Cost-Benefit Analysis and the Environment
Further Developments and Policy Use

PART II

Chapter 8

Discounting

Discounting is both a critical and pervasive issue in CBA, and this is nowhere more so than in environmental applications. On the one hand, this is a technical matter arising from the standard assumption in CBA that the social or shadow price of a unit of consumption in the future is lower than the price of a unit of consumption today. The discount rate simply measures the rate of change of the shadow price. This simplicity is, of course, a matter of extent. While the theory of social discounting shows clearly how the social discount rate should be defined, in practice numerous questions arise especially when considering actions with implications for generations in the far distant future: intergenerational projects and policies. Not only do the assumptions underpinning conventional discounting become problematic but also the ethical underpinnings of discounting become extremely important and influential. As a result, the chapter discusses how the parameters of the discount rate for social CBA are determined as well as their ethical and practical content. This involves a discussion of the problems introduced to the conventional discounting approach by intergenerational projects such as climate change and the strengthening of theoretical and empirical support for schedule of discount rates that decline with time.

8.1. Introduction

Martin Weitzman referred to discounting as one of the most "critical problems in all of economics" (Weitzman, 2001, p. 261). It is a pervasive issue in many economic analyses, particularly in cost-benefit analysis (CBA) and Cost Effectiveness Analysis (CEA). The sensitivity of CBA and CEA to the social discount rate it particularly pronounced when considering public policy or investments with long-lived costs and benefits, such as energy investments (e.g. nuclear power), investments in public health (e.g. eradication of disease), and mitigation of climate change and other long-lived environmental benefits or infrastructure. In this chapter the arguments and social discounting are described in theoretical and empirical terms, and the way in which international practice has interpreted these arguments is described.

In order to place all goods and services in a common metric or numeraire, CBA uses market or *shadow* prices. In this way the social value of apples, oranges, clean air etc., can be compared in terms of Euros or Dollars of consumption. When costs and benefits occur over time, CBA must also place these costs and benefits in a common *temporal* metric to account for changes in the social (real inflation-adjusted) value of the numeraire at future dates. The typical approach is to convert all costs and benefits into present day values, that is, calculate the *present value* of costs and benefits. The process of calculating the present value reflects the idea that there is a price associated with the date at which benefits and costs occur. Typically in CBA it is assumed that the *shadow* price of a unit of consumption in the future is lower than the price of a unit of consumption today. So when one adds up the net benefits of a particular project over time, future costs and benefits receive less weight (lower price) than present ones. The *social discount rate* (SDR) measures the (negative) rate of change over time of the shadow price of the numeraire. A positive discount rate means the shadow price is declining with the time horizon.

This chapter outlines the arguments for using a positive discount rate. There are pure welfare arguments associated with how society values welfare at different points in time, and there are opportunity cost arguments, reflecting the fact that there are alternative projects that a government could invest in. Given that the SDR relates to a price, asset pricing theory can also inform the appropriate price of a claim on a cost or benefit at some future point in time. There remains disagreement among academics and practitioners as to which approach should be taken to social discounting in any given circumstance, and in practice a variety of approaches have been taken by Governments around the world. Among the two main sources of disagreement is whether a normative/prescriptive or a positive/descriptive approach should be taken to the evaluation of public investment and regulations.

The normative approach focusses on the trajectory of social welfare as measured by discounted social welfare (utility) while largely ignoring the trajectory of the rate of return to capital on the opportunity cost side. This approach focusses on the question of the price that *ought* to be placed on future costs and benefits. The positive approach focusses on the trajectory of observable rates of return as its source of information for the SDR. The methodology is positive/descriptive since it focusses on the inter-temporal trade-offs that take place in the economy currently, and selects an SDR for public policy analysis from the rates of return that are observable in the economy. In doing so, the positive approach ignores the trajectory of social welfare.

In the medium term context of many public investments (10-30 years), the arguments surrounding the SDR only make a minor a practical difference to the outcome of CBA. It is when CBA is undertaken over longer time horizons that different positions on the SDR have a material consequence on the type of project that will pass a NPV test. Marginal projects in the realm of energy, climate change mitigation, biodiversity conservation and public health have time horizons of hundreds of years, and consequences for as yet unborn generations. Many argue that in such contexts, the positive approach is constrained by the time horizon of observable assets, which is limited to the 40-50 year duration of government bonds. In such cases the normative arguments for the SDR have become much more prevalent. Ultimately, different governments take different approaches even in the medium term, with the UK government and the EU guidelines on CBA focussing on the normative welfare arguments, whereas the US, Norwegian and Dutch governments, for instance, take a clear positive perspective and embed their SDRs in observable market rates of return. Yet when it comes to project appraisal for long-time horizons, for *intergenerational* projects, many governments now recognise that the standard discounting arguments may need to be augmented, or alternative approaches to appraisal should be considered.

Another important issue that relates to social discounting is risk. This chapter first outlines the theory of social discounting in a risk-free context in which interest rates and growth rates of consumption are certain and projects have sure benefits and costs. This defines a risk-free social discount rate: the rate applicable to risk free projects in a risk free world. The analysis is then extended to deal with risk. First, the impact on the risk free rate associated with the uncertain consumption in the future is described. Second, the implication for the SDR of project risk is described, and the need for risk premium for risky projects outlined. Again, in practice governments differ in their treatment of risk in CBA, with some using a risk-free rate, others adding a risk premium.

When considering the policies, projects and investments with implications for generations in the far distant future i.e. intergenerational projects, conventional discounting leads to a situation in which large costs and benefits that accrue in the distant future become insignificant in PV terms, because the shadow price associated with that time horizon is very small indeed. As this chapter outlines, there are some good theoretical reasons for this apparently myopic outcome. However, not only do the assumptions underpinning conventional discounting theory become problematic when such long time horizons are considered, but also the ethical underpinnings of discounting become extremely more influential. Exactly this kind of ethical debate has taken place in recent years in relation to climate change policy, and it is equally applicable to other important long-term policy questions, e.g. biodiversity conservation and nuclear power. The theoretical arguments for time varying discount rates are outlined, explaining in particular the theoretical arguments for declining risk-free discount rates. Many governments now deploy declining discount rates in their guidance on the basis of these arguments, and the later sections of this chapter discuss the empirical side of operationalising these theories.

Once the discount rate is known, then CBA or CEA of different investments or policies can be undertaken by calculating the present value (PV) and comparing it to the status quo (no other project), or to other potential public investments. A simple numerical example is provided to explain this calculation (See Box 8.1).

Box 8.1. **Discounting and the net present value criterion**

Imagine a consumer can always earn a rate of return *r* per period on funds invested in the bank. This means that an investment of EUR 1 in period 0 will earn EUR 1*(1+*r*) one period in the future. Any alternative investment can now be compared to this baseline by calculating the present value (PV). The baseline rate of return, *r*, becomes the opportunity cost of investing in another project. Given this, one can calculate the relative "price" associated with returns in period 1 rather than in period 0, or the *discount factor* (DF), using *r* as the *discount rate* as follows:

$$DF = \frac{1}{(1+r)}$$

More generally, the discount factor for a benefit (or cost) accruing at any time *t* periods in the future is:

$$DF_t = \frac{1}{(1+r)^t}$$

The discount factor makes it possible to evaluate the desirability of other investments by stating their returns in today's terms taking into account what could have been earned in the bank. Suppose another investment opportunity provides a return **B** at time period 1 for a EUR 1 investment at time 0. One can compare this to the returns with the bank by comparing the benefits at time 1. Since they both cost EUR1 the project is preferred if:

$$\underset{\text{Project payoff}}{B} > \underset{\text{Bank payoff}}{(1+r)}$$

But this is an equivalent criterion to:

$$\frac{B}{(1+r)} > 1$$

Where the LHS is simply the present value of *B* and the RHS is the present value of the returns from the bank. So the evaluation criterion becomes:

$$PV_{\text{Project}} = \frac{B}{(1+r)} > 1 = PV_{\text{Bank}}$$

The PV of returns from investment in the bank is EUR 1 (= EUR 1*(1+r)/(1+r)). So, comparing present values means that the alternative investment yields higher profits than funds invested in the bank if the following criterion holds:

$$PV_{\text{Project}} > 1$$

More generally, the net present value (NPV) is the present value of benefits minus the present value of costs. Since the project cost EUR 1, the NPV of the project is given by:

$$NPV_B = PV_B - 1$$

NPV is therefore another criterion with which to evaluate investments. If NPV > 0 then the project is worthwhile since the present value of cash is higher, otherwise, the returns from the bank are higher. More generally, with benefits and costs at time *t* given by B_t and C_t respectively, the NPV for any project can be calculated as follows:

$$NPV = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+r)^t}$$

The question for CBA is, what is the appropriate social discount rate for calculating the NPV of public projects?

This chapter brings all of these issues together. In order to clarify the welfare/consumption side arguments and the opportunity cost arguments for social discounting, and how they are related to one another, the chapter begins with an introduction to the neoclassical theory of discounting and the Ramsey Rule. This discussion illustrates the welfare significance of the various candidates for the social discount rate (SDR): the social opportunity cost of capital (SOC) and the social rate of time preference (SRTP), as well as hybrid methods. The chapter discusses which rates of return can be used to inform each approach. It also explains what the parameters of the SRTP mean, how they are determined and their ethical content. The chapter then progresses to the issues of inter-generational equity associated with discounting the far distant future. Some of the issues that arise are technical problems, but there are ethical issues at stake too.

The chapter concludes with some examples of international practice and some advice on the issues that have to be borne in mind when deciding on a discounting policy such as when the SOC method should be preferred to the SRTP approach, how to deal with long-term issues, and how to deal with growth and project based risk. The chapter serves as a short summary of the exploding literature in social discounting that followed the Stern Review on the economics of climate change (Stern, 2007).

8.2. Discounting theory

In order to illustrate the welfare arguments for social discounting in CBA, and how the opportunity cost arguments relate to social welfare, this section explains the Ramsey Rule. What this makes clear is the relationship between a positive NPV (see Box 8.1) and an increase in social welfare: if the NPV calculated using the social discount rate is positive, then social welfare is increased. The relationship between the Ramsey Rule and the consumption based asset price theory is also explained.

