho^{c,1}_{k})}\right]^{\frac{1}{1-\rho^{c,1}_{k}}}$$

(6.2.4) $$PC_{k} = \sum_{nf} ac_{nf,k}\, PAc_{nf,k} + aec_{k}\, PEc_{k}$$

(6.2.5) $$cpi = \frac{\sum_{k} PC_{k} C_{k}}{\sum_{k} C_{k}}$$

The first three equations in Table 6.2 determine the price of the different energy components. Equation (6.2.1) determines the CES dual price of the oil+gas bundle, *POGc*. Equation (6.2.2) determines the price of the non-electric bundle, *PNEc*. Equation (6.2.3) defines the price of the aggregate energy bundle, *PEc*, used in producing consumer good *k*. It is the CES dual price using the price of the electric fuel composite and the non-electric bundle. Equation (6.2.4) defines the top level consumer prices, *PC*, for individual consumer good *k*. Given the top level fixed coefficient transformation function, it is simply the weighted sum of the price of the non-energy commodities and the energy bundle, using the fixed coefficients as the weights. Equation (6.2.5) defines the consumer price index, *cpi*.

Table 6.3 provides the data dictionary for this section.

Table 6.3: **Data Dictionary for Subroutine *ConsPrice***
`cprice.c`

$PFUc_{e,k}$	`PFUc[r][e][k][t]`	Price of fuel composites for households (incorporating back-stops)
$POGc_k$	`POilGasc[r][k][t]`	Price of the oil+gas bundle for households
$PNEc_k$	`PN_Elecc[r][k][t]`	Price of the non-electric bundle for households
PEc_k	`PEnerC[r][k][t]`	Price of the energy bundle for households
PC_k	`pc[r][k][t]`	Price of consumption bundles (incorporating transition matrix and energy prices)
cpi	`cpi[r][t]`	Consumer price index

7. Investment and Stock Building

This section (see the subroutine ***InvSt*** in the file `invst.c`) determines investment and stock building demand for goods and services. As described above, aggregate investment is savings driven as the foreign balance is exogenous. The volume of investment is disaggregated into final demand for goods and services using a fixed coefficient Leontief function. Energy is further disaggregated using the same nested structure as that used in production and consumption. Table 7.1 presents the equations for investment final demand for goods and services.

Table 7.1: **Final Demand for Investment Intermediate Inputs**

(7.1.1) $$XAi_{nf} = ai_{nf} TI$$

(7.1.2) $$Ei = aei\ TI$$

Equation (7.1.1) specifies the (Armington) investment derived demand for non-energy goods and services, where XAi represents demand, and ai are the Leontief fixed input coefficients. Equation (7.1.2) determines demand for the energy bundle, Ei, where aei is the input coefficient for aggregate energy demand in investment.

Table 7.2 lists the equations describing the disaggregation of the energy bundle into the fuel composites.

Table 7.2: **Demand for the Fuel Composites in Investment**

(7.2.1) $$FUi_{Elec} = aei1_0 \lambda^{e,i(\rho^{i,1}-1)} \left(\frac{PEi}{PFUi_{Elec}} \right)^{\rho^{i,1}} Ei$$

(7.2.2) $$NEi = aei1_1 \lambda^{e,i(\rho^{i,1}-1)} \left(\frac{PEi}{PNEi} \right)^{\rho^{i,1}} Ei$$

(7.2.3) $$FUi_{Coal} = aeNEi_{Coal} \left(\frac{PNEi}{PFUi_{Coal}} \right)^{\rho^{NEi}} NEi$$

(7.2.4) $$OGi = aeNEi_{OG} \left(\frac{PNEi}{POGi} \right)^{\rho^{NEi}} NEi$$

(7.2.5) $$FUi_{Oil} = aeOGi_{Oil} \left(\frac{POGi}{PFUi_{Oil}} \right)^{\rho^{OGi}} OGi$$

(7.2.6) $$FUi_{Gas} = aeOGi_{Gas} \left(\frac{POGi}{PFUi_{Gas}} \right)^{\rho^{OGi}} OGi$$

(7.2.7) $$FUi_{RefOil} = aeOGi_{RefOil} \left(\frac{POGi}{PFUi_{RefOil}} \right)^{\rho^{OGi}} OGi$$

Equations (7.2.1) and (7.2.2) decompose the energy bundle into, respectively, demand for the electric fuel composite, FUi_{Elec}, and the non-electric bundle, *NEi*.. The CES share parameters are given by *aei1*, and the substitution elasticity is $\rho^{i,1}$. The energy efficiency factor enters at this level of the energy nest. Equations (7.2.3) and (7.2.4) decompose the non-electric bundle into, respectively, demand for the coal fuel composite, and the oil+gas bundle, *OGi*. The CES share parameters are given by *aeNEi*, and the substitution elasticity is ρ^{NEi}. Finally, Equations (7.2.5)-(7.2.7) determine the remaining three – crude oil, natural gas, and refined oil – fuel composite demands in investment as the decomposition of the oil+gas bundle. The CES share parameters are given by *aeOGi*, and the substitution elasticity is ρ^{OGi}.

Table (7.3) lists the equations which disaggregates the fuel composites into their separate components, i.e. conventional fuels and their associated back-stop substitutes.

Table 7.3: **Dis-Aggregation of the Fuel Composites**

(7.3.1) $$XAi_{bf} = aei2^{0}_{bf}\, FUi_{bf}\left(\frac{PFUi_{bf}}{PAi_{bf}}\right)^{\rho^{i,2}_{bf}}$$

(7.3.2) $$XAi_{bfCBS} = aei2^{1}_{bf}\, FUi_{bf}\left(\frac{PFUi_{bf}}{PAi_{bfCBS}}\right)^{\rho^{i,2}_{bf}}$$

(7.3.3) $$XAi_{bfCFBS} = aei2^{2}_{bf}\, FUi_{bf}\left(\frac{PFUi_{bf}}{PAi_{bfCFBS}}\right)^{\rho^{i,2}_{bf}}$$

(7.3.4) $$XAi_{RefOil} = FUi_{RefOil}$$

(7.3.5) $$XAi_{Elec} = aei3^{0}\, FUi_{Elec}\left(\frac{PFUi_{Elec}}{PAi_{Elec}}\right)^{\rho^{i,3}}$$

(7.3.6) $$XAi_{ElecBS} = aei3^{1}\, FUi_{Elec}\left(\frac{PFUi_{Elec}}{PAi_{ElecBS}}\right)^{\rho^{i,3}}$$

Equations (7.3.1)-(7.3.3) define the investment induced intermediate demand for the basic fuels, i.e. coal, crude oil, and natural gas, and their back-stop substitutes, with CES share parameters *aei2* and substitution elasticity $\rho^{i,2}$. Equation (7.3.5) and (7.3.6) define the demand for electricity and its back-stop option with share parameters *aei3* and substitution elasticity $\rho^{i,3}$. *PAi* are the relevant (Armington) prices in investment demand.

The aggregate volume of stock building, *StB*, is exogenous in each period, and normally set to zero in some future year. Final demand for stock building is determined via a fixed coefficient function, including demand for the fuel composites, i.e. the substitution elasticity for splitting the energy bundle into fuel composites is equal to 0. Table 7.4 lists the equations of intermediate demand derived from stock building.

Table 7.4: **Demand for Intermediate Goods and Services, and Fuels, Derived from Stock Building**

(7.4.1) $$XAst_{nf} = ast_{nf}\, StB$$

(7.4.2) $$FUst_{e} = \frac{ast_{e}}{\lambda^{e,st}}\, StB$$

Equation (7.4.1) determines (Armington) demand for non-energy goods and services, *XAst*, derived from stock building, and Equation (7.4.2) defines derived demand for the fuel composites, *FUst*. Table 7.5 lists the equations which determine demand for conventional fuels and the back-stops.

Table 7.5: **Dis-Aggregation of the Fuel Composites**

$$(7.5.1) \qquad XAst_{bf} = aest2^{0}_{bf}\, FUst_{bf}\left(\frac{PFUst_{bf}}{PAst_{bf}}\right)^{\rho^{st,2}_{bf}}$$

$$(7.5.2) \qquad XAst_{bfCBS} = aest2^{1}_{bf}\, FUst_{bf}\left(\frac{PFUst_{bf}}{PAst_{bfCBS}}\right)^{\rho^{st,2}_{bf}}$$

$$(7.5.3) \qquad XAst_{bfCFBS} = aest2^{2}_{bf}\, FUst_{bf}\left(\frac{PFUst_{bf}}{PAst_{bfCFBS}}\right)^{\rho^{st,2}_{bf}}$$

$$(7.5.4) \qquad XAst_{RefOil} = FUst_{RefOil}$$

$$(7.5.5) \qquad XAst_{Elec} = aest3^{0}\, FUst_{Elec}\left(\frac{PFUst_{Elec}}{PAst_{Elec}}\right)^{\rho^{st,3}}$$

$$(7.5.6) \qquad XAst_{ElecBS} = aest3^{1}\, FUst_{Elec}\left(\frac{PFUst_{Elec}}{PAst_{ElecBS}}\right)^{\rho^{st,3}}$$

Equations (7.5.1)-(7.5.3) define the stock building induced intermediate demand for the basic fuels, i.e. coal, crude oil, and natural gas, and their back-stop substitutes, with CES share parameters *aest2* and substitution elasticity $\rho^{st,2}$. Equation (7.5.5) and (7.5.6) define the demand for electricity and its back-stop option with share parameters *aest3* and substitution elasticity $\rho^{st,3}$. *PAst* are the relevant (Armington) prices in stock building.

Table 7.6 provides the data dictionary for the variables determined in this section.

Table 7.6: **Data Dictionary for Subroutine *InvSt***
`invst.c`

XAi_i	`XAi[r][i][t]`	Investment induced (Armington) demand for intermediate inputs
Ei	`EnerI[r][t]`	Investment induced demand for the energy bundle
NEi	`N_Eleci[r][t]`	Investment induced demand for the non-electric bundle
OGi	`OilGasi[r][t]`	Investment induced demand for the oil+gas bundle
FUi_e	`FUi[r][e][t]`	Investment induced demand for the fuel composites
$XAst_i$	`XAst[r][i][t]`	(Armington) demand for intermediate inputs induced by stock building.
$FUst_e$	`FUst[r][e][t]`	Intermediate demand for the fuel composites induced by stock building

8. Investment and Stock Building Prices

In this section (see the subroutine ***InvStPrice*** in the file `istprice.c`) prices in investment and stock building are determined going from the bottom up. Table 8.1 describes the (CES) price of the fuel composites which are determined from the Armington prices.

Table 8.1: **Price of the Fuel Composites in Investment**

(8.1.1) $$PFUi_{bf} = \left[aei2^{0}_{bf}\, PAi_{bf}^{(1-\rho^{i,2}_{bf})} + aei2^{1}_{bf}\, PAi_{bfCBS}^{(1-\rho^{i,2}_{bf})} + aei2^{2}_{bf}\, PAi_{bfCFBS}^{(1-\rho^{i,2}_{bf})}\right]^{\frac{1}{1-\rho^{i,2}_{bf}}}$$

(8.1.2) $$PFUi_{Refoil} = PAi_{Refoil}$$

(8.1.3) $$PFUi_{Elec} = \left[aei3^{0}\, PAi_{Elec}^{(1-\rho^{i,3})} + aei3^{1}\, PAi_{ElecBS}^{(1-\rho^{i,3})}\right]^{\frac{1}{1-\rho^{i,3}}}$$

Equation (8.1.1) in Table 8.1 defines the CES dual price, *PFUi*, for the fuel composite aggregates represented by the index *bf*, i.e. coal, crude oil, and natural gas. Equation (8.1.2) defines the composite price of refined oil which is simply the Armington price since there are no back-stop substitutes for refined oil. Equation (8.1.3) defines the price of the electricity composite for which there is one back-stop substitute.

Table 8.2 provides the aggregate prices for the energy bundle and the aggregate investment price.

Table 8.2: **Price of Energy and Aggregate Investment Price**

(8.2.1) $$POGi = \left[\sum_{e\in\{Oil,Gas,Refoil\}} aeOGi_{e}\, PFUi_{e}^{(1-\rho^{OGi})}\right]^{\frac{1}{1-\rho^{OGi}}}$$

(8.2.2) $$PNEi = \left[aeNEi_{Coal}\, PFUi_{Coal}^{(1-\rho^{NEi})} + aeNEi_{OG}\, POGi^{(1-\rho^{NEi})}\right]^{\frac{1}{1-\rho^{NEi}}}$$

(8.2.3) $$PEi = \left[aei1_{0}\lambda^{e,i(\rho^{i,1}-1)}\, PFUi_{Elec}^{(1-\rho^{i,1})} + aei1_{1}\lambda^{e,i(\rho^{i,1}-1)}\, PNEi^{(1-\rho^{i,1})}\right]^{\frac{1}{1-\rho^{i,1}}}$$

(8.2.4) $$PI = \sum_{nf} ai_{nf}\, PAi_{nf} + aei\ PEi$$

Equation (8.2.1) defines the price of the oil+gas bundle, *POGi*, in investment expenditures. Equation (8.2.2) defines the price of the non-electric bundle, *PNEi*. Equation (8.2.3) defines the price of the energy bundle, *PEi*, it is the CES dual price using the price of the electric fuel composite and the price of the non-electric bundle. Equation (8.2.4) defines the aggregate investment price, *PI*. *PI* is used to determine the volume of investment, the value of investment is determined through aggregate net savings.

Table 8.3 provides similar equations for the stock building sector.

Table 8.3: **Prices in Stock Building**

(8.3.1) $$PFUst_{bf} =$$

$$\left[aest2^{0}_{bf}\, PAst_{bf}^{(1-\rho^{st,2}_{bf})} + aest2^{1}_{bf}\, PAst_{bfCBS}^{(1-\rho^{st,2}_{bf})} + aest2^{2}_{bf}\, PAst_{bfCFBS}^{(1-\rho^{st,2}_{bf})}\right]^{\frac{1}{1-\rho^{st,2}_{bf}}}$$

(8.3.2) $$PFUst_{Refoil} = PAst_{Refoil}$$

(8.3.3) $$PFUst_{Elec} = \left[aest3^{0}\, PAst_{Elec}^{(1-\rho^{st,3})} + aest3^{1}\, PAst_{ElecBS}^{(1-\rho^{st,3})}\right]^{\frac{1}{1-\rho^{st,3}}}$$

(8.3.4) $$PSt = \sum_{nf} ast_{nf}\, PAst_{nf} + \sum_{e} ast_{e}\frac{PFUst_{e}}{\lambda^{e,st}}$$

Equations (8.3.1)-(8.3.3) define the price of the fuel composites, *PFUst*, based on the (Armington) price of the conventional fuels and the back-stops. Equation (8.3.4) defines the aggregate price of stock building, *PSt*. It is the weighted sum of the intermediate input prices, with $\lambda^{e,st}$ being the energy efficiency factor in the stock building sector.

Table 8.4 provides the data dictionary for the variables determined in this section.

Table 8.4: **Data Dictionary for Subroutine *InvStPrice***
`istprice.c`

$PFUi_e$	`PFUi[r][e][t]`	Price of fuel composites in investment goods production (incorporating back-stops)
POGi	`POilGasi[r][t]`	Price of oil+gas bundle in investment goods production
PNEi	`PN_Eleci[r][t]`	Price of non-electric bundle in investment goods production
PEi	`PEnerI[r][t]`	Price of energy bundle in investment goods production
PI	`pi[r][t]`	Aggregate price of investment
$PFUst_e$	`PFUst[r][e][t]`	Price of fuel composites in stock building demand (incorporating back-stops)
PSt	`pst[r][t]`	Aggregate price of stock building

9. Government Expenditures

This section (see the subroutine ***GovExp*** in the file `govexp.c`) determines government expenditures on purchases of goods and services, as well as on labour and capital. Contrary to the other final demand sectors, government is assumed to demand factor services.[25] The top level government expenditure function is a CES, and, as in production, relies on a vintage structure for capital. Final demand by the government is assumed to derive from the minimisation of the following cost function:

$$\min PGx\ Xg + r_g K_g^d + w\ L_g^d$$

subject to the production function[26]:

$$TG = \left(ag_x Xg^{\rho^g} + ag_k K_g^{d\,\rho^g} + ag_l L_g^{d\,\rho^g} \right)^{1/\rho^g}$$

Because of the vintage structure of government capital, government derives its demand for goods, services, and factors using two production functions similar to the above, but with different substitution elasticities. Determination of the amount of output produced with each capital vintage is done using a simple rule: it is assumed that all old vintage capital will be fully used. Any amount of production beyond that level requires the installation of new capital. The first step therefore is to calculate the capital-output ratio for old capital to determine the amount of output old capital is able to produce:

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[25] Note, however, that in most of the SAMs there is no data on labour and capital use in the government sector, and therefore, government expenditures are mostly on goods and services.

[26] As noted earlier, it is assumed that there is no efficiency change in the use of capital, but that there is in labour use. Although this is an ad hoc assumption, in most cases it will have little impact since many of the regions have no data for capital and labour use in the government sector.

$$\frac{KO_g^s}{TGv^{Old}} = \chi_g^{Old} = ag_k^{Old}\left(\frac{PGv^{Old}}{r_g^{Old}}\right)^{\rho^{g,Old}}$$

The amount of output old capital (i.e. capital supply in the government sector at the beginning of the period) is able to produce is given by:

$$TGv^{Old} = \frac{KO_g^s}{\chi_g^{Old}}$$

If this amount is greater than total output (which is determined exogenously), then government is a declining sector, i.e. it has too much initial capital and will desire to disinvest:

$$TGv^{Old} = TG \quad \text{if} \quad \frac{KO_g^s}{\chi_g^{Old}} \geq TG$$

Output from new capital is determined residually:

$$TGv^{New} = TG - TGv^{Old}$$

Equations determining the structure of government output are listed in Table 9.1

Table 9.1: **Structure of Government Output**

(9.1.1)
$$\begin{cases} TGv^{Old} = \dfrac{KO_g^s}{\chi_g^{Old}} & \text{if} \quad \dfrac{KO_g^s}{\chi_g^{Old}} \leq TG \\ TGv^{Old} = TG & \text{if} \quad \dfrac{KO_g^s}{\chi_g^{Old}} > TG \\ \text{where} \quad \chi_g^{Old} = ag_k^{Old}\left(\dfrac{PGv^{Old}}{r_g^{Old}}\right)^{\rho^{g,Old}} \end{cases}$$

(9.1.2)
$$TGv^{New} = TG - TGv^{Old}$$

Equation (9.1.1) determines output produced with old capital, TGv^{Old}, based upon the available supply of old capital at the beginning of the period and relative prices. Equation (9.1.2) determines residually, output produced with new capital.

Equations in Table 9.2 provide the derived reduced form first order conditions for government demand for each vintage structure.

Table 9.2: **Government Demand for Goods, Services, Labour, and Capital**

(9.2.1) $$Xg = \sum_{v} ag_x^v TGv^v \left(\frac{PGv^v}{PGx} \right)^{\rho^g}$$

(9.2.2) $$K_g^{d,v} = ag_k^v TGv^v \left(\frac{PGv^v}{r_g^v} \right)^{\rho^g}$$

(9.2.3) $$L_g^d = \sum_{v} ag_l^v TGv^v \left(\frac{PGv^v}{w} \right)^{\rho^g}$$

(9.2.4) $$K_g^d = \sum_{v} K_g^{d,v}$$

Equation (9.2.1) determines aggregate intermediate demand for goods and services by the government, Xg. TGv represents government output by vintage type, PGv is the aggregate price of government purchases by vintage type, PGx is the aggregate purchase price of goods and services, ag_X is the CES share parameter for goods and services, and ρ^g is the CES substitution elasticity. Equation (9.2.2) determines government demand for capital by vintage type, $K_g^{d,v}$, where r_g^v is the rental rate on government capital by vintage. Equation (9:2.3) determines government's aggregate demand for labour, L_g^d. Equation (9.2.4) determines aggregate demand for capital by government, K_g^d.

The next level of demand disaggregates the Xg bundle into sectoral demand for non-energy goods, and the energy bundle. The energy bundle is further decomposed into demand for the fuel composites.

Table 9.3: **Government Demand for Goods, Services, Energy, and Fuels**

(9.3.1) $$XAg_{nf} = agx_{nf} Xg$$

(9.3.2) $$Eg = aeg\ Xg$$

Equation (9.3.1) determines (Armington) demand for non-energy goods and services, XAg, using the fixed coefficients agx. Equation (9.3.2) determines demand for the energy bundle, Eg.

Table 9.4 lists the equations describing disaggregation of the energy bundle into the fuel composites.

Table 9.4: **Demand for the Fuel Composites in Government**

(9.4.1) $$FUg_{Elec} = aeg1_0 \lambda^{e,g(\rho^{g,1}-1)} \left(\frac{PEg}{PFUg_{Elec}} \right)^{\rho^{g,1}} Eg$$

(9.4.2) $$NEg = aeg1_1 \lambda^{g,i(\rho^{g,1}-1)} \left(\frac{PEg}{PNEg} \right)^{\rho^{g,1}} Eg$$

(9.4.3) $$FUg_{Coal} = aeNEg_{Coal} \left(\frac{PNEg}{PFUg_{Coal}} \right)^{\rho^{NEg}} NEg$$

(9.4.4) $$OGg = aeNEg_{OG} \left(\frac{PNEg}{POGg} \right)^{\rho^{NEg}} NEg$$

(9.4.5) $$FUg_{Oil} = aeOGg_{Oil} \left(\frac{POGg}{PFUg_{Oil}} \right)^{\rho^{OGg}} OGg$$

(9.4.6) $$FUg_{Gas} = aeOGg_{Gas} \left(\frac{POGg}{PFUg_{Gas}} \right)^{\rho^{OGg}} OGg$$

(9.4.7) $$FUg_{RefOil} = aeOGg_{RefOil} \left(\frac{POGg}{PFUg_{RefOil}} \right)^{\rho^{OGg}} OGg$$

Equations (9.4.1) and (9.4.2) decompose the energy bundle into, respectively, demand for the electric fuel composite, FUg_{Elec}, and the non-electric bundle, *NEg*. The CES share parameters are given by *aeg1*, and the substitution elasticity is $\rho^{g,1}$. The energy efficiency factor enters at this level of the energy nest. Equations (9.4.3) and (9.4.4) decompose the non-electric bundle into, respectively, demand for the coal fuel composite, and the oil+gas bundle, *OGg*. The CES share parameters are given by *eaNEg*, and the substitution elasticity is ρ^{NEg}. Finally, Equations (9.4.5)-(9.4.7) determine the remaining three – crude oil, natural gas, and refined oil – fuel composite demands in government as the decomposition of the oil+gas bundle. The CES share parameters are given by *aeOGg*, and the substitution elasticity is ρ^{OGg}.

Table 9.5 presents the by now familiar decomposition of the fuel composites into conventional fuel and demand for the back-stops.

Table 9.5: **Dis-Aggregation of the Fuel Composites**

$$XAg_{bf} = aeg2^{0}_{bf}\, FUg_{bf}\left(\frac{PFUg_{bf}}{PAg_{bf}}\right)^{\rho^{g,2}_{bf}} \tag{9.5.1}$$

$$XAg_{bfCBS} = aeg2^{1}_{bf}\, FUg_{bf}\left(\frac{PFUg_{bf}}{PAg_{bfCBS}}\right)^{\rho^{g,2}_{bf}} \tag{9.5.2}$$

$$XAg_{bfCFBS} = aeg2^{2}_{bf}\, FUg_{bf}\left(\frac{PFUg_{bf}}{PAg_{bfCFBS}}\right)^{\rho^{g,2}_{bf}} \tag{9.5.3}$$

$$XAg_{RefOil} = FUg_{RefOil} \tag{9.5.4}$$

$$XAg_{Elec} = aeg3^{0}\, FUg_{Elec}\left(\frac{PFUg_{Elec}}{PAg_{Elec}}\right)^{\rho^{g,3}} \tag{9.5.5}$$

$$XAg_{ElecBS} = aeg3^{1}\, FUg_{Elec}\left(\frac{PFUg_{Elec}}{PAg_{ElecBS}}\right)^{\rho^{g,3}} \tag{9.5.6}$$

Equations (9.5.1)-(9.5.3) determine demand for the conventional fuels and their back-stops for coal, crude oil, and natural gas. Equations (9.5.5) and (9.5.6) determine demand for conventional electricity and the electricity back-stop, respectively.

The following table provides the data dictionary for variable determined in this section.

Table 9.6: **Data Dictionary for Subroutine *GovExp***
govexp.c

TGv^{v}	tgv[r][v][t]	Government demand according to capital vintage
Xg	xg[r][t]	Aggregate government demand for intermediate goods and services
$K_g^{d,v}$	KapDgv[r][v][t]	Government demand for capital by vintage
K_g^{d}	KapDg[r][t]	Aggregate government demand for capital
L_g^{d}	LabDg[r][t]	Government demand for labour
XAg_{i}	XAg[r][i][t]	Intermediate (Armington) demand by sector derived from final government demand
OGg	OilGasg[r][t]	Government demand for the oil+gas bundle
NEg	N_Elecg[r][t]	Government demand for the non-electric bundle
Eg	EnerG[r][t]	Government demand for the energy bundle
FUg_{e}	FUg[r][e][t]	Government demand for the fuel composites

10. Government Prices

This section (see the subroutine ***GovPrice*** in the file govprice.c) determines prices in government demand. As in all other economic sectors, it starts at the bottom with the price of composite fuels.

