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Harnessing Spatial Thinking to Support Stem Learning

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HARNESSING SPATIAL THINKING TO SUPPORT STEM LEARNING

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ABSTRACT

Spatial intelligence concerns the locations of objects, their shapes, their relations, and the paths they take as they move. Recognition of spatial skills enriches the traditional educational focus on developing literacy and numerical skills to include a cognitive domain particularly relevant to achievement in science, technology, engineering and mathematics (STEM). This report reviews research showing that (a) spatial thinking and STEM learning are related, and (b) spatial thinking is malleable. It evaluates two strategies for exploiting these findings in education. Strategy 1 involves direct training of spatial skills. Strategy 2 involves spatialising the curriculum, using tools suited to spatial thinking including spatial language, maps, diagrams, graphs, analogical comparison, physical activity that instantiates scientific or mathematical principles, gesture and sketching. Existing data support including spatial thinking and tools in designing curricula, training teachers and developing assessments. At the same time research continues to evaluate the effectiveness of the efforts and explore mechanisms.

RÉSUMÉ

L'intelligence spatiale concerne l'emplacement des objets, leurs formes, leurs relations et les chemins qu'ils empruntent lorsqu'ils se déplacent. La reconnaissance des compétences spatiales enrichit l'accent traditionnellement mis sur le développement de la littératie et de la numératie dans l'éducation, afin d'inclure un domaine cognitif particulièrement pertinent pour la réussite en Science, Technologie, Ingénierie et Mathématiques (STEM en anglais). Ce rapport examine les recherches montrant que (a) la pensée spatiale et l'apprentissage STEM sont liés, et (b) la pensée spatiale est malléable. Il évalue deux stratégies pour exploiter ces résultats dans l'éducation. La stratégie 1 implique une formation directe des compétences spatiales. La stratégie 2 consiste à spatialiser le curriculum en utilisant des outils adaptés à la pensée spatiale, y compris le langage spatial, les cartes, les diagrammes, les graphiques, la comparaison analogique, l'activité physique qui instancie les principes scientifiques ou mathématiques, le geste et le croquis. Les résultats de recherche soutiennent l'intégration de la réflexion spatiale et des outils dans la conception des curricula, la formation des enseignants et le développement d'évaluations. En parallèle, des études continuent à évaluer l'efficacité des efforts et explorer les mécanismes.

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1. Introduction

Modern societies place a high priority on excellence in education, aiming to achieve a variety of important national goals. One vital objective is crafting effective education in the science, technology, engineering and mathematics (STEM) disciplines, with two benefits in mind. First, educational policy makers know they need a workforce that includes a critical mass of STEM experts, because scientific progress and technological innovation is now well recognised as an engine for improving the human condition and for driving economic success. Second, STEM-literate citizens are needed as well as experts, because numeracy skills and science literacy are essential for a productive technological economy.

Pursuing the overall objective of effective STEM education requires several kinds of coordinated efforts, including designing effective courses at levels ranging from preschool through university, setting standards, delineating instructional sequences, and monitoring achievement with thoughtfully-designed tests, including the Programme for International Student Assessment (PISA). All of these efforts can profitably build on the large and growing body of evidence on how best to teach and assess STEM learning. This report focuses on one theme from the science of STEM learning, namely the idea that a crucial element in STEM success is *spatial intelligence*. There is growing evidence that spatial intelligence can be used in designing curricula, training teachers, setting goals and developing assessments.

Spatial intelligence concerns the locations of objects, their shapes, their relations to each other, and the paths they take as they move. All of us think spatially in many everyday situations, as when we consider rearranging the furniture in a room, when we assemble a bookcase using a diagram, or when we relate a map to the road ahead of us. We also use spatial thinking to describe non-spatial situations, such as when we talk about being *close to* a goal or describe someone as an *insider*. Recognition of spatial skills enriches the traditional educational focus on developing literacy skills and numerical skills, and the implicit concentration on verbal and mathematical intelligence, by acknowledging that the third broad domain in contemporary models of human intelligence is spatial intelligence (Gray and Thompson, 2004).

2. Spatial skills and STEM achievement

Spatial intelligence may be particularly relevant to STEM achievement. There is a variety of evidence for this assertion, progressing along a continuum from anecdote and intuition to cross-sectional correlations to longitudinal correlations to experimental evaluations. Beginning with the anecdotal, scientists and mathematicians have long speculated that spatial thinking is at the heart of many fundamental discoveries. While some often-cited examples may be fables, such as Kekule's claim that a dream of snakes biting each other's tails inspired his discovery of the structure of the benzene ring, or Tesla's supposed ability to imagine and mentally test all the parts of a working engine independently, other discoveries are well documented. Rosalind Franklin's flat x-ray diffraction images, shown on the left in Figure 1, really were used by Francis Crick and James Watson to envision the three-dimensional structure of DNA. John Snow really did use a map in his investigation of the causes of London's 19th century cholera epidemic, as shown on the right in Figure 1. These examples illustrate the two broad categories of spatial intelligence: spatial skills involving the structure of objects, which are relevant to using

Rosalind Franklin's 2-dimensional images for inferring the 3-dimensional structure of DNA, and navigation skills, which are relevant to the map-like spatial distributions used by John Snow (Newcombe, forthcoming).

Figure 1. X-ray diffraction images of DNA and rendering of the map of cholera cases in London



Source: Ideonex (2009), *NEMO Science Center: Codename DNA*, <http://ideonex.com/2009/01/11/nemo-science-center-codename-dna/> (accessed 01 June 2017) and Snow, J. (n.d.) "Chapter 12 Images", *The John Snow Archive and Research Companion*, http://johnsnow.matrix.msu.edu/book_images12.php (accessed 01 June 2017).

We now have empirical evidence for the link between spatial skills and STEM achievement, evidence that takes us well beyond anecdotal tales of scientific discovery. Research began with a large number of studies showing that successful STEM students have good spatial skills, i.e. cross-sectional correlations. As just one example from this substantial literature, university students who score well on tests of spatial ability are more likely to solve kinematics problems correctly than students with lower scores, even though they score no higher on tests of verbal or mathematical ability (Kozhevnikov, Motes and Hegarty, 2007). (See Box 1 for illustrations of some spatial tests.)

The field has progressed, however, from cross-sectional to longitudinal designs and data sets, which provide stronger evidence and of which there are an increasing number. An initial publication utilised longitudinal data from a follow-up of an intellectually gifted sample first studied in early adolescence, and showed that spatial skills robustly predicted adult STEM interest and success, using an analysis that also accounted for verbal and mathematical scores (Shea, Lubinski and Benbow, 2001). Success included creativity, including scholarly publications and the awarding of patents, with spatial skills adding about 7% to accounting for the variance in these outcomes, even after the 10% that could be attributed to verbal and mathematical skill together (Kell, Lubinski and Benbow, 2013). These correlations do not simply pertain to an elite group, as shown by a later analysis of a much larger sample of students covering a much wider spectrum of ability. Data from follow-ups of a representative sample of around 400,000 American high school students assessed in Project Talent showed that students with higher scores on tests of spatial ability were substantially more likely to enter careers in science and mathematics, controlling for verbal and mathematical ability, gender, and socioeconomic status (Wai,

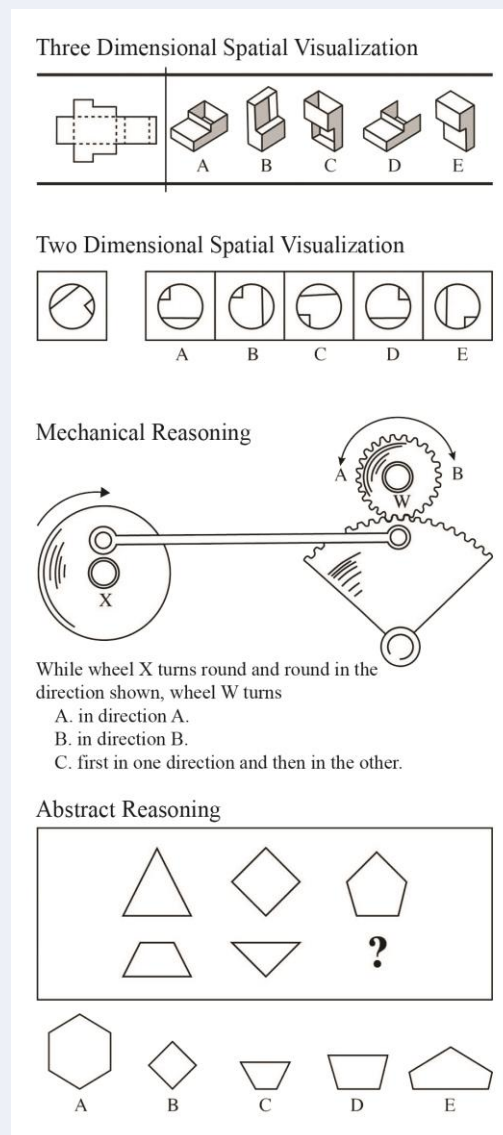
Lubinski and Benbow, 2009). By contrast, there were many occupations (e.g. law) that seemed to place no premium on spatial skills.

We may ask whether these linkages are only evident in high school and university, as students tackle complex topics in STEM. But this speculation turns out not to be true. Beginning with concurrent correlations, there are again many studies. A powerful recent example using a cross-sectional design is a large factor-analytic study showing that children with strong spatial skills do better on mathematics achievement tests across the age range from kindergarten to the end of elementary school, controlling for verbal ability (Mix et al., 2016). Turning to longitudinal data from samples of children, spatial skill at the start of school correlates with number learning and knowledge several years later, with controls for other aspects of cognitive ability such as vocabulary and working memory (Gunderson et al., 2012; Zhang et al., forthcoming) and spatial skills also correlate with an accelerating trajectory of mathematics learning (Carr et al., forthcoming). Spatial attention (a slightly different construct than spatial skill) is also correlated with later scores on mathematics tests (LeFevre et al., 2010; Zhang et al., 2014).

Box 1. How is Spatial Ability Measured?

For the Project Talent study, spatial thinking was defined by these four tests, which were used to assess participants' ability to think spatially. The first test requires imagining folding a two-dimensional shape into a three-dimensional one. The second test involves mental rotation of a two-dimensional shape, the third test imagining mechanical motion, and the fourth test discerning the structure of spatial patterns and progressions. The last test taps abstract analogical thought, realised in a figural fashion. Similar tests have been used in other studies.

Figure 2. Spatial tests used in Project Talent



Source: Wai, J., D. Lubinski and C. Benbow (2009), "Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance", *Journal of Educational Psychology*, Vol. 101, pp. 822. © 2009 American Psychological Association.

Longitudinal linkages between space and mathematics are also evident before school entry. The ability to copy two- and three-dimensional designs at age 3 years is correlated with mathematical skills at age 5 years, controlling for mathematical understanding at age 3, as well as language skills and, importantly, controlling for executive function, which seems to make an independent contribution to mathematical skills (Verdine et al., 2014; Verdine et al., 2017). In addition, growth rate in spatial skill in preschool is related to later arithmetic competence, over and above overall levels of spatial ability, and over and above levels and rate of growth of phonological awareness (Zhang and Lin, 2017). Rate of spatial growth uniquely related to almost 18% of the variance in arithmetic competence.

The number of papers based on longitudinal analyses is growing, and new avenues of inquiry are likely to open. For example, one possibility is that students who thrive in mathematics and science classes may be developing stronger spatial skills that will then stand them in good stead as they tackle new material, i.e. STEM participation may affect spatial skill as well as vice versa. As another example, it is possible that certain spatial skills are more important than other skills at different ages or for different subject matter or for different stages in the development of expertise.

