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Aggregate Supply in INTERLINK: Model Specification and Empirical Results

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This paper describes modifications to, and further developments of, the supply block in the Secretariat's INTERLINK world model as of the autumn of 1985. The objective of the work was to strengthen the role of supply side elements, in particular profitability, in the model. In the process, stockbuilding was endogenised, assigning to inventories an important buffer role between sales and output in the dynamic adjustment process. Price formation has been linked more coherently to the revised supply structure via a dual cost function, and labour supply has been endogenised.

The equations described in this paper have been incorporated in the latest version of INTERLINK, the simulation properties of which will be described in a separate forthcoming working paper, "The Structure and Simulation Properties of the OECD INTERLINK Model: An Overview".

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Dans le présent article, on décrira les modifications apportées par le Secrétariat jusqu'à l'automne 1985 à son modèle INTERLINK de l'économie mondiale en ce qui concerne le bloc de l'offre, ainsi que les nouveaux travaux effectés dans ce domaine en vue de renforcer le rôle de facteurs tels que la rentabilité. Ce faisant, on a rendu endogène la formation de stocks, qui jouent désormais un rôle important de tampon entre la production et les ventes dans la dynamique de l'ajustement. On a en outre lié de manière plus cohérente la formation des prix à la structure révisée de l'offre par l'intermédiare d'une fonction de coût duale, et rendu endogène l'offre de main-d'oeuvre.

Les équations décrites ci-après ont été incorporées dans la version opérationnelle la plus récente d'INTERLINK. Un article prochainement publié, "Structure et caractéristiques des simulations du modèle de l'OCDE INTERLINK: Une présentation générale", décrira les résultats de plusieurs variantes de politiques économiques.

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AGGREGATE SUPPLY IN INTERLINK: MODEL SPECIFICATION AND EMPIRICAL RESULTS

I. INTRODUCTION

1. There is widespread and increasing agreement that aggregate supply deserves a central role in macroeconometric models, especially when such models are used for medium-term analysis. The more traditional demand-oriented models represent supply factors chiefly through price and wage equations, with the unemployment rate providing the main measure of unutilised supply potential, sometimes supplemented by output relative to some measure, usually exogenous, of potential output (1). In 'new classical' models of macroeconomics, the level of output is supply-determined, but little attention is given to supply determination itself (2). A third general class of models seeks to divide macroeconomic experience into two regimes, one being supply-constrained and the other demand-constrained, with sharply different behaviour responses coming into play in the two regimes (3).

2. The previous Secretariat approach to the modelling of aggregate supply was to posit an aggregate three factor production function and, assuming cost minimization by firms, to derive from it consistent factor demand equations for employment, investment and intermediate energy use. A concept of potential output could be derived from the production function. Actual output continued to be proximately demand-determined, but the "gap" between actual and potential output was used as an explanatory variable in price equations. This approach was initially implemented in stand-alone country models and later incorporated into the INTERLINK linked system (4). The system thereby had supply features and endogenous potential output but remained in some respects a refined version of the demand-oriented models referred to above.

3. Continuing Secretariat work in this area has attempted to elevate supply factors to central positions, while still permitting due attention to aggregate demand. This is done by focussing attention on the disequilibrium adjustment processes that are called into play when there is an imbalance between supply and demand. The aim is to embody the key contributions of the three model types mentioned above in a standardized form. The resulting model (or model block) meets the concern of demand-oriented models of the Keynes-Phillips type for modelling domestic and foreign final demand, as well as embodying the impact of unemployment on wage changes. It shares with the 'new classical' models the idea that actual output is basically supply-determined, in the sense that it is the result of explicit choice by producers. It shares with the disequilibrium models the recognition that current choices and hence decisions by firms and consumers are constrained by quantities as well as by prices. However, it is considered preferable to model reality as a "mixed" regime in which supply and demand forces operate simultaneously rather than giving the model distinct regimes in which one or other dominates absolutely. This is thought to be more realistic (the macro-economy does not have "switches") as well as being more tractable in use. The equilibrium regime will then become a special case that results when all exogenous variables follow an anticipated "smooth" path.

4. How is it possible to have final demand determined by income and relative prices, to have producers directly determine aggregate output and to accept at the same time that prices do not move sufficiently in the short-run to provide continuous equilibrium in markets for goods and factors? One answer lies in the use of inventories and utilisation rates as buffers between aggregate supply and demand, and in the use of resulting differences between actual and desired inventory stocks, and between actual and normal utilisation rates, as key factors leading to changes in prices, production and imports. The use of inventories as a key channel in the modelling of macroeconomic dynamics has roots in some of the earliest formal models of the business cycle (5), but was not implemented in early macroeconometric models -- perhaps because of data difficulties (inventories in national accounts often incorporate statistical discrepancies). Only in the last 15 years has this approach started to find empirical application.

5. This paper outlines the empirical implementation of Secretariat work on aggregate supply applied to the country models for the seven largest OECD countries in the INTERLINK system. The equations discussed relate to the description of the underlying long-run production structure, the estimation of the short-term output equation and the derived demands for labour, capital and energy. Specifications and estimation of the price and labour supply equations are considered more briefly.

6. Most importantly, the inclusion of an explicit output equation means that inventory investment (which was exogenous in the previous version of INTERLINK) will now be determined endogenously as production plus imports minus final sales. The respecification of price equations integrates price formation more firmly into the supply side and equations for labour supply complete the logic of the model. Also important, although less likely to influence the main macroeconomic properties of the model, will be the replacement of the existing factor demand equations by new ones. These are derived from a three-factor aggregate production function of nested CES form, as before. However, the vintage nature of the production function has been substantially modified. A rigid putty-clay structure as in the previous model was found in practice to track poorly out of sample and to give rise to instability in certain simulations.

II. THE APPROACH IN OUTLINE

7. At the centre of the proposed supply model is a vector of future output levels that producers anticipate to be profitable and permanent enough to justify assembling factories, offices and work teams of sufficient size and number to produce at normal utilisation rates. This level of output is labelled QBSTAR. While in reality it is a vector of future values, in application matters will be simplified by assuming a particular planning horizon. Given expected output, producers choose the long-run factor mix combination that will minimize the expected cost of meeting the projected output levels and adopt investment and employment plans to implement these decisions. Of course, these plans are subsequently revised as events unfold, but the central idea remains that firms revise their plans to include a consistent set of planned output, planned prices and cost-minimizing factor demands.

8. The second key element of the conception of aggregate supply, which is heavily supported by derived factor demand equations, is that it is costly and time-consuming to adjust the quantities of all the factors of production, which in the present application comprise capital, employed workers and energy (6). Since all three factors are quasi-fixed, firms are in general unable to match final demand and desired output exactly, except in the special case where the future evolution of all variables is smooth and foreseeable.

9. A third key element is the concept of normal or initially expected output (QBSV, from Q for the output measure, B for business, S for supply and V for the vintage structure built into the synthetic bundle of capital and energy), which measures the quantity of output that would be supplied if the existing quantities of employed factors were used at average utilisation rates. It is defined by inserting actual employment and the vintage bundle of capital and energy into the underlying production function. It differs from many customary measures of potential output, since it includes only employed workers and combines capital and energy in a vintage bundle. It is possible to use the same production structure to define a more forward-looking potential output series in which vintage effects have been worked out, and actual employment is replaced by some measure of potential employment. However, for the determination of actual output and prices it is the shorter-term measure QBSV which is most relevant. The actual level of output, QBV, is determined by a behavioural supply equation expressed in terms of the factor utilization rate QBV/QBSV (labelled IFU), as explained in section IV below.

10. How do suppliers respond when demand or cost conditions turn out differently from the way they were expected to evolve? This is one of the most important questions in applied macroeconomics, yet it is not usually treated very explicitly or consistently in applied macro models. In the model of aggregate supply presented here, firms faced with an unexpected increase (for example) in final demand employ several types of response simultaneously, as they:

i) Increase production by raising the utilization rate of employed factors;

ii) Update forecasts of profitable sales, and adjust their factor demands to levels consistent with the revised output expectations. Any increase in the quantities of factors employed in the current period will permit an increase in production at normal utilization rates;

iii) Reduce inventory stocks or lengthen the list of unfilled orders;

iv) Increase imports to help maintain stock levels and meet final demand; and

v) Raise prices.

11. The challenge is to model these responses in an integrated and consistent way. In an accounting sense, inventory change is the residual element in our proposed supply system, since any additional final sales not met by increased output or imports, and not deterred by higher prices, are drawn from inventories. In fact, the responses are mutually dependent, since

any gap between actual and desired inventories should influence changes in production, prices and imports, and abnormal utilisation rates have direct effects on factor demands, prices and possibly imports. Thus there is a simultaneous and joint determination by producers of factor demands, output levels, prices and inventory stocks.

12. In this paper results for four parts of the supply system are presented: the three-factor production structure combining capital, energy and labour to define "normal" output QBSV; the production equations determining the utilization rate IFU; the derived demand equations for labour, investment and energy; and the linkage of the supply structure to the price formation process. In addition, the endogenisation of labour supply via behavioural participation rate equations is discussed. It is appropriate in this initial overview to outline the role of imports in aggregate supply although no empirical work on imports is reported. As explained in the next section, the aggregate output concept used, QBV, is defined as private sector value added plus business energy inputs and is produced by a nested production structure in which capital and energy are bundled together and then combined with labour to define normal output QBSV. Conceptually, there is a still higher level in the supply structure in which domestic output QBV and non-energy imports (MNEV) are combined (e.g. in a CES function) to produce (the utility of) final sales. The derived cost-minimizing import demand ratio is therefore:

$$MNEV/QBSV = (PQB/PMNE)^{e}$$

where e is the long-term elasticity of substitution between domestic and foreign output in final sales, which include private consumption and investment, government spending on goods and services as well as exports. PQB and PMNE are the price indices for QBV and non-energy imports respectively. Following the notion of disequilibrium adjustment, the actual import ratio may reflect some lagged response to changes in expected prices as well as shorter-term variations caused by abnormal utilisation rates or inventory levels.

13. Although imports combine with domestic output at the highest level in the proposed supply structure, this does not imply that imports are typically finished goods rather than raw materials or intermediate products. Almost all imports (except perhaps for tourist services purchased abroad) involve some domestic value-added and hence are intermediate rather than final goods and services. Competition between domestic and foreign value-added takes place at all stages of completion, and the proposed aggregate supply structure does not, and need not, disaggregate imports by commodity or use. This way of nesting the supply structure is broadly though not entirely consistent with the current treatment of imports in INTERLINK. In future work domestic factor utilisation should be tested in import equations; inventories and relative prices already feature.

14. All of the above discussion refers to the modelling of aggregate supply for a national economy. There are additional considerations that should govern the treatment of aggregate supply in a multi-country model such as INTERLINK, especially if account is taken of the OECD's interests in sharing and comparing national experiences as well as in assessing the structure and strength of international economic linkages. These interests imply a high premium on a supply structure which is simple, yet general enough to be

applicable in the same basic form to all member countries. The use of uniform structure and data not only facilitates some international comparisons, but also makes it much easier to model international transmission mechanisms. This is true not only for the standard linkages provided by trade flows, prices, capital flows, exchange rates and interest rates, but also, as will be discussed in section III, for the modelling of the determination and international linkage of technical progress.

15. Therefore an attempt was made to combine simplicity and generality in modelling the structure of aggregate supply and to estimate comparable equations for each country. This is likely to give equations that fit less closely than those that might be obtained by following country-by-country search procedures to find the best-fitting equation forms for each country. However, it is hoped in this way a clearer basis for international comparisons can be provided. The pooling of national experiences in the choice of a common supply structure could also provide a more robust basis for forecasting, since the structure and parameters will be less tuned to a particular period of each country's history. It is possible to go too far in applying a common structure to all countries, since there is also an interest in establishing the size and nature of differences among countries. An effort was therefore made to ensure that there were enough country-specific parameters to identify the most important features of country-specific behaviour.

III. THE UNDERLYING PRODUCTION STRUCTURE (7)

A. The choice of output and input variables

16. An initial choice to be made concerns the appropriate level of aggregation for supply modelling. The highest feasible level of aggregation has three major advantages: first, it permits the use of comparable data for more countries and reduces data problems, model complexity and the need to forecast any exogenous components of output. Second, it permits a more complete and careful modelling of the disequilibrium adjustment processes that come into play when there is any imbalance between aggregate supply and demand. The third reason for high level aggregation is more a reason for avoiding disaggregation. Any model that goes far in explaining the output potential of individual sectors in terms of their separate inputs of materials, capital, labour and technology then faces the formidable task of modelling changes in inter-industry movements of factors. To be sure, an aggregate treatment deals with these questions at the expense of inevitable errors of aggregation. The advantage is that an economic logic can be imposed and tested at the aggregate level with more coherence and completeness than is generally possible or practicable with a constellation of industry supply models (8).

17. It is, however, questionable whether the same behavioural assumptions are applicable for resource allocation by general government (e.g. public employment and investment) as for private sector spending behaviour. Given the purpose of INTERLINK to serve primarily as a short-to-medium term policy simulation model, it was decided to separate the general government from the rest of the economy in the supply side analysis, treating government fixed

investment and employment as exogenous (9). Finally, housing investment was separated from business fixed investment on the assumption that here, too, behavioural characteristics are sufficiently different from other investment components -- and the relative size of housing investment sufficiently large -- to justify disaggregation.

18. Given the analytical advantage of a relatively simple and direct form for the production function, the main question was whether to restrict inputs to aggregate capital and labour or to add energy as a third input. There are good arguments on both sides. Treating energy as a separate factor requires the collection and maintenance of a number of data series, some of which are difficult to obtain, especially in a comparable form over a reasonable span of history, for all OECD countries. It also entails a more complicated production structure and more parameters to estimate. On the other hand, as shown in Chart 1, energy prices moved significantly differently from other prices both before 1973, when they were on a falling trend relative to the price of investment goods, and between 1973 and 1982 (the end of the available energy data used for the estimation of the block), when they were on a sharply rising trend. These changes were large enough, and energy expenditure is sufficiently large in relation to other factors (for example, intermediate energy expenditures in the seven largest OECD economies in 1982 amounted on average to 10 per cent of total labour costs) as to risk important errors if energy were not given special treatment.

19. The suggested reconciliation of these competing arguments has been to treat energy as a separate factor of production in the major seven OECD countries, but to do so in a way that would permit application of the same basic model to smaller countries. This allows separate treatment of energy in the countries with the biggest total impact on OECD output and world energy markets without imposing excessive data difficulties on any subsequent supply modelling for the smaller countries.

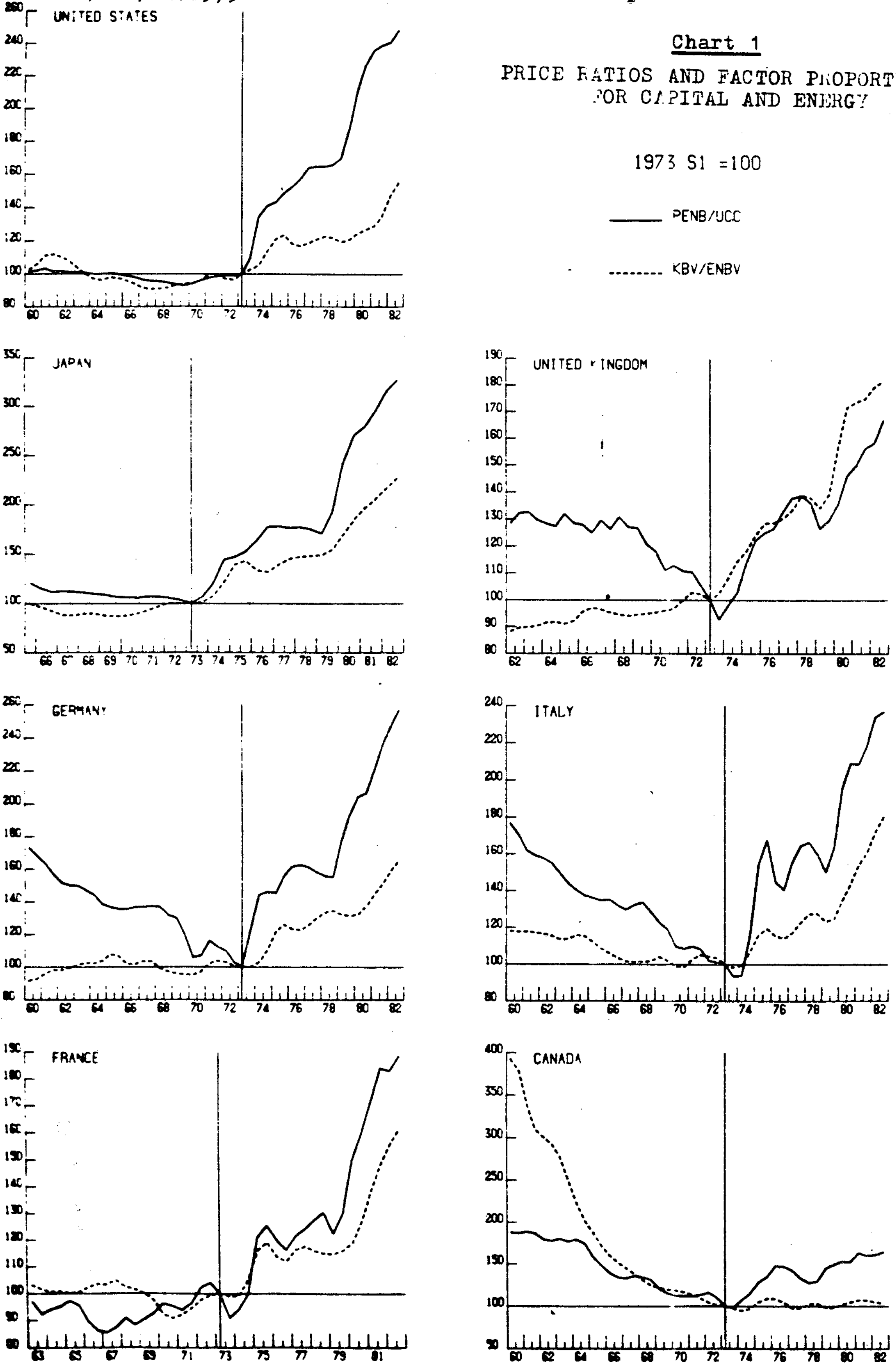
20. Given the decision to include energy, consistent definitions of input and output series had to be constructed. The appropriate energy input and price data were built up from primary data produced by the International Energy Agency and the United States Department of Energy. An annex to this paper containing the definition and describing the derivation of series for total energy use in business and the corresponding energy prices is available from the authors upon request.