Table 10.1: **Price of the Fuel Composites in Government**

$$PFUg_{bf} =$$

(10.1.1)
$$\left[aeg2^{0}_{bf}\, PAg_{bf}^{(1-\rho^{g,2}_{bf})} + aeg2^{1}_{bf}\, PAg_{bfCBS}^{(1-\rho^{g,2}_{bf})} + aeg2^{2}_{bf}\, PAg_{bfCFBS}^{(1-\rho^{g,2}_{bf})}\right]^{\frac{1}{1-\rho^{g,2}_{bf}}}$$

(10.1.2) $$PFUg_{Refoil} = PAg_{Refoil}$$

(10.1.3) $$PFUg_{Elec} = \left[aeg3^{0}\, PAg_{Elec}^{(1-\rho^{g,3})} + aeg3^{1}\, PAg_{ElecBS}^{(1-\rho^{g,3})}\right]^{\frac{1}{1-\rho^{g,3}}}$$

Equation (10.1.1) in Table 10.1 defines the CES dual price for the fuel composite aggregates, *PFUg*, represented by the index *bf*, i.e. coal, crude oil, and natural gas. Equation (10.1.2) defines the composite price of refined oil which is simply the Armington price since there are no back-stop substitutes for refined oil. Equation (10.1.3) defines the price of the electricity composite for which there is one back-stop substitute.

Table 10.2 provides the aggregate prices for the energy bundle, the aggregate price of expenditures on goods and services, the aggregate expenditure price by vintage, and the total government expenditure deflator.

Table 10.2: **Price of Energy and Aggregate Expenditure Prices in Government**

(10.2.1) $$POGg = \left[\sum_{e\in\{Oil,Gas,Refoil\}} aeOGg_e\, PFUg_e^{(1-\rho^{OGg})}\right]^{\frac{1}{1-\rho^{OGg}}}$$

(10.2.2) $$PNEg = \left[aeNEg_{Coal}\, PFUg_{Coal}^{(1-\rho^{NEg})} + aeNEg_{OG}\, POGg^{(1-\rho^{NEg})}\right]^{\frac{1}{1-\rho^{NEg}}}$$

(10.2.3) $$PEg = \left[aeg1_0\, \lambda^{e,g(\rho^{g,1}-1)}\, PFUg_{Elec}^{(1-\rho^{g,1})} + aeg1_1\, \lambda^{e,g(\rho^{g,1}-1)}\, PNEg^{(1-\rho^{g,1})}\right]^{\frac{1}{1-\rho^{g,1}}}$$

(10.2.4) $$PGx = \sum_{nf} agx_{nf}\, PAg_{nf} + aeg\ PEg$$

(10.2.5) $$PGv^{v} = \left[ag_x^{v}\, PGx^{(1-\rho^{g})} + ag_k^{v}\, r_g^{v(1-\rho^{g})} + ag_l^{v}\, w^{(1-\rho^{g})}\right]^{\frac{1}{1-\rho^{g}}}$$

(10.2.6) $$PG = \sum_{v} PGv^{v}\, \frac{TGv^{v}}{TG}$$

Equation (10.2.1) determines the price of the oil+gas bundle, *POGg*. Equation (10.2.2) determines the price of the non-electric bundle, *PNEg*. Equation (10.2.3) determines the price of the energy bundle, *PEg*. The aggregate price of expenditures on goods and services, *PGx*, is given by Equation (10.2.4). The CES dual price of output by vintage, *PGv*, is given by Equation (10.2.5). The aggregate government expenditure price deflator, *PG*, is determined in Equation (10.2.6).

Table 10.3 provides the data dictionary for variables determined in this section.

Table 10.3: **Data Dictionary for Subroutine *GovPrice***
govprice.c

$PFUg_e$	PFUg[r][e][t]	Price of fuel composites in government demand (incorporating back-stops)
$POGg$	POilGasg[r][t]	Price of oil+gas composite in government demand
$PNEg$	PN_Elecg[r][t]	Price of non-electric composite in government demand
PEg	PEnerG[r][t]	Price of energy composite in government demand
PGx	pgx[r][t]	Aggregate price of government demand for goods and services
PGv^v	pgv[r][v][t]	Aggregate price of government consumption by vintage
PG	pg[r][t]	Aggregate price of government consumption

11. Armington Disaggregation

Demand by all economic agents has now been specified at the Armington level of aggregation, i.e. a composite demand of goods produced domestically and imports. We will classify economic activities into 5 categories: production, household consumption, investment, stock building, and government expenditure. We assume each of the five activities represents a different Armington structure, with its own set of share parameters and substitution elasticities. In this section we will describe the first level Armington structure in final demand, leaving the discussion of the Armington structure in production for the section on equilibrium in the goods market. Equations in this section are specified in the subroutine ***Armington*** in the file `trade.c`.

As a reminder, the Armington assumption simply posits that goods are differentiated with respect to region of origin. GREEN has implemented this assumption using a nested structure. At the top level, each domestic agent optimises some objective function (e.g. cost minimisation or utility maximisation). This leads to demand for a composite commodity which has been referred to as the Armington commodity. At the next level (see Figure 3), agents minimise the cost of the Armington bundle, subject to an aggregation function between goods produced domestically and an aggregate import bundle. In the case of GREEN, this is a CES aggregation function. At the next and final level, agents minimise the cost of the aggregate import bundle, again subject to an aggregation function over imports originating in each region of the model. This latter nesting will be described in a later section. The mathematical formulation leads to:

$$\min PD\ D + PM\ M$$

s.t.

$$X = \left[a_d D^{\rho} + a_m M^{\rho}\right]^{1/\rho}$$

where X is the demand for the Armington good, D is demand for domestic production, M is aggregate import demand, PD is the price of domestic sales, and PM is the domestic price of imports (tariff inclusive).

The first order conditions lead to the following demand functions:

$$D = \alpha_d X\left(\frac{PA}{PD}\right)^{\sigma} \quad \text{where} \quad \alpha_d = a_d^{\sigma}$$

$$M = \alpha_m X\left(\frac{PA}{PM}\right)^{\sigma} \quad \text{where} \quad \alpha_m = a_m^{\sigma}$$

and the substitution elasticity is given by:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma - 1}{\sigma}$$

PA is the (Armington) CES dual price determined using *PD* and *PM*:

$$PA = \left[\alpha_d PD^{1-\sigma} + \alpha_m PM^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Table 11.1 specifies the Armington structure for commodities subject to the Armington specification. All goods have been designated as Armington goods except the back-stops, and crude oil. The back-stops are not traded. Crude oil is assumed to be homogeneous, therefore the law of one price holds, and domestic agents do not differentiate between crude oil produced domestically versus crude oil produced abroad.

Table 11.1: **Armington Decomposition of Final Demand**

(11.1.1) $$XDc_{i,k} = \alpha_{i,k}^{c,d} XAc_{i,k} \left(\frac{PAc_{i,k}}{PDc_{i,k}} \right)^{\sigma_{i,k}^{c}}$$

(11.1.2) $$XMc_{i,k} = \alpha_{i,k}^{c,m} XAc_{i,k} \left(\frac{PAc_{i,k}}{PMc_{i,k}} \right)^{\sigma_{i,k}^{c}}$$

(11.1.3) $$XDg_{i} = \alpha_{i}^{g,d} XAg_{i} \left(\frac{PAg_{i}}{PDg_{i}} \right)^{\sigma_{i}^{g}}$$

(11.1.4) $$XMg_{i} = \alpha_{i}^{g,m} XAg_{i} \left(\frac{PAg_{i}}{PMg_{i}} \right)^{\sigma_{i}^{g}}$$

(11.1.5) $$XDi_{i} = \alpha_{i}^{i,d} XAi_{i} \left(\frac{PAi_{i}}{PDi_{i}} \right)^{\sigma_{i}^{i}}$$

(11.1.6) $$XMi_{i} = \alpha_{i}^{i,m} XAi_{i} \left(\frac{PAi_{i}}{PMi_{i}} \right)^{\sigma_{i}^{i}}$$

(11.1.7) $$XDst_{i} = \alpha_{i}^{st,d} XAst_{i} \left(\frac{PAst_{i}}{PDst_{i}} \right)^{\sigma_{i}^{st}}$$

(11.1.8) $$XMst_{i} = \alpha_{i}^{st,m} XAst_{i} \left(\frac{PAst_{i}}{PMst_{i}} \right)^{\sigma_{i}^{st}}$$

Equations (11.1.1) and (11.1.2) determine domestic and import demand for household consumption, *XDc* and *XMc* respectively. The model allows for Armington differentiation across each one of the different consumption sectors. Equations (11.1.3)-(11.1.8) repeat the demand split for each one of the other final demand sectors: government, investment, and stock building.

It is assumed that the backstop fuels are available in unlimited quantities in each region at a given fixed price after a certain date. Because of this assumption, there is no reason for trade in the back-stop fuels, therefore, the Armington demand is identically equal to demand for domestic goods, and import demand is 0:

$$XD = XA \qquad \text{and} \qquad XM = 0 \quad \text{for all back-stop sectors.}$$

for all agents in the economy.

The crude oil market will be discussed in a later section.

Table 11.2 presents the data dictionary for the variables determined in this section.

Table 11.2: **Data Dictionary for Subroutine *Armington***
`trade.c`

$XDc_{i,k}$	`XDc[r][i][k][t]`	Consumer demand for domestic goods
$XMc_{i,k}$	`XMc[r][i][k][t]`	Consumer demand for imports
XDi_i	`XDi[r][i][t]`	Investment demand for domestic goods
XMi_i	`XMi[r][i][t]`	Investment demand for imports
$XDst_i$	`XDst[r][i][t]`	Stock building demand for domestic goods
$XMst_i$	`XMst[r][i][t]`	Stock building demand for imports
XDg_i	`XDg[r][i][t]`	Government demand for domestic goods
XMg_i	`XMg[r][i][t]`	Government demand for imports

12. Equilibrium on the Goods Market

Equilibrium on the domestic goods market requires that output by sector must equal aggregate domestic demand for domestic goods plus export demand (see routine ***prod*** in file `prod.c`). The previous section specified domestic demand for consumption, government, investment and stock building. Table 12.1 specifies the Armington disaggregation for intermediate demand in production.

Table 12.1: **Armington Decomposition of Intermediate Demand**

(12.1.1) $$XDp_{i,j} = \alpha_{i,j}^{p,d} XAp_{i,j} \left(\frac{PAp_{i,j}}{PDp_{i,j}} \right)^{\sigma_{i,j}^{p}}$$

(12.1.2) $$XMp_{i,j} = \alpha_{i,j}^{p,m} XAp_{i,j} \left(\frac{PAp_{i,j}}{PMp_{i,j}} \right)^{\sigma_{i,j}^{p}}$$

Equation (12.1.1) determines intermediate demand for goods produced domestically, *XDp*. Equation (12.1.2) determines intermediate demand for imports, *XMp*.

The equilibrium condition on the goods market is defined in Table 12.2.

Table 12.2: **Equilibrium on the Goods Market**

(12.2.1) $$XP_i = \sum_j XDp_{i,j} + \sum_k XDc_{i,k} + XDg_i + XDi_i + XDst_i + ED_i$$

Equation (12.2.1) states that sectoral output at equilibrium, *XP*, must equal the sum of intermediate demand (across sectors), plus consumer demand (across consumption categories), plus government, investment, and

stock building demand, and export demand. Export demand will be defined below. This equation determines gross output for all commodities except for crude oil (the *XDx* and *XMx* variables are not defined for crude oil since there is no Armington assumption). Output of crude oil will be discussed in a later section.

Equation (12.2.1) determines gross output. Production of gross output must be allocated across vintages. This will be done in a similar way as in the government sector. 1) Calculate the capital output ratio of old capital. 2) Determine if the sector is declining. 3) Allocate output across vintages. These steps are described in Table 12.3.

Table 12.3: **Allocation of Domestic Production Across Vintages**

(12.3.1)
$$\begin{cases} XPv_i^{Old} = \dfrac{KO_i^s}{\chi_i^{Old}} & \text{if} \quad \dfrac{KO_i^s}{\chi_i^{Old}} \le XP_i \\ XPv_i^{Old} = XP_i & \text{if} \quad \dfrac{KO_i^s}{\chi_i^{Old}} > XP_i \end{cases}$$

where
$$\chi_i^{Old} = aklef_i^{Old}\, akef_i^{Old}\, akf_i^{Old}\, \frac{ak_i^{Old}}{\lambda_i^k} \left(\frac{Pklef_i^{Old}}{Pkef_i^{Old}} \right)^{\rho_{1,i}^{k,Old}} \left(\frac{Pkef_i^{Old}}{Pkf_i^{Old}} \right)^{\rho_{2,i}^{k,Old}}$$

(12.3.2)
$$XPv_i^{New} = XP_i - XPv_i^{Old}$$

Equation (12.3.1) determines the level of gross output, XPv_i^{Old}, produced with the old capital vintage where χ^v represents the capital output ratio. (The formulation of the capital output ratio in the code takes due account of zero substitution elasticities, see subroutine ***CalcKXRatio*** in file `prod.c`.) If the beginning of period sectoral capital stock is insufficient to produce total sectoral output, then the sector will demand new capital, otherwise, the sector is contracting and it will disinvest itself of old capital. Equation (12.3.2) determines the level of gross sectoral output which will be produced by new capital.

Note: At this point, the program calculates the complete production structure by vintage as described in Section II.B.2. However, in order to improve the convergence properties of the model, the code implements this procedure in a slightly different manner. The key problem is that intermediate demand is directly linked to production. When Equation (12.2.1) is calculated, and the production structure is determined, the intermediate demand used in the calculation of Equation (12.1.1) is no longer necessarily consistent with output determined in Equation (12.2.1). In a linear model this is no problem since the Leontief inverse matrix is used to determine output given final demand. The same procedure could be performed in a non-linear production structure but this would entail construction of an input-output table which changes with each iteration since it is price dependent. Instead, an iterative procedure has been implemented. The pseudo-code for this iterative procedure is described below:

Initialise iteration ;
    Calculate aggregate sectoral final demand ;
    Calculate the Armington shares for intermediate demand ;
    Calculate the capital output ratio of old capital ;

Loop until convergence ;
    Calculate gross output, based on the current estimate of intermediate demand ;
    Determine the vintage share of output based on the old capital output ratio ;
    Calculate the entire production structure (call subroutine ***Production***) ;
    Determine the Armington structure of intermediate demand ;
    If there is no change in intermediate demand between iterations convergence has been achieved ;

Calculate the remaining equilibrium conditions ;
    Determine gross domestic sales of domestic production ;
    Determine gross domestic sales of imports ;
    Determine aggregate sectoral capital demand ;

The three steps involved in the initialisation process are independent of the iterative procedure. Aggregate sectoral final demand is the sum of demand of households, government, investment, stock building, and exports. The Armington shares in intermediate demand, i.e. XDp_i/XAp_i and XMp_i/XAp_i depend only on prices and can be calculated prior to the iteration. Finally, the capital/output ratios are also only a function of prices. In the actual coding of the iteration procedure, convergence is not tested. Instead the iterative loop is run a fixed number of times. 10 iterations seem to suffice to achieve convergence.

The equations in Table 12.4 determine the remaining identities.

Table 12.4: **Other Identities**

(12.4.1) $$XD_i = \sum_j XDp_{i,j} + \sum_k XDc_{i,k} + XDg_i + XDi_i + XDst_i$$

(12.4.2) $$XM_i = \sum_j XMp_{i,j} + \sum_k XMc_{i,k} + XMg_i + XMi_i + XMst_i$$

(12.4.3) $$K_i^d = \sum_v Kv_i^{d,v}$$

Equation (12.4.1) defines aggregate domestic demand for domestic production, XD, i.e. $XP - ED$ (this last identity is not true if the Constant Elasticity of Transformation (CET) assumption is implemented). Equation (12.4.2) determines aggregate sectoral imports, XM. This will be further disaggregated by region of origin according to the 2nd level Armington structure. Equation (12.4.3) determines aggregate sectoral demand for capital, K_i^d.

Table 12.5 provides the data dictionary for variables determined in this section.

Table 12.5: **Data Dictionary for Subroutine *prod***
`prod.c`

XP_i	`XP[r][i][t]`	Gross domestic output
XPv_i^v	`XPv[r][i][v][t]`	Gross domestic output by vintage
$XDp_{i,j}$	`XDp[r][i][j][t]`	Intermediate demand for domestic goods
$XMp_{i,j}$	`XMp[r][i][j][t]`	Intermediate demand for imports
XD_i	`XD[r][i][t]`	Aggregate domestic demand for domestic production
XM_i	`XM[r][i][t]`	Aggregate domestic demand for imports
K_i^d	`KapD[r][i][t]`	Aggregate capital demand

13. Armington Prices

Armington prices are determined in this section (see ***TradePrices*** in file `tradprice.c`). These prices are calculated immediately after the determination of producer output prices. The base year data incorporates three tax distortions. There is a tax/subsidy on production. There is a tax/subsidy on domestic sales of domestic production. There is a tax/subsidy on imports.[27] Table 13.1 describes the basic sales prices incorporating distortions.

Table 13.1: **Sales Price Including Taxes/Subsidies**

(13.1.1) $PD_i = PP_i(1+\tau_i^d)$

(13.1.2) $PE_i = PP_i$

(13.1.3) $PDp_{i,j} = PD_i(1+dfutx_i)$

(13.1.4) $PMp_{i,j} = PM_i(1+mfutx_i)$

(13.1.5) $PDc_{i,k} = PD_i(1+dfutx_i)$

(13.1.6) $PMc_{i,k} = PM_i(1+mfutx_i)$

(13.1.7) $PDg_i = PD_i(1+dfutx_i)$

(13.1.8) $PMg_i = PM_i(1+mfutx_i)$

(13.1.9) $PDi_i = PD_i(1+dfutx_i)$

(13.1.10) $PMi_i = PM_i(1+mfutx_i)$

(13.1.11) $PDst_i = PD_i(1+dfutx_i)$

(13.1.12) $PMst_i = PM_i(1+mfutx_i)$

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27 The domestic net sales tax or subsidy, on both domestic and imported production, are usually only applied in three energy sectors: coal, crude oil, and gas. These distortions are particularly high in certain countries, notably for oil in the former Soviet Union for coal in China.

Equation (13.1.1) determines the sales price of domestic output on the domestic market, before tacking on the fuel taxes. A price wedge is introduced in this equation between the producer price, *PP*, and the domestic sales price, *PD*.[28] The distortion is designated by τ^d. There is no price wedge introduced between the producer price and the export price, *PE*. Equation (13.1.2) defines the export price. The domestic import price will be defined below.

The fuel taxes are imposed as excise taxes on either carbon emissions or energy content, or both. The excise taxes are converted into ad valorem equivalents, *dfutx* and *mfutx* (this is discussed in more detail below). These taxes are tacked onto the prices *PD* and *PM*. Equations (13.1.3)-(13.1.12) describe the final sales price for all economic agents. *PDp* and *PMp* represent respectively the price of domestic intermediate consumption and import intermediate consumption. *PDc* and *PMc* represent respectively the price of domestic household consumption and import household consumption. The other final demand prices are given by *PDg*, *PMg*, *PDi*, *PMi*, *PDst*, and *PMst*.

Table 13.2 lists the equations determining the Armington prices for each of the economic institutions (the index *i* refers only to the Armington sectors).

Table 13.2: **Armington Prices**

(13.2.1) $$PAp_{i,j} = \left[\alpha_{i,j}^{p,d} PDp_{i,j}^{(1-\sigma_{i,j}^{p})} + \alpha_{i,j}^{p,m} PMp_{i,j}^{(1-\sigma_{i,j}^{p})}\right]^{\frac{1}{1-\sigma_{i,j}^{p}}}$$

(13.2.2) $$PAc_{i,k} = \left[\alpha_{i,k}^{c,d} PDc_{i,k}^{(1-\sigma_{i,k}^{c})} + \alpha_{i,k}^{c,m} PMc_{i,k}^{(1-\sigma_{i,k}^{c})}\right]^{\frac{1}{1-\sigma_{i,k}^{c}}}$$

(13.2.3) $$PAg_{i} = \left[\alpha_{i}^{g,d} PDg_{i}^{(1-\sigma_{i}^{g})} + \alpha_{i}^{g,m} PMg_{i}^{(1-\sigma_{i}^{g})}\right]^{\frac{1}{1-\sigma_{i}^{g}}}$$

(13.2.4) $$PAi_{i} = \left[\alpha_{i}^{i,d} PDi_{i}^{(1-\sigma_{i}^{i})} + \alpha_{i}^{i,m} PMi_{i}^{(1-\sigma_{i}^{i})}\right]^{\frac{1}{1-\sigma_{i}^{i}}}$$

(13.2.5) $$PAst_{i} = \left[\alpha_{i}^{st,d} PDst_{i}^{(1-\sigma_{i}^{st})} + \alpha_{i}^{st,m} PMst_{i}^{(1-\sigma_{i}^{st})}\right]^{\frac{1}{1-\sigma_{i}^{st}}}$$

Equations (13.2.1)-(13.2.5) define respectively the Armington prices using the CES dual price function for production, *PAp*, consumption, *PAc*, government, *PAg*, investment, *PAi*, and stock building, *PAst*.

Determination of crude oil prices, both domestic and traded will be discussed in a later section. Determination of the back-stop prices will also be reserved for a later section.

Table 13.3 provides the data dictionary for the variables determined in this section.

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[28] In the base year, there are three distortionary taxes (or price wedges). There is an indirect tax in production which leads to a wedge between the unit cost of production, *PX*, and the producer price *PP*. This tax is designated by τ^p. There is a wedge between the producer price, *PP*, and the domestic sales price, *PD*. This tax is designated by τ^d. There is no wedge between the producer price and the export price. Finally, there is a wedge between the world import price, and the domestic import price. This tax is designated by τ^m. The indirect tax on production, τ^p, generally comes from the national input/output tables and can be applicable in all sectors. The other two distortions have been estimated from the price wedge between the world price and the domestic price of fossil fuels. In the base year, this wedge is only estimated for coal, crude oil, and natural gas, and the domestic wedge is assumed to be the same as the import wedge (i.e. it is assumed that the producer price is the same as the world price).

Table 13.3: **Data Dictionary for Subroutine *TradePrices***
`tradprice.c`

PD_i	`PD[r][i][t]`	Sales price of domestic production sold domestically
PE_i	`PE[r][i][t]`	Sales price of exports
$PDp_{i,j}$	`PDp[r][i][j][t]`	Price of domestic intermediate consumption
$PMp_{i,j}$	`PMp[r][i][j][t]`	Price of import intermediate consumption
$PDc_{i,k}$	`PDc[r][i][k][t]`	Price of domestic household consumption
$PMc_{i,k}$	`PMc[r][i][k][t]`	Price of import household consumption
PDg_i	`PDg[r][i][t]`	Price of domestic government consumption
PMg_i	`PMg[r][i][t]`	Price of import government consumption
PDi_i	`PDi[r][i][t]`	Price of domestic investment consumption
PMi_i	`PMi[r][i][t]`	Price of import investment consumption
$PDst_i$	`PDst[r][i][t]`	Price of domestic stock building
$PMst_i$	`PMst[r][i][t]`	Price of import stock building
$PAp_{i,j}$	`PAp[r][i][j][t]`	Armington prices for intermediate consumption
$PAc_{i,k}$	`PAc[r][i][k][t]`	Armington prices of household consumption
PAi_i	`PAi[r][i][t]`	Armington prices for investment demand
$PAst_i$	`PAst[r][i][t]`	Armington prices for stock building
PAg_i	`PAg[r][i][t]`	Armington prices for government demand

14. Supply of Old Capital

This section determines the supply of old capital (see routine ***KapSupply*** in file `factsup.c`). If a sector is expanding, its beginning of period supply of old capital, *KO*, is insufficient to produce its expanding output and therefore it will demand new capital. In this case it is assumed that the rental price of the old capital is the same as the rental price of the new capital (though its productivity is different). There is a unique economy-wide rental rate on new capital. If, however, a sector is declining, it will want to disinvest its beginning of period capital stock. In the case of a declining sector, the rental rate on old capital is sector specific. The disinvestment function is based on the relative rates of return of old capital versus new capital. Table 14.1 provides the equations for the supply of old capital in declining sectors.

