Chapter 2

Analytical problem solving: Potentials and manifestations

By

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Being able to solve problems is a crucially important outcome of almost all domains of education, as well as a requirement for future learning and acheivement. This chapter summarises the results from the cross-curricular analytical problem solving element of the 2003 Programme for International Student Assessment (PISA). The results from Germany indicated that schools may not be sufficiently exploiting students' cognitive potential to develop their subject-specific competencies. To investigate this hypothesis and find ways to make use of this potential, the chapter describes research on the cognitive components and the structure of analytical problem-solving competence and its relation to mathematical competence. It concludes with results from two experimental studies aiming to foster students' mathematical competence by training in analytical problem-solving competence.

Introduction

The competence to solve problems is of crucial importance in almost all domains of learning and achievement, both in and out of school. Accordingly, educational standards in various subject areas define it as a subject-specific competence (for example AAAS, 1993; NCTM, 2000; Blum et al., 2006). However, problem solving is also seen as a cross-curricular competence that is acquired and applied in the school context and that is vital for success in the world of work (for example OECD, 2005a). Problem solving is also an object of interest to various research approaches. Educational researchers have developed methods to teach problem-solving skills in diverse learning situations in various school subjects (for example Charles and Silver, 1988; Bodner and Herron, 2002; Törner et al., 2007). In psychology, especially European cognitive psychology, research on problem solving has a long tradition. Its roots lie in the experimental research conducted in Gestalt psychology and the psychology of thinking in the first half of the 20th century (for example Duncker, 1935; Wertheimer, 1945).

Despite differing categorisations of problems and problem solving, there is consensus that a problem consists of a problem situation (initial state), a more or less well-defined goal state, and a solution method that is not immediately apparent to the problem solver (Mayer, 1992). Thus, problem solving requires logically deriving and processing certain information in order to successfully solve the problem. In contrast to a simple task, a problem is a non-routine situation for which no standard solution methods are readily at hand (Mayer and Wittrock, 2006). To use Dörner's (1976) terminology, a barrier between the initial state and the desired goal state makes it unclear how to transform the initial state into the goal state. The solution of problems can thus be defined as "… goal-oriented thought and action in situations for which no routinised procedures are available" (Klieme et al., 2001:185, our translation).

This chapter describes recent results of problem-solving research with a focus on analytical problem-solving competence. From the perspective of a psychometric research approach, a key question is whether, apart from intelligence and other cognitive constructs, problem-solving competence represents a distinct construct, and if so, how it can be assessed and modelled. From an educational psychology perspective the focus lies on the question of how problem-solving competence can be fostered by instructional means.

In the context of school and education systems, research on problem-solving competence has become more relevant following the 2003 Programme for International Student Assessment (PISA) (OECD, 2003, 2005a). PISA 2003 defined cross-curricular problem-solving competence as "… an individual's capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science or reading" (OECD, 2003:156). The PISA problem-solving test showed some remarkable results, especially in Germany (Leutner et al., 2004; OECD, 2005a). While students in Germany only reached average results in mathematics, science and reading, their results in problem solving were above average (compared to the OECD mean of 500 score points; standard deviation = 100). This difference between students' problem-solving and subject-specific competencies, for example in mathematics, was especially pronounced in Germany, at 10 points on the PISA scale. Among all 29 participating countries only 2 countries showed larger differences in favour of problem solving: Hungary, at 11 points, and Japan, at 13 points (OECD, 2005a). This difference is particularly surprising against the background of the rather high correlation between problem-solving and mathematics scores ($r = .89$); the values for science were $r = .80$ and for reading $r = .82$ (OECD, 2005). According to the OECD report on problem solving in PISA 2003, this discrepancy can be interpreted in terms of a cognitive potential exploitation hypothesis: the cross-curricular problem-solving test reveals students' "… generic skills that may not be fully exploited by the mathematics curriculum" (OECD, 2005a:56; see also Leutner et al., 2004)

