INTERNATIONAL MONETARY FUND

Export and Import Price Index Manual

Theory and Practice

International Labour Office

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Statistical Office of the European Communities (Eurostat)

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Foreword

This *Export and Import Price Index (XMPI) Manual* replaces the United Nations' *Strategies for Price and Quantity Measurement in External Trade,* Series M, No. 66, issued in 1981. The development of the *XMPI Manual* has been undertaken under the joint responsibility of six organizations—the International Labour Office (ILO), International Monetary Fund (IMF), Organisation for Economic Co-operation and Development (OECD), Statistical Office of the European Communities (Eurostat), United Nations Economic Commission for Europe (UNECE), and World Bank—through the mechanism of an Inter-Secretariat Working Group on Price Statistics (IWGPS). It is published jointly by these organizations.

The *Manual* contains detailed, comprehensive information and explanations for compiling XMPIs. It provides an overview of the conceptual and theoretical issues that statistical offices should consider when making decisions on how to deal with various problems in the daily compilation of XMPIs, and it is intended for use by both developed and developing countries. The chapters cover many topics; they elaborate on the different practices currently in use, propose alternatives whenever possible, and discuss the advantages and disadvantages of each alternative. Given the comprehensive nature of the *Manual,* we expect it to satisfy the needs of many users.

The main purpose of the *Manual* is to assist producers of XMPIs, particularly countries that are revising or setting up their XMPIs. The *Manual* draws on a wide range of experience and expertise in an attempt to describe practical and suitable measurement methods. It should also help countries to produce their XMPIs in a comparable way, so that statistical offices and international organizations can make meaningful international comparisons. Because it brings together a large body of knowledge on the subject, the *Manual* may be used for self-learning or as a teaching tool for training courses on XMPIs.

Other XMPI users, such as businesses, policymakers, and researchers, make up another targeted audience of the *Manual*. The *Manual* will inform them not only about the different methods that are employed in collecting data and compiling such indices, but also about the limitations, so that the results may be interpreted correctly.

The drafting and revision process has required many meetings over a five-year period, in which XMPI experts from national and international statistical offices, universities, and research organizations have participated. The *Manual* owes much to their collective advice and wisdom.

The electronic version of the *Manual* is available on the Internet at www.imf.org/external/np/sta/tegeipi/. The IWGPS views the *Manual* as a "living document" that it will amend and update to address particular points in more detail. This is especially true for emerging discussions and recommendations made by international groups reviewing XMPIs, such as the International Working Group on Price Indices (the Ottawa Group) and the International Working Group on Service Sector Statistics (the Voorburg Group).

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Preface

Export and import price indices (XMPIs) for a country measure the rate of change over time in the prices of exported and imported goods and services. An *export* price index (XPI) measures the rate of change in the prices of goods and services sold by residents of that country to, and used by, foreign buyers. An *import* price index (MPI) measures the rate of change in the prices of goods and services purchased by residents of that country from, and supplied by, foreign sellers.

This *Export and Import Price Index (XMPI) Manual* provides a detailed account of the theory and practice of compiling such indices. The *Manual* is the result of collaborative work by the International Labour Organisation (ILO), the International Monetary Fund (IMF), the Organisation for Economic Co-operation and Development (OECD), the Statistical Office of the European Communities (Eurostat), the United Nations Economic Commission for Europe (UNECE), and the World Bank, together with experts from a number of national statistical offices, universities, and other international organizations. In addition, these organizations have consulted with a large number of potential users of the *XMPI Manual* to get practical input. The organizations responsible for the *Manual* endorse its principles and recommendations as good practice for statistical agencies in conducting XMPI programs. *Because of practical constraints, however, some of the current recommendations may not be immediately attainable by all statistical offices and, therefore, should serve as guideposts for agencies as they revise and improve their programs.* In some instances, there are no clear-cut answers to specific index number problems, such as making adjustments for particular types of quality changes, treatment of seasonal goods and services, and handling the appearance of new products. The *Manual* provides detailed accounts of the underlying principles and economic and statistical theory that should allow statistical offices to derive practical solutions.

A. Export and Import Price Indices and Unit Value Indices

Many statistical agencies do not use establishment survey-based price indices of well-specified representative items as the building blocks of their XMPIs but compile unit value indices from the more convenient customs source as surrogates for them. However, common index number compilation issues arise for both unit value indices and XMPIs, including the choice of formulas; treatment of seasonal goods, missing values, quality changes, and new goods; organization and management of the index compilation process; and publication and dissemination of the index. There are also commonalities in the needs of valuation, classification, and the scope of the indices and in use of the same data source for weights—that is, relative nominal value shares based on administrative customs documentation. The analysis of, and recommendations for, an appropriate formula at the higher level of aggregation, the subject of Chapters 16 through 18, applies to XMPIs based on both unit value indices and price survey indices. The *Manual* considers all of these issues in detail. The *Manual* treats the component unit value indices as surrogates for price indices and as a result, the issues discussed in the context of XMPIs apply equally to unit value indices. The main difference between the two indices is the source of data and aggregation methods used for the measures of price changes at the elementary level. The distinction between the two approaches appears mostly in Chapter 7 on price collection, and the use of unit value indices is addressed directly in Chapter 2.

International guidelines on choosing between unit value- and price index-based XMPIs were provided by the United Nations (1981)—*Strategies for Price and Quantity Measurement in External Trade*. The strategic case for customs-based unit value indices in United Nations (1981) was based on the relatively low cost of such data. Unit value indices were advised for countries with a tight or medium budget, and well-endowed countries were advised to base their external trade price indices on establishment price survey data. The preference for price survey indices was, in large part, due to a potential bias in unit value indices mainly attributed to changes in the mix of the heterogeneous items recorded in customs documents, but it was also attributed to the often poor quality of recorded data on quantities.[1] The former is particularly important in modern product markets given the increasing differentiation of products and turnover of differentiated products. Unit value indices may suffer further in recent times owing to an increasing irrelevance of the source data: first, increasing proportions of trade are in services; second, countries in customs and monetary unions are unlikely to have intra-union trade data as a by-product of customs documentation; and finally, some trade may not be covered by customs controls, such as of electricity, gas, and water, or may be of "unique" goods, such as ships and large machinery, with profound measurement problems for unit value indices.

Few, including United Nations (1981), deny that narrow specification price indices provide the best measures of relative price change and that, a priori, there are potentially significant biases in using customs unit value indices to measure export and import price changes. Yet unit value proxies are still used because they are by-products of existing customs administration systems and have relatively low incremental cost compared with the price surveys of establishments needed for narrow specification prices. In view of the low cost of the data, the bias in unit value was judged by United Nations (1981) to be tolerable enough that countries were advised to continue compiling them if they do not produce narrow specification price indices. Notwithstanding the putative low cost of obtaining unit values, the *Manual* in Chapter 2 revisits this strategic advice.

The *Manual* recommends that countries using unit value indices with limited resources undertake a staged progression to price indices primarily based on establishment surveys. The initial stage will be to collect price data from establishments responsible for relatively high proportions of exports and imports, particularly those with a relatively large weight and whose unit value indices are at first view inadequate measures of price changes, largely because of the churn in highly differentiated products, or the custom-made nature of the products, such as shipbuilding and oil platforms. It may be that the progression is much quicker, to prepare for the formation of a customs union and loss of intra-union trade data. If the country compiles a producer price index (PPI), much of the technical skills required, and the basis for data collection, will be in place. The rationale for this strategic advice is given in Chapter 2 of the *Manual*.

B. The *Export and Import Price Index Manual*

The *XMPI Manual* serves the needs of different audiences. On the one hand are the compilers of XMPIs. This *Manual* and other manuals, guides, and handbooks are important to compilers for several reasons. First, there is a need for countries to compile statistics in comparable ways so they can make reliable international comparisons of economic performance and behavior using the best international practices. Second, statisticians in each country should not have to decide on methodological issues alone. The *Manual* draws on a wide range of experience and expertise in an attempt to outline practical and suitable measurement methods and issues. Such measurement methods and issues are not always straightforward, and the *Manual* benefits from recent theoretical and practical work. Third, much of the written material in some areas of XMPI measurement covers a range of publications. This *Manual* brings together a large amount of what is known on the subject. It may therefore be useful for reference and training. Fourth, the *Manual* provides an independent reference on methods against which a statistical agency's current methods, and the case for change, can be assessed. The *Manual* should serve the needs of users. Users should be aware not only of the methods employed by statistical offices in collecting data and compiling the indices, but also of the potential such indices have for errors and biases, so that users can properly interpret the results. For example, index number theory presents many issues on formula bias, and the *Manual* deals extensively with the subject.

[1]The advent of computerized systems under the Automated System for Customs Data (ASYCUDA) project of United Nations Conference on Trade and Development (UNCTAD) makes this largely unproblematic.

Collecting data for XMPIs is not a trivial matter. Unit value indices are a readily available by-product of the collection of trade data by customs authorities and, because of this, have served as surrogates for price indices. However, as noted above, unit value indices are recognized as being prone to bias. Survey-based XMPIs are the preferred alternative. Yet in practical terms, these require sampling from a representative sample of establishments, a set of well-defined commodities whose overall price changes are representative of those of the millions of transactions taking place. Statistical offices then monitor the prices of these same commodities on a periodic basis (usually monthly) and weight their price changes according to their trade shares, primarily based on nominal trade value shares from customs data. However, the quality of the commodities produced may be changing, with new establishments and commodities appearing and old ones disappearing on both a seasonal and permanent basis. Statistical offices need to closely monitor potential changes in quality. Yet the index compilers must complete the task of producing a representative index in a timely manner each month.

It is also important to have a well-developed theoretical basis for compiling such indices that is readily accessible for practitioners and users alike. There should be a firm understanding of user needs and how the index delivered fits them. Fortunately, there is a great body of research in this area, much of which is fairly recent. This *Manual* covers the theoretical basis of index numbers to help support some of the practical considerations.

This *Manual* provides guidelines for statistical offices or other agencies responsible for compiling XMPIs, bearing in mind the limited resources available. *Calculating an XPI or MPI cannot be reduced to a simple set of rules or a standard set of procedures that can be mechanically followed in all circumstances.* Although there are certain general principles that may be universally applicable, the procedures followed in practice have to take account of country-specific circumstances. Statistical offices have to make choices. These include procedures for the collection or processing of the price data and the methods of aggregation. Other important factors governing methodology are the main use of the index, the nature of the markets and pricing practices within the country, and the resources available to the statistical office. The *Manual* explains the underlying economic and statistical concepts and principles needed to enable statistical offices to make their choices in efficient and cost-effective ways and to recognize the full implications of their choices.

The *Manual* draws on the experience of many statistical offices throughout the world. The procedures they use are not static, but continue to evolve and improve, for a variety of reasons. First, resource constraints and custom and practice can inhibit innovation; the bias in unit value indices as surrogates for price indices has been well understood for many years. However, the transition of countries to survey-based price indices has been gradual and is still under way. Second, research continually refines the economic and statistical theory underpinning XMPIs and strengthens it. For example, recent research has provided clearer insights about the relative strengths and weaknesses of the various formulas and methods used to process the basic price data collected for XMPI purposes. Third, recent advances in information and communications technology have affected XMPI methods. Both theoretical and data developments can impinge on all the stages of compiling an XMPI. New technology can affect the methods used to collect prices and relay them to the central statistical office. It can also improve the processing and checking, including the methods used to adjust prices for changes in the quality of the goods and services covered. Fourth, improved formulas help in calculating more accurate higher-level indices.

C. Background to the Present Revision

Some international standards for economic statistics have evolved mainly to compile internationally comparable statistics. However, standards may also be developed to help individual countries benefit from the experience and expertise accumulated in other countries. All countries stand to gain by exchanging information about index methods. The United Nations published the *Manual on Producers' Price Indices for Industrial Goods,* Series M, No. 66 (United Nations, 1979), and *Strategies for Price and Quantity Measurement in External Trade,* Series M, No. 69 (United Nations, 1981) about 30 years ago. The methods and procedures presented then are now outdated. Index number theory and practice and improvements in technology have advanced greatly over the past two decades.

C.1 Concerns with current index methods

The *XMPI Manual* takes advantage of the wealth of recent research on index number theory. It identifies many concerns and recommends many new practices instead of just codifying existing statistical agency practices, some of which are listed below.[2]

First, it provides a detailed, if somewhat critical, outline of the use of **unit value indices** based on customs data for XMPI compilation and provides a strategy for countries wishing to establish or develop their XMPIs by making use of price data from establishment surveys (Appendix 1 to Chapter 1 and Chapter 2).

Second, it outlines **how XMPIs fit into the *2008 System of National Accounts*** (*2008 SNA*).[3] Although XMPIs are important economic indicators in their own right, a vital use of XMPIs is as deflators of series of nominal values of exports and imports to contribute to the derivation of volume estimates of GDP by the expenditure approach. Exports and imports are defined by the *2008 SNA*, from a *nonresident's* or rest of the world's perspective: exports are the rest of the world's *uses* of domestic production and imports are the rest of the world's *supply* of goods and services to resident users.

Principles for valuation and time of recording follow from the *2008 SNA*. An important feature of the *2008 SNA* is the use of supply and use tables (SUTs) that balance, at a detailed product group level, the s*upply* of output and imports with the *uses* of intermediate consumption, final consumption, capital formation, and exports. This helps reconcile the major flows in an economy at a detailed level. The *2008 SNA* (Chapter 14) also advises that SUTs be developed in volume terms, for which it is good practice that price deflators be applied at a detailed product group level. The deflation of the aggregates at the level of product groups to provide SUTs in volume terms provides a framework for deflators to be applied in a manner that reconciles the volume estimates, and thus price indices (deflators), for all transactions of goods and services supplied and used. This requires that for *each product group,* each estimate of output, intermediate consumption, final consumption, capital formation, and exports and imports be deflated. Because supply should equal uses, with adjustments for consistent valuation, in volume terms as well as at current prices, the deflators, including the XMPIs, benefit from this reconciliation of the volume estimates. XMPIs as deflators at the detailed level are developed as part of an integrated and consistent system. Chapters 4 and 15 outline this reconciliation and how different valuation systems enable it.

However, if XMPIs are to be used to analyze the transmission of inflation, terms of trade changes and their effect, and productivity changes, it is the resident's perspective that is appropriate, and Chapter 4 explains the valuation basis that results from this. Although the niceties of different valuation systems can be largely observed when using establishment-based survey data, this is not true of the f.o.b. (free on board) and c.i.f. (cost, insurance, and freight) valuations used mainly in customs data for merchandise trade exports and imports. Chapter 4 considers how such customs valuations relate to the desired valuation methods.

Third, the *XMPI Manual* is concerned with appropriate **index number formulas.** The standard methodology for a typical XMPI is based on a Laspeyres price index with fixed quantities from an earlier base period. The construction of this index can be thought of in terms of selecting a basket of goods and services representative of base-period trade values (exports or imports), valuing this at base-period prices, and then repricing the same basket at current-period prices. The target XMPIs in this case are defined as the ratios of these two trade values. Practicing statisticians use this methodology because it has at least three practical advantages. It is easily explained to the public, it can use inexpensive, though untimely, weighting information from the date of the last (or an even earlier) survey or administrative source (rather than requiring sources of data for the current month), and it need not be revised if users accept the Laspeyres premise. One notable advantage of the Laspeyres approach under the

[2]These problems are not ranked in order of importance; they all seem equally important.

[3]The reference to the *2008 SNA* used in this *Manual* is to the final draft of Volume 1 (Chapters 1–17) of the updated *System of National Accounts* adopted by the thirty-ninth session of the United Nations Statistical Commission, February 26–29, 2008, available at: http://unstats.un.org/unsd/sna1993/draftingphase/ChapterList.asp.

ease of explanation heading is its consistency in aggregation. It produces various breakdowns or subaggregates related to one another in a particularly simple way.

Statistical agencies implement the Laspeyres index by putting it into price-relative (price change from the base period) and trade-value-share (from the base period) format. In this form, the Laspeyres index can be written as the sum of base-period trade value shares of the items in the index times their corresponding price relatives. Unfortunately, simple as it may appear, there still are a number of practical problems with producing the Laspeyres index exactly. Consequently, statistical agency practice has introduced some approximations to the theoretical Laspeyres target.

- Accurate trade value shares for the base period down to the finest level of commodity aggregation are not always available, so statistical agencies are often forced to settle for getting base-period trade value weights at a higher than desirable level of aggregation.
- For each of the chosen commodity category aggregates, agencies collect a sample of representative prices for specific transactions from establishments rather than attempting to enumerate every possible transaction. They use equally weighted (rather than traded-value-weighted) index formulas or unit value indices to aggregate these elementary product prices into an elementary aggregate index, which will be used as the price relative for each of the commodity category groups in the final Laspeyres formula. Practitioners recognize that this two-stage procedure is not exactly consistent with the Laspeyres methodology (which requires weighting at each stage of aggregation). However, for a number of theoretical and practical reasons, practitioners judge that the resulting elementary index price relatives will be sufficiently accurate to insert into the Laspeyres formula at the final stage of aggregation.

The above standard index methodology dates back to the work of Mitchell (1927) and Knibbs (1924) and other pioneers who introduced it about 80 years ago, and it is still used today.

Although most statistical agencies have traditionally used the Laspeyres index as their *target index,* both economic and index number theory suggest that some other types of indices may be more appropriate target indices to aim for: namely, the Fisher, Walsh, or Törnqvist-Theil indices. As is well known, the Laspeyres index has an upward bias compared with these target indices. Of course, these target indices may not be achievable by a statistical agency, but it is necessary to have some sort of theoretical target to aim for. Having a target concept is also necessary, so that the index that is actually produced by a statistical agency can be evaluated to see how close it comes to the theoretical ideal. In the theoretical chapters of this *Manual,* it is noted that there are four main approaches to index number theory:

1. Fixed-basket approaches and symmetric averages of fixed baskets (Chapter 16),
2. The stochastic (statistical estimator) approach to index number theory (Chapter 17),
3. Test (axiomatic) approaches (Chapter 17), and
4. The economic approach (Chapter 18).

Approaches 3 and 4 will be familiar to many price statisticians and expert users of the XMPI, but perhaps a few words about approaches 1 and 2 are in order.

The Laspeyres index is an example of a fixed-basket index. The concern from a theoretical point of view is that it has an equally valid "twin" for the two periods under consideration—the Paasche index, which uses quantity weights from the current period. If there are two equally valid estimators for the same concept, then statistical theory tells us to take the average of the two estimators in order to obtain a more accurate estimator. There is more than one way of taking an average, however, so the question of the "best" average to take is not trivial. The *Manual* suggests that the "best" averages that emerge for fixed-base indices are the geometric mean of the Laspeyres and Paasche indices (Fisher ideal index) or using the geometric average of the quantity weights in both periods (Walsh index). From the perspective of a statistical estimator, the "best" index number is the geometric average of the price relatives weighted by the average revenue share over the two periods (Törnqvist-Theil index).

It is usually the case that current period weights are unavailable in real time so a Laspeyres-type index is compiled. The *Manual* allows us to understand the nature of the bias arising from not using the best formula. It also recommends compiling a retrospective analytical series using a Fisher or Törnqvist-Theil price index formula to provide an estimate of the magnitude of bias from the index number formula.

At the final stage of aggregation, the standard XMPI index is *not* a true Laspeyres index, because the trade value weights pertain to a reference base *year* that is different from the base *month* (or quarter) for prices. Thus the trade value weights are chosen at an annual frequency, whereas the prices are collected at a monthly frequency. To be a true Laspeyres index, the base-period trade value should *coincide* with the reference period for the base prices. In practice, the actual index used by many statistical agencies at the last stage of aggregation has a weight reference period that precedes the base-price period. Indices of this type are likely to have some upward bias compared with a true Laspeyres index, as are indices compiled using trade value weights that are price-updated from the weight reference period to the Laspeyres base period. It follows that they must have definite upward biases compared with theoretical target indices such as the Fisher, Walsh, or Törnqvist-Theil indices.

At the early stages of aggregation, unweighted averages of prices or price relatives are used. Until relatively recently, when enterprise data in electronic form have become more readily available, it was thought that the biases that might result from the use of unweighted indices were not particularly significant. However, recent evidence suggests that there is potential for significant upward bias at lower levels of aggregation compared with results that are generated by the preferred target indices mentioned above.

There is one additional result from index number theory that should be mentioned here—the problem of defining the price and quantity of a commodity that should be used for each period in the index number formula. The problem is that the establishment may have purchases or sales *within* a particular product specification in the period under consideration for many transactions at a number of different prices. So the question arises, what price would be most representative of the purchases or sales of these transactions for the period? The answer to this question is obviously the *unit value* for the transactions for the period, because this price will match up with the quantity sold during the period to give a product that is equal to the trade value. Note that the *Manual* does *not* endorse taking unit values over *heterogeneous* items at this first stage of aggregation; it endorses only taking unit values over *identical* items within each period.

The fourth major concern with the standard XMPI methodology is that, although statistical agencies generally recognize that there is a problem with the treatment of **quality change and new goods**, it is difficult to work out a coherent methodology for these problems in the context of a fixed-base Laspeyres index. The most widely recognized good practice in quality-adjusting price indices is “hedonic regression,” which characterizes the price of a product at any given time as a function of the characteristics it possesses relative to its near substitutes. However, there is a considerable amount of controversy on how to integrate hedonic regression methodology into the XMPI’s theoretical frameworks. The theoretical and practical chapters in the *Manual* devote a lot of attention to these methodological problems. The problems created by the disappearance of old goods and the appearance of new models are now much more severe than they were when the traditional XMPI methodology was developed some 30 years ago. For many categories of products, those priced at the beginning of the year are simply no longer available by the end of the year. Thus, there is a tremendous concern with *sample attrition*, which affects the overall methodology; that is, at lower levels of aggregation, it becomes necessary (at least in many product categories) to switch to chained indices rather than use fixed-base indices. Certain unweighted indices have substantial bias when chained.

A fifth major area of concern is related to the first concern: the **treatment of seasonal commodities.** The use of an annual set of products or the use of annual revenue shares is justified to a certain extent if one is interested in the longer-run trend of price changes. If the focus, however, is on short-term, month-to-month changes (as is the focus of central banks), then it is obvious that the use of annual weights can lead to misleading signals from a short-run perspective, because monthly price changes for products that are out of season (i.e., the seasonal weights for the product class are small for the two months being considered) can be greatly magnified by the use of annual weights. The problem of seasonal weights is a big one when the products are not available at all at certain months of the year. There are solutions to these seasonality problems, but the solutions involve the construction of *two* indices: one for the short-term measurement of price changes and another (more accurate) longer-term index that is adjusted for seasonal influences. This may give rise to confusion among users as to which index is correct and thus does not appeal to traditional index number statisticians.

A sixth concern is that the typical XMPI currently produced will generally **exclude services.** A typical XMPI will include merchandise trade and, possibly, electricity, gas supply, and water supply activities. Many countries will also include, or have separate indices for, agricultural and commodity prices. Thus, XMPI coverage includes many more goods-producing activities than services. In a way, this just reflects the historical origins of existing index number theory. Trade in goods was very much more significant than services. Hence, there was not much focus on the problems involved in measuring trade in services. It is only over the past 30 or so years that trade in services has become significant. In addition to inertia, there are some serious conceptual problems involved in measuring the prices of many services, such as insurance, financial, and entertainment services. In many cases, statistical agencies simply do not have appropriate methodologies to deal with these difficult conceptual measurement problems.[4] It is further the case that the source data on weights and unit value indices have been administrative customs documentation based on merchandise trade with a methodology initially dedicated to the collection of merchandise trade statistics and then subsequently, and separately, extended to services.[5] Thus, import and export prices for these service sectors are not widely measured.

A seventh concern arises because the value that multinational enterprises accord to international transactions between divisions operating in different countries, **transfer prices,** may not be market-driven prices, but dictated by a strategic decision to minimize taxes, given that rates of business income taxation differ across countries. Chapter 19 discusses the issues related to transfer prices and offers solutions. The best alternative to a firm's listed transfer price is an internal comparable price for the two periods compared; that is, the average price paid to (for an imported commodity) or received from (for an exported commodity) unaffiliated firms for the same commodity during the reference period, if such unaffiliated purchases or sales exist.

An issue of interest to users is that once XMPIs have been compiled, it is a natural next step to compile **terms of trade indices** as the ratio of the XPI to the MPI. Although the calculation of such ratios is straightforward, a question of natural interest to economists is how changes in the terms of trade of an economy affect the real income of the economy; as Chapter 24 demonstrates, this is more complex. The analysis draws on the economic theory of Chapter 18 and shows how the product of superlative import and superlative export price indices can best account for changes in real income generated by the production sector. Because households frequently directly import consumer goods and services from abroad, without these goods and services passing through the production sector of the economy, the chapter also considers the appropriate measurement of the effect of changes in the prices of imported goods on a household's cost of living. Again, superlative price indices are shown to be appropriate.

Many of the above areas of concern are addressed in this *XMPI Manual*. Frank discussions of these concerns should stimulate the interest of academic economists and statisticians to address these measurement problems and to provide new solutions that can be used by statistical agencies. Public awareness of these areas of concern should lead to a willingness on the part of governments to allocate additional resources to statistical agencies so that economic measurement will be improved. In particular, there is an urgent need to fill in some of the gaps that exist in the measurement of service sector imports and exports.

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[4]The Voorburg Group, which meets annually, has included the expansion of PPIs to services as part of its work program. The OECD, as part of its contribution to this program, conducts periodic surveys on the extension of PPIs in services activities. The latest survey results along with developments in services statistics are available at www.oecd.org/document/43/0,2340,en_2649_34355_2727403_1_1_1_1,00.html.

[5]At its thirty-second session, in March 2001, the United Nations Statistical Commission adopted the draft "Manual on Statistics of International Trade in Services" (United Nations, 2002). A revised version is planned for adoption in 2010.

C.2 Efforts to address the concerns in index number methods

Several years ago it became clear that the outstanding and controversial methodological concerns related to price indices needed further investigation and analysis. An expert group consisting of specialists on price indices from national and international statistical offices and universities from around the world formed to discuss these concerns. It met for the first time in Ottawa in 1994. During 10 meetings between 1994 and 2009, the Ottawa Group presented and discussed more than 160 research papers on the theory and practice of price indices. While much of the research related to consumer price indices (CPIs), many of the issues carry across to XMPIs. It became obvious there were ways to improve and strengthen existing XMPI and CPI methods.

In addition, the Voorburg Group on Service Sector Statistics, with members from many national statistical offices, has held annual meetings for more than a decade. Many agenda topics of the Voorburg Group related to expanding country PPIs to cover service industries and products, though the principles extend readily to XMPIs. The group has provided many technical papers on concepts and methods that serve as documentation other countries can follow.

At the same time, the control of inflation had become a high-priority policy objective in most countries. Policymakers use the CPI, PPI, and XMPIs widely to measure and monitor inflation. The slowing down of inflation in many parts of the world in the 1990s, compared with the 1970s and 1980s, increased interest in XMPI and PPI and CPI methods rather than reducing it. There was a heightened demand for more accurate, precise, and reliable measures of inflation. When the rate of inflation slows to only 2 or 3 percent each year, even a small error or bias becomes significant.

Recent concern over the accuracy of price indices led governments and research institutes in a few countries to commission experts to examine and evaluate the methods used, particularly for the CPI. The methods used to calculate have been subject to public interest and scrutiny of a kind and level that were unknown in the past. One conclusion reached is that existing methods might lead to some upward bias in CPIs. One reason for this was that the methods employed by many statistical agencies made inadequate allowance for changes in the quality of the goods and services priced. The same problem applies to XMPIs. The direction and extent of such bias will, of course, vary between commodity groups, and its total effect on the economy will vary among countries. However, the upward bias has the potential to be large, so this *Manual* addresses adjusting prices for changes in quality in some detail, drawing on the most recent research in this area. There are other sources of bias, including that arising from no allowance, or an inappropriate one, made for changes in the bundle of items produced, when purchases or production switches between commodities with different rates of price change. Further, different forms of bias might arise from the sampling and price collection system. Several chapters deal with these subjects, with an overall summary of possible errors and biases given in Chapter 12.

CPIs are widely used for the index linking social benefits such as pensions, unemployment benefits, and other government payments. The cumulative effects of even a small bias could have notable long-term financial outcomes for government budgets. Similarly, a major use of XMPIs is as an escalator for price adjustments to long-term contracts. Agencies of government, especially ministries of finance, and private businesses have taken a renewed interest in price indices, examining their accuracy and reliability more closely and carefully than in the past.

Developments were also being made in statistics on international trade derived from customs documentation. The United Nations (1998a) guidelines on C*oncepts and Definitions* for international merchandise trade statistics was followed by a *Compilers Manual* in 2004—a draft supplement to the *Compilers Manual* is being prepared at the time of this writing. A further revision (Rev. 3) to *Concepts and Definitions* is planned for 2010.[6]

[6]United Nations (1998a), and to assist in the implementation of the recommendations: United Nations (2004)—a draft supplement has been also prepared at the time of this writing (Statistical Paper, Series F, No. 87, Add. 1).

D. Organization of the Revision

D.1 Background

In response to the various developments outlined above, the need to revise, update, and expand the UN price statistics manuals was gradually recognized and accepted during the late 1990s. The joint UNECE/ILO meeting of national and international experts on CPIs held at the end of 1997 in Geneva made a formal recommendation to revise *Consumer Price Indices: An ILO Manual* (ILO, 1989). The main international organizations interested in measuring inflation agreed to take responsibility for the revision. The United Nations Statistical Commission in 1998 approved this strategy and agreed to set up the *Inter-Secretariat Working Group on Price Statistics* (IWGPS). The terms of reference of the IWGPS, presented to the United Nations Statistical Commission in 1999 at its thirtieth session, emphasized the development of standards and manuals on consumer price indices, producer price indices, and export-import price indices. In its report to the thirty-fourth (2003) session of the Statistical Commission, the Task Force on International Merchandise Trade Statistics stated the need for a manual on international trade price indices to be developed through an inter-agency effort. One of the first decisions of the IWGPS was to produce a new international *PPI Manual* at the same time as the *CPI Manual*. Both manuals were published in 2004, the *CPI Manual* by the ILO and *PPI Manual* by the IMF. The IWGPS decided at its fifth meeting, held in December 2003, to follow with the production of an *XMPI Manual*. The first meeting of the Technical Expert Group for this *Manual* was in June 2004.

The IMF coordinated the work on the *XMPI Manual* with a view of fostering coherence in structure and style with the *CPI* and *PPI Manuals* developed by the IWGPS and adopt, wherever appropriate, consistent contents, terminology, and methodology. A draft version of the *XMPI Manual* was completed in 2006. This initial draft *XMPI Manual* was adapted from the *PPI Manual* (by IMF staff and a few other specialists) and posted on the IMF website, www.imf.org/external/np/sta/tegeipi/index.htm.

The Statistics Department of the IMF (STA) wrote in 2006 to each national statistical office and other interested organizations, including the OECD and World Trade Organisation (WTO), requesting comments on the draft chapters. As a further part of the *XMPI Manual* review process STA organized a seminar on the draft manual held in Washington D.C., during September 25–29, 2006. Participants included a focus group of compilers from selected national statistical offices and experts in the field that included many IWGPS members. Aside from reviewing individual chapters, participants discussed comments from the United Nations Statistics Division (UNSD) and WTO on the need to further embrace unit value indices and the need for evidence-based comments on the subject. An IMF discussion paper was prepared in response to this concern[7] and presented at a meeting hosted by the WTO of the Task Force on Merchandise Trade Statistics in May 2007. At the meeting, the IMF agreed to include a new Chapter 2 on unit value indices. The IMF seminar also included an active discussion of the resident's and nonresident's perspectives to XMPIs. Following the seminar, written comments on each chapter were sent to the originating PPI authors who were asked to update the current versions of their chapters. Further reviews were sought and obtained as relevant, including comments by the WTO and UNSD on earlier chapters. A revised draft was posted on the IMF website in April 2008 and discussed with IWGPS members.

D.2 Agencies responsible for the revision

The following international organizations—concerned with measuring inflation, with policies designed to control inflation, and with measurement of deflators for national accounts—collaborated on revising the *CPI*, *PPI,* and *XMPI Manuals*:

- The International Labour Organization,
- The International Monetary Fund,
- The Organisation for Economic Co-operation and Development,

[7]See www.imf.org/external/pubs/cat/longres.cfm?sk=20943.0.

- The Statistical Office of the European Communities,
- The UN Economic Commission for Europe, and
- The World Bank.

These organizations have provided, and continue to provide, technical assistance on CPIs, PPIs, and XMPIs both to developing countries and to countries in transition from planned to market economies. The group's role was to appoint various experts as authors and to provide substantive advice on the contents of the chapters of the draft manual and organize and manage the process of writing, publishing, and disseminating the *XMPI Manual*.

The experts taking part were invited in their personal capacity as experts and not as representatives, or delegates, of the national statistical offices or other agencies that employed them. Participants were able to give their expert opinions without in any way committing the offices from which they came.

D.3 Links with the new consumer and producer price index manuals

The *XMPI Manual* was a natural progression from the *CPI* and *PPI Manuals* (ILO and others, 2004a and 2004b) and it thus benefited from shared conceptual and practical issues. The manuals have similar contents and are, where applicable, fully consistent with each other conceptually, sharing common text when suitable. The three manuals are each self-contained but share common approaches and conceptual groundings and, as a set, benefit from an internal coherency. However, some features of XMPI compilation are distinct from their CPI and PPI counterparts, and the *XMPI Manual* departs from the *CPI* and *PPI Manuals* in a number of respects, the most notable being the following:

- It includes a new chapter on unit value indices (Chapter 2);
- It includes a new chapter on transfer pricing (Chapter 19);
- It includes a new chapter on terms of trade measurement (Chapter 24);
- It is compatible with *2008 SNA*, especially with regard to the resident's versus nonresident's approach to XMPI measurement, to serve different uses, that is particular to XMPIs (Chapters 4, 15, 18, and 20);
- Attention is given to the use of the information from administrative customs documents as a sampling frame for both commodities and establishments (Chapter 6); and
- The primary source data for weights are administrative data that have particular merits for the frequent updating of weights if sufficiently timely and reliable (Chapters 5, 16, and 20).

E. Acknowledgments

The *XMPI Manual* is the result of a five-year process that involved multiple activities. The starting point was an adaptation of the *PPI Manual* primarily by Kim Zieschang (IMF) and Mick Silver (IMF), posted on the IMF website in 2006. Thomas Alexander (IMF) and Maria Mantcheva (IMF) also contributed by respectively redrafting Chapters 4 and 5 and undertaking additional calculations for some of Chapter 20. Following the September 2006 Washington, D.C., seminar on the draft manual, and request for and receipt of comments from national statistical offices and other interested organizations and individuals on the draft, most of the originating *PPI Manual* authors for each chapter were asked to adapt their chapters to take account of the comments received. The posted chapters were substantially revised and reposted on the IMF website in April 2008.

We are particularly grateful for helpful comments to Bert Balk (Statistics Netherland and Erasmus University) and to the following people: David Collins (Australian Bureau of Statistics (ABS)), Renaud DeCoster, Luis G. González-Morales (UNSD), Carsten Hansen (UNECE), Anne Harrison (IMF/World Bank), Johannes Hoffmann (Deutches Bundesbank), Joost Huurman (Statistics Netherlands), Ronald Jansen (UNSD), Ronald Johnson (U.S. Bureau of Labor Statistics (BLS)), Yann Marcus (WTO), Andreas Maurer (WTO), Věra Petrásková (Czech Statistical Office), Klaus Poetzsch (Statistisches Bundesamt), Marshall Reinsdorf (U.S. Bureau of Economic

Analysis), Ian Richardson (U.K. Office for National Statistics (ONS)), Henk Verduin (Statistics Netherlands), Roberto Vilarrubí (The Barrie School, Silver Spring, Maryland), and Keith Woolford (ABS).

While helpful comments were received from a number of **statistical offices,** special recognition is given for their help to the Statistical Offices of Israel, the Netherlands, Nepal, and the United States and to members of the **Task Force on Merchandise Trade Statistics.**

The participants in the September 2006 Washington, D.C., seminar on the draft manual included the following: Bill Alterman (BLS), Paul Armknecht (expert), Eleonora Baghy (Hungarian Central Statistical Office), Bert Balk (Statistics Netherland and Erasmus University), Brian Costello (BLS), Viviana Depino De Aviles (National Institute of Statistics and Censuses (INDEC), Argentina), W. Erwin Diewert (University of British Columbia), Carsten Hansen (UNECE), Martin Kullendorff (Statistics Sweden), Alberto Lizaola (National Institute of Statistics and Geography (INEGI), Mexico), Rob McClelland (BLS), David Mead (BLS), Marshall Reinsdorf (U.S. Bureau of Economic Analysis (BEA)), Sabrina Sabri (Department of Statistics, Malaysia), Mick Silver (IMF), Tõnu Täht (Statistics Estonia), Bee Yian Tan (Singapore Department of Statistics), Ted To (BLS), Bulent Tungul (State Institute of Statistics, Turkey), Zaki Twalbeh (Department of Economic Statistics, Jordan), Michelle Vachris (Christopher Newport University), Henk Verduin (Statistics Netherlands), and Sharon Willis (Statistical Institute of Jamaica).

Members of the Inter-Secretariat Working Group on Price Statistics (IWGPS), under whose aegis this manual was written, were the following:[8] Paul Armknecht (expert—*PPI Manual* editor), Bert Balk (Statistics Netherlands and Erasmus University), Pam Davies (ONS),* Erwin Diewert (University of British Columbia), Yuri Dikhanov (World Bank),* David Fenwick (ONS), Tarek Harchaoui (Statistics Canada),* Carsten Hansen, (UNECE), Keith Hayes (Eurostat),* Peter Hill (expert—*CPI Manual* editor), Walter Lane (BLS),* Alexandre Makaronidis (Eurostat),* Joaquim Oliveira (OECD),* David Roberts (OECD), Paul Schreyer (OECD),* Mick Silver (IMF), Valentina Stoevska (ILO), Keith Woolford (ABS), and Kimberly Zieschang (IMF).

The editor of the *XMPI Manual* is Mick Silver (IMF).

The author, or authors, of the chapters are a movable feast with, to maintain internal coherency between the manuals, text from the *CPI Manual* carrying over to the *PPI Manual* and, again, to the *XMPI Manual*, as applicable. The attribution of authorship recognizes this and the attribution given below is for, in large part, that of the *PPI Manual,* edited by Paul Armknecht (which carried over from the *CPI Manual,* edited by Peter Hill), with the addition of any authors who made substantive contributions to chapters in the writing of this manual. Generally, the originating PPI authors were responsible for much of the often substantial adaptation of material necessary for the *XMPI Manual*. Chapters 2, 19, and 24 are new to the *XMPI Manual*.

Erwin Diewert merits a special note of appreciation. He has been either sole or joint author of all the chapters concerning theoretical issues that provide the underpinnings of much of the manual.

Preface Paul Armknecht, W. Erwin Diewert, Peter Hill, and Mick Silver.

Reader's Guide Paul Armknecht and Peter Hill.

Chapter 1 *A summary of export and import price index methodology*, Paul Armknecht, David Collins, Peter Hill, and Mick Silver.

Chapter 2 *Unit value indices*, Mick Silver.

Chapter 3 *The price and volume of international trade: background, purpose, and uses of export and import price indices,* Andrew Allen, Paul Armknecht, and David Collins.

[8]Individuals with an asterisk (*) after their name served for only part of the period.

Chapter 4 *Coverage, valuation, and classifications,* Thomas Alexander and Paul Armknecht.

Chapter 5 *Data sources,* Maria Mantcheva.

Chapter 6 *Sampling issues in price collection,* Paul Armknecht and Fenella Maitland-Smith.

Chapter 7 *Price collection,* Andrew Allen, Paul Armknecht, Matthew Berger, David Collins.

Chapter 8 *Treatment of quality change,* Mick Silver.

Chapter 9 *Commodity substitution, sample space, and new goods,* Mick Silver.

Chapter 10 *XMPI calculation in practice,* W. Erwin Diewert, Carsten B. Hansen, Peter Hill, and Robin Lowe.

Chapter 11 *Treatment of specific products and issues,* Dennis Fixler and Michelle Vachris; contributions from Australian Bureau of Statistics, Statistics Canada, Statistics Singapore, and U.S. Bureau of Labor Statistics.

Chapter 12 *Errors and bias in XMPIs,* Mick Silver.

Chapter 13 *Organization and management,* Bill Alterman, David Fenwick, and Yoel Finkel.

Chapter 14 *Publication, dissemination, and user relations,* Paul Armknecht, Tom Griffin, and David Mead.

Chapter 15 *The system of price statistics,* Kim Zieschang.

Chapter 16 *Basic index number theory,* W. Erwin Diewert and Paul Armknecht.

Chapter 17 *Axiomatic and stochastic approaches to index number theory,* W. Erwin Diewert.

Chapter 18 *Economic approach,* W. Erwin Diewert.

Chapter 19 *Transfer prices,* W. Erwin Diewert.

Chapter 20 *Exports and imports from production and expenditure approaches and associated price indices using a simplified example and an artificial data set,* W. Erwin Diewert.

Chapter 21 *Elementary indices,* W. Erwin Diewert and Mick Silver.

Chapter 22 *Quality change and hedonics,* Mick Silver and W. Erwin Diewert.

Chapter 23 *Treatment of seasonal products,* W. Erwin Diewert and Paul Armknecht.

Chapter 24 *Measuring the effects of changes in the terms of trade,* W. Erwin Diewert.

Glossary David Roberts (OECD) and Paul Schreyer (OECD).

Bibliography

The final copyediting and production of the *Manual* in the IMF External Relations Department were coordinated by Michael Harrup.

Reader’s Guide

International manuals in economic statistics have traditionally provided guidance about concepts, definitions, classifications, coverage, valuation, recording data, aggregation procedures, formulas, and so on. They have mainly aided compilers of the relevant statistics in individual countries. This *Manual* shares this same principal objective.

The *Manual* will benefit users of export and import price indices (XMPIs), such as government and academic economists, financial experts, and other informed users. XMPIs are key statistics for policy purposes. They attract much attention from the media, governments, and the public in most countries. Both the export price index (XPI) and the import price index (MPI) are sophisticated concepts that draw on a great deal of economic and statistical theory and require complex data manipulation. This *Manual* is therefore also intended to promote greater understanding of the properties of XMPIs.

In general, compilers and users of economic statistics must have a clear view of what the statistics measure, in principle. Measurement without theory is unacceptable in economics, as in other disciplines. This *Manual* therefore contains a thorough, comprehensive, and up-to-date survey of relevant economic and statistical theory. This makes the *Manual* self-contained in both the theory and practice of XMPI measurement.

The *Manual* is, consequently, large. Because different readers may have different interests and priorities, it is not possible to devise a sequence of chapters that suits all. Indeed, users do not read international manuals from cover to cover in order. Manuals also serve as reference works. Many readers may have interest in only a selection of chapters. The purpose of this Reader’s Guide is to provide a map of the contents of the *Manual* that will aid readers with different interests and priorities.

The *XMPI Manual* has a similar structure and similar material to that of the *Consumer Price Index (CPI) Manual* and *Producer Price Index (PPI) Manual*. The CPI, PPI, and XMPIs in large part have similar theoretical underpinnings and face similar practical problems in their compilation. There are of course important differences and each manual adapts the discussion of principles and practices to meet the needs of the index concerned. In particular the *XMPI Manual* has three new chapters: Chapter 2 on unit value indices, Chapter 19 on transfer pricing, and Chapter 24 on the measurement of terms of trade effects. All three manuals are self-contained.

A. An Overview of the Sequence of Chapters

As mentioned in the preface, the chapters of this *Manual* are arranged so that practical and operational issues (Chapters 1–14 and the Glossary) are supported by theoretical underpinnings (Chapters 15–24). Specifically, the *Manual* is divided into four parts:

Part I (Chapters 1–5) examines XMPI methodology, uses, and coverage;

Part II (Chapters 6–12) covers compilation issues;

Part III (Chapters 13–14) considers operational matters; and

Part IV (Chapters 15–24) explores conceptual and theoretical issues.

The remaining paragraphs in this section give synopses of the individual chapters.

A.1 Part I: Methodology, uses, and coverage

Chapter 1 is a general introduction to the theory and practice of XMPIs. It is intended for all readers. It provides the basic information needed to understand the later chapters and a summary of index number theory, as explained in much more detail in Chapters 16–24. It then provides a summary of the main steps involved in compiling XMPIs, drawing on material in Chapters 4–10. It does not provide a summary of the *Manual* as whole nor does it cover specific topics or special cases that are not of general relevance.

Chapter 2 starts with a strategic decision XMPI compilers must make: their reliance on unit value indices from customs data as surrogates for price indices. Chapter 3 outlines the history of price indices and how XMPIs have changed in response to the demand for broader measures of price change. Chapter 4 presents a few basic concepts, valuation principles, classifications, and the scope or coverage of an index. The scope of XMPIs can vary significantly from country to country.

A.2 Part II: Compilation issues

Chapters 5–10 form an interrelated sequence of chapters describing the various steps involved in compiling XMPIs, from collecting and processing the price data through calculating the final index. Chapter 5 discusses deriving the value weights attached to the price changes for different goods and services. Administrative data from customs authorities and establishment censuses or surveys, supplemented by data from other sources, typically provide the weight data.

Chapter 6 deals with sampling issues. The approach of this manual to the collection of prices is to favor the use of surveys of establishments, as opposed to unit values from customs authorities, though issues relating to the use of unit value indices are outlined in Chapter 2. XMPIs are essentially estimates based on samples of the prices of commodities imported and exported by a sample of establishments. Chapter 6 considers sampling design and the pros and cons of random versus purposive sampling.

Chapter 7 describes the procedures used to collect the prices from a selection of establishments and products. It deals with topics such as questionnaire design, specifying the transactions selected, and methods for collecting data, including the use of electronic media.

Chapter 8 addresses the difficult question of how to adjust prices for changes over time in the quality of the goods or services selected. Changes in value owing to changes in quality count as changes in quantity, not price. Disentangling the effects of quality change poses serious theoretical and practical problems for compilers. Chapter 9 addresses two closely related questions: first, how to deal with goods and services that disappear from the sample; second, how new goods or services not previously traded can enter the sample.

Chapter 10 gives a step-by-step description of editing procedures, calculating elementary price indices from the raw prices collected for small groups of products, and the resulting averaging of the elementary indices to obtain indices at various levels of aggregation up to the overall XMPIs. The chapter also provides a description of the process for the periodic update of the value weights.

Chapter 11 deals with a few cases that need special treatment. It presents methods for handling examples of hard-to-measure goods and services, including agriculture, crude petroleum and gasoline, metals, computers, motor vehicles, clothing, airfreight, air passenger fares, crude oil tanker freight, ocean liner freight, and travel and tourism. The chapter concludes with a discussion of more general issues, including duties, currency conversion, and intra-company transfers. Chapter 12 provides an overview of the errors and biases to which XMPIs may be subject.

A.3 Part III: Operational issues

Chapter 13 deals with issues of organization and management. Conducting the price surveys and processing the results make for a massive operation that needs careful planning, organization, and efficient management. Chapter 14 addresses publication and dissemination standards for the XMPI results.