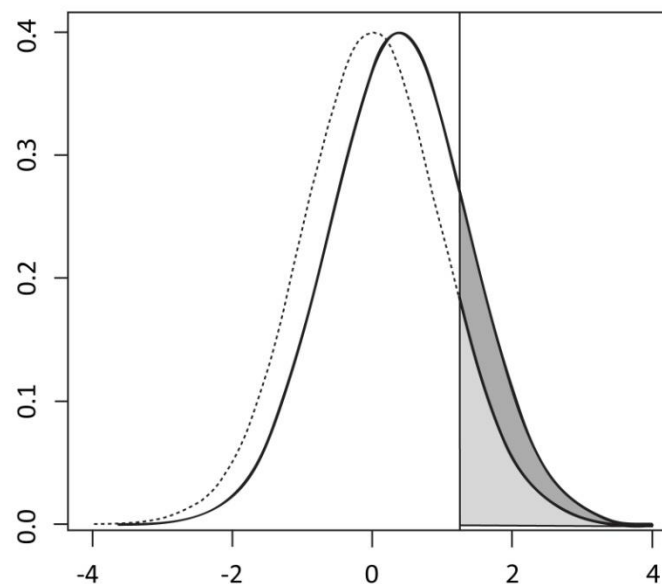
3. Malleability

The findings on longitudinal linkages encourage us to think seriously about education for spatial thinking, beginning in preschool and extending into university. But the data are relevant to educational efforts only if spatial abilities are changeable and trainable mental attributes. Importantly, there is a large literature that supports the idea of malleability. Meta-analysis of these papers shows moderate to moderately large effects of training. Improvement is evident in adults as well as children, and in both men and women (Uttal et al., 2013). Crucially for educational application, the meta-analysis also shows clear evidence of transfer from one kind of spatial skill to other kinds, and indicates that the effects last for at least several months. Generalisation and durability are crucial for efforts to increase the pool of STEM talent. This promising initial evidence from Uttal et al. suggests that future research should be aimed at developing and evaluating training that generalises even more widely, including to better learning of STEM material, and lasts over years as well as months, goals that seemed too bold in the past. Research of this kind is beginning, and is reviewed in later sections.

Uttal et al. (2013) illustrated the potential of a focus on spatial skills with a thought experiment, visualised in Figure 3. Figure 3 shows a normal distribution of spatial skill centred on zero, and a second normal distribution shifted to the right by the amount we know is possible, given the meta-analysis. A vertical line shows the average spatial skill of students in Project Talent who went on to become engineers, one of the occupations that most attracted the high-spatial students. We see in light grey the proportion of students who are normally “spatially qualified” to be engineers, and in dark grey the added proportion of students who would be qualified if the spatial skills of the population had been supported in education. Clearly, training could add considerably to the pool of talent, not only for engineering, but likely also for a wide variety of scientific and mathematical subjects, as well as for architecture, planning and the visual arts. In addition, good spatial thinking could be considered an end in itself, because it is likely to aid in many everyday activities, such as assembling furniture or packing for a household move.

To summarise the basic argument: spatial skills are longitudinally predictive of later STEM achievement, over a very wide age span in development and with good statistical controls, and these skills are malleable. Thus, if improving such skills were a focus of educational effort, there is a reasonable hope that the effort would increase the number of innovative scientists and engineers and augment the size of the technologically-adept workforce needed in modern society. But taking effective action requires more evidence and detail than simply the set of facts reviewed so far and a plausible thought experiment. We need more direct evaluation of causality, asking whether we see effects of spatial training not only on spatial skills but also on STEM outcomes (Stieff and Uttal, 2015). Longitudinal studies with statistical controls that link spatial skills to STEM outcomes are suggestive, but they are not sufficient to infer causality. As Bailey (2017) points out, even the best statistical controls leave open the possibility that unmeasured variables are at work, e.g. aspects of children’s home and school environment may favour the development of both spatial and mathematical skills without the two being inter-dependent.

Figure 3. Distributions of spatial skill before and after training with cut-offs for engineers



Source: Uttal, D. et al. (2013), “The malleability of spatial skills: a meta-analysis of training studies, *Psychological Bulletin*, Vol. 139, pp. 369. © 2012 American Psychological Association.

4. Two educational strategies

This report examines the current state of evidence regarding using spatial skills in the educational system. It is organised around two distinct strategies for taking advantage of the spatial-STEM linkage. One strategy involves aiming directly to improve spatial skills with the hope of seeing later effects on STEM learning. There are several means of doing so, including not only direct spatial training, but also informal or recreational activities and the addition of spatial material to formal lessons in school. This strategy is the one discussed so far and illustrated in Figure 3, for which we have substantial evidence but whose full evaluation requires randomised controlled treatment (RCT) experiments. The report evaluates the evidence for three age ranges: preschool children, elementary school

children and, adolescents and adults. It also takes up the issues of gender and SES differences in these skills. To anticipate, the current evidence base from RCTs is moderately encouraging, but not yet decisive. One reason for this situation is simply a relatively small number of studies, but another reason is probably that we need to know more about what skills to target at what age and how to conduct training. Our examination of Strategy 1 will conclude with a discussion of this issue of the mechanisms by which spatial training might exert effects on STEM outcomes.

The second strategy is a more indirect application of the idea that spatial learning is central to STEM. Rather than focusing on the spatial skills of individual learners, or on inserting spatial activities into the school day as separate elements, this strategy involves using teaching techniques and curriculum designs that make strategic use of a set of tools for spatial thinking, thereby developing both spatial skills and specific knowledge of the scientific and mathematical topics under discussion. This approach was referred to as spatialising the curriculum in the *Learning to Think Spatially* report from the National Academy of Science in the United States (2006). Development and evaluation of this strategy begins with delineation of the tools that can be used in such curriculum redesign, followed by small experimental studies of the efficacy of the techniques that are found promising. The sequence should culminate in RCTs to evaluate the effectiveness of curriculum units that include the multiple spatial tools showing efficacy in smaller studies.

5. Strategy 1: Supporting STEM learning by developing spatial skills

Strategy 1 rests on the idea that children should develop strong spatial skills, or that high school or university students who show spatial weaknesses should receive directed spatial training in order to improve STEM attainment. However, how best to achieve these goals is likely to vary depending on the age of the student. Thus, this section is structured by age ranges: preschool children, children in elementary school and, adolescents and adults. In each section, we first take a look at any available correlational evidence, again emphasising longitudinal studies when available, regarding what informal and educational activities relate to strong spatial skills. We then examine for each age group the experimental evidence regarding whether engaging in these everyday activities or training sessions not only raises spatial skill but also leads to higher STEM achievement. The last sub-section concerning the potential of Strategy 1 focuses on delineating more precisely what mechanisms account for the linkage between spatial skills and STEM. Identifying mechanisms should allow us to hone our interventional strategies to maximise effects.

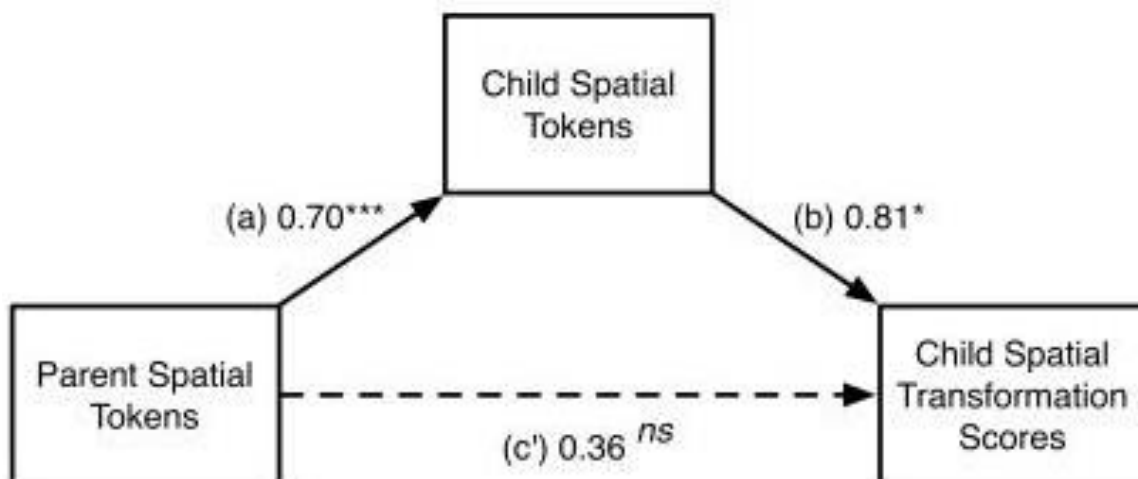
Preschool children

Everyday activities are distinct from the focused training efforts examined in the Uttal et al. meta-analysis, but were also found to correlate with spatial skills in an older meta-analysis (Baenninger and Newcombe, 1989). Because preschool children are too young to engage in lengthy structured training, how spatial play relates to spatial skills is especially important for them. Before young children begin formal schooling, they are rapidly learning new skills and soaking up information, whether at home with their families or in more organised settings such as day-care or preschool. In all these settings, a key ingredient for learning is play, an activity that is very different from the kinds of specific training programmes that were included in the Uttal et al. meta-analysis.

Play can take many forms, ranging across outdoor play on swings and slides, pretend play, construction toys, books, music and so forth. Some kinds of play seem particularly spatial in nature, notably building with blocks and other construction toys, putting together various kinds of puzzles, and spending time on arts-and-crafts activities, such as folding paper to make a flower. We have known for decades that spatial play is correlated with spatial skill in preschool children in concurrent assessments (e.g. Connor and Serbin, 1977). However, most of these studies have involved relatively small and generally middle-class samples. The robustness and generality of this linkage is now clearer, based on an analysis of a large and diverse group of pre-schoolers in the WPPSI-IV standardisation study (Jirout and Newcombe, 2015). Analysis of this data set showed a specific relation between parent-reported spatial play, i.e. play with puzzles, blocks and board games, and WPPSI Block Design scores, controlling for scores on the other WPPSI scales. This relation was not evident for other kinds of play (i.e. it is not just the product of a generally enriching environment) and it appeared for both boys and girls. Crucially, it was evident across a wide range of socioeconomic status.

Of course, any correlation between children and adults is open to the criticism that children may elicit certain kinds of input from adults based on their interests and talents, and, for parent-child dyads, there is the additional possibility of genetic linkages (e.g. Scarr and McCartney, 1983). Thus, other kinds of studies are needed. A preliminary step in bolstering our faith in potential causal linkages between spatial play and spatial skills is provided by studies showing that spatial play predicts skills in longitudinal samples with appropriate controls. Encouragingly, early puzzle play has been found to correlate with pre-schoolers' later mental rotation skills (Levine et al., 2012). In addition, parental use of spatial language is related to children's later spatial skills, in a relation mediated by the fact that children pick up the spatial language used by parents (Pruden, Levine and Huttenlocher, 2011). This mediated effect is shown in Figure 4, illustrating how the effects of playing with spatial materials and learning spatial language go hand in hand.

Figure 4. Parents' spatial language affects children's spatial skill by increasing their spatial language



Source: Pruden, S., S. Levine and J. Huttenlocher (2011), "Children's spatial thinking: Does talk about the spatial world matter?", *Developmental Science*, Vol. 14, pp. 1424. © 2011 John Wiley & Sons Ltd. Reprinted with permission.

These longitudinal studies are, however, still open to the possibility that children with initially stronger spatial skill (possibly based on genetics) might elicit more and higher-quality spatial input from parents, and even from non-related adults such as teachers. Thus, what is needed is RCTs of the effects of enhancements in the amount and quality of spatial play in preschool, based on random assignment. Unfortunately, few such studies seem to have been conducted yet. An initial finding is that parents use more spatial language when interacting with children around building blocks than when they engage in non-spatial play, and more spatial language when given a building goal rather than told to engage in free play or given a pre-assembled structure (Ferrara et al., 2011). Because Ferrara et al. randomly assigned parents and children to these conditions, we know that environment and instruction affects input as well as any pre-existing preferences and skills in either parents or children. However, there was no assessment of whether children's spatial skills improved.

Two subsequent studies focused on pre-schoolers learning about shapes. When an adult experimenter interacted with children in a shape game using guided play, in which adults suggest ideas to children rather than tell them information or direct their activities, pre-schoolers learned more about the shapes than when the experimenter used didactic methods, or simply engaged in free play with the shapes (Fisher et al., 2013). Thus, an experiment using random assignment showed how children's shape learning could be enhanced. More recently, a brief intervention randomly assigning parents visiting a children's museum showed that instructing parents to talk about shapes, rather than about another topic, speeded the children's subsequent puzzle completion (Polinsky et al., 2017).