Table 14.1: **Supply of Old Capital in Declining Sectors**

(14.1.1)
$$\begin{cases} KOld^s_{i,t} = KO^s_{i,t}\left[\dfrac{r^{Old}_{i,t} / r^{New}_{i,t}}{r^{Old}_{i,t-1} / r^{New}_{i,t-1}}\right]^{\eta_i^k} & \text{if} \quad \eta_i^k < \infty \\ KOld^s_{i,t} = Kv^{d,Old}_{i,t} & \text{if} \quad \eta_i^k = \infty \end{cases}$$

(14.1.2)
$$\begin{cases} KOld^s_{g,t} = KO^s_{g,t}\left[\dfrac{r^{Old}_{g,t} / r^{New}_{g,t}}{r^{Old}_{g,t-1} / r^{New}_{g,t-1}}\right]^{\eta_g^k} & \text{if} \quad \eta_g^k < \infty \\ KOld^s_{g,t} = Kv^{d,Old}_{g,t} & \text{if} \quad \eta_g^k = \infty \end{cases}$$

Equation (14.1.1) defines the supply of old capital in declining sectors, $KOld^s$. A sector is determined as declining if it has no demand for new capital, or in other words, it produces no output with new capital. In the code, we use the variable XPv^{New} to determine if a sector is declining. If the disinvestment elasticity, η^k, is finite, supply of old capital will be determined by the relative rental rates between old and new capital. If the disinvestment elasticity is infinite, then supply of old capital equals demand. (Infinite is assumed if the supply elasticity is greater than 98). Equation (14.1.2) describes the same equations for the government sector, where $KOld_g^s$ represents the supply of old government capital. The government sector is declining if the variable TGv^{New} is 0.

The difference between $KOld^S$ and KO^S, i.e. between the end of period and beginning of period capital stocks, is put on the capital market and is assumed to enter the same pool as new capital, with the same price as new capital.

Table 14.2 provides the data dictionary for the variables determined in this section.

Table 14.2: **Data Dictionary for Subroutine *KapSupply***
`factsup.c`

$KOld_{i,t}^s$	`KapSOld[r][i][t]`	Supply of old capital in declining sectors
$KOld_{g,t}^s$	`KapSgOld[r][t]`	Supply of old capital in government (if government is declining)

15. Supply of Fixed Factors

This section describes the determination of the fixed factors (see subroutine ***FFactSupply*** in file `factsup.c`).[29] It will be divided into three sections: fixed factor supply for agriculture, coal, and electricity, fixed factor supply for gas, and fixed factor supply for crude oil. The latter two sectors are treated differently because of natural gas and crude oil reserves.

Table 15.1 describes the supply of fixed factors in the agriculture, coal, and electricity sectors. In these sectors, supply of the fixed factor features an upward sloping supply curve. The supply elasticity is different depending on whether demand for the fixed factor is increasing or decreasing.

Table 15.1: **Supply of Fixed Factors in Sectors other than Gas and Crude Oil**

$$(15.1.1)\quad \begin{cases} FF_{i,t}^s = FF_{i,t}^d & \text{if } \eta_i^{f,\cdot} = \infty \\ FF_{i,t}^s = FF_{i,t-1}^s \left[\dfrac{Pff_{i,t} / PGDP_t}{Pff_{i,t-1} / PGDP_{t-1}} \right]^{\eta_i^{f,Up}} & \text{if } FF_{i,t}^d \geq FF_{i,t-1}^s \\ FF_{i,t}^s = FF_{i,t-1}^s \left[\dfrac{Pff_{i,t} / PGDP_t}{Pff_{i,t-1} / PGDP_{t-1}} \right]^{\eta_i^{f,Down}} & \text{if } FF_{i,t}^d < FF_{i,t-1}^s \end{cases}$$

Equation (15.1.1) describes the supply of fixed factors, FF^s, in sectors other than gas and crude oil. If demand for the fixed factor is increasing (with respect to the previous period), then supply is determined using the so-

[29] It would be more accurate to call these factors sector-specific, rather than fixed, since their supply is allowed to respond to price changes.

called "up" supply elasticity.[30] Supply is determined as a function of the change in the real price of the fixed factor, where the supply elasticity is given by η^f. The same functional form is used on the downside, but with a different elasticity. If the elasticity is infinite (i.e. greater than 98 in the code), supply is set equal to demand.

If the supply elasticity is infinite, the price of the fixed factor is determined in this subroutine, otherwise, it is normally determined by the tâtonnement procedure. The following rules are used to determine the price of the fixed factor in case of infinite supply elasticities:

$$Pff_{i,t} = PGDP_t \qquad \text{if} \quad FF^d_{i,t} \geq FF^s_{i,t-1}$$

$$Pff_{i,t} = \max\left[PGDP_t \frac{Pff_{i,t-1}}{PGDP_{t-1}}, Pff^{min}_{i,t} \right] \qquad \text{if} \quad FF^d_{i,t} < FF^s_{i,t-1}$$

If demand for the fixed factor is increasing and the supply elasticity is infinite, the price of the fixed factor is set to the GDP deflator. If demand for the fixed factor is decreasing, the price of the fixed factor is equal to the previous period's real price of the fixed factor, multiplied by the current period's GDP deflator, i.e. the price of the fixed factor remains constant in real terms. However, a floor is set on the price of the fixed factor in case of decline.[31]

The supply of the fixed factor in the natural gas sector will depend on whether the producer is on the reserve profile curve, or below it. If the producer is not constrained, then the supply of the fixed factor in the natural gas sector is similar to the supply of the fixed factor in the other sectors. If the producer is constrained, then the supply of the fixed factor is the supply potential determined by the depletion profile (which is described below). Table 15.2 describes the supply of the fixed factor in the natural gas sector.

Table 15.2: **Supply of the Fixed Factor in the Natural Gas Sector**

(15.2.1a)
$$FF^{s,Not}_{Gas,t} = \begin{cases} FF^d_{Gas,t} & \text{if} \quad \eta^{f,v}_{Gas} = \infty \\ FF^s_{Gas,t-1}\left[\dfrac{Pff_{Gas,t} / PGDP_t}{Pff_{Gas,t-1} / PGDP_{t-1}} \right]^{\eta^{f,Up}_{Gas}} & \text{if} \quad FF^d_{Gas,t} \geq FF^s_{Gas,t-1} \\ FF^s_{Gas,t-1}\left[\dfrac{Pff_{Gas,t} / PGDP_t}{Pff_{Gas,t-1} / PGDP_{t-1}} \right]^{\eta^{f,Down}_{Gas}} & \text{if} \quad FF^d_{Gas,t} < FF^s_{Gas,t-1} \end{cases}$$

(15.2.1b)
$$FF^s_{Gas,t} = \min\left[FF^{s,Not}_{Gas,t}, XPot_{Gas,t} \right]$$

Equation (15.2.1a) determines the notional supply of the fixed factor in the natural gas sector. If the supply elasticity is infinite, then notional supply is set to fixed factor demand. Price determination of the fixed factor in the natural gas sector in the case of infinite supply will depend on the supply constraint. If the region is on the resource depletion profile, i.e. there is a supply constraint, the price will be determined in the tâtonnement procedure. If the region is not constrained, the price will be set to the GDP deflator. (Similar to the other sectors, there exists a floor price to the fixed factor if the demand for it is decreasing).

In the case of finite supply elasticities, a price sensitive supply function determines notional supply of the fixed factor. The degree of responsiveness will depend on whether demand for natural gas is contracting or expanding.

Equation (15.2.1b) determines the actual supply of the fixed factor, it is the minimum of the notional supply and the natural gas supply potential. In other words, actual supply is never allowed to exceed the supply potential.

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[30] In the implementation of the code, not only must demand for the fixed factor exceed the previous period's supply, but output in the corresponding sector must be greater than 20 per cent of the base period's output ($XP_t > 0.2\ XP_0$).

[31] The current version of GREEN sets the floor at $0.975 Pff_{i,t-1}$, but this may be modified in a subsequent version.

The energy exporting countries (EEX) are assumed to have infinite supply (i.e. they are never constrained), and infinite elasticity. Their supply is therefore always determined by demand and the price is always set to the GDP deflator. (In the code, this is accomplished by setting potential supply to zero.[32])

The fixed factor in the oil market is the easiest to describe: it is always equal to demand for the fixed factor, and the price is set to the GDP deflator. The existence of oil rents allows for the arbitrary allocation of payments to the fixed factor, the determination of the latter will be described below in the section on the oil markets.

Table 15.3 lists the variables determined in this section.

Table 15.3: **Data Dictionary for Subroutine *FFactSupply***
`factsup.c`

$FF^s_{i,t}$	`FFactS[r][i][t]`	Supply of fixed factors

16. Equilibrium on the Factor Markets

This section describes the determination of factor prices (see subroutine ***Equil*** in file `equil.c`). There are three parts to this section: the labour market, capital markets, and fixed factor markets. Note that equilibrium prices cannot be determined directly (e.g. there is no reduced form expression for the determination of wages). Instead, the solution algorithm uses a procedure referred to as tâtonnement. Demand and supply are determined on the relevant market and prices are adjusted in the appropriate direction in order to reduce the imbalance on the market. The size of the adjustment is a critical parameter in determining the convergence properties of the algorithm. The basic formula is:

$$P_{it} = P_{it-1}\left(\frac{D}{S}\right)^{\delta}$$

where P is the market equilibrium price, it is the iteration counter, D is demand, S is supply, and δ is the adjustment (or damping) factor. It is clear from this equation that when supply equals demand, the equilibrium price has been determined. The following example demonstrates the tâtonnement mechanism: assume D is 10 per cent greater than S, i.e. the ratio of demand over supply is 1.1, and assume the current price is 1. If δ is 1, then the new price becomes 1.1. Since demand is greater than supply, and the new price is greater than the old price, demand should decrease, and supply should increase leading to a narrowing of the gap in the market. If δ is 0.3, the new price is 1.03, which leads to the same direction of adjustment, but not as large. If δ is set too high, there can be overshooting leading to large cycles around equilibrium, and even leading to non-convergence of the algorithm. If δ is too small, the number of iterations before achieving convergence can be large leading to long solution times. The choice of δ is more an art than a science. The current version of the model has at least 84 equilibrium prices, i.e. 84 δ parameters, with cross-over effects from one market to another. A listing of the evolution of the market gaps can help the user determine the optimal level of the damping parameters.

The model has only one labour market (per region). Labour demand is summed across all sectors (and government), and there is only one labour supply which is pre-determined at the beginning of each period. This leads to the following equilibrium condition:

Table 16.1: **Equilibrium on the Labour Market**

(16.1.1) $$L^s_t = L^d_{g,t} + \sum_i L^d_{i,t}$$

[32] If supply potential is 0, supply of the fixed factor is set to fixed factor demand.

Equation (16.1.1) determines the wage rate, w. The tâtonnement procedure is used to calculate the wage rate in each iteration.

The fixed factors are sector-specific, and therefore are associated with a sector-specific price. (Note that if the supply elasticities of the fixed factors are infinite, their price is determined in the subroutine ***FFactSupply***, see above).

Table 16.2: **Equilibrium on the Fixed Factor Markets**

(16.2.1) $$FF_{i,t}^{s} = FF_{i,t}^{d}$$

Equation (16.2.1) in Table 16.2 determines the price of the fixed factor, P^{ff}, in each sector. Again, a tâtonnement procedure is used to determine the price of the fixed factor.

The capital market is somewhat more complicated. There is a single rental rate on all capital which is not part of a declining sector. In other words, the single rental rate applies to all new capital, plus old capital in expanding sectors, plus old capital being disinvested by declining sectors. The rental rate in declining sectors is sector specific. This leads to the following market equilibrium conditions:

Table 16.3: **Equilibrium on the Capital Markets**

(16.3.1) $$KOld_{i,t}^{Old} = Kv_{i,t}^{d,Old} \quad \text{if} \quad i \in \{Decl\}$$

(16.3.2) $$KOld_{g,t}^{Old} = Kv_{g,t}^{d,Old} \quad \text{if} \quad TGv_{t}^{New} = 0$$

(16.3.3) $$\sum_{i \notin Decl} K_{i,t}^{d} + \phi K_{g,t}^{d} = K_{t}^{s} - \sum_{i \in Decl} KOld_{i,t}^{s} - (1-\phi) KOld_{g,t}^{s}$$

(16.3.4) $$\begin{cases} r_{i}^{Old} = R & \text{if} \quad i \notin \{Decl\} \\ r_{g}^{Old} = R & \text{if} \quad \phi = 1 \end{cases}$$

(16.3.5) $$\begin{cases} r_{i}^{New} = R \\ r_{g}^{New} = R \end{cases}$$

Equation (16.3.1) determines the rental rate of old capital in declining sectors, i.e. r^{Old}. The set $\{Decl\}$ is the set of all declining sectors, i.e. where $XPv^{New} = 0$. Equation (16.3.2) determines the rental rate of old government capital if the government sector is in decline.

Equation (16.3.3) determines the economy-wide rental rate, R, on all capital which is not used in declining sectors. On the demand side, this is the sum of aggregate capital used in all growing sectors, plus the aggregate demand for capital in government if government is growing. The parameter ϕ is equal to 1 if the government is growing. On the supply side, the total stock of capital is determined as the sum of depreciated capital from the previous period, plus the addition to the capital stock due to the previous period's investment. Capital used in the declining sectors is subtracted from the aggregate stock to give the total net supply of capital available to the non-declining sectors. The rental rates used to determine income and production are differentiated by sector and vintage for the sake of simplifying notation. Equations (16.3.1) and (16.3.2) are used to determine r^{Old} in the declining sectors (r^{New} is irrelevant in declining sectors since new capital demand is 0). In all other sectors, the new and old rental rates are equal to R, and are determined using Equations (16.3.4) and (16.3.5).

Smoothing

Control of the tâtonnement damping factors may not be sufficient to insure proper convergence of the model. The introduction of back-stops with very high CES elasticities, as well as some extreme simulations, such as

radical reductions in emissions, may lead to explosive behaviour on the factor markets. It has proven useful at times to *smooth* some equations, i.e. to slow down the updating of variables. For example, assume variable x is given by:

$$x_{it} = f(y_{it})$$

where f is some function of the variables y, and it is the iteration counter for a specific period of the model. Smoothing the updating of x involves the following step:

$$x_{it} = smooth\, f(y_{it}) + (1 - smooth)x_{it-1}$$

where *smooth* is a number between 0 and 1, i.e. the updated variable is a weighted average of its previous level and its new level. The closer *smooth* is to zero, the slower is the updating process. At equilibrium, the parameter *smooth* is irrelevant. It is best to minimise the use of smoothing because it tends to clutter the code as well as lead to different convergence properties. It has been used mainly for the following circumstances:

- The introduction of the back-stops leads to significant swings in the demand for fuels, particularly electricity. Smoothing has been applied to the output equations for fuels and the demand for fuel intermediates to allow the conventional fuels to adjust slowly to the new substitutes. (Note that the substitutes are introduced with CES elasticities of 10).
- The introduction of carbon taxes can lead to significant changes in the price of fuels. The updating of the (ad valorem) carbon tax has often been smoothed.
- Performing sensitivity analysis on some of the CES parameters, e.g. the Armington elasticities, can lead to big shifts again requiring smoothing.

If a particular simulation is explosive, smoothing may be required, though often times appropriate adjustment of the damping factors suffices. As a rule of thumb it is best to attack the source of the problem, most often prices. However, like the damping factors, appropriate smoothing is more art than science. The code which is being shipped has been fine tuned to handle most of these situations and users should not have to modify these parameters which for the moment are hard coded.

Table 16.4 presents the list of variables determined in this section.

Table 16.4: **Data Dictionary for Subroutine *Equil***
`equil.c`

w	`wage[r][t]`	Nominal wage rate
Pff_i	`pffact[r][i][t]`	Price of fixed factor
r_i^v	`rent[r][i][v][t]`	Rental rates by production sector and vintage
r_g^v	`rentg[r][v][t]`	Rental rate in government by vintage
R	`TRent[r][t]`	Rental rate of "new" capital

17. Tax Revenues

This section will describe net revenues generated by taxes/subsidies on commodities (see subroutine ***Taxes*** in file *taxes.c*). Table 17.1 lists the equations determining tax revenues.

Table 17.1: **Tax Revenue Equations**

(17.1.1) $$PTxY_i = \tau_i^p PX_i XP_i$$

(17.1.2) $$STxY_i = \tau_i^d PP_i XD_i$$

(17.1.3) $$TarY_i = \tau_i^m WPM_i XM_i$$

(17.1.4)
$$\begin{aligned} FuTxY_i &= dfutx_i PD_i \left(\sum_j XDp_{i,j} + \sum_k XDc_{i,k} + XDg_i + XDi_i + XDst_i \right) \\ &+ mfutx_i PM_i \left(\sum_j XMp_{i,j} + \sum_k XMc_{i,k} + XMg_i + XMi_i + XMst_i \right) \end{aligned}$$

Equation (17.1.1) determines sectoral tax receipts, *PTxY*, generated by the production tax/subsidy. Equation (17.1.2) determines tax receipts generated by the sales tax on domestic sales of domestic production, *STxY*. Equation (17.1.3) determines import tariff revenues, *TarY*, where *WPM* is the border price of aggregate imports (to be discussed below).

Fuel taxes by sector, *FuTxY*, are determined by Equation (17.1.4). The equation for fuel taxes in the crude oil sector are slightly modified since imports are not differentiated from domestic production. Fuel taxes in the crude oil sector are determined using aggregate demand (i.e. *XAx*), and there is a single fuel tax denoted *dfutx*.

Table 17.2 lists the variables determined in this section.

Table 17.2: **Data Dictionary for Subroutine *Taxes***
`taxes.c`

$PTxY_i$	`PTxY[r][i][t]`	Net revenue from taxes/subsidies on production
$STxY_i$	`STxY[r][i][t]`	Domestic sales tax income
$TarY_i$	`TarY[r][i][t]`	Tariff income
$FuTxY_i$	`FuTxY[r][i][t]`	Fuel tax income

18. World Trade

This and the following section describe the world trade system. These equations are driven from the subroutine ***World*** in the file `world.c`. This section will describe the world trade system for the Armington commodities (see subroutine ***WorldTrade*** in file `wtrade.c`). The following section describes the world trade system for the crude oil market. Aggregate sectoral imports, XM_i, were determined above (aggregated over all economic agents). The second level of the Armington system distributes the aggregate import bundle across regions, again using a CES aggregation function (see Figure 3). Table 18.1 lists the equations relevant for the second level Armington nest.

Table 18.1: **World Trade and Import Prices**

(18.1.1) $$M_i^{r,r'} = \Phi_i^{r,r'} XM_i^r \left(\frac{WPM_i^r}{PE_i^{r'}} \right)^{\Lambda_i^r}$$

(18.1.2) $$WPM_i^r = \left[\sum_{r'} \Phi_i^{r,r'} PE_i^{r'(1-\Lambda_i^r)} \right]^{\frac{1}{1-\Lambda_i^r}}$$

(18.1.3) $$PM_i^r = (1+\tau_i^r) WPM_i^r$$

Each commodity i is associated with a trade matrix. Along the rows, one determines the imports of country r originating in country r'. Down the columns, one determines the exports from country r' purchased in country r. The sum along a row (in value terms), determines aggregate sectoral imports (in value) for country r. The sum down a column determines exports. (Since exports are not differentiated by region of destination, it is possible to sum down the column of the trade matrix in volume terms.) Equation (18.1.1) defines the sectoral trade matrices (for all commodities except oil and the back-stops). $M^{r,r'}$ represents the imports into region r, originating in region r'. *XM* is aggregate sectoral imports, determined at the top level of the Armington structure. The relevant import prices are the export prices of the originating countries (there are no export taxes or subsidies). An exporting country will lose market share if its export price increases relative to the importing country's import price index, *WPM*. The CES share parameters are given by the matrix Φ, and the substitution elasticity by Λ. Equation (18.1.2) defines the aggregate import price, *WPM*. It is the CES dual price in the export prices of the trading partners of country r. There is no unique world price of imports. Each region's import price will depend on its trading shares and export prices of its trading partners. Equation (18.1.3) determines the domestic price of imports inclusive of tariffs, *PM*. Tariffs are not differentiated by region of origin of the imports.

Table 18.2 lists the export demand equations.

Table 18.2: **Export Demand and Supply**

(18.2.1) $$ED_i^r = \sum_{r'} M_i^{r',r}$$

(18.2.2) $$ES_i^r = ED_i^r$$

Equation (18.2.1) determines sectoral export demand (in volume terms) facing region r, *ED*. It is the sum of imports of the exporting country's trading partners (note the transposition of the indices). Equation (18.2.2) determines trivially export supply, *ES*. Export supply is not a useful variable in GREEN since producers do not differentiate between selling goods domestically or abroad, i.e. there is no specific market behavior associated with exporting. Many CGE models use the constant-elasticity-of-transformation (CET) assumption, which allows exporters to have some market power. In a model with the CET, Equation (18.2.2) would be used to determine the equilibrium price of exports. Without the CET, the export price is simply equal to the producer output price.

The model needs a world price index to evaluate net foreign saving using a consistent world price. A potential candidate is the numéraire. Other indices are possible. The current version uses a weighted average of the regional GDP deflators. This is defined in Table 18.3.

Table 18.3: **World Price Indices**

$$(18.3.1) \qquad P = \frac{\sum_r PGDP_r \left(L_r^s + K_r^s + \sum_i FF_{r,i}^s \right)}{\sum_r \left(L_r^s + K_r^s + \sum_i FF_{r,i}^s \right)}$$

$$(18.3.2) \qquad P^{OECD} = \frac{\sum_{r \in OECD} PGDP_r RGDP_r}{\sum_{r \in OECD} RGDP_r}$$

Equation (18.3.1) determines the price index, P, used to determine the relative value of net foreign saving. Equation (18.3.2) determines a weighted average of the GDP price deflators in the OECD regions. The OECD deflator is used to deflate the world price of crude oil in simulations where the oil price is exogenous.

Two macro closure rules have already been discussed. Government saving is either exogenous (and household tax rates are endogenous) or government saving are endogenous (and household tax rates are fixed). Investment is driven by saving: household, government, foreign, and depreciation allowance.

The third macro closure rule concerns the trade balance which is assumed fixed in each time period. This leads to Equation (18.4.1) in Table 18.4.

Table 18.4: **Trade Closure**

$$(18.4.1) \qquad \sum_i PE_i^r ED_i^r + P\, S_f^r = \sum_i \sum_{r'} PE_i^{r'} M_i^{r,r'}$$

On one side of the balance sheet are exports, evaluated at world prices, and net foreign saving. On the other side of the balance sheet are the sum of imports evaluated at world prices (excluding tariffs). Sf is exogenous, and therefore the net trade balance at world prices is given (in real terms). The variable Sf is specified by the user for each region and time period.[33] For global consistency, it must always be true that the sum of foreign saving globally equals 0.

$$\sum_r S_f^r \equiv 0$$

Due to Walras' Law, one equation in each region is redundant, and Equation (18.4.1) is dropped from the model. It is calculated nonetheless to verify the consistency of the model after convergence has been achieved on the factor markets. Users can specify an absolute limit ($100 million for example) on the gap allowed in Walras' Law. The program will continue to iterate until the gap has been reduced to within the limit, even if convergence has been achieved in the factor markets (within the given convergence tolerance).

Numéraire

Any price in the model can be chosen as the numéraire. However, for reasons of numerical stability, is has been determined (empirically) that it is best to choose a price associated with a large volume. In the current version of the model, the OECD GDP price deflator, P^{OECD}, has been designated as the numéraire, and its value is always set to 1.[34] In order to achieve this (in the solution algorithm), Equation (18.3.2) is used to determine the value of P^{OECD}. Afterwards, all factor prices, in the OECD regions, are divided by this intermediate value of P^{OECD}. At equilibrium, P^{OECD} must equal 1 and hence the division will have no effect on the factor prices. But if, during

[33] In previous versions of the model, each region's foreign saving was held constant at its 1985 level (in real terms).

[34] In past versions of the model, the US wage rate was designated as the numéraire, and set to the value 1.

(raise) all the factor prices in the OECD region, which, all else being equal, should make P^{OECD} converge towards 1.

Table 18.5 lists the variables determined in this section.

Table 18.5: **Data Dictionary for Subroutine *WorldTrade***
wtrade.c

WPM_i^r	WPM[r][i][t]	Aggregate price of sectoral imports excluding import taxes
PM_i^r	PM[r][i][t]	Aggregate price of imports including import taxes
$M_i^{r,r'}$	wtflow[r][rp][i][t]	World trade matrix by sector
ED_i^r	ED[r][i][t]	Export demand
ES_i^r	ES[r][i][t]	Export supply
P	PIndex[t]	Global price index
P^{OECD}	POECD[t]	OECD export price deflator
	Walras[r][t]	Trade closure (Walras' Law)

19. The Crude Oil Market

Crude oil is assumed to be a perfectly homogeneous good across all regions. Therefore the law of one price holds, and all trade occurs at a single world price. The model allows for two different assumptions on the determination of the world oil price. Under either assumption, all regions, except the energy exporting regions (EEX), are assumed to follow their oil reserve depletion path (see below), or in other words, their output is determined mainly by physical constraints (albeit with some variations). The first world oil price assumption is that it is determined exogenously. In this case, the EEX countries are assumed to be the residual suppliers and will produce any amount to assure commodity equilibrium on the crude oil market. (If the oil price is exogenous, it is assumed that the EEX region is never supply constrained. In the reference simulation, the oil price is only exogenous in the initial simulation period (1990), since it has already been observed.)