A.4 Part IV: Conceptual and theoretical issues

Chapter 15 marks a break in the sequence of chapters because it is not concerned with compiling XMPIs. Its purpose is to examine the place of XMPIs in the general system of price statistics. The measurement of series of the volume of GDP (expenditure) requires deflators of the nominal values of the GDP components that include exports and imports. It will be seen that a nonresident's perspective is required in this regard—a perspective that identifies exports as a use by nonresidents and imports as a supply by nonresidents. However, the XMPIs for measuring changes in terms of trade, transmission of inflation via exports and imports, and productivity analysis requires a resident's perspective that may be embedded in the production accounts of the *2008 System of National Accounts (2008 SNA)*. These approaches are outlined in Chapter 15, and their implications for theory and measurement are discussed in Chapters 4, 18, and 20.

Chapters 16–18 provide a systematic and detailed exposition of the index number theory underlying XMPIs. These chapters examine different approaches to index number theory. Collectively, they provide a comprehensive and up-to-date survey of index number theory, including recent methodological developments as reported in journals and conference proceedings.

Chapter 16 provides an introduction to index number theory, focusing on breaking up value changes into their price and quantity components. Chapter 17 examines the axiomatic and stochastic approaches to XMPIs. The axiomatic, or test, approach lists many properties that are desirable for index numbers to have and tests specific formulas to see whether they have them.

Chapter 18 explains the economic approach, using the economic theory of producer behavior for the XPI and consumer behavior for the MPI, to deriving both a theoretical XPI and a theoretical MPI, against which index numbers used in practice can be compared and biases identified. Although these economic indices cannot be calculated directly, a certain class of index numbers, known as "superlative" indices, can be expected to approximate them in practice. From an economic perspective, the ideal index for XMPI purposes should be a superlative index, such as the Fisher index. The Fisher index also is a very desirable index on axiomatic grounds.

Chapter 19 considers the practical issue of transfer pricing between divisions of a multinational corporation that has establishments in more than one country. Tax differentials between countries may provide an incentive for the company to strategically price its international transactions to minimize the overall tax burden of the multinational corporation.

Chapter 20 reconciles XMPIs from GDP production and expenditure approaches and presents a constructed data set to explain the numerical outcomes of using different index number formulas. It shows that the choice of index number formula can make a notable difference, but that different superlative indices approximate one another.

Chapter 21 addresses the important question of what is theoretically the most appropriate elementary price index formula to use at the first stage of XMPI compilation *if* no information is available on quantities or values. This has been a comparatively neglected topic until recently, even though the choice of formula for an elementary index can have a significant impact on the overall XMPIs. The elementary indices are the basic building blocks used to construct higher-level XMPIs. Consideration is also given to the use of unit values as surrogates for prices at this first stage. Such unit values have the benefit of including information on quantities and, for strictly homogeneous commodities, benefit from time aggregation properties *within* the period they are reported, usually a month. However, unit value *indices* for prices of heterogeneous commodities, as is typical of data from customs authorities, are prone to bias. The stance of the *Manual* is to focus on prices of tightly defined commodities from establishment surveys at the elementary level.

Chapters 22, 23, and 24 conclude the *Manual*. They address three conceptually difficult issues. Chapter 22 considers the theoretical issues of adjusting for quality change on the basis of the hedonic approach. Chapter 23 examines the treatment of seasonal products. Chapter 24 provides a framework for the analysis of the contribution of changes in a country's terms of trade to changes in its real income.

A glossary of terms and a bibliography appear at the end of the sequence of chapters.

B. Alternative Reading Plans

Different readers may have different needs and priorities. Readers interested mainly in compiling XMPIs may not wish to pursue all the finer points of the underlying economic and statistical theory. Conversely, readers more interested in the use of XMPIs for analytic or policy purposes may not be interested in the details of the conduct and management of price surveys. Not all readers will want to read the entire *Manual,* or even want to follow the same reading plan.

However, all readers, whether users or compilers, will find it useful to read chapters 1 and 3. Chapter 1 provides a general introduction to the whole subject by providing a review of the XMPI theory and practice appearing in the *Manual*. It provides the basic knowledge needed for understanding later chapters. Chapter 3 explains the need for XMPIs and their uses.

B.1 A compiler-oriented reading plan

Chapters 2 and 5–14 are mainly for compilers. They follow a logical sequence that roughly matches the various stages of compiling XMPIs. They start with data sources for measuring price changes, unit value indices versus survey price indices, deriving the value weights and collecting the price data, and finish with publishing the final index. Chapter 13, on organization and management, is intended for both managers and compilers. It discusses many important issues on the structure and mechanisms that statistical offices need to monitor, control, and ensure the quality of XMPIs and to be efficient in the use of resources. Chapter 14 is on the effective dissemination of the results.

Chapter 15 is for both compilers and users of XMPIs. It places XMPIs in perspective within the overall system of price indices.

The remaining chapters, Chapters 16–24, are mainly theoretical. Compilers may find it necessary to follow certain theoretical topics in greater depth, in which case they have immediate access to the relevant material. It would be desirable for compilers to acquaint themselves with at least the basic index number theory set out in Chapter 16 and the numerical example developed in Chapter 20. The material in Chapter 21 on elementary price indices is also important for compilers.

B.2 A user-oriented reading plan

Although all readers should find Chapters 1–4 useful, and Chapters 5–14 are mainly for compilers, several topics have aroused great interest among many users.

Chapters 8 and 9 discuss the treatment of quality change, item substitution, and new products. Users may also find Chapter 10 helpful because it provides a concise description of the various stages of compiling XMPIs.

Chapter 12, "Errors and Bias in XMPIs," and Chapter 15, "The System of Price Statistics," are also of interest to both users and compilers.

Chapters 16–24 cover the economic and statistical theory underlying XMPIs, and they are likely to be of interest to many users, especially professional economists and students of economics.

C. A Note on the Bibliography

In the past, international manuals on economic statistics have not usually provided references to the associated literature. It was not helpful to cite references when the literature was mostly confined to printed volumes, including academic journals or proceedings of conferences, found only in university or major libraries. Compilers working in many statistical offices were unlikely to have ready access to such literature. However, this has changed with the World Wide Web, which makes all such literature readily accessible. Therefore, this *Manual,* as was the case with the *CPI Manual* and *PPI Manual,* breaks with tradition by including a comprehensive bibliography to the large literature that exists on index number theory and practice that many readers are likely to find useful. In addition, websites are referenced that contain specialist papers on index number theory and practice, including those of the Ottawa Group and the Voorburg Group.

1. A Summary of Export and Import Price Index Methodology

A. Introduction

1.0 A price index number is a summary measure of the proportionate or percentage change in a set of prices over time. Export and import price indices (XMPIs) measure the overall change in the price component of transactions in goods and services between the residents of an economic territory and residents of the rest of the world. The prices of different goods and services all do not change at the same rate. A price index thus summarizes their movement by averaging over them. A price index assumes a value of unity, or 100, in some reference period. The value of the index for other periods of time provides the average proportionate or percentage change in price from the reference period.

1.1 Two basic questions are the focus of this *Manual* and the associated literature on price indices:

- Exactly what sets of prices should be covered by the price index and how should they be collected?
- What is the most suitable way in which to average their movements?

1.2 The answer to the first question depends largely on the purposes of the index, which directly connect with the domain of transactions the index is to cover. Distinct price indices associate with different domains of goods and services, such as household consumption, production, investment, and foreign trade flows. *Export price indices* (XPIs) measure changes in the prices of the goods and services provided by the residents of a given economic territory (usually, country) and used by nonresidents (that is, the rest of the world). *Import price indices* (MPIs) measure changes in the prices of goods and services provided by nonresidents (rest of the world) and used by residents of the economic territory. XPIs and MPIs, or XMPIs, are the concern of this *Manual*.

1.3 Price indices preferably weight the price relative (change) of each specific item they cover by the item's value share. For example, an XPI is a weighted average of the price relatives of its components where the weights are the share of each component in the total value of exports covered by the index. Having collected the appropriate set of prices and the weights, the second question concerns the choice of formula to average the price relatives. Alternative aggregation formulas are considered in Chapters 2, 16–18, and 20–21 of this *Manual*. The price relatives may take the form of ratios of prices between the current and price reference period of specified representative items with detailed commodity descriptions, so that the prices of like items are compared with like items. Such price relatives can generally be obtained only from establishment surveys. However, unit values for commodity groups may be obtained from customs declarations and their ratios used as "plug-ins" for price relatives, a use considered in Chapter 2.

1.4 This chapter provides a general introduction to, and review of, the methods of XMPI compilation. It provides a summary of the relevant theory and practice of index number compilation intended to help with the reading and understanding of the detailed chapters that follow, some of which are inevitably quite technical. The chapter starts, as does the *Manual*, by distinguishing between XMPIs for which the price data are compiled primarily from establishment-survey data and unit value indices that use unit value data from customs documentation as proxies for price data. It considers the merits of each and makes recommendations. Section C then outlines the principal uses of XMPIs. The chapter then describes the various steps involved in XMPI compilation starting in Section D with the basic concepts, scope, and classifications of XMPIs, some of which are dictated by the use to which the XMPIs will be put and much of which are guided by the *2008 System of*

National Accounts (*2008 SNA*).[1] The source data for weights and prices are outlined in Sections E and F, respectively, the latter with an emphasis on sampling procedures and survey methods, used to collect price data from establishments, and data editing procedures. Attention is then directed in Section G to the particular problem of transfer prices and in Section H to methods for dealing with temporarily and permanently missing goods and services, including quality adjustments, and seasonal goods and services. An extreme and problematic form of the problem of dealing effectively with quality change and the churn in goods and services traded is that of new goods and services, as outlined in Section I. With detailed information on weights and prices at hand, there is then a need to aggregate them. The aggregation is usually in two stages. Aggregation with weights follows aggregation, at a more detailed elementary level, without weights. Section J outlines the underlying index number theory at the higher (weighted) level. This includes the properties and behavior of the various kinds of index numbers that compilers might use and factors governing choice. Section K outlines alternative (unweighted) elementary index number formulas and their properties. The *Manual* makes recommendations on choice of formulas at both the elementary and higher levels. Section L is based on Chapter 10, which provides a general overview of the ways in which XMPIs are calculated in practice. Sections J and K provide much of the theoretical basis for the practical examples in Section L. Organizational issues and issues relating to dissemination are briefly outlined in Sections M and N, respectively. An issue for users, once XMPIs have been compiled, is the calculation of terms of trade indices. Section O shows how, although these are easily calculated, determining their contribution to changes in real income is less straightforward. The chapter concludes with an appendix that provides an overview of the steps involved in compiling XMPIs.

1.5 The main topics covered in this chapter are the following:

B. Unit value indices and price indices;

C. Uses of XMPIs;

D. Concepts, scope, and classifications;

E. Source data for weights;

F. Source data for prices;

G. Transfer prices;

H. Missing prices and adjusting prices for quality change

I. Commodity substitution and new goods

J. Basic index number formulas and the axiomatic and economic approaches to XMPIs;

K. Elementary price indices;

L. Basic index calculations;

M. Organization and management;

N. Publication and dissemination; and

O. Terms of trade.

Appendix. An overview of the steps necessary for developing XMPIs.

1.6 It is not the purpose of this introductory chapter to provide a comprehensive summary of the entire contents of the *Manual*. Thus, not all of the topics treated in the *Manual* are included in this chapter. The objective of this general introduction is to give a summary presentation of the core issues with which readers need to be acquainted before tackling the detailed chapters that follow.

B. Unit Value Indices and Price Indices

1.7 Unit values in any period measure, for individual commodity classes in that period, the total value of shipments divided by the corresponding total quantity. Export and import *unit value indices* are based on data from customs documentation and are so named because they take as their building blocks, for individual commodity classes,[2] the ratio of the unit value

[1]The reference to the *2008 SNA* used in this *Manual* is to the final draft of Volume 1 (Chapters 1–17) of the updated *System of National Accounts* adopted by the 39th session of the United Nations Statistical Commission, February 26–29, 2008, available at http://unstats.un.org/unsd/sna1993/draftingphase/ChapterList.asp. The *2008 SNA* is an updated version of the *1993 SNA*—Commission of the European Communities, International Monetary Fund, Organization for Economic Cooperation and Development, United Nations, World Bank (1993), *System of National Accounts 1993* (Brussels/Luxembourg, New York, Paris, Washington: EC, IMF, OECD, UN, World Bank).

[2]The classes used refer to the subheadings of the Harmonized System, which is a complete product classification system designed as a "core" system so that countries adopting it could make further subdivisions according to their particular tariff and statistical needs. At the international level, the Harmonized System consists of approximately 5,000 article descriptions that appear as headings and subheadings. Countries can add more detailed subdivisions for classifying goods for tariff, quota, or statistical purposes so long as any such subdivision is added and coded at a level beyond the six-digit numerical code provided in the Harmonized System. Coding beyond the six-digit level is usually at the eight-digit level and is generally referred to as the "national level"; see Chapter 4 for details.

in the current period to the unit value in the reference period. These elementary-level unit value ratios—also, and hereafter, referred to as (elementary) unit value indices—are subsequently aggregated across commodity classes using standard weighted index number formulas where the weights are the relative shares of the commodity group in total exports or imports. Export and import *price indices* have as their building blocks at the elementary level the price change of well-defined representative items derived from establishment surveys. Export and import unit value indices by necessity differ from price indices because of their source data. A unit value elementary index, P_U, is given for a price comparison between the current period t and a reference period 0 over $m = 1, \ldots, M$ items in period t and over $n = 1, \ldots, N$ items in period 0 by

$$P_U \equiv \left(\frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{m=1}^{M} q_m^t} \right) \bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right), \tag{1.1}$$

where prices and quantities are given, respectively, by p_m^t and q_m^t for period t, and p_n^0 and q_n^0 for period 0.

1.8 Unit value indices are used to represent price changes. The probity of their use relies on the homogeneity of the items transacted within the classification classes for which transactions are aggregated, and the related issue of how tightly the classification classes are themselves defined. Unit value indices work well for the aggregation of identical, homogeneous items, but are biased for the aggregation of different, heterogeneous goods. Consider, for example, the prices of two heterogeneous goods A and B at 10 and 12 in the reference period that remain constant over time, but with a shift in quantities from, say 6, for both A and B in the reference period, to 8 for A and 4 for B in the current period. The correct answer for any price index number formula would give an answer of unity, that is, no overall price change. However, the unit value would *fall* by 3 percent, reflecting the shift in the quantity basket in the current period from the higher price *level* of 12 for B to the lower price *level* of 10 for A. This unit value bias for heterogeneous items arose from a compositional shift in the basket of items transacted. Of note is that if A and B were homogeneous items, there would be no bias. The unit value index would be the correct measure reflecting the fact that the same item has become, on average, cheaper. The problem for XMPI compilers is that unit values from customs documentation have the appeal of a relatively cheap and easily available administrative source of data, compared with pricing representative items from establishments, but the classification classes used are not sufficiently detailed to ensure that the prices of like items in one month are compared with like items in the next. Compositional changes within a classification group in the qualities of goods exported or imported from one month to the next can change, and unit value indices, as can be seen from equation (1.1), do not just measure pure price changes: they are influenced by changes in relative quantities.

1.9 Customs data can usually be reliably used for information on the relative values of goods imported and exported to weight the price changes. Data on the values of goods imported and exported measured in current prices do not suffer from unit value bias. Customs data may be supplemented with data from other sources for weights, including establishment surveys (see Chapter 5). Customs documents can also be used in the development of a sampling frame of establishments and commodities using the details on the customs documents (see Chapter 6).

B.1 Unit value indices and their suitability for aggregation over homogeneous items

1.10 As explained previously, unit value indices are suitable—indeed, they are ideal—for the aggregation of price changes of homogeneous items. They also solve the time aggregation problem for identical items. Consider the case where the exact same item is sold at different prices during the same period, say, lower sales and higher prices in the first week of the month and higher sales and lower prices in the last week of the month. The unit value for the monthly index solves the time aggregation problem and appropriately gives more weight to the lower prices (for which there were higher sales) than the higher prices in the aggregate. Furthermore, if the elementary unit value index in equation (1.2) is used as a price index to deflate a corresponding change in the value, the result is a change in total quantity, which is intuitively appropriate, that is,

$$\frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{n=1}^{N} p_n^0 q_n^0} \Bigg/ \left[\left(\frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{m=1}^{M} q_m^t} \right) \bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right) \right] = \frac{\sum_{m=1}^{M} q_m^t}{\sum_{n=1}^{N} q_n^0}. \tag{1.2}$$

1.11 Note that the summation of the quantities in the top and bottom of the right-hand side of equation (1.2) must be of the exact same type of items for the expression to make sense—quantities of different items cannot meaningfully be added together.

1.12 The *2008 SNA* (Chapter 15, Section B.4) argues that if the price dispersion in a period was not due to quality differences—the homogeneous case—a unit value index should be used. Yet it notes an important exception regarding the case of institutionalized price discrimination. If different importers of the same good or service, say electricity, face different prices, and the individual importers, say commercial customers and private households, are unable to change from one price to another, then price indices should be used. The constraint on the availability to the purchaser of different prices must be institutional and not simply an income constraint. This is because the household importers cannot substitute their purchases for electricity at the commercial rate. Thus for MPIs for identical items purchased by different resident units or groups of units under institutionalized price discrimination, the imports for each unit or group should be treated as separate items, and price indices compiled for these items. For XPIs, the economic theory of producer price index numbers (ILO and others, 2004b, Chapter 17) defines for resident exporters a (fixed-input) output price index as the ratio of the two revenues in the periods compared, assuming fixed technologies and inputs. From the producer's perspective, a shift in the quantities of identical items sold at differentiated prices effects a change in revenue from fixed inputs[3]—the institutional arrangements matter and indeed were likely devised to enable revenue to be maximized. The exports to the different countries for the identical good or service should not be treated as separate items, and unit values should be used. From the purchaser's perspective it makes no difference to the ratio of expenditures for, say, a commercial customer if the producer shifts some of its quantities to private households—the institutional arrangements do not matter and unit values should not be used. In other words, from the viewpoints of the purchasers of the above homogeneous commodity, what counts is his or her (separate from other purchasers) unit value price, not the overall unit value price across all purchasers, which would be the relevant price for the seller.

[3]We assume the costs of serving the different purchasers are not significantly different. Because exports, from a resident producer's perspective, should be valued at basic prices, differential transport margins should not be a consideration.

1.13 Price comparisons may be required for aggregation across comparable, but not identical, items. Consider the case of electricity exported to different countries at different prices and price changes. It may be that some of the price difference can be attributed to the reliability of the supply. If the effect of quality differences on price dispersion was small, unit values may be used as long as the differential quality difference can be stripped from the prices, say, by using explicit estimates of the effect on price of the differences in supply quality. Quality adjustments to prices are a standard part of index number work, and Chapter 8 outlines the methods available to undertake such adjustments—see also Section H.

B.2 Errors and bias in the use of unit value indices

1.14 Unit value indices derived from data collected by customs authorities are used by many countries as surrogates for price changes at the elementary level of aggregation. The following are grounds on which such unit value indices can be deemed to be potentially unreliable or unsuitable:

- Bias arises from compositional changes in the quantities and quality mix of what is exported and imported. Even with best practice stratification, the scope for reducing such bias is limited owing to the sparse variable list—class of (quantity) size of the order and enterprise/country of origin or destination)—available on customs documents. Indeed it does not follow that such breakdowns are always beneficial;
- For unique and complex goods, model pricing can be used in establishment-based surveys where the respondent each period is asked to price a commodity, say a machine with fixed specified characteristics. This possibility is not open to unit value indices;
- Methods for appropriately dealing with quality change, temporarily missing values, and seasonal goods can be employed with establishment-based surveys to an extent that is not possible with unit value indices;
- The information on quantities in customs returns, and the related matter of choice of units in which the quantities are measured, has been found in practice in the past to be seriously problematic, though the advent of computer systems has been a major innovation in mitigating such problems—the Automated System for Customs Data (ASYCUDA)

project[4] of United Nations Conference on Trade and Development (UNCTAD) has applied computerized systems in the customs administrations of the least developed countries;

- Customs union countries may simply have limited or no intra-area trade data to use;
- An increasing proportion of trade is in services and by electronic commerce (e-commerce) and not subject to customs documentation;
- Unit value indices rely to a large extent on outlier detection and deletion. Given the stickiness of many price changes, such deletions run the risk of missing the large price catch-ups when they take place and understating inflation;
- Valuation requirements for the deflation of the aggregates of the *2008 SNA* are determined for unit value indices by customs procedures at the time of crossing a frontier, which are not in accord with the change in ownership principle of the *2008 SNA*.

1.15 A main advantage of the use of unit value indices is held to be their coverage and relatively low resource cost. However, such coverage should not be assumed for all classes because the unit values *used* may effectively be non-random samples and exclude commodities traded irregularly, those that have no quantity reported (especially for parts and machinery), those having low-value shipments, and those with erratic month-to-month changes. The extent of such exclusions may be substantial. Establishment-based surveys can be quite representative. Often a small number of wholesalers or establishments are responsible for much of the total value of imports or exports and, assuming cooperation, will be a cost-effective source of reliable data. Further, good sampling can, by definition, realize accurate price change measures. Finally, the *value* shares of exports and imports, obtained from customs data, which generally have good coverage of merchandise trade, will form the basis of the information used for weights for establishment-based price survey data.

1.16 Alternative index number formulas are usually assessed by determining how well they satisfy a number of reasonable properties, the axiomatic approach. Chapters 17 and 21 outline and apply such tests to compare the performance of several index number formulas used at the higher and elementary level, respectively. Unit value indices fail the identity test—if all prices remain constant the value of the index should be unity—and the proportionality test—a proportional change in all prices should result in the same proportional change in the index; both tests are regarded as important tests in index number theory. Unit value indices also fail the commensurability test—a price index should be invariant to the units of measurement selected. For example, if the measurement of one or more of the items changed from weight in pounds to kilograms, the index should not change. In practice, the units of measurement for an item in a detailed classification group are generally the same for customs documentation; however, quality variations are equivalent to changes in units of measurement. For example, 20 automobiles are not equivalent to 20 automobiles with larger engines, and, in this sense, failure of the commensurability test is an important deficiency of unit value indices derived from customs documentation.

1.17 Alternative index number formulas can also be assessed by the economic approach, as outlined in Chapter 18. Chapter 2 also notes that an index that uses unit value changes as "plug-ins" for price changes will equate to a theoretical economic index number only under restrictive conditions.

1.18 The Fisher index number formula, as will be outlined below and in detail in Chapter 17, has been described as "ideal" on the grounds that it satisfies all reasonable tests required of index numbers. The Fisher index is also "superlative" because it, along with a few other such formulas, well approximates an index well defined in economic theory that has good properties—see Section J below and Chapter 18. An important question is whether the conditions for a unit value index to equal a Fisher index are likely to hold in practice. In Chapter 2, it is shown that such conditions are all highly restrictive. They are that either (1) all prices are equal in each period, or (2) all quantity relatives are equal, or (3) price relatives and quantity relatives are uncorrelated.

B.3 Evidence of errors and bias in using unit value indices based on customs data

1.19 Given the potential for errors and bias in the use of unit value indices based on customs data, it is important to consider the evidence for such errors and

[4] ASYCUDA is functioning in about 90 developing countries. That system verifies declaration entries immediately. Declarations need to be completely filled in order to receive customs clearance. This means among other things that quantity information is required. In addition, customs values are validated using unit values on the declaration matched against a predetermined list of commodity prices.

bias in practice. A number of countries have compiled unit value indices using customs data alongside price indices based on establishment surveys. Establishment-based price indices by their nature are compiled by, first, devising with the responding establishments detailed price-determining specifications of representative commodities, and their prices in the reference period on "price initiation," and then comparing the prices of the same specifications in subsequent periods.[5] In this important regard the cited studies ask how well unit value indices stand up against price indices designed to overcome one of their major failings. Although there are methodological caveats in comparing the two series, including differences in formulas used, the overriding conclusion from the evidence summarized in Chapter 2 is that there are substantial differences between the two. Changes in unit value–based indices of exports and imports do not represent those of their corresponding price indices and, further, can be very misleading as indicators of such price indices. This holds for month-on-month and long-run annual changes with differences compounding for terms of trade indices. Such findings have led the statistical authorities in most of the countries studied to abandon the use of unit value indices.

1.20 As noted in Section B.2, the concern arises not only because of the potential for errors and biases from the use of unit value indices based on customs data, but also because (1) not all customs returns may have suitable quantity data, with the result that the coverage of the unit value is arbitrarily reduced; (2) some unit value changes are often highly volatile and automatic or deletion routines may be unsatisfactory in that they may remove some of the signals as well as the noise; (3) countries joining customs unions may no longer have customs data for much of their trade; (4) customs data do not cover trade in services and e-commerce, as well as trade in electricity, gas, and water, for which establishment surveys are generally used; (5) for the vast majority of commodity classes the turnover in differentiated items each month is substantial; (6) customs data are inappropriate for the treatment of quality changes, new goods, missing goods, seasonal goods, and hard-to-measure goods, such as one-off machines and ships; and (7) the valuation requirements of the *2008 SNA* for trade price indices to be used as deflators for national account aggregates, as outlined in Chapter 4, are better met by data from establishment surveys than by customs data.

B.4 Strategic options for the compilation of XMPIs

1.21 Given there is a serious cause for concern in using unit value indices based on customs data for XMPIs,[6] there is the practical matter of the strategic options open to statistical authorities that use such data. Unit value indices are used by many countries, and a move to price indices based on establishment surveys has resource consequences. One possibility is to identify whether there are particular commodities less prone to unit value bias and utilize unit value indices only for these subaggregates in a *hybrid* overall index. Chapter 2 outlines the methodology for the construction of such indices. The use of hybrid indices has the resource advantage of undertaking price surveys only for commodities for which they are necessary. The efficacy of such advice depends on the extent to which reliable unit value indices will be available at a disaggregated level.

1.22 This *Manual* advises that resource-constrained countries using unit value indices undertake a staged progressive adoption of hybrid indices with, over time, increasing proportions of unit value indices being substituted in favor of establishment-based survey data. An appraisal should be undertaken of each commodity group to determine the source data that are the most resource efficient and methodologically appropriate. Of issue is the homogeneity of subheadings, and there are methods for testing elementary customs aggregates for multiple elementary items. Chapter 6, Section C, provides some guidelines in this regard, though the results should be regarded as indicative, and the long-term goal should be XMPIs that are primarily based on establishment surveys.

1.23 Preference should be given to the use of establishment survey data for the "low-hanging fruit" of establishments responsible for relatively high proportions of exports and imports, some of which may be owned by the state and may have some reporting obligation. Likely examples of such commodity groups include natural gas, petroleum, electricity, and airlines. There

[5]There remains a problem for both types of data when a commodity changes—say, a new improved model is introduced. Unit value indices will be biased upward, even if quantities do not change. A change in the detailed specifications will be noted when using establishment surveys, and the methods in Chapters 8 and 9 are available to help deal with the quality change/new good.

[6]This concern is also apparent in Commission of the European Communities (2008, paragraphs 15.150–151) and the Statistical Office of the European Communities (Eurostat) (2008, pages 258, 260, 266, and 268).

will also be industries in which unit values indices are *prima facie* inadequate measures of price changes, largely because of the churn in highly differentiated commodities, or the custom-made nature of the commodities, such as shipbuilding and oil platforms. Further, there may be industries that account for a substantial proportion of trade and the payoff of reliable data far outweighs the survey costs—for example, the use of surveys of fish-processing plants for major exporters of fish commodities and of agricultural marketing cooperatives for exports of primary commodities.

1.24 Source data for XMPIs other than customs unit values and establishment surveys include **mirror price indices,** that is, the corresponding series from other countries—if your major exports (imports) are to (from) one or more identified countries and these countries have what you believe to be reliable import (export) price indices for these goods, then a weighted (across countries) average of these series may be a suitable proxy. A further alternative is to use **international commodity price indices** to proxy export or import price changes. The assumption is that there is a global market in which countries are price takers with little or no price discrimination between countries. Similar considerations apply to the use of price series produced by a resource-rich country for hard-to-measure goods and services, such as personal computers, that have benefited from quality adjustment procedures. A country may have a program for compiling an establishment-based output (input) **producer price index** (PPI) that is a measure of the price changes of the output from (input to) the domestic economy as a whole to (from) both the resident and nonresident markets. In some cases the establishments may wholly sell to (buy from) nonresident markets, or not practice price discrimination between the two markets (assuming relevant transportation, tax, and other margins are constant or insignificant),[7] in which case a timely series should exist for XMPIs. Or it may be that price changes for a difficult- or costly-to-measure commodity group can be **imputed** from another group.

1.25 A gradualist approach to adopting XMPIs has two potential problems. The first is that its reliance on unit value indices for what may be major commodity classes is unlikely to be soundly based. Chapter 2 examines some evidence on the reliability of unit value indices for particular commodity groups. The evidence is not supportive of there being many subheadings for which unit value indices can be relied upon. The case for adopting hybrid indices is a pragmatic one arising from resource and expertise constraints to the adoption of establishment survey-based XMPIs. A second, potential problem with a gradualist approach is that the measurement of longer-term changes in the index becomes problematic. The user cannot judge how much of the long-run change is due to changes in the indicator series used. A gradualist approach should be accompanied by well-signaled steps to users and, when changes take place, by parallel data for at least 12 months so that 12-month changes can be identified and the new index readily linked to the old. There should be adequate metadata to explain the change. The approach is inferior to a strategy that simply requires the adoption of a primarily establishment-based price index. The culmination of a program of use of hybrid indices should be an index in which unit values have little or no place.

1.26 Of course improvements to unit value indices should be made if possible. These would include more detailed stratification including shipments by/to (major) establishments to/from given countries. However, the absence on customs documentation of highly detailed commodity descriptions by which to stratify unit values precludes any stratification that allows the compiler to be confident that like items in any month are compared with like in the next. Improved outlier detection routines are certainly advocated by the *Manual* when unit value indices are used (see Chapter 6, Section C). However, caution is expressed about the efficacy of such routines unless well applied, and the need for validation prior to deletion with an exporting/importing establishment or other third-party source is strongly recommended. Deletion routines should be used to identify unusual price changes, which then have to be followed up to ensure that they are not real changes—large catch-up price changes under sticky price setting—but owing to wrongly entered numbers or a change in the units for quantities. However, the sheer magnitude of the task of following up the original customs documentation, and then possibly having to refer back to the exporter/importer, may well preclude detailed follow-up with an over-reliance on automatic deletion routines. Second, such routines will be based on past parameters of the dispersion, which may themselves be based on outliers. Further, the parameters may themselves be unstable, say owing to sticky pricing and volatile exchange rates, and past experience may not be useful for future deletion practice. Finally, there is the arbitrary nature of the cutoff values often used in practice for deletion.

[7]From a resident's perspective exports transportation costs should be excluded for export price indices because the pricing basis is the *basic price*—that is, the amount received by the producer, or distributor exclusive of any taxes on commodities and transport and trade margins, whereas from the nonresident's perspective the pricing basis for imports is the basic price—see Chapter 4.

1.27 The main problem with simply introducing a new establishment survey–based program is the resource cost. This includes the training of price collectors, building of sample frames, sample selection of items and establishments, computer routines, data validations, and much more that is the subject of Chapters 3–14 of this *Manual*. However, if a PPI program is already established, there will be synergies with the external trade price index program, including computer routines, price collecting manuals and training, and expertise in sampling items and establishments. There will be some commodity groups for which the PPI results are alone sufficient. However, in other commodity groups for which the current PPI sample is not sufficiently detailed or representative to allow reliable export/import indices to be compiled, the sample of items/establishments will need to be supplemented to include items that are imported/exported. Chapter 13 considers some organizational issues in taking advantage of the synergies between the two programs.

C. The Uses of XMPIs

1.28 The four principal types of price indices in the system of economic statistics—the consumer price index (CPI), the producer price index, and the export and import price indices—are well known and closely watched indicators of macroeconomic performance. They are direct indicators of price inflation for various flows of goods and services. As such, they are also used to deflate series of nominal values of goods and services produced, consumed, and traded to provide measures of changes in their volumes. Consequently, these indices are not only important tools in the design and conduct of the monetary and fiscal policy of the government, but they are also of great utility in informing economic decisions throughout the private sector. They do not, or should not, comprise merely a collection of unrelated price indicators, but provide instead an integrated and consistent view of price developments pertaining to production, consumption, and international transactions in goods and services.

1.29 Like other price indices in the system of price statistics, XMPIs serve multiple purposes. Precisely how they are defined and constructed can very much depend on the data source underlying their construction as well as their intended use.

1.30 Uses of XMPIs can be identified from a *resident unit's perspective*. A monthly or quarterly XMPI with detailed commodity and industry data allows monitoring of price inflation for different types of commodities (henceforth "commodities" or "products" refers to goods and services). MPIs facilitate an understanding of the transmission of inflation through different stages of the resident producer's production process and directly to final products purchased by resident households, government organizations, and other institutions. Measures of changes in the terms of trade of a country, determined as the ratio of the XPI to the MPI, are used in the determination of changes in the real income of residents. The national accounts identify in the production accounts the output and intermediate consumption (inputs) of resident establishments, and these can be decomposed into the output to residents and to the rest of the world (exports) and the inputs from residents and from the rest of the world (imports). An analysis of the productivity of such establishments requires volume measures of such flows that in turn require price deflators for exports and imports. In addition, XMPIs for specific commodities can be used to adjust prices of inputs in long-term purchase and sales contracts, a procedure known as "escalation." Thus an analysis of the transmission of inflation, terms of trade, and productivity of resident establishments, and use for escalation payments by them, is well served by XMPIs.

1.31 Although XMPIs are important economic indicators in their own right, a vital use of XMPIs is as deflators of a series of nominal values of exports and imports to help derive volume estimates of GDP by the expenditure approach. Exports and imports are defined by the *2008 SNA,* from a *nonresident's* or rest of the world's perspective: exports are the rest of the world's *uses* of domestic production and imports are the rest of the world's *supply* of goods and services to resident users. Exports and imports are components of estimates of GDP by expenditure that includes household and government expenditure, capital formation, and net exports (exports *minus* imports) of goods and services. Beyond their use as deflators, the national accounts framework for XMPIs provides insight into the interlinkages between different price measures. Through net exports, XMPIs directly affect the price index (deflator) of GDP by expenditure. The MPI also contributes to the price changes of intermediate consumption by establishments; the household consumption deflator; the government consumption deflator; the capital formation deflator; and, through reexports and goods for processing, the XPI. The XPI contributes to change in the output PPI. As such, the detailed information in XMPIs allows compilers to show the contributions of XMPIs to changes in each index of the system of price statistics. Because the price index (deflator) for GDP by the production approach (value added = output − intermediate consumption) is a function of the output and intermediate consumption PPIs, XMPIs, viewed in this way, contribute to change in the price index (deflator), for not only GDP by expenditure but also GDP by production.

1.32 Any remaining part of exports involves the final uses of goods and services by nonresidents. An example is cross-border shopping by nonresident households, which is exports either as nonresident final consumption if the acquired items are consumer goods, or as capital formation if the acquired items are valuables, such as jewelry. Another example is acquisition of secondhand productive assets by nonresidents for business purposes, which, besides being shown as exports, also enters as negative capital formation in the domestic, supplying economy, and as capital formation in the using economy.[8]

1.33 Unlike the PPI, which involves only establishments, or the CPI, which involves only households, the XMPIs potentially involve all types of units in the world economy—not only establishments, but also the nonbusiness parts of general government, households, and nonprofit institutions serving households—for transactions including the following:

- Intermediate consumption and output by business units;
- Capital formation via acquisition and sales of new and secondhand nonfinancial assets by business units and households if the items transacted are valuables (e.g., works of art and jewelry); and
- Final consumption of services (e.g., vacation accommodations), as well as of goods via exchange of secondhand consumer durables (e.g., automobiles), by nonbusiness parts of nonprofit institutions and government.

1.34 This *Manual* adopts the *2008 SNA* and the *Balance of Payments Manual* (*BPM6*)[9] as comprising the conceptual framework for the value aggregates underlying all macroeconomic statistics, including the XMPIs. The desirability of this conceptual concordance between the price indices permits users to clearly understand the linkages between price series, discussed in more detail in Chapter 15. It is this concordance that makes components of the XMPI useful as deflators for exports and imports in the national accounts.

1.35 The *2008 SNA* equates, as outlined in Chapter 15, the sum of the value of transactions for goods and services supplied to the economy by domestic production (output) and imports with the sum used for intermediate consumption, final consumption, capital formation (including inventories), and exports, that is,

supply: output + imports
+ taxes less subsidies on products
= *uses*: intermediate consumption
+ final consumption
+ capital formation
+ exports.

1.36 For the identity to balance *at the product group level* it is necessary to add trade and transport margins and taxes less subsidies as separate items to the basic supply prices on the left-hand supply to equate with uses at the prices purchasers pay on the right. Such commodity balances are used by national accountants to validate data and, where necessary, to estimate missing values as residuals. Supply and use tables (SUTs) consist of a set of such product balances covering all products in an economy organized in matrix form with product groups in rows. The *2008 SNA* (Chapter 14) also advises that SUTs be developed in volume terms. It is good practice that deflators be applied at a detailed product group level. The deflation of the aggregates at the level of product groups to provide SUTs in volume terms provides a framework for deflators to be applied in a manner that reconciles the volume estimates and, thus, deflators for all transactions of goods and services supplied and used.[10] This requires that for *each product group,* each output, intermediate consumption, final consumption, capital formation, and export and import be deflated, and because the left-hand side should equal the right, in volume terms as well as current prices, the deflators, including the XMPIs, benefit from this reconciliation of the volume estimates. XMPIs as deflators at the detailed level are developed

[8]It is possible, as well, for there to be a generally quite small part of imports not accounted for by nonresident output involving direct change of ownership of secondhand goods and valuables between households resident in different countries. This change of ownership counts as "negative consumption" (consumer durable goods) or negative capital formation (valuables) in the supplying country or territory and positive consumption (consumer durables) or capital formation (valuables) in the using territory. Services imports must be provided by nonresident enterprises, and thus they count as output of establishments rather than as negative consumption or negative capital formation of nonresident households.

[9]IMF, Sixth Edition of the IMF's *Balance of Payments and International Investment Position Manual (BPM6),* available at www.imf.org/external/pubs/ft/bop/2007/bopman6.htm. This is an update of *BPM5*.

[10]The argument is phrased in terms of deflators, but extends to the use of volume/quantity extrapolation, as considered in Chapter 15 of the *2008 SNA*.

as part of an integrated and consistent system. Chapters 4 and 15 outline this reconciliation and how different valuation systems enable it.[11]

1.37 These varied uses often increase the demand for XMPI data. For example, interest in the XMPI as an indicator of externally generated inflation creates pressure to extend its coverage to include more commodities. Although many countries initially develop XMPIs to cover goods in international trade, the XMPIs can and should logically be extended to cover internationally traded services, as noted in Chapters 3, 4, and 15.

D. Concepts, Scope, and Classifications

1.38 The purpose of Chapter 4 of the *Manual* is to define and clarify a number of basic concepts underlying XMPIs and to explain the scope, or domain, of the index: that is, the set of commodities and economic activities that the index is intended to cover. The chapter also discusses the various price concepts and types of prices that are used in XMPI compilation and examines the structure of the classification systems used in the XMPI for commodities and industries.

1.39 The general purpose of XMPIs is to measure changes in the prices of goods and services exchanged in monetary transactions between the residents of an economic territory and the rest of the world. However, an operational definition of a set of XMPIs requires a decision about whether the index is to have the nonresident (national accounts) perspective or the resident perspective because this determines the valuation principles that are suitable. This decision on whether to use a resident's or nonresident's perspective is determined by the analytical needs of the XMPIs, and Chapters 4 and 15 relate the two perspectives to such needs. Price indices for exports and imports from a resident's perspective are suitable for the analysis of the transmission of inflation, terms of trade measurement, and productivity analysis. The counterpart nominal aggregates to these price indices can also, in part, be placed in the production accounts of the *2008 SNA* if the outputs and inputs are disaggregated according to being directed to, or being from, domestic and foreign institutional units. If the analytical need to is to deflate nominal exports and imports within the supply and use system and the goods and services account of the *2008 SNA*, then the nonresident's approach is appropriate. The niceties of the analytical distinction and valuation principles are tempered if the source of data is from customs rather than establishment surveys; Chapter 4 outlines such considerations. Chapter 4 also considers issues of coverage, including (1) whether the index is meant to cover all trade, that is, all commodities regardless of the destination (exports) or source (imports), or just particular commodity groups in transactions with selected parts of the world; (2) for the transactions included, whether the index should cover just "arm's-length" transactions, or so-called transfer prices between related units in different economic territories; and (3) in what geographic territory the defined production is included—for example, does trade include flows through "free zones." The scope of XMPIs is influenced inevitably by what is intended or believed to be its main use, although it should be borne in mind that the index may also be used as a proxy for a general price index, particularly in very open economies, and used for purposes other than those for which it is intended.

D.1 Population coverage

1.40 Many decisions must be made to define the scope and coverage of the XMPIs. These include the economic activities, commodities, and types of buyers and sellers to include in the index. The XMPIs should cover all of a country's international trade in goods and services, which could be the ultimate goal of the price indices. In many countries the XMPIs are limited to the goods trade captured by the customs authorities, and the transport and insurance services provided on imports. This represents a good starting point. However, the share of goods in international trade is becoming smaller, and services such as transport, communication, medical care, trade, tourism, and financial and business services are becoming increasingly more important, depending on the country. Exporters will include producers of goods and services (e.g., manufacturers exporting directly) and export agents (wholesalers). Similarly, importers will include retailers and end users (including manufacturers and households) as well as import agents (wholesalers).

1.41 XMPI's can be compiled and classified by commodity, by destination or source country, and even by the industry of the trading establishment. The XMPIs also can identify commodities by stage of processing and produce measures of commodities for final demand, those for intermediate consumption, and those that are primary commodities.

[11]A detailed account and illustration can be found in Chapter 9 of Statistical Office of the European Communities (Eurostat) (2008).

D.2 Price coverage and valuation

1.42 Chapter 4 considers the two main valuation bases from the *2008 SNA*: the basic price and the purchaser's price. The basic price is what the supplier receives per unit of a good or service exchanged, and the purchaser's price is what the purchaser pays. Basic prices thus pertain to supply flows of goods and services whereas purchasers' prices pertain to use flows of goods and services. The two generally differ because to consummate the transaction the purchaser may be required to pay a third party or parties an amount per unit of the good or service over and above what the supplier is willing to receive. Purchasers may pay a tax on products to (or receive a subsidy on products from) a government unit, pay a transportation charge to another nonfinancial enterprise, pay an insurance charge to an insurance corporation, and/or pay a distribution charge to a retail or wholesale trade enterprise. Data on unit values, or values at current prices to be used as the basis for weights, from customs documentation may be valued as the "free on board" (f.o.b.) price or cost, insurance, and freight (c.i.f.) price. The United Nations (1998a and 2004) recommend that the statistical value of *imported* goods be c.i.f. and *exported* goods be f.o.b. The f.o.b. price is the value of the good or service at the point just prior to departure from the supplying economic territory. The f.o.b. price thus is comprised of the basic price, plus taxes less subsidies on products levied by the general government *of the supplying economic territory,* plus distribution margins, transportation, and insurance services added to get the product from the point of manufacture to the point of departure from the supplying economic territory. The c.i.f. price includes the transaction value of the goods, the value of services performed to deliver the goods to the border of the exporting country, and the value of the services performed to deliver the goods from the border of the exporting country to the border of the importing country.

1.43 Chapter 4 outlines how different analytical needs dictate whether a resident's or nonresident's approach is taken. When we take the *resident's approach to international trade flows*, we value exports of the total economy at basic prices and imports at purchasers' prices. When we take the *nonresident's approach to international trade flows*, we value exports of the total economy at purchasers' prices and imports at basic prices. Chapter 4 further outlines how for balancing SUTs in volume terms, the valuation system has to be consistent across supply and uses. The chapter outlines the pros and cons for using either basic prices or purchasers' prices in this context. Thus the analytical need helps determine the valuation principle. In practice, when using data from customs documentation, the valuation principle is given, whereas for price surveys from establishments the valuation can be better tailored to the analytical need.

1.44 Exports and imports of goods should be recorded in the accounts of the transacting parties at the market price prevailing when *change of ownership* occurs. Exports and imports of services should be recorded at the market price prevailing on the date the services are supplied to the purchaser. The market price is the price a willing buyer would pay to acquire the good or service from a willing seller for one specific exchange. These prices would not necessarily be "list" or "book" prices because they should reflect any applicable discounts, rebates, surcharges, and so on that may apply to their customers for the sampled transactions. These would include contract prices, where they exist, and spot market prices. Care must be taken to make sure the prices reflect those at the time the transaction occurs and not those at the time of order, particularly for major durable goods, such as airplanes and ships, which have a long production period between order and delivery. In practice the process of recording trade in goods by customs authorities may not necessarily coincide with the change of ownership as goods may change ownership without having to cross the customs frontier or may cross the customs frontier without changing ownership. If the difference in price between the time of recording at customs and that at change of ownership is considered to be significant, then price surveys of establishments may be the only source to get accurate price information on the transactions. It may of course be that, for example, an export price deflator is required valued at basic prices and all that is available is an export price index (or components thereof) valued at f.o.b. In such a case it may be reasonable for the industries in question to assume that the price changes of exports at basic prices are similar to those for transport and insurance margins and tax/subsidies changes.

1.45 The treatment of transport margins for valuation depends on when the change in ownership occurs. If, for example, the supplier (exporter) delivers the commodity to the importer without an explicit charge for transportation, the transport margin is part of the basic price—change of ownership takes place on delivery. If the purchaser (importer) is responsible for transport, the basic price excludes transportation—change of ownership takes place on leaving the supplier (exporter). However, the issue is complicated if the transportation is subcontracted to a third party, further depending on whether the third party is subcontracted by the exporter or importer and where the third-party contractor is resident, especially if it is in a third country. The *value of goods* sent abroad for processing should not be included as exports or imports because no change of ownership takes place—there is only trade to the value of the processing service.

D.3 Unit values and shipping quantities

1.46 Average or unit value prices are acceptable in the XMPIs if they represent a strictly *homogeneous* set of commodity transactions. In addition, the measure of quantity should be relevant to the transaction between buyer and seller. The quantity measure for computerized milling machines is not metric tons, but rather number of a narrowly specified type of machine. Often these two criteria for an average price cannot be met by customs data. If average prices are calculated over a large number of transactions with differing quality and/or terms of sale, they are not acceptable in the XMPIs. Changes in such prices will reflect any changes in the mix of quality characteristics of the commodities sold as well as any changes in terms of sale. Where unit values from customs data are used for heterogeneous goods, the country of origin/purchase should be used as part of the price specification. Changes in a heterogeneous mix of transactions lead to what is often referred to as *unit value bias* in the measurement of price changes. As noted above and in Chapter 2, this mitigates against using unit value indices from customs data for very many subheadings of goods.

D.4 Transfer and subsidized prices

1.47 Special care needs to be taken with subsidized prices and intra-company transfer prices. The prices used in the XMPIs should reflect the revenue received by producers from transactions or cost paid. Prices for commodities on which subsidies are received will not reflect the revenue or cost unless the subsidies are included. Also, intra-company transfer prices may not reflect actual market prices and may require special treatment as discussed in Chapter 19.