There is also an RCT with 3- to 5-year-old children that evaluated the effects of a didactic kind of spatial training (Xu and LeFevre, 2016). In each training trial, children received three shapes cut in half to form six pieces, and were encouraged to select the two pieces that would combine to create a whole shape as shown in a standard. They received praise if they succeeded, and some prompts if they did not manage to pick the correct pieces. They completed seven such problems. Compared to a group that received sequential number training, the spatial group improved on a near-transfer spatial task, which tested essentially the same skill as had been trained. However, they did not improve on a number line task. There may be many reasons for this null effect. First, the intervention was very structured; as we have seen, preschool children might respond more fully and in a more generalised way to a more playful intervention. Second, there are more effective training techniques than the one used, which might have resulted in wider generalisation (Ehrlich, Levine and Goldin-Meadow, 2006; Goldin-Meadow et al., 2012). Third, the spatial skill trained may not be the most relevant skill for the number line task. The causal mechanism for the correlation deserves some thought, as discussed in a later section. Fourth, comparison to a group receiving sequential number training may set a very high bar for an effect.

Elementary school children

After children enter formal schooling, opportunities for play are reduced, and more time is spent in structured activities. Nevertheless, play continues to be part of the picture, and has been found to correlate with the development of spatial skills and mathematical proficiency. For example, mothers who supportively guide their 6- to 7-year-old daughters in origami help to build their spatial skills and in turn they tend to do well in mathematics (Casey et al., 2014). There are thus several avenues for improving spatial

skills in elementary school children in an effort to affect STEM learning: play in various settings (at recess, in after-school programmes or at home); structured training sessions, and curriculum modifications that add spatial tasks to everyday lessons. There are starting to be RCTs that bear on each of these approaches, but their numbers are still low. Overall, there are six such studies, with five reporting at least partial success, and another reporting a null result.

Play

There are two RCTs that evaluated the effects of spatial play. In one, kindergarten and first-grade children in an after-school programme in a school district serving a lower-SES population were randomly assigned either to *Minds in Motion*, essentially an arts-and-crafts track, or to regular activities (e.g. theatre, soccer, cooking), and engaged in these activities for 45 minutes a day, 4 days a week, over 28 weeks (Grissmer et al., 2013). The spatial programme had significant positive effects on executive function, spatial skills, and for first graders, on mathematics skills. First graders moved from the 32nd to the 48th percentile on mathematical subtests drawn from the Woodcock-Johnson and Key Math batteries.

In the other study to use play, working with 6- to 8-year-olds, Hawes et al. (2015) gave children iPads over a six-week period. In spatial training, the children played three games, two shape games focused on mirror-image discrimination and one puzzle activity, for a total of 4.5 hours. Children did improve on mental rotation as well as (marginally) on another spatial task. But they did not perform better in mathematics than the control group, which worked on literacy. One reason for this null result may have been the use of iPads. Such devices seem to distract children from learning verbal material (Parish-Morris et al., 2013; Strouse and Ganea, 2017) as well as from shape learning (Verdine et al., forthcoming).

Direct spatial training

There are also two RCTs of this kind of intervention. An initial positive finding was encouraging (Cheng and Mix, 2014). Children from 6 to 8 years of age either practiced mental rotation tasks using items similar to those used by Xu and LeFevre (2016), or did crossword puzzles, in a single 40-minute session. Following training, the mental rotation group did better on missing addend problems, i.e. problems of the form $4 + _ = 7$, although not on other kinds of mathematics tasks. This success was actually somewhat surprising given the brevity of the intervention, and the fact that although the mental rotation skills themselves did show significant improvement, there was no evidence of transfer to another spatial task. It is also surprising when compared to Xu and LeFevre's null finding. However, Cheng and Mix studied older children than did Xu and LeFevre, and also used a more active training, based on a successful prior training study with these materials (Ehrlich, Levine and Goldin-Meadow, 2006).

A larger study with longer and more varied spatial training showed positive effects (Lowrie, Logan and Ramful, 2017). Students in Grade 6 received 20 hours of training in a variety of spatial skills, including mental rotation, spatial orientation and spatial visualisation. Compared to business-as-usual controls, the intervention group improved more in mathematics, a very impressive finding because the control children actually received more hours of mathematics lessons.

Curriculum modifications

There are two RCTs that evaluate the effects of curriculum modifications in elementary school that add spatial activities as units in traditional curricula. The first study introduced spatial modifications into an early mathematics curriculum. Children from 4 to 7 years of age in First Nations schools in Canada either worked with activities derived from a workbook called *Taking Shape* (Moss et al., 2016) or engaged in an innovative environmental studies curriculum, i.e. an active control group, over a period of 32 weeks (Hawes et al., forthcoming). There was some success with this approach. Children in the intervention group improved on three different measures of spatial skills. Both groups improved in vocabulary, indicating that spatial gains were not simply due to the spatial programme being more enjoyable or engaging than the environmental programme. Most importantly, the spatial group improved more than the controls on a symbolic number comparison task, although they did not show differential gains on non-symbolic magnitude comparison or number knowledge. This finding is important because symbolic comparison predicts later mathematics achievement and also dyscalculia (De Smedt et al., 2013).

The second study involved adding new spatially-oriented arts topics and activities to the mathematics and literacy instruction in high-poverty schools in New York City (Cunnington et al., 2014). The focus on the visual arts entailed considerable use of spatially challenging exercises and visual illustration of mathematical concepts, such as fractions. Students in grades 3 through 5 experienced 12 six-week units over these three years. Random assignment was at the school level, not the student level. Students in the arts-integrated schools did better on standardised mathematics tests across all three years than students in comparison schools.

Adolescents and adults

There is a literature that relates playing spatial videogames with the development of spatial skills, and indeed videogame “training” works robustly to this effect (Uttal et al., 2013). However, none of the videogame studies have evaluated whether STEM success is enhanced by play. We also lack studies that encourage adults to engage in the other kinds of recreational activities that may enhance spatial skills (Baenninger and Newcombe, 1989). Hence, the available evidence concerning adolescents and adults centres on direct training and curriculum modification.

Direct spatial training

An early effort to evaluate spatial training in university students involved a chemistry course, and concluded that the treatment led to higher grades (Small and Morton, 1983). Little subsequent research appeared for decades, but more recently, another small-scale study found that spatial training led to better essays in geoscience (Sanchez, 2012). However, probably the greatest excitement in this area has focused on the use of a spatial training workbook designed by Sheryl Sorby (Sorby and Wysocki, 2003). Students complete the workbook as part of a 10-week course, working on hundreds of problems that involve folding, rotations, cross-sections and orthographic projections. Sorby’s initial work involved testing incoming engineering students on spatial skills, and suggesting to low scorers that they might benefit from training using the workbook. Students who did so improved their scores on spatial ability tests, received better grades and were more likely to persist in the major, an extremely-important real-world outcome (Sorby, 2009a). The students were, however, self-selected, and their high motivation to succeed is a

plausible alternative cause of the effects. Subsequent work strengthened the case in a study of calculus learning before and after a spatial training course became mandatory, using a regression-discontinuity design (Sorby et al., 2013). Completing the training led to a small but significant improvement in calculus grades. Another use of the workbook was to improve the spatial skills of university physics students, which led to better grades in physics (Miller and Halpern, 2013), although the effect showed fade-out on follow-up. Thus, the workbook improves spatial skills for adolescent students as well as university students (Sorby, 2009b). However, at this point, there is conflicting evidence regarding whether such improvement affects subsequent STEM achievement. It is possible that the workbook activities are too demanding for some populations, or difficult to execute by some teachers, or not well aimed at the targeted STEM content, as discussed under the section on mechanism.

Curriculum modifications

There is a single RCT that evaluates the effects of a spatially-oriented curriculum modification for adolescents or university students (Stieff et al., 2014). In this study, organic chemistry was taught three times in succession by the same instructor using the same tests, but with one of three instructional strategies. One course emphasised spatial visualisation of the molecules, one emphasised analytic and notational ways to approach the problems, and the last used a mix of the two approaches. (This study is an example of Strategy 1, not Strategy 2, because it focused on notation and visualisation overall, rather than using targeted tools such as gesture or sketching.) For men, course grades were not affected by which of the three emphases they experienced. For women, grades were best with the mixed approach. This result suggests caution about an exclusive focus on spatial thinking; spatial approaches may need to be blended with analytic approaches to ensure comprehension, and a flexible use of different strategies may also be helpful to success. However, a single study alone clearly cannot resolve this question.

6. Gender

Gender differences exist in some (but not all) spatial skills (Levine et al., 2016; Newcombe, forthcoming; Voyer, Voyer and Bryden, 1995). Because spatial skills are linked in term to success in STEM learning, it is common to hypothesise that improving spatial skills might improve the representation of women in STEM fields, and perhaps reduce occupational segregation by gender, given that spatial skills strongly predict occupational choice, even jobs not traditionally considered in analyses of STEM occupations (Baker and Cornelson, 2017). However, this suggestion is controversial. Discussions of how to achieve gender equity in the STEM disciplines take differing positions on whether cognitive differences of this kind can account, even partially, for the under-representation of women (Ceci et al., Williams, 2014; Ceci, Williams and Barnett, 2009; Newcombe, 2007). Many observers emphasise social factors over cognitive ones, including issues of stereotypes, classroom climates, and family-work balance. Additionally, training seems to benefit men as much as women, leaving any initial gender differences intact (Uttal et al., 2013). However, no training programme has yet taken people to asymptotic performance. After a semester of weekly practice, even high-ability men were continuing to improve (Terlecki, Newcombe and Little, 2008). Sometimes girls improve more than boys (Casey et al., 2008), but not usually. In parallel improvement means gender differences remain. Nevertheless, many experts think it is plausible that differing spatial skills play at least some role in STEM career choices for both sexes, and

further, think that improving these skills above threshold levels may support STEM persistence (Sorby et al., 2013).

7. Socioeconomic status

Although it has sometimes been claimed that spatial ability is less affected by poverty than other cognitive skills (Farah et al., 2006), there are frequent findings showing that socioeconomic status (SES) influences spatial skills (Carr et al., forthcoming; Casey et al. correlational SES study; Jirout and Newcombe, 2015; Levine et al., 2005). Furthermore, the neural substrates of spatial skills are affected by income, with especially steep income gradients among the poorest segment of society (Noble et al., 2015). Thus, it seems likely that interventions are especially needed in interventions with low SES children. In one school-based training programme in late elementary school in the Netherlands, low- and high-SES children benefited equally (Vander Heyden, Huizinga and Jolles, 2017). As with gender, this outcome leaves differences remaining at post-test as well as at pre-test. It may be that more intensive efforts are needed to narrow the gaps.

8. Interim summary

The overall picture from this handful of RCTs is promising but mixed, and it hints at complexities in designing the best programmes for various subjects at various ages. There may be a threshold of breadth, intensity or duration of intervention before success can be expected, and many techniques known to be effective in training have yet to be utilised in RCTs. Drawing strong conclusions with any appropriate nuance requires more studies, as also argued by Stieff and Uttal (2015). An additional wrinkle is that spatial skills may be most relevant when new concepts are first introduced (Hambrick et al., 2012; Uttal and Cohen, 2010). Initially, spatial thinking may be helpful to visualise a molecule as well as use standard two-dimensional notational systems, but most experts rely primarily on the notational systems, although they report visualising on occasion in order to check their work on a difficult problem.

More targeted interventions may well be possible given understanding of the mechanisms at work. Anecdotal discussions of spatial thinking in STEM often point rather vaguely to the value of imagery, or the need to understand visualisations of data. Even objective measures of spatial thinking often aggregate over several different tests, as the Project Talent study did, or concentrate on a single well-studied spatial skill, often mental rotation, without probing what precise spatial skill is required for what particular type of mathematical or spatial understanding. However, we know that these approaches are very likely limited. For example, static object-oriented imagery is quite different from dynamic spatial imagery, with only dynamic spatial imagery related to STEM (Kozhevnikov, Kosslyn and Shephard, 2005). Additionally, when geology students must learn the concepts of *strike* and *dip*, distinguishing the angle at which a geologic feature intersects with the horizontal plane in terms of compass directions from the steepest angle of descent into the horizontal plane, they are probably very reliant on their sense of horizontality and verticality (Liben, Kastens and Christensen, 2011). By contrast, the spatial skills needed in organic chemistry to decide if two molecules are *stereoisomers*, that is, molecules that differ *only* in the spatial orientation of their component atoms, seem quite different, probably including mental rotation (Stull, Barrett and Hegarty, 2013). Existing work on the relations between spatial skills and STEM learning has not differentiated sufficiently among the various skills needed for the various subject areas.

