Chapter 6. Designing effective number input: Lessons from cognitive science

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Mastering a basic understanding of numbers is critically important for children's future success in school, the workplace and everyday life. Unfortunately, individual differences in children's understanding of basic number concepts arise early in development and gaps in number knowledge are particularly apparent amongst children of varying socioeconomic statuses. Research in home and school environments suggests that much of the variability in children's number knowledge can be attributed to differences in quantity and quality of the number-related input that children receive from parents, teachers and caregivers. The present chapter explores sources of variability in children's understanding of numbers, factors that make learning about numbers challenging, and characteristics of effective number input. Finally, we discuss how previous and ongoing research can be useful in informing policies aimed at promoting early number knowledge and reducing achievement gaps.

Whether faced with the task of determining the trajectory of a satellite, developing plans for a new skyscraper, calculating a tip, or doubling a recipe, it is clear that developing a firm competency in mathematics is a crucial component of being successful in school, the workplace and everyday life. Mathematical achievement is a cornerstone of the science, technology, engineering and mathematics (STEM) domains, and as countries around the world recognise the need for a robust STEM workforce to fuel economic growth and foster innovation (Lacey and Wright, 2009[1]), it becomes increasingly important to improve students' STEM achievement in school. Basic maths skills are a necessity for the 21st century workplace (e.g. the logic and reasoning skills acquired through maths are essential to lawyers) and everyday life (e.g. cooking, calculating tips). Moreover, maths achievement levels are associated with other important life outcomes including earnings and contributions to society (Rivera-Batiz, 1992[2]).

A critical piece to improving the quality of mathematics education and STEM attainment in the United States and elsewhere is understanding the development of mathematical concepts in children. Individual differences in mathematical abilities emerge as early as the age of four, prior to the beginning of formal schooling, and these differences are predictive of later achievement gaps (MacDonald and Carmichael, 2017[3]). Additionally, these differences are especially apparent between children from lower and higher SES groups (Jordan, Huttenlocher and Levine, 1992[4]). Understanding how and why these differences emerge will enable us to work towards bridging the gap in maths achievement and encouraging a diverse group of students to pursue careers in STEM related fields.

One foundational area where individual differences are stark, is the understanding of the cardinal meaning of the number words. Although number development begins in early childhood, it progresses slowly and presents a challenge to young children. Children often learn to recite a portion of the count list, like the number words "one" through "ten", by their second birthday, but it typically takes around two additional years to learn what these words actually mean (Wynn, 1990[5]). Moreover, children learn each of the first three or four number words one at a time and in order over a period of 18 months or longer. Only then do children come to understand the cardinal principle, that the last number reached when counting a set represents the value of that set.

Although "number" may appear to adults as a single, unified concept, in reality there are many aspects of number and children do not appear to grasp these all at once (Wynn, 1990[5]). Children must learn that number words describe sets of objects (e.g. "three" tables) and do not label objects themselves (like "table") or even a property of an individual object (e.g. wooden) (Bloom and Wynn, 1997[6]). Some researchers have even proposed that children initially lack the concept of "set" and must develop it in the process of learning the meanings of number words (Carey, 2000[7]). Moreover, learning the meanings of number words requires that children learn numbers that refer to exact sets (e.g. "seven") as opposed to approximate sets (e.g. "some" or "few") (Barner and Bachrach, 2010[8]). Even still, children need to learn how to count as well as how the process of counting is related to individual quantities within the count list (Gelman and Gallistel, 1978[9]).

Related to the issue of why number words are challenging but also important in its own right is the question of how we can intervene to improve children's early number knowledge. The characteristics of number words that make them challenging provide insight into the types of input that may be helpful to teach children the meanings of number words. However, further research is necessary in order to translate the problems of learning number words into potential solutions. One body of research that we will review in this chapter are correlational studies that relate the quantity and qualities of number-related input that children receive to their subsequent understanding of number. Although such work is an important first step towards improving early maths education, even this research must be interpreted carefully given its correlational nature. Therefore, we will also review a growing body of research that is testing many of our theories about what constitutes effective number input by measuring the effectiveness of carefully designed number interventions. Such interventions allow researchers to make causal arguments about what types of number input are effective in promoting number development and are therefore the most direct path towards translating cognitive science to educational interventions.

The role of early input

Research examining children's early experiences shows that variations in children's language input has an impact on their language development and later literacy skills (Huttenlocher et al., 1991[10]). For example, early language input is related to variations in children's vocabulary growth and is responsible for strengthening the processing skills that facilitate language growth (Weisleder and Fernald, 2013[11]). Moreover, the richness of parental vocabulary and the overall quantity of parent talk accounted for differences in children's vocabulary growth both within and between SES groups (Hart and Risley, 2003[12]; Huttenlocher et al., 1991[10]). Based on these findings, interventions efforts have encouraged parents to engage their children in literacy related activities (e.g. Reach Out and Read; Too Small To Fail; Reading is Fundamental).