The cognitive potential exploitation hypothesis refers to the international part of the PISA 2003 test which contains analytical problems such as finding the best route on a subway map in terms of time travelled and costs (OECD, 2005a). These kinds of problems can be distinguished from dynamic problems such as a computer simulation of a butterfly swarm which has to be shaped in a certain way by using a controller whose functions are not mentioned in the problem description (Leutner et al., 2004; see also Greiff et al., 2012). In analytical problem solving, all the information needed to solve the problem is explicitly stated or can be inferred; it can thus be seen as the reasoned application of existing knowledge (OECD, 2005a). In dynamic problem solving, in contrast, most of the information needed to solve the problem has to be generated in an explorative interaction with the problem situation, or "learning by doing" (Wirth and Klieme, 2003). Dynamic problems are often characterised by a high number of interrelated variables (complexity), a lack of transparency regarding the causal structure and sometimes several, partly conflicting, goals and subgoals (Funke, 1991). In research, these types of problems are often referred to as complex problems (Funke and Frensch, 2007; Greiff et al., 2013) or, to use the terminology used in PISA 2012, as interactive problems (OECD, 2013, 2014; see also Chapter 6).

Results from a German PISA 2003 subsample show that the analytical and dynamic aspects of problem solving can be well distinguished empirically (Leutner et al., 2004). Analytical problem solving and subject-specific competencies in mathematics, science and reading form a comparably high correlated cluster, whereas dynamic problem solving seems to represent something different (see also Wirth, Leutner and Klieme, 2005). The disattenuated correlation between analytical and dynamic problem solving amounts to only $r = .68$ whereas analytical problem solving correlates with mathematics at $r = .90$ in this sample (Leutner et al., 2004). It can be shown that on the national PISA scale (Mean = 50; Standard deviation = 10) there is a lower mean difference between the lower vocational and the academic tracks of the German secondary school system for dynamic problem solving (14.7 points) than for analytical problem solving (19.0 points). This result points to a relative strength of students from the lower vocational track in dynamic problem solving, which can also be interpreted in terms of the cognitive potential exploitation hypothesis (Leutner et al., 2004). Despite these differences between analytical and dynamic problems, in PISA 2012, problem-solving competence was assessed using both analytical ("static" in PISA 2012 terminology) and interactive problems modelled on the same composite scale and labelled as creative problem solving (OECD, 2013, 2014; see also Chapter 5).

Finally, a subsample of the PISA 2003 students in Germany took part in a repeated measures study, which examined competence development in the 10th grade (from the end of 9th grade to the end of 10th grade) in mathematics and science (N = 6 020; Leutner, Fleischer and Wirth, 2006). A path analysis controlling for initial mathematical competence in 9th grade shows that future mathematical competence in 10th grade can be better predicted by analytical problem-solving competence ($R^2 = .49$) than by intelligence ($R^2 = .41$). In order to further explore the role of analytical problem solving for future mathematics, a communality analysis was conducted, decomposing the variance of mathematical competence (10th grade) into portions that are uniquely and commonly accounted for by problem-solving competence, intelligence and initial mathematical competence. Results showed that the variance portion commonly accounted for by analytical problem solving and initial mathematical competence ($R^2 = .127$) is considerably larger than the variance portion commonly accounted for by intelligence and initial mathematical competence ($R^2 = .042$) whereas the variance portion uniquely accounted for by both intelligence ($R^2 = .006$) and problem solving ($R^2 = .005$) are negligible (Leutner, Fleischer and Wirth, 2006). Further analysis including reading competence as an additional predictor does not change this pattern of results (Fleischer, Wirth and Leutner, 2009). These findings indicate that analytical problem-solving competence consists of several components with different contributions to the acquisition of mathematical competence. Although it is not entirely clear what these components are, it can be assumed that skills associated with a strategic and self-regulated approach to dealing with complex tasks with limited speed

demands play an important role. Intelligence test items, in contrast, do not seem to require these skills, at least not to the same degree (Leutner et al., 2005).

In order to make use of the cognitive potential demonstrated by students in analytical problem solving for domain-specific instruction, three issues have to be dealt with. First, it is crucial to identify the cognitive requirements of analytical problems and the corresponding components of analytical problem-solving competence. Second, the interrelations of these components and their relations with external variables (such as subject-specific competencies) have to be investigated. This means establishing a structural model of analytical problem-solving competence and testing aspects of construct validity. Third, research is needed to determine how students' cognitive potential can be better exploited in domain-specific instruction. This requires developing and evaluating training methods in experimental settings which can then be transferred into school settings.