D.5 Treatment of some specific types of transactions and prices

1.48 The measurement of periodic price changes is not always clear-cut for some commodities, and Chapter 11 includes, as examples of hard-to-measure industries, agriculture (Standard International Trade Classification (SITC) 0), crude petroleum and gasoline (SITC 33), and metals (SITC 68). Commodity areas that experience frequent technological change also present some special problems. Though the trade in the computer industry (SITC 75) and motor vehicles (SITC 78) may be measurable, constructing price indices for these industries is difficult when trying to capture quality change that arises from the technological change. Clothing (SITC 84) presents a similar problem—the trade is measurable, but the measurement of price change is complicated by the change in the quality of the clothing and the influence of seasons. Because service industries generally do not have easily measurable output, it is difficult to apply the concepts set out in the *Manual* to them. Accordingly, Chapter 11 will cover some of the difficulties involved with calculating XMPIs for selected services including airfreight, air passenger fares, crude oil tanker freight, ocean liner freight, and travel and tourism. The chapter concludes with a discussion of various pricing issues that are particularly important for calculating XMPIs. These issues include the country of origin/destination, duties, currency conversion, intra-company transfers, and price bases.

D.6 Statistical units

1.49 The statistical unit in the XMPIs is usually a single, homogeneous, output-generating entity such as the *establishment*, a concept outlined in the *2008 SNA*. Separate auxiliary, sales, or administrative units are not included. This unit is the decision-making unit for all production operations and maintains records on prices and production activities. In some cases records from a clustering of establishments are sent to a single record-keeping unit, the enterprise, from which prices will have to be collected. Transactions can of course also be undertaken directly by households, non-profit institutions serving the households sector (NPISH), and government, for example, tourist/cross-border shopping.

1.50 The rapid rise in e-commerce, globalization, and outsourcing of production is making the identification of the statistical unit, the producing establishment, more difficult. This is particularly the case with the formation of *virtual corporations*. A virtual corporation is the creation of a partnership among several companies sharing complementary expertise and producing a commodity with a very short life cycle. With the conclusion of the commodity's life span, the corporation is disbanded. Also, a considerable volume of business undertaken among corporations is being transacted on the Internet, which is difficult to monitor. These activities will require new approaches to identify and capture such transactions in the XMPIs.

D.7 Classification

1.51 The classification system provides an analytical structure for the XMPIs and facilitates the organization of administrative and survey source information. It forms the index structure and defines which industries, commodities, and aggregate levels will be included. It also determines the publication scheme for the XMPI results. International standard classification systems,[12] discussed in Section D of Chapter 4, are available and should be used to provide a meaningful series for policymaking and analysis, as well as facilitating international comparisons. Data may be published aggregated under more than one classification system and be available at different levels of detail to meet the needs of different users.

1.52 Commodity classifications group commodities into somewhat homogeneous categories on the basis of physical properties and intrinsic nature, as well as on the principle of industrial origin. The physical properties and intrinsic nature are characteristics that distinguish the commodity. These include raw materials from which the goods are made, the stage of production and way in which the goods are produced or service rendered, the purpose or use of the commodities, and the prices at which they are sold. The commodity categories should be exhaustive and mutually exclusive so that a commodity belongs to only one category.

1.53 The categories of commodities (coded, for example, to five-digits) can be aggregated to higher-level groupings (four, three, two, and single digits) of commodities with similar characteristics and uses. Besides the Harmonized System (HS), other international commodity classifications that may be used for XMPIs include the *Central Product Classification* (*CPC,* Ver. 2, draft) and the *EUROSTAT Classification of Commodities by Activity 2008 Version (CPA Ver. 2008* and *PRODCOM 2007)*. In general, each five-digit subclass of the *CPC* consists of goods and services that are predominantly produced in one specific four-digit class or classes of International Standard Industrial Classification of all Economic Activities (ISIC) Revision 4. Additionally there is the Standard International Trade Classification, Revision 4 (SITC, Rev. 4) and Classification by Broad Economic Categories, Revision 3 (BEC Rev. 3).

1.54 Trade data, and thus trade price indices, may be grouped on a regional basis. The main international regional classification is the United Nations Standard Country or Area Codes for Statistical Use. Other analytical systems of grouping countries also are in use. IMF's *World Economic Outlook*, for example, uses a country classification based on the level of economic development and whether the country is a major exporter of oil. For standard publications, XMPI data should be published on the international system first (exports by destination country/area and imports by source country/area), with results on alternative, analytical classifications provided for further information, as necessary.

1.55 Industrial classifications group producer units according to their major kind of activity, based mainly on the principal class of goods or services produced—that is, by an output criterion. At the most detailed four-digit ISIC industrial level, categories are delineated according to what is in most countries the customary combination of activities undertaken by the statistical units, the establishments. The successively broader levels of classification (three-digit, two-digit, one-digit) combine the statistical units according to character, technology, organization, and financing of production. The major international industrial classifications are the *International Standard Industrial Classification of All Economic Activities, Revision 4 (ISIC Rev. 4)*, the *General Industrial Classification of Economic Activities within the European Communities, Revision 2 (NACE Rev. 2)*, the *North American Industrial Classification System, 2007 (NAICS, 2007)*,[13] and the *Australian and New Zealand Standard Industrial Classification, 2006 (ANZSIC, 2006)*.[14]

E. Source Data: Weights

1.56 There are two data needs. First are regularly collected prices aggregated, for a product group at the elementary level, as unweighted (preferably geometric) averages of prices (or using customs data, as unit values) in each, say, month. Ratios of the current period averages to a reference period average provide an elementary aggregate (or unit value) index at the detailed elementary

[12]Available at http://unstats.un.org/unsd/cr/registry/regct.asp?Lg=1 and at http://ec.europa.eu/eurostat/ramon/index.cfm?TargetUrl=DSP_PUB_WELC.

[13]Available at www.census.gov/epcd/www/naics.html.

[14]Available at www.abs.gov.au/ausstats/abs@.nsf/7d12b0f6763c78caca257061001cc588/a77d93484dc49d63ca25712300056842!OpenDocument.

level. The next need is to combine, or average, the elementary price indices to arrive at price indices at higher levels of aggregation up to the overall XPI and MPI. For this purpose, trade share weights are needed for the various elementary aggregate indices. These weights are needed whatever index number formula is used for aggregation purposes at this higher level. This section and Chapter 5 are concerned with the derivation and sources of the trade share weights, and Section F and Chapters 6 and 7 with price collection.

1.57 Customs data are the basic data source for the weights. The regular customs documents (customs declarations) are forms filled in by exporters and importers and submitted to customs officials. In most countries, a customs declaration is required for merchandise imports and exports, whether or not these goods are subject to customs duties. In principle, a customs declaration identifies the importer or exporter, the product code, the value of the shipment, the shipping quantities, the duties paid, the country of origin or destination, the port of entry or exit, the mode of transport, the costs of transport, and the costs of insurance and freight. Customs, the statistical office, or another agency processes copies of the customs documents to compile statistics on foreign trade.

1.58 As explained in Chapter 10, there are two levels of calculation involved in XMPIs. At the lower level, samples of prices are collected and processed to obtain lower-level price indices. These lower-level indices are the elementary indices whose properties and behavior are explained in Chapter 21 and are summarized in Section K below. At the higher level, the elementary indices are averaged to obtain higher-level indices using the relative value of exports for an XPI or imports for an MPI as weights. All the index number theory elaborated in Chapters 16–19 and summarized in Section J below comes into play at this higher level. Customs data form the basis for weights at the higher level irrespective of whether establishment survey-based price indices or customs-based unit value indices are used at the lower level. However, there may be exceptions to this. It may be that data from an establishment survey are considered more reliable for weights than customs data, particularly with regard to (1) categories such as services not covered by customs or others unreliably covered; (2) the valuation principle used by customs is considered inappropriate and the establishment can provide estimates on a more appropriate one; (3) the commodity class used by customs is not sufficiently detailed and the exporting/importing establishment can provide value data at a more detailed level; and (4) the customs data may not always be easily accessible to statistical agencies in a timely and regular manner. Source data for weights are considered further in Section E.1.

E.1 Administrative data: Customs and the International Transactions Reporting System

1.59 For goods, customs export declarations and import tariff (or tariff exemption) forms provide fields indicating the value of export or import by shipment. These data are a primary source of information on the weights of the XMPIs for goods. They are available at a detailed level and on a relatively timely basis. For imports, these data often are compiled only for trade values including not only the f.o.b. cost at the foreign export frontier, but also transportation and insurance, as tariffs are levied on this combined c.i.f. value. As noted earlier and in Chapter 4, the desired valuation concept depends on the purpose of the measure and, thus, the residency perspective adopted, as well as the practical consideration of source data used. Guidelines on valuation are given in Chapter 4.

1.60 The International Transactions Reporting System (ITRS) is present in most countries to implement trade regulations of one type or another, or to allow surveillance on international transactions. It may provide enough information for index weights if the commodity classification code(s) of the items transacted are recorded in the ITRS. Sometimes the ITRS may pick up the commodity code at a less than fully detailed level. This still may be used with other information to construct index weights. Because the ITRS covers all banking system transactions, it includes both goods and services and may be an important source of weights for services in particular, as well as a check on the coverage of customs information on goods.

1.61 A major advantage of the use of customs trade values as the basis for weights is that detailed information on a relatively timely basis is available as a relatively cheap by-product of an administrative process. This has the major advantage of enabling frequent updates of weights and, in particular, the use of a chained index number formula. The coverage of transactions within a category will generally be good. However, as discussed in Chapter 2, there will be some commodities not covered, most particularly services, and in the exceptional case of a customs union, possibly intra-union trade. There may also be commodity groups for which the data are deemed unreliable—for example, where there is known to be substantial illegal cross-frontier transactions. Alternative data sources are thus considered not only for cases where customs data may not be available, but also to reconcile estimates where this is deemed necessary and, in some cases, say, from exporting and importing establishments, to provide more detailed information on weights at the item level. We consider such sources below.

E.2 Survey data: Establishment surveys

E.2.1 Survey frame

1.62 Critical to the accuracy of data from establishment surveys to be used for weights is that the sampling/survey frame be up to date. The survey frame is a list of the universe of statistical units that are the focus of a sample survey. In addition to a name and identifying code, a frame should have a measure of the size of each of the units that is correlated with the concept the survey intends to measure. For the XPI, the focal concept is exports and the frame ideally should have exports for each establishment at least in total, if possible broken down by goods and services or, better still, by major commodity class. For the MPI, the focal concept is imports and the frame ideally should have imports for each establishment at least in total, if possible broken down by goods and services or, better still, by major commodity class. Additional information on the primary source/destination countries may be of use.

E.2.1.1 Customs documents and the ITRS

1.63 For goods, most countries can enumerate all companies filing export declarations or import tariff forms. This is an ideal frame list for the XMPIs for goods, satisfying all of our principal criteria. If there is no smuggling and if customs legislation requires all exporters and importers to file forms, the customs frame is exhaustive for the XMPIs, and it contains exactly the right measures of size of each establishment. Additional customs information on households' direct expenditures on imports can come from compilations of customs debarkation forms, on which travelers report dutiable expenditures on imports.[15] The other source of administrative information on which institutional units and their establishments are engaged in international trade, as well as the approximate amount of that trade, is the ITRS, which is present in one form or another in most countries.

E.2.1.2 Establishment or business censuses

1.64 The establishment or business census covers all establishments that have productive activity within the geographic borders of the country. These censuses may be conducted over a span of years, with different economic activities covered at different times during the cycle. For example, a census of agriculture would be conducted one year, a census of industrial activities (mining, manufacturing, and energy supply) completed during the next year, followed by a census of services. In some instances there may be a size cutoff to exclude very small establishments. For example, some countries exclude establishments with fewer than five employees or with some low threshold of annual production, or complete the census only using a sample of small establishments.

1.65 A business census typically gets detailed accounting data of annual output in value (at basic prices) and quantity terms by detailed commodity classification at the enterprise or establishment level. This would include sales and inventories by commodity, as well as value and quantity of inputs at the prices paid by producers. These data can be used to derive the export weights by detailed commodity classification and establishment, as long as the questionnaire distinguishes between the output of these resident establishments to the domestic and to the export market. If so, this is an excellent source of export weight data, assuming that the coverage of economic activity is essentially complete.

1.66 Censuses also may capture intermediate consumption and capital formation by commodity at some level of detail, including the fraction imported. For those commodities whose uses are largely intermediate consumption or capital formation, this is a good source of import weight data, conditional on good coverage of the business activity.

E.2.1.3 Business register

1.67 Most countries maintain a business register that provides a list of firms that are involved in productive activities. Such registers usually contain information on location, economic activity, size (for example, employment, payrolls, value of annual production, or turnover), contact persons, tax information, and so on. The business register could be an alternative source of weight information, particularly if business censuses are not conducted on a regular basis or if annual surveys do not provide sufficient information for

[15] Of course, the coverage of this source often is incomplete because honest reporting may result in payment of import tax.

establishing weights. This is particularly true if there is an ongoing system for updating and maintaining the information contained in the register and it contains data at the establishment level.

1.68 There are several shortcomings to the use of these registers for weight information. Often the business register is updated only when a firm begins operations. Unless the register is maintained by purging firms that are no longer in business, it will have superfluous information. The information on the size of the firm also needs to be updated on a regular basis. Much of the information may relate to the time at which the firm was introduced into the register. Also, the business register may comprise a list of *enterprises,* which is not completely suitable for the XMPIs, where the concern is to obtain information at the *establishment* level. The register will usually be devoid of information on commodities, which means that additional data collection will be necessary before weights can be established at the commodity level.

E.2.2 Establishment surveys

1.69 These surveys differ from censuses primarily in three respects:

- Coverage is limited to a sample of establishments rather than a full enumeration,
- Commodity detail is limited to higher aggregate levels such as groups, and
- Types of data requested are generally more limited than those requested in a census.

1.70 For example, commodity information in the census may be obtained at the eight-digit commodity code level using PRODCOM, with complete detail on commodity sales and inventories, whereas in the industry survey data are reported at the six-digit level and are requested only for sales. Also, data may be reported only for the enterprise rather than broken down by establishment.

1.71 Thus, for enterprise or industry surveys, the weights that are available will generally be for higher levels in the aggregation structure, such as commodity group and industry, rather than detailed commodity and establishment. The use of these weights for XMPIs will depend on how the XMPI aggregation structure has been established. If multi-tier weights (for example, one set of weights for the commodity group and above, and another set of weights at the establishment level and below) have been set up, the survey results could be used for aggregation at higher levels while the weights at lower levels are determined separately. For example, the survey weights could be used for aggregating from the four-digit industry level to higher levels, while sampling weights (that is, sampling fractions from probability selection procedures) could be used at the establishment and commodity level. In this scheme, the weights at the higher levels would be updated periodically from administrative (customs) and establishment survey data, while the weights at the lower levels would be updated as the samples of establishments and commodities are refreshed. This process is discussed in more detail in Chapter 6.[16]

E.2.3 Household and traveler surveys

1.72 A household expenditure survey (HES) often captures the fraction of expenditure on imported items at some level of commodity detail. In principle, the HES can capture households' direct expenditures on imported goods, whether they were made during travel, or via the Internet or mail order. Further detail and corroborating information on direct household expenditure on imported goods while traveling may be possible with surveys of travelers as they disembark from trips abroad.

E.2.4 Other sources for estimating trade share weights

E.2.4.1 National accounts

1.73 Although much of the same source data described above would also be used in developing the output data for the production account in the national accounts, there can be significant differences. In a number of countries, there may be significant undercoverage in certain types of goods and services owing to the exclusion of informal and illegal activities. National accountants often make adjustments from a variety of sources for this type of undercoverage or for known biases in the survey data. In such instances, the adjusted national accounts information on export and import flows may prove to be a better source of weight information than the original sources by themselves.

1.74 The national accounts often provide additional detail on weights, particularly if supply and use tables or input/output tables are available. The information

[16]In trade price indices, the weighting information for goods may well be more frequent and timely than for services, because the goods data would come largely from customs, whereas services would rely on establishment and traveler surveys.

on commodity flows for various industries and commodities by type of use is an excellent source of weight information. One drawback of national accounts data is that the estimates include imputations for nonmarket activities, and such imputed data may not be appropriate for use as weights in an index whose coverage is primarily market activity.

1.75 Some users may have a special interest in price/volume decompositions of the trade captured by administrative systems. Retaining subaggregates of exports and imports captured by customs and ITRS sources may be of interest for forming subindices of the overall XMPIs that use the national accounts weights with coverage adjustments.

1.76 A wide variety of administrative data on trade in goods and services may be available from public agencies charged with regulating or monitoring certain economic activities. For example, national, regional, or local governmental bodies regulate many public utilities, communication, and transport activities. Some of these commodities may be internationally traded. Typically, these agencies require detailed annual reports that provide information on production value and/or turnover with information broken down by the residency of customer. These sources also have records of all regulated enterprises/establishments, which also are useful for building a sampling frame.

F. Source Data: Prices

1.77 Lower-level indices are calculated for elementary aggregates. Depending on the resources available and procedures adopted by individual countries, these elementary aggregates could be subclasses of the industry and commodity classifications as described in the previous section. If it is desired to calculate XMPIs for different regions, the subclasses have to be divided into strata referring to the different countries or regions depending on the source/destination of the imports/exports. When the subclasses are divided into strata for data collection purposes, the strata themselves become the elementary aggregates. Because a weight must be attached to each elementary aggregate in order to calculate the higher-level indices, an estimate of the value share for each elementary aggregate should be available from separate administrative or survey sources, as outlined in Chapter 5. If no weights can be derived, the elementary indices have to be estimated from price data alone, as explained in Chapter 21.

1.78 Price changes can be measured at the elementary aggregate level using unit values indices from customs data. However, XMPIs based on unit values from customs data are prone to unit value bias unless the items included in the unit value are homogeneous. Chapter 2 outlines in detail the nature of unit value bias and the rationale and preference for using establishment-based survey data.

1.79 Yet if price changes are to be measured using establishment surveys, sampling must be used for both the establishments surveyed and the commodities exported and imported by the establishments sampled. It is simply neither desirable nor feasible for the timely and cost-effective provision of price indices to measure the price changes of all items exported and imported. Such sampling is well established in the compilation of consumer and producer price indices, and the principles of such sampling are given in Chapter 5 of the *CPI* and *PPI Manuals*. The need is to monitor prices of a sample of representative items from a sample of representative establishments. The details of the items whose prices are monitored should be carefully and fully specified so that each month the prices of like items are compared with like items. Chapter 6 is concerned with sampling strategies for price collection. Chapter 7 is concerned with the methods and operational procedures actually used to collect prices. These are considered in turn in Sections F.1 and F.2 for establishment surveys.

F.1 Sampling: Random and purposive sampling

1.80 Prices are collected for commodities from selected establishments in particular industries. The sampling process involves multiple stages of selection. Once the purpose and scope for the XMPIs have been decided (for example, which single-digit commodities will be included), then decisions can be made about the four-digit commodities to be included. After the commodities have been chosen, the establishments within commodity strata must be selected and sampled, and then individual (representative) commodities must be selected or sampled. Finally, individual transactions that represent the sampled commodities in each sample establishment must be selected. The procedures used for selecting the sample at each stage are important.

1.81 In designing the sample for price collection purposes, due attention should be paid to standard statistical criteria to ensure that the resulting sample estimates are not only unbiased and efficient in a statistical sense, but also cost-effective.[17] There is a large literature on sampling survey techniques to which reference may be made and which need not be summarized here. In principle, it would be desirable to stratify the establishments and commodities by criteria that differentiate them according to their relative price changes, and to further select both establishments and commodities using random sampling with known probabilities of selection. This ensures that the sample of commodities selected is not distorted by subjective factors and enables sampling errors to be calculated. However, many countries continue to rely heavily on the purposive selection of establishments and commodities because random sampling may be too difficult and too costly. Purposive selection is believed to be more cost-effective, especially when the sampling frames available are not comprehensive and not well suited for XMPI purposes. It may also be cost-effective to use "cutoff" sampling procedures, discussed in Chapter 6, which are more objective than purposive sampling. Cutoff sampling first establishes a targeted threshold value, and then all establishments/commodities above this value are selected for the sample. It is a simple means, for example, of selecting the representative four-digit industries within a single-digit category, or commodities within an establishment.

1.82 The representative sampling of establishments and commodities requires comprehensive and up-to-date sampling frames. Two separate frames are usually needed for XMPI purposes, one listing the universe of establishments and the other listing the universe of commodities. Customs documentation can be most helpful in devising such frames because they will contain information on the exporters and importers, the commodities exported and imported (classified under the HS), and the size of the trade by value. Further, if a stratified sample design is used and a region or country of origin/destination is used as a stratification factor along with commodity group, then the sampling frame can be compiled with the necessary information on commodity code and country/region to facilitate a stratified sample design. Stratification with selection of establishments using probabilities that are proportional to size increases the efficiency of the sample estimate. Information other than customs documentation can also be used. Examples of possible sampling frames for establishments are business registers, establishment censuses, and central or local government administrative records, particularly customs records. Sampling frames may be supplemented by information from telephone calls to establishments or price survey visits.

1.83 As noted, depending on the information available in the sampling frame, it may be possible to group the establishments into strata on the basis of region, in addition to commodity group, to form the elementary indices. When there is information about size, a random sample of establishments may be selected with probabilities proportional to size. An example of this approach is presented in Chapter 6. Price relatives from preceding periods may further be used as part of the sample allocation, with larger samples being drawn from trade categories whose variance of price relatives is larger. All of this increases the efficiency of the sample estimate. It would also be possible to use cutoff sampling procedures as a simpler, though less efficient, procedure. Cutoff sampling, unlike random sampling, is open to bias, if the excluded smaller establishments have price changes different from the included larger ones. The extent of the bias depends on the threshold cutoff value and the level of aggregation; some of the bias may be offsetting. Estimates using cutoff sampling, when the bias is believed to be significant, should be complemented with estimates from a sample of smaller units, appropriately weighted.

1.84 In most countries, the selection of the individual commodities to be priced within the selected establishments tends to be purposive, being specified by the central office responsible for the XMPI. The central office draws up lists of commodities that are deemed to be *representative* of the commodities within an elementary aggregate. However, if detailed export or import data by commodity are available from customs documents, these data should be used to select the sample through probability proportional to size or cutoff sampling.

1.85 It has been argued that the purposive selection of commodities is liable to introduce only a negligible amount of sampling bias, but this may be no more than speculation or conjecture. In principle, random sampling is preferable, but it may not be feasible for many countries given the additional costs that may be involved. Procedures for selecting transactions are presented in Chapter 6. At this level many countries consult with an official from the establishment to select the most representative transactions for each commodity. Often selecting those with the largest value of exports/imports does this. Such a procedure is analogous to using cutoff sampling. It is also possible to select a

[17]There are two types of bias encountered in the literature on index numbers: *sampling bias*, as understood here, and the *non-sampling biases* in the form of substitution bias or bias owing to inadequate adjustments for quality change, as discussed in Chapter 12 of the *Manual*. It is usually clear in context which type of bias is meant.

probability sample of transactions if the officials can provide estimates of the relative importance of the transactions. But the largest may have price changes that are unrepresentative of other items and if such other items constitute a relatively large proportion of traded goods, then it may be necessary to increase the sample size of items selected or to select an item whose price changes are representative of all items, even if it is not the largest by traded value. The item selected should also be one that is expected to be traded in most months for a relatively long period, though major seasonal commodities should not be left out of the index on this basis.

1.86 As explained in Chapter 6, the universe of establishments and commodities, from which the sample is taken, has several dimensions. That the universe is changing over time is a major problem not only for XMPIs but also for most other economic statistics. Commodities disappear, to be replaced by other kinds of commodities, and establishments close while new ones open. This creates both conceptual and practical problems, given that the measurement of price changes over time requires some continuity in the commodities priced. The matched-models method requires that the price changes recorded should refer to matched commodities that are identical in both time periods, so that price changes are not tainted by quality changes. But this matching creates a new problem; new commodities and new establishments are not introduced and the sample deteriorates. There are further problems created when commodities are no longer produced or establishments close, and these are considered in some detail in Chapters 8 and 9, and are outlined in Sections H and I below.

F.2 Regular price collection

1.87 The previous section focused on the sampling issues that arise when prices have to be collected for a large number of commodities from a large number of establishments. This section is concerned with some of the operational issues relating to price collection, which are discussed in detail in Chapter 7.

F.2.1 Frequency and timing

1.88 Calculating the XMPIs entails collecting prices from businesses relating to particular commodities and time periods. Decisions must be made about the frequency of collection (monthly or quarterly) and the time period covered for the prices (a single point in time, several times during the month, or a monthly average). Usually price collection is monthly and covers the entire month. However, resource considerations may limit collection to a single point in time.

F.2.2 Commodity specifications

1.89 For each commodity in the sample, a detailed list of the specifications needs to be collected. These specifications are those that are important in identifying and determining the price and quality characteristics of the detailed transaction. Details such as commodity name, serial number, description or features, size, units of measure, class of customer, discounts, and so on should be included. The collection of data on such quality characteristics is important to the matched-models method, but it will be seen from Section H that they can serve as a data source for hedonic regressions, which have a similar function—to price-adjust replacement commodities of different quality.

F.2.3 Price collection methods

1.90 The aim of survey collection techniques is to facilitate the transmission of price data from businesses to the statistical office in a secure and cost-effective manner, while minimizing the administrative burden of the respondent. For some commodities, the prices collected may be *estimated* transaction prices because the transaction sampled did not have exports or imports during the reference period. In addition, it is generally neither practical nor cost-effective to try to collect prices each month or quarter directly from establishments by personal visits. Data can effectively be collected using mail questionnaires, telephone contacts, fax, and electronic media. A range of approaches to XMPI data collection are presented in Chapter 7: postal survey, automated telephone response, personal interview, telephone interview, and Internet data provision. All of these methods rely on good questionnaire design, good respondent relations, and good interviewing techniques. The exact methods chosen by countries for particular industries will depend on the special circumstances applicable to each form of collection in their industry/country.

F.2.4 Continuity of price collection

1.91 The commodities whose prices are collected and compared in successive time periods should ideally be perfectly *matched*—that is, they should be identical in respect of their physical and economic characteristics. Identical economic characteristics include the terms and conditions of sale. When the commodities are perfectly matched, the observed price changes are *pure* price changes. When selecting representative commodities, it is therefore necessary to ensure that enough of them can be expected to remain on the market over a reasonably long period of time in exactly the same form or condition as when first selected. Without continuity, there would not be enough price changes to measure.

1.92 Having identified the commodities whose prices are to be collected, the normal strategy is to ask the respondent to continue pricing exactly those same commodities for as long as possible. The respondents can do this if they are provided with very precise, or tight, specifications of the commodities to be priced. Alternatively, they must keep detailed records themselves of the commodities that they have selected to price.

1.93 The ideal situation for a price index would be one in which all the commodities whose prices are being recorded remain on the market indefinitely without any change in their physical and economic characteristics, except of course for the timing of their sale.[18] Most commodities, however, have only a limited economic life. Eventually, they disappear from the market to be replaced by other commodities. Because the universe of commodities is continually evolving, the representative commodities selected initially may gradually account for a progressively smaller share of output and sales. As a whole, they may become less and less representative. Because XMPIs are intended to represent all internationally traded commodities, some way has to be found to accommodate the changing universe of commodities. In the case of consumer durables whose features and designs are continually being modified, some models may have very short lives indeed, being on the market for only a year or less before being replaced by newer models.

1.94 At some point the continuity of the series of price observations may have to be broken. It may become necessary to compare the prices of some commodities with the prices of other new ones that are very similar, but not identical. Statistical offices must then try to eliminate from the observed price changes the estimated effects of the changes in the characteristics of the commodities whose prices are compared. In other words, they must try to adjust the prices collected for any changes in the quality of the commodities priced, as explained in more detail below. At the extreme, a completely new commodity may appear that is so different from those existing previously that quality adjustment is not feasible, and its price cannot be directly compared with that of any previous commodity. Similarly, a commodity may become so unrepresentative or obsolete that it has to be dropped from the index because it is no longer worth trying to compare its price with those of any of the commodities that have replaced it. Similar issues of course arise for establishments, although the focus here is on commodities.

F.3 Resampling

1.95 One strategy to deal with the changing universe of commodities would be to resample, or reselect, at regular intervals the complete set of commodities to be priced. For example, with a monthly index, a new set of commodities could be selected each January. Each set of commodities would be priced until the following January. Two sets have to be priced each January in order to establish a link between each set of 12 monthly changes. Resampling each year would be consistent with a strategy of updating the weights each year.

1.96 Although resampling may be preferable to maintaining an unchanged sample or selection, it is not used much in practice. Systematically resampling the entire set of commodities each year would be difficult to manage and costly to implement. Moreover, it does not provide a complete solution to the problem of the changing universe of commodities because it does not capture price changes that occur at the moment of time when new commodities or new qualities are first introduced. Many producers deliberately use the time when commodities are first marketed to make significant price changes. A more practical way in which to keep the commodity sample up to date is to rotate it gradually by dropping certain commodities and introducing new ones. Commodities may be dropped for two reasons:

- The commodity is believed by the respondent or central office to be no longer representative. It appears to account for a steadily diminishing share of the total weight within the commodity group or industry in question.
- The commodity may simply disappear from the market altogether. For example, among other reasons, it may have become obsolete owing to changing technology or unfashionable owing to changing tastes.

[18]It is worth noting that many theorems in index number theory are derived on the assumption that exactly the same set of goods and services is available in both the time periods being compared.

1.97 At the same time, new commodities or new qualities of existing commodities appear on the market. At some point, it becomes necessary to include them in the list of commodities priced. This raises the general question of the treatment of quality change and the treatment of new commodities.

G. Transfer Prices

1.98 Before turning to quality change, problems arise in collection when the reported price represents a notional book entry between affiliated units in different economic territories. Because there is incentive to set these prices to avoid taxes on imports (possibly also exports in isolated instances) and income in high tax jurisdictions, transfer prices are presumptively different from the market, "arm's-length," transaction prices desired in XMPIs. A large consultancy business has grown to advise business clients how to set inter-subsidiary prices within applicable tax laws. There are some practical rules of thumb in estimating the arm's-length value of international trade between related units, dealt with in Chapter 19. This is a very large problem in many countries.[19] The problem can only be expected to grow as world economies became more integrated, with increasing volumes of trade in intermediate goods and services.

1.99 When there is an international transaction between, say, two divisions of a multinational enterprise that has establishments in two or more countries, the value of the transaction to the exporting division will be equal to the value of the transaction for the importing division. Thus when the multinational enterprise works out its profits worldwide for the period when the transaction took place, the export value will equal the import value and hence will cancel out, leaving the company's overall profits unchanged, no matter at what price it chose to value the transaction. The price chosen to value the transaction is called a *transfer price*. Hence, at first glance, it appears that the multinational firm could choose the transfer price for the transaction to be practically anything. However, in a world where there are taxes on international transactions and where the rates of business income taxation differ across countries, the situation is actually worse: the multinational will have definite financial incentives to *choose strategically* the transfer price to minimize the amount of taxation paid to both jurisdictions. It is this element of strategic choice that casts doubt on the usefulness of simply collecting transfer prices for enterprises as if they were ordinary prices between unrelated parties.

1.100 Strategically chosen transfer prices will generally be very different from economic transfer prices (based on opportunity costs) that would be suitable for an import or export price index. Because international trade between affiliated units is somewhere in the neighborhood of 30 to 40 percent of world trade, it can be seen that this problem of determining appropriate transfer prices is a serious one.

G.1 Alternative prices

1.101 Chapter 19 discusses the issues related to transfer prices and offers solutions. The best alternative to the firm's listed transfer price is internal comparable prices for the two periods compared; that is, the average price paid to (for an imported commodity) or received from (for an exported commodity) unaffiliated firms for the same commodity during the reference period, if such unaffiliated purchases or sales exist. If there are no such unaffiliated purchases or sales, then the use of externally referenced comparable prices is recommended; that is, the price change of the commodity on a recognized exchange that trades in the commodity if such an exchange exists. If no such exchange exists, then attempting to find an external comparable price change based on transactions between unaffiliated traders is recommended. These three methods all focus on using the price change of the same commodity traded by different firms. Where this is impossible, downstream prices, or potentially upstream prices, can be explored to see whether an economically acceptable price can be found. Finally, if there are no internal or external comparable prices, at the same or different levels of the value chain, the international price index should use the multinational's stated transfer price. XMPIs are used to deflate value aggregates in the national accounts, so it follows that the (transfer) prices used to compile the price index should match those used in the value aggregates. Price index compilers will need to discuss such issues with their national accounts counterparts.

H. Missing Prices and Adjusting Prices for Quality Change

H.1 Evaluation of the effect of quality change on price

1.102 It is useful to try to clarify why one would wish to adjust the observed price change between two commodities that are similar, but not identical, for

[19]Alterman, Diewert, and Eden (2005) report estimates for the United States that the fraction of exports at transfer values is as much as 40 percent, while the fraction of imports is about 30 percent.

differences in their quality. A change in the quality of a good or service occurs when there is a change in some, but not most, of its characteristics. For XMPIs, the concepts of the "worth" of quality differences also depend on the residency orientation taken and whether exports or imports are the focus. For illustration we take the resident's perspective here. As explained in Section B of Chapter 8, the evaluation of the quality change is essentially an estimate of the following:

- The per unit change in export revenue that a resident supplier will receive for the new characteristics possessed by the new quality with the same preferences or using the same technology or both; or
- The per unit change in import cost that a resident user will pay for the new characteristics possessed by the new quality with the same preferences or using the same technology or both.

1.103 This amount is not a price change because it solely represents the difference in monetary value of the characteristics of the new variety relative to the old. The value can be estimated on either the basis of the value to the user of the new quality or the cost to the producer.

1.104 The need for quality adjustments arises in practice for establishment survey data because, when using the matched-models method, a carefully specified model is no longer traded and a replacement is needed to be priced in successive periods. If the replacement is noncomparable—of a different quality—an adjustment has to be made to either the replacement model's price or the original model's price for the effect of the quality difference in order that the price of like is compared with like. A quality adjustment in this instance is seen as an adjustment to the price (or price change) of the original or replacement commodity to remove that part having quality differences. A quality adjustment can be seen as a coefficient that multiplies the price of, say, the replacement commodity to make it commensurate with the price of the original.

1.105 Such explicit quality adjustments require information on the characteristics of the old and replacement model. If price data are unit value indices from customs data there is no information on whether the quality of the items in one month's shipments, within a commodity classification, has changed in the next. The change in unit value may be due to both an actual change in price and a change in the quality composition of the shipments and it is only the former that should be measured, as outlined in Chapter 2. Moreover, the problem is not just one of making the appropriate quality adjustment; it is also one of identifying whether a change in quality has taken place, and customs documentation is inadequate for this purpose. Explicit methods of quality adjustment are outlined in Section H.3 and have no real use for unit value indices. Implicit methods of quality adjustment are used in practice but, as will be argued in Chapter 2, their use is very limited. The focus of the discussion in this section is on the use of quality adjustments for establishment survey data simply because an inadequacy of unit value indices is that there is no basis in the data from customs documentation to make sufficiently reliable adjustments.

1.106 For example, if two units of the replacement commodity are equivalent in quality to three of the original, the required quality adjustment to be applied to the price of the replacement commodity is 2/3. Suppose the price of one unit of the replacement is the same as one unit of the original, then the price of the replacement, after adjusting for the change in quality, is only two-thirds that of the price of the original. If one unit of the replacement sells for twice the price of the original, then the quality-adjusted price is $(2 \times 2/3) = 4/3$ that of the original: the price increase is 33 percent, not 100 percent. The XMPIs seek to record the change between the price of the original and the quality-adjusted price of the replacement.

1.107 Of course, it is difficult to estimate the quality adjustment in practice, but the first step has to be to clarify conceptually the nature of the adjustment that is required in principle. In practice, exporters often treat the introduction of a new quality, or new model, as a convenient opportunity in which to make a significant price change. They may deliberately make it difficult for purchasers to disentangle how much of the observed difference in price between the old and the new qualities represents a price change.

1.108 For XPIs, an explicit quality adjustment may be possible using differences in the costs of production between the two qualities. This approach works as long as the assumption is valid that production costs are based on the establishment using the same technology and charging the same margins. Similar principles may be applied to MPIs but the reporting unit's information on the production costs of the overseas exporter may be limited, though the overseas exporter's website may have information on the pricing of models with different options or quality characteristics. Another alternative is to make an implicit adjustment by making an assumption about the pure price change—for example, on the basis of price movements observed for other commodities. The discussion below examines the implicit methods first and then the explicit methods. These approaches are examined in detail in Sections D and E of Chapter 8.

1.109 When the technology changes, there is no comparable basis for comparing costs between the two qualities, and such procedures break down. An alternative approach would be to use hedonic regression techniques, which are also discussed below and in more detail in Section G of Chapter 8.

H.2 Implicit methods

H.2.1 Overlapping qualities

1.110 Suppose that the two qualities overlap, both being produced at time *t*. If both are produced and sold in a competitive market, economic theory suggests that the ratio of the prices of the new to the old quality should reflect their relative cost to producers and value to purchasers. This implies that the difference in price between the old and the new qualities should not indicate any change in price. The price changes up to period *t* can be measured by the prices for the old quality, while the price changes from period *t* onward can be measured by the prices for the new quality. The two series of price changes are linked in period *t*; the differences in price between the two qualities have no impact on the linked series.

1.111 When there is an overlap, simple linking of this kind may provide an acceptable solution to the problem of dealing with quality change. Respondents for an XPI may well have such information if the two models are both produced at the same time. However, the conditions may not be consistent with those assumed in the theory. Even when there is an overlap, the market may not have had time to adjust, particularly when there is a substantial change in quality. When the new quality first appears, the market is liable to remain in disequilibrium for some time. The producers of new qualities may price strategically over the commodity life cycle to, for example, price-discriminate in the early periods following introduction. Obsolete models may be priced very low; they may be dumped to clear the market. There is a case in which the overlap method is used extensively in spite of these difficulties: when the index is rebased or commodities are rotated. The advantage of refreshing the sample is deemed to outweigh such disadvantages.

1.112 There may be a succession of periods in which the two qualities overlap before the old quality finally disappears from the market. If the market is temporarily out of equilibrium, the relative prices of the two qualities may change significantly over time, so that the market offers alternative evaluations of the relative qualities depending on which period is chosen. In general, if the price series for the old and new qualities are linked in a single period, the choice of period can have a substantial effect on the overall change in the linked series.

1.113 The statistician has then to make a deliberate judgment about the period in which the relative prices appear to give the best representation of the relative qualities. In this situation, it may be preferable to use a more complex linking procedure that uses the prices for both the new and the old qualities in several periods in which they overlap. Such information may be available from the respondent's records, although this requires a good relationship with the respondent and good record keeping and retrieval systems by the respondent. In this case, the timing of the switch from the old to the new can have a significant effect on the long-term change in the linked series. This factor must be explicitly recognized and taken into consideration.

1.114 If there is no overlap between the new and the old qualities, the problems just discussed do not arise because no choice has to be made about when to make the link. However, other and more difficult problems take their place.

H.2.2 Nonoverlapping qualities

1.115 In the following sections, it is assumed that the overlap method cannot be used because there is a discontinuity between the series of price observations for the old and new qualities. Adopt the notation that the actual price of the new quality is P_t in period *t* and the price of the old quality is p_{t-1} in the previous period. Because the old quality is not available in period *t*, an imputation is made for its price in period *t* (p^*_t). In order to make the comparison between the prices in periods *t*–1 and *t*, a comparison between commodities of equal quality in the eyes of the producer is needed. The ratio p^*_t/P_t is the required quality adjustment because this ratio provides the estimate of the quality differences at the same point in time. Using lowercase *p*'s for the old quality and upper case *P*'s for the new, it is assumed that the price data available to the index compiler take the following form:

$$\ldots, \quad p_{t-3}, \quad p_{t-2}, \quad p_{t-1}, \quad P_t, \quad P_{t+1}, \quad P_{t+2}, \ldots$$

1.116 The problem is to estimate the pure price change between t–1 and t in order to have a continuous series of price observations for inclusion in the index. Using the same notation as above,

- Price changes up to period t–1 are measured by the series for the old quality;
- The change between t–1 and t is measured by the ratio p^*_t/p_{t-1} where p^*_t is equal to P_t *after* adjustment for the change in quality; and
- Price changes from period t onward are measured by the series for the new quality.

1.117 The problem is to estimate p^*_t. This may be done explicitly by one of the methods described later. Otherwise, one of the implicit methods has to be used. These may be grouped into three categories.

- The first solution is to assume that $p^*_t/p_{t-1} = P_t/p_{t-1}$ or $p^*_t = P_t$. No change in quality is assumed to have occurred so that the whole of the observed price increase is treated as a pure price increase. In effect, this contradicts the assumption that there has been a change in quality. The noncomparable replacement is deemed comparable.
- The second is to assume that $p^*_t/p_{t-1} = 1$, or $p^*_t = p_{t-1}$. No price change is assumed to have occurred, the whole of the observed difference between p_{t-1} and P_t being attributed to the difference in their quality.
- The third is to assume that $p^*_t/p_{t-1} = I$, where I is an index of the price change for a group of similar commodities, or possibly a more general price index.

1.118 The first two possibilities cannot be recommended as default options to be used automatically in the absence of any adequate information. The use of the first option could only be justified if the evidence suggests that the extent of the quality change is negligible, even though it cannot be quantified more precisely. "Doing nothing"—that is, ignoring the quality change completely—is equivalent to adopting the unsatisfactory first solution. There is a very real sense in which unit value indices implicitly treat changes in the quality mix of shipments in this manner. Conversely, the second could only be justified if evidence suggests that the extent of any price change between the two periods is negligible. The third option is likely to be much more acceptable than the other two. It is the kind of solution that is often used in economic statistics when data are missing.

1.119 Elementary indices are typically based on a number of series relating to different sampled commodities. The particular linked price series relating to the two qualities is therefore usually just one out of a number of parallel price series. What may happen in practice is that the price observations for the old quality are used up to period t–1 and the prices for the new quality from t onward, the price change between t–1 and t being omitted from the calculations. In effect, this amounts to using the third option: that is, estimating the missing price change on the assumption that it is equal to the average change for the other sampled commodities within the elementary aggregate.

1.120 It may be possible to improve on this estimate by making a careful selection of the other sampled commodities to include only those whose average price change is believed to be more similar to the commodity in question than the average for the group of sampled commodities as a whole. This procedure is described in some detail in Section D.2 of Chapter 8 where it is illustrated with a numerical example and is described as "targeting" the imputation or estimation.

1.121 The general method of estimating the price on the basis of the average change for the remaining group of commodities is widely used. It is sometimes described as the "overall" mean method. The more refined, targeted version is the "targeted" or "class" mean method. In general, one or the other method seems likely to be preferable to either of the first two options listed above, although each case must be considered on its individual merits.

1.122 Although the overall mean method superficially seems a sensible practical solution, it may nevertheless give biased results, as explained in Chapter 8. It needs to be repeated that the introduction of a new quality is precisely the occasion on which a producer may choose to make a significant price change. Many of the most important price changes may be missed if, in effect, they are assumed to be equal to the average for commodities not subject to quality change.

1.123 Of particular note is the usefulness of short-run comparisons as opposed to long-run ones. For example, a price comparison for an item exported by an establishment between a price reference month of, say, January and February is the simple price relative. For January compared with March it is the result of the previous calculation multiplied by the price relative of February to March. For January compared with April it is the result of the previous calculation, January compared with March, multiplied by the price relative of March to April. The result from the previous month is taken and multiplied by the current month's price relative to that of the preceding one—two-stage short-run comparison. This is at the elementary level before weights are applied. At first glance it may seem that this will yield the same result as a direct long-term comparison for January to April, and so it will if the same item's prices are consistently monitored over time. But, say, in April an old model is no longer available and a new noncomparable replacement is found with price data for March and April. The new model can be easily linked into the series using the overlap method. Further, if the old model was temporarily missing in April, an imputation could be made using the short-run price changes of the commodity group rather than the long-run ones—a more reasonable implicit assumption. Monitoring month-to-month price changes is also useful for data validation checks.

1.124 Of note is that the use of imputed values for overlaps may have a particularly significant shortcoming when used for missing prices that reappear, as is the case for seasonal goods and services. Say the price is 4 at the end of a season and 9 at the start of the new season for the same item, and imputations are used in the intervening months based on the price changes of similar items. The imputed price in the month before the start of the new season may be 8, and this price can be compared with the price of 9 to bring the imputed index to its correct value at the start of the new season. Such a feature is often referred to as "self-correcting," in that the imputed values are brought back to the directly observed price. Were an imputed price and overlap used at the start of the new season, the index would not self-correct. This shortcoming assumes that the item at the start of the season is not different in quality to that at the end, which may be argued for some fruit and vegetables, for example.

1.125 More generally, at least when a significant quality change is believed to have occurred, it is necessary to try to make *explicit* adjustments for the change in quality. Again there are several methods that may be used.

H.3 Explicit quality adjustments

H.3.1 Quantity adjustments

1.126 The quality change may take the form of a change in the physical characteristics of the commodity that can easily be quantified, such as change in weight, dimensions, purity, or chemical composition of a commodity. It is generally a considerable oversimplification to assume that the quality of a commodity changes in proportion to the size of some single physical characteristic. For example, it is very unlikely to rate a refrigerator that has three times the capacity of a smaller one as worth three times the price of the latter. Nevertheless it is clearly possible to make some adjustment to the price of a new quality of different size to make it more comparable with the price of an old quality. There is considerable scope for the judicious, or commonsense, application of relatively straightforward quality adjustments of this kind. A discussion of quality adjustments based on size is given in Section E.2 of Chapter 8.

H.3.2 Differences in production/ option costs

1.127 An alternative procedure may be to try to measure the change in quality by the estimated change in the costs of producing the two qualities. The method is explained in Section E.3 of Chapter 8. The estimates can be made in consultation with the producers of the goods or services, if appropriate. This method, like the preceding one, is likely to be satisfactory only when the quality changes take the form of relatively simple changes in the physical characteristics of the good, such as the addition of some new feature, or option, to an automobile. It is not satisfactory when a more fundamental change in the nature of the commodity occurs as a result of a new discovery or technological innovation. It is clearly quite unacceptable, for example, when a drug is replaced by another more effective variant of the same drug that also happens to cost less to produce. It is also more difficult to apply to MPIs for which the reporting unit's information on the production costs of the overseas exporter may be limited.

1.128 Another possibility when the quality change is more complex or subtle is to seek the advice of technical experts, especially when the respondent may not have the knowledge or expertise to be able to assess or evaluate the significance of all of the changes that may have occurred, at least when they are first made.

H.3.3 Hedonic approach

1.129 Finally, it may be possible to systematize the production/option cost approach by utilizing econometric methods to estimate the impact of observed changes in the characteristics of a commodity on its price. The market prices of a set of different qualities or models are regressed on what are considered to be the most important physical or economic characteristics of the different models. This approach to the evaluation of quality change is known as *hedonic analysis*. When the characteristics are attributes that cannot be quantified, they may be represented by dummy variables (for example, 1 if the characteristic is available on this model or 0 if not). The regression coefficients measure the estimated marginal effects of the various characteristics on the prices of the models and can therefore be used to estimate the effects on price of changes in those characteristics.

1.130 The hedonic approach to quality adjustment can provide a powerful, objective, and scientific method of estimating the effect on price of changes in quality for certain kinds of commodities. It has been particularly successful in dealing with computers. The economic theory underlying the hedonic approach is examined in more detail in Chapter 22. The application of the method is explained in some detail in Section E.4 of Chapter 8. Commodities can be viewed as bundles of tied characteristics that are not individually priced because the producer sells the bundle as a single package. The objective is to try to “unbundle” the characteristics to estimate how much they contribute to the total price. In the case of computers, for example, three basic characteristics are the processor speed, the size of the random access memory (RAM), and the hard drive capacity. An example of a hedonic regression using these and other characteristics is given in Section E.4 of Chapter 8, the actual numerical results being given in Table 8.3.