However, this situation is changing. The richest evidence base concerns early mathematics learning and university-level science education.

9. What mechanisms link what spatial skills to early mathematics learning?

One potential mechanism for the relevance of spatial skills to early mathematics learning involves the ability to accurately place numerals on a number line. Another one is dynamic spatial imagery. There is also a growing literature on mathematics learning at a variety of levels that implicates other mechanisms as well, such as a propensity to sketch word problems before attempting solutions (Newcombe, Booth and Gunderson, forthcoming).

Number line

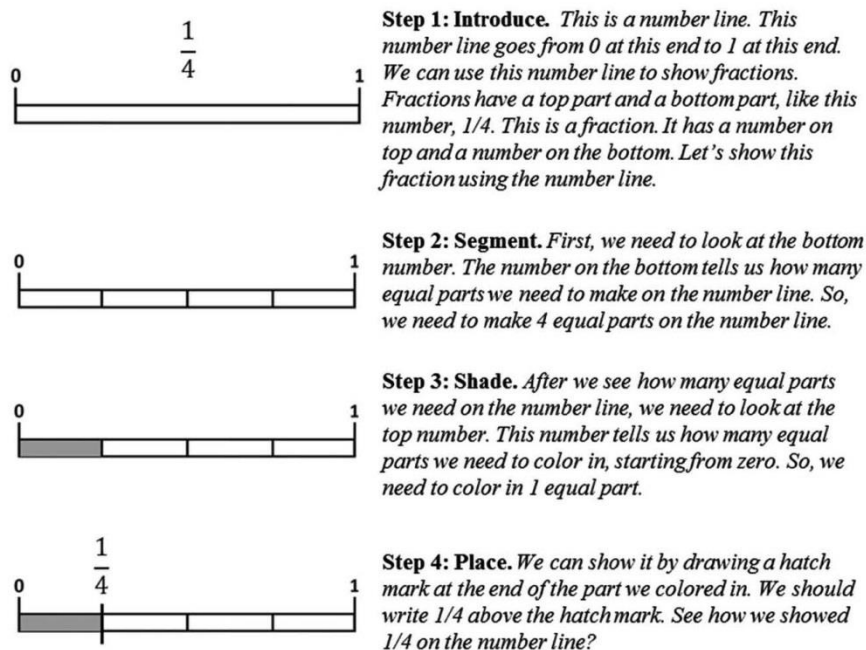
Children initially represent numbers logarithmically within a certain range, e.g. they know that 2 is systematically greater than 1 to the same extent that 3 is greater than 2. But this understanding breaks down for larger numbers so that the function relating actual to estimated magnitude is logarithmic rather than linear (Booth and Siegler, 2006; Siegler and Opfer, 2003). A change from logarithmic to linear functions occurs initially for the range 1-10, then later for 1-100, and still later for 1-1000. Number line estimation is an important predictor of later mathematics achievement (Geary, 2011). Thus, it is extremely interesting that the predictive relation from spatial skills to later mathematics was found to be entirely mediated by number line estimation, in two different longitudinal samples (Gunderson et al., 2012).

The number line task was originally construed as a direct assessment of numerical representation (Siegler and Booth, 2004). Subsequent research has suggested however that children use spatial strategies to perform number line tasks, such as establishing midpoints or quarter points, or reasoning downward from the endpoint as well as upward from the start point (Ebersbach et al., 2008; Slusser, Santiago and Barth, 2013). Strikingly, the pattern of bias in number line estimation tasks resembles the pattern of bias found in spatial estimation tasks (Newcombe, Levine and Mix, 2015). Viewed in this way, establishing a systematic and ordered representation of numbers is a deeply spatial enterprise, throwing light on the mechanism for the connections. Indeed, number line estimation has much in common with spatial scaling, a skill required to read maps. Interpreting a map requires establishing a range of interest (for numbers, a range such as 1-100, for distances, a range such as 0 – 100 metres) and then considering where a particular value should fall (71, or 71 metres) given the particular spatial frame offered (the length of the physical line provided in the number line estimation task, or the dimensions of the map in the mapping task). In accord with this way of thinking, children's ability to perform scaling tasks is related to their number line estimation (Jirout, Holmes and Newcombe, forthcoming; Möhring, Frick and Newcombe, forthcoming) and predicts mathematical achievement in a longitudinal study (Frick, forthcoming). Furthermore, both number line estimation and spatial scaling have much in common with proportional reasoning. In adults, ratio processing relates to mathematical performance including in college-entrance algebra tests (Matthews, Lewis and Hubbard, 2016), and in children ratio processing relates to success on a test of understanding of fractions (Möhring et al., 2016).

As we delineate mechanisms, we may be able to refine our educational interventions. For example, number line training is effective (Ramani and Siegler, 2008), but further efforts

to augment it are underway, e.g. defining just how best to play a number line game (Laski and Siegler, 2014), adapting the game for use in real classrooms (Ramani, Siegler and Hitti, 2012), and applying it to learning fractions (Hamdan and Gunderson, 2017). The intervention used by Hamdan and Gunderson is shown in Figure 5. It involves use of a linear number line; equivalent work with a larger area model was not effective.

Figure 5. Stimuli used to improve children’s understanding of fractions



Source: Hamdan, N. and E. Gunderson (2017), “The number line is a critical spatial-numerical representation: Evidence from a fraction intervention”, *Developmental Psychology*, Vol. 53, pp. 590. © 2016 American Psychological Association.

Dynamic spatial imagery

A second spatial skill that may be involved in number line representations is dynamic spatial imagery, or “zooming” in and out on the scale of a representation (Kosslyn, 1975). Children and adults both use mental transformations to scale distances in space, as shown by linear increases in response times and errors with increasing scaling factor (Möhring, Newcombe and Frick, 2014; 2016). Such an analogue imagery strategy may indicate why mental rotation predicts number line performance. In line with the idea that both scaling and zooming may be involved in number line tasks, different tests of spatial skill make unique contributions to predicting later mathematical performance. In particular, both scaling and mental rotation are predictive of mathematical achievement in a longitudinal study of elementary school children, contributing separate variance (Frick, forthcoming). Dynamic spatial imagery is also implicated by research with adults, for whom mental rotation and number line estimation are correlated (Thompson et al., 2013). Further, in a study of adult mathematicians and non-mathematicians, mathematicians were found to perform better on positive (but not negative) number line estimation tasks, with this relation entirely mediated by scores on a block design test (Sella et al., 2016). Block design involves decomposing a pattern into component shapes and/or assembling a

pattern from component shapes. Some shapes usually need to be rotated to make the specified pattern, suggesting that dynamic mental imagery may be at play, although there are also many differences from mental rotation tasks.

10. What mechanisms link what spatial skills to university science learning?

As pointed out above, a geology student striving to use the concepts of strike and dip (Liben, Kastens and Christensen, 2011) may be using different spatial skills than the organic chemistry student asked to decide if two molecules are stereoisomers (Stull, Barrett and Hegarty, 2013). Thus, substantial efforts involving training many skills, such as the workbook devised by Sheryl Sorby, even if successful, might actually be less cost-effective than targeted just-in-time interventions designed to boost the skills required in the next units or lectures. But doing so requires knowledge of what skills are needed for what tasks.

Spatial skills needed in geoscience

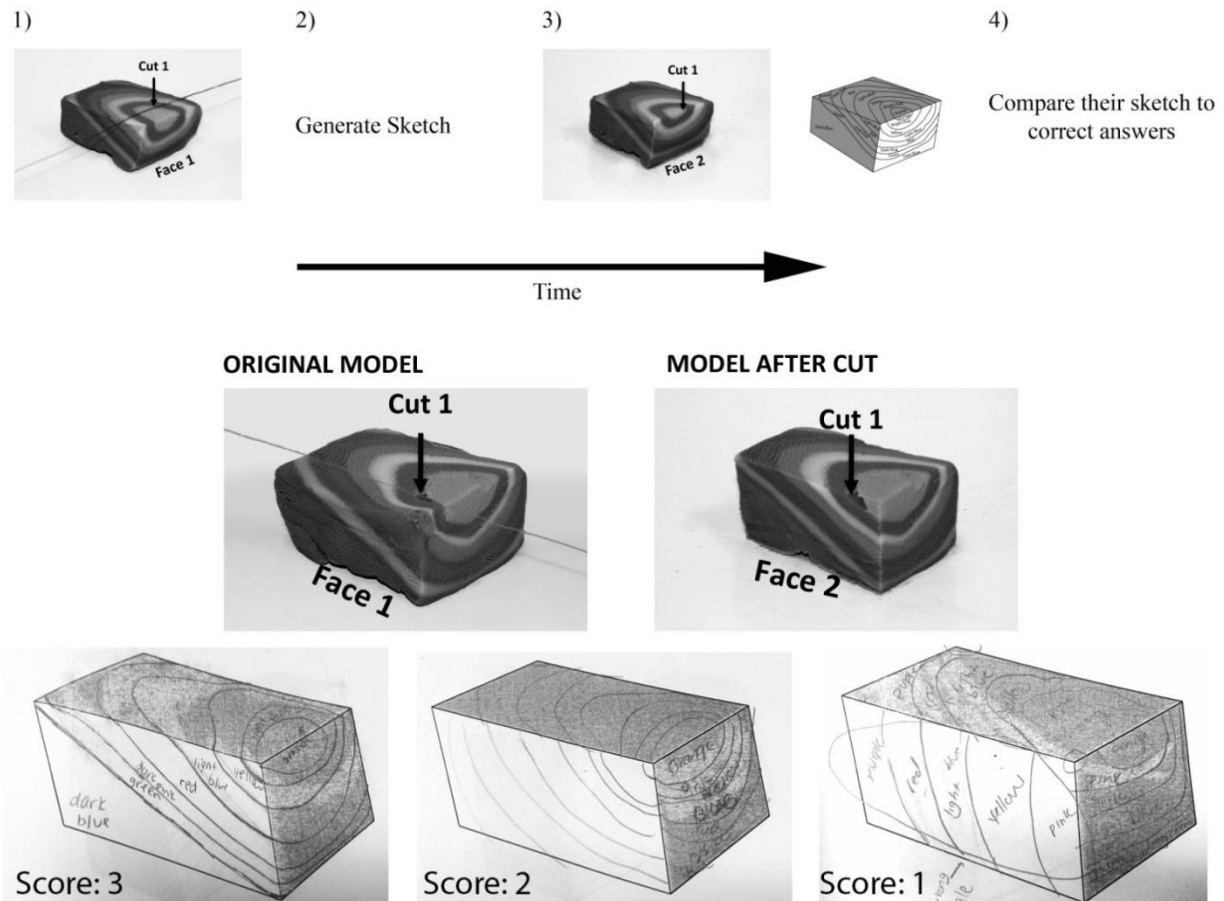
Cognitive scientists have begun to partner with disciplinary specialists to conduct task-specific analyses, and in doing so, have delved into new areas and identified spatial skills not targeted by existing psychometric tests. In one such collaboration, sustained interaction between cognitive science and geoscience has led to new tests and new tools to use in improving education (see review by Gagnier et al., 2016a). For example, geoscientists have to imagine a variety of rigid and non-rigid transformations (Ormand et al., 2014). A variety of the latter is the brittle transformation, in which some spatial region rotates or translates (or both) with respect to others, which may also move. A common example occurs when we break a piece of crockery, but, at a slower time scale, this kind of process occurs constantly in the history of the Earth. Resnick and Shipley (2013) devised a test of this kind of thinking, and showed that expert geologists performed better than comparison groups of organic chemists or English professors. Importantly, organic chemists did just as well as geologists on mental rotation (a skills required by their discipline) although English professors did worse here too. There are also other new assessments, for instance of cross-sectioning and penetrative thinking (Cohen and Hegarty, 2012) and of bending (Atit, Shipley and Tikoff, 2013).