8.2.1. The Ramsey Rule

A conventional analysis of the social discount rate begins by embedding the evaluation of projects in the context of a well-defined measure of inter-temporal social welfare. The standard approach uses the discounted utilitarian social welfare function (DU). The DU approach is a representative agent model in which the well-being of society is measured by the utility of a representative person's utility function: $U(c)$. This was the approach taken by Ramsey (1928) in his seminal analysis of the optimal savings rate. The objective in the Ramsey model is to choose savings and consumption to maximise the discounted sum of utility over an infinite time horizon:

$$\max_{c_t} \sum_{t=0}^{\infty} \frac{U(c_t)}{(1+\delta)^t} \qquad [8.1]$$

given a return to savings/investment equal to the marginal product of capital: $f_k(k) = r$, and where δ is the *utility discount rate*.[1] In CBA, the social discount rate is given by the answer to the question: at what rate should society be compensated in the future for giving up a unit of consumption today such that overall well-being is preserved. The answer is given by the optimality condition known as the Ramsey Rule:

$$r = \delta + \eta g \qquad [8.2]$$

The right hand side is the welfare-preserving rate of return to consumption, often known as the *social rate of time preference* (SRTP). This consists of the utility discount rate, δ, the elasticity of marginal utility, η, and the growth rate of per-capita consumption, g. The left hand side is the social rate of return to capital, *r*, available in the economy. This reflects the opportunity cost arguments for discounting.

Why is [8.2] informative about the social discount rate? If a project funded by a unit of consumption today has a rate of return in the future higher than the SRTP, then it will increase inter-temporal welfare as measured by [8.1], since the SRTP is the rate that just compensates for the unit of consumption foregone. If a project funded by displacing investment has a higher rate of return than the forgone investments, which have a rate of return *r,* then it too will increase [8.1]. Along the optimal path, or if markets are perfect, these rates will be the same. For this reason either *r* or SRTP are valid candidates for the *social discount rate*, SDR. When undertaking CBA in this economy, projects whose consumption valued costs and benefits have a positive (negative) net present value when discounted using the SRTP or *r* will increase (decrease) social welfare. The Annex has a proof of this statement.

There is no uncertainty in the model so far. The rate of growth in consumption and rate of return to capital are known. Furthermore, the proof in the Annex assumes that the benefits and costs of the project are certain: there is no project risk. Therefore, the Ramsey Rule in [8.2] is generally only appropriate for risk-free projects: the SRTP and *r* are risk-free rates.

The Ramsey Rule can also be understood in terms of asset pricing theory. The RHS relates to the asset price to a claim on a risk-free consumption valued benefit with a maturity *t* in the future. The Annex shows the fundamental equation of asset prices and shows how one obtains the RHS of the Ramsey Rule in this context.

The Ramsey Rule is an optimality condition, which also holds in a perfectly competitive, perfect-foresight, decentralised economy. The RHS defines the welfare arguments for social discounting, the LHS relates to the opportunity cost arguments. When the economy is not on the optimal path, or is not perfectly competitive, e.g. distorted by taxation, questions arise as to which side of the Ramsey Rule should inform the SDR. This question is the source of disagreement between those who argue for a normative or prescriptive approach and those who argue for a positive or descriptive approach to the SDR. The former approach involves calibrating the social welfare function and the parameters of the RHS of [8.2]. The latter would search for an appropriate rate of return – a risk-free rate in this case – in the market place.

Related to this debate are the well-known asset-pricing puzzles: the risk-free rate puzzle and the equity-premium puzzle. The risk-free rate puzzle is the observation that if a consumption-based asset pricing model, like the RHS of the Ramsey Rule in [8.2], is calibrated using standard parameters, it over-estimates the observed risk-free rates. The equity-premium puzzle[2] relates to a similar problem in the context of risky assets: the standard model under-estimates the equity premium and hence the rates of return to risky assets. These puzzles are discussed in more detail later in the chapter, but they illustrate the differences that can arise between simple normative models like the DU model, and the observed rates of return that define the positive approach.

The normative and positive approaches as now discussed in detail in the risk-free context, before looking at these broader issues.

8.2.2. A normative approach to the SDR: calibrating the social rate of time preference (SRTP)

The normative approach to the SDR focusses directly on the welfare and consumption side of the Ramsey equation in [8.2], rather than production side. The normative approach answers the question: how *ought* we to discount future societal costs and benefits? The RHS of [8.2], $\delta + \eta g$, is known as the social rate of time preference (SRTP) and reflects the consumption-side motivation for discounting. It indicates the rate at which consumption tomorrow would have to increase to keep social welfare constant given a unit reduction in consumption today, given that the economy is growing at a rate g. Two approaches exist to estimating the SRTP. The *normative* approach calibrates the parameters on the RHS of [8.2]. The *positive* approach uses post-tax returns to saving to reflect the way in which individuals trade-off consumption and saving over time.

The parameters of the Ramsey rule essentially define the form of the welfare function in [8.1]. The following sections will now describe the conceptual meaning of the parameters of the SRTP in more detail, before remarks on how to evaluate them numerically are being presented:

8.2.3. The utility discount rate, δ

This parameter component has typically reflected two distinct concepts:

- *Pure time preference*: A preference for units of social welfare today rather than tomorrow. For social CBA it should reflect society's pure time preference, rather than that of individuals. However, when considering long-term projects, this parameter has an important ethical interpretation and reflects a judgement on intergenerational equity (Beckerman and Hepburn, 2007).
- *Life chances*: It is often argued that another reason to discount future welfare or utility arises because of uncertainty. At an individual level, this would reflect the risk of death. However, for society the appropriate risk to incorporate is the risk of catastrophe eliminating a society. Dasgupta and Heal (1979) stated that positive utility discount rate can be justified because there is a positive probability that society will not exist in the future. Different interpretations have led to different ways of measuring this component.

8.2.4. The elasticity of marginal utility, η

This term also has numerous interpretations depending on the context. In general, it describes the nature of the relationship between consumption, c_t, and welfare/utility in the function, $U(c_t)$. In fact, it is a measure of the curvature of the utility function. Diminishing marginal utility is the typical assumption, which implies $\eta > 0$. In practice η is treated as if it is a fixed parameter, and yet in principal the elasticity could vary with the level of consumption.[3] The elasticity of marginal utility can be interpreted in the following different ways:

- *Consumption smoothing*: The extent to which an individual wishes to smooth consumption over time, i.e. avoid large fluctuations in consumption. Larger values of η indicate a stronger desire for stable consumption.
- *Inter- and intra-generational inequality aversion*: η is often understood to be a measure of both *inter-* and *intra*-generational inequality aversion, that is, the strength of preferences for more equal distributions of income. For instance, if $\eta = 1$, this means that the marginal utility of an additional unit of consumption is twice as much for a person with only a half of the income. With $\eta = 2$, marginal utility is 4 times higher, and for $\eta = X$, 2^X time higher for the person with half the income. So higher values of η reflect greater aversion to income inequality and place higher values on income accruing to the poor.
- *Relative risk aversion*: When consumption or project risks are present, η also measures risk aversion. A high value of η indicates a strong aversion to risk.

Taken together, the SRTP embodies two reasons why one may wish to discount risk-free projects:

Utility discounting, δ: if one values future utilities less for reasons of impatience or hazard: $\delta > 0$;

The wealth effect, ηg: The weight one places on the future depends on what state one will find oneself (or future generations) in the future. If society is richer in the future, $g > 0$, and has a preference for consumption smoothing, or is averse to the income inequality that growth introduces, $\eta > 0$, then less value will be placed on increments to consumption in the future, hence future benefits and costs are discounted. Society values projects that have payoffs in the future less if the future is richer and there is diminishing marginal utility.

8.2.5. *The social opportunity cost of capital, r*

The left hand side of the Ramsey equation broadly reflects the production possibilities in the economy, rather than the consumption possibilities reflected by the SRTP. The term r in [8.2] is the equilibrium social marginal productivity of capital in the economy. This is another candidate for the SDR since it reflects the social opportunity cost of capital (SOC), that is, the alternative rate of return that a government could obtain by investing public funds elsewhere in the economy, or, the cost of financing a public project from the capital markets. The SOC is a natural yardstick against which the use of public funds should be measured. Many countries use the SOC approach for social discounting (See Table 8.4). One complication is that the SOC will depend upon the precise source of funding for the project.

In the deterministic framework that has been presented here, the LHS of the Ramsey Rule as stated in equation [8.2] refers to a *risk-free* rate of return to capital, henceforth, r_f. In theory, in a competitive economy, the risk-free rate of return to capital will equate to the risk-free market interest rate. For this reason, observed rates of return on (relatively) risk-free assets are seen as an appropriate source of information for the social discount rate. As discussed in later sections, the typical asset used to inform the SDR is the return to government bonds as the SDR. These are seen as relatively riskless and of sufficient maturity for use in discounting public projects. They also reflect the cost of government borrowing.[4] For instance, the Norwegian and Dutch governments use the return on relatively risk free assets such as bonds to inform their SDR, albeit with a risk premium added to account for project risks (See Table 8.4).

The SOC is sometimes estimated using some pre-tax rate of return to business, or the post-tax rate of return to consumer saving or foreign finance, or some weighted average of these rates depending on the expected source of funds (Spackman 2017, p. 12). The argument for the former is that funding for government projects crowds out the private sector, and so the opportunity cost to the economy should be represented by some aggregate return in the private sector. This raises several issues concerning the riskiness of these returns which are discussed below. Finally, the SOC is sometimes estimated by looking at the rates of return to public capital (Harberger and Jenkins 2015).

8.2.6. *Discounting in the second-best (risk-free) world*

Only when markets are perfectly competitive and function perfectly both within and between time periods and for all inputs and outputs, will the decentralised economy of utility maximising agents and profit maximising firms equate the rate of return to capital, r, and the SRTP, as in the Ramsey rule (2). Under these circumstances all rates coincide, the source of funding is of no consequence and either *r* or SRTP are in theory valid SDRs. When this assumption fails, which is most of the time, due to externalities and distortionary taxes for instance, then a decision must be made concerning which of these discount rates should be employed for CBA and CEA of public projects. Box 8.2 provides an example of the issue from Lind (1982b). Several solutions to this have been proposed in these circumstances.

One solution is to use a *weighted average of the SRTP and SOC (r)*, where the weights reflect the relative proportions in which investible funds are drawn from consumption and private capital respectively (See Box 8.2 for an example). Arguments of this type have been used in the US guidelines (OMB 1992) and are discussed in Harberger and Jenkins (2015). These recommendations take the view that the SRTP can be reflected by the post-tax returns to consumer saving and that the SOC is given by the pre-tax return in the private sector.

