Annex A1

THE PISA APPROACH TO ASSESSING STUDENT PERFORMANCE IN MATHEMATICS

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The PISA definition of mathematical literacy

The focus of the PISA 2012 assessment of mathematics was on measuring an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals in recognizing the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

The definition asserts the importance of mathematics for full participation in society and it stipulates that this importance arises from the way in which mathematics can be used to describe, explain and predict phenomena of many types. The resulting insight into phenomena is the basis for informed decision making and judgments.

Literacy in mathematics described in this way is not an attribute that an individual has or does not have; rather, it can be acquired to a greater or lesser extent, and it is required in varying degrees in society. PISA seeks to measure not just the extent to which students can reproduce mathematical content knowledge, but also how well they can extrapolate from what they know and apply their knowledge of mathematics, in both new and unfamiliar situations. This is a reflection of modern societies and workplaces, which value success not by what people know, but by what people can do with what they know.

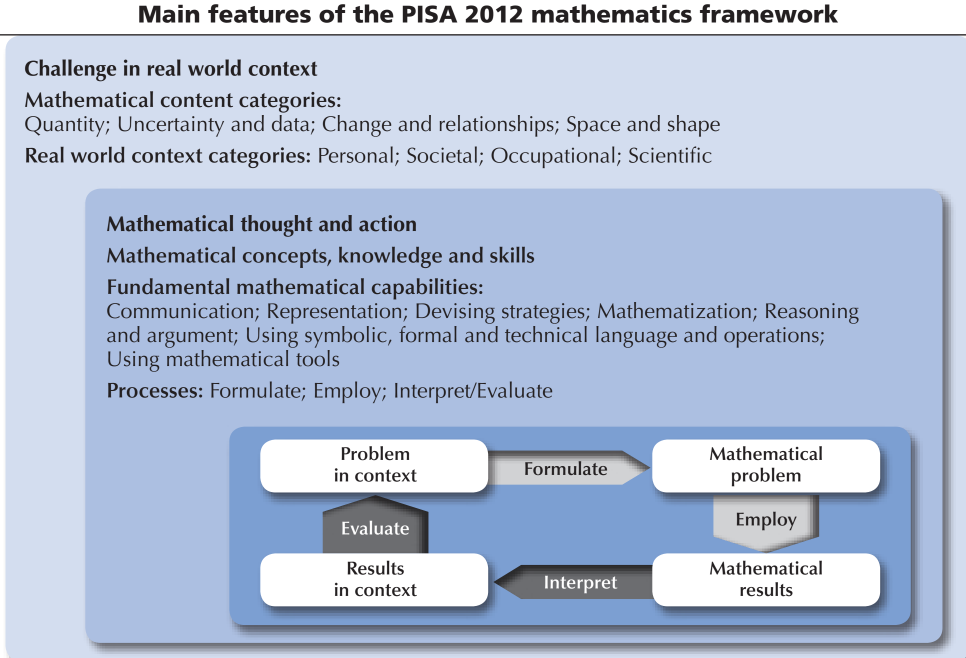
The focus on real-life contexts is also reflected in the reference to using "tools" that appears in the PISA 2012 definition of mathematical literacy. The word "tools" here refers to physical and digital equipment, software and calculation devices that have become ubiquitous in 21st century workplaces. Examples for this assessment include a ruler, a calculator, a spreadsheet, an online currency converter and specific mathematics software, such as dynamic geometry. Using these tools require a degree of mathematical reasoning that the PISA assessment is well-equipped to measure.

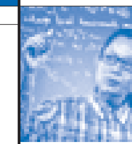
The PISA 2012 framework for assessing mathematics

Figure A1.1 presents an overview of the main constructs of the PISA 2012 mathematics framework that was established and agreed by the participating countries, and how the constructs relate to each other. The largest box shows that mathematical literacy is assessed in the context of a challenge or problem that arises in the real world. The middle box highlights the nature of mathematical thought and action that can be used to solve the problem. The smallest box describes the processes that the problem solver uses to construct a solution.