Penetrative thinking is an excellent example of a spatial skill that was not well-studied by psychologists until they began to engage in interdisciplinary dialogue with STEM experts, for which understanding may lead to targeted interventions. For example, imagine that geology students are about to tackle block diagrams, but that they begin with warm-up exercises in penetrative thinking. They might start with activities designed to convince them that what they see on the surface of a block does not extend uniformly into it, a common misconception identified by Gagnier and Shipley (2016) that extends across a wide variety of materials (including everyday examples such as raisin bread). They might go on to do sketching and prediction exercises (Gagnier et al., 2016b), as illustrated in Figure 6. Such preliminary work might improve their scores on cross-sectioning tests, but they would not have practiced the tests themselves, but rather engaged in course-relevant work. Thus, improving spatial skills in this targeted way can become almost indistinguishable from what we are calling Strategy 2.

Using maps and geographic information systems

Reasoning about large-scale spatial distributions using maps is another area of spatial reasoning that is beginning to be better studied by interdisciplinary collaborations between cognitive scientists and (in this case) geographers. Existing psychometric tests assess small-scale spatial skills that are focused on the encoding and transformation of individual objects (e.g. mental rotation, mental cutting). There is little information regarding the relation of STEM interest and success to extrinsic-dynamic spatial skills like perspective-taking and individual differences in spatial navigation (Kozhevnikov et al., 2006). However, there are reasons to suppose that such thinking may be powerful, especially in disciplines that require thinking about the spatial world as a whole, i.e. all the “geo” disciplines, such as geology, geoscience and geography, and their associated disciplines such as oceanography. In support of this idea, geoscientists report high levels of navigational competence and confidence (Hegarty et al., 2010).

Figure 6. Intervention to help students to understand reasoning about geological formations



Source: K. Gagnier et al. (2016), “Training spatial skills in geosciences: A review of tests and tools”, *Earth, Mind, and Machine: 3-D Structural Interpretation: Earth, mind, and machine: AAPG Memoir*, Vol. 111, pp. 7-25. AAPG © 2016 reprinted by permission of the AAPG whose permission is required for further use.

Geographers have written extensively about a “spatial turn of mind” (e.g. Goodchild and Janelle, 2010), which is a different concept than spatial skills. What they mean is that some people are more likely to use spatial strategies for thinking about problems, and in particular, to use maps to display spatially distributed data. Maps allow for, and indeed often have as their principal aim, the simultaneous display of metric spatial information, in a metric format. Compared to the sequential and categorical nature of language, such displays have compelling advantages for scientific discovery and communication. In line with this argument, some of the most famous examples of the power of spatial thinking, such as John Snow’s discovery of the water-based transmission of cholera, are based on this kind of data display, and modern examples in epidemiology are abundant. More generally, map-like displays and the associated thinking skills may underpin the organisation of many forms of conceptual knowledge (Constantinescu, O’Reilly and Behrens, 2016). There have been several calls for educational reforms aimed at increasing competence with spatial data (National Governors Association Centre for Best Practices and Council of Chief State School Officers 2010; NGSS Lead States 2013). Yet we currently have very little information to firmly support linkages among navigation, map-reading skills and a “spatial turn of mind” (Baker et al., 2015). Nor do we firmly grasp the mechanisms underlying such linkages, if they exist.

An important reason for the lack of research examining large-scale spatial navigation is the difficulty of conducting lengthy and difficult-to-control real-world navigation experiments (Schinazi et al., 2013). Consequently, we know very little about the implications of navigation for STEM learning—it is simply too costly and logistically challenging to gather large samples. A key methodological tool in investigating individual differences in spatial navigation skills is the availability of a standardised measure that can practically be given to large numbers of participants. One such a tool, called Virtual Silton, has now been used with hundreds of participants of varying ages (Weisberg and Newcombe, 2016; Weisberg et al., 2014). Assessment tools of this kind can then be used in studies correlating skills with STEM outcomes, evaluating the efficacy of training techniques, and so on.

Evaluation of Strategy 1

For most people who hear about the link between spatial skills and STEM learning, the implication is obvious: that we should train spatial skills. We have seen that there is moderately good evidence to support this impulse, although more is needed. Future research should not simply proceed in the same vein as prior research, but should rather become more analytic, using what we are learning about the variety of spatial skills needed in a variety of STEM settings to focus our efforts. Furthermore, training techniques need not involve simple practice, or undifferentiated play. The science of learning also provides an array of techniques that can be used for training, such as the sketch-compare-predict framework used by Gagnier and Shipley (2016). Their method blends three powerful learning tools: sketching, analogy, and active engagement. The method is also a just-in-time strategy that could be folded into a standard curriculum. Training interventions of this kind have considerable promise. In fact, they transition smoothly into what we are calling Strategy 2.

11. Strategy 2: Spatialising the STEM curriculum using spatial learning tools

Researchers in the science of learning have now developed a set of powerful tools for teaching. Some techniques are general to any kind of learning, such as spaced study, retrieval practice, multimedia learning or self-explanation; excellent overviews of these findings are available, although research on them continues (Dunlosky et al., 2013). Other techniques, while having wide domains of application, nevertheless have more specifically spatial mechanisms or applications that make them particularly useful in STEM learning. The latter category includes the use of symbolic systems, such as spatial language and a variety of visual systems for communicating information, analogical learning, and learning that is grounded in embodied experience of the world.

Collectively, the various tools in these three categories allow us to pursue a second strategy to leverage the spatial-STEM connection, in which we strive to incorporate spatial skills into the curriculum efficiently and pragmatically. This strategy has a great advantage over Strategy 1, namely that it does not require adding components to an already-crowded school day, or new courses to an already-crowded set of university requirements, although it may require new curriculum materials and professional development for teachers (Liben, 2006). However, it necessitates focused thinking about each of the curriculum components that we wish to teach effectively using these tools. In this section of the report, we survey the available learning techniques and briefly explain their theoretical rationale, including examples of their relevance to STEM learning at a variety of ages and in a variety of settings. We follow by considering how they can be combined, both in small-scale tests of efficacy and in curriculum units that can be evaluated for effectiveness.

Symbolic systems

The use of symbolic systems is a distinctive characteristic of our species (Blaut et al., 2003; Fitch, 2010). Language is the symbolic system that usually springs to mind first, but visual symbols also abound, such as the signs used in sign language, and the representation of information in maps, diagrams, and graphs. These symbol systems differ in how they treat spatial information. Maps, diagrams, graphs and sign language are intrinsically spatial, i.e. they show a set of information so that all of it is simultaneously available, with eye movements easily allowing for a variety of comparisons and linkages among aspects of the display. In addition, they can naturally communicate metric quantities, not simply the categorical information that is generally required by spoken language (Tversky, 2011). By contrast, when using a spoken language, we need to group together spatial locations as regions or relations (*inside*, *on the left*). In addition, we communicate information sequentially, with an order that taxes working memory and that may not proceed in a way best suited to enhancing listener comprehension (Newcombe and Huttenlocher, 2000). Nevertheless, spoken and written language can be extremely useful in spatial thinking. We will discuss each of these symbolic systems in turn.

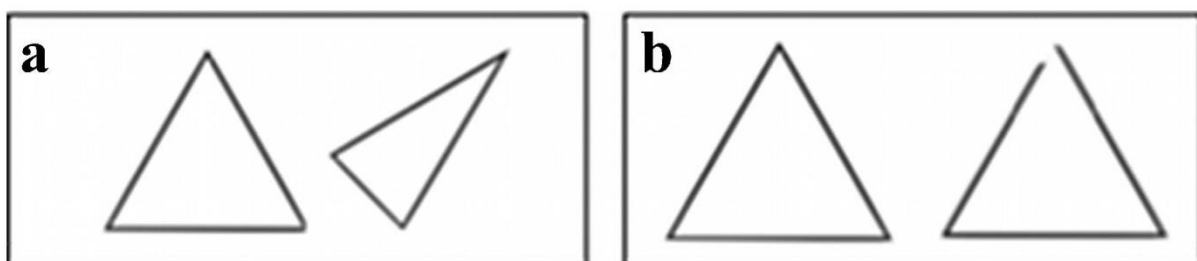
Language

As just mentioned, spoken language typically represents spatial information categorically, as *inside* or *outside*, *near* or *far*, on the *right* or the *left*, *curved* or *straight*. Distances or degree of curvature can be captured only using longer phrases or numerical measurements, e.g. *the road bends 10 degrees*. Such language is rare and often hard to understand. Furthermore, language necessitates choices among spatial reference systems.

We say that an object is to the *left* of another object, but do not also say it is to the *east*. The necessity for such choices, which are made differently in different languages and cultures, has led to fierce controversy and interesting data relevant to the long-standing issue of whether language shapes thought (e.g. Gleitman and Papafragou, 2012; Levinson, 2003), and the role of language in cognitive development (Newcombe, 2017). But thinking about the use of language in supporting spatial thinking does not require resolution of these controversies. Language clearly provides tools for thought (Gentner and Goldin-Meadow, 2003). For one thing, language can highlight dimensions that might otherwise be less likely to be noticed and remembered. For instance, children find it easier to remember left-right relation when someone has drawn attention to the relationship using language, even when the language is not strictly relevant to the precise spatial arrangement, for example when one side is simply said to be *prettier* than the other (Dessaegh and Landau, 2013; Shusterman, Lee and Spelke, 2011; Xu, Regier and Newcombe, 2017). For another thing, the mere use of spatial language by parents, and its acquisition by children, may alert children to the importance of the spatial world and support them in exploring it (Balcomb, Newcombe and Ferrara, 2011; Pruden, Levine and Huttenlocher, 2011). One key educational merit of spatial language lies in the very categorical nature of language. Many mathematical and scientific concepts denote spatial categories that are not used in everyday life, or are used imprecisely. Let's consider examples drawn from both preschool and university STEM learning.

In preschool shape learning, as we have seen, Fisher et al. (2013) used guided play successfully to teach young children the names of shapes, such as *triangle*, *rectangle* and so forth. Being able to name shapes is often considered a central element in school readiness, but it is actually difficult because child-directed input rarely contains a range of instances of the formal concept, e.g. tipped scalene triangles with widely varying side lengths as well as point-side up equilateral triangles (Verdine et al., 2016). Until late elementary school, children may accept shapes that are perceptually distinct from the examples they usually encounter (a tipped triangle as shown at the left of Figure 7) and/or fail to reject examples of rule-violating shapes (such as a triangle with a gap as shown at the right of Figure 7) (Satlow and Newcombe, 1998). A narrow range of input impedes category learning (Nosofsky et al., 2017). But, especially in a playful rather than didactic context, children can learn these words effectively from appropriate instances paired with shape terms. Spatial language is naturally paired with play with toys such as shape sorters, although such language is more common with physically-present traditional toys than with electronic games (Zosh et al., 2015).

Figure 7. Canonical triangle with valid but odd example (left) or invalid example (right)



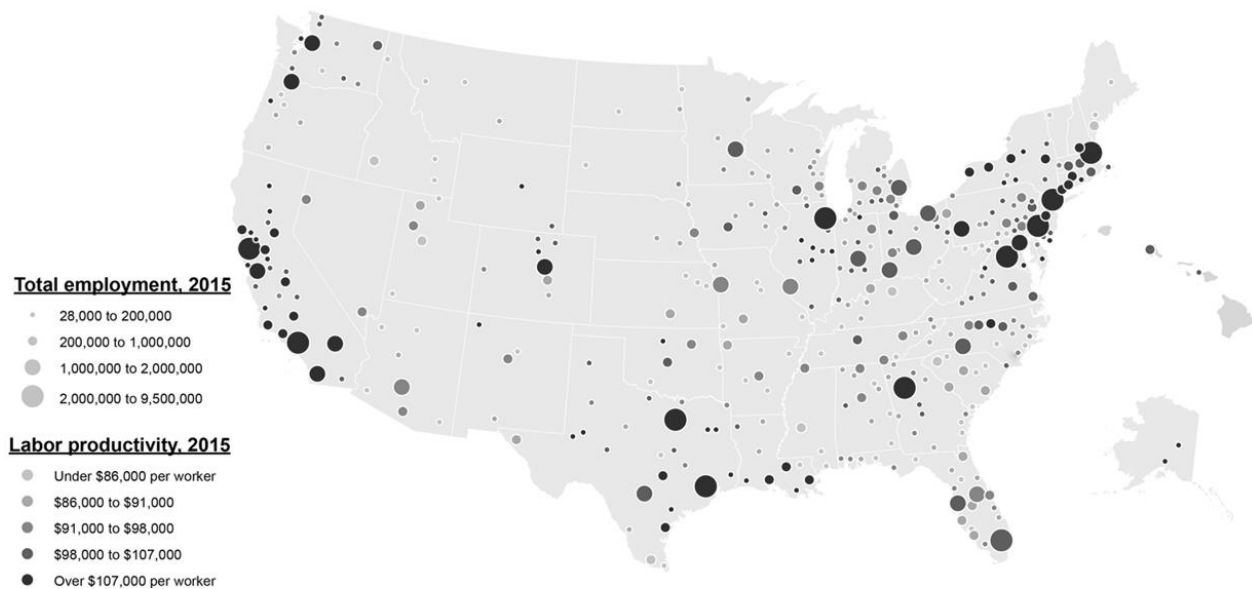
Source: Smith, L. et al. (eds.) (2014), "Mechanisms of Spatial Learning: Teaching Children Geometric Categories", *Spatial Cognition IX. Spatial Cognition 2014. Lecture Notes in Computer Science*, Vol. 8684, pp. 328. © 2014 Springer International Publishing Switzerland. Reprinted with permission.