21. In principle, employment and capital stock data should include all business capital and business labour except that used in the production and distribution of energy. Thus far, the capital stock and employment of the energy-producing sector have not been excluded . Lack of data has prevented making this adjustment, and considerations of model simplicity and data management discourage such adjustments unless they make a considerable difference to the results. Experiments with an earlier Canadian model (Helliwell et. al., 1985) in which the adjustments were made, suggest that the employment adjustment is so small as to be without consequence, but that the energy capital stock is large enough and subject to a different enough growth path that the adjustment is important. This is likely to also be the case for other OECD countries with large and volatile energy investment. Work is planned to explore the importance of the adjustment and the feasibility of integrating it into INTERLINK for the major countries where it is likely to be most important -- the United Kingdom and Canada.

Chart 1

PRICE RATIOS AND FACTOR PROPORTIONS FOR CAPITAL AND ENERGY

1973 S1 =100



23. Corresponding to the three inputs chosen, the output measure is business value added plus the value of business energy inputs (10). To bring the output concept as close as possible to factor cost, non-energy net indirect taxes ought to be subtracted from the output measure at market prices. Since the data required to split energy and non-energy indirect taxes were not readily available, total net indirect taxes were deducted, imparting a downward bias on the business gross output measure. On the other hand the rental value of the housing stock was not deducted from the gross output measure, resulting in a distortion in the opposite direction. It has been assumed that the resulting net distortion is sufficiently small as to be inconsequential for the estimation results described below.

B. Functional forms and nesting

24. The choice of an appropriate functional form depends on several considerations. A simple and highly restricted form poses fewer difficulties in estimation and is likely to be more robust in forecasting, while a more flexible form offers the data more chance to influence the way in which technology is modelled. In the framework presented here the choice is tilted heavily towards simplicity, since the utilisation rate for employed factors can be neither assumed constant nor measured independently of the production structure itself. Thus the three-factor production function cannot be estimated directly, since there is an excluded variable, utilisation, that would bias the parameter estimates. However, if the sample period is sufficiently long and representative, then the average utilisation rate can be taken to be normal. Sample averages and long-run cost minimisation conditions can then be used to identify key parameters, and derived factor demand equations serve to establish the adjustment dynamics.

25. To keep the number of parameters small, a nested production structure was chosen almost identical to that in the previous version of INTERLINK. Capital and energy are bundled together in an inner CES function and that bundle is combined with efficiency units of labour in a CES outer function. This nesting was tested and supported against alternative separability assumptions at the aggregate level by Artus (1983) and for U.S. manufacturing by Berndt and Wood (1979).

26. The general strategy for choosing parameters was to derive them as far as possible from the requirement that the production function should hold on average over the sample period and that the cost-minimizing factor ratios should on average equal the actual factor ratios. This assumption allows the share parameters in the inner and outer CES functions to be determined from observed average factor price and input ratios, given the conventional assumption of constant returns to scale (11).

27. The following section starts with a description of how the form and parameters of the inner CES function for the bundle of capital and energy were developed and estimated, followed by a discussion of the alternative methods used for modelling technical progress and estimating parameters for the outer CES function. The two-level CES structure appears to be necessary for some countries because of evidence of a rising trend in the share of labour cost in total factor payments, a result that is incompatible with the Cobb-Douglas assumption, unless there is persistent lagged adjustment to an increase in the real wage. For countries with a constant labour share an outer Cobb-Douglas

function can be approximated by setting the substitution parameter in the outer CES function equal to 0.99. This permits the maintenance of identical coding for all countries.

C. Vintage structure and the retrofitting parameter

…st important innovation in the bundling of capital and energy presented here lies in its flexible vintage structure, which provides a simple generalisation in which the putty-putty and putty-clay assumptions are both special cases. The earliest empirical attempts to approximate a vintage model, e.g. Bischoff (1971), used separate distributed lags on relative prices and output changes in estimating factor demand equations. This approach has the disadvantage that it does not permit the identification of the underlying production structure. The earlier supply work for INTERLINK (Artus, 1983), which likewise made use of a capital-energy bundle in an inner CES function, applied a rigid putty-clay model which did not permit any substitution between capital and energy after the capital was put in place.

29. The vintage idea has substantial appeal in the modelling of energy use, since many capital goods are built with a particular pattern of energy use in mind and often embody fairly fixed energy requirements. However, it is also clear that by insulation, boiler conversion, process controls and, last but not least, by simple changes in operating procedures, it has been possible to substantially change the amount of energy used to operate capital that is already in place. There are thus good reasons to develop and apply a putty/semi-putty model that permits some change of energy use, or 'retrofitting' after the capital is put in place, but which still maintains a vintage structure. This was done by a simple change to the rigid vintage model, adding a single new parameter that takes the value zero when there is no retrofitting possible and is equal to 1 minus the scrapping rate if putty/putty assumptions are applicable.

30. Given the chosen CES form for combining capital and energy, the cost-minimising ratio of energy (EBSV) to capital (KBV) is

$$(\mathrm{EBSV}/\mathrm{KBV})^* = [\mathrm{XIGAMA.UCC}/(\mathrm{XIBETA.PENB})]^s \qquad (1)$$

where s is the elasticity of substitution between capital and energy, UCC and PENB are the respective factor prices, and XIBETA and XIGAMA are the scaling constants in the inner CES function. In a putty-putty model with immediate adjustment, energy and capital always bear their cost-minimising relationship to each other, and

$$\mathrm{EBSV} = (\mathrm{XIGAMA.UCC}/(\mathrm{XIBETA.PENB}))^s.\mathrm{KBV} \qquad (2)$$

In a strict putty-clay vintage model with a proportionate scrapping rate, the optimal capital/energy ratio can only be applied to gross investment, and energy demand is given by

$$\mathrm{EBSV} = \mathrm{EBSV}(-1).(1-\mathrm{RSCRB}) + \mathrm{IBV}.(\mathrm{XIGAMA.UCC}/(\mathrm{XIBETA.PENB}))^s \qquad (3)$$

where EBSV(-1) is last period's vintage energy requirement. RSCRB is the rate of scrapping, and IBV is business sector gross fixed investment (12).

31. The flexible vintage model, which is a putty/semi-putty model, assumes that some fraction, XR1, of the previous semester's capital stock can be retrofitted to embody the latest cost-minimising capital-energy ratio. In the model, therefore, energy requirements to operate the existing capital stock are defined as:

$$EBSV = EBSV(-1).(1-XR1-RSCRB) + [IBV+XR1.KBV(-1)].[XIGAMA.UCC/(XIBETA.PENB)]^{s} \quad (4)$$

The matching definition of the capital-energy bundle is

$$KEBSV = KEBSV(-1).(1-XR1-RSCRB) + [IBV+XR1.KBV(-1)].[XIBETA+XIGAMA.[XIGAMA.UCC/(XIBETA.PENB)]^{s-1}]^{s/s-1} \quad (5)$$

32. This is equivalent to the putty-putty model if XR1 = 1-RSCRB and to the putty-clay model if XR1 = 0. Given the non-linear form for the vintage energy requirement EBSV, the parameters XR1 and s were found by grid search over all pairs of values over the range .05 to 1-RSCRB for XR1 and from 0.3 to one for s. The parameter pair which maximized the likelihood function of the regression of actual business energy demand on the synthetic energy requirement was chosen. This procedure requires prior knowledge of the ratio XIGAMA/XIBETA, which is obtained, for any value of s, by the assumption that the actual energy/capital ratios are equal to their cost-minimising values on average over the sample period. The resulting production function parameters, are shown for each country in Table 1. Given XR1, s and the ratio XIGAMA/XIBETA, the separate values for XIGAMA and XIBETA are obtained by equalising the means of KBV and KEBSV, thus permitting the definition of a series for CKE, whose value is the proportionate cost of renting the capital stock in place and providing energy for its operation.

$$CKE = [XIBETA^{s}.UCC^{(1-s)} + XIGAMA^{s}.PENB^{(1-s)}]^{1/(1-s)} \quad (6)$$

33. To be consistent with the treatment of the input of capital and labour, for which no attempt is made to adjust for cyclical changes in man-hours or machine-hours, EBSV is chosen as the appropriate measure of energy use in the function for normal output (QBSV). If a measure of utilised capital plus energy were required, one could use the coefficients on QBV/QBSV in the energy demand equations to construct a utilisation rate for the bundle of capital plus energy (cf. section V below). If labour input were adjusted for cyclical changes in hours worked, this adjustment ought to be made for the utilisation rate of the capital-energy bundle also. Since these adjustments to QBSV would still leave a substantial systematic variation in QBV/QBSV, because of other unmeasured and unmeasurable changes in the intensity of factor use, it was preferred to treat all variations in factor utilisation together when modelling the short-run production decision by the equation for QBV/QBSV, as described below in section IV.

D. Specification of technical progress

34. The rate of technical progress is defined as the rate of increase in output if all input quantities as well as the intensity with which they are utilised are held constant. Estimating the rate of technical progress empirically is rendered difficult because of the simultaneous changes in output, inputs and utilisation rates. Any estimate of the rate of technical

Table 1

PARAMETER VALUES OF THE AGGREGATE PRODUCTION STRUCTURE (a)

	OUTER CES FUNCTION			INNER CES FUNCTION			
	Elasticity of substitution	Scale Labour	Parameters capital/energy bundle	Elasticity of substitution	Retrofitting parameter	Scale Energy	Parameters Capital
Country	XTAU	XOBETA	XOGAMMA	s	XR1	XIBETA	XIGAMA
United States	1.01	0.710	0.35	0.5	0.45	0.87	0.003
Japan	0.7	0.001	0.31	0.8	0.68	0.77	0.109
Germany	0.99	0.597	0.34	0.5	0.29	0.82	0.005
France	0.8	0.048	0.38	0.8	0.16	0.86	0.053
United Kingdom	0.6	0.002	0.65	0.3	0.05	0.78	7.E-5
Italy	0.8	0.019	0.25	0.5	0.37	0.77	0.005
Canada	1.01	0.708	0.35	0.9	0.05	0.89	0.065

a) See main text and Annex A for a detailed discussion of how these parameter estimates were derived.

progress which is extracted from the observed input and output data will depend on other key parameters of the assumed production structure and *vice versa*. To reduce the multiple parameter interactions in estimation, various *a priori* constraints were therefore imposed on the production structure:

i) Constant returns to scale;

ii) Technical progress is purely labour augmenting (i.e. "Harrod neutral"); though chiefly conventional, this assumption appears plausible in view of the observed steady increase in the real product wage (or real return to labour) combined with a relatively stable (or declining) real rate of return to capital.

35. The number of possible permutations of parameter estimates is greatly reduced by these assumptions. As a result the determination of the elasticity of factor substitution in the outer CES function and the degree of factor utilisation interact with the estimation of the rate of technical progress. The latter is estimated as the rate of increase in labour efficiency, using the mnemonics ELEFF and DELEFF for the level and annual growth rate of the trend labour efficiency index respectively, while PIM represents the observed labour efficiency index, incorporating cyclical fluctuations, from which DELEFF is estimated (13). The elasticity of substitution between (efficiency units of) labour and the capital/energy bundle (XTAU) was determined from the regression:

$$\ln(QBV/(ETB.ELEFF)) = a_0 + XTAU.\ln(WSSE/(PQB.ELEFF)) + u \qquad (7)$$

where PQB is the output deflator at factor cost, ETB is business sector employment and WSSE is the total labour cost per man year. Equation (7) requires the series ELEFF to be known, while in turn the determination of ELEFF requires knowledge of parameter XTAU (see below). ELEFF and XTAU were therefore determined by an iterative procedure, starting with an assumed value of XTAU = 0.99 (14). This procedure appears to be inefficient in that it neglects information on XTAU to be obtained from the series on capital/energy use and the associated cost. However, given the measurement errors associated with both of these series, the limited information approach was used. This is a pragmatic approach which confines the effect of errors in variables to the estimates of the parameters of the inner CES function.

36. In all countries there was a slowdown in the crudely measured rate of technical progress during the observation period, partly due to cyclical effects on actual total factor productivity. The mutual interdependence of DELEFF estimates and the intensity of factor utilisation (IFU) was dealt with by using an iterative procedure: in the first step a proxy for DELEFF was computed as a simple trend coefficient without any capacity utilisation adjustment. The resulting DELEFF proxy was then used to construct IFUHAT, the natural logarithm of the estimate of the degree of factor utilisation. At the second stage IFUHAT was used jointly with the hypothesised long-run determinants of DELEFF (see below) to obtain a cyclically adjusted estimate of DELEFF. To avoid simultaneous equations bias, two stage least squares and/or instrumental variables estimation procedures were used for both steps. But even after adjustment for cyclical factors was made, the slowdown in the measured rate of growth of DELEFF persisted in most countries (using equation VI), with the notable exception of the United States (cf. Table 2) (15). The latter country is also the one with the highest level of

total factor productivity (if measured at purchasing power parities rather than actual exchange rates) and the lowest trend rate of growth of total factor productivity. This suggests that there is a process of international convergence of growth rates of technical progress, with the speed of growth being positively related to the level difference between the most efficient country and the country concerned. This hypothesis was tested formally by estimating the following equations:

$$\ln(PIM/PIM(-1)) = a_0 + a_1 \ln(ELEFFUS(-1)/PIM(-1)) + u \quad (8a)$$

$$\ln(PIM/PIM(-1)) = b_o + b_1 \ln(ELEFFUS(-1)/PIM(-1)) + b_2\ DIFUHAT + u \quad (8b)$$

$$\ln(PIMADJ/PIMADJ(-1)) = c_o + c_1 \ln(ELEFFUS(-1)/PIMADJ(-1)) + u \quad (8c)$$

where ELEFFUS is the synthetic labour efficiency index for the United States, computed from the cyclically adjusted estimate of DELEFFUS. DIFUHAT is the increase in IFUHAT over its value in the preceding period. PIM represents the empirical value (modified by a multiplicative constant) of ELEFF, while PIMADJ represents the same variable adjusted for cyclical fluctuations in the utilisation rate. This adjustment consisted of replacing QBV by QBV/EXP(IFUHAT) in the equation computing PIM (cf. footnote 13 and Annex A). The estimated coefficients from equation (8) are reported in the first four columns of Table 3.

37. In this specification of the catch-up hypothesis the rate of change of labour efficiency (DELEFF) converges to the same constant rate -- that of the United States -- in all countries. The unconstrained (logarithmic) constant term permits the level of labour efficiency to be different among countries even after convergence in the growth rates has been completed. This seems appropriate, given the likelihood of persistent differences between countries in per capita endowment of resources (including human capital) and cultural characteristics affecting productivity (16). The actually measured labour efficiency index and the synthetic index based on estimation of equation (8b) (i.e. purged of cyclical variation) are juxtaposed in Chart 2.