The cognitive potential exploitation hypothesis assumes that students have unused cognitive potential for science instruction as well as for mathematics (Rumann et al., 2010). In the following sections however, we will focus on the role of cross-curricular analytical problem solving for subject-specific problem solving in mathematics.

Analytical problem solving as a process

Based on early research on problem solving in the first half of the 20th century (e.g., Wertheimer, 1945), Polya (1945) describes problem solving as a series of four steps: 1) understanding the problem; 2) devising a plan; 3) carrying out the plan; and 4) looking back (evaluating the problem solution). In his model of problem solving, Mayer (1992) breaks the process of problem solving down into two major phases: 1) problem representation; and 2) problem solution. Problem representation can be further split into problem translation, in which the information given in the problem is translated into an internal mental representation, and problem integration, in which the information is integrated into a coherent mental structure. Likewise, the problem solution process can be divided into the two sub-processes of devising a plan of how to solve the problem and monitoring its execution.

These conceptualisations of the problem-solving process provide the basis for more recent approaches to both cross-curricular and subject-specific problem solving. The PISA 2003 assessment framework describes a comparable series of necessary steps in order to successfully solve the items of the (analytical) problem-solving test: 1) understand; 2) characterise; 3) represent the problem; 4) solve the problem; 5) reflect; and 6) communicate the problem solution (OECD, 2003:170 f.). A comparable series of steps can also be found in descriptions of the mathematical modelling cycle which constitutes the basis of the PISA mathematics test: 1) starting with a problem situated in reality; 2) organising it according to mathematical concepts; 3) transforming the real problem into a mathematical problem: generalising and formalising; 4) solving the mathematical problem; and 5) making sense of the mathematical solution in terms of the real situation (OECD, 2003:27; see also Freudenthal, 1983).

Analytical problem solving as a competence

Analytical problem solving and intelligence

In the context of research on cognitive abilities, problem solving is sometimes regarded as a component of intelligence (Sternberg and Kaufmann, 1998) or described as a cognitive activity associated with fluid cognitive abilities (Kyllonen and Lee, 2005). Problem solving can, however, be conceptually distinguished from intelligence: problem solving is characterised by a higher degree of domain specificity or situation specificity and it can be acquired through learning (Leutner et al., 2005). Furthermore, test items designed to tap the core domains of intelligence – general or fluid cognitive abilities – are generally decontextualised and less complex than analytical problem-solving

test items, which always require a minimum level of content-specific knowledge (Leutner et al., 2005; see also Klieme, 2004). Results from PISA 2003, as well as results from studies on problem solving as an extension to PISA 2000 in Germany, provide further evidence that problem-solving competence and intelligence are structurally distinct. As shown by Wirth and Klieme (2003), students' scores on items developed to tap dynamic problem solving, analytical problem solving and intelligence are not adequately explained by a single latent factor, despite high correlations. Models with three latent dimensions provide a much better fit to the data (for a comparison between dynamic problem solving and intelligence see also Wüstenberg, Greiff and Funke, 2012).

Components of analytical problem solving

A number of cognitive components of problem solving can be identified from the problem-solving steps listed above, which are needed to successfully solve both cross-curricular analytical and subject-specific problems. Among these components are the availability and application of knowledge of concepts and knowledge of procedures (Schoenfeld, 1985; Süß, 1996). Knowledge of concepts is defined as knowledge of objects and states which can be used to construct an internal representation of the problem situation and the aspired goal state ("knowing that"; see Resnick, 1983). Knowledge of procedures is defined as knowledge of operations which can be applied to manipulate the problem situation, transforming it from its current state into the aspired goal state, as well as the ability to execute cognitive operations or procedures ("knowing how"; see Resnick, 1983). A further component is conditional knowledge which describes the conditions under which specific operations are to be applied ("knowing when and why"; Paris, Lipson and Wixson, 1983). Another important component is the availability and application of general problem-solving strategies which structure the search for relevant information, alternative problem representations, analogies or subgoals (Gick, 1986). Finally, self-regulation ensures that the processes of problem solving are properly planned, monitored, evaluated, and – if necessary – modified (Davidson, Deuser and Sternberg, 1994).