A related approach is to use the *shadow price of capital (SPC)* following e.g. Bradford (1975) or Cline (1992). The SPC approach takes into account the opportunity costs associated with public investment by calculating the *price* (conversion factor) of public investment/capital in terms of consumption and then converting all investment costs (in, e.g., Euros) into units (Euros) of consumption using this price. The NPV of the project is then calculated using the SRTP to discount the adjusted costs and project benefits (See Box 8.2 for an example). The SPC is the present value of consumption displaced by a unit (e.g. 1 Euro) of public investment. A rough approximation of the SPC can be given by *r*/SRTP. The logic (as shown in Box 8.2) is that each unit of public investment displaces *r* Euros of consumption each year for the life of the project. The present value of this stream of consumption is approximately *r*/SRTP for long-horizons.[5] This means that a rule of thumb is that the SPC will raise (lower) the cost of public investment when expressed in units of consumption when r > (<) SRTP.

Using the methodology discussed in Box 8.2, estimates of the SPC range from 1.2 (Bradford, 1979) to 2 (Cline, 1992), meaning that a unit of capital is worth anything between 1.2 and 2 units of consumption. As Harrison (2010, p99-100) notes, the weighted average approach and the SPC approach are identical for the two period time horizon, but typically diverge for longer time horizons. The precise conditions under which the two approaches coincide are discussed at length in Sjaastad and Wisecarver (1977, p. 523).

With a similar principle in mind, i.e. properly evaluating the cost of public investment, an alternative approach is to look at the *marginal cost of taxation* by calculating the deadweight welfare losses from consumer and producer surplus. Using this approach, estimates of 1.3-1.1 are typically obtained. Again these factors would be used to multiply the costs of public investment which would then be discounted using the SRTP. This adjustment is also applicable, using SOC, when projects are financed by taxation rather than the capital markets (see e.g. Spackman, 2017, p. 5-6). Incorporating the SPC into the appraisal of public policy and investment will raise the return required to pass a NPV test if the SOC is greater than the SRTP, and vice versa. Adding the marginal cost of taxation would have similar consequences.

Finally, there are some circumstances in which the SOC approach is not relevant. The first concerns CEA. With CEA or "Choice of Technique" a comparison of the implication for consumption of different solutions to a given problem is required. The opportunity cost of funds will typically not determine the preferred technique (Feldstein, 1970; Spackman, 2017).

Spackman (2017) summarises many of these arguments. In practice, discounting costs and benefits using the SOC, or converting costs into units of consumption using the SPC then discounting using the SRTP, will in general lead to similar public investment advice.

Yet most governments do not use the SPC approach, neither do they make any other adjustments to public investment costs to reflect funding issues such as the cost of increased taxation. The reasons for this absence are essentially twofold: i) Complexity or arbitrariness: the calculation of the SPC introduces a layer of debatable assumptions, and the implication that the SPC will vary from one project to another (e.g. time-horizon of displaced private capital, proportion of project benefits consumed versus reinvested); ii) Impact: the impact of such adjustments is minor in most policy contexts. Yet it remains important to understand the funding implications of CBA and their implications for social discounting and valuation of costs and benefits.

8.2.7. Summary

As the final section shows, some governments use an observed rate of return to capital as the SDR, others use the SRTP approach either in a normative sense by calibrating the Ramsey Rule or in a positive sense by using the post-tax rate of return to saving as a risk-free rate of return as a proxy for the SRTP (See Table 8.4). There are theoretical arguments for using the SPC approach, and yet government guidelines for social CBA typically overlook these arguments. In practice, the theoretical guidance falls foul of its informational requirements and practicalities of policy implementation. Often institutional differences concerning public finance will determine which of these approaches is taken (Groom and Hepburn, 2017; Spackman, 2017). The use of rates of return in the private sector, however, raises the issue of risk, and how this should be incorporated in the discount rate.

8.3. Discounting and risk

So far social discounting has been discussed in a risk-free context, assuming sure pay-offs from projects, no background risk associated with growth or interest rates, and certainly no-correlation of systematic, macro-economic risk with the project benefits. However, each of these aspects are important for project appraisal.

8.3.1. Growth risk and the risk-free rate: Theoretical arguments

The SRTP in [8.2] contains a wealth effect, which is one reason why society might want discount the future. This captures the idea that the discount rate is dependent upon the prediction of the future well-being of society. But what if the growth rate of consumption is uncertain? What does the wealth effect look like in these circumstances, and how should the discount rate be modified to reflect this?

When the project benefits are sure/risk-free and the only source of uncertainty is consumption growth, the typical approach is to use expected utility as the measure of welfare. In this case [8.1] becomes:

$$W = \Sigma_{t=0}^{T} \frac{E\left[U\left(\tilde{c}_t\right)\right]}{\left(1+\delta\right)^t} \qquad [8.3]$$

Gollier (2012) shows that the impact on the SRTP will depend on the diffusion process of growth over time. The basic result was first shown by Mankiw (1980) in the context in which growth follows a Brownian motion process: the growth of consumption is independently identically distributed (i.i.d.) normal with mean μ and variance σ_c^2, and utility is iso-elastic.[6] In words, this means that growth tomorrow is entirely independent of growth today. In this case the SRTP becomes:

$$SRTP = \delta + \eta\bar{g} - 0.5\eta\left(\eta+1\right)\sigma_c^2 \qquad [8.4]$$

Box 8.2. **Private rate of return (*r*) and SRTP in the presence of taxation**

Lind (1982) provides the following example of the impact of corporation and income taxes on the relationship between *r* and the after-tax rate of return to capital, which is used as a proxy for the SRTP in the Ramsey Rule. Imagine that corporation (or profit) tax is 50% and income tax paid on dividends is at 25%. Suppose also that the post tax SRTP is 6%; that is, this is the after tax rate of return that shareholders require to invest. Given the tax regime, what is the private rate of return on capital that is required to provide this minimal rate of return to shareholders?

In order to receive 6% after income tax of 25% the rate must be 8%. In order to have 8% return after a 50% corporation tax, a gross private rate of rate of return of 16% is required. Hence, the presence of this tax regime generates a divergence between the rate of return on private investment and the SRTP: 16% vs 6%. So, what is the SDR in this case?

Weighted average approach: Where a public project displaces both consumption and private investment, some economists have suggested that a weighted average of *r* and SRTP should be used. For instance, if *r* = 16% and SRTP = 6%, and the proportion of funds coming from consumption and private sector investment is 0.8 and 0.2 respectively, then the appropriate SDR should be:

8% = 0.8*6%+0.2*16%

This is a somewhat ad hoc approach that assumes all benefits are consumed rather than invested. Alternative formulae can be used which relax this assumption. Harberger and Jenkins (2015) have a comprehensive discussion of this method.

Shadow price of capital: Cline (1992) provides the following rational for the calculation of the SPC. Suppose a EUR 1 investment in the capital markets provides an annuity benefit *B* measured in units of consumption in each of *N* years, which is solely consumed.[7] The SPC is then given by:

$$SPC = \Sigma_{t=0}^{N} B(1+SRTP)^{-t} \quad [a]$$

The SPC is equal to the present value of the annuity stream of consumption generated from a unit investment. Suppose that this investment has an internal rate of return equal to *r*, the private rate of return on capital, that is:

$$-1+\Sigma_{t=0}^{N} B(1+r)^{-t} = 0 \quad [b]$$

Rearranging (b) to obtain a formula for B and inserting into (a) yields the following formula:

$$SPC = \frac{r}{SRTP} \cdot \frac{1-(1+SRTP)^{-N}}{1-(1+r)^{-N}} \quad [c]$$

Inserting the values from before: r = 16%, SRTP = 6%, and assuming a time horizon of 15 years, one obtains a value of SPC = 1.742. The SPC depends on the time horizon of the project, as well as the disparity between r and SRTP. As the time horizon considered increases, the estimate of SPC approaches r/SRTP, which in this example is 2.67. Cline finds support for values of SPC between 1.5 and 2, compared to 0.98-1.12 for Bradford (1975). Cline's estimates imply that a unit of capital is worth up to 2 units of consumption.

Cline's approach has some advantages over the approach found in Lind (1982b, p. 40), which cannot constrain the SPC to be non-negative or less than infinite for reasonable parameter values. Cline (1992, Annex 6), argues that Lind and Bradford are guilty of double counting the returns from reinvestment.

Source: Cline (1992, Annex 6A), Pearce and Ulph (1999).

In the context of risky growth, the Ramsey rule is extended and reduced by the term $-0.5\eta(\eta+1)\sigma_c^2$. This reflects the fact that although under uncertainty growth could be higher or lower in the future, it is the low growth scenarios that have the main influence. In the face of growth uncertainty, a prudent planner will save more for the future to protect against the possible low growth scenarios. This has the effect of raising the value of additional risk-free consumption in the future, hence lowering the risk-free discount rate. The extension to the Ramsey Rule reflects *prudence* in the face of uncertainty. The reduction of the SDR is higher the larger are volatility of growth, σ_c^2, and the larger are volatility of growth, sigma, and the elasticity of marginal utility, eta.[8]

The theoretical impact of this prudence effect is very small when calibrated to developed country data because the volatility of growth is very small. In developing countries, which have much higher volatility of growth, the prudence term could well be important. Gollier (2012, Ch 4) has a discussion. However, are independent and momentary growth shocks realistic? What if shocks to growth have a persistent component? When growth shocks are persistent, the SRTP varies with the time horizon. The details of time-varying discount rates are discussed below.

The risk-free rate puzzle: An empirical puzzle known as the risk-free rate puzzle exists in relation to the prudence effect and the Ramsey Rule in general. The puzzle stems from the fact that usual calibrations of the theoretical Ramsey Rule in [8.4] predict a risk-free rate that is much higher than that observed in the real world among relatively riskless assets. The puzzle highlights two issues. First, the normative approach may differ from the positive approach. Second, if the standard consumption side approach is to be treated as a positive model that predicts market outcomes, it needs to be augmented.