1.131 The results obtained by applying hedonics to computer prices have had a considerable impact on attitudes toward the treatment of quality change in price index number measurement, though more so in consumer price index numbers. They have demonstrated that for goods with rapid technological change and improvements in quality, the size of the adjustments made to the market prices of the commodities to offset the changes in the quality can largely determine the movements of the elementary price index. For this reason, the *Manual* contains a thorough treatment of the use of hedonics. Reference may be made to Section G of Chapter 8 for further analysis—including a comparison showing that the results obtained by using hedonics and matched models can differ significantly when there is a high model turnover—and Chapter 22.

H.3.4 Conclusions on quality change

1.132 It may be concluded that statistical offices must pay close attention to the treatment of quality change and try to make explicit adjustments whenever possible. The importance of this topic can scarcely be overemphasized. Failure to pay proper attention to quality changes can introduce serious biases into XMPIs.

H.4 Seasonal commodities

1.133 As explained in Chapter 23, the existence of seasonal commodities poses some intractable problems and serious challenges for XMPI compilers and users. Seasonal commodities are commodities that are either

- Not available during certain seasons of the year, or
- Are available throughout the year, but their prices or quantities are subject to regular fluctuations that are synchronized with the season or time of the year.

1.134 There are two main sources of seasonal fluctuations: the climate and custom. Month-to-month movements in XMPIs may sometimes be so dominated by seasonal influences that it is difficult to discern the underlying trends in prices. Conventional seasonal adjustment programs may be applied, but these may not always be satisfactory. However, the problem is not confined to interpreting movements in the XMPIs; seasonality creates serious problems for the compilation of XMPIs when some of the commodities in the basket regularly disappear and reappear, thereby breaking the continuity of the price series from which the XPI and MPI are built up. There is no panacea for seasonality. A consensus on what is best practice in this area has not yet been formed. Chapter 23 examines a number of different ways in which the problems may be tackled using an artificial data set to illustrate the consequences of using different methods.

1.135 One possibility is to impute the price changes for commodities when they are out of season to be the same as those in their commodity group that are in season and those that continue to exist all year round. When the commodity comes back into season the last imputed price is linked to the new in-season price. An alternative imputation procedure is to carry forward the last price, but this induces undue stability into the index and significant catch-up changes when linked to the new in-season prices. Imputation by reference to the price changes of other commodities is preferred to carrying forward prices, especially for high-inflation countries. Simply excluding all seasonal commodities from the index in all periods may be an unacceptable reduction in the scope of the index, because seasonal commodities can account for a significant proportion of total trade (e.g., agricultural imports of perishables such as fruit and vegetables). Assuming seasonal commodities are retained, one solution is to switch the focus from month-to-month movements in the index to changes between the same month in successive years. In some countries, it is common for the media and other users, such as central banks, to focus on the annual rate of inflation between the most recent month and the same month in the previous year. This year-over-year figure is much easier to interpret than month-to-month changes, which can be somewhat volatile, even in the absence of seasonal fluctuations.

1.136 This approach is extended in Chapter 23 to the concept of a rolling year-on-year index that compares the prices for the most recent 12 months with the corresponding months in the price reference year. The resulting *rolling-year indices* can be regarded as seasonally adjusted price indices. They are shown to work well using the artificial data set. Such an index can be regarded as a measure of inflation for a year that is centered around a month that is six months earlier than the last month in the rolling index. For some purposes, this time lag may be disadvantageous, but in Section F of Chapter 23 it is shown that under certain conditions the current month's year-over-year monthly index, together with the previous month's year-over-year monthly index, can successfully predict the rolling-year index that is centered on the current month. Of course, rolling-year indices and similar analytic constructs are not intended to replace the monthly or quarterly XMPIs but to provide supplementary information that can be extremely useful to users. They can be published alongside the official XMPIs.

I. Commodity Substitution and New Goods

I.1 Replacement commodities

1.137 As noted in the previous section, price indices would, ideally, seek to measure pure price changes between matched commodities that are identical in the two periods compared. However, as explained in Chapter 9, the universe of commodities that XMPIs have to cover is a dynamic universe that is gradually changing over time. Pricing matched commodities constrains the selection of commodities to a static universe of commodities given by the intersection of the two sets of commodities existing in the two periods compared. This static universe by definition excludes both new commodities and disappearing commodities, and in both cases their price behavior is likely to diverge from that of the matched commodities. Price indices have to try to take account of the price behavior of new and disappearing commodities so far as possible.

1.138 A formal consideration and analysis of these problems is given in Appendix 9.1 in Chapter 9. A replacement universe is defined as one that starts with the base period universe but allows new commodities to enter as replacements as some commodities disappear. Of course, quality adjustments of the kind discussed in the previous section are needed when comparing the prices of the replacement commodities with those of the commodities that they replace.

1.139 One way in which to address the underlying problem of the changing universe is by sample rotation. This requires a completely new sample of commodities or establishments to be drawn to replace the existing ones. The two samples must overlap in one period that acts as the link period. As noted in Section B.2 of Chapter 9, this procedure can be viewed as a systematic exploitation of the overlap method of adjusting for quality change. It may not, therefore, deal satisfactorily with all changes in quality that occur, because the relative prices of different goods and services at a single point of time may not provide satisfactory measures of the relative qualities of all the goods and services concerned. Nevertheless, frequent sample rotation helps by keeping the sample up to date and may reduce the extent to which explicit quality adjustments are required. Sample rotation is, however, resource intensive.

I.2 New goods and services

1.140 The difference in quality between the original commodity and the one that replaces it may become so great that the new quality is better treated as a new good, although the distinction between a new quality and a new good is inevitably somewhat arbitrary. A distinction is drawn in Section D of Chapter 9 between evolutionary and revolutionary new goods. An evolutionary new good or service is one that meets existing needs in much more efficient, or new, ways; a revolutionary new good or service provides completely new kinds of services or benefits. In practice, an evolutionary new good can be fitted into some subclass of the commodity or industry classification, whereas a revolutionary new good will require some modification to the classification in order to accommodate it.

1.141 As explained in Section D.2 of Chapter 9, a major concern with new goods or services relates to the timing of the introduction of the new commodity into the index. It is often the case that new goods enter the market at a higher price than can be sustained in the longer term, so that their prices typically tend to fall over the course of time. Conversely, the quantities sold may be very small initially but may increase significantly over time. These complications make the treatment of new commodities particularly difficult, especially if they are revolutionary new goods. Because of the tendency for the price of a new good to fall after it has been introduced, it is possible that important price increases in the period of introduction may fail to be captured by XMPIs because of the technical difficulties created by new commodities. The issues are examined in some detail in Section D of Chapter 9. The chapter concludes by expressing concern about the capacity of XMPIs to deal satisfactorily with the dynamics of modern markets. In any case, it is essential that statistical offices be alert to these issues and adopt procedures that take account of them to the maximum extent possible, given the data and resources available.

J. Basic Index Number Formulas and the Axiomatic and Economic Approaches to XMPIs

J.1 Basic index number formulas

1.142 Although the collection of monthly or quarterly data on unit value or price changes at a detailed level is a natural prerequisite for the compilation of XMPIs, as is the collection of data on relative value shares to weight the price changes, an important question to decide on is the kind of index number formula to use when aggregating the data collected. Index number formulas that involve weights are referred to as being used at the *higher* level of aggregation. Thus the subject matter of such weighted index number formulas discussed in this section applies to XMPIs whether compiled using unit value indices derived from customs data at the lower (elementary) level or the price changes of well-defined representative goods and services from establishment surveys at the lower level. Both provide elementary indices at the detailed commodity classification and are aggregated at the higher level using an index number formula that is weighted. The extensive list of references given at the end of this *Manual* reflects the large literature on this subject. Many different mathematical formulas have been proposed over the past two centuries. Nevertheless, there is now a broad consensus among economists and compilers of XMPIs about what is the most appropriate type of formula to use, at least in principle. Although the consensus has not settled for a single formula, it has narrowed to a very small class of *superlative* indices. A characteristic feature of these indices is that they treat the prices and quantities in both periods being compared symmetrically. They tend to yield very similar results and behave in very similar ways.

1.143 However, in some cases, there may not be sufficient information on the quantities and nominal flows (i.e., the weights) of internationally traded goods and services in the current period to calculate a symmetric, or superlative, index. It may be necessary to resort to second-best alternatives in practice, but in order to be able to make a rational choice between the various possibilities, it also is necessary to have a clear idea of the target index that would be preferred, in principle. The target index can have considerable influence on practical matters such as the frequency with which the weights used in the index should be updated.

1.144 The *Manual* provides a comprehensive, rigorous, and up-to-date discussion of relevant index number theory. Several chapters from Chapter 16 onward are devoted to detailed explanations of index number theory from both a statistical and an economic perspective. The main points are summarized in the following sections. Many propositions or theorems are stated without proof in this chapter because the proofs are given or referenced in later chapters to which the reader can easily refer in order to obtain full explanations and a deeper understanding of the points made. There are numerous cross-references to the relevant sections in later chapters.

J.1.1 Price indices based on baskets of goods and services

1.145 The purpose of an index number may be explained from the resident's perspective by comparing the *values* of exports or imports of goods and services in two time periods. For example, knowing that the value of exports has increased by, say, 5 percent is not very informative if we do not know how much of this change is due to changes in the *prices* of the goods and services and how much to changes in the *quantities* produced. *The purpose of an index number is to decompose proportionate or percentage changes in value aggregates into their overall price and quantity* *change components.* XMPIs are intended to measure the price component of the change. One way to do this is to measure the change in the value of an aggregate by holding the quantities constant.

J.1.2 Lowe indices

1.146 One very wide, and popular, class of price indices is obtained by defining the index as the percentage change between the periods compared in the total cost of producing a fixed set of quantities, generally described as a "basket." The meaning of such an index is easy to grasp and to explain to users. This class of index is called a *Lowe* index in this *Manual*, after the index number pioneer who first proposed it in 1823 (see Section B.2 of Chapter 16). Most statistical offices make use of some kind of Lowe index in practice. It is described in some detail in Sections D.1 and D.2 of Chapter 16.

1.147 In principle, any set of goods and services could serve as the basket. The basket does *not* have to be restricted to the basket actually produced or used in one or other of the two periods compared. For practical reasons, the basket of quantities used for XMPI purposes usually has to be based on data from an earlier period than either of the two periods whose prices are compared, simply because it takes time to compile and adopt the data. For example, a monthly XMPI may run from January 2008 onward, with January 2008 = 100 as its price reference period, but the quantities may be derived from customs or establishment-survey value data from an earlier period. The basket also may refer to a year or average of more than one year, whereas the price reference period for the index may be a year, month, or quarter.

1.148 Let there be n commodities in the basket with prices p_i and quantities q_i. Let period b be the period to which the quantities refer and periods 0 and t be the two periods whose prices are being compared. In practice, it is invariably the case that $b \leq 0 < t$ when the index is first published, and this is assumed here. The Lowe index is defined in equation (1.3).

$$P_{Lo} \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \equiv \sum_{i=1}^{n} (p_i^t / p_t^0) s_i^{0b} \text{ where}$$

$$s_i^{0b} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}. \qquad (1.3)$$

1.149 The Lowe index can be written, and calculated, in two ways: either as the ratio of two value aggregates, or as an arithmetic weighted average of the price ratios, or *price relatives*, p_i^t/p_i^0, for the individual commodities using the hybrid value shares s_i^{0b} as weights. The price relatives may in fact be proxied by unit value indices and, hereafter, the term price relatives refers to both possibilities. They are described as *hybrid weights* because the prices and quantities belong to two different time periods, 0 and b, respectively. The hybrid weights may be obtained by updating the actual value shares in period b, namely $p_i^b q_i^b / \Sigma\, p_i^b q_i^b$, by multiplying them by the price relative between b and 0, namely p_i^0/p_i^b. The concept of the *base period* is somewhat ambiguous with a Lowe index, because either b or 0 might be interpreted as being the base period. To avoid ambiguity, b is described as the *weight reference period* and 0 as the *price reference period*. Lowe indices are widely used in XMPIs, though they are sometimes described as Laspeyres indices. The latter description only strictly holds if the *weight reference period* and the *price reference period* are the same.

J.1.3 Laspeyres and Paasche indices

1.150 Any set of quantities could be used in a Lowe index, but there are two special cases that figure prominently in the literature and are of considerable theoretical importance. When the quantities are those of the first of the two periods whose prices are being compared—that is, when $b = 0$—the Lowe is equivalent to the *Laspeyres* index. When quantities are those of the second period—that is, when $b = t$—the Lowe is equivalent to the *Paasche* index. It is useful to consider the properties of Laspeyres and Paasche indices, and also the relationships between them, in more detail.

1.151 Equation (1.4) is the formula for the Laspeyres price index, P_L:

$$P_L = \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \equiv \sum_{i=1}^{n} (p_i^t / p_t^0) s_i^0, \qquad (1.4)$$

where s_i^0 denotes the share of the value of commodity i traded (as exports for an XPI or imports for an MPI) in period 0, that is, $p_i^0 q_i^0 / \sum p_i^0 q_i^0$.

1.152 Equation (1.4) (explained in more detail in Chapter 16) shows the Laspeyres index can be expressed in two alternative ways that are algebraically identical.

The first is the ratio of the values of the basket of producer goods and services traded in period 0 when valued at the prices of periods t and 0, respectively. The second is a weighted arithmetic average of the ratios of the individual prices in periods t and 0 using the traded value shares in period 0 as weights. The individual price ratios, (p_i^t/p_i^0), are described as price *relatives,* though as noted earlier, they may be proxied by unit value indices. Statistical offices often calculate XMPIs using the second formula by recording the changes in the prices of goods and services exported and imported and weighting them by the traded value shares in period 0 or, more often, some earlier period.

1.153 A Young index is similar to the right-hand-side Laspeyres weighted average of price changes given in equation (1.4), but instead of using period 0 weights, it uses earlier period b traded value shares as weights owing to the lack of timely information on the former.

1.154 Equation (1.5) is the formula for the Paasche index, P_P:

$$P_p = \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} \equiv \left\{ \sum_{i=1}^{n} \left(p_i^t/p_i^0\right)^{-1} s_i^t \right\}^{-1}, \tag{1.5}$$

where s_i^t denotes the actual share of traded values of commodity i in period t, that is, $p_i^t q_i^t / \sum p_i^t q_i^t$. The Paasche index can also be expressed in two alternative ways: either as the ratio of two value aggregates or as a weighted average of the price relatives, the average being a *harmonic* average that uses the traded value shares of the later period t as weights.

1.155 If the objective is simply to measure the price change between the two periods considered in isolation, there is no reason to prefer the basket of the earlier period to that of the later period, or vice versa. Both baskets are equally relevant. Both indices are equally justifiable, or acceptable, from a conceptual point of view. In practice, however, XMPIs are calculated for a succession of time periods. Time series of monthly Laspeyres XMPIs based on period 0 benefit from requiring only a single set of nominal trade weights, those of period 0, so that *only the prices* have to be collected on a regular monthly basis. A time series of Paasche XMPIs, on the other hand, requires data on *both* prices *and* quantities (or nominal trade weights) in each successive period. Thus, it is much less costly, and time-consuming, to calculate a time series of Laspeyres indices than a time series of Paasche indices. If detailed data on nominal trade flows are not timely, this is a *decisive practical* advantage of Laspeyres (as well as Lowe) indices over Paasche indices and explains why Laspeyres, Young, and Lowe indices are used much more extensively than Paasche indices. Monthly Laspeyres, Young, or Lowe XMPIs can be published as soon as the price information has been collected and processed, because the base period weights are already available. Of course, weights should be updated regularly, and Laspeyres indices with frequently updated weights, or annual chained Laspeyres indices, are a preferred option to fixed weight ones, as discussed below.

J.1.4 Decomposing current value changes using Laspeyres and Paasche

1.156 Laspeyres and Paasche volume indices are defined in a similar way to the price indices, simply by interchanging the ps and qs in equations (1.4) and (1.5). They summarize changes over time in the flow of quantities of goods and services exported or imported. A Laspeyres volume index values the quantities at the fixed prices of the earlier period, while the Paasche volume index uses the prices of the later period. The ratio of the nominal (current price) traded values in two periods (V) reflects the combined effects of both price and quantity changes. When Laspeyres and Paasche indices are used, the value change exactly decomposes into a price index *times* a volume index only if the Laspeyres price (volume) index is matched with the Paasche volume (price) index. Let P_L and Q_L denote the Laspeyres price and volume indices and let P_P and Q_P denote the Paasche price and volume indices. As shown in Chapter 16, $P_L \times Q_P \equiv V$ and $P_P \times Q_L \equiv V$. It follows that volume series can be defined as V/P_L or V/P_P. This division of a value change by price index to form a volume index is referred to as deflation. If a Paasche deflator is used for comparisons between, say, period 0 and successive periods, this yields, on deflation, a Laspeyres volume series that measures quantities at constant period 0 prices.

1.157 Suppose, for example, compilers want to deflate a time series of imports in the national accounts to measure changes in import volume supplied to the economy at constant prices over time. To generate a series of import values at constant reference period prices (whose movements are identical with those of the Laspeyres volume index), the imports at nominal current prices must be deflated by a series of Paasche price indices. Laspeyres MPIs would not be appropriate for the purpose. If nominal values are available and deflated at a very low level of disaggregation, the resulting volume series for the detailed trade commodities can then be aggregated up using, say, a (chained) Lowe or Laspeyres formula.

J.1.5 Ratios of Lowe and Laspeyres indices

1.158 The Lowe index is transitive. The ratio of two Lowe indices using the same set of q^bs is also a Lowe index. For example, the ratio of the Lowe index for period $t+1$ with price reference period 0 divided by that for period t also with price reference period 0 is

$$\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b \Big/ \sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^{t} q_i^b \Big/ \sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^{t} q_i^b} = P_{Lo}^{t,t+1}. \qquad (1.6)$$

1.159 This is a Lowe index for period $t+1$, with period t as the price reference period. This kind of index is, in fact, widely used to measure short-term price movements, such as between t and $t+1$, even though the quantities may date back to some much earlier period b.

1.160 A Lowe index can also be expressed as the ratio of two Laspeyres indices. For example, the Lowe index for period t with price reference period 0 is equal to the Laspeyres index for period t with price reference period b divided by the Laspeyres index for period 0 also with price reference period b. Thus,

$$P_{Lo} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b} = \frac{P_L^t}{P_L^0}. \qquad (1.7)$$

J.1.6 Lowe indices as indices with price-updated weights

1.161 It is useful to have a formula that enables a Lowe index to be calculated directly as a chain index in which the index for period $t+1$ is obtained by updating the index for period t. Because Lowe indices are transitive, the Lowe index for period $t+1$ with price reference period 0 can be written as the product of the Lowe index for period t with price reference period 0 multiplied the Lowe index for period $t+1$ with price reference period t. Thus,

$$\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}\right]\left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}\right]$$

$$= \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}\right]\left[\sum_{i=1}^{n} \left(\frac{p_i^{t+1}}{p_i^t}\right) s_i^{tb}\right], \qquad (1.8)$$

where the traded value share weights s_i^{tb} are hybrid weights defined as

$$s_i^{tb} \equiv p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^t q_i^b. \qquad (1.9)$$

1.162 Hybrid weights of the kind defined in equation (1.9) are often described as *price updated* weights. They can be obtained by adjusting the original weights $p_i^b q_i^b / \Sigma p_i^b q_i^b$ by the price relatives p_i^t / p_i^b. By price updating the weights from b to t in this way, the index between t and $t+1$ can be calculated directly as a weighted average of the price relatives p_i^{t+1}/p_i^t without referring back to price reference period 0. The index can then be linked to the value of the index in the preceding period t.

J.1.7 Interrelationships between fixed-basket indices

1.163 Consider first the interrelationship between the Laspeyres and the Paasche indices. A well-known result in index number theory is that if the price and quantity changes (weighted by reference period values) are *negatively* correlated, then the Laspeyres index exceeds the Paasche. Conversely, if the weighted price and quantity changes are *positively* correlated, then the Paasche index exceeds the Laspeyres. The proof is given in Appendix 16.1 of Chapter 16.

1.164 This has different implications for purchasers and suppliers. The theory of purchasing behavior indicates that, as *users* of goods and services, producers and consumers typically react to price changes by substituting goods or services that have become *relatively* cheaper for those that have become *relatively* dearer. Thus they purchase smaller quantities of the higher-priced commodities and more of lower-priced ones. This is known as the *substitution effect*, and it implies a negative correlation between the price and quantity relatives. In this case the Laspeyres would be greater than the Paasche index with the gap between them tending to widen over time.[20] That the Laspeyres tends to rise faster than the Paasche is a matter of concern to many analysts and users because it suggests that the widely used Laspeyres index may have an upward bias because it ignores such substitution effects. The extent of substitution bias will vary across commodities with the purchasers' ability to substitute between inputs being limited by technical fixed constraints and the consumers' ability by preferences.

1.165 The theory of the firm indicates the opposite behavior on the part of suppliers of goods and services. As prices for particular commodities begin to rise, suppliers will shift production away from lower-priced, less profitable commodities toward the higher-priced more profitable ones. This type of substitution by producers implies a positive correlation between price and quantity relatives. In this case, the Paasche would be greater than the Laspeyres with the gap between them widening over time. That the Paasche tends to rise faster than the Laspeyres is a matter of concern to many analysts because it suggests that the widely used Laspeyres index may have a downward bias, a point taken up later. As described in Chapter 18 and in Section J.4 below, the nature of the expected substitution bias for XMPIs depends on the behavioral assumptions about the economic agents concerned—whether they substitute toward or away from commodities with above-average price increases—which in turn depends on whether a resident's or nonresident's approach is taken, which, again, in turn depends on the purpose of the measure, as outlined in Section C above.

1.166 In practice, however, statistical offices often do not calculate Laspeyres or Paasche indices but instead calculate Lowe indices as defined in equation (1.3). The question then arises of how the Lowe index relates to the Laspeyres and Paasche indices. It is shown in Section D.1 of Chapter 16 that *if there are persistent long-term trends in relative prices and if the substitution effect for purchasers is dominant, the Lowe index will tend to exceed the Laspeyres, and therefore also the Paasche.* Assuming that the time period b is prior to the time period 0, the ranking under these conditions will be

$$\text{Lowe} \geq \text{Laspeyres} \geq \text{Paasche}.$$

Moreover, the amount by which the Lowe exceeds the other two indices will tend to increase the further back in time period b is in relation to period *0*.

1.167 The positioning of period b is crucial. Given the assumptions about long-term price trends and substitution, a Lowe index will tend to increase (decrease) as period b is moved backward (forward) in time. Whereas b may have to precede 0 when the index is first published, there is no such restriction on the positioning of b as price and quantity data become available for later periods with the passage of time. Period b can then be moved forward. If b is positioned midway between 0 and t, the quantities are likely to be equally representative of both periods, assuming that there is a fairly smooth transition from the relative quantities of 0 to those of t. In these circumstances, the Lowe index is likely to be close to the Fisher and other superlative indices and cannot be presumed to have either an upward or a downward bias. These points are elaborated further below and also in Section D.2 of Chapter 16.

1.168 It is important that statistical offices take these relationships into consideration in deciding upon their policies. There are obviously practical advantages and financial savings from continuing to make repeated use over many years of the same fixed set of quantities to calculate XMPIs. However, the amount by which such XMPIs depart from some conceptually preferred target index, such as the economic index discussed in Section J.4 below, is likely to get steadily larger the further back in time is the period b to which the quantities refer. Large biases may undermine the credibility and acceptability of the indices.

J.1.8 Young index

1.169 Instead of holding constant the quantities of period b, a statistical office may calculate a set of XMPIs as a weighted arithmetic average of the individual price relatives, holding constant the traded value shares of period b. The resulting index is called a *Young* index in this *Manual*, again after another index number pioneer. The Young index is defined in Section D.3 of Chapter 16 as follows:

$$P_{Yo} \equiv \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0}\right) \text{ where } s_i^b \equiv \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}. \tag{1.10}$$

1.170 In the corresponding Lowe index, equation (1.3), the weights are hybrid trade value shares that value the quantities of period b at the prices of 0. As already explained, the price reference period 0 usually is more

[20] If the traded value shares—that is, the weights associated with the price relatives—happen to be the same in both periods, the Laspeyres must be greater than the Paasche because a weighted arithmetic average is always greater than a harmonic average with the same weights. In order to maintain the traded value shares intact, the substitution of the quantities in response to changes in relative prices must be perfect.

current than the weight reference period b because of the time needed to collect and process the trade data. In that case, a statistical office has the choice of assuming that *either* the quantities of period b remain constant *or* the trade value shares in period b remain constant. Both cannot remain constant if prices change between b and 0. If the trade shares actually remained constant between periods b and 0, the quantities would have had to change inversely in response to the price changes. In this case the elasticity of substitution is 1, and the proportionate decline in quantity is equal to the proportionate increase in prices.

1.171 Section D.3 of Chapter 16 shows that the Young index is equal to the Laspeyres index plus the covariance between the difference in annual shares pertaining to year b and month 0 shares ($s_i^b - s_i^0$) and the deviations in relative prices from their means ($r - r_i^*$). Normally the weight reference period b precedes the price reference period 0. The relative magnitudes of the Young and Laspeyres indices depend on the behavioral assumptions by the economic agents concerned, in particular the elasticity of substitution. If the elasticity of substitution is larger than 1—for example, the proportionate decline in quantity is greater than the proportionate increase in prices—the covariance will be positive. Under these circumstances the Young index will exceed the Laspeyres index.[21] Alternatively, if the elasticity of substitution is less than 1, the covariance will be negative and the Young will be less than the Laspeyres. As explained later, the Young index fails some critical index number tests discussed in Section J of this chapter and in Chapter 17, Section C.

J.1.9 Geometric Young, Laspeyres, and Paasche indices

1.172 In the geometric version of the Young index, a weighted geometric average is taken of the price relatives using the traded value shares of period b as weights. It is defined as

$$P_{GYo} \equiv \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right)^{s_i^b}, \tag{1.11}$$

where s_i^b is defined as above. The geometric Laspeyres is the special case in which $b = 0$: that is, the traded value shares are those of the price reference period 0. Similarly, the geometric Paasche uses the traded value shares of period t. Note that these geometric indices cannot be expressed as the ratios of value aggregates in which the quantities are fixed. They are not basket indices and there are no counterpart Lowe indices, that is, no price updating, to be applied.

1.173 It is worth recalling that for any set of positive numbers the arithmetic average is greater than, or equal to, the geometric average, which in turn is greater than, or equal to, the harmonic average, the equalities holding only when the numbers are all equal. In the case of unitary cross-elasticities of demand and constant value shares, the geometric Laspeyres and Paasche indices coincide.

1.174 In this case, the ranking of the indices must be the following:

The ordinary Laspeyres ≥ the geometric Laspeyres and Paasche ≥ the ordinary Paasche.

The indices are, respectively, arithmetic, geometric, and harmonic averages of the same price relatives that all use the same set of weights.

1.175 The geometric Young and Laspeyres indices have the same information requirements as their ordinary arithmetic counterparts. They can be produced on a timely basis. Thus, these geometric indices must be treated as serious practical possibilities for purposes of XMPI calculations. As explained later, the geometric indices are likely to be less subject than their arithmetic counterparts to the kinds of index number biases discussed in later sections. In particular the geometric Laspeyres falls between ordinary Laspeyres and Paasche and this will be seen to be a highly desirable property. If the time period between period b, the weight reference period, and period 0, the price reference period, is short, the geometric Young will approximate the geometric Laspeyres. The geometric Young is preferred to its arithmetic counterpart. Its main disadvantage may be that, because it is not a fixed-basket index, it is not so easy to explain or justify to users. The Lowe index as a fixed-basket index is easier to explain in this respect. An objection to geometric means in general used to be that they were difficult to explain to lay users. However, the widespread adoption of the unweighted geometric mean at the elementary level for CPIs detracts from this objection.

[21]This occurs because commodities with the large relative price increases ($r - r_i^*$ is positive) would also experience declining shares between periods b and 0 ($s_i^b - s_i^0$ is positive), thus having a positive influence on the covariance. In addition, commodities with small relative price increases ($r - r_i^*$ is negative) would experience increasing shares between b and 0 ($s_i^b - s_i^0$ is negative), thus having a positive influence on the covariance.

J.1.10 Symmetric indices

1.176 When the base and current periods are far apart, the index number spread between the numerical values of a Laspeyres and a Paasche price index is liable to be quite large, especially if *relative* prices have changed a lot (as shown in Appendix 16.1 and illustrated numerically in Chapter 20). Index number spread is a matter of concern to users because, conceptually, there is no good reason to prefer the weights of one period to those of the other. In these circumstances, it seems reasonable to take some kind of *symmetric average* of the two indices. More generally, it seems intuitive to prefer indices that treat both of the periods symmetrically instead of relying exclusively on the weights of only one of the periods. It will be shown later that this intuition can be backed up by theoretical arguments. There are many possible symmetric indices, but there are three in particular that command much support and are widely used.

1.177 The first is the *Fisher price index,* P_F, defined as the *geometric* average of the Laspeyres and Paasche indices; that is,

$$P_F \equiv \sqrt{P_L \times P_P}. \tag{1.12}$$

1.178 The second is the *Walsh price index,* P_W, a pure price index in which the quantity weights are *geometric* averages of the quantities in the two periods; that is,

$$P_W \equiv \frac{\sum_{i=1}^{n} p_i^t \sqrt{q_i^t q_i^0}}{\sum_{i=1}^{n} p_i^0 \sqrt{q_i^t q_i^0}}. \tag{1.13}$$

The averages of the quantities need to be *geometric* rather than arithmetic for the *relative* quantities in both periods to be given equal weight.

1.179 The third index is the *Törnqvist price index,* P_T, defined as a *geometric* average of the price relatives weighted by the average traded value shares in the two periods:

$$P_T = \prod_{i=1}^{n} \left(p_i^t / p_i^0\right)^{\bar{s}_i}, \tag{1.14}$$

where $\bar{s}_i$ is the arithmetic average of the traded value shares of commodity i in the two periods,

$$\bar{s}_i = \frac{s_i^t + s_i^0}{2}, \tag{1.15}$$

and where s_i is defined as in equation (1.4) and above. The theoretical attractions of these indices become apparent in the following sections on the axiomatic and economic approaches to index numbers.

J.1.11 Fixed-basket versus chain indices

J.1.11.1 Fixed-basket indices

1.180 This topic is examined in Section F of Chapter 16. When a time series of Lowe or Laspeyres indices is calculated using a fixed set of quantities, the quantities become progressively out of date and increasingly irrelevant to the later periods whose prices are being compared. The base period whose quantities are used has to be updated sooner or later, and the new index series linked to the old. Linking is inevitable in the long run.

1.181 In a chain index, each link consists of an index in which each period is compared with the preceding one, the weight and price reference periods being moved forward each period. Any index number formula can be used for the individual links in a chain index. For example, it is possible to have a chain index in which the index for $t + 1$ on t is a Lowe index defined as $\sum p^{t+1}q^{t-j} / \sum p^t q^{t-j}$. The quantities refer to some period that is j periods earlier than the price reference period t. The quantities move forward one period as the price reference period moves forward one period. If $j = 0$, the chain Lowe becomes a chain Laspeyres, whereas if $j = -1$ (that is, $t - (-1) = t + 1$), it becomes a chain Paasche.

1.182 The XMPIs in some countries are, in fact, annual chain Lowe indices of this general type, the quantities referring to some year, or years, that precedes the price reference period 0 by a fixed period. For example, the monthly indices from January 2005 to January 2006, with January 2005 as the price reference period, could be Lowe indices based on price-updated trade weights for 2004. The monthly indices from January 2006 to January 2007 are then based on price-updated trade weights for 2005, and so on, with annual weight updates.

1.183 The trade share weights lag behind the January price reference period by a fixed interval, moving forward a year each January as the price reference period moves forward one year. Although, for practical reasons, there has to be a time lag between the quantities and the prices when the index is first published, it is possible to retrospectively recalculate the monthly indices, using current period trade value data, or an average of previous and current period data when such data become available. In this way, it is possible for the long-run index to be an annually chained monthly index with contemporaneous annual weights. This method is explained in more detail in Chapter 10.

1.184 A chain index between two periods has to be "path dependent." It must depend on the prices and quantities in all the intervening periods between the first and last periods in the index series. Path dependency can be advantageous or disadvantageous. When there is a gradual economic transition from the first to the last period with smooth trends in relative prices and quantities, chaining will tend to reduce the index number spreads among the Lowe, Laspeyres, and Paasche indices, thereby making the movements in the index less dependent on the choice of index number formula.

1.185 However, if there are fluctuations in the prices and quantities in the intervening periods, chaining may not only increase the index number spread but also distort the measure of the overall change between the first and last periods. For example, suppose all export prices in the last period return to their initial levels in period 0—which implies that they must have fluctuated in between—a chain Laspeyres XPI does not return to 100. It will be greater than 100. If the cycle is repeated, with all the prices periodically returning to their original levels, a chain Laspeyres index will tend to "drift" further and further above 100 even though there may be no long-term upward trend in the prices. Chaining is therefore not advised when the prices fluctuate widely. When monthly prices are subject to regular and substantial seasonal fluctuations, for example, monthly chaining cannot be recommended. Seasonal fluctuations cause serious problems (see Chapter 23). Although a number of countries update their weights annually, the 12 monthly indices within each year are not chain indices but Lowe indices using fixed annual quantities.

1.186 Weights should be updated frequently. Administrative data from customs on weights are less costly than those for CPIs and PPIs. If possible, weights should be updated annually, or less frequently as resources permit, but at least every five years. However, for commodity groups with irregular annual patterns, a rolling average over a suitably short number of years is appropriate. Monthly and quarterly updating of weights is not advised, as noted above, especially so for seasonal commodities or commodities with irregular trade.

J.1.11.2 The Divisia index

1.187 If the prices and quantities are *continuous functions of time,* it is possible to partition the change in their total value over time into price and quantity components following the method pioneered by Divisia. As shown in Section E of Chapter 16, the Divisia index may be derived mathematically by differentiating value (that is, price times quantity) with respect to time to obtain two components: a relative value-weighted price change and relative value-weighted quantity change. These two components are defined to be price and quantity indices, respectively. The Divisia index is essentially a theoretical index. In practice, prices can be recorded only at discrete intervals even if they vary continuously with time. A chain index may, however, be regarded as a discrete approximation to a Divisia index. The Divisia index itself offers no practical guidance about the kind of index number formula to choose for the individual links in a chain index.

J.2 Axiomatic approach to index numbers

J.2.1 The axioms

1.188 The *axiomatic approach* to index numbers is explained in Chapter 17. It seeks to determine the most appropriate formula for an index by specifying a number of axioms, or tests, that the index ought to satisfy. It throws light on the properties possessed by different kinds of indices, some of which are by no means intuitively obvious. Indices that fail to satisfy certain basic or fundamental axioms, or tests, may be rejected completely because they are liable to behave in unacceptable ways. The axiomatic approach is also used to rank indices on the basis of their desirable, and undesirable, properties.

1.189 Twenty axioms or tests (T) are initially considered in Chapter 17. Only a selection of them are given here by way of illustration.

T1—*Positivity:* The price index and its constituent vectors of prices and quantities should be positive.

T3—*Identity Test:* If the price of every commodity is identical in both periods, then the price index should equal unity, no matter what the quantity vectors are.

T5—*Proportionality in Current Prices:* If all prices in period t are multiplied by the positive number λ, then the new price index should be λ times the old price index; that is, the price index function is (positively) homogeneous of degree one in the components of the period t price vector.

T10—*Invariance to Changes in the Units of Measurement* (commensurability test): The price index does not change if the units in which the commodities are measured are changed.

T11—*Time Reversal Test:* If all the data for the two periods are interchanged, then the resulting price index should equal the reciprocal of the original price index.

T12—*Quantity Reversal Test:* If the quantity vectors for the two periods are interchanged, then the price index remains invariant.

T14—*Mean Value Test for Prices:* The price index lies between the highest and the lowest price relatives.

T16—*Paasche and Laspeyres Bounding Test:* The price index lies between the Laspeyres and Paasche indices.

T17—*Monotonicity in Current Prices:* If the only change is that any period t price is increased, then the price index must increase.

1.190 Some of the axioms or tests can be regarded as more important than others. Indeed, some of the axioms seem so inherently reasonable that it might be assumed that any index number actually in use would satisfy them. For example, test T10, the commensurability test, says that if milk is measured in liters instead of pints, the index must be unchanged. One index that does not satisfy this test is the ratio of the arithmetic means of the prices in the two periods (the *Dutot* index—discussed in more detail in Chapter 21, Sections C and F).

1.191 Consider, for example, the average price of salt and pepper. Suppose it is decided to change the unit of measurement for pepper from grams to ounces while leaving the units in which salt is measured (for example, kilos) unchanged. Because an ounce is equal to 28.35 grams, the absolute value of the price of pepper increases by over 28 times, which effectively increases the weight of pepper in the Dutot index by more than 28 times. When the commodities covered by an index are heterogeneous and measured in different physical units, the value of any index that does not satisfy the commensurability test depends on the purely arbitrary choice of units. Such an index must be unacceptable conceptually. However, when the prices refer to a strictly homogeneous set of commodities that all use the same unit of measurement, the test becomes irrelevant. In practice, commodities may differ in terms of their quality characteristics, and there is a sense in which this variation in quality is similar to variation in the units of measurement. Although the quality of individual commodities may not change, the price changes of the higher-price varieties of, say, types of pepper, when aggregated, will be given more emphasis in the calculation.

1.192 Another important test is T11, the *time reversal test.* In principle, it seems reasonable to require that the same result should be obtained whichever of the two periods is chosen as the price reference period: in other words, whether the change is measured forward in time, forward from 0 to t, or backward in time, from t to 0. The Young index fails this test because an arithmetic average of a set of price relatives is not equal to the reciprocal of the arithmetic average of the reciprocals of the price relatives. This follows from the general algebraic result that the reciprocal of the arithmetic average of a set of numbers is the *harmonic* average of the reciprocals, not the arithmetic average of the reciprocals. The fact that the *conceptually* arbitrary decision to measure the change in prices forward from 0 and t gives a different result from measuring backward from t to 0 is seen by many users as a serious disadvantage. The failure of the Young index to satisfy the time reversal test needs to be taken into account by statistical offices.

1.193 Both Laspeyres and Paasche indices fail the time reversal test for the same reasons as the Young index. For example, the formula for a Laspeyres index calculated backward from t to 0, P_{BL}, is

$$P_{BL} = \frac{\sum_{i=1}^{n} p_i^0 q_i^t}{\sum_{i=1}^{n} p_i^t q_i^t} \equiv 1/P_P. \qquad (1.16)$$

1.194 This index is identical with the reciprocal of the (forward) Paasche, not with the reciprocal of the forward Laspeyres. As already noted, the (forward) Paasche tends to register a smaller increase than the (forward) Laspeyres, so that the Laspeyres index cannot satisfy the time reversal test. The Paasche index also fails the time reversal test.

1.195 On the other hand, the Lowe index satisfies the time reversal test *provided* that the quantities q_i^b remain fixed when the price reference period is changed from 0 to t. However, the quantities of a Laspeyres index are those of the price reference period, *by definition,* and *must change* whenever the price reference period is changed. The basket for a forward Laspeyres is different from that for the backward Laspeyres, and the Laspeyres fails the time reversal test as a consequence.

1.196 Similarly, the Lowe index is transitive whereas the Laspeyres and Paasche indices are not. Assuming that a Lowe index uses a fixed set of quantities, q_i^b, whatever the price reference period, it follows that

$$P_{Lo}^{0,t} = P_{Lo}^{0,t-k} \times P_{Lo}^{t-k,t},$$

where $P_{Lo}^{0,t}$ is the Lowe index for period t with period 0 as the price reference period. The Lowe index that compares t directly with 0 is the same as that calculated indirectly as a chain index through period $t–k$.

1.197 If, on the other hand, the Lowe index is defined in such a way that quantities vary with the price reference period, as in the index $\sum p^{t+1}q^{t-j} / \sum p^t q^{t-j}$ considered earlier, the resulting chain index is not transitive. The chain Laspeyres and chain Paasche indices are special cases of this index.

1.198 In reality, quantities do change and the whole point of chaining is to enable the *quantities* to be continually updated to take account of the changing trade patterns and universe of commodities. Achieving transitivity by arbitrarily holding the quantities constant, especially over a very long period of time, does not compensate for the potential bias to index numbers introduced by using out-of-date quantities.

J.2.2 Ranking of indices using the axiomatic approach

1.199 In Section B.6 of Chapter 17 it is shown not only that the Fisher price index satisfies all the 20 axioms initially listed in the chapter but also, more remarkably, that it is the *only* possible index that can satisfy all 20 axioms. Thus, on the basis of this set of axioms, the Fisher clearly dominates other indices.

1.200 In contrast to Fisher, the other two symmetric indices defined in equations (1.13) and (1.14) above do not emerge too well from the 20 tests. In Section B.7 of Chapter 17, it is shown that the Walsh price index fails four tests whereas the Törnqvist index fails nine tests. Although the Törnqvist index does not perform well on these tests, especially compared with Fisher, it should be remembered that the Törnqvist index and Fisher index may, nevertheless, be expected to approximate each other quite closely when the data follow relatively smooth trends, as shown in Chapter 20.

1.201 The Lowe index with fixed quantities emerges quite well from the axiomatic approach. In particular, in contrast to the Laspeyres, Paasche, and Young indices, it satisfies the time reversal test. As already explained, however, the attractiveness of the Lowe index depends very much on the positioning of period b that supplies the quantity weights, rather than its axiomatic properties.

1.202 One limitation of the axiomatic approach is that the list of axioms itself is inevitably arbitrary to some extent. Some axioms, such as the Paasche and Laspeyres bounding test failed by both Törnqvist and Walsh, could be regarded as contrived and dispensable. In particular many of the test properties have an arithmetic basis, whereas the Törnqvist index is a geometric average. Additional axioms or tests can be envisaged, and indeed two further axioms are considered below. Another problem with a simple application of the axiomatic approach is that it is not sufficient to know which tests failed. It is also necessary to know how badly an index fails. Badly failing one major test, such as the commensurability test, might be considered sufficient to rule out an index, whereas failing several minor tests marginally may not be very disadvantageous.

J.2.3 Some further tests

1.203 Consider a further symmetry test. It is reasonable that reversing the roles of prices and quantities in a price index should yield a volume index of the same formula as the price index. A formula that is good enough for a price index should be equally good for a volume index. The *factor reversal test* requires that the product of such a volume index and the original price index should be identical with the change in the value of the aggregate in question. This test is important if, as stated at the outset of this chapter, price and volume indices are intended to enable changes in the values of aggregates over time to be factored into their price and quantity components in an economically meaningful way. Another remarkable result derived from the axiomatic approach, and given in Section B.6 of Chapter 17, is that the Fisher index is the only price index to satisfy four minimal tests: T1 (positivity), T11 (time reversal), T12 (quantity reversal), and T21 (factor reversal).[22] Because the factor reversal test implicitly assumes that the prices and quantities must refer either to period 0 or to period t, it is not relevant to a Lowe index in which three periods are involved, b, 0, and t.

[22] See Funke and Voeller (1978, p. 180).

1.204 It was shown earlier that the product of the Laspeyres price (volume) index and the Paasche volume (price) index is identical with the change in the total value of the aggregate in question. Because Laspeyres and Paasche have different functional forms, this implies that they fail the factor reversal test. However, Laspeyres and Paasche indices may be said to satisfy a weak version of the factor reversal test in that dividing the value change by a Laspeyres or Paasche price index does lead to a meaningful volume index, even though its formula is not identical with that of the price index.

1.205 Another test, discussed in Section C.8 of Chapter 17, is the *additivity test.* A good property for an index is that the changes in the subaggregates add up to the changes in the totals. This is more important from the perspective of volume indices than it is for price indices. Price indices may be used to deflate value changes to obtain implicit volume changes. The results may be presented for subaggregates such as industry or commodity groups. Just as import and export aggregates at current prices are, by definition, obtained simply by summing individual values for import and export transactions or over detailed subaggregate commodity groups, it is reasonable to expect that the changes in the subaggregates of a volume index should add up to the changes in the totals—the additivity test. Volume indices that use a common set of prices to value quantities in both periods must satisfy the additivity test. Similarly, if the Lowe volume index is defined as $\Sigma p^j q^t / \Sigma p^j q^0$ it is also additive. The Geary-Khamis volume index used to make international comparisons of real consumption and GDP between countries is an example of such a Lowe volume index. It uses an arithmetically weighted average of the prices in the different countries as the common price vector $\boldsymbol{p}^j$ to compare the quantities in different countries.

1.206 An alternative solution is to use some *average* of the prices in two periods to value the quantities. If the volume index is also to satisfy the time reversal test, the average must be symmetrical. The *invariance to proportional changes in current prices test* (which corresponds to test T7 listed in Chapter 17 except that the roles of prices and quantities are reversed) requires that a volume index depend only on the *relative*, not the absolute, level of the prices in each period. The Walsh volume index satisfies this test, is additive, and satisfies the time reversal test as well. It emerges as a volume index with some very desirable properties.[23]

1.207 Although the Fisher index itself is not additive, it is possible to decompose the overall *percentage change* in a Fisher price, or volume, index into additive components that reflect the percentage change in each price or volume. A similar multiplicative decomposition is possible for a Törnqvist price or volume index.

J.3 Stochastic approach

1.208 The stochastic approach treats the observed price relatives as if they were a random sample drawn from a defined universe whose mean can be interpreted as the general rate of inflation. However, there can be no single unique rate of inflation. There are many possible universes that can be defined, depending on which particular sets of commodities, industries, or transactions the user is interested in. Clearly, the sample mean depends on the choice of universe from which the sample is drawn. The stochastic approach does not help decide on the choice of universe. It addresses issues such as the appropriate form of average to take and the most efficient way to estimate it from a sample of price relatives, once the universe has been defined.

1.209 The stochastic approach becomes particularly useful when the universe is reduced to a single type of commodity. When there are market imperfections, there may be considerable variation within a country in the prices at which a single commodity is sold by different establishments and also in their movements over time. In practice, statistical offices have to estimate the average price change for a single commodity from a sample of price observations. Important methodological issues are raised, which are discussed in some detail in Chapter 6 on sampling issues and Chapter 21 on elementary indices.

J.3.1 Unweighted stochastic approach

1.210 In Section C.2 of Chapter 17, the unweighted stochastic approach to index number theory is explained. If simple random sampling has been used to collect prices, equal weight may be given to each sampled price relative. Suppose each price relative can be treated as the sum of two components: a common inflation rate and a random disturbance with a zero mean. Using least-squares or maximum likelihood estimators, the best estimate of the common inflation rate is the unweighted

[23]Additivity is a property that is attractive in a national accounts context, where many aggregates are actually defined by processes of addition and subtraction. It is also useful when comparing national accounts data for different countries using purchasing power parities (PPPs), a type of international price index. (See ILO and others 2004a, Annex 4.)

arithmetic mean of price relatives, an index formula known as the *Carli* index. This index can be regarded as the unweighted version of the Young index. This index is discussed further in Section K below on elementary price indices.