The second assumption allows the world oil price to be determined endogenously. While all regions except the EEX follow their reserve depletion profile, supply in the EEX will be price driven leading to an endogenous world oil price.

Table 19.1 presents the equations determining oil output.

Table 19.1: **Crude Oil Output**

(19.1.1) $$XP^r_{Oil,t} = XPot^r_{Oil,t} \quad \text{if} \quad r \neq EEX$$

(19.1.2)
$$\begin{cases} XP^{EEX}_{Oil,t} = \sum_r XA^r_{Oil,t} - \sum_{r \neq EEX} XP^r_{Oil,t} & \text{if} \quad \overline{PW}_{Oil,t} \\ XP^{EEX}_{Oil,t} = \min\left[XP^{EEX}_{Oil,t-1}\left(\dfrac{PW_{Oil,t} / PGDP_t}{PW_{Oil,t-1} / PGDP_{t-1}} \right)^{\eta^f_{Oil,t}}, XPot^{EEX}_{Oil,t} \right] & \text{if} \quad PW_{Oil,t} \end{cases}$$

Equation (19.1.1) describes output in all regions except EEX. Output in these regions is simply equal to potential output, i.e. all of these regions are following their reserve depletion profile. Equation (19.1.2)

determines output in the EEX region. If the world price of oil is assumed to be exogenous, EEX becomes the residual supplier. Total world demand is determined, total supply from all non-EEX regions is determined, and output in the EEX region is determined by subtracting the supply sub-total from the demand total. If the world oil price is to be determined endogenously, then output in the EEX region responds to price signals, with an upward bound given by the reserve depletion constraint.

Trade is simple to determine since it is the difference between domestic supply and domestic demand. Domestic demand is determined at the Armington level for each domestic agent, but it is not broken down into domestic and import component by agent. Table 19.2 lists the equations determining domestic demand for crude oil and the trade equations.

Table 19.2: **Crude Oil Demand and Trade**

(19.2.1) $$XA^{r}_{Oil,t} = \sum_{j} XAp^{r}_{Oil,j,t} + \sum_{k} XAc^{r}_{Oil,k,t} + XAg^{r}_{Oil,t} + XAi^{r}_{Oil,t} + XAst^{r}_{Oil,t}$$

(19.2.2) $$ES^{r}_{Oil,t} = ED^{r}_{Oil,t} = \max\left[0, XP^{r}_{Oil,t} - XA^{r}_{Oil,t}\right]$$

(19.2.3) $$XM^{r}_{Oil,t} = \max\left[0, XA^{r}_{Oil,t} - XP^{r}_{Oil,t}\right]$$

(19.2.4) $$XD^{r}_{Oil,t} = XP^{r}_{Oil,t} - ES^{r}_{Oil,t}$$

Equation (19.2.1) determines total domestic demand for crude oil. (The use of the notation XA, i.e. Armington, is used in order to simplify notation and the code. It is not meant to signify that crude oil is an Armington good.) Aggregate domestic oil demand is the sum over demand from all domestic agents. Equation (19.2.2) and (19.2.3) determine net trade. If output is greater than domestic demand, exports are positive and imports are zero, else imports are positive, and exports are zero (for the sake of consistency with other equations, export demand as usual is identically set to export supply). Equation (19.2.4) determines domestic demand for domestic output, this is trivially determined as the difference between output and export supply (again, though this equation is trivial, it is needed for the consistency of the other equations in the model).

Table 19.3 describes the determination of the world oil price.

Table 19.3: **World Crude Oil Price**

(19.3.1)
$$\begin{cases} PW_{Oil,t} = P_t^{OECD}\,\overline{PW}_{Oil,t} \\ \text{or} \\ \sum_{r} XA^{r}_{Oil,t} = \sum_{r} XP^{r}_{Oil,t} \end{cases}$$

Equation (19.3.1) allows for the two different world oil price assumptions. Under the first assumption, the world oil price is exogenous and determined by the user, and is deflated by the OECD price index to ensure price homogeneity. (The user actually inputs the exogenous price scenario as per annum growth rates between periods.)

Under the second assumption the world oil price is determined endogenously and the second equation reflects the world market equilibrium condition. Because this equation does not determine the world price explicitly, a tâtonnement procedure is used to evaluate changes in the world oil price given the direction of the market disequilibrium.

The next table describes the domestic oil prices.

Table 19.4: **Domestic Crude Oil Price**

(19.4.1) $$PE^{r}_{Oil,t} = WPM^{r}_{Oil,t} = PW_{Oil,t}$$

(19.4.2) $$PD^{r}_{Oil,t} = PM^{r}_{Oil,t} = (1+\tau^{r}_{Oil,t})\,PW_{Oil,t}$$

(19.4.3) $$PAp_{Oil,j} = PDp_{Oil,j} = (1+dfutx_{Oil})\,PD_{Oil}$$

(19.4.4) $$PAc_{Oil,k} = PDc_{Oil,k} = (1+dfutx_{Oil})\,PD_{Oil}$$

(19.4.5) $$PAg_{Oil} = PDg_{Oil} = (1+dfutx_{Oil})\,PD_{Oil}$$

(19.4.6) $$PAi_{Oil} = PDi_{Oil} = (1+dfutx_{Oil})\,PD_{Oil}$$

(19.4.7) $$PAst_{Oil} = PDst_{Oil} = (1+dfutx_{Oil})\,PD_{Oil}$$

Equation (19.4.1) sets the regions' export price and world import price to the global price. Equation (19.4.2) determines the sales price on the domestic markets. (The calibration process insures that the domestic and import price wedges are equal in order to be consistent with the law of one price). Equations (19.4.3)-(19.4.7) determine the *Armington* prices for each one of the corresponding economic sectors.[35] The sector specific import prices are the same (GREEN imposes a uniform carbon tax in the crude oil sector, i.e. *dfutx* = *mfutx*).

There is no mechanism which sets the producer price (i.e. the average cost of production) equal to the sales price, i.e. there exists a price wedge between the producer price and the sales price which determines rents in the crude oil sector. Table 19.5 defines the equation which determines the rent in the crude oil sector.

Table 19.5: **Oil Rents**

(19.5.1) $$OilRents^{r}_{t} = \left[PW_{Oil,t} - PX^{r}_{Oil,t}(1+\tau^{p}_{r,Oil})\right] XP^{r}_{Oil,t}$$

The following table provides the data dictionary for variables determined in this section.

[35] It is only for the simplification of notation (and formulas), that the term Armington is used in the context of crude oil. There is no differentiation between domestic and imported oil, and hence there is only one price.

Table 19.6: **Data Dictionary for Subroutine *OilMarket***
`oilmarkt.c`

$XP^r_{Oil,t}$	`XP[r][Oil][t]`	Domestic crude oil production
$XD^r_{Oil,t}$	`XD[r][Oil][t]`	Domestic oil demand for domestic production.
$XM^r_{Oil,t}$	`XM[r][Oil][t]`	Demand for oil imports
$ED^r_{Oil,t}$	`ED[r][Oil][t]`	Export demand for crude oil
$ES^r_{Oil,t}$	`ES[r][Oil][t]`	Export supply of crude oil
$PW_{Oil,t}$	`PWOil[t]`	Global price of crude oil
$PE^r_{Oil,t}$	`PE[r][Oil][t]`	Domestic world trade oil prices
$WPM^r_{Oil,t}$	`WPM[r][Oil][t]`	
$PD^r_{Oil,t}, PM^r_{Oil,t}$	`PD[r][Oil][t]` `PM[r][Oil][t]`	Domestic sales price of crude oil
$PAx^r_{Oil,t}$	`PAx`	The Armington prices
$OilRents^r_t$	`OilRents[r][t]`	Oil rents

20. Aggregate Capital Stock and Productivity Growth

This section, and the next, provide the key equations for describing the transition from one period to the next. The subroutine ***KapStock*** (in file *kapstock.c*) updates the aggregate capital stock. The aggregate capital stock is not pre-determined because it depends on the current level of investment. The one year gap transition equation is given by:

$$K_t = (1-\delta)K_{t-1} + I_{t-1}$$

where K is the aggregate capital stock, δ is the annual rate of depreciation, and I_{t-1} is the level of real investment in the previous period. A problem appears when the gap between solution periods is greater than 1 year. Since investment in the intervening years is not calculated assumptions must be made in order to integrate the stream of investment. If we expand the transition equation for a multi-period gap, we arrive at:

$$K_t = (1-\delta)\left[(1-\delta)K_{t-2} + I_{t-2}\right] + I_{t-1}$$
$$\vdots$$
$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1} I_{t-j}$$

The model does not calculate investment between periods. A linear growth model is assumed to explain investment in intermediate years, i.e.:

$$I_j = (1+\gamma^i) I_{j-1}$$

where

$$\gamma^i = \left(\frac{I_t}{I_{t-n}}\right)^{\frac{1}{n}} - 1$$

where the annual growth rate of investment is derived from the annualised growth rate of investment in the current period compared to investment in the previous period. We can re-write the multi-year transition equation to be:

$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n} (1-\delta)^{j-1} (1+\gamma^i)^{n-j} I_{t-n}$$

A little bit of algebra yields the transition equation given by Equation (20.1.1) in Table 20.1.

Table 20.1: **Aggregate Supply of Capital**

(20.1.1)
$$K_t = (1-\delta)^n K_{t-n} + \frac{(1+\gamma^i)^n - (1-\delta)^n}{\gamma^i + \delta} I_{t-n}$$

where $$\gamma^i = \left(\frac{I_t}{I_{t-n}} \right)^{\frac{1}{n}} - 1$$

(20.1.2)
$$K_t^s = \frac{K_{t-n}^s}{K_{t-n}} K_t$$

While equation (20.1.1) appears to be independent of the time period t (i.e. it appears K_t is pre-determined), the parameter γ^i is the inter-period growth rate of investment and is a function of the current period's level of investment.

Due to base year normalisation rules (the rental rate is set to 1 in the base year), the aggregate stock of capital, K, is normalised to yield K^S (Equation (20.1.2), which is the level of capital used in determining equilibrium on the capital market.[36]

Efficiency of capital and the fixed factor in the *KF* bundle are assumed to be equal.[37] While in most simulations, this efficiency factor is exogenous, in the reference simulation it is endogenous, and it is calculated in order to yield a pre-determined growth path for the regional GDP.

Only five sectors are assumed to have efficiency growth in capital and the fixed factor: agriculture, refined oil, electricity, energy intensive industries, and other industries and services.[38]

Let us define the set

Ω = { *Agric, RefOil, Elec, EnerInt, OthInd* }

then Table 20.2 defines the accumulation equations for efficiency growth in capital and the fixed factor:

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36 The following numerical example may shed some light on the normalisation rule. Assume the value of capital in a region is 100. Assume, as well, that capital renumeration is 10. Capital remuneration is simply rK where r is the rental rate and K the demand for capital. In this example, rK is equal to 10, which implies a rental rate of 0.1. GREEN uses a different normalisation rule. It assumes that the base year rental rate is 1, and normalises the capital data to be consistent with this normalisation rule, in other words, the normalised capital demand is 10, and it is really an index of capital volume. The non-normalised level of capital is used only in the accumulation function (Equation 20.1.1), and in determining the value of capital depreciation allowance. All other capital stock equations use the normalised value of capital.

37 In other words, the efficiency factor is applied to the *KF* bundle, and not to the capital and fixed factor individually. Furthermore, the efficiency factor is uniform across sectors. Partly this is a target versus instrument problem. In the reference simulation, the target is a single economy-wide real GDP growth rate. Given there is a single target, the single instrument is an economy-wide efficiency factor for the *KF* bundle.

38 The fossil fuel sectors have to be treated differently. Since capital and the fixed factor are perfect complements and the efficiency factor is applied equally to both factors, it would be possible to increase the output of fossil fuels per unit of reserves if we allowed for the possibility of efficiency improvement in these sectors. Clearly, one barrel of oil reserves can only yield one barrel of oil.

Table 20.2: **Productivity Factors for Capital and the Fixed Factor**

(20.2.1) $\lambda^k_{i,t} = \lambda^k_{i,t-n}(1+\gamma^{kf}_t)^n$ if $i \in \Omega$

(20.2.2) $\lambda^f_{i,t} = \lambda^f_{i,t-n}(1+\gamma^{kf}_t)^n$ if $i \in \Omega$

(20.2.1) $\lambda^k_{i,t} = 1.0$ if $i \notin \Omega$

(20.2.2) $\lambda^f_{i,t} = 1.0$ if $i \notin \Omega$

In the reference simulation, the efficiency parameter, γ^{kf}, is calibrated to be consistent with an exogenously determined growth path for the regional economy. The calibration occurs in the subroutine ***CalibKFTrend*** (in `kapstock.c`), and is called by ***KapStock*** if the calibration flag has the value DynamicCalibration. The key concept behind the calculation of the KF efficiency parameter, is the following equation:

$$RGDP_t = (1+\gamma^y)^n RGDP_{t-n}$$

In other words, the KF efficiency factor is calibrated in order to achieve a given growth rate in real GDP (evaluated in efficiency units).[39] The equation above would be sufficient to calculate the KF efficiency factor if the solution algorithm is a Jacobian technique (such as used in the GAMS/MINOS package). In the case of Gauss/Seidel, each endogenous variable must appear at least once on the left hand-side of an equation, and therefore, the equation above is converted into a type of tâtonnement equation to determine the KF efficiency factor.

The first step is to calculate the efficiency factor of aggregate capital and the fixed factors, separating the sectors with efficiency growth from those without efficiency growth. This leads to the following estimate of the efficiency factor:

$$\gamma_{it} = \left[\frac{K^s_t \lambda^{kf}_t + (1-\lambda^{kf}_t)\sum_{i\notin\Omega} K^d_{i,t} + \sum_i \lambda^f_{i,t} FFt^s_{i,t} + K^d_{g,t}}{K^s_{t-n}\lambda^{kf}_{t-n} + (1-\lambda^{kf}_{t-n})\sum_{i\notin\Omega} K^d_{i,t-n} + \sum_i \lambda^f_{i,t-n} FF^s_{i,t-n} + K^d_{g,t-n}} \right]^{\frac{1}{n}} - 1$$

where λ^{kf}_t is the common efficiency factor in the sectors belonging to the set Ω, i.e. the sectors with efficiency improvement.[40] Setting γ_{it} to γ^y in the formula above, and doing a little bit of algebra, would show the equivalence between the two formulas. The efficiency trend for capital and the fixed factor is updated using the following (tâtonnement-type) formula:

$$\gamma^{kf}_{it,t} = \gamma^{kf}_{it-1,t} + \frac{1+\gamma^y_t}{1+\gamma_{it}} - 1$$

where the index *it* refers to the iteration count for the current period. Convergence implies that the average efficiency factor (γ_{it}) over all sectors is equal to the desired growth trend (γ^y) for the economy. If the simulation assumes the γ^{kf} trend is pre-calibrated, the trend is read in from a file.

Table 20.3 lists the variables determined in this section.

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39 Remember, the efficiency of labour is exogenous.

40 In the code, this variable does not exist, instead the efficiency factor for agriculture is used. Also note that the lambda parameters, λ, used in summing up the fixed factors in efficiency terms, have the value 1 for the sectors with no efficiency improvement (as specified in the equations in Table 20.2).

Table 20.3: **Data Dictionary for Subroutine *KapStock***
kapstock.c

K_t	KStock[r][t]	Volume of capital stock
K_t^s	KapS[r][t]	Volume of capital stock (normalised)
$\lambda_{i,t}^k$	lambdak[r][i][t]	Capital efficiency factor
$\lambda_{i,t}^f$	lambdaf[r][i][t]	Fixed factor efficiency factor
γ_t^{kf}	gkf[r][t]	Growth rate of technical progress of KF bundle (if in dynamic calibration mode)

21. Depletion Modules for Oil and Gas Reserves

The subroutine ***Depletion*** (in file *depletn.c*) updates crude oil and natural gas output according to a standard depletion model. This section will first describe the standard depletion model and then explain how it is implemented in GREEN.

The model is based on two key parameters: the rate of extraction from proven reserves, *r*, and the rate of discovery of new reserves, *d*, also called the conversion rate. Potential supply in any given time period is a constant fraction of proven reserves:

$$XPot_t = r\,Res_t \tag{1}$$

where *XPot* is potential supply, *r* is the extraction rate[41], and *Res* is proven reserves (we will drop the sectoral subscript for the moment). New reserves in each time period is determined as a fraction of yet-to-find reserves:

$$NRes_t = d\,YTFR_t \tag{2}$$

The dynamic (or motion) equation for reserves is therefore given by:

$$Res_t = Res_{t-1} + NRes_{t-1} - X_{t-1} \tag{3}$$

where *X* is actual output (not potential output). Reserves in time period *t* are equal to the previous period's reserves, plus new discoveries, less actual output. This equation can be written as:

$$Res_t = (1-r)Res_{t-1} + d\,YTFR_{t-1} \tag{4}$$

under the assumption that actual output is equal to potential output, or in other words, output is occurring along the resource depletion profile. Assuming maximum output, this latter equation can be iterated forwards *n* periods to determine the complete depletion profile given initial reserves and yet to find reserves (and assuming *r* and *d* are constant). Through induction we can expand the equation to yield:

$$\begin{aligned} Res_t &= (1-r)\left[(1-r)Res_{t-2} + d\,YTFR_{t-2}\right] + d\,YTFR_{t-1} \\ &\vdots \\ Res_t &= (1-r)^n Res_{t-n} + d\sum_{j=1}^{n}(1-r)^{j-1} YTFR_{t-j} \end{aligned}$$

41 The extraction rate is region specific, however constant over time. It is invariant with respect to all variables including output price and level of depletion.

Finally, we can use the motion equation for yet-to-find-reserves to simplify the expression above:

$$YTFR_t = (1-d)YTFR_{t-1}$$
$$\vdots$$
$$YTFR_{t-j} = (1-d)^{n-j} YTFR_{t-n}$$

therefore

$$Res_t = (1-r)^n Res_{t-n} + d\, YTFR_{t-n} \frac{(1-r)^n - (1-d)^n}{d-r} \tag{5}$$

This latter equation permits one to easily calculate the entire depletion profile given initial reserves and initial yet-to-find reserves, and the extraction and conversion rates. Figures 4a and 4b depict two different depletion profiles.

Producers are not necessarily going to be on their depletion profile. Current price signals may induce producers to keep the reserves underground and await future higher prices to exploit their reserves. In this case, the depletion profile will change over time. In the extreme case, output may be zero, and reserves will grow until there are no undiscovered reserves. The motion equation in the case of production below potential output can be determined by equation (3). Again, using induction, we have:

$$Res_t = Res_{t-n} + \left[1-(1-d)^n\right] YTFR_{t-n} - \sum_{j=1}^{n} X_{t-j} \tag{6}$$

The model does not calculate the intra-period actual annual output. In this case, a linear growth model is assumed for actual output:

$$X_t = (1+\gamma) X_{t-1}$$
$$\vdots$$
$$X_{t-j} = (1+\gamma)^{n-j} X_{t-n}$$

where

$$\gamma = \left(\frac{X_t}{X_{t-n}}\right)^{\frac{1}{n}}$$

Under this assumption, Equation (6) can be re-written to yield:

$$Res_t = Res_{t-n} + \left[1-(1-d)^n\right] YTFR_{t-n} - \frac{(1+\gamma)^n - 1}{\gamma} X_{t-n} \tag{7}$$

Reserves calculated using Equation (7), are never allowed to fall below reserves calculated using Equation (5), i.e. output can never exceed potential output.

The model relaxes the assumption that the conversion rate or the undiscovered reserves are fixed. Instead a simple price elastic model is assumed to explain changes in the conversion rate or the level of undiscovered reserves:

$$d(\tilde{P}) = \overline{d}\, \tilde{P}^{\omega}$$
$$YTFR(\tilde{P}) = \overline{YTFR}\, \tilde{P}^{\upsilon}$$

where $\tilde{P}$ is the real price of the associated fossil fuel. If the real price goes up, this will induce producers to increase the rate of conversion, or intensify exploration so as to increase the level of new discoveries. Given the calibration procedure only one of the two equations is actually used, i.e. only one of the two parameters is allowed to vary with the real price, the other remaining constant over the entire time framework of the model. This leads to the equations developed in Table 21.1. These equations should be subscripted for both the natural gas and the crude oil sectors, as all the parameters described are sector dependent. (In the model, the identifier *OilRes* is used to identify the oil sector, and *GasRes* is used to identify the natural gas sector.)

Equation (21.1.1) defines the real price of the associated fossil fuel. Equation (21.1.2) describes the level of the conversion rate dependent on the relative real price of the fossil fuel output compared to a reference price scenario. The parameters d_{Lo} and d_{Hi} are calibrated in such a way that the production associated with those values reproduce the consensus low and high projections. (The parameters d_{Lo} and d_{Hi}, and the price scenarios, P_{Lo}, P_{Ref}, and P_{Hi} are read in parameters and are constant over the entire time horizon. These parameters have been set or calibrated to coincide with consensus projections, notably from the IEA. Annex 3 describes in more detail the calibration procedure for the depletion modules.) If the fossil fuel price is increasing, the elasticity is determined (using a numerical derivative) to coincide with the pre-calibrated high conversion rate scenario. The low conversion rate scenario is used in the opposite case. The same equations are used to determine the adjustment factor for the level of yet-to-find reserves. These equations are defined by Equation (21.1.3). (Note that in the current version of the model, D_t, the yet-to-find-reserves shifter, is always set to 1, and only the d coefficient is used to calibrate the production path.)

Table 21.1: **Price Sensitive Conversion Rate and Yet-To-Find-Reserves**

(21.1.1)
$$\tilde{P}_t = \left(\frac{PP_t / PGDP_t}{PP_{t-n} / PGDP_{t-n}} \right)^{\frac{1}{n}}$$

(21.1.2)
$$\begin{cases} d_t = \min\left[d_o \left(\frac{\tilde{P}_t}{P_{Ref}} \right)^{\omega}, d_{Hi} \right] & \text{if} \quad \tilde{P}_t > P_{Ref} \quad \text{where} \quad \omega = \frac{\ln(d_{Hi} / d_0)}{\ln(P_{Hi} / P_{Ref})} \\ d_t = \max\left[d_o \left(\frac{\tilde{P}_t}{P_{Ref}} \right)^{\omega}, d_{Lo} \right] & \text{if} \quad \tilde{P}_t < P_{Ref} \quad \text{where} \quad \omega = \frac{\ln(d_{Lo} / d_0)}{\ln(P_{Lo} / P_{Ref})} \end{cases}$$

(21.1.3)
$$\begin{cases} D_t = \min\left[D_o \left(\frac{\tilde{P}_t}{P_{Ref}} \right)^{\upsilon}, D_{Hi} \right] & \text{if} \quad \tilde{P}_t > P_{Ref} \quad \text{where} \quad \upsilon = \frac{\ln(D_{Hi} / D_0)}{\ln(P_{Hi} / P_{Ref})} \\ D_t = \max\left[D_o \left(\frac{\tilde{P}_t}{P_{Ref}} \right)^{\upsilon}, D_{Lo} \right] & \text{if} \quad \tilde{P}_t < P_{Ref} \quad \text{where} \quad \upsilon = \frac{\ln(D_{Lo} / D_0)}{\ln(P_{Lo} / P_{Ref})} \end{cases}$$

The natural gas and crude oil markets are handled somewhat differently and will be described separately.

Natural Gas

In reference to the discussion above, the model keeps track of two reserve variables. The first relates to the reserve profile, i.e. the level of reserves if production is undertaken at the maximum level. This notion refers to Equation (5) above, and is reproduced, in modified form, in Equation (21.2.1) in Table 21.2. The second reserve variable refers to Equation (7) above. This variable defines the level of reserves given actual production, not potential production. Its value will always be at least as great as the reserves defined by the reserve profile, since production can never exceed maximum potential production (i.e. less production implies greater reserves). Equation (21.2.2) corresponds to this latter notion of reserves.