In a first attempt to examine the role of these components both for cross-curricular and subject-specific problem solving, Fleischer et al. (2010) analysed the cognitive requirement of the PISA 2003 problem-solving items and mathematics items. The aim of this analysis was to reveal the similarities and differences in the conceptualisation and construction of these test items with regard to the cognitive potential exploitation hypothesis (a similar analysis was conducted in order to compare problem solving and science items from PISA 2003; for details see Rumann et al., 2010). All PISA problem-solving items and a sample of the mathematics items (a total of 86 units of analysis) were evaluated by 3 expert raters using a classification scheme consisting of 36 variables (rating dimensions). Table 2.1 shows an excerpt of the results.

The results show, among other things, that the problem-solving items had a more detailed description of the given situation and a more personal and daily life context than the mathematics items. However, the problem-solving items do not clearly show the cross-curricular character which would have been expected from the PISA 2003 definition of problem solving. Even though most problem-solving items could not be associated with one particular subject area, approximately one-third of the items were rated as clearly mathematics-related. Mathematics items were characterised as more formalised and thus requiring greater ability to decode relevant information. This requires the availability and application of content-specific knowledge of concepts, which is also reflected in the higher level of the curricular knowledge needed to successfully solve the mathematics items. In case of problem-solving items, however, knowledge of procedures seems to be more relevant than knowledge of concepts. In particular, the problem-solving items placed greater requirements on students' ability to recognise restrictive conditions and to strategically plan the solution approach. This indicates that conditional knowledge and the ability to self-regulate are more relevant to finding the solution. These seem to be the relevant components of analytical problem solving rather than of mathematics competence (Fleischer et al., 2010).

Table 2.1 **Results of expert rating of PISA 2003 test items**

Rating dimension	Problem solving	Mathematics
Description of given situation	more detailed	less detailed
Item context	personal	global
Subject/domain area	30% mathematical	mathematical
Language	less formalised	more formalised
Recognising restrictive conditions	higher demands	lower demands
Strategic and systematic approach	higher demands	lower demands
Curricular-level of knowledge	low	high
Knowledge of concepts	lower demands	higher demands
Knowledge of procedures	higher demands	lower demands

Fleischer, J., J. Wirth and D. Leutner (2009). "Problem solving as both cross-curricular and subject-specific competence", paper presented at the 13th Biennal Conference of the European Association for Research on Learning and Instruction (Earli), Amsterdam, 29 August 2009.

Different dimensions of analytical problem solving

The PISA 2003 study assessed problem-solving competence using three different problem types: 1) decision making (e.g. finding the best route on a subway map in terms of time travelled and costs); 2) system analysis and design (e.g. assigning children and adults to dormitories in a summer camp considering certain rules and constraints); and 3) troubleshooting (e.g. finding out if particular gates in an irrigation system work properly depending on which way the water flows through the channels). However, the items were subsequently modelled on a one-dimensional scale.

Since these problem types differ with respect to the goals to be achieved, the cognitive processes involved and the possible sources of complexity (for a detailed description of the PISA 2003 problem-solving test items see OECD, 2005a), we looked more thoroughly into these dimensions of analytical problem-solving competence. We examined whether the different problem types used in PISA 2003 represent different dimensions of analytical problem solving, and whether these dimensions have differential relations with external variables (Fleischer, in prep.; see also Leutner et al., 2012). For that reason, the PISA 2003 problem-solving test was re-scaled based on the dataset from Germany (2 343 students with booklets with respective item clusters) according to the Partial Credit Model (Masters, 1982). We then compared the one-dimensional model with a three-dimensional model that allocates each item to one of three dimensions depending on the problem type (decision making, system analysis and design, or troubleshooting). The two models were evaluated by means of item-fit statistics (MNSQ-Infit), correlations of the dimensions of the three-dimensional model, reliabilities and variances of the dimensions, as well as the Bayes information criterion (BIC) and the likelihood ratio test (LRT) (see Wu et al., 2007). Finally, we estimated the correlations between the individual ability estimates from the three-dimensional model with external variables (school track, gender, subject-specific competencies and reasoning) in order to check for differential relations. Differential relations with these variables would provide a strong argument for a three-dimensional structure of analytical problem-solving competence.