One way in which this puzzle has been partially solved is by including catastrophic or "jump-risks", such as the prospect of major depressions, into the uncertainty surrounding growth. Barro (2006) shows that such risks increase the prudence term significantly and may explain consistently low risk-free rates. But perhaps a more immediate aspect of risk for CBA is the presence of project specific risk. Gollier (2012, p. 75-76) provides a simple example of this point. Supposing that the percentage reduction in GDP is given by the parameter λ, and this shock (e.g. a depression due to a financial crash) occurs with a probability p. Together with the other assumptions made in this section, the risk free SDR now becomes:

$$SRTP_{\lambda} = \delta - \ln\left[p(1-\lambda)^{-\eta} + (1-p)\exp\left(-\eta\bar{g} + 0.5\eta^2\sigma_c^2\right)\right] \quad [8.5]$$

which is less than [8.4]. The potential for an economic depression, although unlikely, increases the precautionary motive for saving and lowers the risk-free SDR. This is one of the more intuitive solutions to the risk-free puzzle, and has implications for social discounting.[9]

8.3.2. Project risk

In a number of countries, France, Norway and the Netherlands included, the discount rate is adjusted to account for project risks. There are two basic forms of project risk, one of which is important from the perspective of the SDR, another which is not.

Unsystematic risk is the risk associated with over or under-estimating the costs and benefits of the project. In any given project elements will turn out to be more or less expensive than expected for unforeseen technical or other reasons. These risks are diversifiable across the portfolio of public projects and the theory of asset pricing shows that this kind of risk ought not to affect the price of an asset, and hence the appropriate discount rate.[10] The second type of risk is *systematic risk*, which describes a situation where risky costs and benefits are correlated to returns available in the macro-economy. Systematic risk cannot be diversified across different projects due to the macro scale of the riskiness. Where the project's net benefits are correlated with uncertainty to the wider macro economy, asset pricing theory shows that the discount rate should be augmented by a risk premium which reflects the project specific risk profile of systematic risk, not the diversifiable risk (See Annex 8.A2).

The implications for social discounting of project-based systematic risk are as follows. The asset pricing formula of Annex 8.2A shows that the RHS of the Ramsey risk free rate can be extended to accommodate systematic risks. Indeed, this is an example of the *consumption-based capital asset pricing model* (CCAPM) approach to asset pricing. The CCAPM considers the riskiness of projects and their correlation with returns to societal wealth, where the returns to societal wealth are measured by consumption. In this context, if a project is pro-cyclical, i.e. its benefits are positively correlated with aggregate consumption, then the high payoffs from this project occur in good (high consumption) states of the macro economy. In such states, these payoffs are worth less in welfare terms since marginal utility is lower. Similarly, low payoffs happen in the bad (poor) states when marginal utility is high. Society will want a higher rate of return from such projects in order to bear these extra risks.

Alternatively, some projects might be anti-cyclical, and have high payoffs in bad states of the world, and low payoffs in the good states of the world. Such projects serve an insurance function, and reduce risk in the macro-economy. From a welfare perspective society might be willing to pay for insurance by accepting a lower return from projects like this. In either case, there is a *systematic risk premium* associated with the project, $\pi(\beta)$.

As shown in Annex 8.A2, with the assumptions made in this section regarding utility and growth, coupled with the assumption that the project net benefits and consumption growth follow a bivariate normal distribution, incorporating project risk into the appraisal of projects leads to a simple extension to the RHS of the Ramsey Formula for risky project *i*:

$$\begin{aligned} SRTP_i &= \delta + \eta\mu_c - 0.5\eta^2\sigma_c^2 + \pi(\beta_i) \\ &= \delta + \eta\mu_c - 0.5\eta^2\sigma_c^2 + \eta\beta_i\sigma_c^2 \end{aligned} \qquad [8.6]$$

where the first three terms are the risk free rate in equation [8.4], and the fourth term is the systematic risk premium $\pi(\beta_i) = \eta\beta_i\sigma_c^2$.[11] The parameter β_i is the consumption "beta" which measures the correlation between the net benefits of project *i* and systematic risk associated with consumption growth. E.g. if $\beta = 1$, then a 1% increase in consumption growth will be expected to lead to a 1% growth in the project net benefits. If $\beta > 1$ then the project benefits are expected to increase by more than 1% when consumption grows by 1%, hence introducing proportionally more systematic risk than exists in the economy. If $\beta < 0$, the project reduces risk and has the insurance properties described above.

The consumption CAPM approach social discounting can be thought of as a normative approach since it focusses on the implications of project risk from the welfare perspective of a representative agent who is fully invested in the macro-economy. The CCAPM requires the estimation of normative parameters for this purpose.

However, many models used in finance use market based proxies for marginal utility and in the context of social discounting can be considered *positive* approaches to the SDR. A common example which has been influential in social discounting, is the c*apital asset pricing model* (CAPM). The CAPM pricing formula prices-in the risk associated with an asset by adding a risk premium to the risk-free rate of return in a manner similar CCAPM. The CAPM asset return formula is

$$r_i = r_f + \beta_{i,W}\left(r_m - r_f\right) \quad [8.7]$$

where r_f is the risk free rate of return, r_m is the rate of return on the market/wealth portfolio and $\beta_{i,W}$ is the project "beta" which reflects the correlation between the asset *i* and the market portfolio. The risk premium for this project is given by the market premium $\left(r_m - r_f\right)$ multiplied by the project beta, $\beta_{i,W}$. The risk-premium will be positive when $\beta_{i,W}$ is positive. The logic of this pricing formula is similar to the CCAPM except the covariance is with a market portfolio of assets rather than consumption. This formula for the SDR is project specific, but can be calculated by looking at suitable market returns and calculating the associated project betas.

The equity premium puzzle: Analogous to the risk-free rate puzzle, the CCAPM model calibrated using standard parameter values, based on normative perspectives or actual behaviour, leads to a much smaller equity premium (systematic risk premium) than is observed in real life among risky assets. With a $\beta = 1$, $\sigma_c = 3.6\%$ (volatility of GDPpc in the US) and $\eta = 2$, the systematic risk premium that should be added to the risk free rate is 0.26%. The observed risk premium as calculated as the difference between the return on equities and bonds has on average been much higher in the US between 1970 and 2006, at around 5% (Gollier 2012, p188). Once again, the normative approach leads to very different recommendations for risk free and risky projects compared to the positive approach.

The French CBA guidelines recommend project specific SDRs based on the CCAPM approach. The Dutch government recommends a flat 3% risk premium for all projects based more on the CAPM approach. The Norwegian government follows a CAPM approach with a constant risk premium. The UK does not take project risk into account in the discount rate other than a general 1% addition to pure time preference that represents some kind of generalised catastrophic risk, rather than project specific risk. The main difficulty with incorporating risk into the discount rate is calculating the project specific betas. This partly explains why it is either not applied across all governments, and when it is applied it is done so as a standard risk adjustment, as in the Netherlands. Table 8.5 has more examples.

Yet in France an array of different project specific betas have been calculated. The Gollier Report (Gollier 2011), which led to a risky SDR being recommended in the French guidelines on CBA (Quinet 2013), provides a table of the available estimates of sector specific betas. Table 8.1 shows that most public projects are pro-cyclical and warrant a positive project beta.

Table 8.1. **Sector level betas**

Sector	Estimated consumption Beta
Agriculture, Silviculture and Fisheries	0.85
Industry	2.09
Automobile Industry	4.98
Manufacture of Mechanical Equipment	3.00
Intermediate Industries	2.76
Energy	0.85
Construction	1,45
Transport	1.60
Administrative Services	-0.09
Education	0.11
Health	-0.24
Financial Services	0.15
Financial Intermediation	0.49
Assurance	-0.36

Source: Adapted from Gollier (2011, p. 226-227).

8.4. Declining discount rates

When long time horizons are considered, the theory of social discounting needs to be more precise because long-term CBA becomes ever more sensitive to the choice of SDR. Two issues turn out to be important. First, the persistent uncertainty surrounding some of the primitives of the discount rate, such as growth or the interest rate, need to be modelled more carefully. Second, when intergenerational projects are being evaluated, ethical issues arise.

In this section the issue of declining discount rates (DDRs) is discussed as an example of when more careful consideration of uncertainty can affect the appropriate SDR. This strand of the literature stems from more careful consideration of the term structure of discount rates. The past decade has seen an explosion of research into the term structure of the social discount rate and the applicability of DDRs to CBA. Some arguments stem from the consumption side, and extensions to the Ramsey rule. Others have focussed on uncertainty in the the return to capital and the interest rate.

8.4.1. Persistent growth risks and DDRs

Equation [8.4] presented the Mankiw (1980) result that when growth is i.i.d. normal (as defined above) the SRTP is reduced by a precautionary term: uncertainty in growth reduces the risk-free discount rate, where risk-free here means that one is considering projects with sure returns. But what happens in the more realistic case in which growth shocks persist, and today's growth is highly correlated to tomorrow's? It turns out that persistence of this type makes the prudence term increase with the time horizon considered. The persistence in growth can take many forms, here one such case is considered.

Suppose that growth is still i.i.d normal with mean μ and variance σ_c^2 but there is uncertainty about the mean and variance of growth. In practice what this means is that one is unsure of the regime that one will find oneself in in the future. It could be a high-growth regime, or a low-growth regime, and the volatility around this mean may be uncertain also. Gollier (2008) describes this case in detail. Consider the case when just the mean parameter is uncertain and dependent on some parameter, θ, which represents some uncertain technological or other state of the world upon which the growth regime is dependent. In this case, the extension to the Ramsey rule becomes:

$$SRTP = \delta - \frac{1}{t} \ln E_\theta \left[\exp\left[-\eta t \left(\mu(\theta) - 0.5\sigma_c^2 \right) \right] \right] \qquad [8.8]$$

It is easy to show that the second term on the right hand side is increasing with the time horizon t.[12] In essence, uncertainty over the parameters of the growth distribution introduces further uncertainty which increases with the time horizon. Once again, this has a precautionary savings effect on the prudent social planner, which manifests itself in a declining discount rate. Interestingly, this rationale for DDRs was one of the arguments used to motivate the French government's recommendation to use DDRs (Lebegue, 2005).

There are several other characterisations of growth uncertainty that lead to the same result. In short, provided that growth shocks are persistent over time, so that high growth is more likely to follow high growth, and vice versa, and provided the representative agent is *prudent*, then the outcome is DDRs. Gollier's work, documented in Gollier (2012a), is a powerful set of arguments.

