ANNEX 3.A1

Recent Trends in the Theory of Decision Making: Towards Procedural Rationality

During the 1960s and 1970s the theory of decision making under uncertainty became part of the standard curriculum in all leading Business Schools and Economics Departments. Now, pervasive uncertainty has always been the most obvious feature of decision processes in business and in economic policy making. Why did it take so long to develop a general theory of such processes, and what is the contribution of this theory to a more rational approach to real decision problems? The answer to the first part of the question is that no general conceptual approach to decision making under uncertainty was possible until the twin concepts of subjective (or personal) probability and of probabilistic utility were introduced in a clear and logically consistent way, and this did not happen until the late 1940s. Once these concepts were well understood it became possible to develop a theory based on three simple principles. First, the uncertainties present in the situation must be quantified in terms of values called probabilities. Second, the various consequences of the feasible courses of action must be similarly described in terms of utilities. Third, that decision must be taken which is expected, on the basis of the calculated probabilities, to give the greatest utility: any deviation from this rule is liable to lead the decision maker into procedures which are inconsistent.

As for the practical contribution to better decision making in business and in government: the theory allows us to open up the black box inside which the various ingredients of a decision problem are mixed and synthesised. It may be true, as President J.F. Kennedy once observed, that the essence of decision remains impenetrable to the observer, often even to the decider himself. But in a world where transparency and accountability are viewed as necessary conditions of legitimacy, decision makers in business, and even more those in government, are under an obligation to be as explicit as possible about the steps which led them to their final determination. In turn, this requires a conceptual framework within which the different components of the decision problem can be separately analysed, and then put together in a consistent way. Modern decision theory adds to the notion of "substantive rationality" – which applies to situations where uncertainty can be assumed away – that of "procedural rationality", which is especially relevant when uncertainty is too important to be disregarded. Whereas substantive rationality refers to the extent to which the chosen course of action leads to what, *ex post*, appears to be the optimal outcome, procedural rationality deals with how complex policy issues are structured.

A decision problem can be expressed as a list of alternatives and a list of possible events with the corresponding consequences. On the assumption of consistent comparison of events and of consequences, probabilities can be assigned to events, and utilities to consequences. Each alternative can also be assigned a utility, calculated as the expected value of the corresponding consequences. The best alternative is the one with the highest utility. Thus, the key assumption of the theory is that there is only one form of uncertainty and that all uncertainties can be compared. By saying that there is only one kind of uncertainty, and that therefore all uncertainties can be compared, it is meant that if E and F are any two uncertain events then either E is more likely than F, F is more likely than E, or E and F are equally likely. Moreover, if G is a third uncertain event, and if E is more likely than F, and F is more likely than G, then E is more likely than G. The first requirement expresses the *comparability* of any two events; the second expresses a *consistency* in this comparison.

The comparability and consistency requirements are then used to define the probability of any uncertain event E. This can be done in several, but equivalent, ways. For example, the probability of E can be obtained by comparing it with the probability of a point falling at random within a set S contained in the unit square. Because S is a subset of the unit square, its area is a probability, *i.e.* it is a positive number between 0 and 1, which satisfies all the rules of the probability calculus. Now, consistent comparability implies a unique value for the uncertainty of E, *i.e.* the probability of S (its area), is judged to be as likely as the uncertain event E, in the sense that a prize awarded on the basis of E occurring could be replaced by an equal prize dependent on a random point falling within S. The interested reader can find the details in any good textbook on decision theory, such as the one by Dennis Lindley (1971, pp. 18-26). In addition to a numerical measure of probabilities, we need a numerical measure for the consequences of our decisions.