While neither of the two estimated models could be preferred on basis of the item-fit statistics and the variances of the dimensions, the reliabilities showed a small advantage for the one-dimensional model (r = .78) compared with the three-dimensional model (r = .73; r = .73; r = .69). The dimensions of the three-dimensional model were highly correlated (r = .93; r = .83; r = .80), but

not necessarily higher than the correlations that were usually found between competencies from different domains (OECD, 2005b). Results from the LRT showed a significantly better fit for the three-dimensional model than the more restricted one-dimensional model (ΔDeviance = $184_{(5)}$, p < .001), and the BIC also preferred the three-dimensional model (BIC_{3-dim} = 31 714 vs. BIC_{1-dim} = 31 859). Furthermore, the results displayed in Table 2.2 – although not entirely conclusive – provide some indications of differential relations of the dimensions of the three-dimensional model with external variables and therefore to some extent support the assumption of a three-dimensional structure of analytical problem-solving competence.

Table 2.2 **Intraclass correlation and relations with external variables for the three-dimensional model of analytical problem-solving competence: Mean differences and correlations**

	Decision making	System analysis and design	Troubleshooting
Intraclass correlation	.24	.32	.22
School track (reference = lower vocational track)			
Comprehensive track	0.341 [0.168/0.514]***	0.379 [0.156/0.602]**	0.405 [0.193/0.616]***
Higher vocational track	0.580 [0.446/0.714]***	0.645 [0.492/0.798]***	0.576 [0.438/0.714]***
Academic track	1.163 [1.046/1.280]***	1.293 [1.156/1.429]***	1.067 [0.949/1.184]***
R^2	.82	.76	.74
Gender (1 = female)	-0.045 [-0.117/0.027]	-0.024 [-0.090/0.042]	-0.149 [-0.230/-0.069]***
Mathematics	.611[.578/.644]***	.655[.624/.687]***	.568[.536/.600]***
Science	.582[.551/.612]***	.621[.584/.657]***	.530[.494/.567]***
Reading	.576[.544/.608]***	.608[.570/.646]***	.504[.469/.540]***
Reasoning	.464[.424/.503]***	.481[.439/.523]***	.394[.354/.434]***

Notes: Mean differences of person ability estimates of the three dimensional model for school track and gender. The intraclass correlation provides the proportion of overall between- and within-school variation that lies between schools. R2 provides the proportion of school level variance explained by school track. The person ability estimates are z-standardised weighted likelihood estimates. 95%-confidence intervals are in brackets. For mathematics, science, reading and reasoning the weighted likelihood estimates are correlated with each of the respective five plausible values from PISA 2003 (see OECD, 2005b), and then the mean of these five correlations is calculated per domain.

***p < .001, **p < .01. N = 2 343.

For all three dimensions, a substantial amount of the variance in the ability estimates can be allocated to the school level (between-group variance) but for the system analysis and design dimension this proportion (32%) is higher than for the other two dimensions. With respect to school track, all three dimensions show characteristic achievement differences, with the academic track having the highest mean difference (compared to the lower vocational track) followed by the higher vocational track and the comprehensive track. Even though the difference between the lower vocational track and the academic track is descriptively higher for the system analysis and design dimension (1.293) than for the other two dimensions (1.163 and 1.067), the overlapping confidence intervals prove this difference not to be significant. However, the school track variable explains more variance on the group level for the decision making dimension (82%) than for the other two dimensions (76% and 74%). Results by gender show a significant difference in favour of males (-0.149, CI95 = -0.230/-0.069) for the troubleshooting dimension. Furthermore, results for mathematics,

science, reading and reasoning show substantial correlations with the person ability estimates for all three dimensions. Descriptively, these correlations are more pronounced for the system analysis and design dimension than the other two dimensions but examining the confidence intervals proves these differences to be significant only for the comparison between the system analysis and design and troubleshooting dimension (Fleischer, in prep.).

On the one hand, these three problem types seem to represent different but highly correlated dimensions, due to their rather different cognitive requirements. On the other hand, modelling analytical problem-solving competence as a one-dimensional construct in the context of large-scale assessments does not seem to cause a considerable loss of information. However, the multidimensional structure of analytical problem-solving competence outlined in the results above should be taken into account when assessing this competence on an individual level, especially when it comes to investigating the cognitive potential of analytical problem-solving competence in order to use it to foster subject-specific competencies, for example in mathematics.