While there is much focus on the role of parental input on children's developing language skills, less attention is often paid to the importance of early input for the development of children's maths skills. Parents often believe that reading and social skills are more important for their preschool-aged children to learn than maths (Musun-Miller and Blevins-Knabe, 1998[13]) and that maths instruction is best left in the hands of teachers (Cannon and Ginsburg, 2008[14]). There has been pressure on schools and teachers to improve the maths performance of students, ignoring the fact that parents are the also teachers of their children. While a significant source of early number input may in fact be from day care and preschools, parental maths input also plays a critical role in the development of their children's mathematical knowledge (Bhanot and Jovanovic, 2005[15]; Gunderson and Levine, 2011[16]; Levine, Gunderson and Huttenlocher, 2011[17]). Understanding the role of early maths input on children's academic achievement, and assisting parents and caregivers in providing early maths input to children that is both frequent and qualitatively rich may help close the maths knowledge gap that emerges even before children enter kindergarten.

In fact, parents do engage their children in a wide variety of number activities (Blevins-Knabe and Musun-Miller, 1996[18]). Durkin et al. (1986[19]) conducted a longitudinal observation of ten mother-child dyads, and found that parents commonly used number words when their children were between the ages of 9 to 36 months. Further, the frequency of parents' use of smaller number words, that is "one" through "four", increased between the ages of 9 to 27 months, overlapping with the period when pre-schoolers' counting skills are developing (Gelman and Gallistel, 1978[9]). These naturally occurring maths interactions may provide natural leverage points to increase the quantity and quality of "maths talk".

Importantly, parental maths input varies by SES; when low- and middle-SES parents were asked about the types and frequency of maths activities their children engaged in at home, middle-SES parents reported engaging their children in a wider variety of maths-related activities and provided more support than did lower-SES parents (Starkey et al., 1999[20]). Middle-SES mothers also report engaging their children in more complex mathematical activities than low-SES mothers and these differences were correlated with children's performance on complex mathematical problems (Saxe et al., 1987[21]). It is possible that these SES differences are related to parents' varying education levels, differences in their exposure to mathematics courses, their own maths anxiety, and/or the value they place on their children's mathematical ability (Starkey et al., 1999[20]).

Not only do parents engage their children in maths talk to varying degrees, these differences predict children's later success in mathematics in the United States and elsewhere (Blevins-Knabe and Musun-Miller, 1996[18]; Huang et al., 2017[22]; LeFevre et al., 2009[23]). For example, Levine et al. (2010[24]) found large differences in the amount of number talk that parents engaged in with their children, and these differences related to children's later maths knowledge. They videotaped natural interactions of 44 parent-child dyads in their homes, visiting every 4 months for 9 visits starting at the age of 14 months. In their videos, Levine et al. (2010) found that the number input children received ranged from a low of 4 number words to a high of 257 number words across five 90-minute sessions – extrapolating this 60-fold difference would amount to a range of 28 to 1799 number words over the course of a week. Additionally, this variation in number input was related to children's own talk about numbers and their later understanding of the cardinal values of number words, even when controlling for overall parent talk to the child and SES, suggesting that it was specifically talk about numbers, and not talk in general, that is connected to later number knowledge. Interestingly, in a follow up to this study, Gunderson and Levine (2011[16]) found that parent number talk that referred to visible sets of objects was a significant predictor of cardinal number knowledge at 46 months of age, whereas number talk with no visible sets present did not predict cardinal number knowledge. Furthermore, parental talk that referenced larger sets (e.g. 4-10) was a better predictor of cardinal knowledge than was talk about smaller sets (e.g. 1-3). This suggests that the quality, and not only the quantity, of number input is related to children's mathematical development.

One pitfall of relying on parental report and observations to assess the connection of maths input to children's maths knowledge is that it does not allow us to draw strong conclusions about the causal nature of the relationship between number input and maths skill. Parents who talk more about number may be responding to their children's interest in number, for example. Or parents who provide more input may be better at maths than those who do not, and may be contributing to later variations in their children's maths ability through biological transmission of higher maths-related skills (Braham and Libertus, 2017[25]). Only experimental studies that manipulate the quantity and quality of maths input parents provide can provide the causal evidence needed to link parent maths talk to children's maths knowledge. Further, by manipulating the kinds of maths input provided, we can begin to test the effectiveness of particular kinds of input for children who differ in age and prior knowledge. Below we review the few existing studies that have taken this approach, often drawing on lessons learned from cognitive science research.