8.4.2. Uncertain interest rates

A popular argument for DDRs comes from two contributions by Martin Weitzman which focus on interest rate uncertainty. Compared to the theoretical basis of the consumption-side arguments above, Weitzman (1998, 2001) are much more stylised and ad hoc. Their power lies in the simplicity of the algebraic arguments, making them easy to explain numerically, if not intuitively. Weitzman (1998) can be thought of as follows. Suppose that one has a project that costs EUR 1 today and provides EUR B_t at time t in the future. Suppose that the interest rate, r, that is used to calculate the present value of the project is uncertain. Weitzman proposes an expected net present value (ENPV) criterion to evaluate the project's desirability:

$$ENPV = -1 + E\exp(-\tilde{r}t)B_t \qquad [8.9]$$

The project should be approved if *ENPV* is greater than zero. The decision criterion can be re-framed in terms of the *certainty-equivalent discount rate*, that is, the certain discount rate that if applied over the time horizon t would yield the same *ENPV*. The certainty equivalent discount rate, $r_{CE}(t)$, can be defined as follows:

$$\begin{aligned} &\exp(-r_{CE}(t)t) = E\exp(-\tilde{r}t) \\ &\Rightarrow \\ &r_{CE}(t) = -\frac{1}{t}\ln\left(E\exp(-\tilde{r}t)\right) \end{aligned} \qquad [8.10]$$

Due to the fact that the exponential function is convex, and more so with larger t, it can be shown that the certainty-equivalent decreases with time. In fact, Weitzman (1998) shows that $r_{CE}(0) = E[\tilde{r}]$ and $\lim_{t\to\infty} r_{CE}(t) = r_{\min}$; that is, the certainty-equivalent discount rate should decline from its expected value to the lowest imaginable realisation of the return to capital. The essential insight here is that with the *ENPV* approach, one calculates the expected discount *factor* rather than the expected discount *rate*. The certainty-equivalent discount rate is a DDR. Table 8.2 has a simple numerical example for 10 equally likely interest rate scenarios.

Table 8.2. **Numerical example of Weitzman's declining certainty-equivalent discount rate**

Interest rate scenarios	Discount factors in period t				
	10	50	100	200	500
1%	0.91	0.61	0.37	0.14	0.01
2%	0.82	0.37	0.14	0.02	0.00
3%	0.74	0.23	0.05	0.00	0.00
4%	0.68	0.14	0.02	0.00	0.00
5%	0.61	0.09	0.01	0.00	0.00
6%	0.56	0.05	0.00	0.00	0.00
7%	0.51	0.03	0.00	0.00	0.00
8%	0.46	0.02	0.00	0.00	0.00
9%	0.42	0.01	0.00	0.00	0.00
10%	0.39	0.01	0.00	0.00	0.00
Certainty-equivalent discount factor	0.61	0.16	0.06	0.02	0.00
Certainty-equivalent discount rate	4.73%	2.54%	1.61%	1.16%	1.01%

Source: Adapted from Pearce et al. (2003).

The question is what is the welfare interpretation of the ENPV criterion? While it seems like an intuitive criterion, it is not clear that a project that passes this criterion will contribute to any well-defined notion of social welfare. It turns out that there are some situations in which the ENPV criterion, in which *r* is the risk-free rate of return, has such a theoretical basis. But the informational assumptions and the requirements are quite stringent.

Recent work by Freeman and Groom (2015, 2016) discusses the ENPV criterion in detail. Gollier (2016) highlights in detail the limitations of the ENPV approach and shows that those who use the ENPV approach are likely to be "short-termist" in the sense that the term structure does not decline sufficiently quickly compared to the theoretically sound equivalent. Given all its inadequacies, it is surprising that it has been so influential at the policy level. Objections have not gained too much traction, perhaps because practitioners thought that it is better to be approximately right, than precisely wrong on the issue of DDRs (Groom and Hepburn, 2017).

8.4.3. Ethical issues

The debate between positive and normative approaches to social discounting becomes particularly important when considering inter-generational projects. In such cases the public policy decision affects future unborn generations and the current generation is in the position of custodian of future well-being. One potential problem with the positive approach to social discounting is that there are no obvious assets with sufficient maturity that can be used to price costs and benefits that accrue in the distant future, say 200-300 years hence (Gollier 2012, ch3). Furthermore, any market rate observed today reflects the preferences and behaviours of people today who are most likely not thinking about future generations when they make those decisions (e.g. Beckerman and Hepburn, 2007). While some empirical work does exist on very long-term asset prices (see Giglio et al., 2015) it is not clear that these assets (housing) are relevant to alternative projects in health, transport and climate change mitigation due to their risk characteristics.

The debate in recent years was exemplified in the discussions between Stern (2007) on the one hand and Nordhaus (2007) on the other in the context of climate change and Discounted Utilitarianism (DU). Stern took the view that the DU welfare function should not contain a positive utility discount rate ($\delta = 0$), and that views on intergenerational equity should guide the way in which future welfare should be evaluated. The utility discount rate should only be positive for reasons of catastrophic risk which was estimated to be 0.1%. Nordhaus preferred to calibrate the parameters of the DU social welfare via the Ramsey Rule based on an observable market rate of return on equities. The central discount rate in the Stern Review was 1.4% ($\delta = 0.1\%, \eta = 1, g = 1.3\%$), whereas Nordhaus' calibration of social welfare anchored on the 4-5% witnessed in the equities market in recent history in the United States. Not surprisingly, the Stern Review recommended far more stringent action on climate change than Nordhaus' gradual approach.

Nordhaus used a standard opportunity cost argument: using a low discount rate to analyse climate change mitigation investment means that one is disadvantaging future generations by investing in low return projects now. Better to ensure that investment increases wealth and makes the future better off, than reducing the test discount rate and allowing low return projects (Nordhaus, 2007).[13] Others argue that following such a strategy, and putting off investment in climate change mitigation can subject future generations to catastrophic risks, which would see dramatic reductions in their welfare. This type of risk is unlikely to be captured by current rates of return and standard positive discounting procedures (Weitzman, 2009). Chapter 13 goes into more depth on the specific issues associated with climate change and catastrophic risk. The normative vs. positive debate on inter-temporal welfare analysis and social discounting continues (Drupp et al., 2017).

8.5. Dual discounting

In recent years the issue of dual discounting has resurfaced, that is, applying different discount rates to different classes of commodities. The clearest analytical discussion of dual discounting can be found in Weikard and Zhu (2005), where two classes of goods are considered: consumption goods and "environmental" goods.

Suppose that instantaneous utility depends on consumption *C* and a stock of environmental goods, *E*. Intertemporal social welfare is then given by:

$$W = \Sigma_{t=0}^{T} \frac{U(C_t, E_t)}{(1+\delta)^t} \quad [8.11]$$

where δ is the utility discount rate (which here does not differ between environmental and consumption goods). Now there is an SRTP for each of these arguments of the utility function. These are:[14]

$$\rho_C(t) = \delta + \eta_{CC} g_C + \eta_{EC} g_E \quad [8.12]$$

$$\rho_E(t) = \delta + \eta_{EE} g_E + \eta_{CE} g_C \quad [8.13]$$

where $\eta_{ij} = -x_i \frac{U_{ij}}{U_i}$ for all *i* and *j* and is the elasticity of marginal utility in each case. These should be compared to the standard single good framework of Ramsey in which the social discount rate for consumption goods is simply $\rho = \delta + \eta g$. This is the typical framework for the analysis of dual (meaning separate) discounting of environmental benefits and costs on the one hand, and consumption goods on the other. The practice here is clearly different to the standard approach. But how can this be implemented?

Baumgartner et al. (2014) follows the previous theoretical literature (e.g. Hoel and Sterner, 2007) and assume a constant elasticity of substitution utility function:

$$U(C,E) = \frac{1}{(1-\eta)} \left[(1-\gamma) C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}} \right]^{\frac{(1-\eta)\sigma}{\sigma-1}} \quad [8.14]$$

where σ is the elasticity of substitution between E and C. With this additional structure, the difference between consumption and environmental discount is reduced to:

$$\rho_C - \rho_E = \frac{1}{\sigma}(g_C - g_E) \quad [8.15]$$

which is a matter of estimating three parameters: the growth rate of consumption, g_C, the growth rate of environmental stocks, g_E, and the elasticity of substitution, σ. So in principle, the application of dual discounting is possible, and Baumgartner et al. (2014) illustrate how this might be done.

However, adjusting the discount rate can be a tricky business at the best of times. Fortunately, Weikard and Zhu (2005) showed the difference between the consumption and environmental discount rates has an alternative and practically identical interpretation: it reflects the change in the relative shadow prices for the environment.[15] In practice then, this dual discounting approach could be implemented by making sure that the shadow prices for the environment change according to [8.15] relative to consumption. Accounting for differences in relative prices is a standard piece of guidance in cost-benefit analysis, and so this seems like a more likely approach to be taken up (see e.g. HM Treasury, 2003). Indeed, the Dutch Government is currently investigating this possibility.

What dual discounting highlights in relation to the environment, is the importance of accounting for scarcity of environmental goods when undertaking CBA. This depends on their growth but more importantly on the elasticity of substitution with consumption. If environmental goods are perfectly substitutable with consumption then $\sigma = \infty$, and relative prices are unimportant. If the environment is critical to utility, and not substitutable at all, $\sigma = 0$, and the relative scarcity of environmental resources is paramount to CBA. These are realistic cases, although the middle cases are also likely. It will depend on the resource. Sterner and Persson (2008) make these points in relation to climate change.

8.6. Empirics of the SDR

In this section the empirical estimates of the SDR and its components are discussed. The next section discusses the empirics associated with defining an empirical schedule of DDRs.

8.6.1. Estimating the social rate of time preference

In order to estimate the standard SDR in (1), estimates of three parameters are needed: δ, η and g. The methods used to do this naturally depend on the interpretation of the parameter.

For *the utility discount rate*, δ, several interpretations were described above, and each interpretation provides a potential method for estimation. One way to approach the issue is to disentangle two components of δ: the pure rate of time preference or pure impatience, φ, and some element of hazard, such as life chances or catastrophes, L.

1. *Pure impatience, φ*: The pure desire to have utility sooner, sometimes described as "myopia", rather than later can be estimated by looking at aggregate or individual savings behaviour and estimating an equation based on [8.2]. In the past (some of these applications are quite old) this lead to estimates of around 0.3% to 0.5%. The highest values for this parameter are estimated by Nordhaus (1993). Interestingly, experimental evidence often leads to very high levels of impatience. Some studies record estimates of between 10 and 30% (Warner and Pleeter, 2001; Harrison et al., 2002). Such evidence is typically not regarded as useful for social discount rates.
2. *Life chances, L*: estimation of the hazard that one may not be around to enjoy the benefits of government investment usually focusses on life chances. For example, Ulph and Pearce (1999) estimated life chances as the average probability of death for an average individual. The formula they used for this purpose was:

$$\text{Change in Life Chances} = \frac{\text{Total Deaths in UK}}{\text{Total Population in UK}} = \frac{0.6466mm}{57.56m}$$

   where "mm" refers to mortalities per million. This figure should be interpreted as the risk of death that the average individual faces each year. This lead to an estimate of 1.3%, but in general this type of approach yields estimates ranging from 1% to 1.3% (Kula, 1987; Scott, 1989).
3. *Risk of extinction or catastrophe*, *L*: Many have argued that when considering the appropriate discount rate for evaluating social projects, the SDR, it is inappropriate to use individual life chances. More appropriate, particularly for intergenerational projects is the risk of the extinction of *society* as a whole. Several estimates exist for this alarming concept. Estimates vary between 0.1% and 1.5%, depending on the method of estimation (see Box 8.3). Newbery (1992) for instance, estimates the "perceived risk of the end of mankind in 100 years" as 1%. This is the value that the UK Treasury uses for *L*.