Table 21.2: **Reserves and Supply Potential in the Natural Gas Sector**

$$(21.2.1) \quad Res^{P}_{Gas,t} = (1-r_{Gas})^{n} Res^{P}_{Gas,t-n} + d_{Gas,t} D_{Gas,t} YTF_{Gas,t-n} \frac{(1-r_{Gas})^{n} - (1-d_{Gas,t})^{n}}{d_{Gas,t} - r_{Gas}}$$

$$(21.2.2a) \quad Res_{Gas,t} = Res_{Gas,t-n} + \left[1-\left(1-d_{Gas,t}\right)^{n}\right] D_{Gas,t} YTF_{Gas,t-n} - FF^{S}_{Gas,t-n} G$$

$$(21.2.2b) \quad G = \begin{cases} \dfrac{\dfrac{FF^{S}_{Gas,t}}{FF^{S}_{Gas,t-n}} - 1}{\left(\dfrac{FF^{S}_{Gas,t}}{FF^{S}_{Gas,t-n}}\right)^{1/n} - 1} & \text{if} \quad FF^{S}_{Gas,t} \neq FF^{S}_{Gas,t-n} \\ n & \text{if} \quad FF^{S}_{Gas,t} = FF^{S}_{Gas,t-n} \end{cases}$$

$$(21.2.3) \quad YTF_{Gas,t} = \left(1-d_{Gas,t}\right)^{n} D_{Gas,t} YTF_{Gas,t-n}$$

$$(21.2.4) \quad XPot_{Gas,t} = \begin{cases} r_{Gas} Res^{P}_{Gas,t} + \dfrac{Res_{Gas,t} - Res^{P}_{Gas,t}}{n} & \text{if} \quad Res_{Gas,t} > Res^{P}_{Gas,t} \\ r_{Gas} Res^{P}_{Gas,t} & \text{if} \quad Res_{Gas,t} \leq Res^{P}_{Gas,t} \end{cases}$$

Res^{P}, in Equation (21.2.1) represents the reserve profile (in fact, the minimum level of reserves attainable in the period), and *Res*, in Equation (21.2.2) is the actual level of reserves. Note that the *d* coefficient (the conversion rate) is time subscripted. Also note that the yet-to-find reserve level is adjusted by the time subscripted *D* coefficient. The actual level of reserves is dependent on the production path which is not known with certainty. Production is assumed to grow linearly between two periods, hence the definition of the *G* parameter. Note that in the case of natural gas, production is equivalent to the supply of the fixed factor, not natural gas output (however, since a top-level Leontief production function is used, the ratio of the fixed factor to output is constant). Note that if there is no observed growth, the multiplicative factor on output is simply the number of intermediate years. Actual reserves **cannot** fall below minimum reserves. Therefore the model sets actual reserves to minimum reserves in case the projected actual reserves fall below the lower bound. Equation (21.2.3) determines the level of yet-to-find reserves. Equation (21.2.4) describes the maximum potential supply, *XPot*. If actual reserves are greater than reserves given by the reserve profile – production is less than potential – it is assumed that the natural gas is produced, i.e. taken out of the ground, and added to the potential supply figure. In other words, potential supply is augmented by the (average) amount of natural gas which was extracted, but not sold.

The energy exporting countries (EEX) are assumed to have no resource constraints in the production of natural gas. In other words, their reserves are assumed to be infinite and natural gas production will be determined via market equilibrium conditions (they are assumed to have infinite supply elasticities). (In the model this is coded by setting reserves and supply potential to zero).

Crude Oil

Crude oil is very similar to natural gas. The key difference is that production from reserves is the same as the industry's production (i.e. not the supply of the fixed factor). Table 21.3 provides the equations for determining potential supply, reserves, and the potential reserve profile in the crude oil sector.

Table 21.3: **Reserves and Supply Potential in the Crude Oil Sector**

$$(21.3.1)\qquad Res^{P}_{Oil,t} = (1-r_{Oil})^{n} Res^{P}_{Oil,t-n} + d_{Oil,t} D_{Oil,t} YTF_{Oil,t-n} \frac{(1-r_{Oil})^{n} - (1-d_{Oil,t})^{n}}{d_{Oil,t} - r_{Oil}}$$

$$(21.3.2a)\qquad Res_{Oil,t} = Res_{Oil,t-n} + \left(1-\left(1-d_{Oil,t}\right)^{n}\right) D_{Oil,t} YTF_{Oil,t-n} - XP_{Oil,t-n} G$$

$$(21.3.2b)\qquad G = \begin{cases} \dfrac{\dfrac{XP_{Oil,t}}{XP_{Oil,t-n}} - 1}{\left(\dfrac{XP_{Oil,t}}{XP_{Oil,t-n}}\right)^{1/n} - 1} & \text{if} \quad XP_{Oil,t} \neq XP_{Oil,t-n} \\ n & \text{if} \quad XP_{Oil,t} = XP_{Oil,t-n} \end{cases}$$

$$(21.3.3)\qquad YTF_{Oil,t} = \left(1-d_{Oil,t}\right)^{n} D_{Oil,t} YTF_{Oil,t-n}$$

$$(21.3.4)\qquad XPot_{Oil,t} = \begin{cases} r_{Oil} Res^{P}_{Oil,t} + \dfrac{Res_{Oil,t} - Res^{P}_{Oil,t}}{n} & \text{if} \quad Res_{Oil,t} > Res^{P}_{Oil,t} \\ r_{Oil} Res^{P}_{Oil,t} & \text{if} \quad Res_{Oil,t} \leq Res^{P}_{Oil,t} \end{cases}$$

Equation (21.3.1) determines the minimum attainable reserve level if maximum potential supply is produced. Equation (21.3.2) determines the reserve level given actual production levels. Actual reserves are not allowed to fall below minimum reserves. Equation (21.3.3) determines yet-to-find oil reserves. Equation (21.3.4) determines maximum potential production. All regions, except EEX, are assumed to be on the crude oil resource depletion profile, i.e. there is no gap between the actual reserves and the reserves implied by maximum potential output.

The following table describes the data dictionary for variables determined in this subroutine. (The dot subscript is a holder for either the gas or the oil index.)

Table 21.4: **Data Dictionary for Subroutine *Depletion***
`depletn.c`

$d_{.,t}$	`ConvRate[r][d][t]`	Conversion rate of undiscovered reserves
$D_{.,t}$	`dYTF[r][d][t]`	Yet-to-find reserves "price sensitive" adjustment factor
$Res^{P}_{.,t}$	`ResP[r][d][t]`	Minimum reserves
$Res_{.,t}$	`Reserv[r][d][t]`	Actual reserves
$YTF_{.,t}$	`YTF[r][d][t]`	Yet-to-find reserves
$XPot_{.,t}$	`SupPot[r][d][t]`	Supply potential

22. Back-stop Prices

The introduction of the back-stops occurs under user control. The user determines when the back-stops become available, the price of the back-stops, the degree of penetration (i.e. the back-stop CES share parameters), and the substitutability of the back-stops with the conventional fuels. The price of the back-stop is specified by the

user as a price per terajoule (in 1985 US dollars). The subroutine ***BSPrice*** (in file `BSPrice.c`) converts the terajoule price of the back-stops into a price in the metric of the model, which is 1985 US dollars.

Table 22.1 provides the key equations which determine the price of the back-stops.

Table 22.1: **Determination of the Back-stop Prices**

(22.1.1) $$PD_{CoalCBS,t} = P\,PD_{Coal,1985}\left(\frac{\overline{PTJ^{*}_{CoalCBS,t}}}{PTJ_{Coal,1985}}\right)$$

(22.1.2) $$PD_{CoalCFBS,t} = P\,PD_{Coal,1985}\left(\frac{\overline{PTJ^{*}_{CoalCFBS,t}}}{PTJ_{Coal,1985}}\right)$$

(22.1.3) $$PD_{OilCBS,t} = P\,PD_{Oil,1985}\left(\frac{\overline{PTJ^{*}_{OilCBS,t}}}{PTJ_{Oil,1985}}\right)$$

(22.1.4) $$PD_{OilCFBS,t} = P\,PD_{Oil,1985}\left(\frac{\overline{PTJ^{*}_{OilCFBS,t}}}{PTJ_{Oil,1985}}\right)$$

(22.1.5) $$PD_{GasCBS,t} = P\,PD_{Gas,1985}\left(\frac{\overline{PTJ^{*}_{GasCBS,t}}}{PTJ_{Gas,1985}}\right)$$

(22.1.6) $$PD_{GasCFBS,t} = P\,PD_{Gas,1985}\left(\frac{\overline{PTJ^{*}_{GasCFBS,t}}}{PTJ_{Gas,1985}}\right)$$

(22.1.7) $$PD_{ElecBS,t} = P\,PD_{Elec,1985}\left(\frac{(1+\pi_t)\overline{PTJ^{*}_{ElecBS,t}}}{PTJ_{Elec,1985}}\right)$$

Equations (22.1.1) and (22.1.2) determine the price of the two back-stops for coal, respectively the carbon-based substitute, and the carbon free substitute. The equations convert the relevant prices (provided by the user) from a price per terajoule, into a price in the metric of the model. The price $\overline{PTJ^{*}_{bs}}$ is the user-specified price per terajoule of the back-stop substitute energies.[42] The price PTJ_i is the price, in terajoules, of the relevant conventional fuel. The back-stop price used in the model is, therefore, a price index where the relative price of the back-stop (compared to the price of the conventional fuel) is the same as the relative price of the fuels in terajoules. For example, the price per terajoule of coal in the US is $1,292 in 1985. The price of the two back-stops in the reference scenario are respectively, $8,743 and $18,950. This implies that the price of the back-stops in the model will be respectively, 6.77 and 14.67 (assuming the domestic price of coal is 1). With high CES elasticities, clearly, there will be very little penetration of the back-stop substitutes for coal in the US, unless the price of coal were to increase dramatically.[43] Equations (22.1.3)-(22.1.6) provide the relevant equations for the back-stop prices for the oil substitutes and the gas substitutes. Equation (22.1.7) determines the price of the electric back-stop. The formula is similar to the other formulas, however the price of the electric back-stop is augmented by a transportation margin.

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[42] For most simulations using GREEN, these prices have been hamonized with the assumptions used by the models in the EMF-12 exercise.

[43] Note that the user provides the penetration parameters (i.e. the CES shares). They are implemented using the backstop price indices, i.e. the CES functions for the backstops are not implemented at the level of terajoules and prices in terajoules.

The remaining code in the file `bsprice.c` determines the Armington prices which are the same as the domestic prices since it is assumed that the back-stops are available everywhere at unlimited quantities and same prices and hence there is no trade in the back-stop fuels.

Table 22.1: **Data Dictionary for Subroutine *BSPrice***
`bsprice.c`

PD_{bs}	`PD[r][bs][t]`	Domestic price (index) of the backstops. This routine also updates the Armington prices and the ad valorem carbon taxes for the backstops.

23. Energy Consumption and CO_2 Emissions

Energy consumption occurs in both production and final demand. In the metric of the GREEN model, it is calculated in units of millions of 1985 dollars. To calculate an exajoule equivalent, consumption in millions of 1985 dollars is multiplied by a conversion factor which is determined using the base year data. CO_2 emissions are represented as a constant factor of the exajoule consumption. However, CO_2 emissions are fuel specific since the carbon content of each fuel differs. The following formulas are the basis for calculating energy consumption and emissions:

$$E_f = \varphi_f X_f$$
$$CO_{2,f} = \varepsilon_f E_f = \varepsilon_f \varphi_f X_f$$

where X_f is the volume of consumption of fuel f in millions of 1985 dollars, φ_f converts millions of 1985 dollars into exajoules, E_f is consumption in exajoules, ε_f is the carbon content of fuel f, and CO_2 is the level of carbon emissions, in millions of tons. The following table indicates the relevant data for the US for coal and gas consumption in 1985:

Definition	**Coal**	**Gas**
Demand (10^6 USD)	23,158	50,305
Exajoules per 10^6 USD	773.93×10^{-6}	350.49×10^{-6}
Demand (exajoules)	17.92	17.63
Carbon content (10^6 tons per exajoule)	24.686	13.473
Emissions (10^6 tons of carbon)	442.4	237.5

The table shows that demand for coal and gas are more or less the same in exajoules, but due to the lower carbon content of gas, carbon emissions emanating from gas consumption are only 53 per cent of the carbon emissions emanating from coal.

One of the key parameters of the model, particularly with the introduction of the back-stop, is the price of the energy fuels. This is entered by the user as a price per *terajoule*, not a price per *exajoule*. In the example above, the price of coal, per exajoule, is given by the following formula:

$$P_{EJ} = \frac{23{,}158\ 10^6}{17.92} = 1{,}292\ 10^6$$

Since exajoules are one million terajoules, the price per terajoule is 1,292 (1985 dollars).

Oil, both crude and refined, and electricity are treated different from coal, gas, and the back-stops. The crude oil consumed in oil refineries, is extracted from the energy balance of the crude oil sector, as well as its emissions.

Table 23.1 describes the relevant equations for the coal, gas, and the seven back-stop sectors.

Table 23.1: **Energy Demand and Carbon Emissions for Coal, Gas, and the Back-stop Energies**

(23.1.1) $$E_i = \varphi_i XD_i + \varphi_i XM_i$$

(23.1.2) $$CO_{2,i} = \varepsilon_i E_i$$

Equation (23.1.1) defines the demand for the sector specific energy (coal, gas, and the back-stops), in exajoules, denoted by *E*. The same conversion factor applies to both demand from domestic production, and import demand. To re-iterate, the variables *XD* and *XM* are volumes in units of millions of 1985 US dollars. The conversion factors convert millions of 1985 US dollars into exajoules, and the variable *E* is demand in exajoules. Equation (23.1.2) defines the level of carbon emissions (not CO_2), in millions of tons. The conversion factor ε_f has units millions of tons of carbon per exajoule, and is fuel specific.

Table 23.2 lists the energy and emissions equations for the crude oil sector.

Table 23.2: **Energy Demand and Carbon Emissions for Crude Oil**

(23.2.1) $$E_{Oil} = \varphi_{Oil} XD_{Oil} + \varphi_{Oil} XM_{Oil}$$

(23.2.2) $$CO_{2,Oil} = \varepsilon_{Oil}(E_{Oil} - \tilde{\varphi}_{Oil}^{RefOil} XAp_{Oil,RefOil})$$

Equation (23.2.1) defines the gross energy content of crude oil consumption, including crude oil used as an intermediate input in the refining sector. Equation (23.2.2) reflects the carbon emissions associated with the net consumption of crude oil, i.e. total crude oil consumption net of its intermediate use in the refining sector. The conversion factor on the intermediate input of oil in refineries is not the same as the conversion factor on aggregate demand. (The base year data is given in value terms and there is no information on the price relatives across the row of the input/output table. Therefore, while it is possible to calculate appropriate conversion factors for the row totals, they cannot be applied uniformly across a row. Data from additional sources is used to calculate the appropriate conversion factor for the refined oil cell of the crude oil row.) While the equation uses the value of the Armington aggregate, this is irrelevant in the case of crude oil since it is assumed to be a homogeneous good.

In the case of refined oil, the energy conversion equation is straightforward, and is similar to Equation (23.1.1). The emission content of refined oil is adjusted to compensate for the carbon content of the crude oil used in the imports of refined oil products. The emissions from refined oil consumption are split into two components: emissions from consumption of domestic production, and emissions from consumption of imported production.

Table 23.3 provides the relevant equations for the refined oil sector.

Table 23.3: **Energy Demand and Carbon Emissions for Refined Oil**

(23.3.1) $$E_{RefOil} = \varphi_{RefOil} XD_{RefOil} + \varphi_{RefOil} XM_{RefOil}$$

(23.3.2) $$CO_{2,RefOil} = \varepsilon^{d}_{RefOil} \varphi_{RefOil} XD_{RefOil} + \varepsilon^{m}_{RefOil} \varphi_{RefOil} XM_{RefOil}$$

(23.3.3) $$\varepsilon^{d}_{RefOil} = \varepsilon_{RefOil} \frac{XAp_{Oil,RefOil}}{XAp_{Oil,RefOil} + XAp_{OilCBS,RefOil} + XAp_{OilCFBS,RefOil}}$$

(23.3.4)
$$\varepsilon^{m}_{r,RefOil} = \frac{\sum_{r'} \xi^{r'} M^{r,r'}_{RefOil}}{\sum_{r'} M^{r,r'}_{RefOil}}$$

where

$$\xi^{r'} = \frac{\varepsilon^{r'}_{Oil} XAp^{r'}_{Oil,RefOil} + \varepsilon^{r'}_{OilCBS} XAp^{r'}_{OilCBS,RefOil} + \varepsilon^{r'}_{OilCFBS} XAp^{r'}_{OilCFBS,RefOil}}{XAp^{r'}_{Oil,RefOil} + XAp^{r'}_{OilCBS,RefOil} + XAp^{r'}_{OilCFBS,RefOil}}$$

Equation (23.3.1) converts demand for refined oil from millions of 1985 dollars to exajoules. Equation (23.3.2) defines carbon emissions generated by refined oil consumption. Both the domestic and import component are adjusted. Equation (23.3.3) describes the adjustment of the domestic carbon content factor. The adjustment factor is equal to the share of conventional crude oil in the total intermediate demand for the crude oil composite in the refined oil sector. If there are no back-stops, the adjustment factor is equal to one. If the back-stops squeeze out conventional crude oil demand, the adjustment factor is equal to zero. (The reason for this is that there are no adjustments made in the back-stop sectors. Without the adjustment here, there would be double counting of the emissions.) The second factor in the formula, i.e. the ratio, is calculated in the ***Production*** subroutine in the file `prod.c`. It is stored using the variable name `OilConvShr`. The conversion factor for emissions generated by import demand for refined oil products is an average of the conversion factors across the import partners, using the import shares as weights. This is described in Equation (23.3.4). The conversion factors of the import partners are also adjusted subject to the share of conventional crude oil in the production of imported refined oil. The parameter $\xi^{r'}$ is updated in subroutine ***Production*** in the file `prod.c` and is stored in the variable `CO2_TJRefOil`.

Table 23.4 provides the energy content and emissions for the electric sector. There are no carbon emissions associated with the direct consumption of electricity. The emission variable refers to the emission generated by the electric sector in the production of electricity.

Table 23.4: **Energy Content of Electricity**

(23.4.1) $$E_{Elec} = \varphi_{Elec} XD_{Elec} + \varphi_{Elec} XM_{Elec}$$

(23.4.2) $$E^{ff}_{Elec} = \left[a^{ff}_{Elec} + \frac{FF^{d}_{Elec}}{\lambda^{f}_{Elec} FF^{d}_{Elec} + FUElec} \right] E_{Elec}$$

(23.4.3) $$FUElec = \sum_{e \neq Elec} \left[\frac{\lambda^{e,p}_{Elec,Old,t}}{\lambda^{e,p}_{Elec,Old,t-1}} \right]^{(1-\rho^{p,Old}_{Elec})} FUp_{e,Elec,Old} + \left[\lambda^{e,p}_{Elec,New,t} \right]^{(1-\rho^{p,New}_{Elec})} FUp_{e,Elec,New}$$

(23.4.4) $$CO_{2,Elec} = \sum_{e \neq Elec} \varepsilon_{e} \varphi_{e} (XDp_{e,Elec} + XMp_{e,Elec})$$

Equation (23.4.1) determines the energy content of electricity, and is similar to the formulas for the other energy sources. Equation (23.4.2) is an estimation of the energy (in exajoules) produced by the fixed factor used in the generation of electricity. The term in brackets is a share which is applied to the total energy consumption of

electricity. The share is the sum of two components. The first component is a shift parameter which is calibrated using base year data. The second component is the ratio of the fixed factor to a bundle composed of the fixed factor (adjusted for efficiency improvement), and a fuel bundle input. Equation (23.4.3) defines the fuel bundle input (which is calculated in the subroutine ***Production*** in the file `prod.c`). It is the sum over all energy composites — coal, crude oil, natural gas, and refined oil, but not electricity — over both vintages, and adjusted for fuel efficiency. The fuel composites incorporate both the conventional and back-stop components. (Note: The variable *FUp* in the model is not vintage specific. The values of the vintage specific composite fuel demand for the electric sector are stored in a local variable called `NonElec`.) Equation (23.4.4) defines emissions produced by the generation of electricity. This is different from the other emission variables which relate directly to the consumption of the corresponding fuel, not its production. (Note: This variable is only a crude estimate since the conversion factor from millions of 1985 dollars to exajoules should normally only be applied to the total consumption figure, and is not necessarily uniform across an entire row of the input output table.)

Emission constraints are imposed on total emissions, i.e. they are not fuel specific. The following table describes the emission aggregates.

Table 23.5: **Emission Aggregates**

(23.5.1) $$E_{Liq} = E_{RefOil} + E_{Oil} - \tilde{\varphi}_{Oil}^{RefOil} XAp_{Oil,RefOil}$$

(23.5.2) $$E_{Prim} = E_{Coal} + E_{Gas} + E_{Liq} + \sum_{bs} E_{bs}$$

(23.5.3) $$CO_{2,Liq} = CO_{2,Oil} + CO_{2,RefOil}$$

(23.5.4) $$TotCO_2 = CO_{2,Coal} + CO_{2,Gas} + CO_{2,Liq} + \sum_{bs} CO_{2,bs}$$

Equation (23.5.1) determines the energy consumption of *liquids*, E_{Liq}, i.e. the combination of refined petroleum products and crude oil, netting out the consumption of crude oil in the refining sector. Equation (23.5.2) determines *primary* energy consumption. It is the sum over all non-electric energy consumption, incorporating the combined energy consumption of crude oil and refined oil. Equation (23.5.3) defines the emissions corresponding to liquids consumption. Equation (23.5.4) defines total carbon emissions. It is the sum over all non-electric energy sources of carbon emissions. The variable $TotCO_2$ is used to determine the emissions constraint in the policy simulations.

Finally, the conversion of the back-stop price depends on the terajoule price of the corresponding conventional fuel. Table 23.6 provides the equation which converts the conventional fuel price indices into fuel prices in terajoules (using 1985 US dollars as the basis).

Table 23.6 **Determination of the Conventional Fuel Prices (in 85USD per terajoule)**

(23.6.1) $$PTJ_i = \frac{(1+dfutx_i)PD_i XD_i + (1+mfutx_i)PM_i XD_i}{E_i}$$

Equation (23.6.1) defines the terajoule price of the conventional fuels — coal, crude oil, natural gas, refined oil, and electricity. The numerator is the value of energy consumption, inclusive of taxes, in millions of 1985 US dollars. The denominator is the volume of energy consumption in exajoules, i.e. millions of terajoules. In other words, *PTJ* is the price in (1985 US) dollars of a terajoule, for each one of the fuels. (Note that the variable `P_TJ` in the model is deflated by the GDP deflator. The variable plays no role in the model, it is simply printed out as a statistic at the end of the simulation. In the calculation of the back-stop prices, only the 1985 value is used.)

Table 23.1: **Data Dictionary for Subroutine *CalcEnergy***
`energy.c`

E_i	`Energy[r][i][t]`	Energy consumption in exajoules
E_{Liq}	`EnergyLiq[r][t]`	Combined energy content of crude and refined oil in exajoules
E_{Prim}	`EnergyPrim[r][t]`	Total non-electric energy consumption in exajoules
E^{ff}_{Elec}	`FFEnergy[r][t]`	Energy generated by the fixed factor in the electric sector in exajoules
$CO_{2,i}$	`EmiCO2[r][i][t]`	Fuel specific carbon emissions (in millions of tons)
$CO_{2,Liq}$	`EmiCO2Liq[r][t]`	Net carbon emissions from the consumption of crude and refined oil (in millions of tons)
$TotCO_2$	`EmiCO2Prim[r][t]`	Total cabon emissions (in millions of tons)
PTJ_i	`P_TJ[r][i][t]`	Real price of fuels in 1985 US dollars per terajoule

24. Carbon/Energy Taxes

The Excise Tax

There are two tax instruments related directly to the control of emissions in the GREEN model: an excise tax on the carbon content of fuels – the carbon tax – and a tax on the energy content of fuels – the energy tax (the variables described in this section are determined in the subroutine ***CalcCarbonTax*** in the file `carbtax.c`). The implementation of the carbon/energy tax has been made as flexible as possible. The following lists some of the features of the taxes:[44]

- Carbon taxes are defined at a *zone* level where a zone is defined as the aggregation of one or more regions. The definition of the zones is under user control.
- Carbon taxes can be determined endogenously – corresponding to the shadow price of the corresponding carbon emission constraint – or alternatively, they can be exogenously specified by the users.
- Alternatively, an energy tax can be imposed which taxes the energy content of fuels. The energy tax can also be determined endogenously or specified exogenously. Both carbon and energy taxes can be implemented simultaneously. If the taxes are determined endogenously, the carbon tax represents the shadow price of the carbon emission constraint and the energy tax is determined using a fixed weight with respect to the carbon tax.
- The carbon tax system allows for the allocation of region specific emission quotas with a corresponding system of tradable permits.
- Direct transfer schemes can be implemented which re-allocate a share of the tax receipts with user specified donor and recipient shares.
- The carbon tax system is time variant which allows the user to define a new tax system in each time period.

In the case of an endogenous tax, the tax (or the shadow price of the emission constraint), is calculated using a tâtonnement procedure, with the emission constraint equation providing the basis for the tâtonnement process. Table 24.1 contains the emission constraint equation.

44 A complete description of the user implementation of the tax system can be found in the GREEN User Manual.

Table 24.1: **Emission Constraint Equation**

$$\sum_{r \in Z} TotCO_{2,t}^{r} = \chi_t^{Z} \sum_{r \in Z} TotCO_{2,t'}^{r} \quad \forall Z \tag{24.1.1}$$

Equation (24.1.1) defines the emission constraint for each (user-specified) zone, where a zone is an aggregation of one or more regions. On the left hand side is the sum, over all the regions in the zone Z, of total regional carbon emissions in the current time period t. On the right hand side, is the sum, over the same corresponding regions in the zone, of total regional carbon emissions in a user-specified previous period t' (very often t' corresponds to 1990). The aggregate emissions from the previous period are scaled by the factor χ, in other words, current emissions must equal emissions from a previous period multiplied by a scalar factor. For example, stabilization, with respect to period t', implies a scaling factor of 1. A twenty percent reduction implies a scaling factor of 0.8.