■ Figure A1.1 ■

Main features of the PISA 2012 mathematics framework





Context categories

Real-world challenges or situations are categorized in two ways: their context and the domain of mathematics involved. The four context categories identify the broad areas of life in which the problems may arise: personal, which is related to individuals' and families' daily lives; societal, which is related to the community – local, national or global – in which an individual lives; occupational, which is related to the world of work; or scientific, which is related to the use of mathematics in science and technology. According to the framework, these four categories are represented by equal numbers of items.

Content categories

As seen in Figure A1.1, the PISA items also reflect four categories of mathematical content that are related to the problems posed. The four content categories are represented by approximately equal proportions of items. For the assessment of 15-year-olds, age-appropriate content was developed.

The content category *quantity* incorporates the quantification of attributes of objects, relationships, situations, and entities in the world, which requires an understanding of various representations of those quantifications, and judging interpretations and arguments based on quantity. It involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results.

The content category *uncertainty and data* covers two closely related sets of issues: how to identify and summarize the messages that are embedded in sets of data presented in different ways, and how to appreciate the likely impact of the variability that is inherent in many real processes. Uncertainty is part of scientific predictions, poll results, weather forecasts and economic models; variation occurs in manufacturing processes, test scores and survey findings; and chance is part of many recreational activities that individuals enjoy. Probability and statistics, taught as part of mathematics, address these issues.

The content category *change and relationships* focuses on the multitude of temporary and permanent relationships among objects and circumstances, where changes occur within systems of interrelated objects or in circumstances where the elements influence one another. Some of these changes occur over time; some are related to changes in other objects or quantities. Being more literate in this content category involves understanding fundamental types of change and recognizing when change occurs so that suitable mathematical models can be employed to describe and predict change.

The content category *space and shape* encompasses a wide range of phenomena that are encountered everywhere: patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, navigation, and dynamic interaction with real shapes and their representations. Geometry is essential to space and shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas, such as spatial visualization, measurement and algebra. Mathematical literacy in *space and shape* involves understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes.

Process categories

The smallest box of Figure A1.1 shows a schema of the stages through which a problem-solver may move when solving PISA tasks. The action begins with the "problem in context". The problem-solver tries to identify the mathematics relevant to the problem situation, formulates the situation mathematically according to the concepts and relationships identified, and makes assumptions to simplify the situation. The problem-solver thus transforms the "problem in context" into a "mathematical problem" that can be solved using mathematics. The downward-pointing arrow in Figure A1.1 represents the work undertaken as the problem-solver employs mathematical concepts, facts, procedures and reasoning to obtain the "mathematical results". This stage usually involves mathematical manipulation, transformation and computation, with and without tools. The "mathematical results" then need to be interpreted in terms of the original problem to obtain the "results in context". The problem-solver thus must interpret, apply and evaluate mathematical outcomes and their reasonableness in the context of a real-world problem. The three processes – formulate, employ and interpret – each draw on fundamental mathematical capabilities, which, in turn, draw on the problem-solver's detailed mathematical knowledge.

However, not all PISA tasks engage students in every stage of the modeling cycle. Items are classified according to the dominant process and results are reported by these processes, formally named as:

- Formulating situations mathematically
- Employing mathematical concepts, facts, procedures and reasoning
- Interpreting, applying and evaluating mathematical outcomes

Fundamental mathematical capabilities

Through a decade of experience in developing PISA items and analyzing the ways in which students respond to them, a set of fundamental mathematical capabilities has been established that underpins performance in mathematics. These cognitive capabilities can be learned by individuals in order to understand and engage with the world in a mathematical way. Since the PISA 2003 framework was written, researchers (e.g. Turner, 2013) have examined the extent to which the difficulty of a PISA item can be understood, and even predicted, from how each of the fundamental mathematical capabilities is used to solve the item. Four levels describe the ways in which each of the capabilities is used, from simple to complex. For example, an item involving a low level of communication would be simple to read and require only a simple response (e.g. a word); an item involving a high level of communication might require the student to assemble information from various different sources to understand the problem, and the student might have to write a response that explains several steps of thinking through a problem. This research has resulted in sharper definitions of the fundamental mathematical capabilities at each of four levels. A composite score has been shown to be a strong predictor of PISA item difficulty. These fundamental mathematical capabilities are evident across the content categories, and are used to varying degrees in each of the three mathematical processes used in the reporting. The PISA framework (OECD, 2013c) describes this in detail.