1.211 If the random component is multiplicative, not additive, the best estimate of the common inflation rate is given by the unweighted *geometric* mean of price relatives, known as the *Jevons* index. The Jevons index may be preferred to the Carli on the grounds that it satisfies the time reversal test, whereas the Carli does not. As explained later, this fact may be decisive when deciding on the formula to be used to estimate the elementary indices compiled in the early stages of XMPI calculations.

J.3.2 Weighted stochastic approach

1.212 As explained in Section F of Chapter 17, a *weighted* stochastic approach can be applied at an aggregative level covering sets of different commodities. Because the commodities may be of differing economic importance, equal weight should not be given to each type of commodity. The commodities may be weighted on the basis of their share in the total traded value of exports or imports, or other transactions, in some period or periods. In this case, the index (or its logarithm) is the expected value of a random sample of price relatives (or their logarithms) with the probability of any individual sampled commodity being selected being proportional to the traded value of that type of commodity in some period or periods. Different indices are obtained depending on which period's weights are used and whether the price relatives or their logarithms are used.

1.213 Suppose a sample of price relatives is randomly selected, with the probability of selecting any particular type of commodity being proportional to the traded export or import value of that type of commodity in period 0. The expected price change is then the Laspeyres price index for the universe. However, other indices may also be obtained using the weighted stochastic approach. Suppose both periods are treated symmetrically, and the probabilities of selection are made proportional to the arithmetic mean value shares in both periods 0 and t. When these weights are applied to the logarithms of the price relatives, the expected value of the logarithms is the Törnqvist index. From an axiomatic viewpoint, the choice of a symmetric average of the value shares ensures that the time reversal test is satisfied, whereas the choice of the arithmetic mean, as distinct from some other symmetric average, may be justified on the grounds that the fundamental proportionality in the current prices test, T5, is thereby satisfied.

1.214 The examples of the Laspeyres and Törnqvist indices just given show that the stochastic approach in itself does not determine the form of the index number. There are several stochastic indices to choose from, just as there are many possible universes. However, as already noted, the elementary prices from which most aggregate price indices are constructed usually have to be based on samples of prices, and the stochastic approach may provide useful guidance on how best to estimate them.

J.4 Economic approach

J.4.1 An overview

1.215 The economic approach differs from the previous approaches in an important respect: quantities are no longer assumed to be independent of prices. The behavioral assumptions underlying the economic theory applied to XMPIs depend on whether we take a resident unit's or nonresident unit's perspective. In the former case exporters behave to maximize revenue and importers to minimize cost. In the latter, the exports are purchased by cost minimizers and imports are produced by revenue maximizers.

1.216 We consider here import and export price indices from a *resident unit's* perspective.[24] If, for example, it is assumed that resident institutional units behave as revenue maximizers in *supplying* exports to a given economic territory, it follows that they would produce more of commodities with above-average price changes. As a result, the revenue shares in period 1 from such commodities will increase, and therefore, their weights will also. This behavioral assumption about the firm, as it switches production to higher-priced commodities, allows something to be said about what "true" indices should be and the suitability of different index number formulas. For example, the Laspeyres output or supply price index uses fixed period 0 export revenue shares to weight its price relatives and ignores the substitution of production toward commodities with higher relative price changes. It will thus understate

[24]For a nonresident's perspective see Dridi and Zieschang (2004). Note that the findings for exports from the resident's approach apply to those of imports from the nonresident's approach, and findings for imports from the resident's approach apply to those of exports from the nonresident's approach.

aggregate price changes—be biased downward against its true index. The Paasche supply price index uses fixed period 1 weights and ignores the initial revenue shares in period 0. It will thus overstate aggregate price changes—be biased upward against its true index.

1.217 Again, taking a resident unit's perspective, we can observe a like phenomenon in the opposite direction for import price indices, which represent the price change for a resident institutional unit's *uses* of commodities supplied to them as imports by the rest of the world. If resident institutional units behave as cost minimizers in purchasing the exports from the rest of the world, it follows that they would use less of commodities with above-average price changes compared with period 0. As a result, the import shares in period 1 of such commodities will decrease, and therefore, their weights do too. This behavioral assumption about cost-minimizing agents, as they switch uses to lower-priced commodities, allows something to be said about what "true" indices should be and the suitability of different index number formulas. For example, the Laspeyres input price index uses fixed period 0 import value shares to weight its price relatives and ignores the substitution of imports away from commodities with higher relative price changes between periods 0 and 1. It will thus overstate aggregate price changes—be biased upward against its true index. The Paasche input price index uses fixed period 1 weights, which although representative of period 1 import value shares ignore, for the period 0 to 1 comparison, the initial import value in period 0. It will thus understate aggregate price changes—be biased downward against its true index.

1.218 The economic approach thus is very powerful, because it identifies a type of bias in Laspeyres and Paasche indices not apparent from other approaches: *substitution bias*. Laspeyres and Paasche indices ignore the change in weights as producers substitute their production toward, and substitute their uses away from, commodities with above-average price increases. Yet the nature of the bias arises from an assumption about the behavior of producers and other economic units engaged in international trade—that they are revenue maximizers and cost minimizers. Chapter 18 shows that Laspeyres and Paasche indices can under certain conditions act as bounds on a more generally applicable "true" economic theoretic index. The axiomatic approach in Section J led to an index number formula that used an average of the Laspeyres and Paasche indices, and even at this early stage in the discussion, the economic approach seems to provide further support.

1.219 The economic approach also identifies the circumstances under which the conventionally used Laspeyres index is appropriate. This would require, for exports by resident producers, that the exporting unit does not change its production configuration in response to relative price changes, at least over the short term of the price index comparisons. Economic theory thus argues that the Laspeyres index may be appropriate for industries in which quantities are known not to respond to relative price changes over the period of the price comparisons. But this may be the exception rather than the norm, and the theory points to a requirement for a more generally applicable index number formula, one that also encompasses rigidities in substitution. The economic approach, as shown in Chapter 18, demonstrates the following:

- A substitution bias can exist when using Laspeyres and Paasche formulas. The nature of the bias depends on the behavioral assumptions of the firm, which will vary depending on the residency orientation and whether we consider exports or imports.
- Laspeyres and Paasche indices act as bounds on their true indices and, under certain conditions, also are bounds for a more generally applicable true index.
- Some symmetric averages of these bounds can be justified from economic theory.

1.220 The approach from economic theory is thus first to develop theoretical index number formulas based on what are considered to be reasonable models of economic behavior by the resident unit supplying exports and purchasing imports. This approach is very different from the others considered here. Consider an establishment supplying exports and purchasing imports for use as intermediate consumption (inputs). A mathematical representation of the production activity—whereby capital and labor conjoin to turn intermediate inputs into outputs—is required for an output index, and a representation of the input cost is required for an input index.[25] Also required is an assumption of optimizing behavior (cost minimization or revenue maximization), along with assumptions relating to the form or nature of the mathematical representation of the production or cost function, so that a theoretical index can be derived that is "true" under these conditions.

[25]If our concern was with a model for consumers purchasing imports, a mathematical function of the utility derived from purchasing different quantities is required, a utility function; see ILO and others (2004a), Chapter 17.

The economic approach then examines practical index number formulas such as Laspeyres, Fisher, and Törnqvist and considers how they compare with "true" formulas defined under different assumptions. Two theoretical formulations will be examined. Neither can be practically calculated (for reasons that will be explained), but both can be closely approximated. The resident producer's export price index is supported by the concept of the fixed-input *output price* index. This index is a ratio of hypothetical revenues over the two periods being compared, say periods 0 and 1, that the revenue-maximizing resident establishment could realize, where the technology and inputs to work with were fixed to be the same for both periods. An establishment that, for example, doubles its revenue using a fixed technology and inputs effectively doubles its price. The theoretical index is a ratio of revenues, so it incorporates substitution effects as more revenue is obtained as firms substitute toward higher-priced commodities. The theoretical index wishes to have as its period 1 quantities the results of the firm changing the mix of output it produces in response to relative price changes. But there is a dilemma: only price changes should be reflected, and by allowing quantities to change, pure price changes would not be measured. So the theoretical index fixes the amount that can be produced by holding the technology and inputs at some constant level. The firm can change its output mix but must use constant inputs and technology. Note that there is a *family* of theoretical price indices depending on which period's reference technology and inputs are held constant: fixed period 0 technology and primary inputs, fixed period 1 technology and primary inputs, or some average of the two.

1.221 We can define the conceptual foundation of the resident producer's import price index as a theoretical fixed-output *input price* index. This is the ratio of hypothetical intermediate input costs that the cost-minimizing resident establishment must pay in order to produce a set of outputs, again with technology and primary inputs fixed to be the same for the comparison in both periods.

1.222 The key to understanding the economic approach is the appreciation that theoretical index numbers have a strong conceptual basis. As noted above, a family of such index numbers can be derived, some of which are based on more restrictive assumptions than others. Actual index number formulas can then be appraised in terms of whether they correspond to or approximate theoretical index number formulas with good foundations, as considered in more detail below.

J.4.2 Theoretical output export price indices

1.223 The theoretical *output export price index* between periods 0 and 1 is the ratio of the maximum revenues (from exports) that the establishment could attain when faced with period 0 and 1 prices using a fixed, given technology and a fixed set of inputs. Although the discussion is phrased in terms of the output and revenue from a producer, our concern hereafter is with the output or revenue of a resident producer *for export*. We assume such decisions are separable from those for the output to domestic markets. Consider a theoretical index in which period 0 technology and inputs are held constant, the theoretical counterpart to the Laspeyres index. What is required for the numerator of the ratio is to generate what the period 1 quantities would be, holding the production process and inputs constant in period 0 after the change in relative prices from the period 0 technology and inputs. This in turn requires a mechanism to generate these hypothetical period 1 quantities from the fixed period 0 technology and inputs. In the economic approach the technology of a firm or industry is described in terms of a production (possibility) function, which tells us the maximum amount of output(s) that can be produced from a given set of inputs. If the values of all the inputs to a firm or industry were given, the production function would be able to generate all possible combinations of output of commodities from the technology—it would be a mathematical representation of the technology that converts inputs to outputs. The prevailing relative prices would dictate exactly how much of each commodity is produced. The economic approach to output price indices relies on the assumption of *optimizing behavior* on the part of producers in competitive, price-taking markets so that they respond to relative price changes. In this approach, while actual prices are considered for both periods, the quantities in each period may not be the observed ones. They are generated from a given period's production function (with fixed technology) and level of inputs, using assumptions of maximizing behavior and dictated by relative prices, which may be the ones in another period. This is a powerful analytical framework because it allows us to consider, at least in theory, how quantities would respond to different price regimes (say, period 1 prices) under constant (say, period 0) reference technologies and inputs. They are hypothetical quantities that cannot be observed, but are generated in a mathematical model so that their formulation can be compared with real index number formulas based on observable prices and quantities.

1.224 "Pure" price index number formulas (based on observed data) and theoretical price indices both are defined as the ratios of revenues or costs in two periods. However, by definition, although the quantities are fixed in pure price indices, they vary in response to changes in relative prices in theoretical indices. In contrast to the axiomatic approach to index theory, the economic approach recognizes that the quantities produced are actually dependent on the prices. In practice, rational producers may be expected to adjust the *relative* quantities they produce in response to changes in *relative* prices. A theoretical set of XMPIs assumes that an institutional unit or establishment seeking to maximize revenues or minimize cost will make the necessary adjustments. The baskets of goods and services in the numerator and denominator of a theoretical XMPIs are not, therefore, exactly the same.

J.4.3 Upper and lower bounds on a theoretical output export price index

1.225 The theoretical output export price index between periods 0 and 1 is the ratio of export revenues in those periods using fixed technology and inputs. Consider an index that held the technology and inputs constant in period 0. The revenue generated in period 0 from period 0 prices using period 0 technology and inputs is what actually happened: the denominator of the theoretical ratio is the observed revenue, assuming the producer was optimizing (export) revenue. The numerator is period 1 prices multiplied by the hypothetical quantities that would have been produced using the same period 0 technology and inputs, had period 1 prices prevailed. It is **not,** as in the Laspeyres index, period 1 prices multiplied by the actual quantities produced at period 0 prices using period 0 technology and inputs. Both the theoretical and the Laspeyres indices use the same period 0 technology and inputs, but the theoretical index generates quantities from it as if period 1 prices prevailed, whereas the Laspeyres index uses the actual period 0 quantities. In practice, relative prices may change between the two periods, so the quantities generated will be different. Higher revenue could be achieved by substituting, at least marginally, some commodities that have relatively high price changes for some that have relatively low ones. The theoretical index based on period 0 technology and inputs will take account of this and will increase by more than the Laspeyres index. The theoretical index will be at least equal to or greater than the Laspeyres, because the producer has the possibility of, at worst, producing the same set of commodities as in period 0. Being a revenue maximizer, it is assumed the producer will substitute commodities with relatively high price changes—the Laspeyres index thus incurs a "substitution bias."

1.226 By a similar line of reasoning, it can be shown that when relative prices change, the theoretical output price index based on period 1 technology and inputs will increase by less than the Paasche index. In other words, as shown in Chapter 18, Section D.1, from the resident (producer) exporter's perspective the Laspeyres index provides a lower bound to its (period 0) theoretical index, and the Paasche an upper bound to its (period 1) theoretical index. Note that these inequalities are in the opposite direction for imports purchased by the resident producer or consumer; the theory and findings for import price indices is given in Section J.5.[26]

1.227 The practical significance of these results stems from the fact that the Laspeyres and Paasche indices can be calculated directly from the observed prices and quantities, whereas the theoretical indices cannot, thus giving some insight into the bias involved in the use of these two formulas. Suppose the official objective is to estimate a reference period theoretical output price index from the *resident* producer's perspective as an import price index, MPI, but that a Laspeyres index is calculated instead for practical reasons. One important conclusion to be drawn from this preliminary analysis is that the Laspeyres MPI may be expected to have an upward bias. Similarly, a series of downward biased Paasche MPIs from a resident producer's perspective used to deflate a series of import values at current prices generates a volume series of constant price import values (Laspeyres volume index), which in turn will also suffer from an upward bias. The approach informs us that there are *two* equally valid theoretical economic price indices, and that the bounds, although useful, only show how Laspeyres and Paasche indices compare with their own theoretical counterparts. What we require are *two-sided* bounds on the theoretically justified index.

J.4.4 Estimating theoretical export output indices by superlative indices

1.228 The next step is to establish whether there are special conditions under which it may be possible to exactly measure theoretical XMPIs. In Section D.2

[26]Importers wish to minimize costs and purchase more of commodities with below-average price increases. A Laspeyres import price index thus overstates its true index by not reflecting the shift in quantities to lower prices commodities. Similarly a Paasche import price index understates its true index.

of Chapter 18 theoretical indices based on weighted "averages" of the period 0 and period 1 technology and similarly weighted averages of the period 0 and 1 inputs are considered. These theoretical indices deal adequately with *substitution effects;* that is, when, for example, a resident producer's (output) export price increases, the producer's supply increases, holding inputs and the technology constant. Such theoretical indices are argued to generally fall between the Laspeyres (lower bound) and Paasche (upper bound) indices. The Fisher index, as the geometric mean of the Laspeyres and Paasche indices, is the only symmetric average of Laspeyres and Paasche that satisfies the *time reversal test.* Thus, economic theory was used to justify Laspeyres and Paasche bounds, and axiomatic principles led to the Fisher price index as the best symmetric average of these bounds.

1.229 In Section D.3 of Chapter 18 the case for the Törnqvist index number formula is presented. It is assumed (for a resident's output index) that the revenue function takes a specific mathematical form: a translogarithmic function. If the price coefficients of this translog form are equal across the two periods being compared, then the geometric mean of the economic output price index that uses period 0 technology and the period 0 input vector, and the economic output price index that uses period 1 technology and the period 1 input vector, are *exactly equal* to the Törnqvist output price index. The assumptions required for this result are weaker than other subsequent assumptions; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period. The ability to relate an actual index number formula (Törnqvist) to a specific functional form (translog) for the production technology is a powerful analytical device. Statisticians using particular index number formulas are in fact replicating particular mathematical descriptions of production technologies. A good formula should not correspond to a restrictive functional form for the production technology.

1.230 Diewert (1976) described an index number formula to be *superlative* if it is equal to a theoretical price index whose functional form is flexible—it can approximate an arbitrary technology to the second order. That is, the technology by which inputs are converted into output quantities and revenues is described in a manner that is likely to be realistic of a wide range of forms. Relating a class of index number formulas to technologies represented by flexible functional forms is another powerful finding, because it gives credence to this class of index number formulas. Note also that the translog functional form is an example of a *flexible* functional form, so the Törnqvist output price index number formula is *superlative.*

1.231 In Section E.3 of Chapter 18 the Fisher index is revisited from a purely economic approach. An additional assumption is invoked, that outputs are homogeneously separable from other commodities in the production function: if the input quantities vary, the output quantities vary with them, so that the new output quantities are a uniform expansion of the old output quantities. It is shown that a homogeneous quadratic production or utility function is flexible and corresponds to the Fisher index. The Fisher output price index is therefore also *superlative.* This is one of the more famous results in index number theory. Although it is generally agreed that it is not plausible to assume that a production technology would have this particular functional form, this result does at least suggest that, in general, the Fisher index is likely to provide a close approximation to the underlying unknown theoretical XMPI—and certainly a much closer approximation than either the Laspeyres or the Paasche indices can yield on their own.

1.232 This intuition is corroborated by the following line of reasoning. Diewert (1976) noted that a homogeneous quadratic is a flexible functional form that can provide a second-order approximation to other twice-differentiable functions around the same point. He then described an index number formula that is exactly equal to a theoretical one based on the underlying aggregator function as *superlative* when that functional form is also flexible—for example, a homogeneous quadratic. The derivation of these results, and further explanation, are given in detail in Sections E.1–3 of Chapter 18. In contrast to the theoretical index itself, a superlative index is an actual index number that can be calculated. The practical significance of these results is that they provide a theoretical justification for expecting a superlative index to provide a fairly close approximation to the unknown underlying theoretical index in a wide range of circumstances.

J.4.4.1 Superlative indices as symmetric indices

1.233 The Fisher index is not the only example of a superlative index. In fact, there is a whole family of superlative indices. It is shown in Section E.4 of Chapter 18 that any quadratic mean of order r is a superlative index for each value of $r \neq 0$. A quadratic mean of order r price index P^r is defined as follows:

$$P^r = \frac{\sqrt[r]{\sum_{i=1}^{n} s_i^0 \left(\frac{p_i^t}{p_i^0}\right)^{r/2}}}{\sqrt[r]{\sum_{n=1}^{n} s_i^t \left(\frac{p_i^0}{p_i^t}\right)^{r/2}}}, \qquad (1.17)$$

where s_i^0 and s_i^t are defined as in equation (1.4) above.

1.234 The symmetry of the numerator and denominator of equation (1.17) should be noted. A distinctive feature of equation (1.17) is that it treats the price changes and value shares in both periods symmetrically whatever value is assigned to the parameter r. Three special cases are of interest:

- When $r = 2$, equation (1.17) reduces to the Fisher price index;
- When $r = 1$, it is equivalent to the Walsh price index;
- In the limit as $r \to 0$, it equals the Törnqvist index.

1.235 These indices were introduced earlier as examples of indices that treat the information available in both periods *symmetrically*. Each was originally proposed long before the concept of a superlative index was developed.

J.4.4.2 Choice of superlative index

1.236 Section E.5 of Chapter 18 addresses the question of which superlative formula to choose in practice. Because each may be expected to approximate to the same underlying theoretical output index, it may be inferred that they ought also to approximate to each other. That they are all symmetric indices reinforces this conclusion. These conjectures tend to be borne out in practice by numerical calculations. It seems that the numerical values of the different superlative indices tend to be very close to each other, but only so long as the value of the parameter r does not lie far outside the range 0 to 2. However, in principle, there is no limit on the value of the parameter r, and in Section E.4 of Chapter 18, it is shown that as the value of r becomes progressively larger, the formula tends to assign increasing weight to the extreme price relatives and the resulting superlative indices may diverge significantly from each other. Only when the absolute value of r is very small, as in the case of the three commonly used superlative indices—Fisher, Walsh, and Törnqvist—is the choice of superlative index unimportant.

1.237 Both the Fisher and the Walsh indices date back nearly a century. The Fisher index owes its popularity to the axiomatic, or test, approach, that Fisher (1922) himself was instrumental in developing. As shown above, it appears to dominate other indices from an axiomatic viewpoint. That it is also a superlative index whose use can be justified on grounds of economic theory suggests that, from a theoretical point of view, it may be impossible to improve on the Fisher index for XMPI purposes.

1.238 However, the Walsh index has the attraction of being not merely a superlative index, but also a conceptually simple *pure* price index based on a fixed basket of goods and services. That the Walsh index is both a superlative and a pure index throws light on the interrelationships between the theoretical output price index and pure price indices. The distinctive feature of a Walsh index is not just that the basket of goods and services is a simple (geometric) average of the quantities in each of the two periods; by being a geometric average, it also assigns equal importance to the *relative*, as distinct from the absolute, quantities. Such an index clearly treats both periods symmetrically.[27] Pure price indices do not have to diverge from the theoretical output price index and are not inherently biased as estimators of the theoretical index. Bias is only likely to arise when the relative quantities used in a pure price index favor one of the periods at the expense of the other, as in a Laspeyres or Paasche index.

J.4.4.3 Representativity bias

1.239 That the Walsh index is a Lowe index that is also superlative suggests that the bias in other Lowe indices depends on the extent to which their quantities deviate from those in the Walsh basket. This can be viewed from another angle.

1.240 Because the quantities in the Walsh basket are *geometric* averages of the quantities in the two periods, equal importance is assigned to the *relative*, as distinct from the absolute, quantities in both periods.

[27]The Marshall-Edgeworth index (see Chapter 15) uses a simple arithmetic average of the quantities, but the resulting basket will be dominated by the quantities for one or other of the periods if the quantities are larger, on average, in one period than the other. The Marshall-Edgeworth is not a superlative index.

The Walsh basket may therefore be regarded as being the basket that is most representative of *both* periods.[28] If equal importance is attached to the production patterns in the two periods, the optimal basket for a Lowe index ought to be the most representative basket. The Walsh index then becomes the conceptually preferred target index for a Lowe index.

1.241 Suppose that period b, whose quantities are actually used in the Lowe index, lies midway between 0 and t. In this case, assuming fairly smooth trends in the relative quantities, the actual basket in period b is likely to approximate the most representative basket. Conversely, the farther away that period b is from the midpoint between 0 and t, the more the relative quantities of b are likely to diverge from those in the most representative basket. In this case, the Lowe index between periods 0 and t that uses period b quantities is likely to exceed the Lowe index that uses the most representative quantities by an amount that becomes progressively larger the farther back in time period b is positioned. The excess constitutes "bias" if the latter index is the target index. The bias can be attributed to the fact that the period b quantities tend to become increasingly unrepresentative of a comparison between 0 and t the farther back period b is positioned. The underlying economic factors responsible are, of course, exactly the same as those that give rise to bias when the target index is the economic index. Thus, certain kinds of indices can be regarded as biased without invoking the concept of an economic index. Conversely, the same kinds of indices tend to emerge as being preferred, whether or not the objective is to estimate an economic index.

1.242 If interest is focused on short-term price movements, the target index is an index between consecutive time periods t and $t + 1$. In this case, the most representative basket has to move forward one period as the index moves forward. Choosing the most representative basket implies chaining. Similarly, chaining is also implied for the target economic index t and $t + 1$. In practice, the universe of commodities is continually changing as well. As the most representative basket moves forward, it is possible to update the set of commodities covered as well as take account of changes in the relative quantities of commodities that were covered previously.

[28]The Walsh basket is the one that minimizes the sum of the squares of the logarithmic deviations between the quantities in the two actual baskets and those in the index basket.

J.4.4.4 Data requirements and calculation issues

1.243 Because superlative indices require price and trade share value data for both periods and such share data may not be available for the current period, it may not be feasible to calculate superlative XMPIs, at least at the time XMPIs are first published. In practice, it may be necessary for the official index to be a Laspeyres-type index, such as the Lowe or Young index. To the extent that regular trade share data are available from customs and/or establishment surveys, chain-linked XMPIs with frequently updated weights, preferably annually, should be produced. The advantage of the annual updating is that chaining helps to reduce the spread between the Laspeyres and Paasche indices. Chained indices are highly desirable not only because they reduce spread, but they facilitate the incorporation of new commodities into the index and the dropping of old ones. However, in the course of time more trade share data may become available, enabling chained superlative XMPIs to be calculated subsequently. Some statistical offices may find it useful to do so, without necessarily revising the original official index. Comparing movements in the official XMPIs with those in a retrospectively calculated chained superlative version may be helpful in evaluating and interpreting movements in the official XMPI. It may be that value data can be collected from establishments alongside price data or some customs data with little time lag, and this is to be encouraged so that Fisher XMPI indices may be calculated in real time for at least some commodity sectors.

1.244 Section E.6 of Chapter 18 notes that, in practice, XMPIs are usually calculated in stages (see Chapters 10 and 21) and addresses the question of whether indices calculated this way are consistent in aggregation—that is, have the same values whether calculated in a single operation or in two stages. The Laspeyres index is shown to be exactly consistent, but superlative indices are not. However, the widely used Fisher and Törnqvist indices are shown to be approximately consistent.

J.4.4.5 Allowing for substitution

1.245 One further index proposed recently, the Lloyd-Moulton index, P_{LM}, is defined as follows:

$$P_{LM} = \left\{ \sum_{i=1}^{n} s_i^0 \left(\frac{p_i^t}{p_i^0} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \quad \sigma \neq 1. \tag{1.18}$$

1.246 The parameter σ, which must be nonpositive for the price index of the nonresident supply of imports, is the elasticity of substitution between the commodities covered. It reflects the extent to which, on average, the various commodities are believed to be substitutes for each other. The advantage of this index is that it may be expected to be free of substitution bias to a reasonable degree of approximation, while requiring no more data, except for an estimate of the parameter σ, than the Laspeyres index. It is therefore a practical possibility for XMPI calculation, even for the most recent periods. However, it may be difficult to obtain a satisfactory, acceptable estimate of the numerical value of the elasticity of substitution, the parameter used in the formula.

J.4.5 Intermediate input price indices

1.247 Having considered the theory and appropriate formula for *output* export price indices, Chapter 18 turns to *input import* price indices (Section F). It is implicit in the description of the export price index as an output one and the import price index as an input one that the resident's perspective is being taken here, but it is apparent from the preceding discussion in Section J.4.1 that a nonresident's perspective may reverse the Laspeyres bounds. It is also apparent from the discussion in Section J.4.4 that the Fisher superlative index, as a symmetric average of Laspeyres and Paasche, will be unaffected by the direction of the bounds and thus whether a resident's or nonresident's perspective is taken.

1.248 The behavioral assumption behind the theory of the output export price index is one of producers maximizing a revenue function. An input import price index *for producers* is concerned with the price changes of intermediate inputs, and the corresponding behavioral assumption is the minimization of a conditional cost function. The producer is held to minimize the cost of intermediate inputs of imported goods and services in order to produce a set of outputs, given a set of intermediate import inputs prices, and given that the primary inputs and technology used on the imported inputs are fixed. Of course cost-minimizing decisions regarding intermediate inputs are based on both domestically supplied and imported inputs; however, we assume that they are separable and, hereafter, "inputs" refers to imported ones. The primary inputs and technology are assumed to be fixed so that hypothetical input quantities can be generated from a fixed setup that allows the input quantities in period 1 to reflect the producer buying more of those inputs that have become cheaper. Theoretical intermediate input import price indices are defined as ratios of hypothetical intermediate input import costs that the cost-minimizing producer must pay in order to produce a fixed set of outputs from technology and primary inputs fixed to be the same for the comparison in both periods. As was the case with the theory of the output export price index, theoretical input import indices can be derived on the basis of either fixed period 0 technology and primary inputs, or fixed period 1 technology and primary inputs, or some average of the two. The observable Laspeyres index of intermediate input import prices is shown to be an *upper bound* to the theoretical intermediate input import price index based on period 0 technology and inputs. The observable Paasche index of intermediate input import prices is a *lower bound* to its theoretical intermediate input import price index based on period 1 fixed technology and inputs. Note that these inequalities are the reverse of the findings for the output export price index, but that they are analogous to their counterparts in the CPI for the theory of the true cost-of-living index, which is also based on an expenditure (cost) minimization problem. Thus the same findings for input import price indices apply to consumers purchasing imported goods and services.

1.249 Following the analysis for the output export price index, a family of intermediate input import price indices can be shown to exist based on an *average* of period 0 and period 1 technologies and inputs, leading to the result that Laspeyres (upper) and Paasche (lower) indices are bounds on a reasonable theoretical input import index. A symmetric mean of the two bounds is argued to be applicable given that Laspeyres and Paasche indices are equally justifiable, with the Fisher index having support on axiomatic grounds. If the conditional intermediate input import cost function takes the form of a translog technology, the theoretical intermediate input import price index is exactly given by a Törnqvist index, which is superlative. If separability is invoked, Fisher and Walsh indices are also shown to be superlative, and the three indices closely approximate each other.

J.5 Illustrative numerical data

1.250 Chapter 20 presents numerical examples using an artificial data set. The purpose is not to illustrate the methods of calculation as such, but rather to demonstrate how different index number formulas can yield very different numerical results. Hypothetical, but economically plausible prices, quantities, and revenues are given for six commodities over five periods of time. In general, differences between the different formulas tend to increase with the variance of the price relatives. They also depend on the extent to which the prices follow smooth trends or fluctuate.

1.251 The numerical results are striking. For example, the Laspeyres index over the five periods registers an increase of 49 percent while the Paasche only rises 19 percent. The two commonly used superlative indices, Törnqvist and Fisher, register increases of 40 percent and 33 percent, respectively, an index number spread of only 7 points compared with the 30-point gap between the Laspeyres and Paasche. When the indices are chained, the chained Laspeyres and Paasche indices register increases of 47 percent and 28 percent, respectively, reducing the gap between the two indices from 30 to 19 points. The chained Törnqvist and Fisher indices register increases of 37.7 percent and 37.1 percent, respectively. These results show that the choice of index formula and method is crucial.

1.252 Chapter 15 shows how the nominal values of, and thus price indices for, exports and imports fit into the *2008 SNA*. Particular emphasis is given to the role of price indices as deflators for estimating volume changes in GDP by the expenditure approach. Chapter 20 also outlines how price indices for exports and imports can be defined and reconciled from the expenditure and production approaches to estimating GDP. Indeed, the illustrative data used to outline and demonstrate differences in the results from different index number formulas are applied not only to export and import price indices, but also to price indices for the constituent aggregates of GDP from both the expenditure and production approaches.

J.6 Choice of index formula

1.253 The index theory developed in Chapters 16–18 demonstrates that the Fisher and the Törnqvist indices are equally good alternatives. Further, the Fisher may be preferred to Walsh on axiomatic grounds, given that the two indices will tend to give almost identical results for comparisons between successive time periods.

1.254 By drawing upon the index number theory surveyed in Chapters 16–18 it is possible to decide on the type of index number in any given set of circumstances. However, there is little point in asking what is the best index number formula for XMPIs. The question is too vague. A precise answer requires a precise question. For example, suppose that the principal concern of most users of XPIs is to have the best measure of the *current rate* of factory gate inflation for exports. The precise question can then be posed: what is the best index number to use to measure the change between periods $t-1$ and t in the prices of the producer goods and services leaving the factory between periods $t-1$ and t for export? If the objective of the XPI is to measure the current rate of change in export revenues from using a technology and set of inputs that are fixed for a representative (symmetric average of the reference and current) period, the theory elaborated in Chapter 18 shows that the best estimate will be provided by a superlative index. The three commonly used superlative indices are Fisher, Törnqvist, and Walsh. All produce similar answers under small to moderately large changes in prices and quantities, because all approximate one another to the second order.

1.255 The question itself determines both the coverage of the index and the system of weighting. The establishments in question have to be those of the country in question and not those of some foreign country. Similarly, the question refers to exporting establishments in periods $t-1$ and t, not to establishments 5 or 10 years earlier. Sets of establishments 5 or 10 years apart are not all the same and their inputs and production technologies change over time.

1.256 Because the question specifies goods and services exported in periods $t-1$ and t, the basket of goods and services used should include *all* the quantities exported by the establishments in periods $t-1$ and t, and *only* those quantities. One index that meets these requirements is a pure price index that uses a basket consisting of the total quantities produced in both periods $t-1$ and t. This is equivalent to an index that uses a simple arithmetic mean of the quantities in the two periods, an index known as the Edgeworth-Marshall index. However, this index has a slight disadvantage in that if export production is growing, the index gives rather more weight to the quantities produced in period t than those in $t-1$. It does not treat both periods symmetrically. It fails tests T7 and T8 listed in Chapter 17 on the axiomatic approach, the tests for invariance to proportional changes in quantities. However, if the arithmetic mean quantities are replaced by the geometric mean quantities, as in the Walsh index, both tests are satisfied. This ensures that the index attaches equal importance to the *patterns* of production, as measured by *relative* quantities produced in both $t-1$ and t.

1.257 The Walsh index therefore emerges as the pure price index that meets all the requirements. It takes account of every single product produced in the two periods. It utilizes all the quantities produced in both periods, and only those quantities. It gives equal weight to the patterns of production in both periods. In practice, it may not be feasible to calculate a Walsh index, but it can be calculated and used retrospectively as the standard by which to evaluate other indices.

1.258 As already noted, for practical reasons XMPIs often are calculated as a time series of Lowe indices based not even on period $t-1$ quantities, but on some much earlier period *b*. A Laspeyres index based on period $t-1$ quantities between $t-1$ and t will be subject to substitution bias. We can infer, then, by reasoning along lines explained in Chapter 16, that the monthly change Laspeyres output export price index will tend to be less than the Walsh output export price index. If the XMPIs are intended to measure inflation, therefore, the monthly change Laspeyres export price index could have a downward bias. But the bias will tend to get worse as the current period for the Lowe index moves further away from the base period: in our case, the further period *b* precedes $t-1$, which for some countries can be a few years. This is the kind of conclusion that emerges from the index theory presented in Chapters 16 and 18. It is a conclusion with considerable policy and financial implications. It also has practical implications because it provides an argument for rebasing and updating a Lowe index as often as resources permit, preferably on an annual basis as many countries are now doing.

K. Elementary Price Indices

1.259 As explained in Chapters 10 and 21, the calculation of XMPIs typically proceeds in two or more stages. In the first stage, *elementary price indices* are estimated for the *elementary aggregates* of an XPI or an MPI. In the second stage, these elementary indices are combined to obtain higher-level indices using the elementary aggregate indices with export or import trade weights. An elementary aggregate consists of the traded values for a small and relatively homogeneous set of commodities defined within the commodity classification used in international trade. Samples of prices are collected within each elementary aggregate, so that elementary aggregates serve as strata for sampling purposes. There are two main sources of data for these prices. The first is data derived from customs documentation: the values of, for example, exports for a detailed classification group within the Harmonized System are aggregated in a given month and divided by the corresponding aggregate quantities to form a unit value. A similarly defined unit value for the next month is defined and the price change for goods in this classification proxied by the unit value index change. Each unit value index, derived for a detailed commodity classification, has a weight attached to it for aggregation to a higher level of classification. It may be that the unit values compared within the detailed classification are further refined and stratified to relate to, say, a single country of export, but the principle remains the same. The aggregation of transactions within the unit value indices benefit from quantity information, and although it is usual to define the elementary-level indices as being unweighted, this is not strictly so for unit value indices. However, unit value indices do relate to aggregation that is practically at the lowest level of aggregation for the period in question. The index number properties of unit value indices and their suitability as proxies to represent price changes have been the subject of Section B and, in detail, are the subject of Chapter 2. These lower-level unit value indices can be meaningfully defined for only strictly homogeneous goods, and any change in their quality composition between the months compared renders them subject to errors. Given that the use of unit value indices at the lower level has been discussed in this *Manual*, this chapter addresses itself to the second data source for elementary aggregate price indices—establishment surveys.

1.260 Data on the traded value shares, or quantities, of the representative goods and services whose prices are monitored and compared each period are not usually available within an elementary aggregate from an establishment survey. Because it has been shown that it is theoretically appropriate to use superlative formulas, data on trade value shares should be collected alongside those on prices whenever possible. Given that this may not be possible in order to meet the timeliness requirements of users—that is, there are no quantity or value share weights—most of the index number theory outlined in the previous sections is not applicable. An elementary price index is a more primitive concept that relies on price data only. It is something calculated when there is no explicit or implicit quantity or value data available for weights. Implicit quantity or value data may, however, arise from a sampling design whereby the selection of commodities is with probability proportionate to quantities or sales value.

1.261 The question of what is the most appropriate formula to use to estimate an elementary price index is considered in Chapter 21. This topic was comparatively neglected until a number of papers in the 1990s provided much clearer insights into the properties of elementary indices and their relative strengths and weaknesses.

Because the elementary indices are the building blocks from which XMPIs are constructed, the quality of a set of XMPIs depends heavily on them.

1.262 As explained in Chapter 7, compilers have to select *representative and well-defined items* within an elementary aggregate and then collect a sample of prices for each of the representative items, usually from a sample of different establishments. The individual items whose prices are actually collected are described as the *sampled items*. Their prices are collected over a succession of time periods. An elementary price index is, therefore, typically calculated from two sets of matched price observations. It is assumed in this section that there are no missing observations and no changes in the quality of the items sampled, so that the two sets of prices are perfectly matched. The treatment of new and disappearing items, and of quality change, is a separate and complex issue that is discussed in detail in Chapters 8, 9, and 22 of the *Manual*.

1.263 The price quotes for the elementary aggregate indices are assumed to be collected from price surveys of establishments and relate to matched specified commodities whose quality characteristics are well specified.

K.1 Heterogeneity of commodities within an elementary aggregate

1.264 If a number of different representative items are selected for pricing, the set of items within an elementary aggregate cannot be homogeneous. Even a single representative item may not be completely homogeneous, depending upon how tightly it is specified. The degree of heterogeneity of the sampled items must be explicitly taken into account in the calculation of an elementary index.

1.265 When the quantities are not homogeneous, *they cannot be meaningfully added from an economic viewpoint, and their prices should not be averaged.* Consider again the example of salt and pepper, which might be representative items within an elementary aggregate. Pepper is an expensive spice sold in very small quantities, such as ounces or grams, whereas salt is relatively cheap and sold in much larger quantities, such as pounds or kilos. A simple arithmetic average of, say, the price of a gram of pepper and the price of a kilo of salt is an arbitrary statistic whose value depends largely on the choice of the quantity units. Choosing the same physical unit of quantity, such as a kilo, for both does not resolve the problem, because both the average price and the change in the average price would be completely dominated by the more expensive commodity, pepper, even though exporters may obtain more revenue from salt. In general, arithmetic averages of prices should be taken only when the corresponding quantities are homogeneous and can be meaningfully added.

K.2 Weighting

1.266 As already noted, it is assumed in this section that there are no quantities or trade share value data available to weight the prices, or the price relatives, used to calculate an elementary index. If they were available, it would usually be preferable to use them to decompose the elementary aggregate into smaller and more homogeneous aggregates.

1.267 However, some system of weighting may have been implicitly introduced into the selection of the sampled items by the sample design used. For example, the establishments from which the prices are collected may have been selected using probabilities of selection that are proportional to their sales or some other variable.

K.3 Interrelationships between different elementary index formulas

1.268 Valuable insights into the properties of various formulas that might be used for elementary price indices may be gained by examining the numerical relationships between them, as explained in Section D of Chapter 21. There are two basic options for an elementary index:

- To average the price relatives—that is, the ratios of the matched prices; and
- To calculate the ratio of average prices in each period.

1.269 It is worth recalling that for any set of positive numbers the arithmetic average is greater than or equal to the geometric average, which in turn is greater than or equal to the harmonic average, the equalities holding only when the numbers are all equal. Using these three types of average, the ranking of the results obtained by the first method are predictable. It should also be noted that the ratio of geometric averages is identical with the geometric average of the ratios. The two methods give the same results when geometric averages are used.

1.270 As explained in Section C of Chapter 21, there are several elementary price indices that might possibly be used. Using the first of the above options, three possible elementary price indices are the following:

- The arithmetic average of the price relatives, known as the Carli index, or P_C;
- The geometric average of the price relatives, known as the *Jevons* index, or P_J;
- The harmonic average of the price relatives, or P_H.

As just noted, $P_C \geq P_J \geq P_H$. And using the second of the options, three possible indices are the following:

- The ratio of the arithmetic average prices, known as the Dutot index, or P_D;
- The ratio of the geometric averages, again the Jevons index, or P_J;
- The ratio of the harmonic averages, or R_H.

1.271 The ranking of *ratios* of different kinds of average are not predictable. For example, the Dutot, P_D, could be greater or less than the Jevons, P_J.

1.272 The Dutot index can also be expressed as a weighted average of the price relatives, in which the prices of period 0 serve as the weights:

$$P_D \equiv \frac{\sum_{i=1}^{n} p_i^t \Big/ n}{\sum_{i=1}^{n} p_i^0 \Big/ n} = \frac{\sum_{i=1}^{n} p_i^0 \left(\dfrac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n} p_i^0}. \tag{1.19}$$

1.273 As compared with the Carli, which is a simple average of the price relatives, the Dutot index gives more weight to the price relatives for the items with high prices in period 0. However, it is difficult to provide an economic rationale for this kind of weighting. Prices are not revenues or expenditures. If the items are homogeneous, very few quantities are likely to be purchased at high prices if the same items can be purchased at low prices. If the items are heterogeneous, the Dutot should not be used anyway, because the quantities are not commensurate and not additive.

1.274 Noting that $P_C \geq P_J \geq P_H$, it is shown in Section D of Chapter 21 that the gaps between these indices widen as the variance of the price relatives increases. The choice of formula becomes more important the greater the diversity of the price movements. Moreover, both P_D and P_J can be *expected* to lie *approximately* halfway between P_C and P_H. Although it is useful to establish the interrelationships between the various indices, they do not actually help decide which index to choose. However, because the differences between the various formulas tend to increase with the dispersion of the price relatives, it is clearly desirable to define the elementary aggregates in such a way as to try to minimize the variation in the price movements within each aggregate. The less variation, the less difference the choice of index formula makes. Because the elementary aggregates also serve as strata for sampling purposes, minimizing the variance in the price relatives within the strata will also reduce the sampling error.

K.4 Axiomatic approach to elementary indices

1.275 One way to decide between the various elementary indices is to exploit the axiomatic approach outlined earlier. A number of tests are applied to the elementary indices in Section E of Chapter 21.

1.276 The Jevons index, P_J, satisfies all the selected tests. It dominates the other indices in the way that the Fisher tends to dominate other indices at an aggregative level. The Dutot index, P_D, fails only one, the commensurability test. This failure can be critical, however. It reflects the point made earlier that when the quantities are not economically commensurate, their prices should not be averaged. However, P_D is satisfactory when the sampled items are homogeneous. The key issue for the Dutot index is, therefore, how heterogeneous are the items within the elementary aggregate. If the items are not sufficiently homogeneous for their quantities to be additive, the Dutot index should not be used.

1.277 The Carli index, P_C, is widely used, but the axiomatic approach shows that it has some undesirable properties. In particular, it fails the product reversal, the time reversal, and the transitivity tests. These are serious disadvantages, especially when month-to-month indices are chained. A consensus has emerged that the Carli may be unsuitable because it is liable to have a significant upward bias. This is illustrated by numerical example in Chapter 10. Its use is not sanctioned for the Harmonized Indices of Consumer Prices used within the European Union. Conversely, the harmonic average of the price relatives, P_H, is liable to have an equally significant downward bias, although it does not seem to be used in practice anyway.

1.278 On the axiomatic approach, the Jevons index, P_J, emerges as the preferred index. However, its use may not be appropriate in all circumstances. If one observation is zero, the geometric mean is zero. The Jevons is sensitive to extreme falls in prices, and it may be necessary to impose upper and lower bounds on the individual price relatives when using the Jevons.

K.5 Economic approach to elementary indices

1.279 The economic approach, explained in Section F of Chapter 21, seeks to take account of the economic behavior of producers and their economic circumstances. Price differences may be observed at the same point of time for two quite different reasons.

- Exactly the same items may be sold by different categories of producers at different prices.
- The sampled items are not exactly the same. The different prices reflect differences in quality.
- Both phenomena may occur at the same time.

1.280 *Pure* price differences can occur when the items sold at different prices are exactly the same. Pure price differences imply differing technologies or market imperfections of some kind, such as local monopolies, price discrimination, consumer or producer ignorance, or rationing. If all consumers had equal access, were well informed, and were free to choose, and all producers produced using the same technologies in price-taking markets, all sales would be made at a single price, the lowest on offer.

1.281 On the other hand, if markets were perfect, exporters would be able to supply and importers be prepared to purchase at different prices only if the items were qualitatively different. Included in the term "item" are the terms and conditions surrounding the sale, including the level of service and convenience. It is tempting to assume, therefore, that the mere existence of different prices implies that the items *must* be qualitatively different in some way. For example, even units of the same physically homogeneous item produced at different locations or times of the day may be qualitatively different from an economic viewpoint. For example, electricity supplied during peak (hours) may carry a price premium, owing to higher demand and limited supply, even though it is essentially the same service. In this instance, the higher price is arguably not a pure price difference. However, the relative prices charged by, for example, different exporters for an almost identical item cannot usually be fully explained by differences in inputs and technologies and may be, in part, pure price differences. In practice, almost all markets are imperfect to some extent.

1.282 If there is, say, only a single homogeneous item produced by an establishment for export on a "normal" day, the price differences must be pure. The average price is equal to the *unit value*, defined as the total value sold divided by the total quantity. The unit value is a quantity-weighted average of the different prices at which the item is sold. It changes in response to changes in the mix of quantities sold (of the same homogeneous item) at different prices as well as to any changes in the prices themselves. It is most satisfactory in this regard. In practice, however, the change in the unit value has to be estimated from a sample of prices only. Unit values exist at two levels. The first is for a production run i at the establishment level where a batch of, say, q_i items may be exported for revenue p_iq_i, the price recorded being the unit value. There may be more than one production run at different batch sizes, and the unit values may vary with batch size. The recorded "price" for these items may then be the revenue from several batches divided by the quantity supplied, $\Sigma p_iq_i/q_i$. If the mix of batch sizes varies over time, then there will be unit value bias when dividing the unit value in one period by that in a preceding period. The second aggregation of unit values is across establishments producing the same item. Again, any difference in the relative quantities sold from different establishments will lead to unit value bias if the items are not strictly homogeneous.

1.283 This requirement of price indices that unit values must be taken only over sets of homogeneous items speaks directly to the problem of unit value bias in customs-source international trade data on goods (see Chapter 2). To use changes in unit values from customs data as a proxy for price relatives, the goods in the customs class must be homogeneous. The class must contain a single undifferentiated type of good sold at the same terms, or if it contains more than one type of good or is sold at different terms, the characteristics or terms must be such that they do not have an effect on price. Otherwise, any changes in the mix of goods in the customs class will result in index changes unrelated to price changes. In fact, unit values over an aggregate containing multiple items with different price levels but changing relative quantities of trade will change even if none of the prices of the items has changed, giving a false price movement signal.

K.6 Sets of homogeneous items

1.284 The economic approach views, for example, exported items as if they were a sample from a basket produced by a group of resident, rational, revenue-maximizing suppliers. One critical factor is how much item variation there is within an elementary aggregate, bearing in mind that it should be as narrowly defined as possible, possibly even consisting of a set of homogeneous items.