While Equation (24.1.1) is a binding constraint in the case of an endogenous carbon or energy tax, it is converted into a tâtonnement process in order to determine the tax itself (see Section II.B.16 for a discussion of the tâtonnement process). Let $\tau_{Z,t}^{CO_2}$ represent the zone specific carbon tax. This tax is updated at each iteration using the following formula:

$$\tau_{Z,t,it}^{CO_2} = \tau_{Z,t,it-1}^{CO_2} \left[\frac{\sum_{r \in Z} TotCO_{2,t}^{r}}{\chi_t^{Z} \sum_{r \in Z} TotCO_{2,t'}^{r}} \right]^{\delta_Z^{CO_2}}$$

The formula implies that if currently determined emissions exceed the emission constraint, the level of the tax (the shadow price) is raised, with δ being a user-specified damping factor (the index *it* is the iteration counter in the current period).[45] In the case of an endogenous energy tax, the same process is used to determine the energy tax.

The region specific taxes follow immediately once the zone specific tax is calculated. Table 24.2 provides the equations for the region specific taxes under the possible tax configurations.

Table 24.2: **Regional Carbon/Energy Taxes**

$$\tau_{r,t}^{CO_2} = \begin{cases} \tau_{Z,t}^{CO_2} & \text{if Endogenous or Mixed} \\ PGDP\,\overline{\tau_{Z,t}^{CO_2}} & \text{if Exogenous} \\ 0 & \text{Otherwise} \end{cases} \tag{24.2.1}$$

$$\tau_{r,t}^{E} = \begin{cases} \tau_{Z,t}^{E} & \text{if Endogenous} \\ \tau_{Z,t}^{CO_2} \dfrac{\varsigma_{Z,t}^{E}}{\varsigma_{Z,t}^{CO_2}} & \text{if Mixed} \\ PGDP_t\,\overline{\tau_{Z,t}^{E}} & \text{if Exogenous} \\ 0 & \text{Otherwise} \end{cases} \tag{24.2.2}$$

The user can specify four different values for the tax flag. They are:

45 The calibration routine arbitrarily sets an initial value for the carbon and/or energy tax if they are endogenous. The carbon tax is set to \$50 (per ton), and the energy tax is set to \$250 (per terajoule). If starting from a previous run, these initialisation values may be over-ridden with the values from the previous run.

- *CarbonTax.* In this case, the carbon tax is endogenous and is calculated using Equation (24.1.1) at the zone level. The top equation of Equation (24.2.1) is used to determine the regional carbon tax, which is identical to the zone-based carbon tax. The energy tax is set to zero.
- *EnergyTax.* In this case, the energy tax is endogenous and is calculated using Equation (24.1.1) at the zone level. The top equation of Equation (24.2.2) is used to determine the regional energy tax, which is identical to the zone-based energy tax. The energy tax is also imposed on the fixed factor in the electricity sector. The fixed factor is a proxy for electricity produced by non-fossil fuel sources such as nuclear power and hydro. The carbon tax is set to zero.
- *MixedTax.* A mixed tax implies both a carbon tax and an energy tax. The regional carbon tax is equal to the zone-specific carbon tax which is determined from the emission constraint, i.e. the constraint determines the carbon tax. The energy tax is set to the carbon tax using user-specified weights. The second sub-equation of Equation (24.2.2) shows how the energy tax is determined in the case of a mixed tax. Note that a mixed tax also implies a tax on the fixed factor in the electricity sector.
- *ExogTax.* In this case the regional taxes are set to the user-specified zone-specific carbon and/or energy taxes and are multiplied by the region-specific GDP deflator.

The carbon tax is specified as a dollar amount (in 1985 dollars) per ton of carbon emitted. The following table indicates how a $100 tax translates into a fuel tax for the different types of fuels:

Table 24.3: **Price Effect of $100 Carbon Tax**

	Energy Content[a]	Carbon Content[b]	Tax per Unit	Energy Tax Equivalent[c]	Unit Price[d]
Ton of Coal	2.931 10^{-2}TJ	0.72355TC	$72.35	$2468	$46.50
Barrel of Oil	5.736 10^{-3}TJ	0.11855TC	$11.86	$2073	$18.67
10^3 cu. ft. of Gas	1.055 10^{-3}TJ	0.01421TC	$1.42	$1346	$1.91

Notes:

a) In terajoules (TJ) per unit.
b) In tons of carbon (TC) per unit.
c) In dollars per TJ to get the same tax per unit.
d) Coal: Steam Coal Import Costs (excluding intra-EEC trade) in USD/metric ton, OECD weighted average, average unit value (CIF), second quarter 1992 (US import price $35.44). Oil: Average unit import value (CIF), fourth quarter 1992, USA. Gas: Costs for Natural Gas Imports by Pipeline in USD/cubic foot of gas, Average unit value (CIF), USA, 3rd quarter 1992 (based on 75.64 USD/ton of oil equivalent. European prices are more in the $115-120 range per ton of oil equivalent). Source: OECD (1993). *IEA Statistics: Energy Prices and Taxes*, Fourth Quarter, 1992, Paris.

Column 1 in Table 24.3 gives the quantity of terajoules (10^{12} joules) per unit of the corresponding fuel. Column 2 gives the carbon content per unit of fuel. Column 3 is determined by multiplying Column 2 by 100. Column 4 gives the amount of the energy tax (in dollars per TJ) which would correspond to the same unit tax as appears in Column 3. For example, a $100 carbon tax on oil, leads to a per barrel tax of $11.86. A $2073 tax per TJ on oil, would lead to the same unit tax on a barrel of oil (i.e. 2073 x 5.736 10^{-3} = 11.86). The fifth column gives an indication of *international* prices in 1992 in several different economies for the fuels. To get the price per terajoule of each fuel, divide column 5 by column 1 (coal: $1586, oil: $3255, gas: $1810). Note, the prices are border prices and there are significant variations across regions which take into account such factors as the quality of fuel and transportation costs. A standardised measure of the carbon content can be determined by dividing column 3 and by column 2 to get the carbon content per terajoule (coal: 24.69TC/TJ, oil: 20.67TC/TJ, gas: 13.47TC/TJ).

The ad Valorem Tax

The carbon tax is calculated as an excise tax per ton of carbon emitted. This tax is transformed into an ad valorem tax on the price of the relevant fuel. An example may help clarify the formulas. Assume the fuel price is P, and total domestic consumption is C. Let CO_2 be the relevant emission level (i.e. $CO_2 = \varepsilon \varphi C$), and let τ be the carbon excise tax. The ad valorem tax must satisfy the following formula:

$$P\,C + \tau\ CO_2 = P(1+t)C$$

where t is the ad valorem tax. The ad valorem tax can be determined by the following formula:

$$t = \frac{\tau\ CO_2}{P\ C} = \frac{\tau\ \varepsilon\ \varphi}{P}$$

For example, if C equals 23080 (10^6 USD), P equals 1 (USD), τ equals 100 (USD), φ = 0.000774 (Exajoules per 10^6 USD), and ε = 24.686 (10^6 tons per exajoule), the ad valorem tax equivalent of $100 tax per ton of carbon is 1.91 (i.e. almost a tripling of the price of carbon). Energy consumption in exajoules is 17.86EJ. Carbon emissions are 441 10^6 tons. The carbon tax generates 44.1 10^9 USD which is equal to 1.91 x 23.08 10^9.

Table 24.4 gives the equations which determine the ad valorem equivalents for the fuel taxes.

Table 24.4: **Ad Valorem Carbon/Energy Taxes**

(24.4.1a) $$dfutx_i = \frac{\left[\tau_r^{CO_2}\varepsilon_i + \tau_r^{E}\right] E_i - \tau_i^{nd} PM_i XM_i}{PD_i XD_i + (1+\tau_i^{nd}) PM_i XM_i}$$

(24.4.1b) $$\tau_i^{nd} = \begin{cases} \dfrac{\sum_j \left[dfutx_j PD_j XDp_{ji} + mfutx_j PM_j XMp_{ji}\right]}{PP_i XP_i - \sum_j \left[dfutx_j PD_j XDp_{ji} + mfutx_j PM_j XMp_{ji}\right]} & \text{if} \quad i \in \{Coal, Gas\} \\ 0 & \text{Otherwise} \end{cases}$$

(24.4.2) $$mfutx_i = (1+dfutx_i)(1+\tau_i^{nd}) - 1$$

The equations in Table 24.4 are more complicated than the numerical example given above. There are several differences. The first difference is that there is an adjustment to the ad valorem fuel tax on domestic consumption of domestic production in order to compensate domestic producers for the lack of an energy tax on the (foreign based) production of imports. The introduction of the energy tax domestically, raises the cost of production relative to the cost of production of imports. The price wedge is determined by the parameter τ^{nd}, which is given by the ratio of energy taxes paid on intermediate consumption to the cost of production net of the taxes. This wedge only exists for coal and natural gas. Because of the homogeneity of crude oil, there is no price wedge between domestic production and imports (there will be a change in the value of the oil rents, however). There is no intermediate consumption in the production of back-stops. The second difference is that the energy tax has also been incorporated into the calculation. The third difference is that demand is broken down into its domestic and import component.

Table 24.5 provides the remaining tax equations.

Table 24.5: **Tax on Imported Refined Oil and Non-Fossil Fuel Electricity**

(24.5.1) $$\tau_{RefOil}^{m} = \frac{\left(\tau_r^{CO_2}\varepsilon_{r,RefOil}^{m} + \tau_r^{E}\right)\varphi_{RefOil} XM_{RefOil}}{PM_{RefOil} XM_{RefOil}}$$

(24.5.2) $$\tau_{Elec}^{ff} = \frac{\tau_r^{E} E_{Elec}^{FF}}{Pff_{Elec} FF_{Elec}^{s}}$$

Equation (24.5.1) determines an import price wedge for imported refined oil. (Note: the code assumes that there are no other tariffs on imported refined oil). Equation (24.5.2) describes the energy tax on the non-fossil fuel component of electricity production. In the case of a carbon tax, this tax is obviously zero.

Transfers

There is user flexibility in describing carbon tax revenue recycling schemes. For each zone with a carbon tax, the user can specify the percentage of the tax to be re-cycled, the incidence (or the shares) of the transfer of the donor regions, and the shares allocated to the recipient regions (the transfers are calculated in the subroutine ***WorldTrade*** in the file `wtrade.c`). The transfer equations are listed in Table 24.6

Table 24.6: **Re-Cycling of Carbon/Energy Tax Revenues**

(24.6.1) $$TaxTr_Z = \chi_Z^{tr}\left[\sum_{r\in Z}\left\{\sum_i FuTxY_i^r + \tau_{Elec}^{ff} Pff_{Elec} FF_{Elec}^d\right\}\right]$$

(24.6.2) $$FT_{r,r'} = \pi_{Z,r}^d \pi_{Z,r'}^r TaxTr_Z$$

Equation (24.6.1) determines the amount of total tax revenue which is to be re-cycled in each zone. First, total carbon/energy taxes are summed over all sectors and all regions within the zone. This amount is then multiplied by a share factor (which must be less than 1). The amount to be transferred is given by *TaxTr* and the share parameter by χ. Equation (24.6.2) determines a bilateral financial flow matrix. Each row of the matrix determines the positive flows from region r to region r'. The parameter π^d gives the donor shares. The shares within a zone must add up to 1. The parameter π^r gives the recipient shares. The recipient shares must add up to one across all regions.

The following example illustrates the mechanism. Assume there is an OECD region composed of the USA, Japan (JPN), the European Community (EEC), and the rest of the OECD countries (OOE), and that the aggregate carbon tax receipts are $250 billion. Let the transfer share be 80 per cent. Assume the donor shares are 25 per cent for the USA and Japan, 30 per cent for EEC, and 20 per cent for OOE. Next, assume the recipient shares are 30 per cent for China (CHN), 20 per cent for the former Soviet Union (FSU), 10 per cent for Eastern and Central Europe (EET), 15 per cent for India (IND), 5 per cent for Brazil, and 20 per cent for the rest of the world (ROW). The bilateral transfer matrix is (leaving out zeros):

	CHN	FSU	EET	IND	BRA	ROW	Total
USA	15.0	10.0	5.0	7.5	2.5	10.0	50.0
JPN	15.0	10.0	5.0	7.5	2.5	10.0	50.0
EEC	18.0	12.0	6.0	9.0	3.0	12.0	60.0
OOE	12.0	8.0	4.0	6.0	2.0	8.0	40.0
Total	60.0	40.0	20.0	30.0	10.0	40.0	200.0

For the donor countries, the transfer comes from households. The relevant equations are given in the file `ydist.c`. In essence, the transfer implies a reduction in the after tax disposable income of households. For the recipient countries, the transfer accrues to the investment sector, i.e. it is added to the foreign saving variable. The total transfer for the donor country is:

$$OutFlow_r = \sum_{r'} FT_{r,r'}$$

The total transfer for the receiving country is:

$$InFlow_{r'} = \sum_{r} FT_{r,r'}$$

The transfer scheme affects international financial flows and this is duly incorporated in the calculation of Walras' Law.

Tradable Permits

Another optional feature of the carbon tax system in GREEN is the implementation of a scheme of tradable permits (see routine ***WorldTrade*** in `wtrade.c`). Table 24.7 presents the equation which determines the revenue generated by quotas on emission rights.

Table 24.7: **Revenues from Emission Quotas**

(24.7.1) $$EmiQY_r = \tau_r^{CO_2}(\overline{TotCO_2^r} - TotCO_2^r) \quad \forall r \in Z$$

Equation (24.7.1) determines the revenues generated by a system of emission quotas for each region in the zone Z. The quota is specified in the GREEN model as a multiplicative factor of a previous period's emissions. The formula for the quota is given by:

$$\overline{TotCO_{2,t}^r} = \chi_{r,t}^{Q} TotCO_{2,t'}^r$$

Because the user inputs both the multiplicative factor by region and the overall aggregate emission constraint, there exists a constraint on the χ factors. The constraint is:

$$\chi_t^Z \sum_{r \in Z} TotCO_{2,t'}^r = \sum_{r \in Z} \chi_{r,t}^{Q} TotCO_{2,t'}^r$$

The GREEN User Manual provides a complete description of the constraints and how the code handles any inconsistencies in the user-specified parameters.

The revenues generated by the tradable permits are allocated to the government sector (see the code in `ydist.c`). In the case of endogenous government saving, the revenue simply shifts the government savings/deficit value. In the case of exogenous government saving, the revenue will ultimately be rebated back to households in a lump sum. Similar to the transfer scheme, the tradable permits affect the balance of payments, and the calculation of Walras' Law is modified to reflect the change in international financial flows.

The following table provides the data dictionary for the variables updated in the subroutine *CalcEnergy*. The bilateral transfers and the emission quota revenues are updated in the subroutine *WorldTrade*.

Table 24.8: **Data Dictionary for Subroutine *CalcEnergy***
`energy.c`

$\tau_{r,t}^{CO_2}$	`CO2Tax_TJ[r][t]`	Carbon tax (1985 US dollars per ton of carbon)
$\tau_{r,t}^{E}$	`ETax_TJ[r][t]`	Energy tax (1985 US dollars per terajoule)
$dfutx_i$	`dfutx[r][i][t]`	Ad valorem carbon/energy tax equivalent on domestic production
$mfutx_i$	`mfutx[r][i][t]`	Ad valorem carbon/energy tax equivalent on imports
τ_{Elec}^{ff}	`taxff[r][t]`	Energy tax on the fixed factor in the electric sector
τ_{RefOil}^{m}	`tariff[r][RefOil][t]`	Compensating tariff on imported refined oil
$FT_{r,r'}$	`BilatTrans[r][rp][t]`	Bilateral inter-regional transfers if carbon/energy tax revenues are recycled (see `wtrade.c`)
$EmiQY_r$	`CarFlow[r][t]`	Income (or expenditure) from tradable emission quotas (see `wtrade.c`)

C. Model Statistics

This section provides a definition for the variables which are in the summary file but which are not model variables. These variables are statistics which are calculated using the model variables (they are calculated in the file `endprog.c`).

Macro Aggregates

$$TCons_t = \sum_i \sum_k PAc_{i,k,0} XAc_{i,k,t}$$

$$TGov_t = \sum_i PAg_{i,0} XAg_{i,t} + w_0 L^d_{g,t} + R_0 K^d_{g,t}$$

$$TInv_t = \sum_i PAi_{i,0} XAi_{i,t}$$

$$TStocks_t = \sum_i PAst_{i,0} XAst_{i,t}$$

$$TExp_t = \sum_i PE_{i,0} ES_{i,t}$$

$$TImp_t = \sum_i WPM_{i,0} XM_{i,t}$$

$$BalPP^r_t = \frac{1}{P_t}\left(\sum_i PE^r_{i,t} ES^r_{i,t} - \sum_i WPM^r_{i,t} XM^r_{i,t}\right)$$

$$TotFF_t = \sum_i FF^d_{i,t}$$

where *TCons* is the aggregate volume of household consumption, *TGov* is the aggregate volume of government expenditures (including demand for labour and capital), *TInv* is the aggregate volume of investment expenditures, *TStocks* is the aggregate volume of expenditures on inventory changes, *TExp* is the aggregate volume of exports, *TImp* is the aggregate volume of imports (evaluated at base year **world** prices), *BalPP* is the real balance of payments deflated by the world price index, and *TotFF* is the aggregate volume of the demand for the fixed factors.

Welfare Measures

This section provides several different measures of welfare or real income.

$$E_t = \sum_k PC_{k,0}\theta_k Pop_t + \exp\left(\sum_k \mu_k \ln\left(\frac{PC_{k,0}}{\mu_k}\left(C_{k,t} - \theta_k Pop_t\right)\right) + \mu_s \ln\left(\frac{cpi_0}{\mu_s}\frac{S_{h,t}}{cpi_t}\right)\right)$$

where

$$\mu_s = 1 - \sum_k \mu_k$$

E is the consumer expenditure function (derived from the *ELES* utility function), evaluated at the current period's utility level, but at the base year's prices, i.e. it is $E(p^0, u^1)$. Note that it is assumed that the consumer price index, *cpi*, is a proxy for the price of household saving. A measure of welfare change is:

$$E(p^1, u^1) - E(p^0, u^1)$$

The first term can be evaluated directly by calculating consumer expenditure (and saving) in current prices, i.e.

$$E(p^1, u^1) = \sum_k PC_{k,t} C_{k,t} + S_{h,t}$$

The following are some additional measures of real income.

$$RealY_t = E_t + TGov_t + TStocks_t$$

$$Absorption_t = TCons_t + TGov_t + TInv_t + TStocks_t$$

$$CPX_t = 100 \sum_k PC_{k,t} C_{k,t} \,/\, TCons_t$$

$$RealYd_t = 100\, Yd_t \,/\, CPX_t$$

RealY is in some sense the regional expenditure function, the sum of household expenditure, the government's real expenditure, plus the change in stocks. Investment expenditures are already incorporated in the household expenditure function because it includes household saving. *Absorption* is the sum of real expenditures by households, the government, investment, and stock building. *CPX* defines a consumer price index, different from *cpi*. *RealYd* is real disposable income using *CPX* as the deflator (the scale factor of 100 is necessary to make the units of *RealYd* millions of dollars).

Price Indices

This section provides the definition of some price indices.

$$EPI_t = 100 \sum_i PE_{i,t} ES_{i,t} \,/\, TExp_t$$

$$MPI_t = 100 \sum_i WPM_{i,t} XM_{i,t} \,/\, TImp_t$$

$$ToT_t = 100\, EPI_t \,/\, MPI_t$$

$$GPI_t = 100 \frac{\sum_i PAg_{i,t} XAg_{i,t} + w_t L^d_{g,t} + r^{Old}_{g,t} K^t_{g,t}}{TGov_t}$$

$$PA_{i,t} = \frac{\sum_j PAp_{i,j,t} XAp_{i,j,t}}{\sum_j PAp_{i,j,0} XAp_{i,j,t}} \quad \text{if} \quad i \in \{Agric, EnerInt, OtherInd\}$$

$$PA_{e,t} = \frac{\sum_j PFUp_{e,j,t} FUp_{e,j,t}}{\sum_j PFUp_{e,j,0} FUp_{e,j,t}} \quad \text{if} \quad e \in \{Coal, Gas, Oil, RefOil, Elec\}$$

$$PEner_{i,t} = \frac{\sum_v PEp^v_{i,t} Ep^v_{i,t}}{\sum_v PEp^v_{i,0} Ep^v_{i,t}}$$

EPI is an export price index, *MPI* is an import price index, *ToT* is an index for the terms of trade, in this case the relative price of exports to imports, and *GPI* is a government expenditure price index. *PA* is the average input

price of intermediates across sectors. In the case of non-fuels it is the average Armington price, i.e. aggregated across sectors and between domestic and import consumption. In the case of fuels, it is the average price of the fuel composites (Coal, Oil, Gas, RefOil, and Elec), in other words it aggregates across sectors, between conventional and backstop fuels, and between domestic and import consumption. *PEner* is the average energy price across vintages by sector. Note that the price index *pi* is scaled by 100 in the summary file as compared to its simulation value.

Aggregate Statistics

Five sectoral vectors are aggregated into four sectors (from the original fifteen). The mapping of the sectors is given by the following table:

Aggregate Sector	***Original Sector***
Agric	Agric
FossFuels	Coal, Oil, Gas
ProcEnergy	RefOil, Elec, CoalCBS, CoalCFBS, OilCBS, OilCFBS, GasCBS, GasCFBS, ElecBS
Other	EnerInt, OtherInd

The following aggregation matrix is constructed to calculate the aggregates:

$$A=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The aggregates are defined by the following matrix equations.

$$\overline{AggXP_t} = A\,\overline{XP_t}$$

$$\overline{AggLab_t} = A\,\overline{L_t^d}$$

$$\overline{AggExp_t} = A\,\overline{ES_t}$$

$$\overline{AggImp_t} = A\,\overline{XM_t}$$

$$\overline{AggVA_t} = A\,\overline{VA_t} + \boldsymbol{B1}\,OilRents_t + \boldsymbol{B2}\,BSRents_t$$

where

$$VA_{i,t} = w_t L_{i,t}^d + \sum_v r_{i,t}^v K_{i,t}^{d,v} + Pff_{i,t} FF_{i,t}^d$$

$$\boldsymbol{B1}=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B2}=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

where the variables with an overscore are column vectors of the appropriate dimension (4 on the left hand side, and 15 on the right hand side). *AggXP* is aggregated output, *AggLab* is aggregated labour demand, *AggExp*, is aggregated exports, *AggImp* is aggregate imports, and *AggVA* is aggregated value added. The rents generated in the crude oil sector are allocated to fossil fuels, whereas the rents generated by the backstop sectors are allocated to the processed fuels sector.

Energy Statistics

The model variables — E_i, E_{Liq}, E_{Prim}, and E^{ff}_{Elec} — are scaled by 1000 prior to printing in the summary file, which makes their units thousands of terajoules (rather than exajoules).

Carbon/Energy Taxes

The following formulas define the various real tax variables in the summary table.

$$RealTax_{o,t} = \tau_t^{CO_2} / PGDP_t$$

$$RealTax_{1,t} = \varepsilon_{Coal}\, \tau_t^{CO_2} / PGDP_t$$

$$RealTax_{2,t} = \varepsilon_{Oil}\, \tau_t^{CO_2} / PGDP_t$$

$$RealTax_{3,t} = \varepsilon_{Gas}\, \tau_t^{CO_2} / PGDP_t$$

$$RealTax_{4,t} = \tau_t^{E} / PGDP_t$$

$RealTax_0$ is the real level of carbon tax which is specified in units of dollars per ton of carbon. $RealTax_1$ converts the real carbon tax into units of dollars per terajoule of coal, and $RealTax_2$ and $RealTax_3$ similarly define the carbon tax in terms of dollars per terajoule of oil and terajoule of natural gas, respectively. $RealTax_4$ defines the real level of the energy tax specified in units of dollars per terajoule of energy.

Global Variables

At the end of the summary file, after the printing of the regional statistics, there are a few global statistics. These are defined below. A few of the global statistics are regional aggregations. The following table defines the aggregation:

Aggregate Regions	***Original Regions***
OECD	USA, JPN, EEC, OOE
FSU+EET	FSU, EET
Other_Regions	EEX, CHN, IND, DAE, BRA, ROW
Global	USA, JPN, EEC, OOE, EEX, CHN, FSU, IND, EET, DAE, BRA, ROW

The aggregation matrix is defined by:

$$\boldsymbol{RA} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The aggregate regional variables are defined by the following matrix formulas:

$$\overline{TotEner_t} = \boldsymbol{RA}\ \overline{E_{Prim,t}}$$

$$\overline{TotEmi_t} = \boldsymbol{RA}\ \overline{TotCO_{2,t}}$$

where variables with an overscore are regional column vectors with appropriate dimensions (4 on the left hand side, and 12 on the right hand side). *TotEner* is the aggregated level of primary energy consumption, and *TotEmi* is the aggregated level of carbon emissions.