The seven fundamental mathematical capabilities used in the PISA 2012 assessment are described as follows:

Communication is both receptive and expressive. Reading, decoding and interpreting statements, questions, tasks or objects enables the individual to form a mental model of the situation. Later, the problem-solver may need to present or explain the solution.

Mathematizing involves moving between the real world and the mathematical world. It has two parts: formulating and interpreting. Formulating a problem as a mathematical problem can include structuring, conceptualizing, making assumptions and/or constructing a model. Interpreting involves determining whether and how the results of mathematical work are related to the original problem and judging their adequacy. It directly relates to the *formulate* and *interpret* processes of the framework.

Representation entails selecting, interpreting, translating between and using a variety of representations to capture a situation, interact with a problem, or present one's work. The representations referred to include graphs, tables, diagrams, pictures, equations, formulae, textual descriptions and concrete materials.

Reasoning and argument is required throughout the different stages and activities associated with mathematical literacy. This capability involves thought processes rooted in logic that explore and link problem elements so as to be able to make inferences from them, check a justification that is given, or provide a justification of statements or solutions to problems.

Devising strategies for solving problems is characterized as selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context, and guiding and monitoring its implementation. It involves seeking links between diverse data presented so that the information can be combined to reach a solution efficiently.

Using symbolic, formal and technical language and operations involves understanding, interpreting, manipulating and making use of symbolic and arithmetic expressions and operations, using formal constructs based on definitions, rules and formal systems, and using algorithms with these entities.

Using mathematical tools involves knowing about and being able to use various tools (physical or digital) that may assist mathematical activity, and knowing about the limitations of such tools. The optional computer-based component of the PISA 2012 mathematics assessment has expanded the opportunities for students to demonstrate their ability to use mathematical tools.

Paper-based and computer-based media

PISA 2012 supplemented the paper-based assessment with an optional computer-based assessment, in which specially designed PISA units were presented on a computer and students responded on the computer. Thirty-two of the 65 participating countries and economies took part in this computer-based assessment. For these countries and economies, results are reported for the paper-based assessment scale and supplemented with a computer-based scale and a combined paper-and-computer scale ((see Annex B3 in OECD, 2013).

The design of the computer-based assessment ensures that mathematical reasoning and processes take precedence over mastery of using the computer as a tool. Each computer-based item involves three aspects:

- the mathematical demand (as for paper-based items);
- the general knowledge and skills related to information and communication technologies (ICT) that are required (e.g. using keyboard and mouse, and knowing common conventions, such as arrows to move forward). These are intentionally kept to a minimum; and
- competencies related to the interaction of mathematics and ICT, such as making a pie chart from data using a simple "wizard", or planning and implementing a sorting strategy to locate and collect desired data in a spreadsheet.

Response types

The response types distinguish between selected response items and constructed response items. Selected response items include simple multiple choice, complex multiple choice, in which students must select correct answers to a series of multiple-choice items, and, for computer-based items, "selected response variations", such as selecting from options in a drop-down box. Constructed response items include those that can be scored routinely (such as a single number or simple phrase, or, for computer-based items, those for which the response can be captured and processed automatically), and others that need expert scoring (e.g. responses that include an explanation or a long calculation).

References

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Turner, R., J. Dossey, W. Blum and **M. Niss** (2013), "Using mathematical competencies to predict item difficulty in PISA", in M. Prenzel, M. Kobarg, K. Schöps and S. Rönnebeck (eds.), *Research on PISA: Research Outcomes of the PISA Research Conference 2009,* Dordrecht, Springer, pp. 23-37.