38. An alternative explanation of the observed slowdown in labour efficiency growth is based on the assumption that technical progress is embodied in new capital goods, i.e. related to gross fixed investment. Since there has been a trend decline in the rate of growth of the gross fixed capital stock in virtually all countries in the sample over the observation period, the embodiment hypothesis is consistent with the observed productivity slowdown and thus represents a competing explanation for the latter. This hypothesis was formally tested by estimating the following equations:

$$\ln(PIM/PIM(-1)) = d_0 + d_1 \ln(AIBV/KBV(-5)) + u \quad (9a)$$

$$\ln(PIMADJ/PIMADJ(-1)) = e_0 + e_1 \ln(AIBV/KBV(-5)) + u \quad (9b)$$

$$\ln(PIM/PIM(-1)) = f_0 + f_1 \ln(AIBV/KBV(-5)) + f_2 \ln DIFUHAT + u \quad (9c)$$

where AIBV is a moving average of business gross fixed investment over ten half years (the average length of a full cycle). The resulting coefficient estimates, reported in columns five to eight of Table 3, are disappointing: the estimated embodiment coefficients are insignificant and in several countries have the wrong sign. Invariably the explanatory power of the

Table 2

TESTS FOR CHANGES IN THE RATE OF LABOUR EFFICIENCY GROWTH
Estimated coefficients (t-statistics)

Country	Estimation Period	I	II		III		IV			V	VI	
		a_1	b_1	b_2	c_1	c_2	d_1	d_2	d_3	e_1	f_1	f_2
United States	60:S1-82:S2	0.012 (9.7)	0.166 (8.6)	-0.001 (-7.9)	0.012 (47.4)	1.547 (31.9)	0.184 (1.9)	-0.0000 (-0.6)	1.510 (20.3)	0.012 (9.4)	0.162 (7.9)	-0.001 (-7.4)
Japan	65:S1-82:S2	0.044 (21.3)	0.359 (9.1)	-0.002 (-7.9)	0.044 (96.5)	1.387 (24.6)	0.056 (2.4)	-0.0001 (-0.5)	1.353 (15.5)	0.042 (21.5)	0.323 (7.3)	-0.002 (-6.4)
Germany	60:S1-82:S2	0.036 (37.8)	0.141 (8.4)	-0.001 (-6.2)	0.036 (71.0)	1.508 (11.2)	0.080 (5.4)	-0.0003 (-3.0)	1.206 (7.6)	0.036 (41.9)	0.124 (7.4)	-0.006 (-5.3)
France	63:S1-82:S2	0.041 (26.6)	0.297 (20.6)	-0.002 (-17.8)	0.041 (106.0)	1.483 (23.8)	0.130 (5.3)	-0.0006 (-3.7)	1.027 (7.6)	0.041 (26.9)	0.286 (16.2)	-0.002 (-13.9)
United Kingdom	63:S1-82:S2	0.015 (11.9)	0.180 (7.4)	-0.001 (-6.8)	0.014 (19.0)	1.528 (9.0)	0.073 (2.3)	-0.0004 (-1.9)	1.181 (4.7)	0.016 (14.6)	0.160 (7.6)	-0.001 (-6.8)
Italy	60:S1-82:S2	0.044 (24.5)	0.306 (18.1)	-0.002 (-15.6)	0.044 (92.4)	1.268 (24.4)	0.125 (5.5)	-0.0006 (-3.6)	0.941 (9.2)	0.043 (24.7)	0.297 (15.1)	-0.002 (-12.9)
Canada	60:S1-82:S2	0.020 (13.3)	0.233 (12.8)	-0.001 (-11.7)	0.020 (66.2)	1.549 (33.3)	0.044 (2.8)	-0.0002 (-1.5)	1.423 (15.0)	0.020 (12.8)	0.240 (12.8)	-0.002 (-11.8)

Reported coefficients are from the following six equations:

I: $\ln PIM = a_0 + a_1\ time$

II: $\ln PIM = b_0 + b_1\ time + b_2\ time^2$

III: $\ln PIM = c_0 + c_1\ time + c_2\ IFUHAT$

IV: $\ln PIM = d_0 + d_1\ time + d_2\ time^2 + d_3\ IFUHAT$

V: $\ln PIMADJ = e_0 + e_1\ time$

VI: $\ln PIMADJ = f_0 + f_1\ time + f_2\ time^2$

Table 3

TESTS OF THE CATCH-UP AND EMBODIMENT HYPOTHESES

	Catch-up equations				Embodiment equations				Mixed Equations				
	I	II		III	IV	V	VI		VII		VIII		
	a_1	b_1	b_2	c_1	d_1	e_1	f_1	f_2	g_1	g_2	h_1	h_2	h_3
USA	n.a.	n.a.	n.a.	n.a.	-1.930	-0.014	-0.465	1.183	n.a.	n.a.	n.a.	n.a.	n.a.
(S2 60 to S2 82)					(2.0)	(-0.0)	(-0.6)	(5.6)					
JAP	0.058	0.054	0.990	0.053	0.015	0.154	0.01	0.183	0.103	-0.173	0.231	-0.684	0.343
(S2 65 to S2 82)	(3.0)	(3.0)	(3.9)	(2.6)	(0.1)	(1.3)	(0.1)	(0.9)	(1.5)	(-0.7)	(3.3)	(-2.7)	(1.9)
GER	0.033	0.031	0.649	0.047	0.479	0.579	1.360	0.286	0.123	-0.736	0.198	-0.481	0.665
(S2 60 to S2 82)	(1.6)	(1.4)	(1.6)	(1.7)	(1.0)	(1.2)	(1.4)	(1.0)	(1.9)	(-0.9)	(3.5)	(-0.5)	(2.5)
FRA	0.058	0.053	1.062	0.069	0.950	0.591	0.945	-0.116	0.072	0.099	0.223	-0.635	0.979
(S2 63 to S2 82)	(3.3)	(3.5)	(4.1)	(3.9)	(1.6)	(1.2)	(1.6)	(-0.5)	(1.9)	(0.2)	(4.4)	(-1.1)	(3.2)
UKM	0.167	0.137	0.610	0.327	2.117	1.441	2.433	0.212	0.368	2.149	0.449	3.370	1.145
(S2 63 to S2 82	(2.1)	(1.6)	(1.2)	(2.8)	(0.7)	(0.5)	(0.8)	(0.7)	(2.6)	(0.8)	(4.0)	(1.3)	(3.3)
ITA	0.050	0.047	1.031	0.054	0.649	0.659	0.744	0.179	0.075	-0.139	0.263	-1.676	0.548
(S2 60 to S2 82)	(3.1)	(3.2)	(4.0)	(3.5)	(1.7)	(2.2)	(2.0)	(1.4)	(1.2)	(-0.2)	(4.5)	(-2.7)	(4.1)
CAN	0.050	0.036	1.051	0.102	-2.189	-0.186	-1.868	0.203	0.186	0.807	0.34	0.464	0.647
(S2 60 to S2 82)	(1.4)	(1.2)	(4.8)	(1.8)	(-1.5)	(-0.2)	(-1.3)	(2.3)	(2.1)	(0.6)	(4.9)	(0.4)	(5.7)

n.a. Not applicable.

Catchup

I $\ln(PIM/PIM(-1)) = a_0 + a_1 \ln(ELEFFUS(-1)/PIM(-1))$

II $\ln(PIM/PIM(-1)) = b_0 + b_1 \ln(ELEFFUS(-1)/PIM(-1)) + b_2 DIFUHAT$

III $\ln(PIMADJ/PIMADJ(-1)) = c_\theta + c_1 \ln(ELEFFUS(-1)/PIMADJ(-1))$

Embodiment

IV $\ln(PIM/PIM(-1)) = d_0 + d_1 \ln(AIBV/KBV(-5))$

V $\ln(PIMADJ/PIMADJ(-1)) = e_0 + e_1 \ln(AIBV/KBV(-5))$

VI $\ln(PIM/PIM(-1)) = f_0 + f_1 \ln(AIBV/KBV(-5)) + f_2 DIFUHAT$

Mixed

VII $\ln(PIMADJ/PIMADJ(-1)) = g_0 + g_1 \ln(ELEFFUS(-1)/PIMADJ(-1)) + g_2 \ln(AIBV/KBV(-5))$

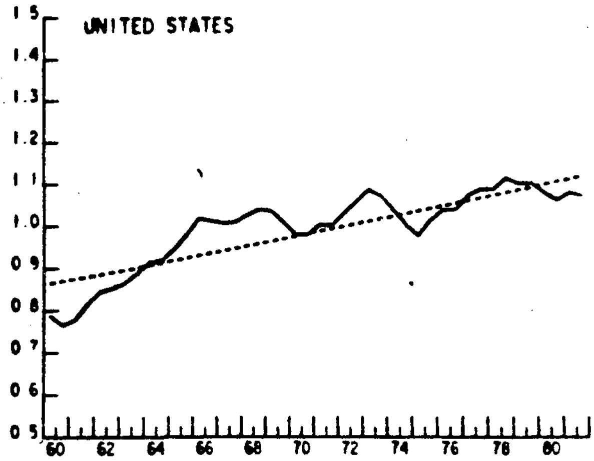
VIII $\ln(PIM/PIM(-1)) = h_0 + h_1 \ln(ELEFFUS(-1)/PIM(-1)) + h_2 \ln(AIBV/KBV(-5)) + h_3 DIFUHAT$

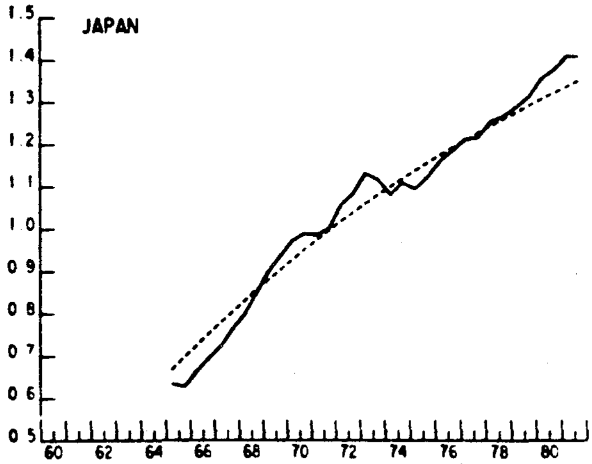
Chart 2

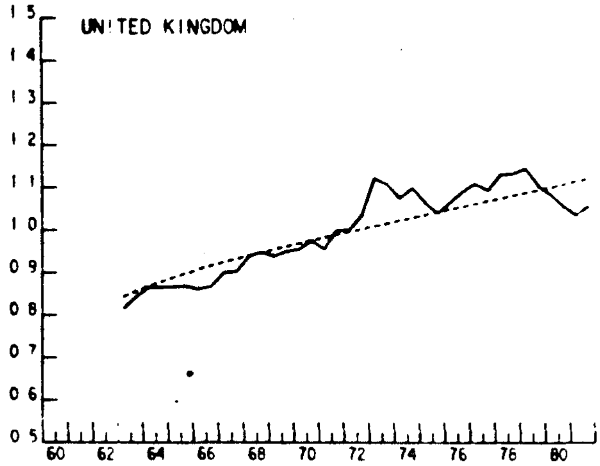
LABOUR EFFICIENCY INDEX

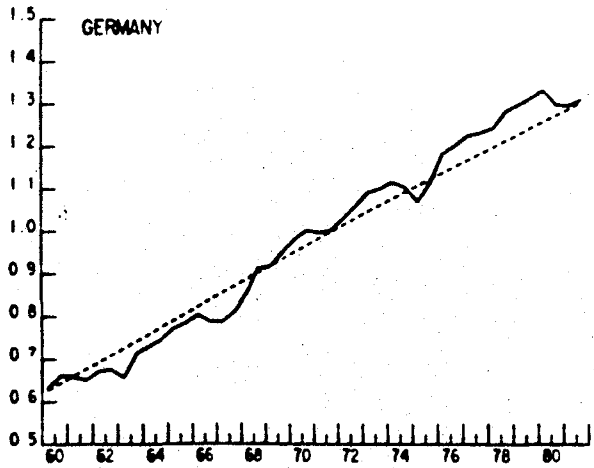
——— MEASURED

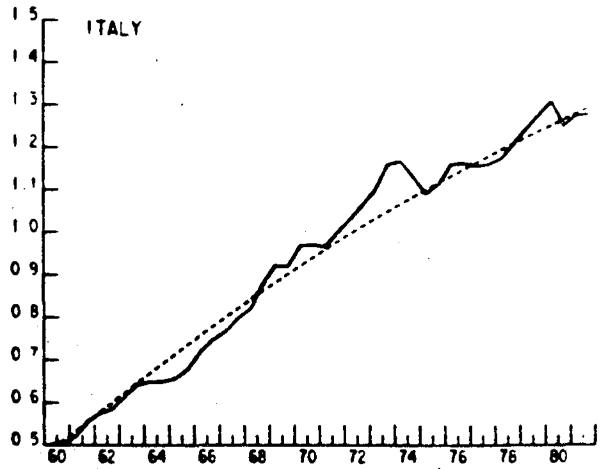
-------- MODELLED

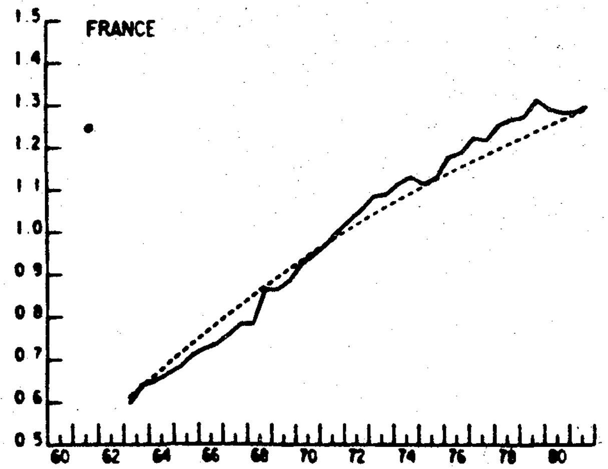


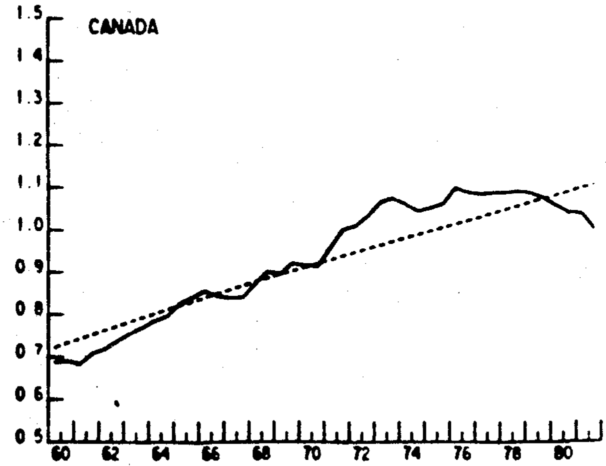












catch-up formulation (equations (8)) exceeded that of the embodiment equations (equations (9)).

39. The catch-up and the embodiment hypothesis are not mutually exclusive, and their relative explanatory power was tested by estimating the equations:

$$\ln(PIMADJ/PIMADJ(-1)) = g_0 + g_1 \ln(ELEFFUS(-1)/PIMADJ(-1)) + g_2(AIBV/KBV(-5)) + u \quad (10a)$$

$$\ln(PIM/PIM(-1)) = h_0 + h_1 \ln(ELEFFUS(-1)/PIM(-1)) + h_2 \ln(AIBV/KBV(-5)) + h_3 DIFUHAT + u \quad (10b)$$

The estimates of g_2 and h_2 were insignificant and/or had the wrong sign while the catch-up coefficient remained significant in at least one (and usually both) of the two equations with the right sign for all countries. On the basis of these results it was decided to include the catch-up formulation for the specification of labour efficiency in INTERLINK and to omit terms for embodied technical progress. This means that technical progress will remain invariant to simulated shocks to other variables. Since the main behavioural parameters are very similar with and without the catch-up effect (cf. Helliwell, Sturm and Salou, 1985), the choice between versions only makes a material difference for medium-term simulations where the difference in labour efficiency growth accumulates substantially.

IV. THE PRODUCTION DECISION

A. The theoretical framework

40. Once the parameters of the production function are determined, they can be used to obtain a series for normal or expected current output (called QBSV). This series is defined by the production function using as inputs the current values for the capital-energy bundle and for employment, measured in efficiency units. It represents the production level that would be forthcoming at average utilisation rates for employed factors. Firms are presumed to make investment and employment plans sufficient to assemble working teams adequate to meet their expectations (as of the time the plans were adopted), of what would be their target levels of output for the current period. Thus QBSV is also a measure of past expectations, partially adjusted in the meantime, of the level of profitable output in the current period.

41. The ratio of actual to normal output, QBV/QBSV, is the utilisation rate for employed factors. It is also, and equivalently, a measure of total factor productivity, after adjusting for long-term increases in labour efficiency. If primary factor inputs could be instantaneously and costlessly adjusted there would be no movement in QBV/QBSV even under conditions of uncertain demand and costs. Similarly, if future demand and cost conditions could be forecast with certainty, and if they were not subject to temporary movements, then there would be no variation in QBV/QBSV even if all of the measured factors were costly to adjust. In the model QBV/QBSV is called "IFU", since conceptually it is a measure of the intensity with which actually employed factors are used (i.e. a short-term utilisation rate).

42. Given costs of adjustment, recognition lags and uncertainty about future demand and cost conditions, it is inevitable that there are systematic variations in the utilisation rate, to an extent determined by the costs of abnormal utilisation relative to those of the other alternative adjustments to changes in demand or cost conditions: changes in imports and inventories, adjustment of other factors and foregone sales. The costs of abnormally high utilisation rates cannot be measured directly, since they only eventually show up as higher repair expenditures, breakdowns, accidents or inadequate planning for future projects. However, it can be presumed that the normal utilisation rate is deliberately chosen to minimise expected costs. When deciding, in any particular circumstances, to what extent to respond to an unexpected change in demand or cost conditions, the immediate binary choice facing firms (taking as given, for the moment, the related adjustments of prices, imports, investment and employment) is whether to meet any change in demand by a change in production or a change in inventories. The key factors influencing the choice are sales (representing a shift in the demand function, at given prices), profitability (represented inversely by the the ratio of unit costs to the output price) and the ratio of actual to desired inventories, all entering multiplicatively. Thus, the extent to which, e.g., any increase in demand will be satisfied by increased production is influenced by the cost ratio and the adequacy of the current stock of inventories. The cost variable captures to some extent the marginal costs of holding inventories as well as the frequency of firms operating at reduced capacity, or even suspending operations, because of low current profit rates. There is more to be done in separating and specifying the channels by which aggregate costs and profits influence the production and investment decision. For example, the ratio of current to long-run costs (after factor proportions have been optimally adjusted) might have a different impact, especially on investment, than would the ratio of long-run costs to the output price.

43. How does this framework compare with others used for explaining interrelated factor demands? There is a growing literature on investment and the demand for capital under conditions of uncertainty combined with adjustment costs. Under most assumptions about the nature of the future demand and cost uncertainty and about the nature of the costs of adjustments, the theoretical results indicate an increased stock demand for the fixed factor and more variation in the demand for the variable factor (cf. Pindyck (1982) and Abel (1983, 1984)). The models referred to treat only capital as being subject to adjustment costs, and labour is adjusted freely to permit the underlying production function to hold exactly. In the framework presented here, both labour and the capital-energy bundle are quasi-fixed factors, and there is an additional factor, the utilisation rate, which is free to vary in the short-term but approaches its normal cost-minimising value on average in the longer-run. The result is that uncertainty and costly adjustment combine to ensure that changes in the utilisation rate provide for a large proportion of short-term changes in output. Earlier uses of general factor utilisation as a separate factor of production include the Canadian RDX2 Model (Helliwell et. al., 1971) and the interrelated factor demand studies of Nadiri and Rosen (1973). The framework presented here is like that of RDX2 in ensuring that the utilisation and factor demand decisions are mutually consistent and also satisfy the constraints of the underlying production function.