We proceed as follows. Let c_{ij} be the consequence if we choose alternative A_i and event E_j occurs, $i = 1, 2,\dots n$; $j = 1, 2,\dots m$. Note that the consequences may be qualitative as well as quantitative. Denote by c and C two consequences such that all possible consequences in the decision problem are better than c and less desirable than C (it can be shown that the precise choice of c and C does not matter, as long as the condition of inclusion is satisfied; thus, we could choose as c the worst possible outcome in the payoff table, and C as the best outcome). Now take any consequence c_{ij} and fix on that. Consider a set S of area u in the unit square (the reason for using "u" will be clear in a moment; also, keep in mind that the area of S is a probability). Suppose that if a random point falls in S, consequence C will occur, while c will occur if the random point falls elsewhere in the unit square. In other words, C occurs with probability u and c with probability 1 – u. We proceed to compare c_{ij} with a "lottery" in which you receive C with probability u and c with probability 1 – u. Thus, if u = 1, "C with probability u" is better than (or at least as good as) c_{ij}, while if u = 0 then "C with probability u" is worse than c_{ij} . Furthermore, the greater the value of u the more desirable the chance consequence "C with probability u" becomes.

Using again the principle of consistent comparisons it can be shown that there exists a unique value of u such that the two consequences, c_{ij} and "C with probability u", are equally desirable in that you would not mind which of the two occurred. The argument consists in changing the value of u, any increase making the "lottery" more desirable, any decrease, less desirable, until "C with probability u" is as desirable as c_{ij}. We indicate this value with u and call it the *utility* of c_{ij}: $u_{ij} = u(c_{ij})$. We repeat the process for each of the possible consequences in the payoff table, replacing each consequence by its utility. The crucial point to remember is that all these utilities are probabilities and hence obey the rules of the probability calculus.

The final step consists in calculating the (expected) utility of each of the alternatives: $u(A_1)$, $u(A_2)$,… $u(A_n)$. Using the basic rules of probability, it is easy to show that $u(A_i)$ is simply the expected value of the utilities of all the consequences corresponding to A_i: $u(A_i) = u(c_{i1})p_1 + u(c_{i2})p_2 + \dots u(c_{im})p_m$. A moment's reflection will show that the expected utility of A_i is simply the probability of obtaining C, when this particular alternative is chosen. It follows that the best alternative is the one with the highest utility, being the one which maximises the probability of getting C. This is the principle of maximisation of expected utility, the major result of decision theory. Note that this principle, or decision rule, has nothing to do with the notion of an indefinite repetition of the same decision, as in some interpretations of expected gain in repeated games of chance. The principle follows directly from the rules of probability and hence can be applied to any decision situation, whether repetitive or unique.

One final point. Any decision under uncertainty, even one which does *not* make explicit use of probabilities, in fact implies at least a partial probability assessment. There is nothing mysterious in this statement, which is only a straightforward application of a line of reasoning frequently used also in elementary game theory (see for example Morrow 1994). Suppose a decision maker has to choose between two alternatives with the consequences indicated below:

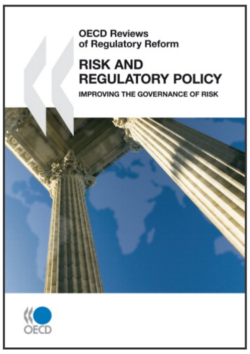
	E_1	E_2
A_1	10	1
A_2	3	2

Without attempting to estimate the probabilities of the uncertain events E_1 and E_2, but only taking the consequences in the payoff table into account, she chooses alternative A_2. This choice suggests that our decision maker is very risk-averse. In fact, she has used the "maximin" decision rule, according to which one should take the worst consequence for each alternative, and then select the alternative which offers the maximum of these minima; hence the name of the decision rule. Although the maximin does not use probabilities, the choice of A_2 indicates that the decision was taken *as if* the probability of E_1 was less than 1/8. In fact, letting p be the unknown probability of E_1, hence $1 - p$ the probability of E_2, the expected values M of the two alternatives are:

$$M(A_1) = 10p + 1\,(1 - p) = 9p + 1$$

$$M(A_2) = 3p + 2\,(1 - p) = p + 2$$

Thus, our decision maker is indifferent between the two alternatives if $9p + 1 = p + 2$, *i.e.* if $p = 1/8$. Any value less than 1/8 makes A_2 preferable to A_1. Since A_2 was chosen we infer that the decision maker implicitly assumed that the probability of E_1 is less than 1/8, q.e.d.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