Methods i) – iii) above are all *positive* methods in that they generally use revealed preference or observed outcomes as their empirical basis. Often a more *normative* or *prescriptive* stance is taken towards the pure rate of time preference, particularly when projects have intergenerational consequences. Normative approaches address the question of what one ought to do, positive approaches look at observed behaviour. In the intergenerational case, some argue that the normative and ethical case is more important for social discounting than observed market behaviour. In recent years, differences of opinion emerged on this matter between Stern and Nordhaus after the Stern Review. Stern believed for ethical reasons that $\delta = 0$ and the well-being of all generations should count equally in the evaluation of social welfare. Nordhaus believed that δ should be imputed from market behaviour so that the discount rate should reflect the returns available in the market, which is a rough measure of how people actually make inter-temporal trade-offs. The latter approach led to $\delta = 3\%$, which places a lot less weight on future utilities. Nordhaus' approach leads to radically less stringent recommendations for action on climate change than the Stern Review (Stern, 2007).

So opinions differ on the matter of the pure rate of time preference, and the general motives for discounting future generations' utilities. A recent of survey of experts (as judged by their publications on the matter) by Drupp et al. (2017) casts more light on this issue. Table 8.3 shows that on the matter of δ the pure rate of time preference, the experts mean (median, mode) was 1% (0.5%, 0%). These responses reflect a variety of different approaches to estimating δ for long-term (> 100 years) projects.

Box 8.3. **Estimates of the utility discount rate, η**

Source	Estimate	Theoretical Basis
Scott (1977)	1.5%	Myopia: 0.5%, destruction of society: 1%
Kula (1987)	1.2%	Average annual survival probability in UK 1900-1975
Scott (1989)	1.3%	Myopia: 0.3%, risk of total destruction of society: 1%
Newbery (1992)	1%	Risk of end of mankind in next 100 years
Nordhaus (1993)	2-3%	Calibration of DICE model to actual data on savings etc.
Evans (2004)	1-1.5%	Catastrophic risks: 1% for EU, 1.5 for non-EU countries
Stern Review (2006)	0.1%	Probability of extinction of the human race per year, based on likelihood of catastrophe in next 100 years

Source: Adapted from ADB (2007).

The *elasticity of marginal utility of consumption*, η, also has a number of different interpretations. As described above, this parameter can reflect several features of societal preferences: i) Consumption smoothing; ii) Intertemporal or intratemporal inequality aversion; iii) Societal risk aversion. Since this parameter captures many dimensions of behaviour, there are numerous empirical strategies that can be employed to estimate it. Box 8.4 has a summary of the estimates from the literature. Not only is the underlying rationale important, but so is the source of data for estimation. The following examples contain estimates from both *revealed* and *stated* preferences.

1. *Consumption smoothing*: Many of the estimates of this parameter use econometric techniques applied to individual or aggregate (revealed) data on savings and consumption behaviour

over time. The estimates range between values of 1 and 10 and depend upon the behavioural assumptions underpinning the econometric models, as well as the country in question. Stern (1977) and Pearce and Ulph (1999) review these estimates. For the United Kingdom, Groom and Maddison (2017) estimate the elasticity of intertemporal substitution, which is the inverse of η, and find that $\eta = 1.5$.

2. *Inter- and intra-generational inequality aversion*: Inequality aversion has been estimated using both revealed and stated preference methods. When students are asked, estimates of inequality aversion vary from 0.2 to 0.8. *Revealed social values* can be imputed under some assumptions from the extent of redistribution from progressive taxation systems. Evans (2005) suggests values between 1 and 2 for EU countries, while Atkinson and Brandolini (2007) suggest lower values. Groom and Maddison (2017) find that whether looked at over time or in the cross section, $\eta = 1.5$. Tol (2008) looks at the values of international inequality aversion implied by transfers between developed and developing countries and finds very low levels of inequality aversion.
3. *Risk aversion*: Estimates of η which capture risk aversion have a wider range in general. These estimates are obtained from revealed behaviour in markets for insurance, or from stated preference surveys in which individuals are asked for their preferences over a set of gambles. Indeed Gollier (2006) reviews some of the evidence and suggests that values lie in the range of 2-4. This reflects individual risks, and it is arguable whether these values are directly relevant to social decisions.

8.6.2. The social opportunity cost of capital: which rates?

There is also a debate as to which rates of return should be used if one thinks that *the social opportunity cost of capital (r)* is the correct SDR? The typical source of information for the risk-free rate would be government bonds. In fact, these assets are not entirely risk free since, compared to risk-free short term Treasury Bills for instance, bonds contain inflation risk. The use of treasury bills is not recommended however due to their short maturity. Most governments use the returns on government bonds as their (relatively) risk free SDR.

Dimson et al. (2017) have collated historical interest rate data and find that over the period 1900-2016 the global average real interest rate for relatively risk-free assets was approximately 0.8% and -0.5% for bills since 2000. For bonds the rates were 1.8% and 4.8% respectively. Each country will have its own appropriate rate of return.

It has long been argued that argued that the government can pool risks across many different projects, hence the appropriate SDR is a risk-free rate (Lind, 1982). The usual theoretical justification for this is the Arrow-Lind theorem (Arrow and Lind, 1972), which some interpret as meaning that public investment is inherently less risky than private investment. However, as shown by Baumstark and Gollier (2015), this is not necessarily the implication of the *Arrow-Lind theorem*, and in fact ignoring the risks associated with public investment may lead to the government taking on too many risky projects. When project risk is a significant factor, Baumstark and Gollier (2015) argue that the appropriate discount rate should reflect the returns available from a project with a similar risk profile, as discussed in the context of the CCAPM above.

The US government, for instance, proposes a SDR of 7% based on the observed rates of return to equities, for projects which are expected to draw upon or displace private business capital. The motivation for this discount rate stems from the source of funding rather than issues of risk. Yet such rates reflect a premium for bearing risk. This leads to the question of whether SDRs that embody risky returns are appropriate for the evaluation of public projects. This speaks to the broader question of how to deal with project specific risks in social discounting discussed above.

Box 8.4. **Estimates of η**

Source	Estimate	Theoretical Basis
Stated Preference		
Barsky et al. (1995)	4.0	Risk aversion of middle aged people in the US
Amiel et al. (1999)	0.2-0.8	Inequality aversion of US students
Gollier (2006)	2 – 4	Risk aversion in gambling
Individual Revealed Preference		
Kula (1989)	1.89	US data, constant elasticity of demand
Blundell (1993)	1.06-1.37	UK data: Aggregate and micro (QUAIDS) models
Gollier (2006)	2 – 4	Risk preference revealed in insurance markets
Social Revealed Preference		
Atkinson and Brandolini (2007)	< 1	Public decision making on redistribution and taxation
Evans (2005)	1.25-1.45	20 OECD countries' tax schedules
Tol (2008)	-0.1 – 0.9	Redistribution of OECD countries to developing countries via

Source: ADB (2007), Dietz et al. (2008), Pearce and Ulph (1999).

Table 8.3. **Estimates of the elasticity of marginal utility**

Methodology	η	Standard error
Inequality Aversion (Equal sacrifice, Weighted)	1.515	0.047
Inequality Aversion (Equal sacrifice, Historical)	1.573	0.481
Inter-temporal substitution (Euler equation)	1.584	0.205
Elasticity of Marginal Utility (Additive preferences, Rotterdam model)	3.566	2.188
Elasticity of Marginal Utility (Additive preferences, CEM)	2.011	1.337
Risk Aversion (Demand for insurance)	2.187	0.242
Subjective well-being (Happiness data)	1.320	0.168
Pooled estimate (Fixed Effects)	1.528	
Pooled estimate (Random Effects)	1.591	
Parameter homogeneity	Chi-sq(6) = 10.10 (p = 0.121)	

Source: Groom and Maddison (2017).

Table 8.4. **Survey results for intergenerational discounting**

Variable	Mean	StdDev	Median	Mode	Min	Max	N
Real growth rate per capita (g)	1.70	0.91	1.60	2.00	-2.00	5.00	181
Rate of societal pure time preference (δ)	1.10	1.47	0.50	0.00	0.00	8.00	180
Elasticity of marginal utility of consumption (η)	1.35	0.85	1.00	1.00	0.00	5.00	173
Real risk free interest rate (r)	2.38	1.32	2.00	2.00	0.00	6.00	176
Social Discount Rate (SDR)	2.27	1.62	2.00	2.00	0.00	10.00	181
SDR lower bound	1.12	1.37	1.00	0.00	-3.00	8.00	182
SDR upper bound	4.14	2.80	3.50	3.00	0.00	20.00	183
Social Rate of Time Preference (SRTP)	3.48	3.52	3.00	4.00	-2.00	26.00	172

Note: The SRTP is imputed from the individual determinants: the rate of societal pure time preference, and an interaction term of the real growth rate of per-capita consumption and the elasticity of marginal utility of consumption. See Drupp et al. (2017) for details. This equates to the RHS of equation [8.2].

8.7. The empirics of declining discount rates

8.7.1. Consumption side DDRs

In principle, when using the consumption-side approach to DDRs, such as those described by Gollier (2012a), there are two empirical steps that are required:

1. Empirical estimation of the long-term growth process
2. Estimation of the parameters of the theoretical term structure

In practice, only one of these steps is undertaken. The typical approach is to decide on a particular model of diffusion: e.g. the i.i.d. normal growth discussed above, or parameter uncertainty model in Section 8.3., and then obtain estimates of its underlying parameters: e.g. mean growth, μ, and volatility, σ_c^2. In some cases empirical models of diffusion are estimated and the SDR is calibrated this way. Groom and Maddison (2017) estimate a diffusion model with persistence for the UK economy, and construct a term structure for the United Kingdom in this way. But to date, a serious attempt to compare models of growth by some measures of goodness of fit is lacking in this area, and so the rigour with which the theory is being implemented falls short of that of the theory itself.