Interventions

Insights from how children learn relational language broadly, as well as analyses of children's errors when learning maths terms specifically, suggest that children may have difficulty attending to the features of a scene that are labelled by relational terms, including maths terms (Bloom and Wynn, 1997[6]). Therefore, maths instruction may be most effective when it clearly draws children's attention to features of children's experience that are relevant to maths vocabulary and concepts. In this vein, researchers have developed interventions that draw on principles from cognitive science to better support the development of mathematical concepts.

Efficacy studies: Interventions in controlled laboratory settings

Analogical reasoning is a powerful tool for teaching children about abstract properties. For instance, previous studies have shown that giving children multiple examples of a concept can help highlight a key similarity or difference and prevent children from focusing on erroneous features of the objects that do not correspond to the target concept but that are particular salient to children (Christie and Gentner, 2010[26]). Interestingly, commercially available tools for teaching children about number such as picture books often use a different strategy – mainly, pairing examples of sets that differ on multiple dimensions (e.g. two houses versus three dogs). To explore whether structural alignment could be a useful resource in teaching children about number, we provided children with evidence designed to either broaden their understanding of a given number word by comparing sets of multiple kinds (low aligned) or to narrow children's definition by contrasting multiple sets of only one kind (high aligned). Findings suggest that children may initially benefit more from a situation which limits the context in which they learn a new number word (e.g. holding the types of objects that comprise different sets constant) before broadening their definition of new number words to include sets of unfamiliar objects (Gibson, Congdon and Levine, 2015[27]).

There are numerous other ways that the context in which a child hears a number word can help children develop an understanding of number. There is evidence that labelling the objects within a set prior to labelling the number of items in the set can help children learn the meanings of number words (Ramscar et al., 2011[28]). In other words, saying, "balls, there are three" was more effective than "there are three balls", perhaps because by labelling "balls" first, children are less likely to assume that "three" means "balls" and more likely to focus on the number of items.

Mix et al. (2012[29]) looked at the relative benefits of four training approaches that manipulated the context in which number words were embedded: counting sets only, labelling cardinal values of sets only, labelling cardinal values of each set and then immediately counting, and alternating between counting and labelling cardinal values at each session. Children were trained once a week for six weeks and then tested on their understanding of the cardinal principle. They found that children only showed significant improvements in their understanding of the cardinal principle in the combined counting and cardinal labelling training condition, demonstrating that providing a clear link between counting and cardinality provides better information for children to learn the number words than either piece of input individually.

Other research has also started to examine how the types of objects used to teach children number words might be differentially beneficial to children as they attempt to break into an understanding of number. Petersen and McNeil (2013[30]) compared performance in counting tasks that used either perceptually rich or simple objects, and varied by whether children had prior established knowledge of the objects or not. They found that using perceptually rich objects was only beneficial to children's performance when children did not have established knowledge of the objects. In fact, perceptually rich objects hindered performance on the counting tasks when children were familiar with the objects, likely because children were focused on the objects themselves rather than on the set size. In a related study, Petersen et al. (2014[31]) trained children on number words using either pictures of sets or three-dimensional objects and found that practicing counting with pictures helped improve children's understanding of cardinality while practicing counting with objects did not. They suggest that the high representational status of the pictures makes them less distracting than the objects, and that the objects might have afforded more off-task distractions than did the pictures.

Finally, manipulating the type of maths input children receive is not only important early on, before formal schooling begins, but can be beneficial to children once they are in school and dealing with more formal mathematics. For example, children's numerical magnitude estimation abilities are related to later mathematical abilities, but children from low-SES backgrounds tend to have poorer knowledge of numerical magnitudes (Siegler and Ramani, 2008[32]). To address this inequality, Siegler and Ramani (2008[32]) had low-SES preschool-aged children play a simple numerical board game in four 15-minute sessions over the course of two weeks. Playing this simple game drastically boosted the accuracy of children's number line estimations, suggesting that exposure to linear numerical board games helps build children's intuitive understanding of numbers.

All of these findings highlight the importance of considering the quantity and quality of exposure to mathematical concepts that parents and teachers provide. Yet, many of these interventions are only first steps towards developing more effective maths interventions that can be implemented in schools and homes. The interventions we have just described took place under semi-controlled conditions, conducted by trained researchers. However, it is important to take into account that it may not be possible for homes to reflect the best practices discerned in the lab (e.g. Mix et al. (2012[29]) found that parents rarely provide cardinal labels immediately followed by counting). To develop effective ways to support children as they learn challenging mathematical words and concepts, we need to have a better idea of what it looks like when parents, or teachers, implement the kinds of interventions that have been designed and tested in laboratory studies. Recently, researchers have started to think about how we can encourage and support parents to provide better quality input to their children, by focusing on interventions that target parent/child interactions rather than solely focusing on content provided to the child alone.