Word learning biases also need to be understood and harnessed to help children learn key geometric concepts, such as understanding *angle* and angle size. Preschool children often assume that a new word refers to a whole object, a bias that helps them in many word learning situations. However, when they hear the word *angle*, they may falsely conclude that the word encompasses the length of the lines that constitute the figure they see. Blocking this interpretation helps them learn the correct concept (Gibson, Congdon and Levine, 2015).

Learning in STEM subject areas in fact requires learning a great number of novel words and concepts and/or requires old words to be used in different and technical ways, such as the meaning of *work* in physics. The acquisition of these concepts and words affects perception and memory, as shown by a study of location memory in geology experts and non-experts looking at pictures in which geology concepts competed with natural concepts to constrain possible locations of a point (Holden et al., 2015). One example of a challenging concept is that denoted by the word *elevation*, which must be differentiated from words like *height* and *slope*, and used to interpret topological maps (Atit et al., 2017).

Maps

Maps represent metric information naturally, and make many relations available simultaneously (Uttal, 2000). In addition, maps highlight spatial relations that can be difficult or impossible to perceive in direct experience. For example, looking at a map allows us to see the relative spatial position of cities across the United States. Maps can also be used to convey non-spatial information as a function of locations in space (e.g. productivity and employment as a function of location, as shown for the United States in 2015 in Figure 8). Understanding the spatial conventions needed to interpret maps was basic to John Snow's work on cholera, and continues to be essential to becoming proficient in the STEM disciplines. For example, developing skill in geoscience depends on knowing how to understand and use complex maps, including Geographic Information Systems (GIS), which represent three-dimensional topography. GIS is also used extensively in fields as diverse as criminology and marketing.

Figure 8. Maps effectively convey information

Source: Parilla, J. and M. Muro (2017), “Understanding US productivity trends from the bottom-up”, *The Brookings Institution*, www.brookings.edu/research/understanding-us-productivity-trends-from-the-bottom-up/, accessed 01 June 2017. Reprinted with permission.

Children’s abilities to construct internal spatial representations of their world and to interpret maps develop hand in hand. Between the ages of 2 and 3 years, children start to use external landmarks to mark location (Balcomb, Newcombe and Ferrara, 2011) and about the same time, they begin to succeed in using scale models to find hidden objects (DeLoache, 2004). The development of navigation and way-finding skills continues into late childhood and adolescence (Newcombe and Huttenlocher, 2006) and is marked by substantial individual differences (Weisberg and Newcombe, 2016). In parallel, the ability to interpret scaled representations first appears in the preschool years (Huttenlocher, Newcombe and Vasilyeva, 1999). Children as young as 4 years can use maps to guide navigation in simple situations (Scholnick, Frin and Campbell, 1990), but have difficulty dealing with misalignment (Bremner and Andreasen, 1998). Children of 6 years can use maps in larger and more complex spaces, where there are many alternative ways to go and where rescaled distance information must be used (Sandberg and Huttenlocher, 2001). However, children of 5 and 6 years rarely turn the map into physical alignment with the space (Vosmik and Presson, 2004). Instead, they can succeed when they are able to “look ahead”, planning what they will do for several turns, while looking at an aligned map (Sandberg and Huttenlocher, 2001; Vosmik and Presson, 2004). Children in early elementary school have difficulty with scaling when dealing with maps or models of large spaces (Liben and Yekel, 1996; Uttal, 1996). Map use continues to gain in sophistication and scope over the school years (Liben et al., 2013).

These facts about development suggest that map reading can be taught from preschool onwards, and used to convey information even as these reading skills are taught. In fact, education in map reading and education in geography more broadly have been suggested as central to any movement towards spatialising the curriculum (Liben, 2006). Explicit methods for accomplishing this goal are being developed. Books for pre-schoolers are now available that engage children in understanding how maps can be both enjoyable and useful, such as *Mapping My World* or *Follow That Map!* (For a useful list of such books,

see ww2.kqed.org/mindshift/2015/12/18/15-picture-books-that-support-childrens-spatial-skills-development/). There are specific educational units to support map interpretation in elementary school instruction (Liben, Kastens and Stevenson, 2002), and the teachATLAS project involves upper elementary school in GIS projects (Hund et al., 2015; see www.teachatlas.com/). By adolescence, students can engage fully in GIS, and the Geospatial Curriculum has been shown to enhance spatial skills and to allow high school seniors to use spatial thinking to solve real-world problems (Jant, Uttal and Kolvoord, 2014). But some kinds of maps remain challenging for university students. Geology students are required to learn to interpret topological maps, but often find them very challenging to understand and use (Rapp et al., 2007). New techniques for instruction are currently being devised, which involve combining several of the tools discussed in this section (Atit, K. et al., 2017).

Diagrams

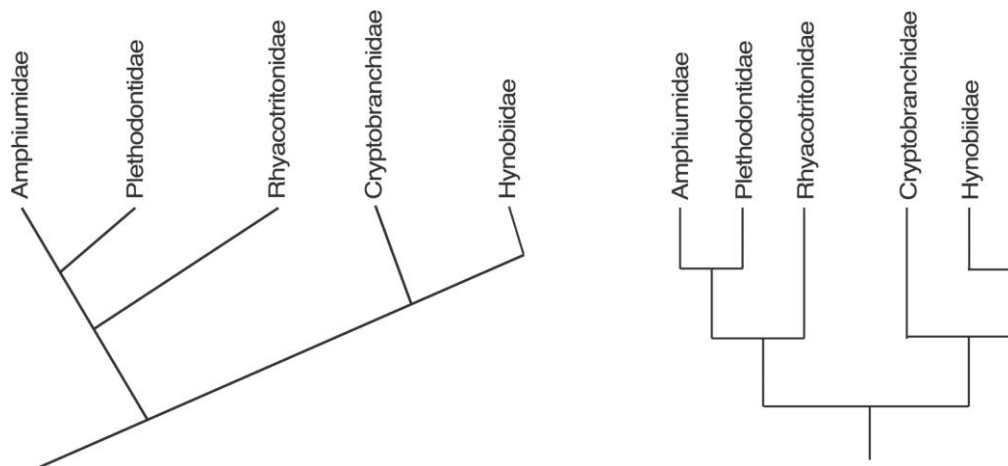
Diagrams are common in science and math textbooks and in assessments, illustrating key scientific concepts, such as Newton's laws, plate tectonics, and DNA replication (Schmidt, Wang, and McKnight, 2005; Slough et al., 2010). Diagrams are recommended tools for math instruction (National Council of Teachers of Mathematics [NCTM], 2000), and for good reason. Students provide more correct answers to word problems when the problems contain accompanying diagrams (Hembree, 1992), and representation comprehension skills predict student learning in mathematics (Pantzaria, Gagatsis and Elia, 2009) as well as in science (Madden, Jones, and Rahm, 2011). Unfortunately, students often fail to understand diagrams (e.g. Hegarty and Kozhevnikov, 1999; Kozhevnikov, Motes, and Hegarty, 2007; Wainer, 1992). For example, students do not always follow the path of a diagram correctly (Kozhevnikov et al., 2007) or form an accurate mental model of the represented system or object in the diagram (Kriz and Hegarty, 2007; Bodemer et al., 2005). Nor do they always succeed at identifying conceptual relations between multiple representations or components of representations, or inferring information that is not explicitly represented in complex graphs or diagrams (Bertin, 1983; Pinker, 1990). Overall, it is clear that diagrams are useful spatial tools but that their use in education could be more effective.

One way to improve the effectiveness of diagrams in instruction is to improve the diagrams. Such improvement is best grounded in strong knowledge of perception and cognition, and may require sustained cooperation with disciplinary specialists. An excellent example comes from a decade of work on depictions of evolutionary relations (Novick and Catley, 2017; Novick, Shade and Catley, 2011). Displays of these relations are called *cladograms*. An initial insight was that evolutionary biologists prefer the rectangular displays shown at the left in Figure 9, but high school and university texts often used the diagonal format. However, the diagonal format encourages students to scan left to right along the diagonal, inferring a straight path in evolutionary change with occasional branches, which is not the way evolutionary biologists conceptualise speciation. A series of studies led eventually to the formulation of a full curriculum, shown to be effective for university education, and also to changes in the practices of publishers.

However, improving all of the diagrams in current STEM instruction poses logistical challenges. There are simply too many diagrams on too many disciplinary topics, and publishers lack financial incentives to improve them. An alternative strategy is to instruct students in how best to read diagrams. In fact, such instruction is likely necessary even with the best-designed diagrams, because diagrams are never simply pictures. They use

conventions that must be taught and practiced, such as zoom-outs or zoom-ins, or the use of false colour to heighten contrasts. The arrows in diagrams have multiple meanings, many very different from each other (Tversky et al., 2007). There is now good evidence that diagram instruction can often help students in middle and high school learn science (Bergey, Cromley and Newcombe, forthcoming; Cromley et al., 2013a; Miller, Cromley and Newcombe, forthcoming) and improve their scores on diagram items on important assessment instruments (Cromley et al., 2016). However, such effects are not always obtained (Bergey et al., 2015).

Figure 9. Two ways of drawing evolutionary relationships



Source: Novick, L. and K. Catley (2007), "Understanding phylogenies in biology: The influence of a gestalt perceptual principle", *Journal of Experimental Psychology: Applied*, pp.204. © 2007 American Psychological Association.

Another tack to take in using diagrams is to support students in working with them, for instance by asking them to fill in verbal labels, finish partially-completed drawings, or discuss them in groups (Tippett, 2016). However, the impact of such interventions on science learning is sometimes positive (McCrudden, McCormick and McTigue, 2011; Rotbain, Marbach-Ad and Stavy, 2006; Schwamborn et al., 2010), but not always (Cromley et al., 2013b; Schwamborn et al., 2011). Overall, it is likely that the simplest and most effective educational intervention is to instruct students in the conventions of reading diagrams, beginning with first use in mathematics and science classes, likely in late childhood.

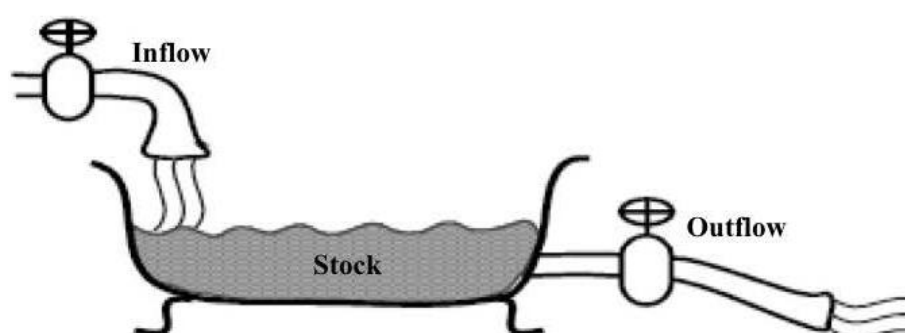
Graphs

Graphs are a standard tool in mathematics and science. They are a cognitively natural means for representing quantities and quantitative relations, because numerical and spatial relations are deeply intertwined (Dehaene, Bossini and Giraux, 1993). As with maps and diagrams, because they present multiple spatial relations simultaneously, they allow for multiple comparisons and the drawing of novel inferences. However, as with maps and diagrams, graphs require instruction in how to construct and interpret them.