8.7.2. Production side DDRs

Where interest rate uncertainty is concerned, a great deal more empirical work has been undertaken in order to calibrate the schedule of DDRs coming from the theoretical structure of Weitzman (1998). Two sources of data that have been used for this purpose are expert opinions (Weitzman, 2001) and historical interest rates (Newell and Pizer, 2003; Groom et al., 2007; Hepburn et al., 2008; Freeman et al., 2015).

In determining which empirical approach is best between the production- and consumption-side approaches, the empirical rigour of those methods using historical interest rate data needs to be balanced against the theoretical rigour of the consumption-side approaches, which as yet have fallen short empirically.

8.8. Social discounting in practice

As the previous theoretical and empirical discussions have made clear, there are many decisions to be made when it comes to choosing the SDR. One approach is to take an opportunity cost perspective. This leads to a discussion about the appropriate rates of return that should inform the SDR. The alternative is to take a consumption side approach, which leads to a discussion about the nature of the social welfare function, the estimation of its parameters, and the empirics of consumption growth. Governments around the world have made different decisions in this regard and in this section we summarise the various approaches that have been employed.

Table 8.5 summarises some of the approaches recently taken in OECD countries on social discounting. Since 2003, many policy changes and reviews have taken place in the United Kingdom, United States, France, Norway and the Netherlands. Other countries have followed suit in some cases, like Denmark and Germany. These changes have happened in concert with an ever expanding literature on social discounting which has accompanied the focus of public policy on very long-term projects, such as nuclear power, biodiversity conservation, public health, transport and, in particular climate change and the social cost of carbon (See Chapter 13). Furthermore, at the time of writing, the World Bank is reviewing its guidance on social discounting for is assistance programmes.

Table 8.5. **Discounting guidance in several OECD countries**

Country	Risk-free discount rate (%)	Rationale	Risk premium (%)	Overall discount rate (%) (short to medium term)	Long-term discount rate
United Kingdom	3.5%	Simple Ramsey Rule, SRTP. Growth risk not incorporated, project risk is minor	0%, although 3.5% contains 1% for "catastrophic risk"	For all projects and regulatory analysis: 3.5%	The forward rate (%) for time horizon in years (H) is respectively: H = (0-30, 31-75, 76-125, 126-200, 201-300, 301+), SDR = (3.5%, 3%, 2.5%, 2%, 1.5%, 1%)
United States	For CBA: 3%, with sensitivity up to 7%	3% = consumption rate of interest, risk-free. (SOC/SRTP) 7% = average corporate returns (SOC)	7% is a risky rate of return, but no project specific risk premia,	Depending on source of funding, projects and regulatory analysis: 3 7%	OMB (2003) recommends lower rate for 'intergenerational' projects, for USEPA (2010) recommends 2.5%.
United States	For CEA: 2%	SRTP	None	2%	No guidance on long-term CEA
France	2.5%	Quinet (2013), Risk free rate of return. (note, Lebegue (2005), Ramsey Rule)	β * 2%, 2% comes from the estimated risk of "deep recession", see Barro (2006).	For risky projects: 2.5%+ β * 2%	Risk free rate: declining to 1.5% after 2070 time horizon. Risky premium: 2% for β = 1 rising to 3.5% after 2070 time horizon.
Norway	2%	CAPM approach, risk-free return to government bonds.	1%: systematic risk premium of 1%, aggregate β = 1, fixed for all projects	Risky projects and regulatory analysis: 3%	Risk free rate declining to 1% after 100 years.
Netherlands	0%	CAPM, opportunity cost approach.	3% systematic risk premium, fixed for all projects.	All projects and regulatory analysis: 3%	Accepts DDRs, but with real interest rates < 0% opted for fixed risk free rate of 0%, and fixed systematic risk premium.

8.9. Conclusions

The social discount rate is central to the appraisal of public policy and public investment using CBA and CEA. During the 60s and 70s, when CBA and CEA were first being introduced to government appraisal, there was much debate about the merits of using the Social Opportunity Cost (SOC) of capital or the Social Rate of Time Preference (SRTP) to inform the Social Discount Rate (SDR). The variety of different approaches to discounting in practice today reflect, in part, different conclusions drawn by different governments on which is the most appropriate (see Table 8.5). Each has its difficulties.

With regard to the SOC, it is not always easy to find the appropriate rate that reflects the cost of government funds. With externalities and poorly functioning capital markets, it is not clear that an observable market rate reflects the welfare trade-offs that society faces when making inter-temporal decisions. The pre-tax business rate, as used in the US in some cases, may not the appropriate opportunity cost for social CBA, particularly as it embodies risks faced by the private sector which may not be relevant to the public sector. Others argue that the cost of borrowing is the appropriate rate of return to inform the SDR, which would recommend the rate of return on government bonds as the SDR. Yet when all of these rates differ, and the sources of funding for a given investment of regulation are diverse, care is required in selecting the appropriate SDR.

With regard to the SRTP, the normative approach embeds the SDR directly in the social welfare function and reflects the welfare aspects of inter-temporal trade-offs. The merit of this approach is that the NPVs that provide a direct statement of whether a public policy and pubic investment increases social welfare, which is the objective of public appraisal. The downside is that the SRTP has to be calibrated in some way, typically following the

Ramsey Rule, and this parameterisation is regarded by some as somewhat arbitrary in some circles. Neither does the use of the SRTP take account of the opportunity cost of government funds or the cost of raising public funds.

The SRTP approach focusses on the consumption side and ignores the production side, which the SOC approach does the opposite. Using the Shadow Price of Capital approach (SPC) is advisable when using the SRTP, so that the opportunity cost of public capital can be reflected in the NPV calculation. This rarely happens in practice due to onerous informational requirements, and the lack of a generally accepted approximation, although Cline (1992, Annex 6A) does offer a way forward.

Recent advances in discounting have focussed on the evaluation of extremely long-horizon projects, such as climate change mitigation and nuclear power. This upsurge in interest has stemmed from the need to analyse intergenerational projects, and the sensitivity of these analyses to the selection of the discount rate. The fact is that large costs on future generations may appear insignificant in a cost-benefit analysis. Similarly, actions now that will benefit future generations for a long-time may not be undertaken in light of a cost-benefit analysis. While there are good reasons for discounting the future, these outcomes are often seen as being unfair to future generations.

The social cost of carbon for instance, which is one of the most important policy parameters for the public appraisal of carbon mitigation projects, is highly sensitive to the discount rate with which it is evaluated. Estimates of the SCC appear in current US evaluations of fuel efficiency regulations, and are incorporated in the CBA of transport projects in many OECD countries (OECD, 2015).

The use of declining discount rates to evaluate public projects is now commonplace in several OECD countries: United Kingdom, France, Norway and Denmark, and has strong theoretical and empirical support when considering risk-free discount rates.

For projects that have an important environmental component, the use of DDRs may or may not be a positive development with regard to conservation. It certainly means that the long-run is more important in standard CBA and CEA. More important from this perspective is the concept of dual discounting which places more emphasis on environmental costs and benefits when environmental quality is becoming scarce. While the theory is not new, the applications are becoming more frequent. The Netherlands has specific guidance on discounting environmental quality changes. However, rather than adjusting the discount rate for environmental quality, the guidance typically focusses on ensuring that changing relative prices for are properly accounted for. Such guidance can also be found more generally in UK Treasury Green Book.

Finally, there is the question of project-risk and whether this should be reflected in the SDR. This time old debate is yet to be generally resolved. The French guidance recommends the use of a systematic, project specific risk premium in the discount rate. The UK Green Book guidance considers project risk to be an issue of relatively insignificant importance. These are the issues that governments need to decide for themselves, being mindful that governments may take on too much risk if they fail to evaluate risk somewhere in their appraisal of projects.

There have been many developments in the world of social discounting in recent years. Groom and Hepburn (2017) chart the changes in 4 OECD countries over the past 20 years, and emphasise the role not just of the technical economic advances, but also the political economy of the times.

In summary, this chapter shows that the following factors are pertinent when choosing the SDR:

First, for non-intergenerational projects of medium time horizons up to 50 years or so:

a) What type of evaluation is taking place? I) CBA of public investment; ii) CBA of regulatory change; or, iii) CEA of Choice of Technique Analysis?

b) The way in which the project is funded is important: i) capital markets; ii) general taxation; iii) foreign loans; iv) user fees; or, v) a mixture of these options.

c) The SDR can be informed by the social opportunity cost of capital (SOC), risk-free rate of return or social rate of time preference (SRTP)? There are strong arguments for using the SRTP for CEA, SOC when private capital will be displaced and risk-free rates when government borrowing funds the project.

d) Theoretically, the cost of public funds should be taken into account via estimates of: i) Weighted SOC/risk-free SRTP approach; ii) Shadow price of capital (SPC) approach; or, iii) Deadweight loss of raising funds via taxation.

e) The SDR can reflect project risk and the uncertainty in growth. A consumption CAPM approach would add a consumption risk premium to the SDR to reflect project risk, alternatively certainty equivalent net benefits could be estimated and discounted using a risk free SDR (risk-free interest rate or SRTP).

f) Dual discounting/relative price effects for environmental scarcity: The government of the Netherlands are currently investigating this approach.

When discounting for long-term intergenerational projects:

a) The SDR for risk-free projects should decline with the time horizon due to uncertainty in the discount rate itself or due to uncertainty in secular consumption growth.

b) The SDR for risky projects: what is the likely term structure of the systematic risk premium for long-term projects? Theoretical work exists (Gollier, 2012; 2016) showing that the same forces that argue for a declining discount rate for risk free rate lead to argument for risk premiums that increase with the time horizon for pro-cyclical projects. Such arguments are included in the French guidelines on CBA.

c) To what extent should ethical consideration inform the SDR and is the SDR a sufficient device for evaluating very long-run projects? The latest UK Green Book will propose to truncate the calculation of the NPV at 120 years, and invoke a separate decision-making approach for evaluating the long-run, such as looking at the distribution of costs and benefits in the long-run.

These are the main issues that should be considered when thinking about social discounting for public policy appraisal.

Notes

1. *f(k)* is a neoclassical production function which assumes that per capita output/income, y, is a function of per-capita capital stocks, y = *f(k)*, where $f_k(k) = \frac{\partial f(k)}{\partial k} > 0$ and there is diminishing returns to capital: $f_{kk}(k) = \frac{\partial^2 f(k)}{(\partial k)^2} < 0$
2. The SRTP is sometimes referred to as the Consumption Rate of Interest.