1.285 If the sampled items are all identical, the observed price differences must be due to establishments using different production technologies and market imperfections such as price discrimination, consumer ignorance, or rationing, or some kind of temporary disequilibrium. Informed suppliers with unrestricted production and transaction possibilities would not sell at a lower price if they had the opportunity to sell exactly the same item at a higher price. It is tempting to assume, therefore, that the items are not really homogeneous and that the observed price differences *must* be due to quality differences of some kind or another, but imperfections in producer and consumer markets are widespread and cannot be assumed away a priori.

1.286 As explained in Section B of Chapter 21, when a single item is sold at different prices, the price of that item for XMPI purposes is the unit value, defined as total value of transactions divided by total quantities—that is, the quantity-weighted average price. The price relative for the item is the ratio of the unit values in the two periods. This may be affected by a change in the pattern of items that sell at high and low prices as well as by changes in the individual prices.

1.287 Consider the case in which the representative sampled items are selected with probabilities proportional to the quantities sold at the different prices in the first period. Such sampling assumes homogeneous items because quantities can only be meaningfully added and compared for homogeneous items. For homogeneous items, a unit value index is the appropriate measure of overall price change. Given sampled items are selected with probabilities proportional to the quantities sold in the first period, a simple (unweighted) arithmetic average of their prices will provide an estimate of the unit value in the first period. The Dutot index is the ratio of the simple arithmetic average prices in the two periods. However, given that the two sets of prices are perfectly matched—that is, geared to the pattern of production or purchases in the first period only—the Dutot cannot take account of any changes in the patterns of production or purchases between the two periods and may not provide an unbiased estimate of the ratio of the unit values. As shown in Section F of Chapter 21, the sample Dutot index with probabilities proportional to quantities sold in the first period may be expected to approximate to a Laspeyres-type index in which the quantity weights are fixed, by definition. It does not provide a satisfactory estimate of a unit value index in which the relative quantities do change. Moreover, this approximated Laspeyres-type index is not a conventional Laspeyres index because the quantities do not refer to different items, or even different qualities, but to different quantities of exactly the same item sold at different prices.

1.288 Consider a resident exporting producer. In practice, even though producers' choices may be restricted because of their production technology, buyer-seller relationships, market ignorance, and other market imperfections, they may switch production toward items sold at high prices and away from those at low prices, as market conditions change and restrictions on choice are eased. The Dutot index, based on matched prices, cannot take account of such switches and may tend to understate the *rise* in the unit values for this reason. Alternatively, it may be that the demand side dictates market behavior, with establishments responding to demand by increasing production of low-priced items. For example, when the ratio of the unit values changes because nonresident purchasers of exports, or at least some of them, succeed in switching from resident establishments selling at high prices to resident establishments selling at low prices, the failure of XPIs to take account of such switches leads to the Dutot index overstating the *fall* in the unit value index.

K.7 Heterogeneous elementary aggregates

1.289 In practice, most elementary aggregates are likely to contain a large number of items that are similar but not identical. Assuming producers are informed and have a perfectly flexible set of production possibilities, the relative prices may then be expected to reflect a producer's marginal rates of substitution. Within the same elementary aggregate, the different items will often be close substitutes for each other, often being no more than marginally different qualities of the same generic item, so that the quantities produced may be expected to be quite sensitive to changes in relative prices.

1.290 Using an economic approach, it is possible to ask what the best estimate of the "true" economic index for the elementary aggregate is. Bearing in mind, however, that no information on quantities and values is available within the aggregate, it is necessary to resort to considering certain hypothetical special cases. Suppose that resident exporters react to importers' preferences; as demand increases for a relatively low-priced item, exporters produce more of it. Assume nonresident importers have so-called Cobb-Douglas preferences, which imply that the cross-elasticities of substitution between the different items are all unity. The quantity relatives vary inversely with the price relatives, so that their value shares and the suppliers' revenues remain constant. The true economic index can then be shown to be a weighted geometric average of the price relatives, the weights being the export value shares—which, as just noted, are the same in both periods. Now, suppose that the items whose prices are sampled are randomly selected with probabilities proportional to their export value shares in the first period. As shown in Section F of Chapter 21, with this method of selection, the simple geometric average of the sample price relatives—that is, the Jevons index—may be expected to provide an approximation to the underlying economic index. It should be noted that the Jevons index does not imply, or assume, that the trade value shares remain constant. Obviously, the Jevons index can be calculated whether or not changes occur in the value shares in practice. What the economic approach shows is that *if* sampling is with probabilities proportionate to population value shares, and *if* the value shares remain constant, or roughly constant, *then* the Jevons index can be expected to provide a good estimate of the underlying ideal economic index.

1.291 However, for XPIs the assumption of unit cross-item elasticities of substitution with equal values in both periods is *not* consistent with resident producer (export) economic theory. Revenue-maximizing suppliers will produce *more* of the sampled items with above-average price increases, so their share of revenue cannot be expected to be constant. Indeed the Jevons index, in assuming constant revenue shares, will understate price changes under such revenue-maximizing behavioral assumptions. The Jevons index allows implicit quantities to fall as relative prices increase, to maintain equal revenue shares, rather than allowing an increase. There is not an accepted unweighted price index number formula that incorporates such substitution behavior.

1.292 Alternatively, suppose that the production technology is such that, at least in the short term, there is no substitution in response to relative price changes, and the relative quantities remain fixed. In this case, the true economic index would be a Laspeyres-type index. If the items were sampled with probabilities proportional to the revenue shares in the first period, a simple arithmetic average of the price relatives—that is, the Carli index—would approximate it.[29] However, assuming no substitution is unreasonable and counterfactual in general, although it may occur exceptionally.

1.293 Thus, using the economic approach, under one set of conditions the Jevons index would provide an approximation to the underlying economic index, whereas under another set of conditions the Carli index would do so. In most cases, the actual conditions seem likely to be closer to those required for the Jevons to estimate the underlying index than for the Carli, because the cross-elasticities of substitution seem much more likely to be close to unity than zero for industries whose pricing behavior is driven by demand. Thus, the economic approach provides some support for the use of Jevons rather than Carli, at least in most situations. However, if resident exporting revenue-maximizing behavior is believed to dominate an industry, use of the Jevons index is not supported. Yet a cautionary note should be applied to such conclusions. They require that a specific sample design is used—that is, sampling is with probabilities proportionate to population value shares for the reference period. There is no quantity or value information used in the construction of Jevons and Carli indices, so conclusions from economic theory can be derived for them only if quantity/value is introduced by the sample design used. If such a sample design is not used, the conclusions do not hold. In such circumstances, the axiomatic approach should form the basis for guidelines as to which formula is appropriate and this approach favors the Jevons index.

1.294 Another alternative is suggested in Section G of Chapter 21. If items are sampled according to fixed-revenue shares in each period, then the resulting sample can be used with the Carli formula (P_C) to estimate the Laspeyres index, and the harmonic mean formula (P_H) to calculate the Paasche index. By taking the geometric average of these two formulas, as suggested by Carruthers, Sellwood, and Ward (1980) and Dalén (1992), a Fisher index would result:

$$P_{CSWD} = \sqrt{P_C \times P_H}. \tag{1.20}$$

[29]Notice that the Dutot index cannot be used when the items are not homogeneous, because an arithmetic average of the prices of different kinds of items is both arbitrary and economically meaningless. If a Laspeyres index is estimated as a simple average of the price relatives—that is, assuming equal trade value shares—the implied quantities cannot be equal because they vary inversely with the prices.

1.295 However, because statistical offices would not have the trade value shares to use for weights for the current period, an approximation to the Fisher index is obtained by assuming they are not too different from those used in the base period 0. A similar assumption would justify the use of a Jevons index (P_J,) as an approximation to a Törnqvist index. Again recall that these approximations result when the observations are sampled in proportion to trade value shares.

L. Basic Index Calculations

1.296 Chapter 10 provides a general overview of the ways in which XMPIs are calculated in practice. The methods used in different countries are by no means all the same, but they have much in common. There is clearly interest from users as well as compilers in knowing how most statistical offices set about calculating their XMPIs. The various stages in the calculation process are illustrated by numerical examples.

1.297 Chapter 10 is descriptive and not prescriptive, although it does try to evaluate the strengths and weaknesses of existing methods. It makes the point that, because of the greater insights into the properties and behavior of indices gained in recent years, it is now recognized that not all existing practices are necessarily optimal.

1.298 Because the various stages involved in the calculation process have, in effect, already been summarized in the preceding sections of this chapter, they are not repeated in this section. However, given below is an indication of the nature of the contents of Chapter 10. Illustrations of a number of useful stages in compilation are given, including calculating elementary and higher-level price indices, price-updating weights, rebasing and changing the reference year, use of long-term and short-term links, decomposition of index changes (contribution analysis), chaining, and data editing.

L.1 Elementary price indices

1.299 Chapter 10 describes how the elementary price indices are calculated for the elementary aggregates. It reviews the principles underlying the delineation of the elementary aggregates themselves. Elementary aggregates are relatively small groups of commodities that are intended to be as homogeneous as possible, not merely in terms of the physical and economic characteristics of the commodities covered, but also in terms of their price movements. They may also be broken down by the source or destination country or region of the trade. Samples of prices are collected for a number of representative transactions across establishments within each elementary aggregate in order to estimate the elementary price index for that aggregate, with each elementary price index providing a building block for the construction of the higher-level indices.

1.300 Section B of Chapter 10 considers the consequences of using alternative elementary index formulas to calculate the elementary indices. It proceeds by means of a series of numerical examples that use simulated price data for four different commodities within an elementary aggregate. The elementary indices and their properties have been explained in some detail in Section K above. An elementary price index may be calculated either as a chain index[30] or as a direct index—that is, *either* by comparing the price each month, or quarter, with that in the immediately preceding period *or* with the price in the fixed price reference period. Table 10.1 uses both approaches to illustrate the calculation of three basic types of elementary index—Carli, Dutot, and Jevons. It is designed to highlight a number of these indices' properties. For example, it shows the effects of "price bouncing," in which the same four prices are recorded for two consecutive months but the prices are switched among the four commodities. The Dutot and Jevons indices record no increase, but the Carli index registers an increase. The table also illustrates the differences between the direct and the chain indices. After six months, each of the four prices is 10 percent higher than at the start. Each of the three direct indices records a 10 percent increase, as the chained Dutot and Jevons indices also do because they are transitive. The chained Carli, however, records an increase of 29 percent, which is interpreted as illustrating the systematic upward bias in the Carli formula resulting from its failure to satisfy the time reversal test.

1.301 Section B.3 of Chapter 10 notes that the chaining and direct approaches have different implications when there are missing price observations, quality changes, and replacements. It concludes that the use of a chain index can facilitate the estimation of missing prices and the introduction of replacement commodities, as also argued in Chapter 8.

[30]The term "chain index" is used in the context of elementary index numbers to refer to a process in which fixed weights are adopted, but the price change is calculated for the unweighted relatives not as price reference period to current period, but as the previous to current period, linked by successive multiplication to the elementary index for the previous period.

1.302 Section B.5 of Chapter 10 examines the effects of missing price observations, distinguishing between those that are temporarily missing and those that have become permanently unavailable. Table 10.2 contains a numerical example of the treatment of the temporarily missing prices. One possibility is simply to omit the commodity whose price is missing for one month from the calculation of indices that compare that month with the preceding and following months and also with the base period. Another possibility is to impute a price change on the basis of the average price for the remaining commodities using one or other of the three types of average. The example is a simplified version of the kind of examples that are used in Chapter 8 to deal with the same problem.

1.303 The possibility of using other elementary index formulas is considered in Section B.6 of Chapter 10. The harmonic mean of the price relatives, P_H, and the ratio of the harmonic means, R_H, are examined. The P_H has the inverse properties of the Carli index, P_C, and can therefore be assumed to have an opposite bias. Because it is also a rather difficult concept to explain, it is not recommended. The Jevons index, P_J, has attractive axiomatic properties. The geometric mean of the P_C and the P_H, a kind of elementary Fisher index, remains a possibility with some theoretical attractions, though, because it provides close results to the Jevons index, P_J.

1.304 A *prerequisite* for the application of the economic approach to elementary index number formulas is that prices are sampled with probabilities proportionate to reference period population quantity or value shares. Given a particular sample design, as a means by which quantity or value information is "attached" to an unweighted index, unweighted index number formulas can be justified in cases of particular forms of substitution behavior, as outlined above. However, it is unusual that such formal sample designs are used for the sampling of establishments and items in survey-based XMPIs at the elementary level and, in their absence, choice of formulas is best guided by the axiomatic approach.

1.305 Section C of Chapter 10 discusses the issue of consistency in aggregation between lower- and higher-level indices that may arise if different formulas are used at different levels. Consistency of aggregation means that if an index is calculated stepwise, by calculating intermediate indices that are themselves subsequently aggregated, the same result should be obtained as if the calculation were made in a single step without the intermediate indices. This can be an advantage for purposes of presentation. If a Young or Laspeyres index is used for the higher-level indices, including the overall XMPIs themselves, then the Carli index is the form of elementary index that is consistent with it.[31] Given that the Carli does not emerge as the preferred elementary index from the axiomatic and economic approaches to elementary indices, this creates a dilemma when the Laspeyres or Young formula is used. It is suggested that consistency in aggregation may not be so important if there are different degrees of substitution within elementary aggregates at the lower level, as compared with the degree of substitution between commodities in different elementary aggregates at the higher level.

1.306 It is not necessary to use the same index formula for every elementary index. The characteristics of the price behavior within each elementary aggregate should be examined to identify the most appropriate formula. However, it may be decided to use a single formula throughout if resources are limited and computational procedures need to be kept as simple as possible. If sample design considerations and economic behavioral assumptions do not favor the economic approach, there is much in the axiomatic approach to support a general use of the Jevons index.

L.2 Calculation of higher-level indices

1.307 Section C of Chapter 10 considers the calculation of the higher-level indices utilizing the elementary price indices and the weights provided for the elementary aggregates. In many instances, statistical offices do not use a true Laspeyres index, but rather a Lowe or Young index (discussed in Section J.1 above). These two indices use price reference periods and weight reference periods that differ, whereas in the Laspeyres index the price and weight reference periods are one and the same. Typically the weight reference period precedes the price reference period in the version of the Young and Lowe indices used by statistical offices owing to the time it takes to develop weights from administrative sources and establishment surveys in earlier periods. It is at this stage that the traditional index number theory discussed in Chapters 16–18 comes into play. Because this theory is explained in detail and in depth in these chapters, which are also summarized in Sections J and K of this chapter, it is not repeated here.

1.308 At the time the monthly or quarterly XMPIs are first calculated, particularly if the weights come

[31] Also recall that the Jevons index would be consistent with a geometric Laspeyres at higher levels.

from survey sources, the only weights available must inevitably refer to some earlier period or periods. As mentioned above, this predisposes the XMPIs to some form of fixed-basket index (Laspeyres, Lowe, or Young index, or chained Laspeyres index), though annual chaining should be applied to such indices. However, at some later date estimates must become available of the values of imports and exports in the current period, so that retrospectively it becomes possible to calculate a Paasche-type index and superlative indices such as Fisher or Törnqvist.[32] There is some interest in calculating such indices later, if only to see how the original indices compare with the superlative indices. Some countries may wish to calculate retrospective superlative indices for this reason. Thus, although most of the discussion in Chapter 10 is based on the assumption that some type of fixed-basket index is being calculated, this should not be interpreted as implying that this is the only possibility in the long term. It may be that annually chained Lowe indices are compiled, in which case the compilation, for analytical purposes, of annually chained superlative, fixed-base superlative and fixed-base Lowe indices may be compiled for comparison to identify how the results differ from employing the desirable features of chaining and use of a superlative formula.

L.3 Production and maintenance of higher-level indices

1.309 In practice, the higher-level indices up to and including the overall XMPIs are often calculated as Young indices: that is, as weighted averages of the elementary price indices using weights derived from traded value shares in some earlier weight reference period. This is a relatively straightforward operation, and a numerical example is given in Table 10.5 of Chapter 10, in which, for simplicity, the weight and price reference periods are assumed to be the same. Table 10.6 illustrates the case in which weight and price reference periods are not the same and the weights are price-updated between weight reference period *b* and the price reference period 0. This yields a Lowe index with quantities fixed for period *b*. It illustrates the point that statistical offices have two options when a new price reference period is introduced: they can either preserve the relative quantities of the weight reference period or they can preserve the relative value shares. They cannot do both. Price updating the value share weights preserves the quantities and produces a Lowe index. A Lowe index with quantities fixed in period *b* might be preferred, because it has better axiomatic properties compared with a Young index with value shares from period *b*.

1.310 The weights in the XMPIs need to be updated periodically or problems will result when a fixed set of weights is used for a very long period of time. For example, the prices of consumer durables, especially when adjusted for quality, have been falling relative to other goods, although the quantities purchased and value shares have increased. An out-of-date set of weights would give insufficient weight to these falling prices. In the presence of rapid changes in technology or tastes, the weights need to be updated frequently and not allowed to continue for too long.

1.311 Section C.7 of Chapter 10 notes that the introduction of new weights is a necessary and integral part of the compilation of XMPIs over the long run. Weights have to be updated sooner or later, and some countries actually update their weights each year. Whenever the weights are changed, the index on the new weights has to be linked to the index on the old weights so that the XMPIs inevitably become chain indices over the long term. Chapter 10 also discusses the techniques for linking series together by developing a set of linking factors (coefficients) that can be used for either forward linking or backward linking. This involves calculating the higher-level indices on both the old and new weights during an overlap period.

1.312 In producer price index (PPI) and consumer price index (CPI) number work, the weights may be updated less frequently than annually. This is a result of the relative high costs of establishment and household expenditure surveys. However, there are a sizable number of countries that have for some time annually updated the weights of their CPI, effectively running a continuous household budget survey. For XMPIs, customs data on weights from administrative sources provide the opportunity for a regular updating of weights, and the recommendation is to utilize such information and update and chain-link if possible annually. Apart from the technicalities of the linking process, the introduction of new weights provides an opportunity to undertake a major review of the whole methodology. New commodities may be introduced into the index, classifications may be revised and updated, and even the index number formula might be changed. Annual chaining facilitates the introduction of new commodities and other changes on a more regular basis, but in

[32]In fact, if a Laspeyres index is used and the weights do not change much through time, a geometric Laspeyres index will approximate a Törnqvist index (Chapter 10, Section C.6).

any case some ongoing maintenance of the index is needed whether it is annually chained or not.

L.4 Data editing

1.313 Chapter 10 concludes with Section D on data editing. It is included in Chapter 10 because data editing is a process that is closely linked to the actual calculation of the elementary price indices. Data editing involves two steps: the detection of possible errors and outliers, and the verifying and correction of the data—see also Chapter 6 Section C.[33] Effective monitoring and quality control are needed to ensure the reliability of the basic price data fed into the calculation of the elementary price indices on which the quality of the overall index depends. However, extreme values arise for traded goods and services because price changes are undertaken infrequently. There is much theory and evidence on this. For example, cost or exchange rate changes may not be immediately passed through to prices but stored up and delivered as a large price increase rather than a series of smaller ones. Harsh data editing may take the increase when it occurs to be noise, rather than the signal of an actual price change. It is advised that automatic outlier detection routines be used in conjunction with a system that allows, at least for commodities with substantial trade, an external validation, say by phone contact with the establishment responsible. This is facilitated when the source of the price data is the establishment. Unit value indices are by their nature volatile, and automatic outlier routines may well distort the indices.

M. Organization and Management

1.314 Collecting price data is a complex operation involving extensive work by a large number of statistical office staff and respondents. The whole process requires careful planning and management to ensure that data collected conform to the requirements laid down by the central office with overall responsibility for the XMPIs. The *Manual* describes appropriate management procedures in Chapter 13. Statistical offices may compile unit value indices from customs data and then change the methodology to price indices or hybrid indices. Assuming an existing producer price index exists, the organization and management of the XMPIs should benefit from synergies between the two programs.

1.315 When surveys are the source of price data, staff should be well trained to ensure that they understand the importance of helping respondents select the right transactions for pricing on initiation of the sample. As already explained, one issue of crucial importance to the quality and reliability of XMPIs is how to deal with the slowly evolving set of commodities. Commodities may disappear and have to be replaced by others, but it may also be appropriate to drop some commodities before they disappear if they have become quite unrepresentative. Staff members need appropriate training to give very clear instructions and documentation to guide respondents on how to proceed. Clear instructions are also needed to ensure that respondents report the correct prices when there are discounts, special offers, or other exceptional circumstances.

1.316 The price data reported also have to be subjected to careful checking and editing. Computers using standard statistical control methods can carry out many checks. It may also be useful to send out auditors to verify the quality and accuracy of reported price data. The various possible checks and controls are explained in some detail in Chapter 13.

1.317 Improvements in information technology should obviously be exploited to the fullest extent possible. For example, responding establishments can use some form of electronic data transfer to report their prices or use an Internet-based reporting system set up by the statistical office.

N. Publication and Dissemination

1.318 As noted here and in Chapter 3, the XMPIs are important statistics whose movements can influence the central bank's monetary policy, affect stock markets, influence wage rates and contract settlements, and so on. They are used to deflate their counterpart estimates for national accounts, which are constituent parts of GDP expenditure estimates, and thus can potentially bias estimates of growth and productivity. The analysis of inflation transmission and terms of trade movements requires reliable XMPIs. The public must have confidence in their reliability and the competence and integrity of their compilers. The compilation methods must therefore be fully documented, transparent, and open to public scrutiny. Many countries have an official price statistics advisory group consisting of both experts and users. Its role is not just to advise the statistical office

[33]Systems for outlier detection tailored to the needs of unit value indices are outlined in Technical Annex B, pages 190–198 of the World Trade Organization, International Trade Centre, and UNCTAD (2007).

on technical matters but also to promote public confidence in the index.

1.319 Users of the index also attach great importance to having the index published as soon as possible after the end of the month or quarter, preferably within two or three weeks. On the other hand, most users do not wish the index to be revised once it has been published, and there can be some trade-off between timeliness and the quality of the index. For example, it would be possible to revise the index retrospectively as an analytical series by calculating a Fisher index when the requisite data on updated weights become available, without impacting on the timeliness of the current index.

1.320 Publication must be understood to mean the dissemination of the results in any form. In addition to publication in print or hard copy, the results should be released electronically and be available through the Internet on the website of the statistical office.

1.321 As explained in Chapter 14, good publication policy goes beyond timeliness, confidence, and transparency. The results must be made available to all users, within both the public and the private sectors, at the same time and according to a publication schedule announced in advance. There should be no discrimination among users in the timing of the release of the results. The results must also not be subject to governmental scrutiny as a condition for their release, and the results must be seen to be free from political or other pressures. There are many decisions to be made about the degree of detail in the published data and the alternative ways in which the results may be presented. Users need to be consulted about these questions. These issues are discussed in Chapter 14.

O. Terms of Trade

1.322 Once XMPIs have been compiled and disseminated, there is the question of their uses, something considered in Section C above. One such use is the analysis of the terms of trade of a country. A terms of trade index is calculated for a country as the ratio of its XPI to its MPI—a simple enough calculation. But many users' interest is with the effect of changes in a country's terms of trade on its real income. One framework for such analysis is given in Section C of Chapter 15 of the *2008 SNA*. Chapter 24 of this *Manual* provides an alternative framework that benefits from being derived from the economic theory of price indices, as outlined in Chapter 18. It shows how the product of a superlative export price index and a superlative import price index can account for changes in the country's real net income.

1.323 The chapter builds the analysis in steps. First, the effect of changes in real *export* prices facing the economy on the real income (generated by the market-oriented production sector) of the economy is considered. The analysis takes a production-based theoretical framework to derive theoretical Laspeyres- and Paasche-type indices. Actual Laspeyres and Paasche indices that can be compiled from observed data are then considered as approximations of their theoretical counterparts and directions of bias established. It is shown that an actual Laspeyres index generally understates the hypothetical change in the real income defined by the theoretical Laspeyres index owing to *substitution bias;* that is, the change in the real price of exports will induce producers to substitute away from their base period production decisions in order to take advantage of the relative increase in real export prices. A Paasche index generally overstates the hypothetical (theoretical) change in the real income because an increase in the real price of exports will induce producers to substitute away from their period 1 production decisions. Both the Laspeyres and Paasche theoretical indices are equally plausible, and Fisher as a symmetric average of the two is argued to have much to commend it. The case for superlative indices of export prices, to measure the effects of changes in the price of exports on real income, is then made. Such superlative indices include the Fisher price index. The effect of a change in the real *import* price on real income is similarly considered. Finally, the measurement of the combined effects of changes in real import and export prices on the real income generated by the production sector is appropriately considered in terms of the product of superlative export and import price indices.

1.324 Because households frequently directly import consumer goods and services from abroad, without these goods and services passing through the production sector of the economy, Chapter 24 also considers the appropriate measurement of the affect of changes in the prices of imported goods on a household's cost of living. Theoretical Laspeyres indices are defined as the hypothetical expenditure a household would undertake, using the household preferences of period 0, to attain the same level of utility that it attained in period 0 if it faced the period 0 domestic consumption price but the period 1 import price. The required cost of living index is this hypothetical expenditure compared to the period 0 actual expenditure level. Theoretical Paasche indices are similarly defined in terms of period 1 utility levels. However, there is a change in the direction of bounds in this case, with Laspeyres generally overstating the hypothetical change in the cost of living owing to a change in import prices and Paasche understating it. Again, the Fisher index in particular, and superlative price indices in general, is shown to be appropriate. Moreover, the overall cost of living index is shown to be exactly equal to the product of the (partial) domestic consumption price cost of living index and the (partial) import price cost of living index.

Appendix 1.1 An Overview of the Steps Necessary for Developing XMPIs

1.325 This appendix provides a summary overview of the various steps involved in designing export and import price indices (XMPIs), deriving the index structures and weighting patterns, designing the samples, establishing price collections, calculating indices, and disseminating the results. It also outlines procedures for ensuring that the price samples, index structures, and weighting patterns remain representative. These issues are discussed in more detail in subsequent chapters.

1.326 In following the steps described below, it is important to be mindful of the resources available for the index. Sources of data are of primary importance. It is assumed that data on export and import nominal *values* are available at a detailed level of classification, possibly, where relevant, by country or region of origin or destination. Such data will be the primary source for the weights of the index. That source will need to be supplemented by other data sources when it is limited, unreliable, or simply absent. However, if resources are limited XMPIs may be initially compiled only for merchandise goods for which data on traded values are available from customs documentation. Administrative source data from customs in the form of unit value indices at a detailed level of classification—possibly, where relevant, by country or region of origin or destination—are often used to represent price changes. However, in Chapter 2, the *Manual* cautioned against the use of such indices and advised that establishment survey price indices be compiled. Again resource constraints may be a factor. Such surveys require resources and expertise. If a producer price index (PPI) exists, there are many synergies XMPIs may benefit from. In some cases the PPI establishment price surveys for major establishments that are significant importers and/or exporters may be used as long as it is possible to identify whether—and, if so, the extent to which—price changes for/from the domestic market differ from those for/from the rest of the world. If there is not a PPI and resources are likely to be constrained, the "low-hanging fruit" of establishments responsible for relatively large proportions of trade should be surveyed and unit values or world commodity or mirror price indices relied upon. This gradualist approach requires, as a first step, a rigorous evaluation of each commodity group of the relative payoff and cost of abandoning unit value indices. A good starting point would be a listing by commodity group that includes the weight, the perceived reliability of the unit value series, the likely source and reliability of alternative series, and a grade for the relative cost of obtaining such data. The initial aim would be to identify important commodity groups whose current series are deemed unreliable but for which there are readily available alternative sources. International commodity prices and mirror prices, discussed above, may be usefully employed in some cases.

1.327 However, the use of such hybrid indices should be considered as the initial stage in the development of survey-based XMPIs. A hybrid index should be identified as a very much second-best strategy, part of a gradualist strategy to the development of primarily establishment-based XMPIs. The steps below will be outlined primarily for an establishment-based survey for price collection, though much applies to a unit value–based one.

1.328 Given an adequate resource base, important prerequisites for the construction and compilation of accurate XMPIs are the following:

B. The prices recorded in the indices over time must relate to

i. Commodity specifications that are price determining;

ii. Constant quality commodities with fixed specifications; and

iii. Actual market transactions inclusive of all discounts, rebates, surcharges, and so on;

C. The weights need to be representative of the relevant pattern of transactions over the period for which they are used for index aggregation; and

D. The aggregation formulas used must be appropriate to the needs of the particular index and not yield significant bias or drift.

Basic Steps in XMPI Development

1.329 Ten basic steps can be defined for the design, construction, dissemination, and maintenance of the XMPI indices:

- **1** Determining the objectives, scope, and conceptual basis of the indices;
- **2** Deciding on the coverage and classification structures of the indices;
- **3** Deriving the weighting patterns of the indices;
- **4** Designing the samples for the indices;
- **5** Collecting and editing the prices;
- **6** Adjusting for changes in quality;
- **7** Calculating the indices;
- **8** Disseminating the indices;
- **9** Maintaining samples of reporters and commodities; and
- **10** Reviewing and reweighting the indices.

A summary of the issues involved with each of these steps follows.

Step 1. Determining the objectives, scope, and conceptual basis of the index

1.330 Decisions made following close consultation with users (both external users and internal national statistical agency users such as national accounts) about the objectives of the proposed XMPIs, and hence their scope, are fundamental. Ideally, scope is predetermined by the definitions of exports and imports of goods and services given in the *2008 System of National Accounts (2008 SNA)* and the sixth edition of the *Balance of Payments Manual* (*BPM*6). Practically, countries start with covering goods using primarily customs administrative sources, a less ambitious subaggregate of the target scope. Compilers still will have to decide trade-offs when, as is likely, they find through initial testing that customs unit values are unusable as prices for a large number of commodity strata. A decision is necessary on the extent to which prices of narrowly specified items will be collected in a survey of establishments engaged in international trade and the extent to which the price movements of other items will be imputed with the indices for others whose prices are thought to be correlated with them, or unit value indices used.

1.331 Uses range from economic policy (for example, inflation analysis), to business applications, such as contract price escalation and monitoring of relative performance, industry policy formulation, and volume estimation (for example, national accounts growth estimates). All key stakeholders need to be consulted early in the index design stage to ascertain what their needs are (i.e., what are the questions they are aiming to answer and, hence, what are the characteristics of the required statistics). Stakeholder interest can strongly influence which commodity strata with inadequate customs unit values will use more expensive survey sources for prices, or which services will be covered, again very likely using survey sources for prices.

1.332 Having decided on the objectives and scope of the new XMPIs, it is then necessary to formulate the detailed conceptual basis of the indicators, again in consultation with users as necessary. Conceptual characteristics to be determined include the point of pricing, valuation basis, coverage, and classification structure.

1.333 Decisions on the point of pricing and on the valuation basis of the index largely fall into place once the objectives and scope have been determined. As a rule of thumb, for an output (supply-based) index, the pricing point is ex-producer (e.g., ex-factory, ex-farm, ex–service provider) with a valuation basis of "basic prices" (i.e., reflecting the amount received by the producer exclusive of any taxes on commodities and transport and trade margins). On the other hand, for an input (demand-based) index, the pricing point is "delivered into store" with a valuation basis of "purchaser's price" (i.e., reflecting the amount paid by the purchaser inclusive of any taxes on commodities and transport and trade margins). In trade data the statistical standards for national accounts and balance of payments accounting specify uniform valuation and timing of free on board (f.o.b.) export frontier for both supply and use/demand prices.

Step 2. Deciding on the index coverage and classification structure

1.334 The issue of the actual coverage of the domain of transactions defined by the economic scope of XMPIs can be viewed from several perspectives.

1.335 Of immediate concern will be the desired geographical residency coverage and the coverage of product groups. Weights for merchandise trade should be readily available based on customs sources, and unit value indices may be appropriate—at least in the short term—as surrogates for price indices for some product groups. Decisions have then to be made as to the resource needs to extend the coverage to services and any goods not covered by merchandise trade and to adopt establishment surveys or other source data for the likely many product groups for which customs unit value indices are unsuitable.

1.336 Choices need to be made as to whether *non-market* transactions should be included or excluded. The decision will be based on a consideration of the primary objective of the index and on practical pricing considerations. Similarly, practical decisions need to be made about whether efforts should be expended on trying to capture price changes of goods and services transacted in the nonobserved (hidden) economy. Issues such as the relative size of the non-observed economy and its accessibility for price measurement need to be considered. The nonobserved economy shows itself in international trade as, for example, smuggling and under- or overvaluation of transactions to avoid taxes.

1.337 Other coverage issues include the treatment of intracompany transfer prices and capital work on own account. This looms very large in the international trade of many countries, in view of the increasingly international and interdependent organization of production. A decision needs to be made on whether these flows are to be included or excluded in trade statistics and their price and volume decomposition. If they are to be included, an assessment needs to be made about whether the book entry valuations recorded in the company accounting records are realistic in terms of being contemporary market-based estimates, or are merely notional estimates. If the latter, the preferred approach would be to assign the weight associated with these transfers to the prices obtained from businesses engaging in arm's-length trading. For many countries, omitting international trade occurring between affiliates is not viable because it is a large fraction of total trade.

1.338 Compilers can construct XMPIs under alternative classification structures. The most common constructs are based on commodity, geographical destination (exports) or source (imports), and industry of reporting establishments. International commodity classifications (e.g., the Harmonized System [HS] of Commodity Description and Coding, the Standard International Trade Classification [SITC], and Central Commodity Classification [CPC]), a geographical classification (Standard Country or Area Codes for Statistical Use), and industry classifications (International Standard Industrial Classification [ISIC] of All Economic Activities) are available for use in index construction to ensure adherence to accepted statistical standards and facilitate international comparisons. Many countries or regions have developed local adaptations of these classifications that still conform to the underlying principles. Data may be published aggregated under more than one classification system and be available at different levels of detail to meet the needs of different users.

1.339 Formal classifications are hierarchical in nature. For example, CPC covers all goods and services produced in an economy and provides for the progressive aggregation of data from a fine level of detail (e.g., soft drinks), through successively broader levels of aggregation (e.g., beverages; food, beverages, and tobacco; total). In designing an index classification structure, it is important to consider issues such as the following:

- *Publication goals.* In particular, the level of detail to be released, whether the indices will be national only or include regional series, and the needs of internal users; and
- *Potential bias in the index owing to commodity replacement and new goods.* There are opportunities to minimize such bias through grouping commodities that are close substitutes.

1.340 Having determined the index classification structure, the weighting pattern needs to be derived and issues of sample design and price collection addressed.

Step 3. Deriving the weighting pattern

1.341 A price index can be considered as being built up from samples of prices of individual transactions (or their price relatives) that are progressively weighted together through successive levels of aggregation within a classification framework.

1.342 In considering the development of an index-weighting pattern, two different categories of indices need to be considered: lower-level indices (sometimes referred to as elementary aggregates) and upper-level indices.

1.343 The lower-level indices are built up by combining together the individual prices using one of a range of available price index formulas. At this initial level of aggregation, the internal weighting can be either *explicit* or *implicit*. If *explicit* weights are used, then, as part of the price collection activity, it is necessary to obtain relevant value data (e.g., commodity exports). This is discussed further under Step 5 below. On the other hand, if *implicit* weights are used, then the design features of the sampling techniques employed to select the commodity specifications for pricing need to result in the prices being "self-weighted." Such a result would be achieved, for example, by using probability sampling proportional to size.

1.344 Upper-level indices are formed through weighting together lower-level indices through progressive levels of aggregation defined by the classification structure, usually employing weights that are fixed for a period (say, one, three, or five years) between index reweighting.

1.345 The selection of the level in the index hierarchy at which the structure and weights are fixed for a period is particularly important. The main advantage of setting the level relatively high (e.g., at the four-digit commodity group level) is that the price statistician then has greater discretion to update the lower-level price samples (at the establishment and commodity levels), their structure, and their internal weighting on a needs basis as market activity changes. New commodities and establishments can be introduced easily into the samples, and the weights at the lower level reestablished on the basis of more recent market conditions. That is, there is greater opportunity to keep the index representative through an ongoing program of sample review (see Step 9).

1.346 On the other hand, if the level is set relatively low in the index structure, there is less freedom to maintain the representativeness of the index on an ongoing basis, and there will be a greater dependence on the periodic index review and reweighting process (see Step 10). In such circumstances, the argument for frequent reweighting becomes stronger.

1.347 Assume an export price index is to be developed with the broad index structure based on CPC. In order to derive the upper-level weighting pattern, a data source is required; potential sources include industry surveys, economic censuses, input-output tables, and international trade statistics.

1.348 The relevant values need to be assigned to each of the industry groupings, taking a top-down approach. It may be appropriate to assign the values associated with international trade that is not going to be directly priced in the index (either because it is too small, or because of practical pricing difficulties) to a related commodity in order to maintain the correct broad weighting relativities. The assumption underlying this practice is that the price movements of the unpriced commodities are more likely to be similar to those of related commodities than to those of the aggregate of all the commodities priced in the index.

1.349 Weights aim to be representative of the pattern of transactions expected to prevail during the period for which they are used in the index construction (perhaps one year, or five years, depending on the frequency of reweighting). It may therefore be necessary to adjust some of the values to *normalize* them and overcome any irregularities in the data for the particular period from which it is being sourced (e.g., as a result of a one-off increase in exports of a commodity in response to a temporary increase in demand). Alternatively, the weights may be *smoothed* by basing them on data from a run of years (say, three years). Other adjustments may be needed to overcome problems of seasonality that are discussed in Chapter 23.

1.350 If the price reference (base price) period of the index is different from the period from which the value weights are derived, then the weights can be *revalued* to the prices of the price reference (base price) period using relevant price indices in order to ensure that the weights are effectively based on the underlying quantities or volumes.

1.351 Having assigned weights to the upper-level index structure that are to be fixed for a period of one or more years, the next step is to consider the lower-level index construct and the sample design.

1.352 If explicit lower-level weighting of price samples is to be incorporated, then international transactions data will need to be obtained directly from reporters during the process of establishing price collections (Step 5). Often businesses will have data on the value of transactions as well as their price, and such value data allows for further lower-level weighting for the items selected to be priced along with the items they represent. As noted above, the price data may be derived from existing sources, primarily unit value indices, but also from world commodity prices and mirror indices. Steps 4 to 6 are concerned with collecting prices of representative items from representative establishments.

Step 4. Designing the sample

1.353 Consider the example of soft drink exports in Step 2, and assume that this is an elementary aggregate product group with a fixed weight of, say, 2.45 out of 100, within the upper-level index structure. The compilers in a country know virtually all exported soft drinks involve domestic producers (rather than sales from stocks held by households, for example, possibly in contrast with used cars). It is now necessary to choose techniques for selecting samples of businesses (statistical units) to provide transaction prices of a selection of representative commodities on an ongoing basis. The prices, or price movements, collected from different businesses will be aggregated to form indices.

1.354 To select a sample of businesses, the first step is to identify the sample frame (that is, a listing of the population of units from which to select). A very good source for a frame would be the set of companies having filed export declarations for soft drinks in the past two years. Other possible frame sources include registers of businesses maintained by national statistical agencies, trade organizations, commercially maintained lists (e.g., as used for marketing mailings), company registers, taxation records, telephone directory "yellow pages," and so on, or some combination of such sources.

1.355 Next is the selection of a sample of such businesses. Either probability (scientific) sampling or nonprobability (judgmental) sampling techniques can be used,[34] and the choice may be based largely on practical resource considerations. Some agencies use a combination of techniques—for example, scientific sampling to select the businesses and judgment sampling to select the commodity specifications for pricing.

1.356 In deciding how to select the sample of businesses, the degree of industry concentration is a relevant consideration. For example, in a highly concentrated industry dominated by, say, three businesses producing more than 90 percent of the output, it may be acceptable to aim for high, rather than complete, coverage, and to select only the three largest businesses.

1.357 However, as the degree of concentration decreases, the need is greater for the sample to include a selection of smaller businesses. If, for example, the three largest businesses account for less than 70 percent of the industry exports, with the remaining 30 percent being produced by a large number of small businesses, it may not be possible to achieve adequate representation of price movements by relying only on prices reported by the three largest businesses. That is, it may not be reasonable to assume that the pricing behavior of the small businesses mirrors that of the large ones, because, for example, they may target separate niche markets and direct their pricing strategies accordingly. Therefore, it would be prudent to select a sample of the small businesses to represent the markets they serve.

1.358 The less concentrated the industry structure is, the stronger the case is for using probability sampling techniques. Experience has shown that, although many manufacturing and mining industries may be dominated by a few large businesses, many service industries have a very large number of small businesses and, if there are any large businesses, they produce a relatively small proportion of the output, and this may apply well apply to services directed to the export market. An added advantage of probability sampling techniques is that they enable sampling errors to be calculated, which provide some guide to the accuracy of the resultant indices.

1.359 Procedures need to be implemented to ensure that samples of businesses remain representative through, for example, regularly augmenting the sample by enrolling a selection of new businesses as they enter the market. Also, a sample rotation policy needs to be considered in order to spread the business reporting load.

1.360 Once the sample of businesses has been selected, they need to be contacted to agree on a sample of representative commodity specifications for ongoing price reporting. This is discussed further under Step 5.

Step 5. Collecting and editing the prices

1.361 The main source of ongoing price data is usually a sample of businesses. The sample can relate to either buyers or sellers, or a combination of both. The choice will be influenced by the pricing point of the index (input/export or output/import) and practical considerations such as the relative degree of concentration of buyers, and of sellers, and the implications for sample sizes and costs.

[34]Judgmental sampling should be avoided, if possible. Often cutoff sampling, as discussed in Chapter 5, Section D, can be used in place of judgmental sampling.

1.362 The statistical units to be sampled may be head offices reporting national data, establishments reporting regional data, or a mixture. Decisions on the units to be surveyed may be based largely on pragmatic grounds, such as efficiency of collection, location of relevant business records, and so on.

1.363 The aim of the price collection is to enable the calculation of reliable indicators of period-to-period—say, monthly—price change. As such, choices need to be made as to the type and frequency of pricing. For example, point-in-time prices may be the easiest to collect and process (e.g., transaction prices prevailing on a particular day, say the 15th of the month) and commonly prove to be reliable indicators. For workload management, it may be decided to spread pricing over the reference period with, say, three or four pricing points and different commodities priced on different days.

1.364 For commodities with volatile prices, it may be necessary to price them on several different days of the month and calculate time-weighted averages; alternatively, businesses can be asked to provide weighted average monthly prices (usually derived by dividing the monthly value of commodity sales by the quantity sold). This approach should be avoided because it is susceptible to the unit value "mix" problem, where commodities of different qualities are included.

1.365 The most appropriate pricing methodology to use is *specification pricing,* under which a manageable sample of precisely specified commodities is selected, in consultation with each reporting business, for repeat pricing. In specifying the commodities, it is particularly important that they be fully defined in terms of all the characteristics that influence their transaction prices. As such, all the relevant technical characteristics need to be described (e.g., make, model, and features) along with the unit of sale, type of packaging, conditions of sale (e.g., delivered, payment within 30 days), and so on. This technique is known as *pricing to constant quality*. When the quality or specifications change over time, adjustments must be made to the reported prices (see Step 7).

1.366 Another important consideration in establishing and maintaining price collections is to ensure that the prices reported are *actual market transaction prices*. That is, they must reflect the prices paid for commodities imported by resident units inclusive of all discounts applied to the transactions whether they be volume discounts, settlement discounts, or competitive price-cutting discounts, as well as any premiums, which are likely to fluctuate with market conditions. Any rebates also need to be considered. List or book prices do not reflect actual transactions, are unlikely to yield reliable price indices, and may produce quite misleading results because they do not capture fluctuations in market prices. Care should be taken to ensure that the currency of the returned price is clearly denoted, so that prices over time in different currencies are not compared. This is particularly important when respondents are providing, for example for exports, price information for sales to both the domestic and export markets. Procedures should be in place to convert all returned prices to home currency values. The data supplied by the contributor should be in the currency in which the transaction took place; then the currency conversion, undertaken in the national statistical office, should ideally follow the principles in the *2008 SNA*. The midpoint between the buying and selling rates should be used and the timing should be at the rate prevailing at the time the transaction takes place, which may differ from the time the payment is made. In practice some rule may be used, such as using mid-month exchange rates.

1.367 The principles underlying the selection of the sample of commodity specifications from a particular business, whether using probability or nonprobability sampling, are similar. That is, the international sales (purchases) of the business and the commodity markets are stratified into categories with similar price-determining characteristics. For example, in selecting a sample of specific motor vehicles in consultation with the manufacturer, the first dimension may be the broad category of vehicle (e.g., four-wheel-drive recreational vehicles, luxury cars, family cars, and small commuter cars). These categories will reflect different pricing levels as well as different pricing strategies and market conditions. A further dimension may be to cross-classify by the type of market (e.g., destination country of exports).

1.368 Then, from each of the major cells of the matrix of vehicle category by market, a sample of representative vehicles can be selected, with each one representing a broader range of vehicles.

1.369 If *explicit* internal weights are to be used in the construction of the lower-level indices (e.g., for motor vehicle exports), then the relevant sales data for (1) the individual vehicles in the sample, (2) the wider range of vehicles being represented (i.e., as defined in the matrix of vehicle category by market), and (3) all vehicles should be collected from the business for a recent period. This will enable internal weights to be calculated for combining the prices of individual commodity specifications and the prices of different producers.

1.370 Ideally, a statistical agency will initialize collection from a business through a personal visit. However, this is an expensive exercise, and budgetary considerations may necessitate a compromise. Alternative, though less effective, approaches to initialization include telephone, Internet, fax, and mail contact, or some combination of approaches. At a minimum, the larger businesses and those producing complex (e.g., high-tech) commodities and operating in changing markets should be visited.

1.371 In cases where the commodities are unique and not reproduced over time—for example, machinery, ships, and many customized business services—specification pricing is not feasible, and alternative pricing techniques must be used, often involving compromise. Possibilities include model pricing, input cost pricing, and weighted averages of rate changes for profiles of customer needs (e.g., for a banking service).

1.372 Most national statistical agencies use mail questionnaires to collect their export and import prices, though there is an increasing use of electronic communication. Collection procedures include designing tailored forms incorporating the particular commodity specifications for each sampled business and controlling collection to facilitate dispatch, mark-in, and follow-up of price reports with the participating businesses.

1.373 It is important that rigorous *input editing* techniques are employed, and that any price observations that do not appear credible are queried (usually by telephone) and either confirmed with an acceptable reason or amended. Input editing involves analyzing the prices reported by an individual business and querying large changes (editing tolerances may be built into processing systems) or inconsistent changes across commodity lines. An important objective of the editing process is to ensure that actual transaction prices are reported, inclusive of all discounts, and to detect any changes in the specifications.

1.374 If the price of a commodity has not changed for several months it may be appropriate to contact the business to make sure the prices reported are not being automatically repeated.

1.375 *Output editing*, which is often an integral part of calculating the lower-level indices (see Step 7), involves comparing the price levels, and price movements, of similar commodities between different businesses and discretely querying any outliers.

1.376 In undertaking these editing processes, reference to other supporting price information is often valuable. Examples include international commodity prices (e.g., London Metal Exchange), exchange rates, press and wire service reports, and general market intelligence obtained during the sample maintenance activities described under Step 9.

1.377 Alternatives to the traditional mail questionnaire include telephone, e-mail, Internet, telephone data entry, fax, and the use of electronic data transfer from company databases. Several national statistical agencies have had experience with at least some of these alternatives. Important factors to be considered are data security, the convenience of reporting for the business, cost, and effectiveness.