B. Empirical estimates of the utilisation rate equation

44. The supply equation was specified in log-linear form with the factor utilisation rate as the dependent variable:

$$\ln(QBV/QBSV) = a_0 + a_1\ln(CQB) + a_2\ln((SALES-a_4*(MGSV-MESV))/QBSV) + a_3\ln(STOCKV(-1)/QBSV) + u \quad (11)$$

where SALES represents final sales (final domestic demand plus exports), STOCKV represents the end-of-period level of inventories, MGSV-MESV is real non-energy imports of goods and services and CQB is the ratio of actual unit costs to output price (17)(18). Equations for all countries were estimated using two-stage least squares on semi-annual data, over a sample period that starts in the second semester of 1960, or as soon as possible thereafter, and ends in the second semester of 1982. All of the equations estimated embody the catch-up model results (but no embodiment term) for the labour efficiency index as reported in column 2 of Table 3. Results reported in Table 4 are based on the production structure parameters reported in Table 1, featuring an elasticity of substitution below unity for most countries.

45. Unconstrained estimation of the output equation produces highly significant coefficient estimates (of correct sign) for the cost and sales variables. When any of latter were imposed prior to estimation it was either due to convergence or other problems in simulation. The import share subtracted fell strictly within the zero-one range for only three countries in free estimation. For the United States and Canada nominal values of 0.25 for a_4 and for Japan and Italy maximum values of unity were imposed prior to final estimation. The inventory coefficient, however, is insignificant in three or four out of the seven equations and had a wrong (positive) sign in the case of the United States and Italy. For these countries, the output equation was therefore re-estimated, imposing a coefficient of -0.05 on the inventory variable, corresponding to the size of that coefficient from a pooled regression including all countries. The closeness of fit of the utilisation rate equation is depicted in Chart 3; in general, the fit is quite tight, supporting the behavioural hypotheses described above. The United Kingdom model is the least satisfactory, probably because of the imposition of a_2=.7 which was made necessary in order for the model to converge in simulation (the freely estimated parameter was about 0.84).

V. DERIVED DEMAND EQUATIONS FOR CAPITAL, LABOUR AND ENERGY

46. In keeping with the view that labour and capital are both relatively fixed factors of production, comparable derived demand equations are specified for both of them. Starting with demand for capital and simply put, the desired capital stock is defined by

$$KBSTAR = QSTAR \cdot KQBSTAR \quad (12)$$

where QSTAR is the desired future output level and KQBSTAR is the expected cost-minimising ratio of capital to normal output QBSV, derived from the underlying production structure and relative expected input prices. More exactly, 2447E

Table 4

OUTPUT SUPPLY EQUATIONS

$$\ln(QBV/QBSV) = a_0 + a_1 \ln(CQB) + a_2 \ln((SALES - a_4(MGSV\text{-}MESV))/QBSV) + a_3 \ln(STOCKV(-1)/QBSV)$$

Non-linear estimation: 2SLS

	Estimated coefficients (t-statistics)						Regression statistics		
	a_0	a_1	a_2	a_3	a_4	Sample	R^2	DW	SEE
United States	-0.08 (-94.0)	-0.30 (-20.0)	0.96 (i)	-0.05 (i)	0.25 (i)	60S2-82S2	0.98	0.9	0.0055
Japan	-0.21 (-3.4)	-0.25 (-10.3)	0.75 (i)	-0.20 (-3.9)	1.00 (i)	66S1-82S2	0.96	0.7	0.0096
Germany	-0.14 (-1.1)	-0.15 (-1.6)	0.93 (7.1)	-0.14 (-1.3)	0.93 (9.3)	60S2-82S2	0.84	0.9	0.0112
France	-0.12 (-4.0)	-0.24 (-9.0)	0.83 (13.2)	-0.04 (-4.4)	0.40 (3.0)	63S2-82S2	0.94	1.1	0.0060
United Kingdom	-0.15 (-1.5)	-0.13 (-2.4)	0.70 (i)	-0.09 (-0.9)	0.44 (2.8)	63S2-82S2	0.76	0.7	0.0144
Italy*	-0.13 (-22.9)	-0.11 (-4.9)	0.71 (13.4)	-0.05 (i)	1.00 (i)	61S1-82S2	0.85	0.6	0.0123
Canada	-0.26 (-6.2)	-0.32 (-11.2)	0.64 (27.7)	-0.12 (-3.6)	0.25 (i)	60S2-82S2	0.96	1.3	0.0077

(i) Coefficient imposed.

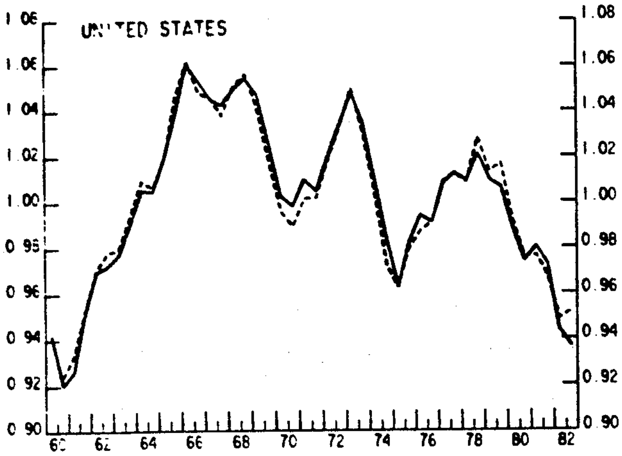
* The Italian equation was actually estimated and these are the results which are given for a_4=0.25; it was only at the last minute that a_4=1 was imposed, but no further estimation was subsequently undertaken.

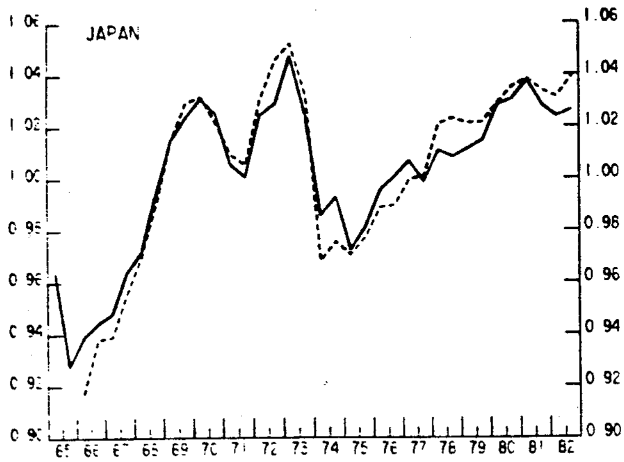
Chart 3

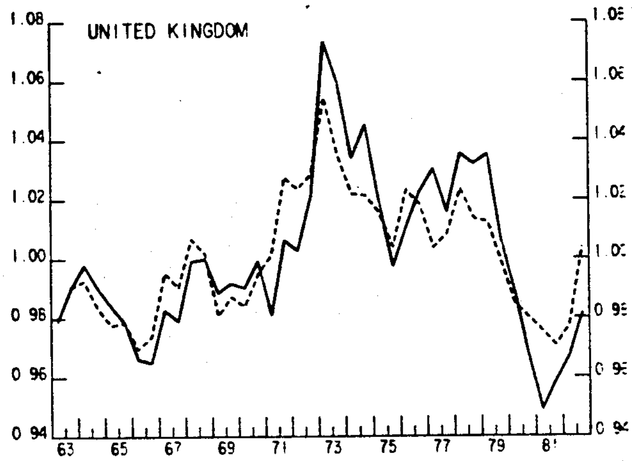
INTENSITY OF FACTOR UTILISATION EQUATION

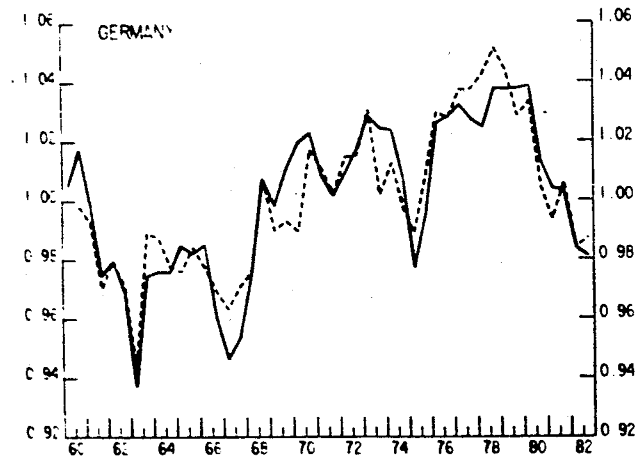
——— ACTUAL

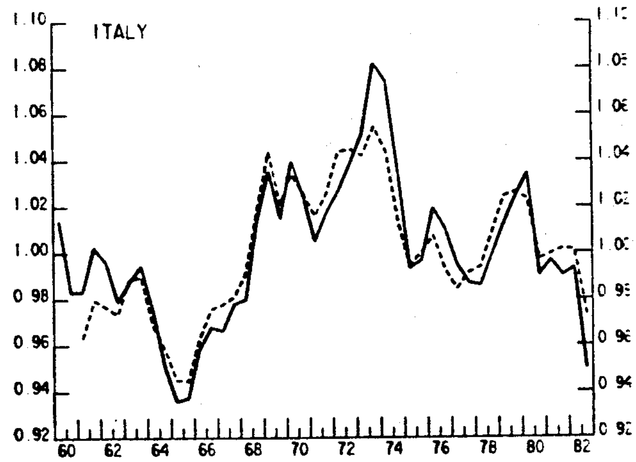
-------- PREDICTED

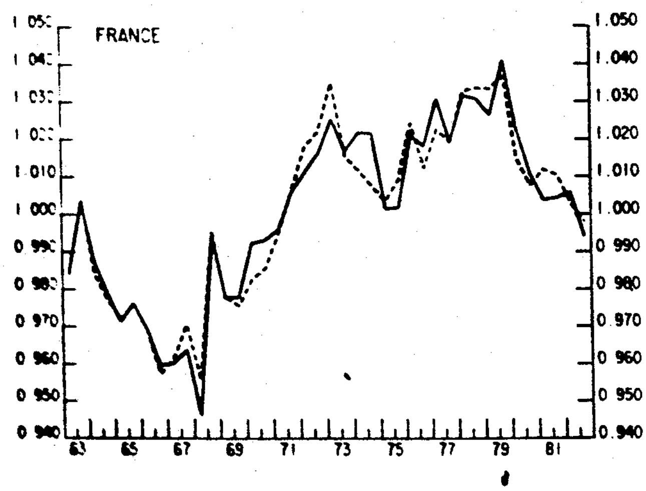


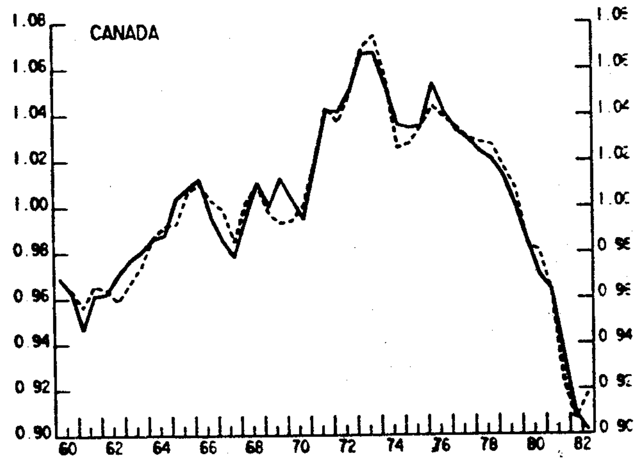












$$KBSTAR = ((XIBETA + XIGAMA. (XIGAMA/XIBETA)^{s-1} . (PENB/UCC)^{1-s})^{s/1-s}) . QSTAR . ((XOGAMA + (XOBETA^{XTAU} . ((XOGAMA . (WSSE/ELEFF)/(XIBETA^{s} . (UCC^{1-s}) + (XIGAMA^{s} . (PENB^{1-s}) ^{1/1-s}))) ^{1-XTAU}))) ^{XTAU/1-XTAU})$$

47. The expectation formation mechanism generating QSTAR is a modified adaptive expectations process:

$$QSTAR = QBV^{W} . QBSV^{(1-W)} . LFG^{4}.(PQB/CKEL)^{.3} \tag{14}$$

Where: QSTAR is the expected future output relevant for planning input quantities;
W is the weight attached to actual output;
LFG is the growth rate of the labour force in efficiency units over the past five years; and
CKEL is the cost dual coming out of the production structure.

The inclusion of the ratio PQB/CKEL in the definition of expected profitable future (or "desired") output strengthens the response of aggregate supply to profitability: the higher the sales price relative to normal cost, the larger desired output will be. W is a parameter in the model. Its empirical value was chosen with a view to minimizing the standard error of the corresponding factor demand equations as well as to giving reasonable simulation properties. The final values of W are given below:

United States 1.8	Japan 1.25	Germany 1.0	France 1.0
United Kingdom 1.0	Italy 1.0	Canada 1.5.	

It should be pointed out that the QSTAR equation can be rewritten with a term in IFU to the power W-1 replacing the QBSV term to the power 1-W. Therefore values of W in excess of unity just mean that values of IFU in excess of unity raise output expectations (and thereby factor demands) given the current level of QBV. However, such values do mean that output expectations do not return to previously anticipated normal output in level terms.

48. For the investment function a dynamic adjustment specification was used which assumes that actual factor quantities adjust gradually to desired factor inputs. This specification has the attractive property that the equality of actual to desired input can be imposed as a long-run equilibrium condition, while the precise short-run adjustment dynamics are determined by the observed (disequilibrium) data. Additional variables can easily be added to either modify the imposed steady state equilibrium or to influence the dynamic adjustment path. For investment demand the hypothesis that the speed of adjustment is influenced by the unit cost or profitability variable and/or the prevailing factor utilisation rate was tested which led to the following investment equation:

$$\begin{aligned} \ln(IBV/IBV(-1)) = a_0 &+ a_1 \ln(KBSTAR/KBSTAR(-1)) \\ &+ a_2 \ln(KBSTAR(-1)/IBV(-1)) \\ &+ a_3 \ln(KBSTAR(-1)/KBV(-1)) \\ &+ a_4\, PROFR \\ &+ a_5 \ln(IFU) + u \end{aligned} \tag{15}$$

where KBV represents the business gross fixed capital stock, and PROFR is a transformation of the unit cost indicator CQB into a capital profitability measure such that its sample mean is zero, i.e. PROFR , =, 0 depending on whether the gross operating surplus per unit of gross fixed capital is bigger, equal to, or smaller than its average over the sample period (19). Specification (15) is, intentionally, consistent with the increasing theoretical and applied literature suggesting that Tobin's q theory of investment is especially likely to be applicable under conditions of uncertainty combined with costly factor adjustment (20).

49. The investment equation presented above implies that when profitability is at its normal value (and thus PROFR equals zero) the capital stock will converge to the values sufficient to produce QSTAR at normal rates of capacity utilisation. When profitability is abnormally low, the capital stock converges to values that would require abnormally high utilisation rates to produce QSTAR. This would not in general happen, of course, because the preferred utilisation rate is below 1.0 when CQB is above 1.0, given equality between actual and expected sales and no discrepancy between actual and desired inventory levels. What this particular factor demand specification in fact does is to add CQB as a modifier of the desired factor input.

50. Estimation results for the investment demand equations are presented in Table 5, and graphs of estimated vs. actual growth rates of business gross fixed investment are displayed in Chart 4. Unfortunately, in no case was the coefficient estimate for a_3 correctly signed and the integral adjustment term was therefore omitted. Thus, the model represents an error correction specification where the adjustment is one of investment towards desired capital stock. Since this stock is related to expected output the model still implies a constant capital-expected output ratio in the long run under general assumptions. The IFU effect was present in free estimation in all countries except France and Canada although significant only for Germany and Italy. The profitability hypothesis receives support in four cases and the effect was imposed in the United Kingdom and Canadian equations; for the United States a_4 was competitive with a_5, and the latter was judged more important. The tracking performance of the equations for Canada and the United Kingdom is the least satisfactory, probably owing to the fact that in those two countries energy investment has played an important role. Remember that the investment series have not yet been properly adjusted to remove energy investment from total investment, as required by the logic of the supply block structure.

51. The labour demand function can then be specified as an error correction equation (21):

$$\begin{aligned} LN(ETB/ETB(-1)) = a_0 &+ a_1 \ln(EBSTAR/EBSTAR(-1)) \\ &+ a_2 \ln(EBSTAR(-1)/ETB(-1)) \qquad (16) \\ &+ a_3 \ln(CQB) \\ &+ a_4 \ln(IFU) + u \end{aligned}$$

EBSTAR, or desired employment, is defined as the number of workers required to produce the expected future profitable output QSTAR with the desired capital stock KBSTAR and the corresponding energy input. It is calculated by inverting the aggregate production function:

$$EBSTAR = (((QSTAR^{XTAU-1/XTAU} - (XOGAMA.((QSTAR.(XOGAMA + (XOBETA^{XTAU} . (XOGAMA. ((WSSE/ELEFF)/CKE))^{1-XTAU}))^{XTAU/1-XTAU}))^{XTAU-1/XTAU}))) /XOBETA)^{XTAU/XTAU-1})/ELEFF$$

52. Estimation results for the employment equations are presented in Table 6, and graphs of estimated vs. actual employment growth rates are displayed in Chart 5. The integral adjustment parameter had to be imposed for Italy, and its estimate is disconcertingly small for Canada. Profitability effects are identified only for Germany and the United Kingdom. IFU effects are found for all countries except Japan and Italy; in the latter a non zero parameter was imposed for overall model properties. The apparent weak performance of the equation for Italy is mainly due to the failure to capture some of the large semi-annual employment growth fluctuations, especially during the first half of the sixties. These large periodic fluctuations look peculiar, however, and may be due to deficient seasonal adjustment procedures.

53. The means and variances of the dependent variables for the factor demand equations for each country are shown in Table 7. One striking feature of this table is the low variance of employment change in Japan. This suggests that in the Japanese case the labour market may be flexible enough for employment to be determined by slowly moving demographic factors, with variations in output and profitability leading to redeployment and changes in the utilisation rate rather than to changes in employment.