3. Note that the elasticity of marginal utility is defined as: $\eta(c_t) = -c_t u''(c_t) / u'(c_t)$, which is constant when $u(c_t) = (1-\eta)^{-1} c_t^{1-\eta}$, known as iso-elastic utility.
4. Some argue that this rate is a good proxy for the SRTP since it reflects the savings rate available to individuals. Others argue that the risk-free rate is a candidate for the SDR since it reflects the cost of public borrowing (Spackman, 2017; CEA, 2017).
5. This assumes: i) the internal rate of return of stream of consumption the project is zero: $\Sigma_t^N B(1+r)^{-t} - 1 = 0$, which for long time horizons means that $B / r \approx 1$; hence, ii) $B \approx r$ so one can use r Euros rather than B Euros for the approximation of the SPC.
6. Intuitively, i.i.d. normal this means that the growth in any given year is a random number, and each year a new random growth level is drawn. If these draws are i.i.n.d. then each draw comes from the same (normal) probability distribution and is uncorrelated with the previous year's random growth. More formally, if: $c_t = c_0 \exp(\Sigma_{\tau=0}^{t} x_\tau)$, where x_τ is annual growth and: $x_\tau \sim N(\mu, \sigma_c^2)$. If in addition: $U(c_t) = (1-\eta)^{-1} c_t^{1-\eta}$ and $\bar{g} = t^{-1}E[(c_t - c_0)/c_0]$, which is annualised expected growth, then one obtains [8.4].
7. The weighted average approach and the SPC approach can accommodate situations when benefits are consumed or reinvested. See Cline (1992, Annex 6A).
8. This result only occurs if societal preferences reflect *prudence* in the formal sense. Iso-elastic utility (utility with constant elasticity of marginal utility with respect to consumption: $U(c_t) = (1-\eta)^{-1} c_t^{1-\eta}$) ensure this, but any set of preferences for which $U'''(c_t) > 0$ has this property.
9. For more on the asset price puzzles see Mehra and Prescott (1985) and Weil (1989).
10. Chapter 9 on Uncertainty has more detail.
11. The simplifying assumption here is that project returns and consumption are bivariate normal with correlation coefficient ρ_i. In this context $\beta_i = \rho_i \sigma_B / \sigma_c$, which defines the covariance of project returns and consumption, divided by the volatility of consumption.
12. Defining the certainty equivalent M_t of $\mu(\tilde{\theta}) - 0.5\eta\sigma_c^2$ as follows that $\exp(-t\eta M_t) = E_\theta\left[\exp\left(-\eta t\left(\mu(\tilde{\theta}) - 0.5\eta\sigma_c^2\right)\right)\right]$ it can be seen that M_t is decreasing in t since the exponential function becomes more concave in t. Therefore, the SDR is declining with the time horizon since it can be written as: $SRTP_t = \delta + \eta M_t$. See Gollier (2008).
13. For similar arguments see also Harrison (2010).
14. For each of these, the social discount factor with which to "price" changes in the quantities of each of the arguments, consumption and environment, from the perspective of today (t = 0) is given by:
$$P_i(t,0) = \frac{U_i(C_t, E_t)}{U_i(C_0, E_0)} \exp(-\delta t) \quad \text{for} \quad i = C, E$$
The social discount rate in each case is simply the rate of change of this shadow price which leads to (9) and (10). See Traeger (2010).
15. The shadow price of the environment is the marginal rate of substitution: $p = \frac{U_E}{U_C}$. It is easy to show that the rate of p change of this over time leads to (12) in the CES case, and more generally the difference between (10) and (11) (Weikard and Zhu, 2005)

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ANNEX 8.A1

Positive NPV using SDR increases social welfare

Suppose social welfare is given by Equation [8.1] for two periods:

$$W = U(C_0) + \exp(-\delta)U(C_1)$$

Suppose that a project costs 1 unit of consumption today for sure, but provides sure benefits in consumption ε at time *t* in the future. That is, there is no project risk. Given that the changes are small, the overall change in welfare from such a project is given by:

$$\Delta W = -U'(C_0) + \exp(-\delta t)U'(C_t)\varepsilon$$

The internal rate of return of the project is given by r_p, so that $\varepsilon = \exp(r_p t)$. If this return just offsets the welfare cost of the project (1 unit of consumption) then:

$$-U'(C_0) + \exp(-\delta t)U'(C_t)\exp(r_p t) = 0$$

$$\Rightarrow$$

$$\exp(r_p t) = \frac{U'(C_0)}{U'(C_t)}\exp(\delta t)$$

$$\Rightarrow$$

$$r_p = \delta - \frac{1}{t}\ln\left[\frac{U'(C_t)}{U'(C_0)}\right]$$

If utility has a constant elasticity of marginal utility: $U'(C) = C^{-\eta}$, then:

$$r_p = \delta - \frac{1}{t}\ln\left[\left(\frac{C_t}{C_0}\right)^{-\eta}\right] = \delta - \frac{1}{t}\ln\left[\left(\exp(gt)\right)^{-\eta}\right]$$

$$= \delta + \eta g$$

So the rate of return that a project must have to just compensate for a marginal investment at time zero is given by $r_p = \delta + \eta g$, where g is the (continuously compounded) growth rate of consumption. Any project that beats this rate of return will increase social welfare W. Hence, $\delta + \eta g$ is the social discount rate. At the optimal defined by [8.2] this will be equal to the return to capital, *r*.

ANNEX 8.A2

Fundamental equation of asset pricing and the Ramsey Rule[1]

In the basic consumption based model of asset pricing under uncertainty, the individual consumer's utility is defined over current and future utility: $U(C_0, C_t)$. Typically intertemporal utility is represented by the additive, time-separable form: $U(C_0, C_t) = U(C_0) + \beta^t U(C_t)$, where $\gamma^t = \exp(-\delta t)$, and δ is the utility discount rate. When the future is uncertain, the expected utility model gives:

$$U(C_0, C_t) = U(C_0) + \gamma^t E\left[U(C_t)\right]$$

An investor decides how much to consume today, and how much to save for the future of an asset *i* which has maturity t and an uncertain payoff x_{it}, which has a price today (t = 0) of $p_i(0,t)$. The first order condition for this consumption-saving problem is given by:

$$\Delta U = -U'(C_0)\, p_i(0,t) + \gamma^t E_0\left[U'(C_t)\, x_{it}\right] = 0$$

Rearranging yields and expression for the asset price:

$$p_i(0,t) = E_0\left[m_{0,t}\, x_{it}\right] \qquad [8.A2.1]$$

where $m_{0,t}$ is the stochastic discount factor which reflects the value of expected marginal utility at time *t* relative today (t = 0):

$$m_{0,t} = \beta^t \frac{U'(C_t)}{U'(C_0)}$$

Noting that $E[XY] = E[X]E[Y] + \text{cov}(X,Y)$, [8.A2.1] can be re-written as:

$$p_i(0,t) = \frac{E_0\left[x_{it}\right]}{r_t^f} + \frac{\text{cov}\left(\gamma^t U'(C_t), x_t\right)}{U'(C_0)} \qquad [8.A2.2]$$

where r_t^f is the risk-free rate of return defined by: $\exp\left(-r_t^f t\right) = \gamma^t \frac{E\left[U'(C_t)\right]}{U(C_0)}$ so that:

$$r_t^f = \delta - \frac{1}{t}\ln\frac{E\left[U'(C_t)\right]}{U(C_0)} \qquad [8.A2.3]$$

Equation [8.A2.2] has two terms: 1) the asset price for a risk-free asset; 2) an adjustment for a risky asset. Equation [8.A2.3] is the expression for the risk free rate of return. The correspondence of [8.3] with the expression for the Ramsey Rule in Annex 8.A1 is clear. The consumer is thought of as a representative agent in that case.

The risk adjustment in [8.A2.2] reflects the covariance of the asset returns with the stochastic discount factor. [8.A2.2] can be re-written in terms of rates of return by normalising the asset price to 1 and treating the expected payoff $E[x_t]$ as the expected annualised return on asset *i*, r_i. This yields the following expression for the annualised rate of return:

$$r_i = r^f - \frac{\text{cov}\left(\gamma U'(C_t), r_i\right)}{U'(C_{t-1})} \quad [8.A2.4]$$

As Gollier (2012, p. 190-193) shows, if utility is iso-elastic: $U(C) = (1-\eta)^{-1} C^{1-\eta}$, growth is a Brownian motion (as in section 8.3.1), and r_i and ln C_t are jointly normally distributed with a correlation parameter ρ, then [8.A2.4] becomes:

$$r_i = \delta + \eta\mu_c - 0.5\eta^2\sigma_c^2 + \gamma\beta\sigma_c^2 = r^f + \pi(\beta) \quad [8.A2.5]$$

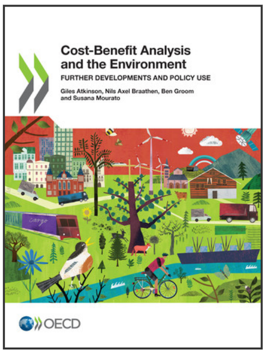
where $\pi(\beta) = \rho\beta\sigma_c^2$ is the consumption risk premium, and the project "beta" is given by:

$$\beta = \frac{\rho\sigma_r}{\sigma_c} = \frac{\text{cov}\left(\ln x_t / x_{t-1}, \ln C_t / C_{t-1}\right)}{\sigma_c^2} \quad [8.A2.6]$$

The numerator of the project beta captures the correlation of the growth of the payoffs of the asset, x_t, with the growth rate of consumption. It is this covariance that determines the risk adjustment. If the covariance is positive, then the asset pays off when consumption is high and marginal utility is low. Such assets require a positive risk premium to account for this correlation. The opposite is true for insurance type assets for which the correlation is negative, and which pay out when consumption is low.

Note

1. This Annex follows Gollier (2012, p. 190) and Cochrane (2005, p. 3-17).



From:

Cost-Benefit Analysis and the Environment

Further Developments and Policy Use

Access the complete publication at:

https://doi.org/10.1787/9789264085169-en

Please cite this chapter as:

OECD (2018), "Discounting", in *Cost-Benefit Analysis and the Environment: Further Developments and Policy Use*, OECD Publishing, Paris.

DOI: https://doi.org/10.1787/9789264085169-11-en