Table A1.1 [Part 1/2]

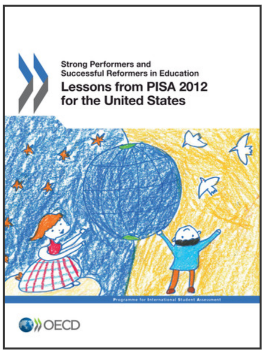
Items by solution rates for United States and comparator countries

		Solution rates (percentage of students providing the correct answer)							Framework categories	
Name	Item Code	United States	OECD average	Canada	Germany	Netherlands	Korea	Shanghai-China	Content	Process
Apartment purchase	PM00FQ01	37.60	44.67	51.25	42.78	59.47	62.02	70.48	Space and shape	Formulate
An advertising column	PM00GQ01	5.37	8.78	9.24	11.26	15.47	15.13	18.00	Space and shape	Formulate
Wheelchair basketball	PM00KQ02	7.01	14.86	19.15	21.26	17.80	14.57	45.83	Space and shape	Formulate
A View Room	PM033Q01	73.04	75.79	77.84	81.60	80.25	78.22	78.73	Space and shape	Interpret
Bricks	PM034Q01T	32.69	42.40	48.07	48.17	46.07	55.57	54.04	Space and shape	Formulate
Population Pyramids	PM155Q01	69.39	67.68	74.31	69.58	79.92	77.51	72.66	Change and relationships	Interpret
Population Pyramids	PM155Q02D	62.09	61.59	73.20	69.57	79.55	73.49	76.15	Change and relationships	Employ
Population Pyramids	PM155Q03D	14.82	18.69	25.63	22.07	29.22	34.08	42.97	Change and relationships	Employ
Population Pyramids	PM155Q04T	53.92	55.88	63.73	58.62	51.02	64.44	69.20	Change and relationships	Interpret
Containers	PM192Q01T	35.86	42.45	50.22	50.77	59.48	58.18	64.88	Change and relationships	Formulate
Pipelines	PM273Q01T	49.22	51.48	53.41	57.02	62.21	64.14	73.40	Space and shape	Employ
Map	PM305Q01	46.45	60.37	63.47	66.34	67.05	62.46	71.95	Space and shape	Employ
Running Tracks	PM406Q01	16.69	25.63	35.52	30.90	29.79	40.12	57.83	Space and shape	Employ
Running Tracks	PM406Q02	8.87	16.90	24.32	19.40	22.91	30.06	45.13	Space and shape	Formulate
Lotteries	PM408Q01T	33.40	39.41	47.91	49.43	36.86	39.02	53.74	Uncertainty and data	Interpret
Diving	PM411Q01	47.16	51.11	58.31	55.60	63.10	61.24	70.77	Quantity	Employ
Diving	PM411Q02	48.94	45.73	51.92	49.66	62.04	56.45	71.11	Uncertainty and data	Interpret
Transport	PM420Q01T	57.82	50.03	64.12	50.70	57.67	28.19	60.67	Uncertainty and data	Interpret
Tossing Coins	PM423Q01	74.59	79.06	80.05	81.24	83.79	81.91	94.43	Uncertainty and data	Interpret
Braille	PM442Q02	34.66	38.28	43.11	42.09	39.28	56.06	58.58	Quantity	Interpret
Thermometer Cricket	PM446Q01	74.74	68.59	77.93	65.00	73.19	79.33	88.07	Change and relationships	Formulate
Thermometer Cricket	PM446Q02	5.38	6.82	8.87	7.24	13.73	16.22	41.05	Change and relationships	Formulate
Tile Arrangement	PM447Q01	65.06	68.36	73.37	74.17	79.16	85.13	83.35	Space and shape	Employ
Third Side	PM462Q01D	10.16	12.20	7.62	12.49	7.32	44.38	76.82	Space and shape	Employ