There is an increasingly rich literature on how to make useful and clear graphs, which started with the impressions and judgments of Edward Tufte (2001), but has continued to be the focus of research in the science of learning. For example, people associate exact quantities with bar graphs but trends and relation with line graphs (Zacks and Tversky,

1999). They describe information drawn from bar and line graphs differently, even when the content is familiar and meaningful and even when some descriptions are actually nonsensical. Furthermore, people are more likely to gather information about interactions from line graphs (Shah and Freedman, 2011), although such conclusions were also supported by graphic literacy and familiarity with the content. Interactions are hard enough to grasp that a well-designed display, a skill in graph reading, and content knowledge were all needed. At an earlier level in instruction, consider the case of a very simple bar graph displaying two magnitudes. Elementary school children may be asked questions about these displays, such as “which is more, blueberries or oranges?” They have natural tendencies to start at the left, look to the highest, or look first at the fruit first mentioned, and these tendencies seem to conflict (Michal et al., 2016). Designing introductory bar graphs with these issues in mind may smooth the path to graph reading.

Figure 10. Cartoon illustrating the structure of stock-and-flow problems



Source: Smith, L. and D. Gentner (eds.) (2012), “Using spatial analogy to facilitate graph learning”, *Spatial Cognition VIII*, pp. 199. © 2012 Springer-Verlag Berlin Heidelberg. Reprinted with permission.

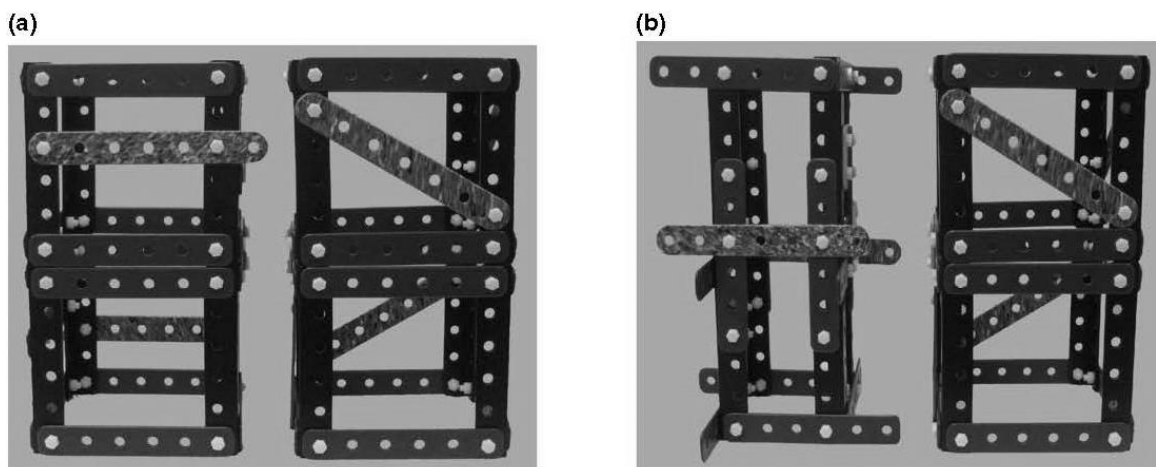
Students and readers are, however, often forced to interpret graphs that have not been drawn so as to optimise cognitive processing. In this case, instruction may help to deal with the issues. One method shown to work well takes us back to the tool of analogical learning. Students often find it difficult to interpret stock-and-flow graphs, representing the situation shown in Figure 10, in which amounts are incoming and outgoing in temporal relation to each other. They perform better when they study two examples side by side and are encouraged to compare them than when they study the same examples sequentially, and they also do better when the two examples have a perceptual similarity that facilitates structural mapping (Smith and Gentner, 2012).

Analogical learning

Thinking in terms of comparison is a powerful way to learn. We can leverage knowledge in one situation to acquire new information, both by seeing what elements and relations are similar across situations, and also by focusing on crucial differences. Analogical learning is central to human intelligence and can be expressed in verbal, mathematical or spatial ways (Gray and Thompson, 2004). Even verbal or mathematical analogies, however, have a spatial aspect, in that they involve *structure mappings* between structurally aligned representations (Gentner, 1983). There is abundant theoretical background and empirical evidence for using analogies in STEM education, including a rationale for linkages to conceptual change in science (Goldwater and Schalk, 2016), a

meta-analysis that shows educational effectiveness (Alfieri, Nokes-Malach and Schunn, 2013), and overviews that explain the central concepts for educational practitioners (Richland and Simms, 2015) and that include information on the neural bases of analogical learning (Vendetti et al., 2015). In this section, we will review three examples of the successful use of analogy at various age levels and in various domains: preschool shape learning, elementary school learning of a principle of physics and engineering, and university geology learning.

Figure 11. Pairs of stable and unstable buildings that are aligned (left) or not (right)



Source: Gentner, D. (2016), “Rapid Learning in a Children’s Museum via Analogical Comparison,” *Cognitive Science*, Vol. 40, pp. 227. © 2015 Cognitive Science Society, Inc. Reprinted with permission.

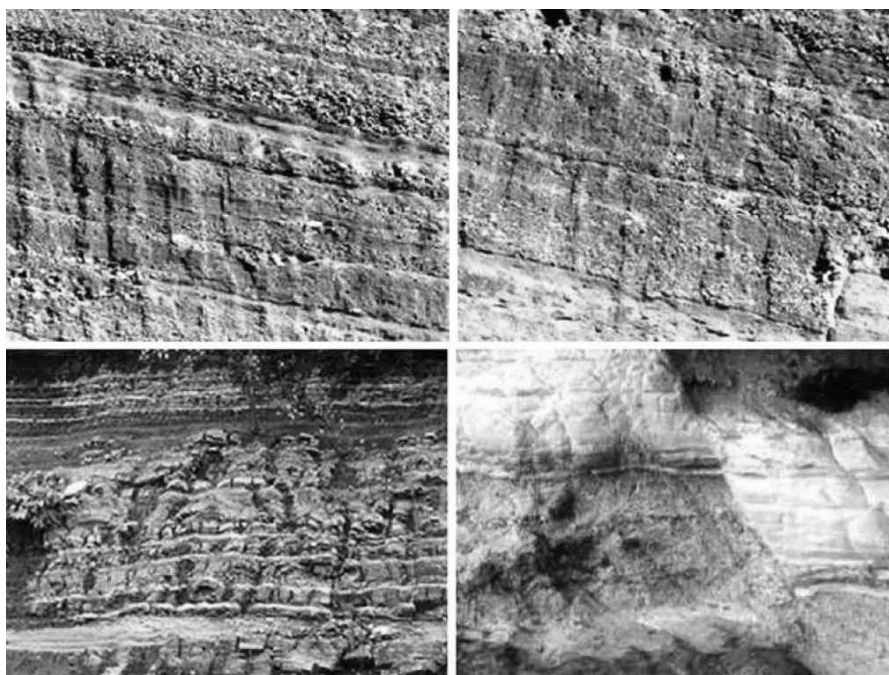
We have already discussed the importance of playful demonstration of spatial vocabulary for shapes. Analogy provides another arrow in the quiver, an exceptionally powerful arrow in that it targets the precise problems children encounter, namely what shapes they should generalise a term over, and which perceptually similar symbols are actually non-exemplars. Three- and four-year-old children were engaged either in a brief within-category comparison task or a between-category comparison task. Each task fostered category learning, but within-category comparison had a distinct impact on generalisation to new examples, as predicted, whereas between-category comparison reduced generalisation to non-exemplars (Smith et al., 2014).

Analogy can also be easily integrated into learning in brief experiences in children’s science. One example comes from a brief intervention conducted in children’s museums that helped 6- to 8-year-old children learn to use the diagonal brace principle to achieve a stable construction when building with play materials (Gentner et al., 2016). As shown in Figure 11, children either saw an aligned set of constructions or similar constructions that were made in a way that made it harder to compare and thus to notice a crucial difference, i.e. the presence of a diagonal brace. They were invited to shake the structures and determine which wobbled more. On testing after they completed their visit to the wider museum, children who tested the aligned pair of buildings were more likely to use the brace principle on a variety of transfer tests.

Alignable comparisons also help adult learners. When searching for anomalies in skeletal structures, people catch more anomalies more quickly with an alignable comparison, speeding perceptual learning of the kind valuable in medical training (Kurtz and Gentner,

2013). Similarly, in geoscience, students are more able to detect faults when comparing alignable diagrams (Jee et al., 2013). As shown in Figure 12, comparing the two slides in the top row makes it easier to identify the fault in one of the pictures (lower right area of the image on the right) than does comparing the two slides in the bottom row, where the two pictures cannot be aligned.

Figure 12. Pairs of aligned (top) or unaligned (bottom) images



Source: Jee, B. et al. (2013), “Finding faults: analogical comparison supports spatial concept learning in geoscience”, *Cognitive Processing*, Vol. 14, pp. 179. © 2013 Marta Olivetti Belardinelli and Springer-Verlag Berlin Heidelberg. Reprinted with permission.

Action-to-abstraction

In contrast to traditional views of the mind as an abstract information processor, recent theories of embodied cognition suggest that our representations of objects and events are often grounded in the sensorimotor systems we use to perceive and act on the world (Wilson, 2002). Involving the action systems in learning is increasingly seen as helpful in deepening students’ knowledge of abstract concepts by tying them to sensorimotor brain systems that are good at capturing spatial/action relationships. However, this action information might also harm performance by tying students’ representations too closely to the physical world. Action representations likely need to be converted into abstract representations. There are action-based tools such as gesture and sketching that may serve as a bridge between concrete physical relations and more abstract knowledge (Beilock and Goldin-Meadow, 2010; Novack and Goldin-Meadow, 2017). These tools involve acting in space with the hands but the actions are abstracted away from the physical realities. Thus, they may drive the action-to-abstraction process.

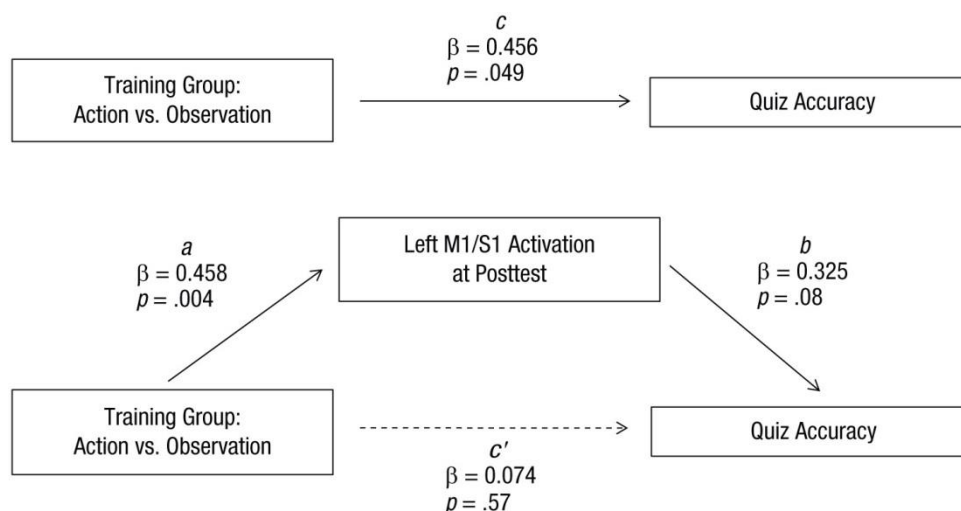
It is possible that these tools might be particularly important for younger children, for whom many educators advocate hands-on learning. But there are reasons to be cautious about this conclusion. First, as we shall see, action, gesture and sketching are also very

important for adults. Second, very young children may be less able to learn using these means than older ones, exactly because abstraction is required (Novack, Goldin-Meadow and Woodward, 2015). Third, young children may become distracted by manipulatives, focusing on them as objects rather than remembering they are symbols (Uttal, Scudder and DeLoache, 1997).