Training in analytical problem-solving competence

The research results described so far indicate that analytical problem-solving competence, as it was assessed in PISA 2003, consists of different components which to some extent require different cognitive abilities. Therefore, systematic training in a selection of these components should have an effect on analytical problem-solving competence in general. In accordance with the cognitive potential exploitation hypothesis, and based on the assumption that both cross-curricular and subject-specific problem-solving competence share the same principal components, training in components of analytical problem-solving competence should also have transfer effects on mathematical competencies.

Based on the analysis of relevant components of analytical problem-solving competence and mathematics described above, three components – knowledge of procedures, conditional knowledge and planning ability (as part of self-regulation) – were chosen for two training studies (for details see Buchwald, Fleischer and Leutner, 2015; Buchwald et al., 2017).

Study I: Method

In a between-subjects design, a sample of 142 9th grade students (mean age 15 years; 44% female) was randomly assigned to one of two experimental conditions. The experimental group received 45minutes of computer-based multimedia training in problem solving focusing on knowledge of procedures, conditional knowledge and planning. The training was a mainly task-based learning environment with feedback. Students in the control group received a software tutorial dealing with an introduction to the graphical user interface of a computer-based geometry package.

Following the training phase, a post-test was administered with three parts: 1) test instruments assessing the three problem-solving components (knowledge of procedures, conditional knowledge and planning ability) as well as a global analytical problem-solving scale consisting of an item selection from the PISA 2003 problem-solving test; 2) test instruments assessing knowledge of procedures, conditional knowledge, and planning ability in the field of mathematics as well as a global mathematics scale consisting of an item selection from the PISA 2003 mathematics test; and 3) a scale assessing figural reasoning (Heller and Perleth, 2000) as an indicator of intelligence as a covariate.

We expected the experimental group to outperform the control group in all three components they had been trained in (treatment check) and on the global problem-solving scale. We further expected a positive transfer of the training on the three components in the field of mathematics as well as on the global mathematics test.

Study I: Results and conclusions

Due to participants' self-pacing in the training phase and the first part of the post-test, the control group spent less time on training and more time on the first part of the post-test than the experimental group – although pre-studies regarding time on task indicated equal durations of the experimental and control group treatment. Therefore, the first part of the post-test was analysed by means of an efficiency measure (post-test performance [score] per time [min]).

Analysis of (co-)variance controlling for school track and intelligence showed that the experimental group worked more efficiently on the items testing planning (η^2 = .073), conditional knowledge (η^2 = .200), and knowledge of procedures (η^2 = .020) than the control group, indicating a positive treatment check. The experimental group also outperformed the control group on the global problem-solving scale (η^2 = .026) However, regarding transfer on the mathematical scales, the results showed no significant differences between the performances of the two groups (η^2 = .005).

In conclusion, the problem-solving training proved to be successful at enhancing students' efficiency when working on items assessing knowledge of procedures, conditional knowledge and planning ability as components of analytical problem-solving competence. The training was also successful at raising students' efficiency when working on problem-solving items from PISA 2003, which further underlines the importance of these components for successful analytical problem solving in general. However, no transfer of training on the mathematical scales could be found (Buchwald et al., 2017).

Study II: Method

The second study implemented an extended problem-solving training with a longitudinal design, additional transfer cues and prompts to enhance transfer on mathematics. This field experimental study used 173 9th grade students (60% female; mean age 14 years) participating as part of their regular school lessons. The students in each class were randomly assigned to either the experimental or the control group and were trained in a weekly training session of 90 minutes. Including pre-test, holidays and post-test, the training period lasted 15 weeks. The experimental group received a broad training in problem solving with a focus on planning, knowledge of procedures, conditional knowledge and meta-cognitive heuristics. The control group received a placebo training (e.g. learning to use presentation software) consisting of exercises that were important within and outside the school setting.

The problem-solving and mathematics pre-test and post-test consisted of an item selection from the relevant PISA 2003 tests. All items were administered in a balanced incomplete test design with rotation of domains and item clusters to avoid memory effects between pre- and post-test. We expected the experimental group to outperform the control group on the problem-solving test (near transfer) as well as on the mathematics test (far transfer).