54. The basic structure of the business energy demand (ENBV) equation is different from the one chosen for the employment and investment demand equations since energy inputs can be adjusted to optimal levels without delay: the vintage energy requirement (EBSV) is given by equation (3) in section III.C above. This is the optimal energy input subject to the existing and partially retrofitted vintage capital stock. Although there are no adjustment lags in the demand for energy, actual energy demand (ENBV) may deviate from "normal" energy requirements because of abnormal factor utilisation rates. This leads to the following energy demand equation:

$$\ln(ENBV/ENBV(-1)) = a_0 + a_1 \ln(EBSV/EBSV(-1)) + a_2 \ln(IFU/IFU(-1)) + u \qquad (17)$$

55. In all countries outside North America a weak negative constant term implies some autonomous restraint on energy demand growth, probably reflecting energy saving technical progress or the effects of administrative (non-price) measures to save energy. Estimation results for the energy demand equations are presented in Table 8, and graphs of estiamated vs. actual growth rates of energy demand are displayed Chart 6. The coefficient (a_1) was constrained to unity for all countries, forcing the instantaneous response of actual energy demand to an increase in vintage energy requirements to be proportional.

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Table 5
BUSINESS FIXED INVESTMENT EQUATIONS

$$\ln(IBV/IBV(-1)) = a_0 + a_1 \ln(KBSTAR/KBSTAR(-1)) + a_2 \ln(KBSTAR(-1)/IBV(-1)) + a_3 \ln(KBSTAR(-1)/KBV(-1)) + a_4\, PROFR + a_5 \ln(IFU) + u$$

	Estimated coefficients (t-statistics)							Regression statistics		
	a_0	a_1	a_2	a_3	a_4	a_5	Sample	R^2	DW	SEE
United States	-0.26 (-50.2)	0.74 (3.1)	0.10 (i)	-	-	0.15 (0.8)	66S1-82S2	0.64	0.8	0.026
Japan	-0.56 (-5.1)	0.19 (0.8)	0.28 (5.4)	-	0.97 (7.1)	0.16 (0.7)	66S2-82S2	0.75	1.1	0.029
Germany	-	0.26 (0.6)	4E-3 (1.3)	-	1.14 (2.7)	0.73 (2.6)	66S1-82S2	0.53	1.4	0.035
France	-0.73 (-3.2)	1.16 (3.5)	0.30 (3.2)	-	0.33 (2.0)	-	66S1-82S2	0.49	1.5	0.026
United Kingdom	-1.44 (-2.7)	0.94 (3.4)	0.50 (2.7)	-	0.30 (i)	0.02 (0.1)	67S1-82S2	0.49	1.7	0.028
Italy	-0.60 (-3.5)	0.20 (0.9)	0.25 (3.6)	-	0.73 (4.0)	0.49 (1.8)	67S1-82S2	0.62	2.0	0.033
Canada	-	0.27 (1.2)	0.01 (2.0)	-	0.30 (i)	-	66S1-82S2	0.14	1.1	0.039

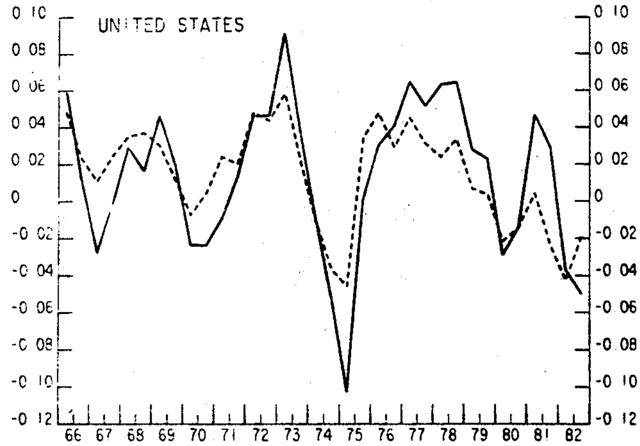
(i) Coefficient imposed.

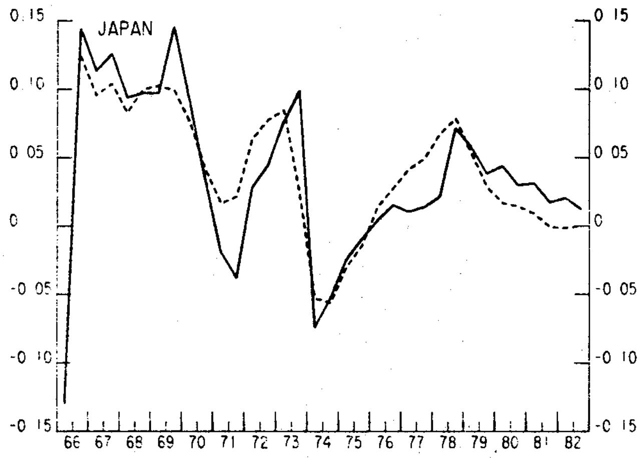
Chart 4

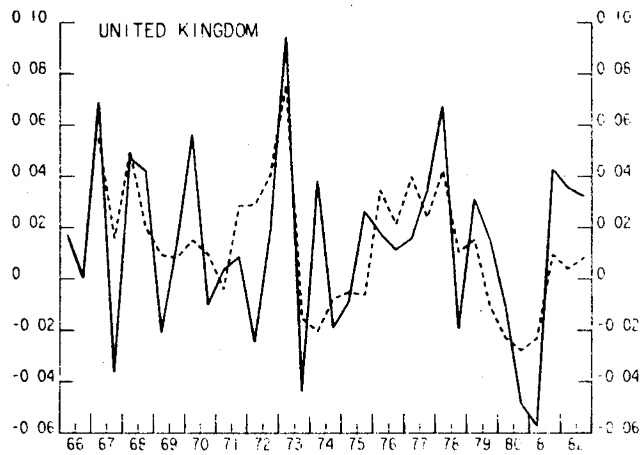
BUSINESS FIXED INVESTMENT (GROWTH RATE)

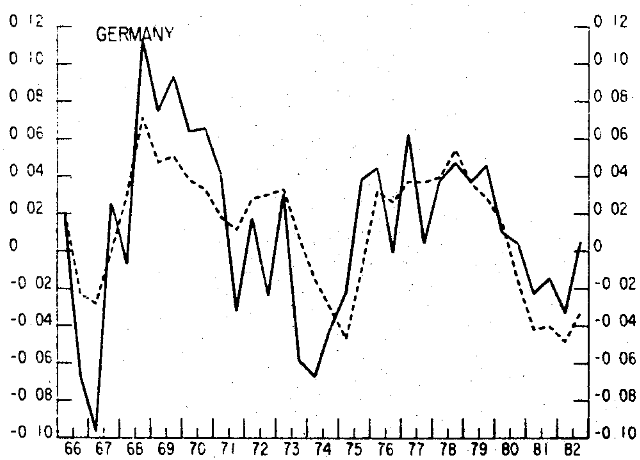
——— ACTUAL

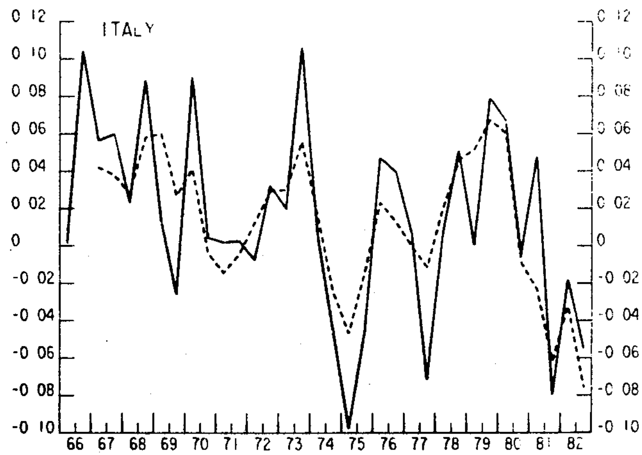
-------- PREDICTED

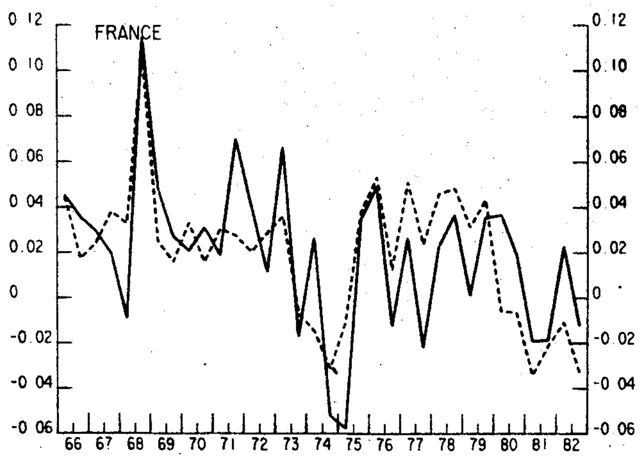












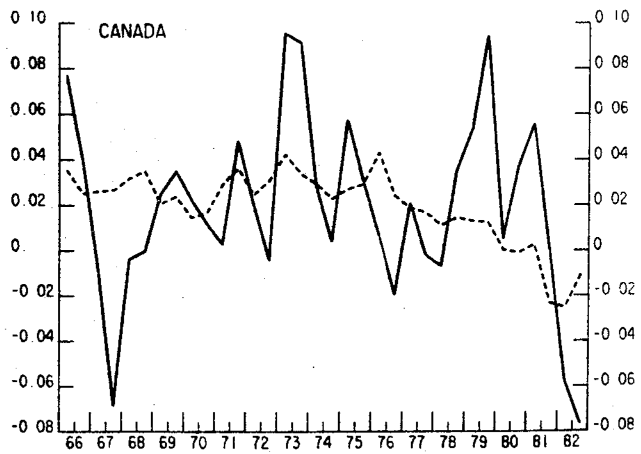


Table 6

BUSINESS EMPLOYMENT EQUATIONS

$$\ln(ETB/ETB(-1)) = a_0 + a_1 \ln(EBSTAR/EBSTAR(-1)) + a_2 \ln(EBSTAR(-1)/ETB(-1)) + a_3 \ln(CQB) + a_4 \ln(IFU) + u$$

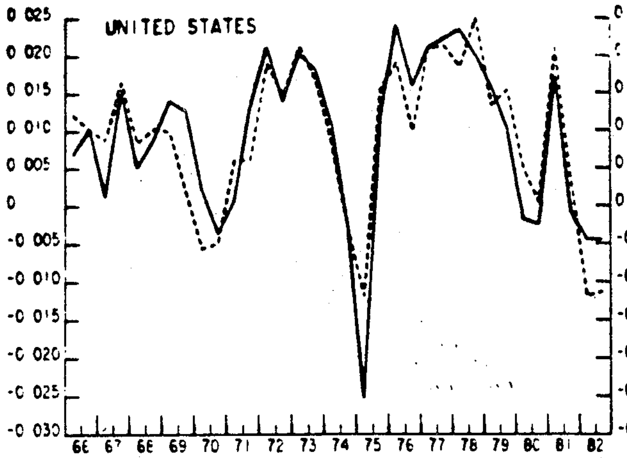
	Estimated coefficients (t-statistics)						Regression statistics		
	a_0	a_1	a_2	a_3	a_4	Sample	R^2	DW	SEE
United States	-0.01 (-2.8)	0.60 (7.7)	0.20 (3.5)	-	0.09 (2.6)	66S1-82S2	0.77	1.4	0.0055
Japan	-	0.28 (5.2)	0.02 (4.2)	-	-	66S2-82S2	0.38	2.1	0.0049
Germany	-0.01 (-4.4)	0.23 (4.3)	0.13 (3.6)	-0.15 (-6.5)	0.15 (5.3)	66S1-82S2	0.89	1.6	0.0031
France	-0.01 (-3.6)	0.29 (5.2)	0.14 (3.9)	-	0.06 (2.1)	66S1-82S2	0.54	1.8	0.0033
United Kingdom	-0.02 (-4.0)	0.23 (3.9)	0.23 (3.2)	-0.04 (-3.4)	0.06 (1.1)	67S1-82S2	0.80	2.0	0.0045
Italy	-0.1 (7.0)	0.29 (2.8)	0.10 (i)	-	0.10 (i)	67S1-82S2	0.25	1.5	0.0089
Canada	0.00 (0.5)	0.40 (5.0)	0.01 (0.1)	-	0.15 (1.4)	66S1-82S2	0.74	1.2	0.0080

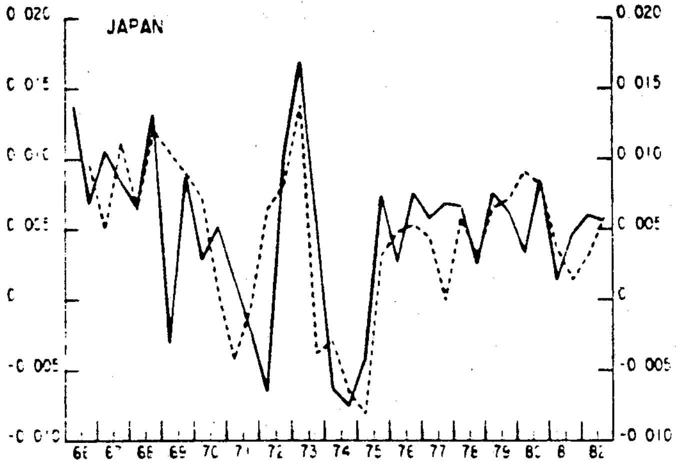
<u>Chart 5</u>

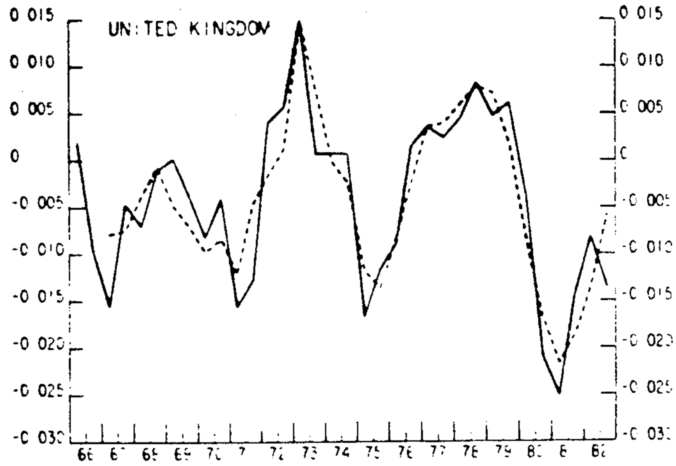
BUSINESS EMPLOYMENT (GROWTH RATE)

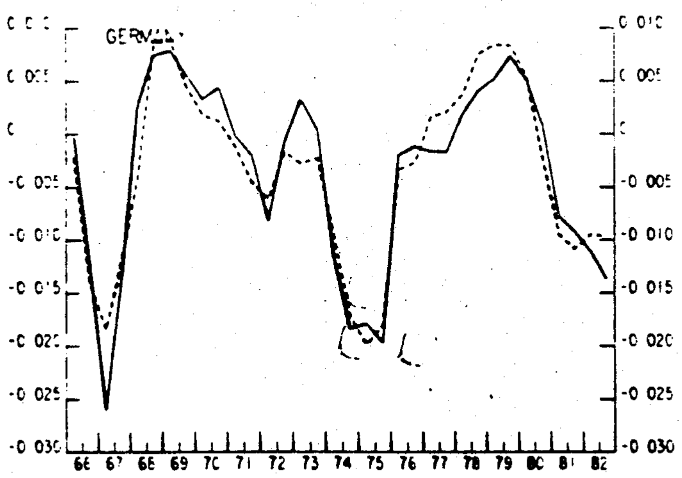
——— ACTUAL

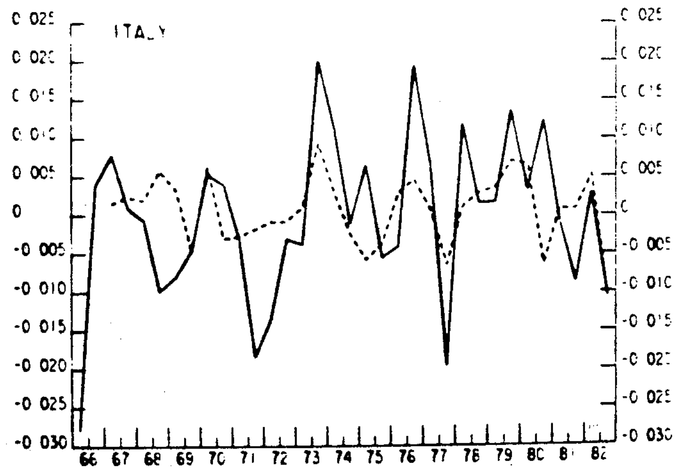
-------- PREDICTED

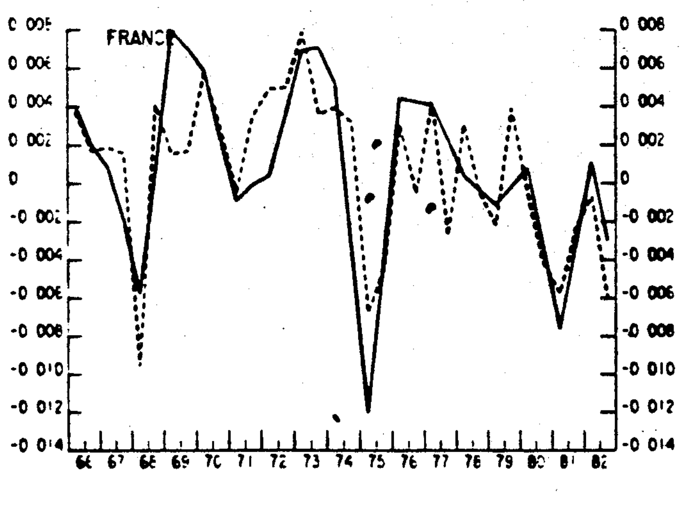












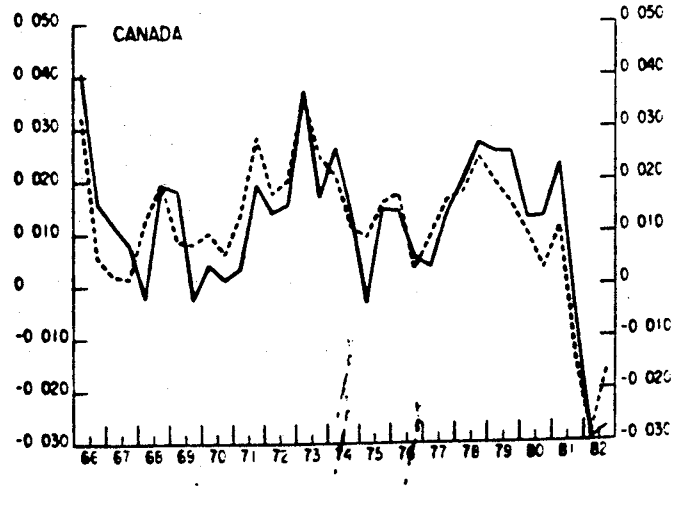


Table 7

MOMENTS OF CHANGES IN FACTOR INPUT VARIABLES

Country	Sample Period	LN(ETB/ETB(-1)) Mean	LN(ETB/ETB(-1)) Standard deviation	LN(IBV/IBV(-1)) Mean	LN(IBV/IBV(-1)) Standard deviation	LN(ENBV/ENBV(-1)) Mean	LN(ENBV/ENBV(-1)) Standard deviation
United States	S2.60-S2.82	0.0084	0.0099	0.0195	0.0410	0.0101	0.0302
Japan	S2.66-S2.82	0.0050	0.0057	0.0465	0.0870	0.0205	0.0500
Germany	S2.60-S2.82	-0.0025	0.0082	0.0148	0.0455	0.0111	0.0308
France	S2.63-S2.82	0.0008	0.0042	0.0214	0.0331	0.0132	0.0351
United Kingdom	S2.60-S2.82	-0.0032	0.0089	0.0165	0.0388	-0.0011	0.0291
Italy	S2.60-S2.82	-0.0011	0.0110	0.0130	0.0543	0.0165	0.0379
Canada	S2.60-S2.82	0.0115	0.0135	0.0229	0.0426	0.0541	0.0428