Step 6. Adjusting for changes in quality

1.378 The technique of *specification pricing* was outlined under Step 5. The objective is to *price to constant quality* in order to produce an index showing *pure price change*. This is the most common technique employed by national statistical agencies in compiling XMPIs.

1.379 To the extent that pricing is *not* to constant quality, then, over time, the recorded prices can incorporate a nonprice element. For example, if a commodity improves in quality and its recorded price does not change, there is an effective price *fall* because an increased volume of commodity is being sold for the same price. Conversely, if the quality of a commodity declines without a recorded price change, there is an effective price *rise*. In such circumstances, the recorded price of the new commodity of changed quality needs to be adjusted so that it is directly comparable with that of the old commodity in the previous period.

1.380 Failure to make such adjustments can result in biased price indices and consequently biased constant price, or volume, national accounts estimates.

1.381 It is possible to identify fairly readily the main price-determining characteristics of many goods (e.g., a washing machine) that are mass produced to fixed technical specifications and can be readily described in terms of brand names, model codes, and so on. However, specification pricing cannot be used for customized goods such as the output of the construction industry. Nor can it be used for much of the output of business service industries (such as computing, accounting, and legal services) because it is unique in nature (each transaction is commonly tailored to the needs of an individual client). Further, it is far more difficult to identify all the price-determining characteristics of many services because some are intangible. In such cases, other approaches to pricing to constant quality may be employed, for example, model pricing (see Step 5).

1.382 Even in areas that do lend themselves to specification pricing, problems arise when there are *changes* to the specifications, and hence the quality, of the commodities over time. Examples of possible commodity changes would include the following:

- Presenting it in new packaging;
- Selling it in different size lots (e.g., 1 kg. packets of sugar replaced with 1.2 kg. packets); and
- Replacing it with a commodity with different technical and design characteristics (e.g., a new model of motor vehicle).

1.383 The first step, in consultation with the provider, is to fully identify the changes and assess whether they are, in fact, quality changes.

1.384 The first example above (new packaging) may be deemed to be cosmetic only; alternatively, it could be assessed as being substantive if, for example, it led to a reduction in the damage to the contents. In the latter case, a value would need to be placed on the improvement on the basis of some estimate of the value of reduced damage.

1.385 The second example (change in size lot) would be likely to involve an office adjustment based on matching the new and old prices per a common unit of measure (e.g., price per kilogram).

1.386 The third example (new model of motor vehicle) is the most complex. Possible techniques include using an assessment of the difference in the production costs of the old and new models to adjust the price of the new model. Alternatively, the different commodity characteristics can be identified and a value placed on them. The valuation can be based on consultation with the producer or, if the new model has features that were available as options on the old one, market prices will exist for those options. In cases where the old and new model are sold (in reasonable volume) in parallel, the difference in the overlapping transaction prices may be taken as a guide to the value of the quality difference.

1.387 Increasingly, national statistical agencies are researching and selectively implementing hedonic regression techniques as a means of placing a market value on different characteristics of a commodity—for example, the value of an additional unit of RAM on a personal computer. When the characteristics of a particular commodity change, these techniques enable its price to be adjusted to make it directly comparable with that of the old model. Unfortunately, hedonic techniques tend to be costly, involving extensive research and analysis, and the collection of large volumes of data.

Step 7. Calculating the index

1.388 Under Step 3, the two categories of indices were described: lower-level and upper-level indices. Having established the structure and weighting pattern of the index, constructed a processing system, and established the regular price collection, the first step in the routine production cycle is to aggregate the input-edited prices to form the lower-level indices. There is a range of micro-level index formulas available for use, each being based on different assumptions about the relative behavior of prices and quantities in the economy (see Chapters 16 and 18).

1.389 The initially compiled lower-level indices should be scrutinized for credibility in terms of the latest period movement, the annual movement, and the long-term trend. *Output editing,* involving comparisons of price levels and movements between different businesses, is an integral part of the credibility checking. Reference to the type of supporting information described under Step 5 will be valuable for this analysis.

1.390 Despite the most rigorous collection processes, there are often missing prices that need to be imputed. Prices may be missing either because the provider failed to report on time or because there were no transactions in that commodity specification in the relevant period. Imputation techniques include applying the price movements of like commodities to the previous period's price observations. The like commodities may be reported either by the same business or by other businesses. Another approach is to simply repeat the previous period's prices, but this approach should be generally avoided and used only if there is reasonable certainty that the prices have not changed.

1.391 Once the price statistician is satisfied with the lower-level index series, the series should be aggregated to form the hierarchy of upper-level indices, including the total measure. This aggregation is undertaken using the classification structure and weighting pattern, determined in Step 2, and an appropriate index formula.

1.392 Extensive studies have concluded that the theoretically optimal formulas for this purpose satisfy a range of tests and economic conditions, and as a class are known as superlative formulas (Chapter 16). A basic characteristic of such formulas is that they employ weights symmetrically based on volume data from both the current period and the period of index comparison. In practice, because the volume data from the current period are not available at the time of index construction, the use of a superlative formula would necessitate the estimation of the current period's volume data in order for timely indices to be produced. When the current period volumes subsequently became available, the index numbers would need to be recompiled using the actual volumes, and the earlier index numbers revised. This ongoing cycle of recompilation and revision of published index numbers may initially encounter resistance from users unaccustomed to revised price series (as explained under Step 8). Therefore, most national statistical agencies compromise and use a base-weighted Laspeyres-type formula, such as Lowe or Young. Agencies can acclimate users to the revised, but more accurate, chain superlative series by producing these series alongside the unrevised fixed-basket series, measuring the bias in the unrevised series against the more accurate superlative one.

1.393 The upper-level indices are aggregated across commodities, regions of the world, industries of reporters, and/or commodity stages of processing, as defined in Steps 2 and 3, to produce the aggregates required for publication (Step 8).

1.394 Finally, annual average index numbers and the suite of publication and analytical tables should be produced and the commentary on main features prepared for publication (see Step 8). It is prudent to apply broad credibility checks to the aggregates before release. Are the results sensible in the context of the prevailing economic conditions? Can they be explained?

Step 8. Disseminating the indices

1.395 During the initial user consultation phase described under Step 1, and the formulation of the index classification structure under Step 2, broad publication goals will have been formulated. It is now time to refine and implement these goals, probably involving further user contact.

1.396 As well as releasing time series of index numbers for a range of industries or commodities or stages, and aggregate measures (e.g., all groups), user analysis can be enhanced by the release of time series of percentage changes, as well as tables presenting the contribution that individual components have made to aggregate index point changes. This latter presentation is particularly important to help gain an understanding of the sources of inflationary pressure.

1.397 Different tabular views of the data can be provided. For example, classification could be done by the following:

- Destination (source) country of exports (imports);
- Economic destination—consumer or capital goods;
- Commodity and industrial activity of the reporting establishments.

1.398 Some form of analysis of the main movements and, ideally, the causes of those movements, should be provided. These will be based on the percentage change and point-contribution tables described above.

1.399 In addition to the summary tables, analytical tables, and detailed tables, explanatory notes should outline the conceptual basis of the index including the objectives, scope, coverage, pricing basis, sampling techniques, and data sources. The weighting patterns should also be published. Any caveats or limitations on the data should be included to caution users.

1.400 As well as release in hard copy form, electronic delivery and access through the Internet website of the national statistical agency should form part of the overall dissemination strategy.

1.401 In terms of timeliness of release, there will be a trade-off between accuracy and timeliness. In general, the faster the release, the lower the accuracy of the data, and hence its reliability, as the need for revisions increases. Price index users—whether they be public policy economists, market analysts, or business people adjusting contract payments—place a high value on certainty (i.e., the nonrevisability of price indices). Accordingly, some compromise in the timeliness of release will probably need to be made in order to achieve a high degree of certainty and user confidence.

1.402 Policies need to be developed in relation to the following:

- Security of data through the uses of a strict embargo policy;
- Publication selling prices and electronic access charges based on relevant principle—for example, free, commercial rates, cost recovery, or rationing of demand; and
- Community access to public interest information—for example, through free provision to public libraries.

1.403 Ongoing consultation with users should be maintained to ensure that the indices, and the way they are presented, remain relevant. The establishment of a formal user group, or advisory group, should be considered.

Step 9. Maintaining price series and samples of businesses and commodity specifications

1.404 Some of the necessary prerequisites for the production of an accurate price index are to incorporate prices that, over time, relate to the following:

- Commodity specifications that are representative indicators of price change;
- Constant quality commodities with fixed specifications; and
- Actual market transactions inclusive of all discounts, rebates, and so on.

1.405 Step 5 above expanded on these principles and outlined the methodology for selecting the sample of commodity specifications from a business at initialization, preferably by personal visit.

1.406 However, the initial evaluation of suitable data sources and balance of price data from establishment surveys, unit value indices, mirror indices, international commodity markets, and PPI sources will need to be reviewed on a regular basis, especially if resources change and expertise develops. It may be that what was once considered a homogeneous product group that had relatively stable price changes with relatively limited dispersion is no longer so owing to market developments and unit value indices may be abandoned, or that data from an establishment survey is no longer reliable or available.

1.407 Given the dynamics of many marketplaces in terms of changing commodity lines and marketing strategies, it is important that procedures are put in place to ensure that the commodity samples remain representative and have fixed specifications, and that the prices reported incorporate all discounting.

1.408 Further, if explicit internal weighting is used in the lower-level index aggregation, these weights need to be monitored and updated as necessary, on a component-by-component basis.

1.409 Ideally, a rolling program of evaluating data sources and, for establishment surveys, regular interviews of the sampled businesses should be established to undertake these reviews on a fairly frequent basis. Costs may prohibit regular visits to all of the businesses, so it may be necessary to prioritize them according to factors such as their weight in the index, the extent of technical change in the industry, and the volatility of the markets. A program may be devised such that the high-priority businesses are visited frequently and the lower-priority ones visited less frequently or contacted by telephone.

1.410 In addition to these structured proactive reviews, resources should be made available to enable a quick reaction to changed circumstances in relation to a particular commodity or industry and to undertake specific reviews on a needs basis. For example, competitive pressures resulting from deregulation of a particular industry may quickly, and radically, transform the commodity lines and methods of transacting and produce substantial market volatility. Examples in recent years include the deregulation of the electricity supply, telecommunications, and transport industries in many countries.

1.411 The samples of businesses also need to be reviewed, either through a formal probability-based sampling process incorporating a rotation policy or by some more subjective approach that includes initialization of price collections with substantial new businesses as they enter the market.

Step 10. Reviewing and reweighting the index

1.412 Other necessary prerequisites for the production of an accurate and reliable price index are that

- The weights need to be representative of the relevant pattern of transactions over the period for which they are used for index aggregation; and
- The aggregation formulas used must be appropriate to the needs of the particular index and not yield significant bias or drift.

1.413 Studies have concluded that, in practice, price indices are often not highly sensitive to small errors in weighting patterns. However, the greater the variation in price behavior across different commodities is, the more important are the weights in the production of an accurate measure of aggregate price change.

1.414 Assuming that a rolling sample review program is in place for the maintenance of price samples and the lower-level internal weights (see Step 9), then the question of the frequency of reweighting of the upper-level indices (which were established under Step 3) needs to be considered. Alternatively, if no such sample review program is in place, a strategy needs to be put in place for the periodic reweighting of the entire index (lower and upper levels) along with a complete review of the commodity samples.

1.415 Practices in this regard vary among national statistical agencies. Some agencies update the upper-level weights on an annual basis and link the resultant indices at the overlap period such that there is no break in continuity of the series. That is, if the link was at June 2000, then the "old" weights would be used to calculate the index movements between May and June, and the new weights used to calculate the index movements between June and July (and subsequent months), with the July movements "linked" onto the June level. This process is termed annual *chaining* or *chain linking.*

1.416 For XMPIs in which timely and reliable administrative customs data form the basis of the data source for the weights, it should be possible to reweight annually with a relatively short lapse in time between the price reference and weight reference period. If annual chaining is not possible the indices can be reweighted on a less frequent basis, perhaps once every three or five years. Considerations in making decisions on the frequency of reweighting other than the availability of source data include the following:

- Changes over time in the pattern of transactions covered by the index: the greater the volatility in the transaction patterns, the greater the need for frequent reweighting to maintain the representativeness of the weights;
  - If the trading patterns are highly volatile, it may be desirable to "normalize" or smooth them by using data from a run of years in order to mitigate against *chain linking bias* or *drift;*
  - If the trading patterns are relatively stable and tend to shift on a trend basis, very frequent reweighting is of little benefit, and it may be assessed that reweighting every three, five, or more years is adequate;
- The availability of reliable and timely weighting data sources; and
- Resource constraints.

1.417 If reweighting is done on an infrequent basis using data from a single year, it is important that a *normal* year is selected in terms of providing weights that can be expected to be representative of the period (say five years) for which they are used in the index. Again, the use of data from a run of years may be prudent.

1.418 In addition to developing a reweighting strategy, it is desirable to undertake thorough periodic (say every 5 or 10 years) reviews of the XMPIs to ensure that the conceptual basis is still relevant to the needs of users.

Summary

1.419 Early consultation with users and decisions on the scope and conceptual basis of the XMPIs are fundamental to the production of *relevant* indices. In order for an index to be *accurate*, it must be constructed using indicative transaction prices, measured to constant quality, and use representative weights.

1.420 The issue of the reporting burden is an important consideration in seeking the cooperation of businesses, and, along with resource constraints facing national statistical agencies, heavily influences decisions on sampling strategies and other methodological matters. Ensuring the security of often commercially sensitive price data is another essential prerequisite to building good business relationships.

1.421 A dissemination strategy that meets the needs of the wide variety of users must be developed, and ongoing consultation maintained, to ensure that users' requirements continue to be met.

1.422 It is important to appreciate that a price index seeks to provide contemporary information in relation to dynamic markets. As such, it is not sufficient to develop a new index framework, establish the collection of the price samples, and simply aggregate them over time. Mechanisms need to be put in place to ensure the ongoing integrity and representativeness of the measure. That is, the price samples and weights need to be systematically reviewed and updated periodically.

2. Unit Value Indices

A. Introduction

2.1 Export and import price indices (XMPIs) are compiled by three general methods, the nature of which largely depends on the source data used. The first and predominant method,[1] in terms of the number of countries using it, uses unit value indices compiled from detailed import and export merchandise trade data derived from administrative customs documents. Unit value indices are not price indices because their changes may be due to price *and* (compositional) quantity changes. However, they are used by many countries as surrogates for price indices.[2] The second method is to compile price indices using data from surveyed establishments on the prices of representative items exported and imported. The surveyed prices should be of items that are defined according to detailed specifications, so that the change in price of the same item specification can be measured over time. Third, there is a hybrid approach that involves compiling establishment survey–based price indices for some product groups and customs-based unit value indices for others. It is usually the case with hybrid indices that one type of index is the exception, used when the principal type of method is considered less suitable than the alternative. For example, some oil-producing countries use unit value indices, but because detailed reliable data are readily available from the oil-producing establishments for this important sector, the unit value indices are complemented by survey-based price indices or price quotations from international markets. Alternatively, price indices may be the norm, but unit value indices are used for exceptions such as relatively homogeneous product groups whose composition of traded goods and services, in terms of the quality mix of items traded each period, is considered to be unlikely to change significantly.

2.2 International guidelines on choosing among these alternatives methods—*Strategies for Price and Quantity Measurement in External Trade*—were provided by the United Nations (1981) and are briefly outlined in Section B of this chapter. The strategic case for customs-based unit value indices in United Nations (1981) was based on the relatively low cost of such data. Unit value indices were advised for countries with a tight or medium budget, and well-endowed countries were advised to base their external trade price indices on establishment survey data. The preference for price survey indices was, in large part, owing to errors in unit value indices attributed mainly to changes in the mix of the heterogeneous items recorded in customs documents, but it was also attributed to the often poor quality of recorded data on quantities. The former is particularly important in modern product markets given the increasing differentiation of products and turnover of differentiated products. Statistical offices working with customs authorities can make some improvements in the quality of quantity data, and commodity descriptions used by customs (that is, the Harmonised System outlined in Chapter 4) are periodically updated in response to changes in product markets. In almost all countries both the customs administration and the national statistical office make extensive use of computer equipment. Practically all customs declarations today are captured electronically. This implies that customs data are verified, available in great detail, and available to the statisticians in a very timely manner. Even in the least-developed countries, customs administrations are computerized owing to the Automated System for Customs Data (ASYCUDA)

[1]The United Nations (2007) survey on country practice—see also United Nations (2005)—found that for 88.4 percent of countries, customs declarations remain the main source of data. However, there was a considerable difference in country practices because only 55.6 percent of developed countries confirmed that customs declarations are the main source of data, whereas 97.9 percent of developing and transitional countries did.

[2]It may be argued that unit value indices are not a method for compiling price indices in that they are a distinct concept. However, they are used in economic analysis as surrogates for price indices, and there is no distinct conceptually useful area of analysis for which they are designed and solely used.

project[3] of the United Nations Conference on Trade and Development (UNCTAD). Yet unit value indices have suffered further in recent times owing to an increasing irrelevance of the source data, first, with increasing proportions of trade being in services and by electronic commerce (e-commerce) and hence not covered by customs data. Second, countries in customs and monetary unions are unlikely to have intra-union trade data as a by-product of customs documentation. Finally, some trade may not be covered by customs controls, such as electricity, gas, and water, or be of "unique" goods, such as ships and large machinery, with profound measurement problems for unit values.

2.3 Few, including United Nations (1981), deny that price indices based on surveys of narrowly specified products provide the best measures of relative price change and that there are potentially significant biases in using customs unit value indices to measure export and import price changes. Yet, unit value data are readily available from customs administration systems at a relatively low incremental cost (compared with obtaining price surveys of establishments needed for narrow specification prices). In view of the low cost of the data, the bias in unit value was judged by United Nations (1981) to be tolerable enough that countries were advised to continue compiling them if they do not produce narrow specification price indices. Notwithstanding the putative low cost of obtaining unit values, in this chapter the *Manual* revisits this strategic advice. More than 25 years has passed since the United Nations (1981) publication and there have been major developments that impose on this strategic issue of choice of method, including developments in the analytical tools by which such methods are evaluated; further evidence regarding the validity of unit value indices as surrogates for price changes; the increasing extent of product differentiation, and associated developments in the classification system and documentation upon which customs data are based; the automation of customs data records; and developments in the principles and practice of price index number methodology.[4]

2.4 Unit value indices as measures of price changes of imported and exported goods serve economic analysis in many important ways. They are used as short-term indicators of inflation transmission, to measure changes in a country's terms of trade (effect), and as deflators of export and import values to yield measures of changes in export and import volumes. An issue of strategic concern is whether unit value bias misleads economists in their analysis to the extent that their compilation and use for countries with resource constraints should not be deemed a sensible alternative but as only the first step of a program of hybrid indices that increasingly limits, over time, any reliance on unit value indices.

2.5 Index numbers are generally calculated in two stages. The first stage is the building blocks of price indices, the measurement of price changes of similar "elementary" items exported or imported by one or more establishment (or institutional unit)—say, a front-loading washing machine of given quality characteristics including size, spin speed, and energy saving. The resulting indices from these elementary aggregates are referred to as *elementary aggregate indices* or, more simply, *elementary indices*. Data on weights are not available at this level of aggregation, and Chapter 21 considers the issue of appropriate unweighted formulas for elementary indices. At the next stage of aggregation, weights are applied to the elementary indices, and weights are again applied to the resulting indices at higher stages of aggregation, until an overall index is derived. Unit value indices derived from customs data use unit value indices to compile elementary indices, the elementary indices then being weighted at higher levels. Standard index number theory applies to the issue of weighted formulas at the higher levels, as outlined in Chapters 10, 16–18, and 20, irrespective of whether the elementary indices are compiled from unit values derived from customs data or price indices based on establishment surveys. Compilers of unit value indices as well as of price indices are referred to these chapters for higher-level aggregation formulas. *The concern here is with the suitability of unit value indices as measures of price changes at the lower elementary level.*

2.6 This chapter first, as background, in Section B briefly outlines the recommendations on export and import price measurement given in the United Nations' statistical manuals and handbooks. The nature of unit value indices and the circumstances under which they may be reliably used is next considered in Section C. If unit value indices are to be used it is essential that compilers and users have a sound understanding of their properties. Given the potential for concern arising from the conclusions of Section C, Section D considers the

[3]ASYCUDA is functioning in about 90 developing countries. That system verifies declaration entries immediately. Declarations need to be completely filled in to receive customs clearance. This means among other things that quantity information is required. In addition, customs values are validated—to avoid undervaluation—using unit values on the declaration that are matched against a predetermined list of commodity prices.

[4]These prompted the writing of the consumer price index (CPI) and producer price index (PPI) manuals by ILO and others (2004a and 2004b, respectively).

evidence on the suitability of such indices. Such evidence is by its nature limited to countries that compile both price indices and unit value indices. It is also limited to the fact that the deficiencies in unit value indices are not measured against a perfect benchmark because the price indices themselves have deficiencies. Yet, as will be outlined, unit value indices suffer from not comparing the prices of like with like when the commodity description used for customs purposes is too broad and the relative share of each kind of item it covers shifts over time. Establishment-based price indices by their nature are compiled by, first, determining with the responding establishments detailed price-determining specifications of representative commodities, and their prices in the reference period on "price initiation," and then comparing the prices of the same specifications in subsequent periods.[5] In this important regard the cited studies ask how well unit value indices stand up against price indices designed to overcome one of their major failings. In Section E strategies are outlined for the development of a country's external trade price indices.

B. International Recommendations

2.7 The potential limitations of unit value indices were recognized in both the United Nations (1979) manual on producer price indices (PPIs) and United Nations (1981).[6] Yet the use of unit value indices was recommended in the latter publication as a pragmatic approach for statistical authorities with resource constraints.[7] Indeed, United Nations (1983) included case studies on the development and implementation of the two main approaches—the survey pricing approach as used by the Federal Republic of Germany and the unit value approach as used by Norway—to assist countries in initiating and developing their trade price change measures. The vast majority of countries produced unit value indices then, as many do today, as the only available information on trade price changes and thus as surrogates for price indices.

2.8 United Nations (1981) recognized that the budget available to statistical authorities constrains the set of feasible data sources and methods for compiling external trade price indices. Thus, countries with "tight budgets" were advised to use the unit values compiled by customs authorities to construct price relatives, provided that such unit values are defined for commodities in the narrowest sense possible (using customs documentation). Resources permitting, unit values for the same commodity should be compared across different countries of origin or destination and, if they differ markedly, they should be treated separately. Subject to the need to achieve broad representation, unit value indices that exhibit exceptional price changes should be excluded. The caveat as to the need for achieving broad representativity was dropped for "machinery and transport equipment" and "miscellaneous manufactures" owing to the likely widespread contamination of the unit value changes by changes in product mix.

2.9 For a country with an "average budget," the strategy advised in United Nations (1981) was for the individual statistical agency to access the individual documents of the customs authorities, legal considerations permitting. Within each product category, it was advised to study the statistical properties of detailed unit value data (e.g., exploring the arithmetic mean and dispersion of unit values from individual transactions, in addition to the unit value for the product category as a whole and other indicators such as the number of transactions). Explicit criteria should then be established to determine whether the category's unit value changes could be further considered as a suitable indicator of price changes. These criteria might require, for example, the mean and variability of unit values being inside specific limits, no reversal in the direction of change, limited deviations between the mean of the unit values of a category and the aggregate category unit value, and so on. The product categories used were recommended to be broken down as far as possible to include country of origin or destination and size group of commodity. Statistical authorities on an average budget were advised to put more emphasis on the validation of transactions using specific editing criteria than statistical authorities on a tight budget. In case a unit value indicator failed the validation procedures and was rejected on further investigation, it was advised that the gap could be filled

[5]There remains a problem for both types of data when a product changes—say, a new improved model is introduced. Unit value indices will be biased upward. A change in the detailed specifications will be noted when using establishment surveys, and the methods in Chapters 8 and 9 are available to deal with the quality change/new good.

[6]It is stressed that the concern of this chapter is with the use of unit values derived from customs documentation to proxy price changes. Data on the traded value shares of commodity groups in total imports or exports, derived from customs documentation, are generally most suitable for use as weights for both unit value and price indices.

[7]The advice is reiterated in United Nations (2004, paragraph 150, page 97): "Two kinds of indices may be produced to reflect prices: unit value indices based primarily on customs documents and price indices based on survey data. The relative strengths and weaknesses of those two approaches to index number compilation are described in...United Nations (1981). Although price indices are generally preferred, in practice countries may not have the resources available to compile that information. **It is recommended that all countries produce and publish volume (quantum) indices and either unit value or price indices for their total imports and exports on a monthly, quarterly and annual basis."** [Emphasis in the original].

by the judicious selection of unit values from domestic or partner-country price indicators.

2.10 Two variants were suggested for the strategy of a statistical authority endowed with considerable budgetary resources. The first option was to pursue a *dual strategy*, according to which a comprehensive price survey of importers and exporters is conducted while keeping track of unit values from customs authorities. Alternatively, a *hybrid strategy* might be followed, in which the price relatives from establishment surveys and unit value indices from customs data complement each other. The dual approach involves using both methods and thus allows price relatives from a survey to be compared, at a detailed level, with the corresponding unit values and for unit value bias to be identified. It was to give insights into potential errors in coverage, leads and lags between changes in the transaction prices, and flows of merchandise in and out of the country. Yet it remains highly resource-intensive and it is not easy to investigate what causes the differences between the results of the two. The hybrid approach is a more useful allocation of resources. It is explained in United Nations (1981), pages 58–59:

> For the dovetailing of the two methods of collecting data, the commodity universe is divided into two portions. The first includes those commodities which are not susceptible to quality change at all, or only to small variations in quantifiable price-determining characteristics. These commodities are measured primarily by unit values. The other segment of the commodity universe corresponds to those commodities for which there are no quantity measures to speak of, to those commodities that are unique because of their size and complexity and to those commodities that are the object of significant change in their physical characteristics. This segment is dealt with primarily by direct survey. The overall index is, in effect, derived as a weighted average of the two kind of indicators.

2.11 With regard to unit value indices based on customs data, the *2008 System of National Accounts*, paragraph 15.15, notes that

> Unfortunately, it may sometimes happen, especially in the field of foreign trade statistics based on customs documentation, the data on which price and volume indices have to be calculated are insufficient or otherwise not adequate for the purpose. For example, the basic information available may be limited to the total number of units of some group of products imported or exported, or their total weight: for example, the total number of pairs of shoes, or total weight of equipment of a certain type. Indices built up from information of this kind are not volume indices when the numbers, or weights, cover different items selling at different prices. They are sometimes described as "quantity indices" for this reason.

C. Unit Value Indices and Their Potential Bias

2.12 In this section the nature of the potential bias in a unit value index, arising from changes in the compositional product mix, is first outlined in Section C.1, and then again considered more formally in Sections C.2 and C.3 when looking at the properties of unit value indices in relation to the main axiomatic tests and economic theory used in index number work to justify formulas. Section C.4 considers limitations in the coverage of their use, and Section C.5 provides a summary of issues of concern in relation to unit value indices. Given that unit value indices are widely used it is important that compilers and users be fully aware of their properties, so that a strategic decision to move to hybrid or establishment-based indices can be appropriately made.

2.13 The attention in this chapter to unit value bias is not to disavow the potential problems inherent to the use of price indices. The subject matter of Chapter 12 of this *Manual* is errors and bias in price indices and many of the remaining chapters are concerned with the principles of, and good practice for, mitigating such errors and bias. However, there remains an important deficiency of unit value indices. Unlike price survey indices, where it is possible to collect matched prices over time for detailed item descriptions, the nature of customs documentation makes this problematic at best and impossible for trade within customs unions and trade in services.

2.14 A unit value index, P_U, compares the weighted average price of $m = 1, \ldots, M$ individual transactions that took place a given period (denoted period 1) with the weighted average price of $n = 1, \ldots, N$ individual transactions that took place in another reference period (denoted period 0). All the transactions in both periods are of commodities that belong to the same commodity classification and are characterized by prices and quantity pairs (p_m^1, q_m^1) and (p_n^0, q_n^0), respectively. More specifically, the unit value index for period 1 relative to the reference period 0 at the most disaggregated commodity classification is given by

$$P_U \equiv \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1} \right) \Bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right), \tag{2.1}$$

where the prices of individual transactions are weighted by their corresponding quantities.

Table 2.1. Illustration of Unit Value Bias

	Size of Refrigerator											
	Small			Medium			Large			All Sizes		
Period	*q*	*p*	*v*	*q*	*p*	*v*	*q*	*p*	*v*	*q*	*uv*	*v*
Now	5	2	10	3	4	12	2	6	12	10	3.4	34
Then	2	1	2	3	2	6	5	3	15	10	2.3	23

C.1 Unit value bias illustrated

2.15 United Nations (1981, p. 15) illustrated the potential unit value bias with a hypothetical example of trade in refrigerators, which we include here as Table 2.1—also see Table 6.1 for a further example. With the exception of the "size" of the refrigerator, assume that the mix of all price-determining characteristics—including brand, frost-free technology, color, energy efficiency, and possession of ice-making feature, drink dispenser, and so forth—are adequately proxied by the "size" of a refrigerator. Assume further that there is a meaningful division of "size" into the three groups of "small," "medium," and "large" and that there is a trend in purchasing patterns toward larger refrigerators. In the illustrative example below, refrigerator prices, p, double for each size group and there is a shift to the quantities, q, sold from "then"—in proportion to 2, 3, and 5 going from smallest to largest—from what is "now"—5, 3, 2—though total quantity remains the same over time. The value, v, is given as $p \times q$.

2.16 Because prices in each size group have doubled, a weighted average of these price changes, $\sum_i w_i(p_i^{Now}/p_i^{Then})$, over the three size groups is 2.0. But the change in the unit value (uv) is 4.6/1.7 = 2.71. Thus, *if transactions of all size categories are classified together under the same commodity description (e.g., "refrigerator")*, there will be an upward bias in the unit value index owing to the change in the product mix toward more expensive refrigerators. In other words the measurement of price changes will be confounded with the effect of a shift toward purchases of larger refrigerators.[8]

[8]The Harmonized System at six digits includes four subheadings for refrigerators: combined refrigerator-freezers fitted with external door (8418.10), compression-type household refrigerators (8418.21), absorption-type electrical household refrigerators (8418.22), and other household-type refrigerators (8418.29)—there is no coding by size.

C.2 Unit value indices: The test approach

C.2.1 Homogeneous items

2.17 The test or axiomatic approach to choice of index number formula is well established and outlined in Chapter 17. If the items whose prices are being aggregated are identical—that is, perfectly homogeneous—a unit value index has desirable properties. If, for example, exporters shift some their sales of an identical item to purchasers willing to pay a higher price, then the average price paid for the item increases, as would the revenue received from exporting the same amount. A unit value index captures this shift to higher absolute price levels. Consider further the case where the same item is sold at different prices during the same period—say, lower sales and higher prices in the first week of the month and higher sales and lower prices in the last week of the month. The unit value for the monthly index solves the time aggregation problem and appropriately gives more weight to the lower prices than the higher ones in the aggregate. If the unit value index in equation (2.1) is used as a price index to deflate a change in the value, the result is a change in total quantity, which is intuitively appropriate, that is,

$$\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{n=1}^{N} p_n^0 q_n^0} \Bigg/ \left[\left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1} \right) \Bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right) \right] = \frac{\sum_{m=1}^{M} q_m^1}{\sum_{n=1}^{N} q_n^0}. \tag{2.2}$$

2.18 Note that to be homogeneous, all price-determining quality characteristics, including the conditions of sale, must be the same. The summation of quantities in the top and in the bottom of the right-hand side of equation (2.2) must be of the exact same items for the expression to make sense. Thus, for example, if the very same type of item is exported by an establishment to two purchasers in the same country with similar transport charges[9] under the same conditions of sale, but one is sold at a higher price than the other, then if the quantities sold shift in favor of the higher-price purchaser, the unit value index would show an increase irrespective of whether prices changed. This would be an appropriate increase.[10] However, if the items are not identical—for example, the purchasers were in different countries with different terms of sale and/or qualities of items—then the unit value index would be wrong. Prices would not have changed but the index would have. It is as if they export apples to one country and pears to another with prices unchanged.

C.2.2 Heterogeneous items

2.19 The very nature of an index number problem is that the desired measure is the change in price or volume of an aggregate over different items, or items of different quality. Chapters 16, 17, and 18 of this *Manual* clearly show that the superlative class of index number formulas, which includes the Fisher, Törnqvist, and Walsh index number formulas, is appropriate.

2.20 Unit value indices *derived from customs data* cannot a priori be taken as being the result of an aggregation over homogeneous items. Note that to be homogeneous all price-determining quality characteristics, including the conditions of sale, must be the same. The item descriptions on customs documents do not have sufficiently detailed descriptions to provide for this, and the Harmonised System (HS) subheadings used, even at a detailed level, are too broad. The axioms established to test index number formulas apply when items are not strictly homogeneous, as would be the case for unit values from customs data. They are considered in detail in Chapter 17, though their main properties with regard to unit value indices are outlined below.

2.21 The *proportionality test* requires that if all prices in period 1 are the result of multiplying the corresponding price in period 0 by a positive number, λ, then the price index should be equal to λ. More formally, the proportionality test requires that $P(p, \lambda p, q^0, q^1) = \lambda$ for $\lambda > 0$. The unit value index satisfies the proportionality test only in the unlikely event that relative quantities do not change (preferences are linearly homogeneous and identical—Bradley, 2005). Changes in the index can thus reflect both changes in price and changes in the product mix over the two periods compared. Because the index should only measure price changes, the index number formula has a potential bias.

2.22 It follows from the failure of the proportionality test that the unit value index also fails the *identity or constant prices test:* $P(p, p, q^0, q^1) = 1$; that is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the quantity vectors are. It satisfies the identity test only if relative quantities—that is, the composition of the products compared—do not change.

2.23 That a price index can be affected by changes in relative quantities is a serious deficiency. The essence of the fixed-basket concept of price indices is the need to hold quantities constant over time. There is a very real sense in which a unit value index should not be properly described as a price index unless applied to transactions for homogeneous products, and thus, by definition, the composition of products cannot change.

2.24 The unit value index, however, satisfies the *proportionality in current period prices test,* $P(p^0, \lambda p^1, q^0, q^1) = \lambda P(p^0, p^1, q^0, q^1)$ for $\lambda > 0$; that is, if all period 1 prices are multiplied by the positive number λ, then the new price index is λ times the old price index.

2.25 The unit value index fails the *invariance to changes in the units of measurement (commensurability) test:* $P(\alpha_1 p_1^0, \ldots, \alpha_n p_n^0; \alpha_1 p_1^1, \ldots, \alpha_n p_n^1; \alpha_1^{-1} q_1^0, \ldots, \alpha_n^{-1} q_n^0; \alpha_1^{-1} q_1^1, \ldots, \alpha_n^{-1} q_n^1) = P(p_1^0, \ldots, p_n^0; p_1^1, \ldots, p_n^1; q_1^0, \ldots, q_n^0; q_1^1, \ldots, q_n^1)$ for all $\alpha_1 > 0, \ldots, \alpha_n > 0$; that is, the price index does not change if the units of measurement for each product are changed. For example, if the measurement of one of the products changed from pounds as the measure of weight to kilograms, the index should not change.

2.26 In addition, where there is more than one product variety, it is necessary to have a natural unit of measurement common to both goods in order to allow for meaningful unit value calculations. For example, say an index

[9]From a resident's perspective export transportation costs are not an issue; they should be excluded for export price indices because the pricing basis is the *basic price*—that is, the amount received by the producer or distributor, exclusive of any taxes on commodities and transport and trade margins, while from the nonresident's perspective the pricing basis for imports is the basic price.

[10]That this is appropriate is apparent from the economic theory of an export price index established in Chapter 18 as a ratio of hypothetical revenues over the two periods being compared that the revenue-maximizing exporting establishment could realize, where the technology and inputs to work with (and thus quantities) are fixed to be the same for both of the periods. An establishment that, for example, doubles its revenue using a fixed technology and inputs—say, by shifting some output to higher-paying customers—effectively doubles its prices.

covered two products, product A measured in terms of the number of items imported/exported and product B in terms of the weight of the items. Then a change in the units of product B, *for both periods*—from, say, pounds to kilograms—would affect the results of the unit value index. That is, different results would arise if pounds were used to measure total quantity in each period as opposed to kilograms. That the results of the index depend on the units adopted results in a quandary as to which units are correct. Units of measurement are, however, standardized for detailed customs categories, something facilitated by the ASYCUDA computer system for customs documentation noted in Section A.

2.27 However, a particular instance of the effect of the failure in the commensurability test impacting adversely on the unit value index is one in which the quality of products imported/exported changes. When this occurs, the actual units of measurement may not change, but the implicit unit of productive service or utility would change and bias the index. Accounting for the effects of quality changes on prices is difficult enough for index number work based on price surveys (ILO and others, 2004a, and Chapter 8). Customs data on quality characteristics are likely to fall well short of the corresponding information that would normally be available from establishments producing for export, or purchasing as imports.

2.28 Unit value indices pass the other main index number tests, including the time reversal test, the circularity test, and the product test. However, the fact that they are affected by changes in the composition of products and (changes in) their units of measurement—that is, they fail the proportionality and commensurability tests—is critical to concluding that they are an inappropriate measure.

2.29 Unit value bias can be considered as the ratio of a unit value index to an acceptable number formula. Párniczky (1974) used the Paasche index in this latter respect whereas Balk (1998b) used the superlative Fisher index. The notation for p and q is as price and quantity vectors, respectively, for a variety of m commodities where ι is an m-dimensional vector of ones $\Sigma_m p^t_m q^t_m / \Sigma_m p^t_m = p^t q^t / \iota q^t$ for (t = 0, 1). Following Balk (1998b)—see also Silver (2008b)—the unit value bias is given by

$$B = P_U(p^1, q^1, p^0, q^0)/P_F(p^1, q^1, p^0, q^0)$$
$$= \left[\left(1 + \text{relcov}_0\left(p^1_m, q^1_m/q^0_m\right)\right) \times \left(1 + \text{relcov}_0\left(p^0_m, q^1_m/q^0_m\right)\right)\right]^{½}, \quad (2.3)$$

where the relative covariances are defined by

$$\text{relcov}_0\left(p^t_m, q^1_m/q^0_m\right) \equiv \textstyle\sum_m q^0_m\left(p^t_m - p^t.q^0/\iota.q^0\right)\left(q^1_m/q^0_m - \iota.q^1/\iota.q^0\right)/ \left(p^t.q^0\left(\iota.q^1/\iota.q^0\right)\right) \quad (t = 0,1). \quad (2.4)$$

2.30 Balk (1998b) drew the following conclusion: the unit value bias will be equal to zero for a comparison between periods 0 and 1 if one or more of the following conditions are met:

- All base period prices p^0_m are equal to each other and all current period 1 prices p^1_m are equal to each other;
- All quantity relatives q^1_m/q^0_m are equal to each other; and
- There is no correlation between p^0_m and q^1_m / q^0_m, and also no correlation between p^1_m and q^1_m / q^0_m.

2.31 These are all highly restrictive conditions. The first condition requires product homogeneity to an extent that defeats the purpose of a price index, in that if all prices were equal in each period, then there is no index number problem; the price change of a single product would suffice. The second condition is the assumption required above for satisfaction of the identity and proportionality tests. If all quantity relatives were equal, and this were known, the price index number problem would be solved by dividing the change in the total value by this single quantity relative. At the heart of the index number problem is that such quantity changes cannot be assumed to be the same. The third condition arises from the fact that if price relatives and quantity relatives are uncorrelated, then a change in prices would not affect quantity relatives, and vice versa, and there will be no change in the composition mix owing to relative price changes. There may be some markets in which there is market failure or temporal inconsistencies, but for the large part the laws of economics cannot be assumed away.

C.3 Unit value indices: Economic theory

2.32 Economic theory allows a theoretically "true" index to be defined under assumptions of (competitive) *optimizing behavior* on the part of economic agents and related (not independent) prices and quantities. Actual index number formulas can be evaluated against their theoretical counterparts and also against a class of superlative indices that have good approximation properties to a well-defined theoretical economic index. This approach is based on the theory of the cost-of-living index developed by Konüs (1924) and used in the consumer price index (CPI), as outlined in the CPI counterpart to this manual, ILO and others (2004a), and the theory of output index developed by Fisher and Shell (1972) and Archibald (1977), as outlined in the producer price index (PPI) counterpart to this manual, ILO and others (2004b), and it follows closely the exposition of the export price index (XPI) by Alterman, Diewert, and Feenstra (1999), as outlined in Chapter 18 for XMPIs. Bradley (2005) took a cost-of-living index defined in economic theory and compares the bias that results from using unit values as "plug-ins" for prices. In Chapter 18 it is shown that the economic theory of cost-of-living indices can be applied to resident economic agents purchasing imported goods and services. As a result of this, the findings of Bradley (2005) carry over to the use of unit value indices for import price indices.[11]

2.33 Bradley (2005) found that if there is no price dispersion in either the current or reference period compared, the unit value (plug-in) index will not be biased against the theoretical index.[12] But this is equivalent to the case of a single item, and the index number problem arises because we are aggregating across more than one item. He also finds that if there is price dispersion in the current period, but not the reference period, a unit value plug-in would have a downward bias; if prices are dispersed in the reference period, but not in the current period, there will be an upward bias in the index; and if there is price dispersion in both periods, there is a guarantee (i.e., there is a zero probability that the condition of no bias will hold for any arbitrary data-generating process) that there will be a bias in the plug-in unit value index, but one cannot sign that bias. Economic theory thus does not support the use of unit value indices. It does, however, support a number of index number formulas considered in Chapter 18.

C.4 Unit value indices: Coverage limitations

2.34 United Nations (1981, paragraph 2.31–32, p. 21) acknowledged that the balance could shift toward survey pricing if, as a result of tariff-reduction agreements, customs controls were to all extents abolished. For example, on January 1, 1993, the physical frontiers and customs checks, including the use of the single administrative (customs) document, were abolished for movements of goods between member states of the European Union (EU). Since then, under the Intrastat system, intra-community trade data are collected directly from firms.[13] The system has a close link with the value-added tax (VAT) system, under which most cross-border transactions are taxed and recorded on acquisition. The coverage of the surveys is akin to a census with only a minor proportion of trade excluded owing to nonresponse or falling below the VAT threshold. Establishment-based sources for external trade price data are the only practical option in these cases.

2.35 Furthermore, with services and e-commerce making up an increasing share of trade, customs data on merchandise trade will be unsuitable as the sole data source. Some goods, such as electricity, gas, and water, should be included in external trade, but are excluded from customs documentation. United Nations (2004, paragraph 255, pp. 44–45) advised that enterprise surveys be used to collect such data.

2.36 Finally, there are unique commodities, such as ships and oil platforms, and commodities highly differentiated by quality characteristics, including brand and conditions of sale, such as personal computers. For such commodities, it is often the case that the differing characteristics are responsible for a high degree of price dispersion, and there is much quality change and change in the month-to-month churn in the mix of the characteristics of the differentiated items traded.[14] For such highly differentiated commodities, establishment price surveys based on tightly specified representative items are most suitable for price index number measurement,[15] and unit value indices based on administrative customs documents are most unsuitable.

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[11]Similar conclusions could also be derived for indices defined in economic theory for revenue-maximizing producers exporting goods and services. Such indices are defined in Chapter 17.

[12]The theoretical cost-of-living index is defined for homogeneous preferences that are identical across all consumers and a negative (compensated) demand and price relationship.

[13]New Intrastat legislation was introduced on January 1, 2005 (the European Parliament and Council Regulation (EC) 638/2004, the Commission Regulation (EC) 1982/2004, and amending Commission Regulation (EC) 1915/2005). The European Commission Short-Term Statistics Council Regulation 1165/98 amended by 1158/2005 introduced requirements for the compilation of import and export price indices based on price surveys.

[14]There is much theory and evidence in economics, based on search costs, menu costs, and signal extraction models, of substantial price dispersion, even for similar items, including Sheshinski and Weiss (1977), Varian (1980), Bénabou and Gertner (1993), Sorensen (2000), Silver and Ioannidis (2001), and Lach (2002).

[15]Chapters 7 and 8 of this *Manual* provide detailed guidelines on adjustments to prices for quality changes and incorporating the effects of new goods of different quality. Chapter 10 provides examples of how the price changes of hard-to-measure commodities, such as ships, can be measured.

C.5 Unit value indices: The cause for concern

2.37 The concern over bias in unit value indices is not new. Early critical studies of unit value bias as measures of import and export price changes and terms of trade include Kravis and Lipsey (1971 and 1985). The United States discontinued publication of unit value trade indices in 1989 owing to concern over bias and introduced trade price indices based on establishment surveys.

2.38 Unit value indices derived from data collected by customs authorities are mainly used by some countries as surrogates for price changes at the elementary level of aggregation. The following are grounds upon which unit value indices might be deemed unreliable, some of which are considered in more detail in later sections of this chapter:

- Bias arises from compositional changes in quantities and quality mix of what is exported and imported. Even with best-practice stratification, the scope for reducing such bias is limited because of the sparse variable list—class of (quantity) size of the order and country of origin/destination—available on customs documents. Most promising, however, is the inclusion of an establishment number on customs declarations that, when used as a stratification factor, should reduce the heterogeneity of items included in a unit value. However, Párniczky (1974) showed—see Appendix 2.1—that such breakdowns are not always beneficial to a unit value index;
- For unique and complex goods, model pricing can be used in establishment-based surveys where the respondent is asked each period to price a product—say, a machine with fixed specified characteristics. This possibility is not open to unit value indices;
- Methods for appropriately dealing with quality change,[16] temporarily missing values, and seasonal goods can be employed with establishment-based surveys to an extent that is not possible with unit value indices;
- The information on quantities in customs returns, and the related matter of choice of units in which the quantities are measured, has been found in practice to be seriously problematic, though the former has improved with the adoption of ASYCUDA;
- Intra-area trade data for customs unions countries may simply be too limited to use;
- An increasing proportion of trade is in services and by electronic trade (e-trade) and not subject to customs documentation;
- Unit value indices rely to a large extent on outlier detection and deletion. Given the stickiness of many price changes, such deletions run the risk of missing the large price catch-ups when they take place and understating inflation;
- Valuation issues can be addressed at the establishment level to an extent not possible using custom data.

2.39 A main advantage of the use of unit value indices is their coverage and relatively low resource cost. However, the unit values used are drawn as nonrandom samples in the sense that they exclude products traded irregularly and those that have no quantity reported—something less likely now given the widespread adoption of ASYCUDA, have low-value shipments, and, where outlier detection is automatically and badly applied,[17] have erratic month-to-month changes. For some commodities, the extent of such exclusions may be substantial.[18] Establishment-based surveys can be quite representative, especially if a small number of wholesalers or establishments are responsible for much of the total value of imports or exports and, with the important assumption of cooperation, will be a cost-effective source of reliable data. Further, good sampling, can, by definition, realize accurate price change measures, and finally, the *value* shares of exports and imports, obtained from customs data, will form the basis of information for weights for establishment-based surveys.

2.40 Errors and biases are recognized in price index measurement. Chapter 12 of this *Manual* outlines such sources of error and bias, and this chapter, along with Chapters 3–11, provide detailed accounts of methods to mitigate them—methods that statistical authorities are well versed in for compiling consumer and producer price indices. These methods include the use of detailed specifications of representative goods and services so each month the prices of like items are compared with

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[16] Von der Lippe (2007) shows that adjustment for quality change is one reason why price indices are less volatile than unit value indices.