The Fence	PM464Q01T	14.08	23.67	30.46	32.58	23.51	41.98	59.82	Space and shape	Formulate
Running Time	PM474Q01	69.64	74.33	74.61	77.50	75.19	81.77	80.75	Quantity	Employ
Cash Withdrawal	PM496Q01T	49.60	52.99	59.87	55.42	58.05	63.96	70.98	Quantity	Formulate
Cash Withdrawal	PM496Q02	60.92	66.76	70.43	69.97	68.77	80.00	82.53	Quantity	Employ
Telephone Rates	PM559Q01	56.81	63.16	65.55	66.75	67.53	81.35	88.22	Quantity	Interpret
Chair Lift	PM564Q01	41.28	46.12	48.48	51.56	57.97	63.37	71.27	Quantity	Formulate
Chair Lift	PM564Q02	39.30	45.82	51.86	48.39	58.58	52.25	74.74	Uncertainty and data	Formulate
Stop The Car	PM571Q01	44.03	47.68	53.06	56.11	57.65	58.10	63.80	Change and relationships	Interpret
Number Check	PM603Q01T	39.30	45.08	47.51	57.76	44.68	47.06	59.42	Quantity	Employ
Computer Game	PM800Q01	79.49	88.41	87.28	84.64	88.33	96.45	96.88	Quantity	Employ
Labels	PM803Q01T	25.78	29.19	36.01	36.06	44.29	43.23	45.89	Uncertainty and data	Formulate
Carbon Dioxide	PM828Q01	29.87	28.48	38.32	33.09	18.20	49.06	62.09	Change and relationships	Employ
Carbon Dioxide	PM828Q02	58.68	55.97	62.50	59.80	69.44	66.28	76.04	Uncertainty and data	Employ
Carbon Dioxide	PM828Q03	21.10	28.03	31.55	36.65	37.28	14.19	52.42	Quantity	Employ
Drip rate	PM903Q01	19.24	22.24	24.69	27.71	24.09	43.32	70.67	Change and relationships	Employ
Drip rate	PM903Q03	29.92	25.72	37.09	24.79	30.62	46.27	70.77	Change and relationships	Employ
Tennis balls	PM905Q01T	78.25	77.71	82.53	79.57	82.84	86.18	86.18	Quantity	Interpret
Tennis balls	PM905Q02	38.70	50.07	57.38	56.35	61.69	63.12	80.62	Quantity	Employ
Crazy ants	PM906Q01	50.70	60.67	62.11	67.66	75.23	61.07	79.18	Quantity	Employ
Crazy ants	PM906Q02	42.93	42.15	56.73	49.03	50.76	59.18	84.05	Quantity	Employ
Speeding fines	PM909Q01	92.91	89.35	94.40	92.44	95.72	93.51	94.43	Quantity	Interpret
Speeding fines	PM909Q02	56.32	63.14	68.04	72.49	77.16	71.25	74.25	Quantity	Employ
Speeding fines	PM909Q03	28.21	35.71	40.01	44.93	47.43	55.00	62.10	Change and relationships	Interpret
Carbon tax	PM915Q01	43.56	40.20	40.72	31.91	49.44	66.66	72.63	Uncertainty and data	Employ
Carbon tax	PM915Q02	52.94	68.26	76.10	80.46	75.40	74.02	90.10	Change and relationships	Employ
Charts	PM918Q01	91.93	87.27	86.89	87.95	90.43	90.76	92.55	Uncertainty and data	Interpret
Charts	PM918Q02	76.97	79.56	82.55	85.19	83.47	86.88	89.70	Uncertainty and data	Interpret
Charts	PM918Q05	76.66	76.68	83.58	77.00	81.88	85.05	89.04	Uncertainty and data	Employ
Z's fan merchandise	PM919Q01	78.76	84.53	85.81	88.68	86.30	89.06	94.13	Quantity	Employ
Z's fan merchandise	PM919Q02	39.28	44.73	49.38	46.86	49.51	52.37	56.01	Quantity	Formulate