Table 8

BUSINESS ENERGY DEMAND EQUATIONS

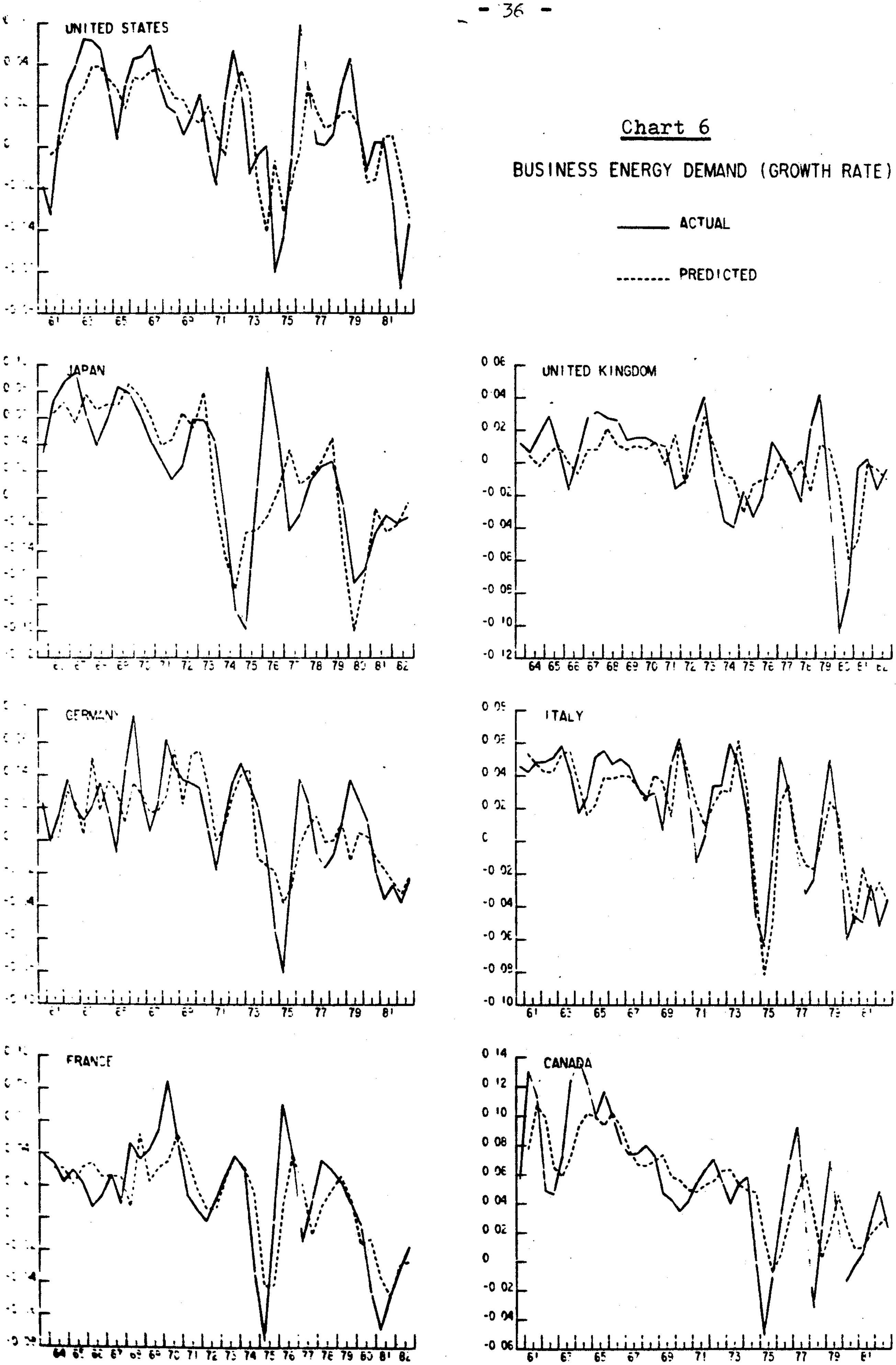
$$\ln(ENBV/ENBV(-1))=a_0 + a_1 \ln(EBSV/EBSV(-1))+a_2 \ln(IFU/IFU(-1))+u$$

Country	Estimation Period	Estimated Coefficients (t-statistics) a_0	a_1 (imposed)	a_2	Regression Statistics R^2	SEE	DW	RHO
USA	1960S2 1982S2	0.0009 (0.1)	1	0.19 (0.5)	0.35	0.023	1.7	0.5 (4.6)
JAP	1965S2 1982S2	-0.0025 (-0.3)	1	0.49 (1.1)	0.16	0.034	1.5	0.34 (2.2)
GER	1960S2 1982S2	-0.0068 (-1.2)	1	0.52 (2.6)	0.31	0.022	1.6	0.43 (3.3)
FRA	1963S2 1982S2	-0.0031 (-0.4)	1	0.17 (0.7)	0.27	0.024	1.6	0.49 (3.4)
UKM	1963S1 1982S2	-0.0145 (-1.9)	1	0.32 (1.4)	0.32	0.025	1.6	0.47 (3.3)
ITA	1960S2 1982S2	0.0072 (-1.3)	1	0.15 (1.1)	0.32	0.019	1.7	0.51 (4.0)
CAN	1960S2 1982S2	0.0083 (1.0)	1	0 (0.0)	0.23	0.029	1.3	0.46 (3.6)

a) Coefficient imposed.

Chart 6

BUSINESS ENERGY DEMAND (GROWTH RATE)



VI. LINKING THE SUPPLY BLOCK TO PRICE DETERMINATION

56. The production structure outlined in the preceding section has been linked to the price formation process in INTERLINK to complete the supply side of the model. The unit price of gross business output corresponding to the level of aggregation in the supply structure (gross value added plus intermediate energy inputs) is the aggregate output deflator PQB. Actual output prices (including supra-normal mark-ups, positive or negative) will depend inter alia on costs of inputs and prices of foreign competitors, approximated by the country's unit import prices. The latter influences the cost mark-up that producers are able to charge. The disaggregated import price series currently in INTERLINK have therefore been reweighted according to the industrial structure of the individual importing country, since the commodity structure of imports may differ substantially from that of domestic output (witness Japan). The resulting series is PMQ.

A cost index CKEL is computed from the dual to the aggregate production function, i.e.

$$CKEL = (XOBETA^{XTAU}.(WSSE/ELEFF)^{(1-XTAU)}+XOGAMA^{XTAU} . CKE^{(1-XTAU)})^{(1/(1-XTAU))} \quad (18)$$

where CKE is the cost index of the capital-energy bundle, computed from the dual to the inner (marginal) CES function which aggregates capital and energy into the increment to the input bundle KEBSV:

$$CKE = (XIBETA^{s}.UCC^{(1-s)}+XIGAMA^{s} .PENB^{(1-s)})^{(1/(1-s)} \quad (19)$$

An additional domestic cost measure which does not assume that prices are set based on full adjustment of factor inputs to changes in relative factor prices is also included. It is defined as:

$$COST = (WSSE.ETB+UCC.(KBV+KBV(-1))/2+PENB.ENBV)/QBSV \quad (20)$$

where WSSE is private sector compensation per employee.

57. In addition, cyclical effects from inventory disequilibrium as well as from changing rates of factor utilisation were assumed to influence the price formation process. However, proxying the former by the ratio of actual inventories to the product of normal output and the mean of the historical average stock-output ratio led to very little success. Thus, the effects of aggregate demand on prices are limited to the direct IFU channel. Long-run homogeneity was imposed ex ante with respect to costs and import prices and, where necessary, dummy variables and time trends were included in the estimation but not in the model code. Rather free estimation of dynamics was allowed. Therefore, the general form of the equation was:

$$\ln(PQB_t) = a_0 + \sum_{i=1}^{b} a_{1i}\ln(PQB_{t-i}) + \sum_{j=0}^{c} a_{2j}\ln(CKEL_{t-j}) +$$

$$\sum_{k=0}^{d} a_{3k}\ln(COST_{t-k}) + \sum_{l=0}^{e} a_{4l}\ln(PMQ_{t-l}) + \sum_{m=0}^{f} a_{5m}\ln(IFU_{t-m}) + u_t \quad (21)$$

where $\sum_{i=1}^{b} a_{1i} + \sum_{j=0}^{c} a_{2j} + \sum_{k=0}^{d} a_{3k} + \sum_{l=0}^{e} a_{4l} = 0$

In practice, b = 2, c = d = e = 4, f = 2. All insignificant parameters were eliminated. Estimation results are given in Table 9, long-run elasticities in Table 10 and graphs of estimated vs. actual growth rates of PQB in Chart 7.

58. In general, the fits are fairly good. The averge elasticity of prices with respect to domestic costs is 0.73 in the long run, while the remaining 0.27 emanates from import prices. Import price elasticities vary from 0.13 for France and 0.16 for the United States to 0.50 in Japan. The Japanese result is somewhat strange, but it should be noted that the mean lag is in excess of 21 semesters and that the short- and medium-term import price feed-through is relatively small. The COST variable tends to dominate in Japan, France, Italy and Canada, while CKEL dominates for the other three countries. Significant IFU effects are in evidence for the United States, Germany, France and Canada. Parameter impositions had to be effected for Japan and the United Kingdom; a large value was chosen for the latter in order for the model to generate reasonable overall simulation properties: not much inflation is generated by the United Kingdom wage equations.

59. The business output price PQB and the business value added deflator (at factor cost, PGDPB) are connected by the following identity:

PGDPB = (PQB-PENB.ENBV/QBV)/(GDPBV/QBV) (22)

The GDP deflator can in turn be computed from the business value added deflator by identity:

PGDP = (PGDPB.GDPBV+TIND-TSUB+CGW)/GDPV (23)

where CGW is nominal government expenditure on wages and salaries. Individual demand component deflators were computed as weighted averages of the business value-added deflator, import price deflators and net indirect taxes, using aggregation weights roughly corresponding to the appropriate input-output coefficients. The weights have been imposed, and no deflator-specific IFU terms have been included, but work is underway to remedy these deficiencies using mixed regression techniques.

Table 9

BUSINESS OUTPUT DEFLATOR EQUATION

$$\ln(PQB_t) = a_0 + \sum_{i=1}^{b} a_{1i} \ln(PQB_{t-i}) + \sum_{j=0}^{c} a_{2j} \ln(CKEL_{t-j}) +$$

$$\sum_{k=0}^{d} a_{3k} \ln(COST_{t-k}) + \sum_{l=0}^{e} a_{4l} \ln(PMQ_{t-l}) + \sum_{m=0}^{f} a_{5m} \ln(IFU_{t-m}) + u_t$$

	USA[1]	JAP[2]	GER[3]	FRA[4]	UKM[5]	ITA	CAN[6]
a_0	.0069 (4.64)			.0236 (7.45)			
a_{11}	.7960 (22.89)	.9550 (i)	1.0216 (8.98)	.5446 (5.75)	1.1999 (16.18)	.7691 (12.17)	1.0642 (9.16)
a_{12}			-.3019 (3.84)	.3089 (3.10)	-.3094 (4.58)		-.2903 (2.89)
a_{20}	.5108 (i)		.2230 (i)		.3297 (i)	.7396 (6.59)	
a_{21}	-.3398 (4.08)			.7367 (1.82)		-.7396 (6.59)	-.4201 (4.70)
a_{22}				-.3773 (9.46)	-.2484 (4.80)		.1443 (3.40)
a_{23}				-1.0832 (3.04)			
a_{24}						-.2692 (3.45)	
a_{30}		.6232 (9.97)		.5053 (i)			.4201 (4.70)
a_{31}		-.3971 (3.25)		-.7367 (1.82)			
a_{32}		-.3113 (2.74)					

Table 9 (continued)

	USA[1]	JAP[2]	GER[3]	FRA[4]	UKM[5]	ITA	CAN[6]
a_{33}		.1921 (1.75)		1.0832 (3.04)		.3580 (3.69)	
a_{34}		-.0844 (1.45)					
a_{40}	.0330 (7.82)						.0818 (i)
a_{41}			.0573 (8.92)			.1421 (i)	
a_{42}					.0283 (2.03)		
a_{43}		.0225 (1.45)					
a_{44}				.0184 (1.13)			
a_{50}							
a_{51}		.02 (i)	.1320 (2.72)	.0972 (1.96)	.14 (i)	.1293 (1.26)	.1367 (3.05)
a_{52}	.1109 (3.95)						
RSQ	.9999	.9995	.9998	.9999	.9999	.9997	.9998
SEE	.0029	.0077	.0045	.0036	.0078	.0118	.0069
DW/h							
Sample	63S1-82S2	67S1-82S2	64S2-82S2	66S1-82S2	65S1-82S1	63S2-82S2	63S1-82S2

(i) Imposed

1. Also includes a dummy variable equal to unity for 66S1-69S2 and 76S1-78S1 as well as a high-order term in time.
2. Also includes a dummy variable equal to unity for 74S2.
3. Also includes a dummy variable equal to unity for 70S2 and minus unity for 71S1.
4. Also includes a dummy variable equal to unity for 74S1 and minus unity for 75S1.
5. Also includes dummy variables equal to unity for 71S1, 73S1, 73S2, 79S2 and 80S1.
6. Also includes dummy variables equal to unity for 75S2 and for 76S1-78S1.

Table 10

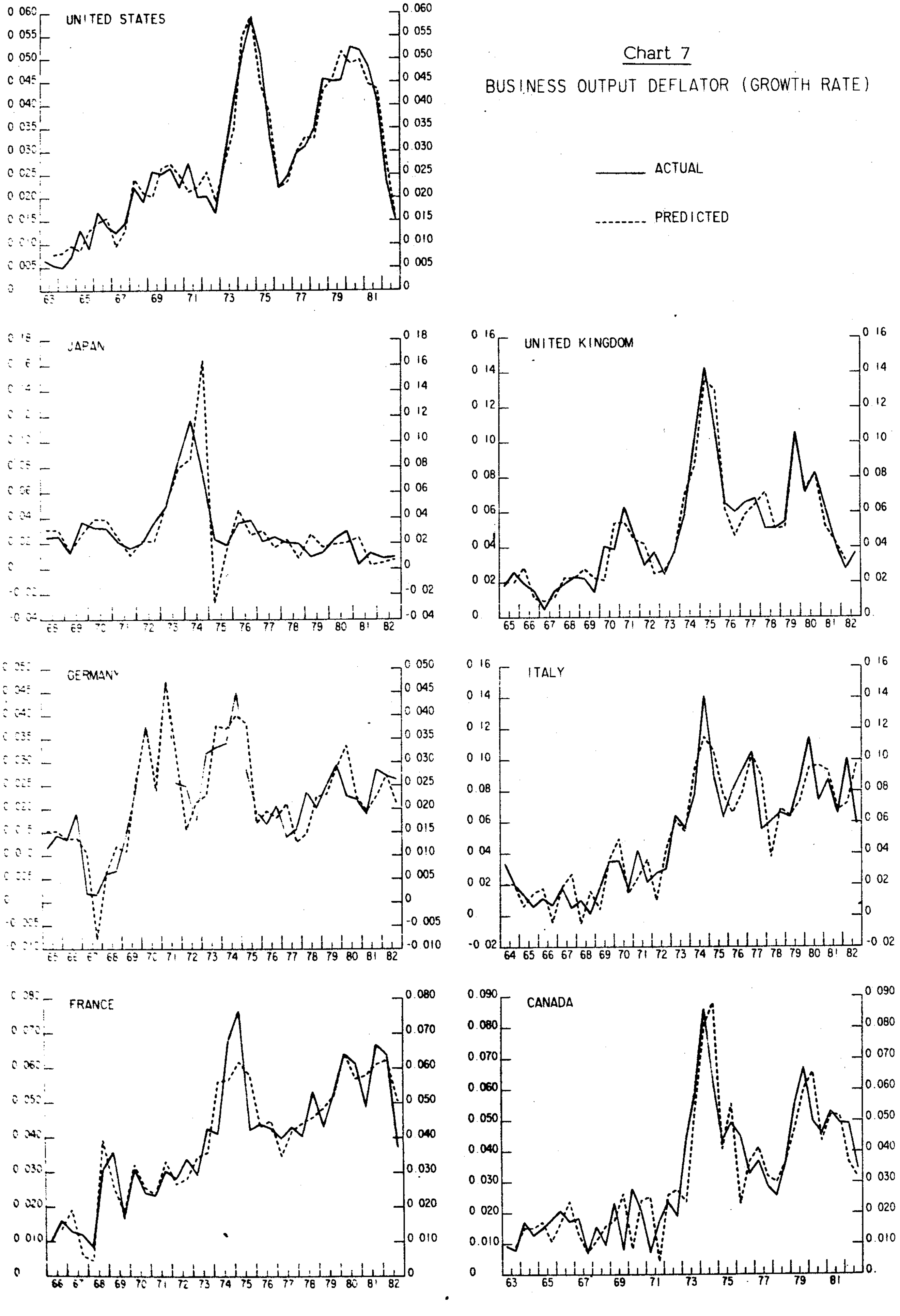
LONG-RUN PQB ELASTICITIES

United States			**United Kingdom**		
CKEL	0.84		CKEL	0.74	
COST	0.00		COST	0.00	
PMQ	0.16		PMQ	0.26	
IFU	0.54		IFU	1.28*	
Japan			**Italy**		
CKEL	0.00		CKEL	-1.17	0.74 (CKEL + COST)
COST	0.50		COST	1.91	
PMQ	0.50		PMQ	0.26	
IFU	0.44*		IFU	0.56	
Germany			**Canada**		
CKEL	0.80		CKEL	-1.22	0.64 (CKEL + COST)
COST	0.00		COST	1.86	
PMQ	0.20		PMQ	0.36	
IFU	0.47		IFU	0.60	
France			**Unweighted mean**		
CKEL	-4.94	0.87 (CKEL + COST)	CKEL/COST	0.73	
COST	5.81		PMQ	0.27	
PMQ	0.13		IFU	0.55**	
IFU	0.66				

* Imposed.
** Excluding the United Kingdom.