Global Trade Statistics

The following definitions define global trade statistics.

$$TotImp_{i,t} = \sum_r WPM^r_{i,0} XM^r_{i,t}$$

$$TotExp_{i,t} = \sum_r PE^r_{i,0} ES^r_{i,t}$$

$$ImpPr_{i,t} = \frac{\sum_r WPM^r_{i,t} XM^r_{i,t}}{\sum_r WPM^r_{i,0} XM^r_{i,t}}$$

TotImp is the aggregate volume of imports by sector, and *TotExp* is the aggregate volume of exports by sector. *ImpPr* is a world import price index.

III. Data Management

Data management in GREEN is one of the key departures from more traditional code, particularly code written in FORTRAN. This part of the reference manual describes the data management structure in GREEN.

A. Multi-dimensional arrays in C

This section contains a brief overview of arrays in C. Users are referred to C reference books for a more detailed analysis.

One Dimensional Arrays in C

A one dimensional array in C can be declared using the following statement:

```
double x1[10] ;
```

which declares a double precision array with ten cells. An individual cell can be addressed the following way:

```
x1[i]
```

where *i* is an integer between 0 and 9 (array addresses in C start at 0). To sum all the elements of an array, the following code could be used:

```
sum = 0.0 ;
for(i=0 ; i < 10 ; i++)
   sum += x[i] ;
```

An alternative way to create one-dimensional arrays is to allocate the memory explicitly. The standard C syntax for doing this is the following:

```
double *x1 ;
x1 = (double *) calloc(sizeof(double), 10) ;
```

The first statement declares the variable *x1*. However, the value of *x1* is undetermined, the declaration simply says that *x1* may eventually point to a space in memory containing contiguous double precision numbers. The second statement does two things. First, it asks the operating system for a contiguous space in memory (which is determined at runtime). The amount of space to allocate is 10 items of size double precision. For example, if the size of a double is 8 bytes, it will ask for at least 80 bytes (some machines will round the number up because of alignment constraints). The second thing it does is to make the variable *x1* point to this allocated space in memory (if there was enough memory). The nice feature of C is that it is still possible to address a single cell of *x1* as if it were an array, i.e. `x1[i]`.

Two Dimensional Arrays in C

Things get a bit trickier with two dimensional arrays. The easy way to declare two dimensional arrays is as follows:

```
double x2[5][10] ;
```

This declares a two dimensional array with 5 rows and 10 columns. The following statement would address the third row and the seventh column:

```
x2[2][6] ;
```

There are some disadvantages to declaring arrays using this method.

- The (maximum) dimensions of the array must be known at compile time.
- In C, it is difficult to pass variable dimension arrays to subroutines (this is quite a technical point but is relevant to simplifying much of the GREEN code).
- The memory which will be allocated has to be contiguous. On some computers, particularly computers with segmented memory, it might be difficult to allocate big contiguous blocks of memory.

An alternative method is the following:

```
double **x2 ;
x2 = (double **) calloc(sizeof(double *), 5) ;
for(i=0 ; i < 5 ; i++)
   x2[i] = (double *) calloc(sizeof(double), 10) ;
```

The first statement declares *x2* to be a variable which will point to a space in memory which contains an array of pointers to single arrays of doubles. The second statement allocates space for an array of five cells, each cell

will contain a pointer to a single array of doubles. The for-loop allocates the single dimension array for each row of the matrix, and each array has ten cells. Again, a cell can be addressed using `x2[2][6]`, for example.

There are some inconveniences. First, space must be allocated by the programmer, rather than by the compiler and linker. Second, it is easy to access memory which hasn't been allocated, for example `x2[5][10]`, with unpredictable results. (This can occur using the traditional array declaration, however, many debuggers can easily identify these bound problems in this case.) But the advantages far outweigh the inconveniences.

The three and four dimensional arrays are even more complicated, but follow more or less the same structure.

From the user point of view, only two things need to be retained from this discussion. The first is how to identify variable declarations and definitions, and the second is how to use the variables.

All variable declarations are in header files, i.e. in files with the extension '.h' (this is a C convention). For example, most of the regional endogenous variables are contained in the file `vars.h`. A one dimensional variable will have one asterisk before its name, a two dimensional variable will have two asterisks, a three dimensional variable will have three asterisks, and so forth. The actual space allocated to these variables is undetermined until the program is initialised.

A declaration in C just means the variable exists. All variables must also have a definition, i.e. the compiler reserves space for the variable. (In the case of pointers, i.e. arrays, it simply reserves space for a pointer, typically four bytes). All variable definitions are also in header files. These files have the same structure and the same name as the variable declaration headers, but with the extension '.hh'. These header files can only be included once, whereas declaration files can be included many times.

The use of the variables is very straightforward. Simply use the standard array notation for arrays in C. For example, rent is a four dimensional array in GREEN: region x sector x vintage x time (i.e. 12 x 15 x 2 x 10). Its declaration is:

```
extern double ****rent ;
```

Its definition is:

```
double ****rent ;
```

To access cell r, sector i, vintage Old, in time t, one would use:

```
rent[r][i][Old][t] ;
```

Time

A note on the time dimension of variables. There are 8 time periods in the standard version of GREEN. However, 10 time cells are allocated. The cell 0 contains the value of the variable as it is read in or calibrated. Cells 1-8 correspond to the simulation periods of the model. Cell 9 (i.e. the 10th cell), which has the identifier *last*, contains the value of the variable from the previous iteration (within a period). This is useful in smoothing equations or when convergence problems arise and it could be useful to look at the changes in variables from one iteration to the next. For example, the wage smoothing equation could be:

```
wage[r][t] = 0.4*wage[r][t] + 0.6*wage[r][last] ;
```

where the variable `wage[r][t]` on the right hand side has just been updated using the tâtonnement procedure for the labour market.

All space for variables are allocated in a subroutine named *AllocVars* in the file vars.c. There is a different routine for each variable depending on its dimensionality.

B. Variable structure

The section above described the use of variables in GREEN. The variables exist as independent entities, e.g. there is no physical link between the variable rent, and the variable wage. This can make many repetitive operations tedious, and it also reduces the non-programmers access to the model variables. For example, to set all variables to zero, one would need to code the following:

```
for(r=0 ; r < nr ; r++) {
   wage[r][t] = 0.0 ;
   for(i=0 ; i < ns ; i++)
      for(v=0 ; v < nKap ; v++)
         rent[r][i][v][t] = 0.0 ;
```

It would be easier to simply say:

```
SetAllVars(0.0) ;
```

However, this requires the variables to somehow be physically connected. This is done via a symbol table, which will be described in the next section. The symbol table contains a list of all the variables including their attributes. This section will describe the variable attributes, or their structure.

Each variable of the model has a pointer, e.g. *rent*, but space is also allocated for all the attributes of the variable. The attributes are contained in a C object known as a structure (in Pascal, this is known as a record). The following C code defines the structure (see `green.h`):

```
typedef struct varlist {
   enum vardim dim ;                /* Dimension of variable */
   enum vartype type ;              /* Price, volume, value, etc. */
   enum varstat stat ;              /* Status of variable */
   void *data ;                     /* Generic data holder */
   char *name ;                     /* Variable name */
   int  dim1 ;                      /* First dimension size */
   int  dim2 ;                      /* Second dimension size */
   int  dim3 ;                      /* Third dimension size */
   int  dim4 ;                      /* Fourth dimension size */
   char **lab1 ;                    /* Labels for first dimension */
   char **lab2 ;                    /* Labels for second dimension */
   char **lab3 ;                    /* Labels for third dimension */
   char **lab4 ;                    /* Labels for fourth dimension */
   struct varlist *next ;           /* Pointer to next variable */
} *VARLIST ;
```

The name of the structure is varlist (i.e. variable list). The structure has fourteen items, not all of which will contain useful information. The following describes each item:

1. *Dimension*. The first item contains the dimension size of the variable. This can take one of three values (for variables): nodim, onedim, and twodim. *wage* is an example of a nodim variable, i.e. it has no other dimension other than region and time. *rent* is an example of a twodim variable. Other than region and time, it has a sector and vintage dimension. All variables are of one of the three types. Coefficients can also be three dimensional, with the identifier threedim. In this latter case, the coefficients have three dimensions other than the region dimension. Coefficients have no time dimension.

2. *Type*. Variables can be of several types. Some of the more common types are PRICE, VOLUME, VALUE, POLICY, etc. These types can be used at different times in the code to produce specific behaviour. For example, to set all prices to unity, it is easy to loop through the symbol table and see if a variable is of type PRICE, and then set the corresponding data to the value 1.

3. *Status*. The status of a variable indicates whether it has been read in or somehow initialised.

4. *Data* (or pointer to data). This item holds the pointer to the variable itself. For example, the structure for the variable wage, points to the data contained in the wage variable. If this item did not exist, there would be no way to link the variables together in the symbol table.

5. *Name*. This item contains the name of the variable. This is critical to give the non-programmer access to the internal data. For example, one could let the user define a list of variables to print in the input control file, and the code could loop through that list and match the names with the names in the symbol table. Another particularly useful area has been in the input routines. Since the variables in the GAMS input files are identified by their names, the management of the symbol table has no problems in associating the incoming data with the appropriate internal variable. This means that the GAMS input files can be set up with very little structure. Further, if a user adds a new variable to the symbol table, the input routines need no modification. The variable's data can simply be added to the GAMS input files.

6-9. *Dimensions*. Items 6-9 contain the size of the different dimensions. If a variable has no dimensions, all these items contain the value 0. If a variable has one dimension then item 6

contains the value of that dimension, and so forth. This feature allows many subroutines to be called without needing to pass the dimensions of the variables.

10-13. *Labels*. Items 10-13 contain pointers to labels which are associated with the corresponding dimensions. If the variable has no dimensions, none of these items have any values. A one dimensional variable will have a pointer to its labels in item 10, and so forth. Again, this is a useful feature, for example in print routines.

14. *Link*. This is the key item in a linked list, or in the case of GREEN, to the symbol table. This item points to the next variable in the symbol table. If its value is zero, then there are no items left in the symbol table. (If the symbol table was constructed as a contiguous space in memory, it would be possible to use array symbols, i.e. [], to access sequential members in the symbol table. However, as noted above, there are significant advantages to using pointers, rather than arrays, to access items in memory.) An example in the next section shows how to use the symbol table.

C. Symbol Tables

The GREEN code contains four symbol tables. Two relate to variables. This means that all the variables in these symbol tables have at least two dimensions: region and time. The two variable symbol tables are called *VarSymTab* and *ExogSymTab*. Originally, *VarSymTab* contained only endogenous variables, and *ExogSymTab* contained exogenous variables. Over time, the distinction between exogenous and endogenous has occasionally become blurred, as some variables are endogenous in certain simulations, and exogenous in others. From a practical point of view, the two are treated in much the same way. The declarations for variables span several header files, the key header file being `vars.h` (and `vars.hh`), but also includes `tech.h` and `exog.h`. Many of the routines to manage the variable symbol tables are in the file `vars.c`.

One symbol table contains all the coefficients, *CofSymTab*. The coefficients are declared in the header file named `cofs.h`, and are handled by the various subroutines in the file `cofs.c`. All coefficients have at least one dimension: region. None have the time dimension. Coefficients with a time dimension are in the *ExogSymTab* symbol table. The fourth symbol table is known as *GblSymTab*. This symbol table contains all global variables, i.e. variables which have a time dimension, but no regional dimension. The global variable declarations are in the header file `global.h`, and the management routines are in the file `global.c`.

The following code provides an example of how to search for a variable in the symbol table:

```
VARLIST LookUp(VARLIST ptr, char *name)
{
   /* Search the variable "ptr", for the variable named "name" */

   for( ; ptr != (VARLIST) NULL ; ptr = ptr->next) {
      if (!stringcmp(ptr->name, name))
         return ptr ;
   }

   /* Not found */

   return (VARLIST) NULL ;
}
```

The function is called LookUp. It takes two parameters. The first parameter is a pointer to a symbol table, for example *VarSymTab*. The second parameter is a name, for example *rent*. The for-loop is very instructive. First there is no initialisation of the for-loop, since the search for the variable starts at the beginning of the symbol table which is already initialised to ptr. The for-loop stops when ptr points to the NULL value, i.e. 0. The iteration step updates the variable ptr to point to the next variable in the symbol table, that is the meaning of the statement:

```
ptr = ptr->next ;
```

Finally, the only statement in the for-loop calls for a comparison (of strings), between the name of the variable ptr currently points to, and the name of the variable that is being searched for. If the strings are identical, the function returns the pointer to the variable's structure, i.e. it returns ptr. If the for-loop continues to the end, i.e. until ptr points to NULL, then the variable was not found, and the function returns the NULL pointer, i.e. it failed to find the variable called *name*.

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Annex 1: Model Dimensions

This annex lists the dimensions of the GREEN model and the standard abbreviations used in the model. All user input files are required to use the same abbreviations as those listed in this annex. (The letters in parentheses in the table titles refer to the default index label used in the GAMS input files, and most often in the code.)

Dynamic Dimension

The reference version of GREEN spans the period 1985-2050.[46] The model solves for eight years within this time range of 65 years. Initially, the model has an inter-year gap of five years, i.e. the solution years are 1985, 1990, 1995, 2000, 2005, and 2010. After 2010, the inter-year gap is 20 years, i.e. the solution years are 2030 and 2050.

Table A1: **Time Dimension of GREEN *(t)***

Period Index	Year	Period Index	Year
1.	1985	5.	2005
2.	1990	6.	2010
3.	1995	7.	2030
4.	2000	8.	2050

Regional Dimension

The current version of GREEN partitions the world economy into 12 economic regions, listed in the following table. The second column denotes the standard abbreviations which must be used in all user input files.

Table A2: **Regions of GREEN *(r)***

Region Index	Abbreviation	Description
1.	USA	United States
2.	JPN	Japan
3.	EEC	European Economic Community
4.	OOE	Other OECD
5.	EEX	Energy Exporting Countries
6.	CHN	People's Republic of China
7.	FSU	Former Soviet Union
8.	IND	India
9.	EET	European Economies in Transition
10.	DAE	Dynamic Asian Economies
11.	BRA	Brazil
12.	ROW	Rest of the World

Sectoral Dimension

The current version of GREEN has 15 sectors. 5 sectors are related to so-called conventional fuels: Coal, Crude Oil, Gas, Refined Oil, and Electricity. 3 sectors are related to other economic activities: Agriculture, Energy Intensive industries, Other industries and services. The remaining seven sectors are related to the so-called back-

[46] Users can specify a different time framework as long as all the input files are consistent with the different time framework. See the GREEN User Manual.

stop technologies. These technologies are substitutes for conventional fuels. They differ both in their carbon content and in their cost of production. Coal, crude oil, and gas each have two back-stop substitutes: one which emits carbon, and the other carbon free. The carbon free substitute is typically more expensive than the carbon based substitute. The model assumes that there are no substitutes for refined oil. Finally, electricity has one back-stop substitute.

The next table lists the sectors of the GREEN model with the official abbreviations.

Table A3: **Sectors of GREEN *(i,j)***

Sector Index	Abbreviation	Description
1.	Agric	Agriculture
2.	Coal	Coal Mining
3.	Oil	Crude Petroleum
4.	Gas	Natural Gas
5.	RefOil	Refined Oil Products
6.	Elec	Electricity, Gas, and Water Distribution
7.	EnerInt	Energy-Intensive Industries
8.	OtherInd	Other Industries and Services
9.	CoalCBS	Carbon-based Coal Back-stop
10.	CoalCFBS	Carbon-free Coal Back-stop
11.	OilCBS	Carbon-based Oil Back-stop
12.	OilCFBS	Carbon-free Oil Back-stop
13.	GasCBS	Carbon-based Gas Back-stop
14.	GasCFBS	Carbon-free Gas Back-stop
15.	ElecBS	Back-stop Electric Option

Consumption Goods Sectors

There are four consumer goods which are listed in the table below. Each component of consumer demand is transformed into demand for intermediate inputs using a nested CES-Leontief structure.

Table A4: **Consumption Sectors of GREEN *(k)***

Consumption Good Index	Abbreviation	Description
1.	FoodBev	Food and Beverages
2.	Energy	Fuel and Power
3.	TrnspComm	Transport and Communication
4.	Other	Other Goods and Services

Energy Sectors

The energy sectors are nominally a subset of the production sectors. Each economic agent formulates demand for an energy bundle which is disaggregated into five energy components. When the back-stops are active, these five energy demands are composite demands (except for refined oil). When the back-stops are inactive, the demand are for the corresponding conventional fuels. Table A5 lists the five energy components:

Table A5: **Energy Sectors of GREEN *(e)***

Energy Sector Index	Abbreviation	Description
1.	`Coal`	Coal
2.	`Oil`	Crude Oil
3.	`Gas`	Natural Gas
4.	`RefOil`	Refined Oil
5.	`Elec`	Electricity

Basic Fuels

The five energy sectors are partitioned into three sets for determining back-stop penetration: basic fuels, refined oil, and electricity. The basic fuels are coal, crude oil, and gas. Each of these fuels have two back-stop substitutes: a carbon based back-stop and a carbon free back-stop. The file containing the back-stop specification contains some parameters which are only relevant to these three basic fuels.

Table A6: **Basic Fuel Sectors of GREEN *(bf)***

Index	Abbreviation	Description
1.	`Coal`	Coal
2.	`Oil`	Crude Oil
3.	`Gas`	Natural Gas

Back-stop Sectors

Some of the parameters in the back-stop specification file are valid only for the back-stop sectors. The following table contains the mandatory abbreviations for the back-stop sectors (note that the names are the same as in Table A3).

Table A7: **Back-stop Sectors of GREEN *(bs)***

Index	Abbreviation	Description
1.	`CoalCBS`	Carbon-based Coal Back-stop
2.	`CoalCFBS`	Carbon-free Coal Back-stop
3.	`OilCBS`	Carbon-based Oil Back-stop
4.	`OilCFBS`	Carbon-free Oil Back-stop
5.	`GasCBS`	Carbon-based Gas Back-stop
6.	`GasCFBS`	Carbon-free Gas Back-stop
7.	`ElecBS`	Back-stop Electric Option

Vintages

Many production parameters are indexed by vintage. The valid abbreviations for vintage are *Old*, for old capital, and *New*, for new capital.

Table A8: **Capital Vintages *(v)***

Index	Abbreviation	Description
1.	`Old`	Index for old capital
2.	`New`	Index for new capital

Resource Depletion Indices

The resource depletion module applies to two sectors: natural gas and crude oil. The indices *OilRes* and *GasRes* are used to distinguish the respective sectors.

Table A9: **Resource Depletion Indices *(d)***

Index	Abbreviation	Description
1.	`OilRes`	Index for oil depletion
2.	`GasRes`	Index for gas depletion

Trend Indices

The resource depletion module is calibrated to a reference scenario which includes bounds. The parameters are indexed by the bounds which are specified using the notation *low* and *high*.

Table A10: **Trend Indices *(tr)***

Index	Abbreviation	Description
1.	`Low`	Index for low trend scenario
2.	`High`	Index for high trend scenario

Regime Indices

The fixed factor supply functions have two elasticities. The supply elasticity is a function of the demand of the fixed factor (with respect to the previous period). If demand is decreasing, the "down" elasticity is used. If demand is increasing the "up" elasticity is used. In the United States, for example, the land supply elasticity is 2 if demand for land is increasing, and it is 0.5 if demand is decreasing.

Table A11: **Regime Indices *(s)***

Index	Abbreviation	Description
1.	`Down`	Index for downward supply elasticities
2.	`Up`	Index for upward supply elasticities

CES Share Parameter Indices

Certain parameters, notably CES share parameters are indexed by digits (rather than symbolic variables). In such cases, the user input files should contain the appropriate numeric index. Because of the way the C programming language uses indices, all indices start with '0' as opposed to '1'. Therefore in a 3-argument CES, the share parameter of the first argument will be indexed by '0', the second by '1', and the third by '2'.

Table A12: **Numeric Indices *(dig)***

Index	Abbreviation	Description
1.	`0`	Index for the first argument
2.	`1`	Index for the second argument
3.	`2`	Index for the third argument

Annex 2: The CES Function

This annex provides a brief overview of the constant-elasticity-of-substitution (CES) function since it used extensively in the GREEN model.

The CES function can be formulated as a cost minimisation problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$

subject to

$$V = \left[\sum_i a_i (\lambda_i X_i)^{\rho} \right]^{1/\rho}$$

where V is the aggregate volume (of production, for example), X are the individual components ("inputs") of the production function, P are the corresponding prices, and a and λ are technological parameters. a are most often called the share parameters. λ are technology shifters. In GREEN the share parameters are usually constant over time. The technology shifters are most often constant in any given time period, but are allowed to change over time, for example, the energy efficiency improvement is implemented via the technology shifter parameter in the appropriate CES functions. The parameter ρ is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

A bit of algebra produces the following derived demand for the inputs, assuming V and the prices are fixed:

(1) $$X_i = \alpha_i \lambda_i^{\sigma-1} \left(\frac{P}{P_i} \right)^{\sigma} V$$

where we define the following relationships:

$$\rho = \frac{\sigma - 1}{\sigma} \quad \Leftrightarrow \quad \sigma = \frac{1}{1-\rho} \quad \text{and} \quad \sigma \geq 0$$

$$\alpha_i = a_i^{\sigma}$$

and

(2) $$P = \left[\sum_i \alpha_i \lambda_i^{\sigma-1} P_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

The parameter σ, is called the substitution elasticity. This term comes from the following relationship which is easy to derive from Equation (1):

$$\frac{\partial \left(X_i / X_j \right)}{\partial \left(P_i / P_j \right)} \frac{\left(P_i / P_j \right)}{\left(X_i / X_j \right)} = -\sigma$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note, we are assuming that the substitution elasticity is a positive number). For example, if the price of input i increases by 10 per cent with respect to input j, the ratio of input i to input j will decrease by (around) σ times 10 per cent.

The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero, in other words, there is no substitution between inputs, no matter what the input prices are. Equations (1) and (2) become:

(1') $$X_i = \frac{\alpha_i V}{\lambda_Y}$$

(2') $$P = \sum_i \alpha_i \frac{P_i}{\lambda_Y}$$

The aggregate price is the weighted sum of the input prices. The Cobb-Douglas function is for the special case when σ is equal to one. It should be clear from Equation (2) that this case needs special handling. GREEN does not use the Cobb-Douglas function, so its derivation is left for the interested reader. If a user would like to implement a Cobb-Douglas, it is suggested to us 1.01 as the input parameter.

Calibration

Typically, the base data set along with a given substitution elasticity are used to calibrate the CES share parameters. Equation (1) can be inverted to yield:

$$\alpha_i = \frac{X_i}{V}\left(\frac{P_i}{P}\right)^{\sigma}$$

assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalised to 1, simplifying the above expression to a true value share. Let's take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalised to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by α, not the share parameters represented by a. This saves on compute time since the a parameters never appear explicitly in any equation, whereas a raised to the power of the substitution elasticity, i.e. α, occurs frequently.)

Annex 3: Calibration of the Resource Depletion Modules

The extraction rates are assumed to be constant over time and estimated from base year data. The conversion rate, i.e. the rate of conversion of yet-to-find reserves to proven reserves is calibrated using medium term projections of output from energy experts. These projections are drawn from several sources, including the International Energy Agency (IEA) forecasts for the year 2005. The reference scenario of the medium term projections are conditioned by a price scenario. The calibration of the reference d, is conditioned on this price scenario which generates a given production target for the year 2005. Using the production target, the base year reserves and yet-to-find reserves and the extraction rate, it is possible to generate the value of d which is consistent with all these inputs. The key equation is set out in section II.B.21 of the GREEN Reference Manual, Equation (5), which is reproduced below:

(1) $$Res_t = (1-r)^n Res_{t-n} + d\ YTFR_{t-n} L(d)$$

where

(2) $$L(d) = \frac{(1-r)^n - (1-d)^n}{d-r}$$

Given the base year level of reserves and the yet-to-find reserves, and a projection of output (which is the same as a projection of reserves, see Equation (1) in Section II.B.21), it is possible to calculate via iteration a value for d using equations (1) and (2). It is necessary to find the root of a non-linear equation of the following form:

(3) $$f(d) = d\ L(d) + \left(\gamma - (1-r)^n\right)\frac{Res_{1985}}{YTFR_{1985}} = 0$$

where n is equal to 20, if the projection is for the year 2005, and

(4) $$\gamma = \frac{Res^{Ref}_{2005}}{Res_{1985}}$$

where γ is the projected change in reserves (output) in the year 2005. A classic method of finding the root is known as Newton-Raphson which implies the following iterating equation:

(5) $$\tilde{d} = d - \frac{f(d)}{f'(d)}$$

After some tedious calculus, it is possible to derive the derivative of f as:

(6) $$f'(d) = \frac{d\ n\ (1-d)^{n-1} - r\ L(d)}{d-r}$$

There is nothing which guarantees equation (3) has a real root, or that it has a single real root. In practice, if more than one root has been calculated, it has always been possible to throw one of them away because of its implausibilty. It is easy to programme this formula into a spreadsheet and to play around with different starting values. If the process does not converge, it has been possible to make small perturbations in the extraction rate, r, or the base year level of undiscovered reserves, to achieve convergence.

The same process can be used to calibrate the conversion rates for the low and high scenarios. Tables 1 and 2 provide the estimates used to determine the values of the conversion rates for the reference scenario, and the low and high scenarios.