■ Table A1.1 [Part 2/2] ■

Items by solution rates for United States and comparator countries

		Solution rates (percentage of students providing the correct answer)							Framework categories	
Name	Item Code	United States	OECD average	Canada	Germany	Netherlands	Korea	Shanghai-China	Content	Process
Sailing ships	PM923Q01	50.09	59.50	58.03	64.10	78.08	69.10	73.91	Quantity	Employ
Sailing ships	PM923Q03	43.69	49.81	57.49	53.59	61.97	56.27	81.84	Space and shape	Employ
Sailing ships	PM923Q04	11.48	15.28	20.72	19.61	24.83	21.16	47.04	Change and relationships	Formulate
Sauce	PM924Q02	51.32	63.48	60.64	66.49	81.32	73.22	85.13	Quantity	Formulate
Arches	PM943Q01	51.17	50.04	53.80	52.01	45.06	62.81	78.57	Change and relationships	Formulate
Arches	PM943Q02	2.48	5.29	4.14	5.82	3.25	19.51	50.93	Space and shape	Formulate
Roof truss design	PM949Q01T	57.68	67.54	75.97	73.06	77.77	85.38	89.75	Space and shape	Formulate
Roof truss design	PM949Q02T	27.81	31.75	38.49	26.59	21.14	73.62	86.04	Space and shape	Employ
Roof truss design	PM949Q03	31.45	32.57	35.93	32.00	39.54	68.39	83.20	Space and shape	Formulate
Flu test	PM953Q02	48.70	49.78	61.13	51.83	47.50	63.03	74.27	Uncertainty and data	Interpret
Flu test	PM953Q03	44.29	51.82	56.27	56.60	63.86	67.13	81.34	Uncertainty and data	Formulate
Flu test	PM953Q04D	16.52	18.22	24.74	20.22	23.67	35.38	53.08	Uncertainty and data	Formulate
Medicine doses	PM954Q01	70.33	65.42	71.93	68.42	58.88	81.18	92.45	Change and relationships	Employ
Medicine doses	PM954Q02	31.73	33.57	36.89	44.77	35.36	40.04	56.23	Change and relationships	Employ
Medicine doses	PM954Q04	23.54	26.36	33.00	36.01	32.63	38.76	54.49	Change and relationships	Employ
Migration	PM955Q01	73.54	72.13	79.06	79.03	70.39	75.15	84.35	Uncertainty and data	Interpret
Migration	PM955Q02	19.61	34.23	34.95	36.35	48.47	56.27	57.59	Uncertainty and data	Interpret
Migration	PM955Q03	5.97	11.99	14.97	14.35	22.29	23.96	42.90	Uncertainty and data	Employ
Employment data	PM982Q01	87.23	87.30	87.76	89.08	88.66	85.34	81.65	Uncertainty and data	Employ
Employment data	PM982Q02	33.19	30.74	35.81	28.72	30.06	44.53	37.59	Uncertainty and data	Employ
Employment data	PM982Q03T	58.54	64.96	64.97	69.84	70.77	73.80	73.29	Uncertainty and data	Interpret
Employment data	PM982Q04	43.32	51.46	51.98	60.01	66.01	58.89	68.10	Uncertainty and data	Formulate
Spacers	PM992Q01	75.45	77.61	84.69	83.29	79.63	86.78	90.98	Space and shape	Formulate
Spacers	PM992Q02	14.20	18.26	23.55	16.53	15.63	40.50	45.54	Space and shape	Formulate
Spacers	PM992Q03	5.43	8.11	8.72	9.67	6.87	27.34	38.07	Change and relationships	Formulate
Revolving door	PM995Q01	46.96	57.71	57.44	63.99	59.09	79.08	89.70	Space and shape	Employ
Revolving door	PM995Q02	2.26	3.47	4.26	3.32	4.09	6.35	13.62	Space and shape	Formulate
Revolving door	PM995Q03	44.62	46.44	54.21	49.38	56.58	52.65	65.20	Quantity	Formulate
Bike rental	PM998Q02	80.71	71.59	84.93	76.06	89.07	80.06	87.63	Change and relationships	Interpret
Bike rental	PM998Q04T	32.26	40.45	36.68	45.42	43.40	49.90	57.15	Change and relationships	Employ



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