Movement

Some scientific concepts clearly are bound with physical actions in the spatial world. For example, the concept of angular momentum involves the generation of forces from spinning objects, formalisation of these forces as vector quantities with magnitude and direction, and calculations involving these vectors. Of course, spinning objects can be held, and the forces generated by the spin can be felt. Thus, if the idea that sensorimotor learning can undergird learning abstract scientific concepts has any validity, supporting evidence should certainly be seen in problems of this kind. In fact, learning concepts like angular momentum by physically experiencing that angular momentum by holding spinning bicycle wheels enhances physics learning on quizzes, including tests in real-world classrooms, relative to control conditions in which students observe but do not feel the spin (Kontra et al., 2015). For some participants, the involvement of sensorimotor brain areas while thinking about these problems was measured using fMRI. Intriguingly, improvements in quiz accuracy were fully mediated by sensorimotor activation, as shown in Figure 13.

Figure 13. Action improves quiz performance by creating embodied representations



Source: Kontra, C. et al. (2015), "Physical experience enhances science learning", *Psychological Science*, Vol. 29/6, pp. 745. © 2015 Sage Publishing. Reprinted with permission of SAGE Publishing.

But are there sensorimotor components to thinking about more abstract problems that do not have initial action components? Indeed, there may be some components of this kind, which can be usefully engaged in instruction. For example, negative numbers can be hard to understand because they are ineffable. But they are symmetric with positive numbers around zero. In one study of fourth graders learning about negative numbers, some students used techniques in which a physically-present number line could literally be folded at the zero point to emphasise this symmetry. Compared to two active control

conditions, students who experienced folding did better on problems, even ones beyond the scope of the instruction (Tsang et al., 2015).

Gesture

Gesture is inherently spatial, as it involves continuous movements of the arms and hands in a spatial world (Goldin-Meadow, 2014; 2015). Gesture can capture the imagistic and continuous aspects of space that are often lost when a spatial situation is described in language (e.g. saying *turn right* indicates the direction the listener should take but does not convey whether the turn is a hard or soft right, information that can easily be conveyed in gesture) and thus add continuous information to the categorical information in language. But there are many other advantages for gesture in learning and education. First, gesture can indicate what children are thinking and their readiness to learn (Church and Goldin-Meadow, 1986). Second, gestures from teachers can effectively communicate information to learners. Gesturing while solving spatial problems helps young children more than watching gesture (Goldin-Meadow et al., 2012), and learning to solve mathematical equivalence problems via gesture promotes transfer of the knowledge gained to new types of problems and promotes generalisation better than learning via manipulating objects (Novack et al., 2014). Third, gesture helps students to solve problems by lightening the load on working memory (Goldin-Meadow et al., 2001). Fourth, gesture is part of the action-to-abstraction continuum and hence has been found to create learning that is more likely to transfer to other tasks and to consolidate into lasting gains (Nocak and Goldin-Meadow, 2017). For example, when learners are encouraged to gesture while explaining their solutions to a math problem, they are subsequently more likely to profit from a lesson in how to solve the problem than if they are not told to gesture (Broaders et al., 2007).

Much of the research on gesture in education concentrates on elementary school mathematics, but gesture continues to be useful for older students learning more complex mathematical subjects, including geometry (Nathan and Walkington, 2017) and statistics (Rueckert et al., 2017). Gesture is also useful in science learning. STEM practitioners routinely use gesture when they talk about objects in their area of expertise. For example, expert geologists routinely gesture when discussing rock formations and landscape changes (Atit, Shipley and Tikoff, 2014). Similarly, organic chemists gesture a great deal when discussing molecular structure and chemistry students benefit from having specific gestures to model and enact (Stieff, Lira and Scopelitis, 2016). These gestures are shown in Figure 14.

Figure 14. Learning to gesture in chemistry



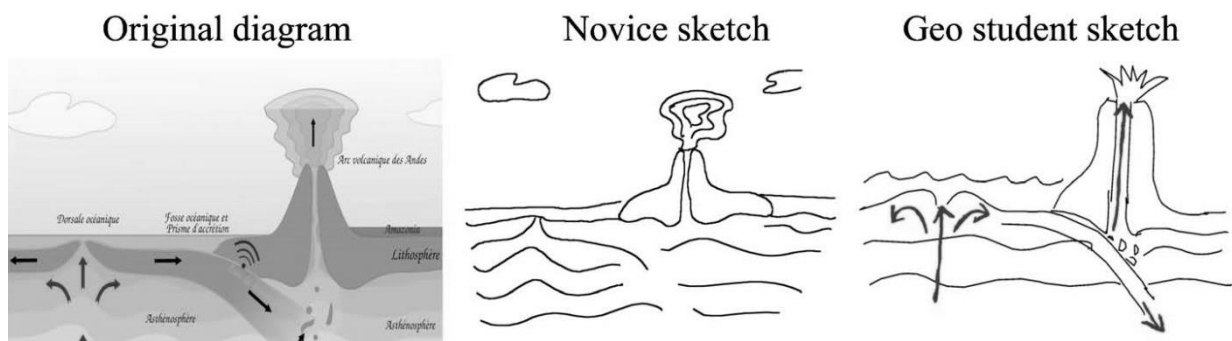
Source: Stieff, M., M. Lira and S. Scopelitis (2016), “Gesture supports spatial thinking in STEM”, *Cognition and Instruction*, Vol. 34/2, pp. 85. © 2016 Taylor & Francis Group, LLC. Reprinted with permission.

Sketching

Sketching is somewhat like reading diagrams, but sketches are diagrams actively produced by students, and hence they might be expected to be associated with more active learning. Sketching is also somewhat like gesturing, in that the hands move to create a representation of what the student is thinking, but in the case of sketching, enduring marks are made. Producing such marks has several advantages, in that more complex representations can be created with gesture, and they endure. On the other hand, the fact that they endure (unless erased) may also be a problem, in that revision is more difficult as problem-solving proceeds. In sum, sketching may be a complex learning activity, benefiting from feedback and scaffolding (Van Meter and Garner, 2005).

Research on sketching in elementary mathematics education supports the idea of not only encouraging sketching but also scaffolding its use. Some students do spontaneously sketch to solve math word problems (Uesaka and Manalo, 2012), and spontaneous use of formal representations (e.g. Venn diagrams; Zahner and Corter, 2010) is associated with better solutions. But students can be taught to make sketches, and such instruction improves performance on math word problems for 5th grade students, although not for students in 1st-2nd grade (van Essen and Hamaker, 1990). Likewise, undergraduate students who make errors in their sketches can be trained to make correct sketches, leading to better performance on word problems at post-test (Lewis, 1989). One issue is avoiding unnecessary detail. Spontaneous student sketches of math problems often include extraneous or decorative features, which are associated with worse performance (Edens and Potter, 2008), whereas production of sketches that preserve spatial relations is associated with better performance (Hegarty and Kozhevnikov, 1999).

Figure 15. Novices sketch detail; experts sketch to show principles



Source: Jee, B. et al. (2014), "Drawing on Experience: How Domain Knowledge Is Reflected in Sketches of Scientific Structures and Processes", *Research in Science Education*, Vol. 44, pp. 868. © 2014 Springer Science + Business Media Dordrecht. Reprinted with permission.

Sketching is also useful in teaching science (Ainsworth, Prain and Tytler, 2011). For example, middle-school students learning about plate tectonics were found to perform better on inferential test items when they sketched, although not on literal items (Gobert and Clement, 1999). Leopold and Leutner (2012) find similar effects for sketching across two experiments with 10th grade students, although Leutner, Leopold, and Sumfleth (2009) found significant disadvantages on comprehension scores perhaps due to the high cognitive load self-reported by students in the drawing conditions.

One practical problem with using sketching in teaching is that correcting sketches and showing more accurate representations can be time consuming. An alternative would be

to input sketches to a cognitive tutor that could provide automated feedback. However, sketch understanding is a difficult problem in artificial intelligence. One way to short circuit this problem is to have students label components of their sketches as they work, using an interface called CogSketch (Forbus et al., 2011). Sketches produced using this system can also be used for assessment, gauging the expertise level of the sketcher (Jee et al., 2014). Students who know more focus on crucial elements and omit nonessential elements and decorative detail, as shown in Figure 15. However, they may still produce errors and such mistakes can be corrected in a timely way by CogSketch.

Evaluation of Strategy 2

We have reviewed a range of spatial learning tools for which there is some evidence that they support STEM learning. However, the list of techniques does not exhaust the potential for doing cognitive science on spatial-STEM intersections. For example, spatial and numerical ideas are sometimes intertwined and confused in preschool and elementary school (Newcombe, Levine and Mix, 2015) with a variety of educational implications. One area of concern is measurement, a skill that is basic to learning in mathematics and science alike, but about which many children are confused, due in part to their focus on integers as opposed to extent (Solomon et al., 2015). Thus, they sometimes read off the rightmost value on a ruler aligned with an object they are measuring, even if the left side is not aligned with the start point, or they count the numbers on the ruler rather than the units of measurement. Understanding these confusions leads to the invention of techniques to address them in the classroom.

Another consideration to keep in mind is the idea that techniques need not and probably should not be used in isolation. Combination can be powerful, although there needs to be thought about what tools to combine exactly how. One good example comes from research on mathematics education, showing that spatial alignment of visual representations is augmented by teachers' use of linking gestures pointing out the analogies (as well as any differences) between two problems (Begolli and Richland, 2015; Richland, 2015). Of course, simultaneous visual access is prerequisite to the possibility of linking gestures, and sequential presentations may actually impede comprehension. Another example comes from research already discussed on university science learning, in which Gagnier et al. (2016b) combined sketching, comparison and prediction to improve students' penetrative thinking and ability to complete block diagrams.

Combination can also be necessary because each technique sometimes has a selective impact. A case in point comes from research on teaching students to read topological maps, already mentioned briefly. Experts use certain gestures to refer to hills, valleys and slope, and it might be thought that use of these gestures by both teachers and students would improve map-reading skills. However, it turns out that students also do not understand the concept of elevation, and exactly what is denoted by the use of contour lines (Atit, K. et al., 2017). Given this problem, pointing at contour lines and tracing them is actually somewhat more helpful, and use of the relevant spatial language also can speed understanding. But crucially, the gestures and the language affect somewhat different elements of topological understanding. They are probably best used together, although we have yet to ascertain the optimal sequence.

In sum, we see that cognitive science can provide valuable ideas about how best to teach specific STEM material at specific ages, and is well equipped to conduct small-scale efficacy studies. However, children need to be educated now, and we cannot afford to

wait for scientific evidence to accumulate regarding every decision made by curriculum designers, policy makers and practicing teachers. Thus, a science of learning needs to proceed in parallel with designing, improving and evaluating effective curriculum units in a continuing design cycle, units such as the Connected Chemistry Curriculum (Stieff and Ryan, 2016). Of course, it is always possible that techniques that work well in small studies show smaller or no effects when implemented at scale (Stieff et al., 2016; Stull et al., 2012). The overall message that active learning is best (Freeman et al., 2014) needs to be augmented by this cycle, which can provide clues as to what method is needed when and for whom.

12. Conclusion

Research now supports the conclusions that spatial thinking and STEM learning are correlated longitudinally as well as cross-sectionally, and that spatial thinking is malleable. This report evaluated two strategies for exploiting these findings in education. Strategy 1 involves direct training of spatial skills. There is emerging evidence that interventions that increase spatial skills have downstream effects on STEM learning, although more studies are needed, especially work that takes a more analytic look at various spatial skills and the mechanisms linking each skill to particular aspects of STEM learning. Strategy 2 involves spatialising the curriculum, using tools suited to spatial thinking including spatial language, maps, diagrams, graphs, analogical comparison, physical activity that instantiates scientific or mathematical principles, gesture and sketching. There is extensive research on each of these tools, but less than is needed on how to combine them and how to use them collectively to mould curriculum. The overall conclusion of this report is that it is likely enough that spatial intelligence is an important element in STEM success that we should use this idea in designing curricula, training teachers, setting goals and developing assessments, while simultaneously evaluating the effectiveness of the efforts and continuing basic research on the mechanisms.

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