Table 1: **Calibration of the Conversion Rates**

Oil Conversion Rates

	USA	JPN	EEC	OOE	EEX	CHN	FSU	IND	EET	DAE	BRA	ROW
Res0 ($)	1,446	3	646	428	37,201	546	1,849	91	36	6	53	1,172
YTFR0 ($)	2,087	3	568	2,055	22,423	2,307	4,184	141	71	86	190	4,184
Extraction Rate	0.0625	0.0417	0.0450	0.0539	0.0130	0.0455	0.0556	0.0400	0.0667	0.0500	0.0600	0.0600
Projections (Res/Res0)												
Lo	56.8	90.2	71.7	85.4	121.9	138.8	88.4	124.1	72.2	99.4	170.9	170.9
Ref	60.2	90.2	75.5	90.6	121.9	146.6	93.2	127.9	76.2	99.4	176.2	176.2
Hi	62.1	90.2	77.4	93.8	121.9	151.0	96.0	129.4	78.4	99.4	178.6	178.6
Projections (p.a.)												
Lo	-2.8	-0.5	-1.6	-0.8	1.0	1.7	-0.6	1.1	-1.6	0.0	2.7	2.7
Ref	-2.5	-0.5	-1.4	-0.5	1.0	1.9	-0.4	1.2	-1.4	0.0	2.9	2.9
Hi	-2.4	-0.5	-1.3	-0.3	1.0	2.1	-0.2	1.3	-1.2	0.0	2.9	2.9
Calibrated Conversion Rate												
Lo	0.0226	0.0698	0.0423	0.0098	0.1000	0.0225	0.0277	0.0839	0.0293	0.0034	0.0737	0.0737
Ref	0.0263	0.0698	0.0520	0.0109	0.1000	0.0249	0.0312	0.1051	0.0332	0.0034	0.0861	0.0861
Hi	0.0285	0.0698	0.0583	0.0116	0.1000	0.0263	0.0334	0.1233	0.0355	0.0034	0.0950	0.0950

Gas Conversion Rates

Res0 ($)	181	7	663	180	0	84	2,692	10	224	8	0	40
YTFR0 ($)	568	36	870	237	0	1,306	5,311	151	224	146	10	1,044
Extraction Rate	0.0547	0.0171	0.0273	0.0273	0.0000	0.0167	0.0160	0.0167	0.0273	0.0167	0.0547	0.0547
Projections (Res/Res0)												
Lo	91.2	172.9	119.7	119.7	0.0	644.5	191.3	644.5	104.9	765.0	524.6	524.6
Ref	107.5	205.2	121.1	121.1	0.0	644.5	191.3	644.5	105.9	765.0	660.7	660.7
Hi	120.2	231.4	128.9	128.9	0.0	644.5	191.3	644.5	111.9	765.0	766.8	766.8
Projections (p.a.)												
Lo	-0.5	2.8	0.9	0.9	0.0	9.8	3.3	9.8	0.2	10.7	8.6	8.6
Ref	0.4	3.7	1.0	1.0	0.0	9.8	3.3	9.8	0.3	10.7	9.9	9.9
Hi	0.9	4.3	1.3	1.3	0.0	9.8	3.3	9.8	0.6	10.7	10.7	10.7
Calibrated Conversion Rate												
Lo	0.0186	0.0126	0.0495	0.0495	0.0000	0.0283	0.0622	0.0283	0.0495	0.0283	0.0186	0.0186
Ref	0.0257	0.0174	0.0515	0.0515	0.0000	0.0283	0.0622	0.0283	0.0515	0.0283	0.0257	0.0257
Hi	0.0322	0.0217	0.0657	0.0657	0.0000	0.0283	0.0622	0.0283	0.0657	0.0283	0.0322	0.0322

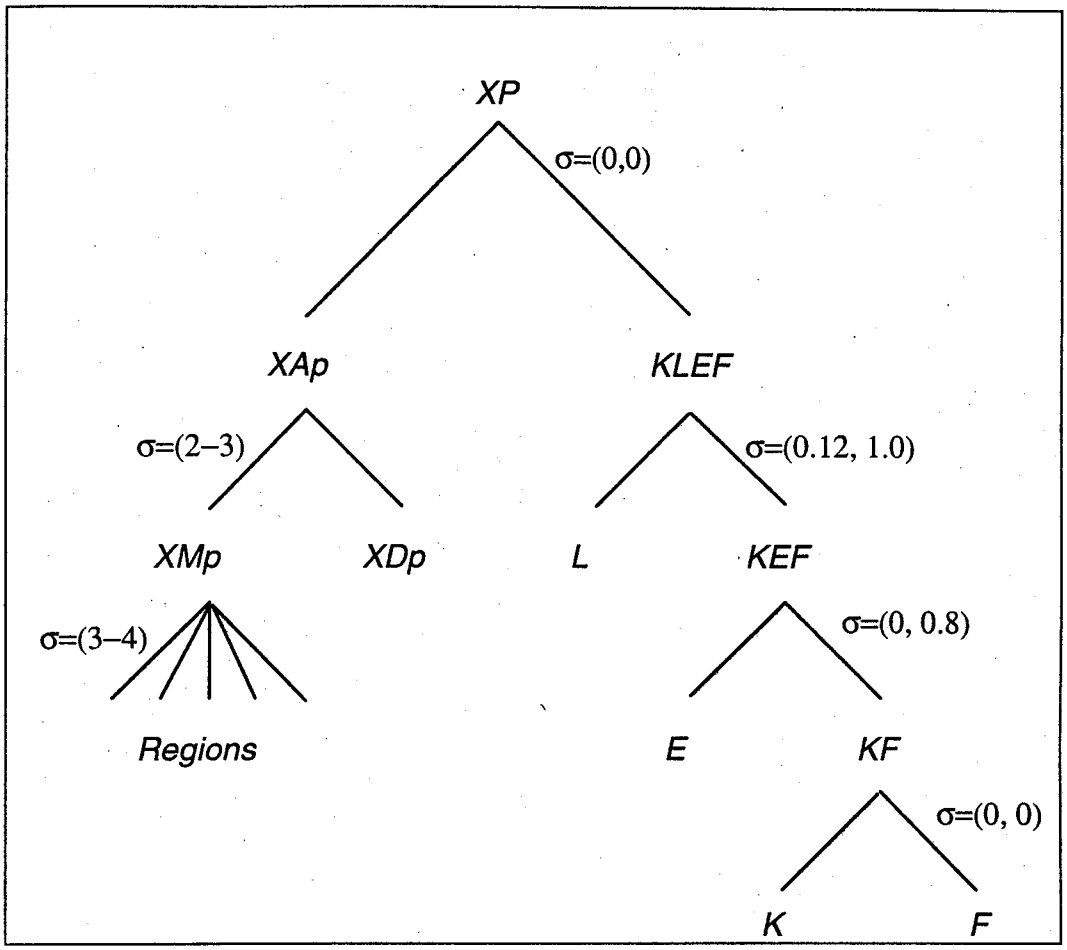
Notes:

a) Dollar amounts are in billions of 1985 dollars. The Res/Res0 ratio is in per cent, and represents a projection in the year 2005 of the production of oil. The figures in the rows below the label p.a. represent (in percent) the annual average growth rate in production.

b) The conversion rates have been calibrated using the iterative technique described above.

Figures

Figure 1a: **Nested CES Structure of Production in GREEN**

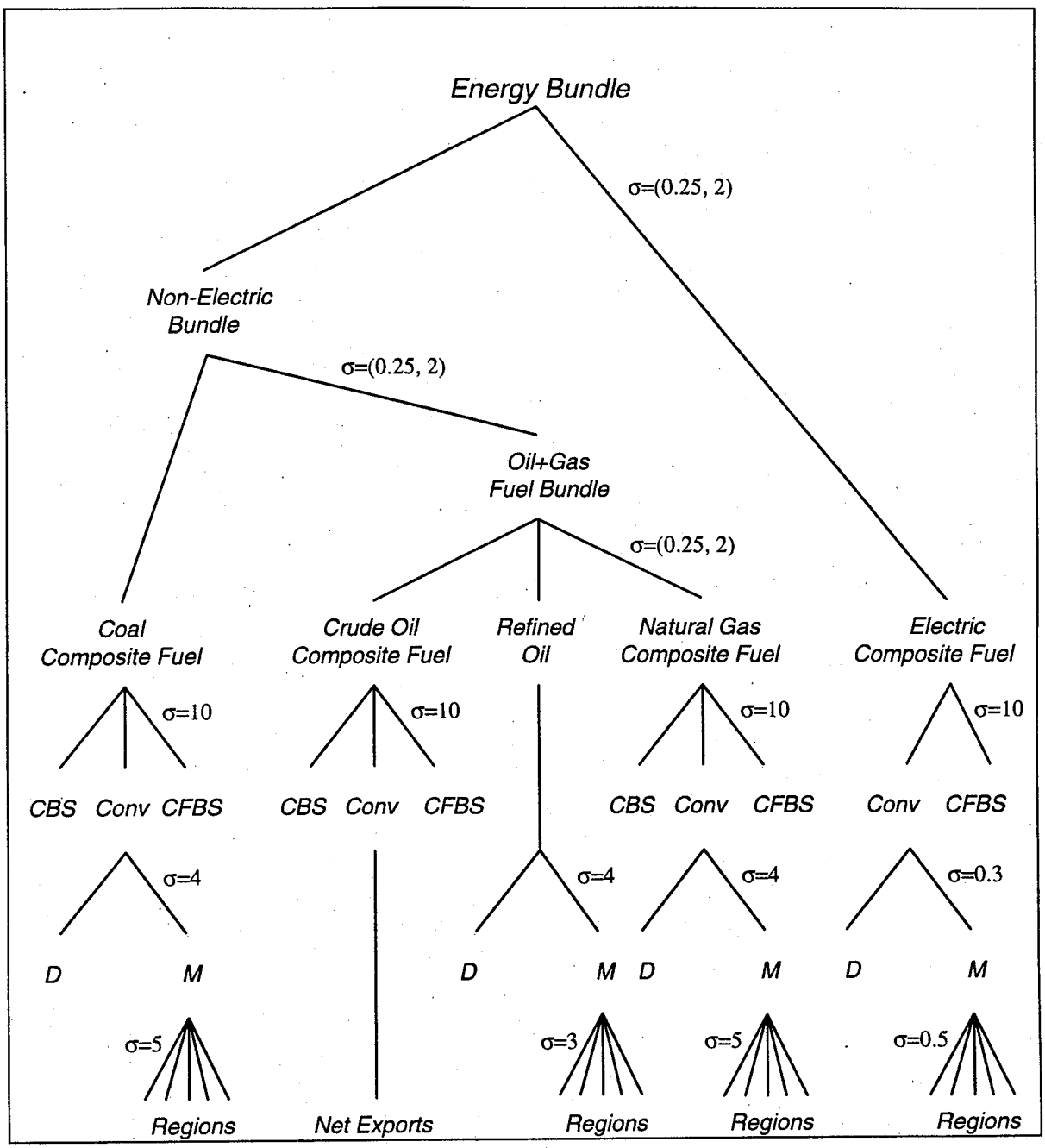


XP:	Output (by vintage)
XAp:	Intermediate (Armington) demand for non-energy goods
XMp:	Intermediate demand for (aggregate) import of non-energy goods
XDp:	Intermediate demand for domestic non-energy goods
K:	Capital input demand
L:	Labour input demand
E:	Energy (bundle) demand
F:	Fixed factor input demand
KLEF:	Composite input – *Capital+Labour+Energy+Fixed Factor*
KEF:	Composite input – *Capital+Energy+Fixed Factor*
KF:	Composite input – *Capital+Fixed Factor*

Notes:

1) The elasticities on the right side of the figure indicate the vintage-specific substitution elasticities. On the left side, they indicate the range of the Armington elasticities.

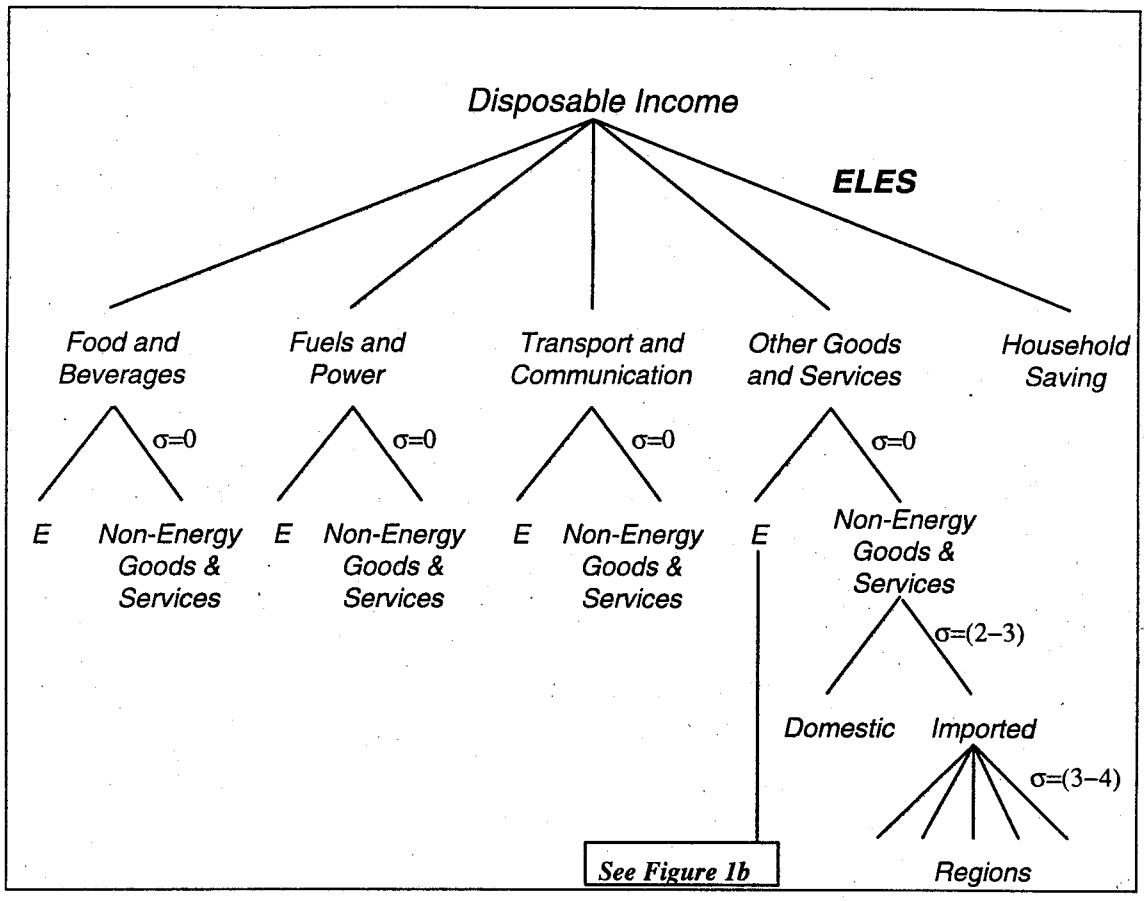
Figure 1b: **Energy Nesting in GREEN**



Notes:

1) When two elasticities are given, the first is the elasticity for old capital, the second for new capital.
2) Demand for electricity is composed of demand for conventional electricity, and demand for an electric back-stop.
3) Demand for coal, crude oil, and natural gas is decomposed into three components: the conventional fuel component (*Conv*), a carbon-based back-stop (*CBS*), and a carbon-free back-stop (*CFBS*). Refined oil has no back-stop substitutes.
4) Demand for conventional electricity, coal, natural gas, and refined oil is further decomposed using a two-tiered Armington approach. *D* and *M* represent respectively the domestic component and the aggregate import component.
5) Crude oil is assumed to be homogenous throughout the world. Hence, only net exports are calculated at the trade level.

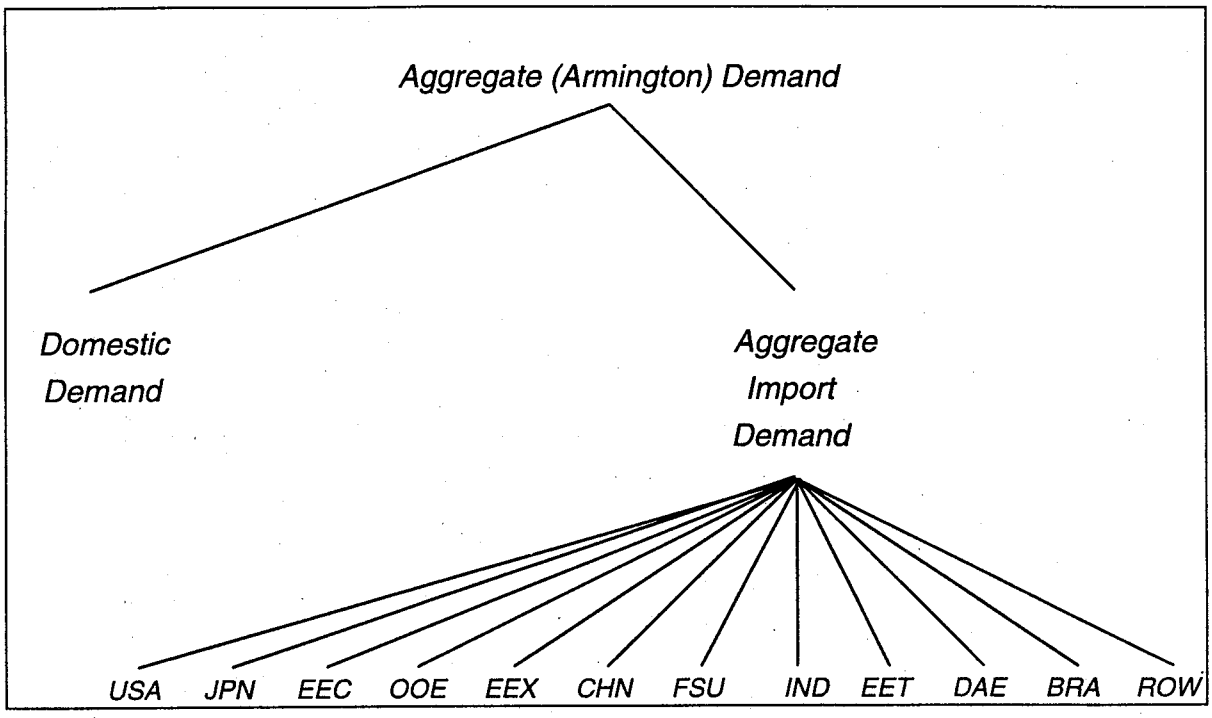
Figure 2: **Structure of Consumer Demand in GREEN**



Notes:

1) While the full diagram is only drawn for the fourth consumer commodity, the structure of the nest is identical for the first three commodities as well.
2) The letter *E* indicates consumer demand for the aggregate energy bundle. The nested structure of the energy bundle is the same as the structure of the energy bundle in production depicted in Figure 1b. Given the absence of vintage capital in consumption, a single inter-fuel elasticity is specified. The default value is 1.2. However, the back-stop elasticities are identical to those in production.
3) The top tier *ELES* structure has base year income elasticities which range from 0.5 to 1.5. The income elasticities are region and commodity specific.
4) The energy efficiency parameter (AEEI) is applied in consumption at the level of the split between energy and non-energy demand. It has the same value as in production.

Figure 3: **Trade (Armington) Structure in GREEN**



Notes:

1) GREEN uses a two-tier Armington trade structure. Each agent in the economy determines a demand for an aggregate composite commodity, often referred to as the Armington commodity. The first stage of the Armington structure disaggregates the Armington demand into a domestic component (i.e. demand for domestic production), and an aggregate import component. The second stage further disaggregates the aggregate import demand into demand for imports from each individual region. This latter second stage determines the world trade flow matrices. The first level of the Armington structure is agent specific, i.e. both the share parameters and the substitution elasticities are specific to each agent of the economy – production, consumption, government expenditure, investment expenditure, and stock building. The second level of the Armington structure uses an economy-wide demand function.
2) The Armington structure does not apply to the crude oil market since crude oil is assumed to be an homogenous good, i.e. the implicit assumption is that the Armington elasticity is infinite (at both levels). There is no trade in the back-stop commodities. These are assumed to be available in infinite supply in each region.

Figure 4a: **Resource Production Profile**

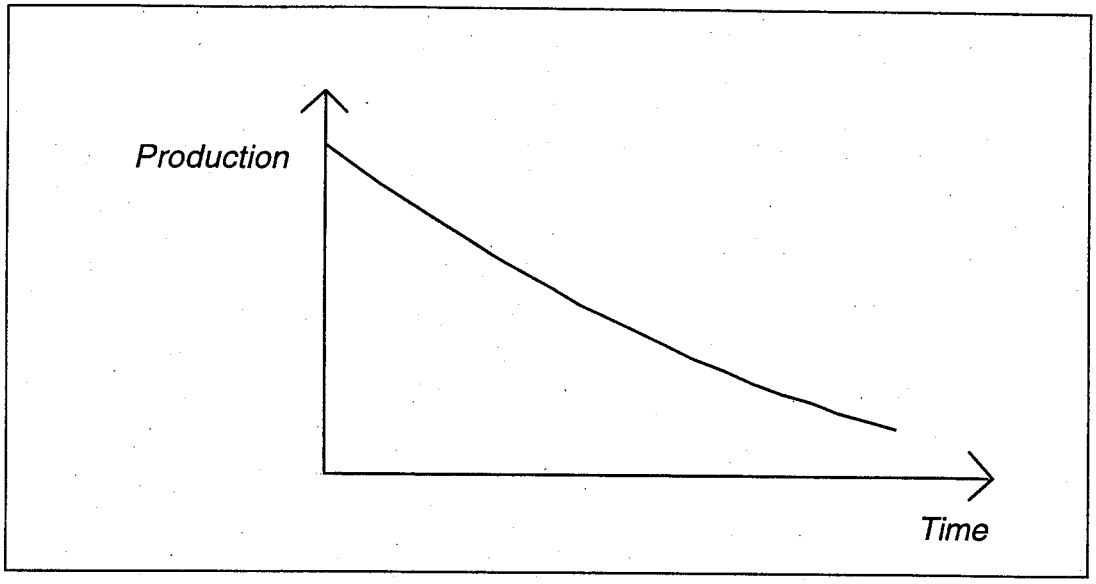
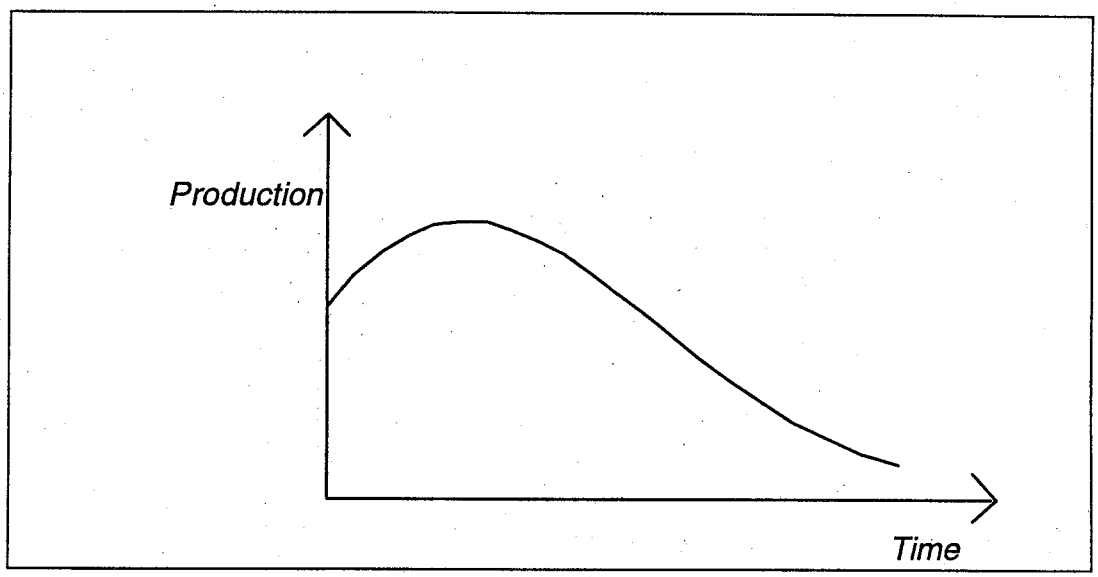


Figure 4b: **Resource Production Profile**



Notes:

1) The shape of the production curves will depend on the extraction rate from proven reserves (r), the rate of conversion of yet-to-find reserves into proven reserves (d), and the initial level of the yet-to-find reserves. Figure 4a represents the situation of the USA. The rate of extraction exceeds new reserves starting in the base year (the curve is based on the following data: r=0.0625, d=0.06, *Res*=1446, *YTF*=2087). Figure 4b represents the situation of the Rest of the World region. The initial value of the yet-to-find reserves, along with the given extraction and conversion rates, leads proven reserves (and hence production) to increase in the initial years, followed by a decline in production in later years (the curve is based on the following data: r=0.06, d=0.0861, *Res*=1172, *YTF*=4184).

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