Effectiveness studies: Interventions in the wild

One such intervention focused on exploring how providing children with greater exposure to talk about the numbers 4 through 6 might promote an understanding of cardinality above and beyond exposure to talk about the numbers 1-3. As previously mentioned, in a correlational study, Gunderson and Levine (2011[16]) found that parents' labelling and counting of larger sets (4-10) of present objects was a better predictor of children's later cardinality scores than was talk about smaller sets (1-3). To test this finding, we created two different versions of a number book – one that contained only the numbers 1-3 and the other than focused on the numbers 4-6 – to look at which book promoted greater gains in children's cardinal understanding of the values of numbers, and whether this might vary depending on children's pre-existing number understanding as assessed by their "knower level" on the Give-N task. Following a brief pre-test, children were randomly assigned, within knower level, to one of three conditions (small number, large number, and a control, adjective book), where parents were asked to read these books to their children frequently over the course of a month. After the month-long period, children were invited back to the lab and again tested on their understanding of the cardinal values of numbers. Findings showed that children who were 1 and 2 knowers benefited more from the 1 to 3 books than from either the 4 to 6 books or the control adjective books. In contrast, children who were

3 or 4 knowers benefited from both the 1 to 3 books and the 4 to 6 books, compared to children in the control adjective condition.

In somewhat older children, we assessed the effectiveness of a maths app in promoting 1st grade children's maths knowledge. By this age, we know there is a negative relationship between parent maths anxiety and children's performance, where students with high-maths-anxious parents learn less maths over the school year than students with a low-maths-anxious parent. Moreover, when high-maths-anxious parents help their children more often with homework, the children learn less over the school year, controlling for parents' education and classroom level variables (Maloney et al., 2015[33]). Therefore, in a recent study, we asked whether engaging parents and children in scripted, high-quality maths interactions might help ameliorate the negative effects of being a maths-anxious parents. Indeed, we found that when first-grade children in households with a high-maths-anxious parent used a maths-related iPad app as little as one or two times per week, they made gains in maths equal to those of their peers with low-maths-anxious parents (Berkowitz et al., 2015[34]). Moreover, these benefits persisted: being assigned to the maths group continued to have beneficial effects for children of higher maths-anxious parents well into second grade, when the children were rarely using the app, if at all (Schaeffer et al., 1999[35]). Creating a context for parents and children to have positive maths interactions led to significant gains for those children who might not otherwise have received high-quality maths input at home, partly because parents in the intervention group began to increase their value of maths achievement for their children, as well as their expectations of their children's achievement in maths. These few studies demonstrate how the lessons learned from cognitive science can be applied and tested in real world settings. Findings suggest that research-based interventions can lead to positive maths learning gains. There are of course many remaining questions having to do with the mechanisms that undergird these gains, and whether or not the interventions lead to long-term benefits in achievement.

Policy implications

Evidence that gaps in maths achievement are already apparent as early as kindergarten drive home the need for early interventions that reach homes as well as schools. In creating these interventions, it is important to consider the obstacles that parents and teachers face in talking effectively about numbers with young children. Maths anxiety and SES are just two factors that we know are related to parent-child maths interactions, and so interventions should be designed to provide adequate support to adults from a variety of backgrounds and with a variety of their own feelings about maths. By ensuring that students are better prepared for maths at the time of school entry, we can improve students' academic outcomes. Given the growing body of research on the factors that are important for developing children's early mathematical thinking, researchers and policymakers should work to develop effective ways of broadening the reach of research-based mathematics interventions.

The research discussed above suggests that technology can be a useful tool to improve parent-child interactions around maths. As computers and smartphones with internet access become a staple in every household, websites can be an increasingly effective way to demonstrate positive ways to incorporate maths into the home and classroom. Parents can look to websites such as Maths 4 Mums and Dads (maths4mumsanddads.co.uk) and Becoming a Maths Family (becomingamathsfamily.uchicago.edu) for ideas of how to talk to their children about maths and use numbers in everyday activities. Similar resources are being developed to support teachers as well. For example, DREME TE (http://prek-maths-te.stanford.edu) provides early maths resources for teacher educators to use in their classrooms. As research on effective early maths input grows, and technological advances make it increasingly possible to reach people in their homes, we are excited about the potential impact research can have on early mathematics education and the benefits this can reap over time in terms of STEM outcomes.

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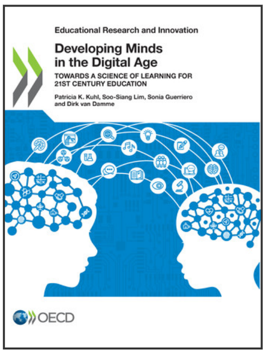

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