Chart 7

BUSINESS OUTPUT DEFLATOR (GROWTH RATE)



VII. ENDOGENISING LABOUR SUPPLY

60. In order to capture additional supply-side effects in the labour market, the new labour demand equations discussed above and the existing wage equations in INTERLINK have been supplemented by labour force participation equations. Previously, labour force was treated as exogenous. This rounds out modelling of the labour market and endogenises unemployment and thence wages more satisfactorily. Equations were estimated for males and females separately, given the contrary developments of participation rates frequently seen for the two sexes.

61. The initial specification was ad hoc. One group of variables proxies social/demographic factors which cannot be adequately endogenised in a macroeconomic model. Nevertheless, such variables were included at the estimation stage to preclude biased estimates of coefficients on "economic" variables. The age structure of the population is a potentially important influence on participation rates. It was tested in two alternative ways: first by a variable AGE, defined as the logarithm of the ratio of actual and age-constant participation rates with an indeterminate sign expectation (22); secondly the share of those under the age of 25 (YOUTH) and those over 55 (OLD) in the working-age population were included as explanatory variables. Initially AGE, YOUTH and OLD were included jointly in the equation. Participation rates are also likely to be affected by rates of family formation and childbearing. The ratio of children under the age of 5 to women in the 20 to 34 age bracket (DEP) is thus included in both equations with a weak positive sign expectation for males and a negative expectation for females. Continuing education is an alternative to labour force participation, so the more people in the 20 to 24 age bracket enrolled in post-secondary institutions (ENR), the lower should be their probability of labour force participation. Finally, employment opportunities for women differ across sectors with services displaying the highest female representation. It is possible that women will enter the labour force in response to the long-term structural shift of OECD countries toward the production of services and resultant enhanced employment opportunities. The share of services employment (SERF) is included, with a positive sign expectation in the female equation and an uncertain sign expectation in the males equation.

62. A second group of explanatory variables comprises the economic factors which are endogenous in INTERLINK. First, there are possible cyclical effects: potential labour force participants probably base their entry decision on the probability of success from job search -- high unemployment yields the well-known "discouraged worker effect". However, some secondary workers may enter and exit based on the (un)employment status of their spouses. Unemployed workers can cause other household members to seek work -- the so-called "added worker effect". The net cyclical effect is therefore to some extent unclear, although it seems likely that the discouraged worker effect will dominate and that the participation rate will be procyclical. In general, the specification includes the aggregate unemployment rate (UNR) with a (qualified) negative sign expectation, especially for males, as well as the factor utilisation rate from the new supply block in INTERLINK (IFU), with a positive sign expectation. In the case of Japan, however, aggregate employment itself was directly inserted into the equation, in response to the well-known paucity of variance in the historical series for UNR in Japan.

63. The real wage is expected to have a positive effect on participation rates, by increasing the (opportunity) cost of leisure and/or other unremunerative activity. Considering that participation decisions are made within a family unit there may also be a negative income effect on participation rates from higher real wages, i.e. a family member leaving the labour force because of increased earnings by another member. The earnings concept used is wages per employee net of household direct taxes (TYH) and social security contributions (TRSSH), deflated by the consumption deflator, i.e. WAGE = ((WSSS-TYH-TRSSH)/EE)/PCP.

64. Non-wage incomes can be expected to be negatively related to participation decisions since they have no associated substitution effects. The aggregate non-wage income is scaled by working-age population, and the resulting series is deflated using the private consumption deflator. Where data are available interest on consumer debt (INTDBT) is removed, but government transfers are not included in the resulting term (NWY). Government transfers (TRRH) are separately treated, since in addition to the negative income effects, there are possible negative incentive effects as well. Here too, the scale variable used is working-age population, and deflation is by the private consumption deflator, yielding the proxy GT.

65. A double-log specification is used. While the dependent variable is limited to a range between zero and one, experience shows that usually only trivial differences in parameter estimates are found when probit/logit procedures are used, and thus "ordinary" estimation techniques were applied. However, since the equations for the two sexes will obviously have cross-correlated error structures, both OLS and Zellner-efficient "seemingly unrelated regression estimators" are used. The simultaneity problem has, however, been neglected to this point, as has the possibility of cross-nationally correlated error structures. The preliminary specification is therefore:

$$
\begin{aligned}
LFPR_{ijt} = {} & M_{0ij} + M_{1ij}.AGE_{ijt} + M_{2ij}.YOUTH_{ijt} + M_{3ij}.OLD_{ijt} + M_{4ij}*DEP_{jt} \\
& + M_{5ij}.ENR_{ijt} + M_{6ij}.SERF_{jt} + M_{7ij}.UNR_{jt} + M_{8ij}.IFU_{ijt} \\
& + M_{9ij}.WAGE_{jt} + M_{10ij}.NWY_{jt} + M_{11ij}.GT_{jt} + M_{12ij}.LFPR_{ijt-1} + e_{ijt}
\end{aligned}
$$

where i = females, males

j = 1, ..., 7 (the Big Seven)

t = time, by semesters

e is a conventionally-defined error term, with $r(e_{fjt}, e_{mjt}) \neq 0$ assumed

and (M_{4m}), M_{6f}, M_{8}, (M_{9}), M_{12} 0

M_{2}, M_{3}, M_{4f}, M_{5}, (M_{7}), M_{10}, M_{11} 0

(parentheses indicate caution).

The estimation strategy was to begin with the full specification but to exclude variables with insignificant and/or perverse and marginally significant coefficients. However, where necessary to whiten residuals, various polynomial functions of time were included as were longer lags on the dependent variables and Cochrane-Orcutt corrections for first-order serial correlation of the error terms.

66. A summary of estimation results is given in Table 11. The Canadian equation is for the aggregate labour force because of convergence problems with the sex-disaggregated equations. The average standard error of estimates is one-third of 1 per cent.

67. Many of the "social" variables are significant. The age variable has a significant coefficient in about half the equations with all but one sign being positive. In general, it appears to be more important in the equations for women than for men which is reasonable given the more substantial changes that have occurred in women's social and economic roles over the estimation period. The youth proportion has the expected negative elasticity in only five of thirteen cases with three perversely positive estimates as well. The share of the elderly similarly takes a negative coefficient in five cases and a positive one in two, although the Japanese males effect is more than wiped out by the effect from Japanese women. The effect is by far the most substantial in the United States. The number of infants per woman of child-bearing age takes the expected negative sign in only three of the six females equations and has a perverse positive parameter estimate for French women. There are also positive parameters for males in Italy and Canada. The enrolment rate variable was available only for the United States for this project and influences only males' participation decisions. Last among the "social" regressors is the service industry variable which has the expected positive effect in all female cases except France where it comes indirectly from the male cross effect. It is strongest for Italy and the United Kingdom and weakest for France and Canada.

68. Among the "economic" variables the cyclical proxies are significant determinants of the participation rate in all cases except American, French and Italian men. In five of the six cases where a sex comparison is possible the female rate is more procyclical than the male rate; only in Italy does the added-worker effect dominate the discouraged worker effect for women. The unemployment rate seems to be a better proxy for the effect of the cycle in the labour market than the factor utilisation rate coming from the supply block. However, for British and French women the two are both individually significant. For Japan the use of the employment rate gave apparently reasonable estimation results: the long-run elasticity of the labour force with respect to total employment is 1.37 for women and 0.45 for men with a weighted average of about 0.8. However, the Japanese model proved too inflation-prone in simulations, and it was decided to remove the lagged employment term and the lagged dependent variable from the LFM equation, thereby raising the male elasticity to 0.58 (1.22 - 0.47*1.37) and the weighted average to about 0.89.

69. The real wage turns out to be the most difficult economic variable to interpret. Wages disaggregated by sex are not available so the same aggregate average earnings rate was used in both equations. It has a significant influence in only about half the equations estimated, and of these almost half are in a negative direction. A negative effect for women could be justified by the second-earner nature of many women in the labour force: when their spouses' incomes increase their income effect may indeed dominate their substitution effect in the determination of their demand for leisure, and they

Table 11

PARTICIPATION RATE EQUATIONS

	United States		Japan		Germany		United Kingdom		France		Italy		Canada
	Females	Males	Females	Males (b)	Females (c)	Males	Females (d)	Males	Females (e)	Males	Females (g)	Males	Total (h)
AGE			+0.06	+0.002	-0.02	+0.01			+0.06	+0.01	+0.07		
YOUTH		-0.05	-0.41		-0.10	-0.06			+0.32	+0.12		+0.44	-0.49
OLD	-0.94	-0.59	-0.20	+0.01	-0.24				+0.25		-0.17		
DEP	-0.05		-0.29				-0.13		+0.26			+0.23	+0.60
ENR		-0.03											
SERF	+0.76		+0.69		+0.49		+1.48			-0.25	+2.64		+0.23
UNR			+1.37(a)	+0.99(a)(i)	-0.01	-0.004	-0.004	-0.001	-0.05		+0.04		-0.03
IFU	+0.07						+0.14		+0.12				
WAGE		+0.06			+0.06	-0.03	+0.03		+0.14			-0.09 (i)	-0.07
NWY		-0.06	-0.05					-0.03		-0.006			-0.04
GT		-0.05	-0.08										
LDV1	+0.56	+0.40				+0.82		+0.75	+0.33	+0.41(f)	-0.25	+0.20	
LDV2		-0.21		-0.31 (i)									
RHO	-0.34					-0.33							
Weighted SEE	0.0036		0.0019		0.0021		0.0036		0.0017		0.0067		0.0040

a. For Japan the regressor is actually total employment scaled by the working age population in the females equation; in the males equation it is both current and one-semester lagged scaled employment with elasticity estimates of 1.22 and -0.23, respectively. (See (i) below.)
b. The equation also includes the female participation rate with an elasticity of -0.47.
c. The equation includes a second-order polynomial on TIME prior to 1975 as well as an inverse on TIME thereafter.
d. The equation includes an inverse on TIME.
e. The equation includes the male participation rate with an elasticity of -0.88 and a second-order polynomial and an inverse on TIME.
f. The equation actually includes four lags on the dependent variable with elasticity estimates as follows: 1.19, -1.31, 0.76, -0.24.
g. The equation actually includes a second-order polynomial and an inverse on TIME.
h. The equation actually includes a second-order polynomial on TIME.
i. After estimation it was decided to (1) remove the perverse WAGE coefficient in the Italian LFM equation; (2) remove the negative second-order lagged dependent variable in the Japanese LFM equation; and (3) remove the lagged employment term in the Japanese LFM equation. No re-estimation was undertaken.

may not only reduce their desired hours of work but may indeed drop out of the labour force altogether. However, the German and Italian results manifest negative coefficient estimates in their males equations which is harder to justify, suggesting as it does that the aggregate labour supply curve is backward-bending. Yet, the Italian and Canadian models show a negative effect in the aggregate which was eventually removed in the Italian case. Further work in this area must aim to overcome this deficiency.

70. Non-wage income other than government transfers leads to an observable negative income effect on the demand for leisure but in a fairly limited number of cases. There is no effect at all for Germany and Italy. Furthermore, an additional disincentive effect from government transfer income can be empirically detected only in the case of Japanese females. For British and French men as well as Canadians in general even the pure income effect is not in evidence. Adjustment of actual to equilibrium participation rates is seen to be quite rapid, except in the cases of German and British men where the mean lags reach 2.3 and 1.5 years, respectively.

71. The procedure used to insert the equations into the experimental version of INTERLINK, given the desire to avoid inclusion of additional exogenous variables in the model, was to drop the social/demographic variables from the equations, to constrain the other elasticities to equal their values estimated in the first stage, to include a high-order polynomial function on time and its inverse and to estimate the parameters of that function in a second round of Zellner-efficient estimation (23). In all cases except the United States, the estimated time polynomial would imply implausible movements in either the male or female participation rates in the near future. Finally, therefore, the equations were implemented with the estimated elasticities of the economic regressors, but trend effects having been omitted and the constant terms readjusted to make the average error over the recent past equal to zero. This implies that only the U.S. equations can be used for unconditional forecasting without extensive add-factoring. When used for counterfactual simulations, the absence of projections of exogenous "social" variables will not have any effects on simulation results.

VIII. CONCLUSIONS

72. This paper presents production and factor demand equations for the seven largest OECD economies. The linkage of this supply structure to the price formation process and the endogenisation of labour supply are also discussed. In these concluding comments, some of the main features of the estimation results are summarized, indicating some areas for possible further development, and suggesting ways in which other parts of the INTERLINK model could be adapted to mesh more effectively with the integrated supply model presented in this paper.

73. For the underlying longer run production function, significant support for the idea of using a vintage bundle of capital and energy in combination with efficiency units of labour was found . Several countries showed quite high long-run elasticities of substitution between capital and energy. Most showed important vintage effects that supported a putty/semi-putty view of technology, whereby energy use is fully flexible before capital is put in place and partially flexible thereafter.

74. Evidence exists for most countries that there has been international convergence in the rate of increase in labour efficiency for the faster growing countries during the 1960s and early 1970s -- especially Japan, Germany, France and Italy. This implies a substantial slowing down of the rate of growth of labour efficiency at the end of the 1970s and in the early 1980s in these countries. The size of this estimated catch-up effect may be overstated in our current results, because the end of the sample period was marked by low utilization rates, and it is difficult to find an adequate means of simultaneously estimating the cyclical and catch-up effects. The hypothesis that technical progress is embodied in new capital equipment was tested, but results were dominated by the performance of the catch-up specification which has therefore been integrated into the model as the maintained hypothesis.

75. Because there is evidence of increasing trends in the wage share that are inconsistent with the assumption of Cobb-Douglas technology for the outer function combining labour with the bundle of capital plus energy, a more general CES form was developed. However, the derived equations for production and factor demands revealed very little power to choose a value for the newly-freed elasticity of substitution. The evidence suggests that in most countries it could easily be as low as 0.65 or as high as 1.0, and that the choice has virtually no effects on the fit or the parameter values of the estimated production and factor demand equations. Of course, the effects of wage flexibility or rigidity on the demand for labour, and hence on the unemployment rate, depend importantly on the elasticity of substitution between labour and other factors. Furthermore, changes in the derived demand for labour are an important part of the overall macroeconomic effects of alternative patterns of wage behaviour. So the uncomfortable fact is that important simulation properties of the model are not very well determined on the basis of analysis of historical data.

76. In terms of future development of the long-term production structure, it would probably be useful to take account of longer-term movements in average hours worked when defining the labour input to the production function. Above (cf. Part III) the advantages were pointed out of treating cyclical variation in man-hours, and in machine hours, as part of our overall measure of capacity utilisation, QBV/QBSV, because so many other features of utilisation cannot be separately measured. In the present version of the model, any trend changes in hours worked are automatically treated as part of the aggregate index of labour efficiency thus reducing the measured growth rate of total factor productivity. As long as the downward trend in hours is at a constant rate, and likely to remain so, this procedure does not cause problems. However, there is some evidence that the downward trend in hours is slowing down, and in several countries there are discussions of substantial legislative changes in the number of weekly hours. In addition, the levels and trends of average hours differ among countries, and this may disturb international comparisons of labour efficiency, and affect estimates of the international catch-up hypothesis. For all of those reasons, a redefinition of the labour input to include long-term changes in average hours merits examination.

77. The short-run production equations provide strong and consistent evidence of systematic and economically explicable variations in the utilisation rate. The use of utilisation rates as the dependent variable for the short-term production decision provide a means of consistently integrating

the demands for capital, labour and energy (all combined in the normal output series QBSV) with the factors influencing the intensity of factor use: sales, profitability, and the level of opening inventories. Of the three factors, the sales and profitability effects were empirically the most important.