Study II: Results and conclusions

In order to investigate near transfer training effects, a linear model was calculated. The predictors were group membership and problem solving in the pre-test (T1), and the criterion was problem solving in the post-test (T2). This model showed a significant effect of problem solving at T1 (f^2 = .37) as well as a significant interaction of problem solving at T1 and group membership (f^2 = .22). To investigate far transfer training effects, a linear model was calculated with group membership and problem solving at T1, and mathematics at T1 as predictors and mathematics at T2 as criterion. This model showed significant effects for mathematics at T1 (f^2 = .55), problem solving at T1 (f^2 = .21) as well as a significant interaction effect of problem solving at T1 and group membership (f^2 = .23).

These aptitude treatment interaction (ATI) result patterns indicate the extended problem-solving training to be effective for both near and far transfer for students with low initial problem-solving competence, but not for those with high initial problem-solving competence (Buchwald, Fleischer and Leutner, 2015; Buchwald et al., 2017).

Summary and discussion

The development of problem-solving competence is a crucial goal of various educational systems (OECD, 2013) from two perspectives: first, problem-solving competence is an important outcome of educational processes and second, it is also an important prerequisite for successful future learning both in and out of school. The German results from the PISA 2003 study of cross-curricular (analytical) problem solving suggested that students in Germany possess generic skills or cognitive potential which might not be fully exploited in subject-specific instruction in mathematics (OECD, 2005a; Leutner et al., 2004),

To investigate this hypothesis and to make use of students' cognitive potential for subject-specific instruction, it is necessary to gain a better understanding of the cognitive requirements of the PISA 2003 problem-solving items and their relationship to other cognitive constructs. This understanding could be used to develop and evaluate training and instructional tools aimed at improving problem-solving competence.

There are strong similarities between the steps and cognitive resources needed to solve mathematics and problem-solving items. Knowledge of concepts, knowledge of procedures, conditional knowledge, general problem-solving strategies, and the ability to self-regulate are all to different degrees relevant for both problem solving and mathematics (Fleischer et al., 2010). The three problem types used in PISA 2003 – decision making, system analysis and design, and troubleshooting – represent distinct but highly correlated dimensions of problem-solving skills with somewhat different relations to external variables such as subject-specific competencies and reasoning ability. The substantial correlations of these dimensions with subject-specific competencies, particularly for mathematics, suggest that a training in some aspects of problem solving should also improve subject-specific skills. Results from the German PISA 2003 repeated measures study also support this assumption (Leutner, Fleischer and Wirth, 2006).

A first experimental study into training on the components of knowledge of procedures, conditional knowledge and planning ability (as part of self-regulation) showed improved efficiency on the trained components and improved problem solving in general. This result further underlines the relevance of these components for analytical problem-solving competence, although the training failed to show transfer effects in mathematics (Buchwald et al., 2017). A second training study using an extended problem-solving training programme showed some evidence for the cognitive potential exploitation hypothesis. Among low-achieving problem solvers the training improved both problem-solving and mathematical competence, but not among students with high initial problem-solving competence, indicating a compensatory training effect (Buchwald, Fleischer and Leutner, 2015; Buchwald et al., 2017). These results are in line with the interpretation of the German PISA 2003 results that emphasised unused cognitive potential especially for low-achieving students (Leutner et al., 2005).

Students in Germany performed above the OECD average in both problem solving and mathematics in PISA 2012 (OECD, 2014). However, since the test focus has changed from analytical problem solving in PISA 2003 to creative problem solving combining both analytical and complex problems in PISA 2012 (OECD, 2013, 2014), future research has to elaborate whether this result indicates a better exploitation of students' cognitive potential. Future research should also address potential reciprocal causal effects between problem solving and mathematics in order to determine how cross-curricular problem-solving competence could be used to foster mathematical competence (Leutner et al., 2004; OECD, 2005a) and vice versa (Doorman et al., 2007; OECD, 2014).

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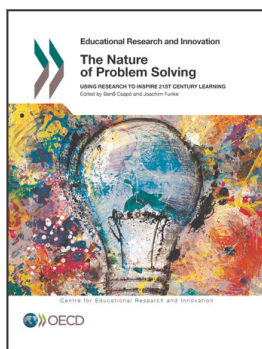
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