78. In all countries, the estimated speed of the production response to an inventory gap is not well determined. However, this does not mean that the inventory level itself will be ill-determined, or that it will wander indefinitely far from its desired value when sales change. This is because the production response to the inventory gap is only one of several possible channels through which inventory equilibrium can be restored. The inventory gap should also influence trade flows, and future work on trade equations in INTERLINK could test this hypothesis. However, initial re-estimations of those equations has not yielded promising results. In addition, any discrepancy between expected sales and desired production that is not matched by desired imports will lead to changes in factor demands, and hence in production capacity and actual output that will choke off the original pressure to build up or run down inventories.

79. The estimated equations for investment showed substantial lags in the adjustment of capital to changes in desired levels. To some extent these partial adjustments, and the sometimes substantial unexplained variance in the factor demands, may be due to inevitable difficulties in measuring the desired factor levels. However, the results provide evidence that both capital and labour are properly regarded as quasi-fixed factors though with the degree of fixity for capital much exceeding that for labour. This provides further support for treating factor utilization as a short-term adjustment variable used to help mediate between demand conditions and production capacity when the former are uncertain and the latter is costly to adjust.

80. Recent theoretical work suggests that the investment rate should be directly affected by profitability in conditions of uncertainty and costly adjustment, so that the empirical strength of PROFR in the investment equations (other than for the United States) is welcome and helps to provide another important element in the macroeconomic adjustment process.

81. The dual cost function to the aggregate production function has been used to link the production structure to the price formation process. As can be expected the long-term link between factor cost and output prices detected empirically is very strong, as is the link between competitors' prices and domestic price levels in most countries. However, the cyclical impact of variations in factor utilisation rates and inventory/output ratios on output prices was somewhat more difficult to detect empirically.

82. Overall, the results while variable, are moderately encouraging. Certainly they do not lead to a revision of the view that an integrated model of production, factor demands and price formation as outlined in this paper provides a suitable way of building the supply side of an econometric model.

83. The production, factor demand, price, and labour supply equations reinforce supply considerations in the logic of INTERLINK, thus making the model a handier tool in dealing with medium-term issues. In principle, the endogenous determination of production, and hence of inventory changes, should improve the model's short-term performance, although the empirical robustness of this innovation is not yet established. The integration of the short-term

and longer-term supply factors should permit the model to make the transition from the short-run conjuncture to the medium-term growth path. If these attractive possibilities should be supported by experience with the supply structures for the major seven OECD countries, this might justify extending comparable treatment, with simplifications where required, to more OECD economies.

NOTES

1. Ball (1973) provides a survey of the structure of the various national models used in project LINK, all of which treat output as essentially demand-determined. Ball notes (1973, p.90) that inventory demand equations can be thought of as renormalized output equations. However, all of the models he surveys have inventory demand determined by a flexible accelerator, with no role given to supply factors. This contrasts with the RDX2 (Helliwell et al, 1971) procedure, wherein the inventory change equation is recognised as the link between aggregate supply and demand, with the difference between final sales and the output level desired by firms being the key determinant of the change in inventories.

2. For example, Sargent's (1976) "new classical" macroeconometric model for the United States determines output in terms of current and lagged employment plus a time trend.

3. In METRIC (1981) aggregate supply in most periods is determined by expected final sales and some fraction of the gap between actual and desired inventories. However, whenever the margin of unused capacity drops below 16.5 per cent, there is a reduction in the fraction of expected final sales that is supplied by domestic output and imports; the gap therefore is made up from inventory reductions. Since the resulting inventory discrepancy does not directly influence price formation, the modelling of the supply-constrained regime is not complete.

4. This work is described in Artus (1983).

5. For example, Lundberg (1937) and Metzler (1941), as characterized by Allen (1968).

6. Employment is a quasi-fixed factor because of costs of hiring and firing and the costs of assessing the quality of heterogeneous workers. Energy is quasi fixed because it is complementary with fixed capital with only limited possibilities for adapting the latter after installation.

7. See also Annex A for a complete annotated list of equations pertaining to the production structure. This Annex also describes how the various parameters characterising the production structure were estimated.

8. It is for these reasons, as well as overall considerations of model management, international comparability and resource implications, that a suggestion of Klein (1978) has not been followed, namely that the most appropriate way of modelling the interplay of macroeconomic supply and demand is to combine a Keynesian set of demand-side equations with a disaggregated supply side based on a Leontief inter-industry model.

9. This statement refers to general government only; investment and employment by public enterprises are part of the "business sector", in line with standard SNA definitions.

10. In principle, the output measure should exclude value added in the energy sector, but as in the case of capital and labour inputs, this adjustment has not yet been implemented.

11. See the following section and Annex A for a detailed exposition of the algebraic transformations involved.

12. As in the previous supply block, this assumes that scrapping proportionally affects all vintages of capital rather than being concentrated on marginal vintages.

13. PIM is defined as follows:

$$PIM = (QBV^{-r} - XOGAMA.KEBSV^{-r})^{-1/r} / ETB$$

where r = (1-XTAU)/XTAU

14. In practice, PIM turned out to be rather insensitive to changes in XTAU, and the procedure converged rapidly.

15. The slowdown effect in the United States evident from the significant coefficient in Table 2 (last column) depends entirely on the rapid cyclical recovery of productivity from the 1961 recession : if the first three years are excluded from the relevant regression, the slow-down coefficient remains insignificant.

16. This holds a fortiori for output per employed person which -- apart from labour efficiency -- will be influenced by capital and energy input per worker.

17. The precise definition of the inverted profitability variable is:

CQB=(WSSE.ETB+ENBV.PENB+KBV.PIB/100.(RSCRB+RES+DFR.IRL))/(PQB.QBV)

where DFR is the average debt financing ratio and RES is computed as a residual, so that on average all factor payments exhaust total output over the sample period and the sample mean of CQB equals 1.0.

18. Work is currently underway to respecify the output supply equation in a way which would (1) add ENBV, subtract only MGSV and possibly add "normal" inventory growth to the numerator of the demand term, and (2) allow for dynamics in the supply response by including one-semester lags on each of the independent variables.

19. The precise formula is:

PROFR = PQB.QBV.(1-CQB)/(PIB.(KBV + KBV(-1))/2)

20. To the (substantial) extent that the stock market uses current profits as a proxy for the discounted present value of expected future profits, our variable CQB is closely related to the inverse of Tobin's q (1969). Several recent papers (e.g. Yoshika (1980), Hayashi (1982) and Abel (1984)) have confirmed that q, or equivalently CQB and PROFR, ought to influence the rate of investment when there are adjustment costs. Poterba and Summers (1983) apply a "q" model to U.K. investment expenditures.

21. This approach to the modelling of dynamic adjustment paths follows Hendry and Mizon (1978), but for simplicity and comparability we considerably restrict the number of possible lags, thus reducing the complexity of the implied dynamic adjustment path.

22. The age-constant participation rate is calculated by assuming the age structure of the population is constant at some base year pattern. The (fixed) shares of different age groups in that base year population are then used to weight the participation rates of the age groups in subsequent periods. This yields an aggregate participation rate equal to what it would have been if the population age structure had remained constant.

23. The standard errors of estimate were all much increased by this procedure, the average increase being about 80 per cent. It can be concluded that there is therefore substantial information contained in the social/demographic variables.

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ANNEX A

THE PRODUCTION STRUCTURE: SPECIFICATION AND PARAMETER DERIVATION

The production structure is based on a nested double CES production function, combining capital, labour and energy to define gross output.

A. The Inner CES function.

The inner function which combines energy and capital in a vintage bundle has the form

$$KEBSV = KEBSV(-1).(1-XR1-RSCRB) + [IBV+XR1.KBV(-1)] . [XIBETA+XIGAMA.(XIGAMA.UCC/(XIBETA.PENB))^{s-1}]^{s/(s-1)} \quad (A1)$$

where KEBSV = vintage capital-energy bundle
XR1 = retrofitting parameter
RSCRB = scrapping rate
IBV = gross fixed investment
XIBETA, XIGAMA = scale parameters in the inner CES function
UCC = capital cost variable
PENB = energy price index (for final users)
s = elasticity of substitution between energy and capital

In this equation the business gross fixed capital stock (KBV), business gross fixed investment (IBV), the price index of energy used by business (PENB) and the scrapping rate (RSCRB) are variables available from official statistics.

The user cost of capital UCC is computed as

$$UCC = PIB.(RSCRB + IRLRE + XRHOR). (1-RITC-RTYB.PVDEP)/(1-RTYB) \quad (A2)$$

where PIB is the (observed) business gross fixed investment deflator and XRHOR (the "real supply price of capital") was defined as a constant such that on average total factor earnings exhaust total output over the sample period and IRLRE is the expected real long-term interest rate defined as

$$IRLRE = W.IRLRE(-1) + (1-W).IRLR$$

IRLR being the "actual" long-term real interest rate.

RITC is the rate of investment tax credit, RTYB is the effective (marginal) tax rate on business income and PVDEP is the unit present value of tax depreciation allowances.

Assuming that the capital-energy ratio (EK) is optimal (subject to prevailing relative prices PENB/UCC) on average over the sample period implies that

$$XIGAMA/XIBETA = (MEAN(EK)/MEAN [(UCC/PENB)^{s}])^{1/s} \quad (A3)$$

which allows direct computation of XIGAMA/XIBETA from observed variables for any given value of s, the elasticity of substitution between capital and energy. Normalising KEBSV such that MEAN(KEBSV)=MEAN(KBV) allows XIBETA to be computed as

$$XIBETA = 1/(1+(XIGAMA/XIBETA)^{s}\ MEAN\ ((PENB/UCC)^{1-s}) \qquad (A4)$$

The elasticity of substitution (s) and the retrofitting parameter (XR1) are determined by estimating the energy demand function

$$\ln(ENBV) = a_1 \ln(EBSV)+u \qquad (A5)$$

where u represents the stochastic error term and EBSV is the vintage energy requirement needed to operate the capital stock KTV subject to prevailing relative energy prices (PENB/UCC), defined as

$$EBSV = EBSV(-1).(1-XR1-RSCRB) + (IBV+XR1.KBV(-1)).((XIGAMA.UCC)/(XIBETA.PENB))^{s} \qquad (A6)$$

To obtain a starting value, EBSV is set equal to ENBV at the beginning of the sample period, on the assumption that no large and surprising changes in energy prices have occurred over the preceding few years.

The parameter pair (s, XR1) which maximised the likelihood function of regression (A5) was chosen as the preferred parameter combination.

This completes the estimation of all the relevant parameters of the inner CES function.

B. The outer CES function

The outer function which bundles labour and the capital-energy aggregate into gross output has the form

$$QBSV =(XOBETA.(ELEFF.ETB)^{-r} + XOGAMA.KEBSV^{-r})^{-1/r} \qquad (A7)$$

where QBSV = potential output (at normal rates of factor utilisation)
XOBETA, XOGAMA = scale factors in the outer CES function
ELEFF = labour efficiency index
ETB = total employment, business
r = substitution parameter in the outer CES function, with 1/(1+r)=elasticity of substitution (XTAU)
KEBSV =vintage capital-energy bundle (see above)

Cost-minimising behaviour by producers implies that on average

$$KEBSV/ETB = ((WSSE/ELEFF)/CKE)^{1/(1+r)} \qquad (A8)$$

where WSSE is observed labour cost per man year and CKE is the cost of the capital energy bundle computed from the dual cost function to the inner CES function

$$CKE = (XIBETA^{s} . UCC^{1-s} + XIGAMA^{s} . PENB^{1-s})^{1/(1-s)} \qquad (A9)$$

The parameter XTAU (=1/(1+r)) was determined from the regression

$$\ln(QBV/(ETB.ELEFF)) = a_{o} + XTAU.\ln(WSSE/(PQB.ELEFF))+u \qquad (A10)$$

where PQB is the deflator for gross output (QBSV) at factor cost. Equation (A10) requires ELEFF to be known, while in turn the determination of ELEFF requires knowledge of XTAU (see below). ELEFF and XTAU were therefore determined by an iterative procedure, starting with an assumed value of XTAU=1.

Equation (A7) can be inverted and solved for the labour efficiency bundle

$$XOBETA.ELEFF^{-r} = (QBSV^{-r} - XOGAMA.KEBSV^{-r})/ETB^{-r} \qquad (A11)$$

Assuming that on average the observed input ratios are cost minimising (subject to observed relative factor prices) allows the parameter XOGAMA to be determined as follows:

$$XOGAMA = MEAN(PKQ)/(MEAN(NKS) + MEAN(PKK)) \qquad (A12)$$

Where:

$$MEAN(PKQ) = MEAN((CKE/WSSE) . (QBV/ETB)^{-r})$$
$$MEAN(NKS) = MEAN((ETB/KEBSV)^{(1+r)})$$
$$MEAN(PKK) = MEAN(CKE/(WSSE.ETB).(KEBSV/ETB)^{-r})$$

A labour efficiency bundle can now be computed as:

$$XOBETA^{-1/r} . PIM = ((QBV^{-r} - XOGAMA.KEBSV^{-r})/ETB^{-r})^{-1/r} \qquad (A13)$$

where PIM is the observed labour efficiency index which includes cyclical variations and is obtained by substituting actual output (QBV) for potential output (QBSV) in the inverted production function (A11).

This bundle was used as the dependent variable for the various tests of the time invariance of the rate of technical progress and the testing of the embodiment and catch-up hypotheses. Parameter XOBETA can be determined from this bundle by normalising the calculated labour efficiency index such that

$$ELEFF_{1971S1} = 1.0$$

and imposing the constraint that mean (QBSV) equals mean (QBV) over the sample period.

This completes the determination of the parameters for the outer CES function.

The values of the structural production function parameters for the inner and outer CES function thus derived are reported in Table 1 in the text.

GLOSSARY

AIBV	FIVE YEARS MOVING AVERAGE OF BUSINESS INVESTMENT (IBV)
CGW	GOVERNMENT CONSUMPTION, WAGES
CKE	DUAL COST , CAPITAL-ENERGY (INNER FUNCTION)
CKEL	DUAL COST , CAPITAL-ENERGY-LABOUR (OUTER FUNCTION)
COST	AVERAGE NORMAL COSTS
CQB	INVERSE PROFITABILITY VARIABLE (BUSINESS)
DELEFF	ANNUAL GROWTH RATE OF THE TREND LABOUR EFFICIENCY INDEX
EBSTAR	OPTIMAL BUSINESS EMPLOYMENT
EBSV	BUSINESS ENERGY REQUIREMENT
EE	DEPENDENT EMPLOYMENT
ELEFF	TREND LABOUR EFFICIENCY INDEX
ELEFFUS	US LABOUR EFFICIENCY INDEX
ENBV	ENERGY USED BY BUSINESS SECTOR
ETB	BUSINESS SECTOR EMPLOYMENT , TOTAL LESS GOVT
GDPBV	GROSS DOMESTIC PRODUCT (AT FACTOR COST) LESS GOVT WAGES VOL
GDPV	GROSS NATIONAL/DOMESTIC PRODUCT AT MARKET PRICES,VOLUME
IBV	INVESTMENT BY THE BUSINESS SECTOR, VOLUME
IFU	INTENSITY OF FACTOR UTILISATION ,INDEX
IFUHAT	LOGARITHM OF THE ESTIMATE OF THE DEGREE IF FACTOR UTILISATION
INTDBT	INTEREST ON CONSUMER DEBT
IRLR	LONG TERM INTEREST RATE, REAL
IRLRE	EXPECTED REAL LONG TERM INTEREST RATE
KBSTAR	OPTIMAL BUSINESS CAPITAL STOCK
KBV	CAPITAL STOCK BUSINESS
KEBSV	VINTAGE CAPITAL-PLUS-ENERGY BUNDLE (C.E.S INNER FUNCTION)
LF	LABOUR FORCE - TOTAL
LFM	LABOUR FORCE - MALES
MESV	IMPORTS OF ENERGY , VOLUME
MGSV	IMPORTS OF GOODS AND SERVICES, VOLUME, N.A. BASIS
MNEV	NON ENERGY IMPORTS
PCP	DEFLATOR FOR PRIVATE CONSUMPTION
PENB	DEFLATOR FOR ENERGY USED BY BUSINESS SECTOR
PGDP	DEFLATOR GDP AT MARKET PRICES
PGDPB	BUSINESS OUTPUT DEFLATOR
PIB	DEFLATOR FOR BUSINESS INVESTMENT
PIM	OBSERVED LABOUR EFFICIENCY INDEX
PIMADJ	OBSERVED LABOUR EFFICIENCY INDEX ADJUSTED FOR CYCLICAL VARIATIONS
PMNE	PRICE OF NON-ENERGY IMPORTS
PMG	OUTPUT WEIGHTED IMPORT PRICES
PQB	BUSINESS GROSS OUTPUT DEFLATOR
PROFR	PROFIT RATE
PVDEP	DISCOUNTED PRESENT VALUE OF DEPRECIATION ALLOWANCES
QBSV	NORMAL GROSS OUTPUT BUSINESS SECTOR, VOLUME
QBV	BUSINESS GROSS OUTPUT
QSTAR	GROSS OUTPUT EXPECTATIONS
RSCRB	SCRAPPING RATE OF CAPITAL (BUSINESS)
RTYB	RATE OF BUSINESS PROFIT TAX
SALES	FINAL SALES, (FINAL DOMESTIC DEMAND PLUS EXPORTS)
STOCKV	TOTAL STOCK, VOLUME
TIND	INDIRECT TAXES
TRSSH	SOCIAL SECURITY CONTRIBUTIONS BY HOUSEHOLDS
TSUB	SUBSIDIES
TYH	DIRECT TAXES, HOUSEHOLDS
UCC	USER COST OF CAPITAL
UNR	UNEMPLOYMENT RATE
WAGE	WAGES AND SALARIES EXCL.SOC.SEC.CONTRIB.
WSSE	COMPENSATION PER EMPLOYEE, PRIVATE SECTOR
WSSS	COMPENSATION OF EMPLOYEES