[17] Though it recognized that the appropriate response is good outlier detection. However, this in turn requires collaboration with establishments for confirmation as to whether a large change is real or not.

[18] For an illustration of the substantial extent of unit value dispersion and outliers, see WTO, UNCTAD, and ITC (2007), Technical Appendix B also discussed below.

like, something customs data do not allow for. CPI and PPI compilation practices have benefited from much research and experience since the publication of United Nations (1981), and extensive guidelines on good practice are available in the *CPI* and *PPI Manuals* (ILO and others, 2004a and 2004b, respectively), the benefits of which should carry over to XMPI practice—see Ruffles and Williams (2004, Chapter 5) for an example of country practice for the United Kingdom.

2.41 There is next the need for any strategic decisions as to the adoption/continued use of unit value indices by a compiler, or evaluation of such statistics by a user, to be evidence based. Because unit value indices may not be suitable surrogates for price indices, it is necessary to consider the empirical evidence available on the nature and extent of any differences.

D. Evidence of Unit Value Bias

2.42 We briefly outline comparisons of price and unit value indices when they coexist. The evidence is presented below first at the **aggregate level.** Results at a more disaggregated level will be considered in Section E.4.

2.43 Angermann (1980) compared price index number changes with unit value changes for exports from and imports to the Federal Republic of Germany. Between 1970 and 1976 the price index (Paasche version, to be consistent with the unit value index) for exports increased by 38.6 percent, compared with 34.3 percent measured by (Paasche) unit value indices. The discrepancy between such import price indices and unit value indices was greater, at 45.8 and 33.1 percent, respectively. Angermann also finds that when unit value indices were used to calculate the terms of trade effect there was a gain in 1976 of DM 1.4 billion to real income, at 1970 prices, compared with a loss of DM 6.6 billion when using a Paascheized price index.

2.44 Alterman (1991) compared export and import price changes between March 1985 and June 1989 for the United States as measured by unit value indices and by the price indices based on establishment surveys that replaced them.[19] For imports over this period, the price index increased by 20.8 percent, and the unit value index increased by 13.7 percent. For exports, the figures were much closer, 13.0 and 12.2 percent for the price and unit value indices, respectively. Some of the difference between the two series may be attributed to their use of different periods for weights. However, when price indices were recalculated using the same weights as the unit value indices, the differences were exacerbated: a 20.6 percent and 16.4 percent increase for the import and export price indices, respectively. The average (absolute quarter-on-quarter) unit value index changes for imports and exports, respectively, were 27 and 70 percent larger than the corresponding price index changes. One method of considering whether such differences matter is to evaluate the implications of such discrepancies for the deflation of the foreign trade component of the national accounts. Alterman (1991) found that the annualized second quarter 1989 “real” trade deficit in March 1985 dollars would have been \$128.4 billion if deflated by a unit value index, but just \$98.8 billion, 23 percent less, if deflated by a price index.

2.45 A review in 1992 of the unit value methodology used by the United Kingdom led to that country’s change in May 1996 to establishment-based trade price indices, following similar changes in methodology by the United States, Japan, and Germany (Ruffles and Williamson, 1997). The annual averages of export prices in 1995, compared with 1994, increased by 6.6 percent using price indices compared with 8.1 percent using unit value indices.

2.46 Such findings are not new. Kravis and Lipsey (1971) found that the prices of manufactured goods exported by developed countries to developing countries had risen over about 20 years by 75 percent, as compared with the 14 percent shown using unit value indices. Kravis and Lipsey (1985) found a decrease in the terms of trade of manufactures relative to all primary products between 1953 and 1976 of more than 36 percent, using price indices, almost a quarter greater than that suggested by the unit value indices (28 percent). With a further correction for quality change, the price data suggested a fall in manufactures terms of trade of more than 45 percent, more than 50 percent greater than unit value indices.

2.47 Silver (2008a) compared unit value indices and price indices for both Germany and Japan for exports and imports using monthly data for July 1996 to September 2006. Unit value indices were found to seriously misrepresent price indices in the sense that discrepancies between unit value indices and the price index were substantial; changes could not be relied upon to have the same sign; there was no evidence

[19]The official U.S. unit value indices were discontinued by the Bureau of the Census in 1989, so these figures are the latest available estimates.

of long-run (cointegrating) relationships between price index and unit value indices; and unit value indices were of little help in predicting price index. The findings held both for month-on-month and month-on-12-month changes. The marked unreliability of unit value indices as measures of export and import price inflation was surpassed by the unreliability of the terms of trade (TOT) indices based upon them. For example, the mean month-on-month discrepancy for TOT changes for Germany was 1.3 percent, compared with discrepancies of 1.1 and 0.9 percent, respectively, for imports and exports. A discrepancy of, for example, 1.1 percent implies that if the month-on-month change in the price index was zero, no change, then the unit value index would take a value of a 1.1 percent change on average. Such discrepancies can be regarded as seriously misleading for economists. The discrepancy for individual months can, of course, be much larger than this mean discrepancy, as reflected by an associated standard deviation of 1.0 percent and maximum of 7.3 percentage points for Germany's import month-on-month index changes. The problem is not just of the magnitude of error, but also of the direction. For about 25 percent of month-on-month comparisons the signs differed; that is, in one quarter of the comparisons the economist would read that prices were rising (falling) when they were falling (rising).

2.48 Silver (2008a) also found the discrepancy between use of unit value indices to calculate the terms of trade *effect*, as against price index, was most marked. For example, Japan's trade balance in 2005 of 6,956 billion yen was eliminated by the adverse change in its terms of trade when using a price index, but only halved when using unit value indices. The values of exports and imports of Germany and Japan were deflated over the period from 1999 to 2005 by corresponding unit value indices and price indices and the results compared. For example, the volume of exports by Japan increased by 50 percent when a unit value deflator was used, but the increase was halved when a price index was used.[20] Silver (2008a) noted,

> The evidence . . . is that export and import unit value indices are inadequate surrogates for their price index counterparts when used in economic analysis. Such analysis includes their use in the measurement of short- and long-run inflation, prediction, terms of trade, terms of trade effects, and as deflators. Indeed, the evidence is that they are seriously misleading.

E. Strategic Options: Compilation of Hybrid Indices

2.49 There is the practical matter of what should be done. Unit value indices are used by many countries, and a move to price indices has resource consequences. One possibility is to identify whether there are particular commodity classes less prone to unit value bias and utilize unit value indices only for these subaggregates in a hybrid overall index and price indices elsewhere—Chapter 6, Section C, provides some distributions and summary measures to help identify commodity classes more suited to unit value indices. Hybrid indices have resource advantages, yet the efficacy of their use depends on the extent to which reliable unit value indices will be available at a disaggregated level.

E.1 The case for hybrid indices

2.50 The account given in this section identifies hybrid indices as primarily based on unit value indices with the use of alternative sources, considered in Section E.3, as the exception when unit value indices are considered to be subject to unacceptable levels of bias. However, in the case of countries whose primary source of price change information is establishment-based price surveys, unit value indices are exceptionally used for goods whose characteristics are considered to be homogeneous. The account here will be from the former position, external trade price indices primarily based on customs data. This is because this *Manual* takes the stance that there is sufficient cause for concern, established in Sections C and D, to advise countries using unit value indices to move to establishment-based surveys. A strategic response to this would be a staged progressive adoption of

[20]Price index and unit value indices may be compiled using different formulas at the higher level, so differences in the results may in part be due to formula differences. Some insights are available for Germany, a country that has import price indices, import deflators of the national accounts, and unit value indices. The import price indices are of the Laspeyres type and refer to the year 2000. The Laspeyres principle is applied, however, only to the basket of goods, but not to the countries of origin, meaning that any shifts to low-cost producers will be captured by the import price index. The national accounts deflators are annually chained Paasche indices, and the unit value indices are Paasche indices referring to the year 2000. The product-specific price indices used for the compilation of the national accounts deflators are taken from the price statistics. Hence, the main difference between the import price index and the import deflator is to be found in the index formula. In the years 2000 to 2005, the unit value index displayed a decline of 1.8 percent per year, whereas the import price index increased slightly and the import deflator decreased less strongly (+0.3 and −0.8, respectively). Taking the geometric average of the change in the import price deflator and the import price index gives an estimate of −0.2 percent as the "true" annual change in import prices, implying that the German unit value index is significantly distorted downward.

hybrid indices with, over time, increasing proportions of unit value indices being substituted in favor of establishment-based survey data and other sources outlined in Section E.3, though issues arising in the adoption of such a policy are the subject of Section F. First is the need to outline the characteristics of hybrid indices.

E.2 Using unit value subindices for homogeneous product groups only

2.51 Hybrid indices can be readily calculated. A worked example is given in Table 6.2, and the principles outlined below. Say a Laspeyres index between period 0 and 1 is to be compiled for a Harmonised System commodity code from $i = 1,...,4$, lower-level HS codes for which unit value indices are available. Then the unit value index given by equation (2.1) for a commodity group i may be defined as $P_{Ui}^{0,1}$, where the m and n prices and quantities in respective period 0 and 1 are allowed to vary with commodity classification i:

$$P_{Ui}^{0,1} = \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1} \right) \Bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right). \tag{2.5}$$

2.52 However, say the unit value index is considered to be unreliable for the first commodity group; its nature is that it is composed of, say, equipment whose quality varies from shipment to shipment, and the resulting index is highly volatile. Then the measured price change of this commodity group is, say, the unweighted geometric mean of the matched price changes of $j = 1,...,J$ representative items from an establishment survey:[21]

$$P_1^{0,1} = \prod_{j=1}^{J} \left(p_j^1 / p_j^0 \right)^{1/J}. \tag{2.6}$$

2.53 The overall index for the four commodity groups is then a period 0[22] weighted average of the price index and unit value indices:

$$I^{0,1} = \frac{p_1^0 q_1^0 P_1^{0,1} + \sum_{i=2}^{4} p_i^0 q_i^0 P_{Ui}^{0,1}}{\sum_{i=1}^{4} p_i^0 q_i^0}. \tag{2.7}$$

[21] In Chapter 21, the advantages of using a geometric mean of price ratios—equivalent to the ratio of geometric means—rather than arithmetic means, are outlined in the context of elementary aggregate indices—that is, when there is no information on weights.

[22] As outlined in Section F.3, alternative and better formulas exist to this base period-weighted Laspeyres, which is used here for illustration.

E.3 Sources of alternative measures of price changes

E.3.1 Establishment surveys

2.54 Establishment surveys are considered the most appropriate data source for the measurement of price changes. However, customs documentation can still provide contact details for the importers and exporters concerned and the amount of the shipment, so that a representative sample of major importers and exporters can be contacted. Note that this in turn raises particular challenges for price measurement, the subject of much of the remaining chapters of this *Manual*. Chapter 6 considers issues of ensuring the establishments and items selected for the measured price changes are representative. Guidance on the collection of prices of detailed specified items over time is provided in Chapter 7, with further detail on the treatment of quality change and inclusion of new goods in Chapters 8 and 9, respectively. Price measurement is more problematic for some commodities, such as telecommunications and shipbuilding, and the issues relating to specific hard-to-measure commodities are considered in Chapter 11, along with more general problem areas. That there will be errors and biases as a result of this process is acknowledged in Chapter 12, in which is an outline of their nature, though the previously mentioned chapters have as their purpose the reduction of such errors and biases. Such issues are not new. They are for the most part similar to those that arise and are dealt with in PPI and CPI construction, guidelines on the principles and practice of which can be found in ILO and others (2004a and 2004b).

E.3.2 Global and mirror price indices

2.55 An alternative to using unit value indices is to use the corresponding series from other countries; that is, if your major exports (imports) are to (from) one or more identified countries and these countries have what you believe to be reliable import (export) price indices for these goods, then a weighted average of these series may be a suitable proxy. Alternatively, there may be the case where there are substantial quality changes in the goods imported and exported—for example, personal computers—and unit values, measured in terms of the quantity (by number or weight) traded, will provide inadequate measures of price change. However, some countries may compile detailed estimates of price and quality change, and their estimates of quality-adjusted—say, export price—changes in an international competitive market with little price differentiation by country may be the most suitable measure. A further alternative is to use international commodity price indices to proxy exports or imports price changes. The assumption is that there is a global market in which countries are price takers with little to no price discrimination between countries. In advocating stratification by country of origin/destination, United Nations (1981) implicitly argued against this as a general strategy. However, there may well be product areas for which this is useful.

E.3.3 Producer price indices

2.56 A country may well have a program for compiling an establishment-based output (input) PPI for which the price changes of a representative sample of tightly specified outputs (inputs) are monitored and compiled into a price index. The resulting price index is a measure of the output from (input to) the domestic economy as a whole to (from) both the resident and nonresidents. In some cases the establishments may wholly sell to (buy in from) nonresident markets, in which case a timely series should exist for the external trade price index. In other cases output may be to both markets, but the contact at the establishment should be able to provide information as to the proportion of output that goes to nonresident markets and whether there is any price discrimination in the prices, or price changes. If only a small proportion goes to (is bought by) the domestic market or if prices or price changes are, and are expected to continue to be, similar, then such price indices may be usefully employed for commodity groups of the external trade price indices.

E.3.4 Imputation

2.57 It may well be that data from establishment-based surveys or other sources are unavailable, in which case, in the absence of hard information, a default option would be to impute the price change for commodity group 1, in the example in Section E.2 above, from the price change of commodity groups 2–4. The measured price change from this calculation will be the same for all four groups, as that from the three groups. However, the overall index will benefit from the selective basis on which the imputations are undertaken. Imputations should be undertaken on the basis of following what are considered to be reliable price changes of groups expected to have a similar price change. The reliability will be based on the sample size, variability of price changes within the product group—less variable means more reliable—and robustness of the data source. If, for example, commodity group 2 was based on measured price changes of carefully specified representative items from an establishment survey, while commodity groups 3 and 4 were based on ill-defined unit value indices, it would be better to impute the price changes of commodity group 1 from those of commodity group 2 alone. The case would be even stronger if the price changes of commodity group 1 were expected to be more similar to those of group 2, say, for export price index, because they both utilized similar raw material inputs and other inputs and targeted their output to the same markets.

2.58 Imputation also has a wider use. For unit value or price indices, should some proportion of the weight of the commodity group not be covered—say, owing to poor data for unit value indices or sampling for price indices—then imputation is a means by which the coverage is factored up to the weight for the commodity group as a whole. Further, as outlined in Chapter 8, imputations may be used for temporarily missing items.

E.4 Hybrid indices: The reliability of unit value subindices

2.59 The extent to which unit value indices are included in a hybrid index depends on the resources of the country's statistical authority (customs data from administrative sources are generally cheaper); the availability of alternative sources (establishment response may be very poor for some industries); and the reliability of the unit value indices for the goods considered. Much of this will require specialized knowledge of the goods and services markets concerned. This *Manual* cautions against uninformed judgments as to what might be deemed homogeneous. In informal discussions with unit value index compilers as to which product groups give reliable series, the anecdotal information confirmed the care required. For example, for one country, coal was initially considered to be homogeneous, but they discovered that the unit value indices were completely unpredictable and uncorrelated with price changes. The explanation was that coal is not just coal; it varied in quality in terms of the amount of energy it produced, the extent of cleaning/filtering required, and residual use, say for road construction. Ruffles and Williamson (1997), in arguing the case for a change in the United Kingdom from unit value indices based on customs data to establishment survey–based price indices, cited the case Adipic acid, an organic chemical that is an important precursor in the manufacture of nylon. Unit values from customs data increased from 0.74 in April 1995 to 4.06 in May 1995, to return to 0.77 in June 1995. The price returns from individual companies had average quotes of 1.56, 1.56, and 1.55 for these three respective months. Individual customs returns from different companies had a wide variation in prices with a significant number of occasional traders whose unit values varied considerably from the average for the month in question, owing mainly to quality differences.[23]

2.60 The price dispersion of unit values within detailed groups of the Harmonized System is substantial. WTO and others (2007, Technical Appendix B), using data for 2003–2005, cite as an example the Harmonized System product code 04041006, imported by the EU. Before the exclusion of extreme values, the 65 unit values in this group ranged from US$39/tonne to US$111,250/tonne with an average of US$4,021/tonne. After the elimination of six extreme values, the range reduced to a still substantial US$39/tonne to US$3,212/tonne, with an average of US$1,282/tonne. WTO and others (2007) also provides summary information on the dispersion of unit values for goods subject to non-ad valorem duties (NAV). There were about 28,000 NAVs present in the tariff schedule in 2006 for which unit values were calculated. The summary results for the ratio of the lower (Q1) to upper (Q3) quartiles for the about 15,000 unit values for NAVs calculated at the more tightly specified tariff line level are given in WTO and others (2007, Technical Appendix B). About 46 percent of such unit values had a ratio that was more than 0.5, and that can be improved to 66 percent if outliers were removed. Bear in mind that this tells us that the 75th observation in a ranked ordering of unit values, for what is supposed to be the same good, had a price that was at the very least twice that of the 25th price in the ordering (Q3 <= 2 * Q1).

2.61 Silver (2008a) analyzed disaggregated monthly data for Germany, comparing unit value indices with their price index counterparts. The data were for export and import unit value indices and price indices for the period from January 2000 to November 2006 at the four-digit level of the Statistical Classification of Products by Activity in the European Economic Community, 2002 version (CPA). The relative discrepancies between month-on-month changes between the unit value indices and price indices were calculated for each of the 150 class series available for both unit value indices and price indices at this level. The 15 classes in the lowest percentile were then identified and their mean, maximum, minimum, and standard deviation month-on-month change over the period January 2000 to November 2006 calculated. These were the best product classes to use for unit value indices in the sense that they had the fewest average discrepancies. The results identified the magnitude of the discrepancies of what should be the most homogeneous product classes, with the intention of assessing whether they could be used in a reliable way.

2.62 The best two product classes, the manufacture of motor vehicles and the manufacture of pulp, had mean month-on-month discrepancies of 2 percent; that is, if the price index showed no change the unit value index would show a 2 percent change. At the bottom of the best percentile range was the manufacture of fertilizer and nitrogen compounds with a discrepancy on average of 4 percent. These averages of course have standard deviations that, at 6 and 7 percent for the best two classes and 15 percent for the worst end of the best percentile, demonstrated serious cause for concern. Had the results been more favorable, it would have been useful to attempt to characterize these "best" product classes for use in the compilation of unit value indices in hybrid unit value/price indices. Unexpectedly the best percentile groups included three heterogeneous classes composed of "other" and "not elsewhere classified" products. There is also some concentration around plastic products and motor vehicle–related activities, but given the size of the discrepancies these were not useful groupings.

2.63 Also of particular help in examining unit value and price index discrepancies at a disaggregated level are the extensive reports by PLANISTAT Europe (Decoster, 2003a and 2003b) commissioned by the Statistical Office of the European Communities (Eurostat) to examine the methodology for export and import price indices by EU member states. In particular, the second report provided a comparative analysis of import price and import unit value indices for Finland, Germany,[24] the Netherlands, and Sweden. The monthly import indices used were those provided by these countries to Eurostat. The unit value indices were extracted from the Comext database.[25] The series were available at a three-digit level CPA

[23]For further information on the wide dispersion and extreme outliers in customs-based unit values, see WTO, UNCTAD, and ITC (2007), Technical Appendix B.

[24]Data were not readily available at a detailed level of aggregation for Japan.

[25]Unit value indices are subject to outlier detection and revision, and the series available in Comtext may differ from those available from the individual countries in this regard.

The series are monthly from January 1995 (=100) to September 2001.[26]

2.64 For Finland, of the 77 product groups at the three-digit level CPA for which import data were available, 17 percent had an average discrepancy between unit value and price indices of less than 2.5 percent, and about another 40 percent between 2.5 and 5 percent. There was less difference between unit value and price indices for Sweden, with about one-third of three-digit product groups having a discrepancy of less than 2.5 percent. Bear in mind that a discrepancy of 0.025 implies that if the month-on-month change in the price index were zero, no change, then the unit value index would take a value of a 2.5 percent change on average. Such discrepancies can be regarded as misleading for economists, and 83, 87, and 66 percent of three-digit groups in Finland, the Netherlands, and Sweden, respectively, had discrepancies that on average exceeded this value. The average discrepancies for the three countries were 5.3, 5.4, and 4.1 percent, respectively. The minimum discrepancy for a three-digit group was between 1.5 and 2 percent—still significant potential to mislead economists.

2.65 The figures cited above are for the mean discrepancy over the 68 month-on-month comparisons for the period January 1995 (=100) to September 2001. The standard deviation of the discrepancy for each group was calculated for the 68 comparisons over time to quantify the volatility of the discrepancies. The average dispersion was high: for each country the mean standard deviation over the groups exceeded the mean of the groups. The very lowest dispersion over time of the month-on-month discrepancy for a three-digit group was for Finland at 1.9 percent, allowing an approximately 95 percent plus or minus range of $2 \times 1.9 = 3.8$ percentage points around the mean discrepancy—and this was the lowest dispersion.

2.66 Decoster (2003a) analyzed the data in considerably more detail. He found that price indices are more stable over time than unit value indices, and that unit value indices often display erratic behavior and that price indices do not; he therefore concluded in the Final Report (2003b, p. 9) that

> Any list of CPA categories for which UVIs [unit value indices] are a priori acceptable as proxies for SPIs [import price indices] would be very short, especially as regards monthly data. It would include almost only aggregates and raw materials, even if sizable discrepancies between unit value indices and SPIs are deemed acceptable. Apparently, any list of product categories for which short-term UVIs are acceptable proxies for SPIs seems country specific. For a few low-tech products, for which quality changes are slow, UVI changes over the long term (several years) may be acceptable proxies for SPIs.

F. Strategic Options: Improve Unit Value Indices

2.67 A response to concern over the unreliability of unit value indices is to improve them. This may be undertaken, first, by more detailed stratification of the customs data and, second, by better data validation routines. These are considered in turn.

F.1 Use a more detailed stratification of unit values

2.68 United Nations (1981) emphasized the need to stratify unit values to the (limited) extent possible and drew attention to doing so where possible by country of destination and size of batch, but see Párniczky (1974) on the limitations to this. Stratification is also possible for shipments by/to (major) establishments to/from given countries. The absence on customs documentation of highly detailed product descriptions by which to stratify unit values precludes any benchmark as to what is a reliable unit value index. However, such experiments can be undertaken for consumer goods using highly detailed bar-code scanner data. Bradley (2005) examined the issue in some detail and found that even for detailed data of sales of cereal in 169 selected stores by 1,369 brands, aggregating unit values that distinguish a brand of tuna according to the week of purchase and store in which it is sold, as against simply aggregating unit values for the self-same brand and item, leads to substantial differences in the results. Silver and Webb (2002) took (brand and) model numbers for washing machines, dishwashers, vacuum cleaners, television sets, cameras, and personal computers and compared unit value changes for the same models over different store types, finding quite different results when aggregating with and without store type as a variable. De Haan and Opperdoes (1997) undertook a similar study

[26]The PLANISTAT report was undertaken by Renaud Decoster. The results provided here are based on the worksheets of summary measures for the individual series, provided to the author of this chapter by Mr. Decoster. The author acknowledges Mr. Decoster's help and advice. The results do not appear in the report, but are based on the data series used for the report. The conclusions drawn here and in the report are very similar and differ only in that less favorable consideration is given in the report to unit value indices than here.

on coffee, further apportioning their data according to the week of the month the data relate to, again finding unit value bias. Given such bias at this fine level of detail for aggregating identical items, it is hard to imagine disaggregated unit values based on customs returns being robust to unit value bias.

2.69 However, as noted in Section C.2.1, if, say, an exported commodity is homogeneous, and there is price discrimination according to the country or group of countries exported to, then, for example, a shift in quantities to, say, countries in which export prices are lower would result in a fall in the average price at which the commodity is sold, and stratification by country would be inappropriate. If the commodities are broadly comparable and the quality difference can explicitly be stripped out using a suitable method outlined in Chapter 8, then unit values for these quality-adjusted prices would be appropriate.

F.2 Use deletion routines for unusual price changes

2.70 Of widespread use in the compilation of unit value indices are deletion routines. This is because much of the data from customs records on unit value changes are highly volatile and often discarded. Some of this volatility arises from absent data or a poor quantity of data—something less likely now given the widespread adoption of ASYCUDA. In other cases it is due to unit value bias. The problem of such deletions is twofold. First is the implicit effect on the sample representativity and coverage. Price indices are based on selected items from selected establishments with the purpose in mind that they are representative. The second problem is that the deletion removes signal as opposed to noise. There is much evidence in CPI compilation—for example, Hoffmann and Kurz-Kim (2006)—that price changes can be substantial and irregular, with long periods of constant prices followed by relatively large catch-up price changes. The problem is aggravated with external trade statistics when price changes can become volatile as a result of exchange rate fluctuations (see, e.g., Nakamura, 2008).[27] Large catch-up price changes may be deleted by automatic outlier detection routines, resulting in unit value indices that are unduly stable, and volatile price changes, owing to exchange rate fluctuations, may lead to unduly high dispersion parameter values, used in deletion routines, and deletion rates.

2.71 Improved deletion routines are certainly advocated by this *Manual* when unit value indices are used. However, caution is expressed about the efficacy of such routines and the need for validation with an exporting/importing establishment or other third-party source prior to deletion. Deletion routines should be used to identify unusual price changes, which then have to be followed up to ensure that they are not real changes—large catch-up price changes under sticky price setting—but owing to wrongly entered numbers or a change in the units for quantities. However, the sheer magnitude of the task of following up the original customs documentation, and then possibly having to refer back to the exporter/importer, may well preclude detailed follow-ups with an over-reliance on automatic deletion routines. Deletion routines will be based on past parameters of the dispersion, which may themselves be based on outliers. Further, the parameters may themselves be unstable and past experience not be useful for future deletion practice. Finally, there is the arbitrary nature of the cutoff values often used in practice for deletion. Routines for outlier detection are outlined in Chapter 6, Section 6; Chapter 10, Section D; WTO, UNCTAD, and ITC (2007);[28] and specialized conferences such as those held by the United Nations Economic Commission for Europe (UNECE).[29]

2.72 The very nature of establishment-based price surveys, for which good practice is well established (ILO and others, 2004a and 2004b), militates against such problems. A contact is made in an establishment on price initiation, in which the characteristics and conditions of sale of well-specified representative items are established. The contact is responsible for providing a price each month for these items and is able to explain why unusual price changes take place, which in turn facilitates adjustments to prices for quality changes.

F.3 Use better formulas

2.73 Chapters 15–19 advise the use of a superlative index number formula, such as the Fisher, Törnqvist, and Walsh index number formulas. These formulas make symmetric use of reference and current period

[27]Exchange rate effects will pass-through to commodity prices with varying rates in terms of their extent and time lags.

[28]Systems for outlier detection tailored to the needs of unit value indices are outlined in Technical Annex B, pages 190–98, of WTO, UNCTAD, and ITC (2007).

[29]The UNECE holds conferences on data editing that include papers on outlier detection. Papers for the 2008, 2006, and 2005 meetings can be found on the following websites:
www.unece.org/stats/documents/2008.04.sde.htm
http://www.unece.org/stats/documents/2006.09.sde.htm and
http://www.unece.org/stats/documents/2005.05.sde.htm.

quantity information and can be justified as providing a good approximation to a "true" index defined in economic theory; in particular, the Fisher index has good axiomatic properties. Chapters 9 and 15 also provide a case for using annually chained index number formulas. Customs data benefit from the possibility of obtaining current period information to enable superlative index number formulas and/or chaining to be employed. Chaining of, say, a Laspeyres index has its own advantages, in terms of facilitating the introduction of new goods/establishments, but also has the advantage of reducing the gap or spread between Laspeyres and Paasche index numbers and thus better approximating a superlative index.[30]

G. Strategic Options: Move to Establishment-Based Price Surveys

2.74 Resources permitting, countries should adopt establishment-based price surveys. United Nations (1981) recognized the superiority of price indices by recommending that well-endowed countries compile them, while advising countries with limited resources to compile unit value indices. Countries require price indices not only for trade flows but also for the deflation of output and intermediate and final consumption goods and services by resident units. In particular, an output producer price index is required that measures the changes in the prices of output of resident establishments. Price indices are compiled by selecting representative items from major/representative establishments and comparing the prices of like items with like over time. Such output covers the domestic and export market (ILO and others, 2004b). For a self-standing export price index, there would be a need to identify price changes from such establishments for foreign markets as well as overall output and, as necessary, expand the sample size to ensure those establishments serving foreign markets are included in a representative manner. Similar arguments apply to imports. Establishment-based trade price indices are but an extension of establishment-based price surveys for producer prices. If a PPI program exists there are synergies and cost savings to starting an XMPI program (see Chapter 13). Poorer countries have fewer establishments serving foreign markets, with large proportions of exports usually being the responsibility of a relatively small number of establishments. There are also resource costs. But reliable XMPIs have their benefits, namely, the proper measurement of two major economic flows affecting the country, to allow for appropriate policy responses when necessary.

[30]Chaining is not advised for high-frequency, say monthly or quarterly, data subject to seasonal/volatile fluctuations.

G.1 The gradualist approach

2.75 A gradualist approach has major resource benefits. There will be some "low-hanging fruit" establishments responsible for relatively high proportions of exports and imports, some of which may be owned by the state and may have some reporting obligation. Likely examples of such commodity groups include natural gas, petroleum, electricity, and airlines. There will also be industries in which unit value indices are *prima facie* inadequate measures of price changes, largely because of the churn in highly differentiated products, or the custom-made nature of the products, such as shipbuilding and oil platforms. Further, there may be industries that account for a substantial proportion of trade, and the payoff of reliable data far outweighs the survey costs—for example, the use of surveys of fish-processing plants for major exporters of fish products and of agricultural marketing cooperatives for exports of primary products.

2.76 The gradualist approach requires as a first step a rigorous evaluation of each commodity group of the relative payoff and cost of abandoning unit value indices. A good starting point would be a listing by commodity group that includes the weight, the perceived reliability of the unit value series, the likely source and reliability of alternative series, and a grade for the relative cost of obtaining such data. The initial aim would be to identify important commodity groups whose current series are deemed unreliable and for which there are readily available alternative sources. International commodity prices and mirror prices, discussed above, may be usefully employed in some cases. For some commodities, a small number of traders or establishments, say marketing cooperatives, may be responsible for a relatively large proportion of external trade, and exploratory contacts with such establishments can be initiated to obtain price data. The price data obtained may be considered to be representative of other commodity classes, and imputations may be used as outlined in Section E.3. A further alternative, also outlined in Section E.3, is to use subindices from a PPI, if compiled, in instances where the share of the market to/from nonresident producers is very high or if there is limited price discrimination or difference in price changes between output to (input from) nonresident and resident markets. In other cases, data from customs documentation may be considered, at least as a short-run expedient, to be sufficiently reliable to be used for a commodity group as whole, or for that part of the group not covered by the establishment/global/mirror prices. Chapter 6, Section C, outlines some simple distributions and summary measures that may indicate classes for which the use of unit value indices would be less problematic. Where there is doubt, especially for groups with high weights, a coding for the priority for future change should be attributed to the group. Where unit value indices are the only available series, at least in terms of currently available resources, then the strategy must be to improve them by ensuring that stratification as detailed as possible is undertaken and employing automatic outlier detection, but not automatic outlier deletion.

2.77 A potential problem with a gradualist approach is that longer-term changes in the index become problematic. The user cannot judge how much of the long-run change is due to changes in the price change indicator series used. For example, say unit value indices on average had an upward bias as the mix of items moved to better quality ones. And say in year 1 about 20 percent of the index by weight was replaced by a more reliable price index that cut the annual increase in the unit value index of, say, 5 percent to 3 percent. Then the overall index would increase by only 4.6 percent; if in the next year a further 20 percent was changed under the same assumptions, the index would increase by 4.2 percent, and subsequently, with replacements of a further 20 percent each time, by 3.8, 3.4, and 3.0 percent when the whole index finally becomes a price index. Such a gradualist policy would yield misleading results. A gradualist approach should be accompanied by well-signaled steps to users and, when changed, be accompanied by back data for at least the past 12 months so that 12-month changes can be identified and the new index readily linked to the old. There should be adequate metadata to explain the change.

G.2 The introduction of a new establishment-based price index

2.78 The reason for moving to establishment-based price surveys may be outside the control of the statistical authority. Customs data may become unsuitable for measuring trade flows for some countries if a country joins a customs or monetary union.

2.79 The introduction of a new external trade index, distinct from unit value indices, has advantages. First, there is no confusion over the measurement of external trade price changes over a period that encompasses both types of indices, as would be the case with a unit value index that evolves through a hybrid index to become an establishment survey–based index. Second, contacts with establishments can arise from a well-publicized and well-marketed process embedded in a legislative framework having a response burden that is identified as fairly spread across establishments, as opposed to grace and favor requests.

2.80 The main problem with simply introducing a new program is the resource cost. This includes the training of price collectors, building of sample frames, sample selection of items and establishments, computer routines, data validations, and much more that is the subject of the remaining chapters of this *Manual*. However, if a PPI program is already established, there will be synergies with the external price index program, including computer routines, price-collecting manuals and training, and expertise in sampling items and establishments. There will be some commodity groups for which the PPI results alone are sufficient. However, in other commodity groups for which the current PPI sample is not sufficiently detailed to allow reliable export/import indices to be compiled, the sample of items/establishments will need to be supplemented to include items that are imported/exported. Of course, an import price index will include transactions by households mainly (cross-border shopping is an exception) through a chain of resident wholesalers and retailers who will need to be sampled. Appendix 1 to Chapter 1 provides a step-by-step guide to developing export and import price indices. Chapter 1 itself provides an overview of this *Manual:* the theory and practice of export and import price indices.

H. Summary

2.81 Section B briefly outlined the rationale for the United Nations (1981) strategy advising that unit value indices based on data from customs administrative documents be used by countries with limited resources, as surrogates for narrow specification price indices. In the absence of a systematic examination of the evidence and in view of the low cost of the data, the bias in unit value indices was judged by United Nations (1981) tolerable enough to advise their use. This *Manual* provides an opportunity to review this strategy.

2.82 Unit value indices were evaluated against the axiomatic and economic theoretic approaches to index numbers and found to have major shortcomings (Sections C.2 and C.3). Unit values were also found wanting because they are applicable only to trade subject to customs administration and thus can neither be compiled for trade within economic customs unions where there is insufficient documentation, nor be used for trade in services. Unit value indices also have insurmountable problems when used for "unique" goods such as ships (Section C.4). The case for concern over the use of unit values was summarized (Section C.5); there were strong reasons for advising that they not be used. Evidence for the extent of the bias was then outlined (Section D). Unit value indices were found to have

- No well-defined relationship over time to the desired narrow specification price indices; indeed there are substantial discrepancies in direction and magnitude; and
- No predictive power for narrow specification price indices.

For terms of trade indices the discrepancies are much worse. Measures of the terms of trade effect (for real national income) and deflated volume changes were vastly different when measured by unit value indices as opposed to price indices.

2.83 The evidence in Section D went beyond the fact that export and import unit value indices are inadequate surrogates for their price index counterparts when used in economic analysis; the evidence was that they were seriously misleading.

2.84 The *Manual* turned to the question of what can be done. Sections E, F, and, G examined strategic options available in light of the conclusions of Sections C and D. Countries whose resources are limited and use unit value indices were advised to compile hybrid indices with, over time, a staged progressive program decreasing the reliance on unit value indices. The nature of hybrid indices was outlined (Sections E.1 and E.2)—a worked example is given in Table 6.2—and alternative data sources to unit value indices were considered, including establishment-based price surveys, global/other country (mirror) price indices, producer price indices, and imputations (Section E.3). There remained the issue of establishing which commodity groups might remain as being best served by unit value indices. Evidence on the reliability of subgroups was presented (Section E.4), it being concluded that there is no evidence of particular subgroups for which the unit value indices reliably represent price indices. The culmination of a program of use of hybrid indices should be an index in which unit values have little or no place.

2.85 Strategies of improving unit value indices were considered (Section F). Increased stratification, by country of origin/destination, size of shipment, and type of commodity (Section F.1) and improved detection and deletion (after follow-up) of outliers (Section F.2) will improve unit value indices and should be adopted—Chapter 6, Section 6, and Chapter 10, Section D, contain outlines of methods a statistical office might use, and Section F.2 footnotes references to other material. However, the nature of customs documentation and natural volatility of sticky price changes and pass-through rates constrain the benefits of increased stratification and unduly robust outlier detection. Better formulas (Section F.3) can be employed, particularly the adoption of chaining and use of superlative indices, but such advantages also carry over to price indices.

2.86 The strategic issue of a staged gradualist approach through hybrid indices or a one-off switch to establishment-based surveys was finally considered (Section G). A gradualist approach (Section G.1) using hybrid indices has major resource benefits. The gradualist approach requires, as a first step, a rigorous evaluation of each commodity group of the relative payoff of abandoning unit value indices. The initial aim would be to identify important commodity groups whose current series are deemed unreliable and for which there are readily available alternative sources: the "low-hanging fruit." However, changes should be in a limited number of staged steps with good metadata and backdated series so that analysts do not confuse price changes with changes in methodology.

2.87 A preferable, though resource-intensive, approach is a one-off switch to an index based on establishment-based price surveys, possibly supplemented by global/mirror series as applicable (Section G.2). This may be prompted by a country joining a customs or monetary union. While the main problem with simply introducing a new program is the resource cost, there will be natural synergies between the administrative offices responsible for XMPIs and PPIs, if a PPI program is already established. Chapters 3–14 of this *Manual* provide practical advice on the compilation of export and import price indices based on establishment surveys, and Chapter 16 onward provides a detailed account of the theory of price indices and issues in their construction, including choice of formula, the treatment of transfer prices, seasonal goods and services, and quality changes. Chapter 1 provided an overview of the *Manual,* and, more particularly, its Appendix is a step-by-step account of how to start compiling XMPIs.

Appendix 2.1 On Limitations to the Benefits of Stratification

2.88 Consider a unit value index that is disaggregated into $k = 1, \ldots .K$ subclasses is given by

$$P_{DU} = \sum_{k=1}^{K} w_k P_{Uk}, \qquad \text{(A2.1)}$$

where P_{Uk} are the unit value indices defined by equation (2.1) for each subclass k, and w_k are their respective weights. If the only limit to the disaggregation were a reduction to perfectly homogeneous commodities, then the unit value index would be ideal, as argued in Section B. There will, however, be practical limits to the extent to which a class can be broken into detailed subclasses, as dictated by the data source. Given these limits, there is the question as to whether a strategy of disaggregation into smaller commodity groups to reduce bias should be undertaken. The analysis that follows is that by Párniczky (1974). The total price variation of the unit value class can be decomposed into the variation within the subclasses, and the variation between the subclasses, that is,

$$\sigma_w^2 (p_0) + \sigma_b^2 (p_0) = \sigma^2 (p_0), \qquad \text{(A2.2)}$$

where the three terms are, successively, weighted[31] variances of prices in period 0 within subclasses, between subclasses, and overall. The weighted variances for quantity relatives $g = q_1/q_0$ between periods 1 and 0 can be similarly decomposed as $\sigma_w^2\ (g)$ and σ_b^2 (g), as can weighted correlation coefficients r_w and r_b between p_0 and q_1/q_0. The relative bias, B, can be stated as an additive decomposition:

$$B = \frac{\sigma_w (p_0).\sigma_w (g_0).r_w}{\bar{p}_0 \bar{g}} + \frac{\sigma_b (p_0).\sigma_b (g_0).r_b}{\bar{p}_0 \bar{g}}. \qquad \text{(A2.3)}$$

[31]The form of the weighting depends on the nature of the bias measure—that is, whether the unit value index is related to a, say, Paasche index, Laspeyres index, or an index that makes use of a symmetric mean of base and current period weights.

2.89 The first term at the right-hand side is the component of the bias associated with using a weighted average of the unit values of disaggregated subclasses. Only the within-subclass component generates bias; the between-subclass component is removed by the disaggregation. The sum of the two terms would represent the bias from using a unit value index without disaggregation. If, a priori, between-group bias is considered to be the dominant effect—the compositional changes between the subclasses is considered to generate the largest part of the bias—disaggregation is worth pursuing. However, say the two sources of bias are considered to be of the same order of magnitude. If the *direction* of the two sources of bias is the same, then again, disaggregation is worth pursuing. However, say the direction of the bias from the two components is opposite and they were of equal magnitude, then the component biases in a unit value index that is *not* disaggregated would cancel. A disaggregated unit value index would be more biased. Disaggregation would not the best strategy.

2.90 The directions of bias are dictated by the correlations of prices in the base period with relative quantities; the expectation is that the price increases of lower-priced commodities in period 0 would be higher—a negative correlation. But say this is true of the relationship within subclasses, but not between them. Equation (A2.3) warns us that a strategy of disaggregation may increase unit value bias. Price and quantity data within subclasses are often unavailable. Indeed, were the data available they should be used to compile weighted unit value indices at a further level of disaggregation. Unfortunately, empirical studies as to the relative signs for r_w and r_b are not feasible and reliance must be put on a priori expectations. It is further apparent from equation (A2.3) that if the within-subclass component of bias is the dominant one, then there is less to gain from a disaggregated unit value index, which removes the between-subclass component of bias. Note the multiplicative nature of the terms in the components of bias in equation (A2.3). If either $\sigma_b\ (p_0)$, or $\sigma_b\ (g_0)$, or r_b is close to zero, then the between-subclass component of bias will be small and there would be little to be gained from the disaggregation of a unit value index.

3. The Price and Volume of International Trade: Background, Purpose, and Uses of Export and Import Price Indices

A. Background and Origins of Price Indices

3.1 The purpose of this brief historical survey is to place the contents of this *Manual* in a longer-term perspective and to show that the measurement of price changes, or inflation, has long been recognized to be theoretically challenging as well as practically important.

3.2 Price indices are used for a variety of different purposes. There is general public interest in knowing the extent to which the prices of goods and services have risen. Also, it has long been customary in many countries to adjust levels of wages, pensions, and payments in long-term contracts in proportion to changes in relevant prices, a procedure known as index linking or contract escalation. Price indices have a long history for this reason.

3.3 A very early example is a simple index compiled by William Fleetwood in 1707, which was intended to estimate the average change in the prices paid by Oxford University students over the previous two and half centuries. Another 18th century example is an index compiled by the legislature of Massachusetts in 1780 to index the pay of soldiers fighting in the Revolutionary War against England (see Diewert, 1993a, for an account of the early history of index numbers).

3.4 During the 19th century, interest in price indices gathered momentum. In 1823, Joseph Lowe published a study on agriculture, trade, and finance in which he developed the concept of a price index as the change in the monetary value of a selected set, or basket, of goods and services, an approach still used today. He also noted the various uses for a price index, such as the linking of wages and rents, and the calculation of real interest. Diewert (1993a) argued that Lowe can be considered "the father of the consumer price index." Later in the 19th century, further important contributions were made, including those of Laspeyres (1871) and Paasche (1874), whose names are associated with particular types of price indices that are still widely used. Marshall (1887) advocated the use of chain indices, in which indices measuring price movements from one year to the next are linked together to measure price movements over longer periods of time.

3.5 During the 1920s, several important developments occurred. In 1922, Irving Fisher published his monumental work, *The Making of Index Numbers*. This was prompted by Fisher's interest in inflation and his advocacy of the quantity theory of money, in which changes in the money supply were held to lead to corresponding changes in the price level. A good measure of changes in the price level was needed—that is, a good price index—which led him into a systematic investigation of the properties of hundreds of different kinds of possible formulas for price indices.

3.6 Fisher's preferred index, the geometric average of the indices advocated by Laspeyres and Paasche, is now known as the Fisher index. As summarized in Chapter 1 of this *Manual*, the Fisher index (or the closely related Törnqvist index) remains the preferred measure from a theoretical point of view for most purposes. From the perspective of the *economic approach to index number theory*, these indices have been shown in most circumstances to provide an unbiased estimate of changes in the cost of living for consumers and for price changes for firms that maximize revenue and minimize costs. The full details of the economic approach to the export and import price index (XMPI) are discussed in Chapter 18. The Fisher index number formula can also be justified from the perspective of averaging two equally plausible fixed-basket index number formulas (the Laspeyres and Paasche formulas), and this justification is presented in Chapter 16. The Fisher index also has a strong justification from the viewpoint of the *test approach to index number theory*, which is discussed in Chapter 17. The Törnqvist formula can also be justified from the viewpoint of the *stochastic* *approach to index number theory*, which is also discussed in Chapter 17.

3.7 In 1924, Konüs published a seminal paper laying down the foundations for the economic theory of the cost-of-living index, or COL index. A COL index is designed to measure the change in the cost of maintaining a given standard of living (or utility or welfare) as distinct from maintaining sufficient purchasing power to buy a fixed set of goods and services. In reality, consumers do not go on purchasing the same set of goods and services over time but adjust their expenditures to take account of changes in relative prices and other factors. The producer counterpart to the consumer's cost-of-living index is the *fixed-input output price index*. This *economic approach* to the theoretical foundations for the XMPI was not fully developed until the 1970s: see Fisher and Shell (1972), Samuelson and Swamy (1974), and Archibald (1977). This approach is pursued in Chapter 18.

3.8 In 1926, Divisia published a paper in which he proposed price and quantity indices that factor the change in the monetary value of some aggregate flow of goods and services over time continuously and instantaneously into its price and quantity components. Although Divisia's approach to index number theory is not immediately applicable, because price and quantity data are not available on a continuous basis, the Divisia index is useful conceptually when one has to choose between fixed-base indices or chained indices. The Divisia index and its connection with the chain principle for constructing index numbers are discussed in Chapter 16, Section E.

3.9 Thus, by 1930 the theoretical foundations (from all of the above perspectives) for the compilation of price indices, including XMPIs, had been laid. Although there were many refinements to index number theory from both an economic and a statistical viewpoint during the mid- and late 20th century, the essential elements were already in place early in the century. Developments in index number theory and practice over the past few decades are dealt with in detail in various chapters in this *Manual* and will not be summarized here, except to note that all of the above approaches lead to a very small number of index number formulas as being designated "best." In particular, the Fisher formula emerges as being "best" from the perspectives of the economic, test (axiomatic), and averaging of fixed-basket indices approaches, whereas the Törnqvist formula emerges as being "best" from the perspectives of the economic and stochastic approaches.

B. Official Price Indices

3.10 As noted, there has always been considerable interest in, and demand for, price indices from the general public as well as governments. The 1780 index, referred to in the previous section, was specifically commissioned by a government agency in order to adjust the pay of soldiers in its employment. It is now generally acknowledged that governments have an obligation to provide the community and not merely themselves with information about price movements in the economy. A price index is a public good.

3.11 The practice of index-linking wages has a long history. Index linking means that the wage rate or material costs are adjusted in proportion to the change in some specified price index, the purpose being to maintain the real purchasing power of wages over the kinds of goods and services typically consumed by wage earners. As explained later in this chapter, an important use of XMPIs is to make adjustments in long-term contracts for changes in material costs. For such applications the specification of the index that is to be used can be a matter of some controversy. Whatever the exact formula used, index linking has important financial implications both for those making and for those receiving the payments in question. This in turn implies that there is a need for impartial, independent, objective, reliable, and credible price indices. The responsibility for compiling price indices must therefore be entrusted to a statistical agency that has both sufficient resources and the necessary independence from pressure groups of various kinds. This provides a second reason governments find themselves under an obligation to compile and publish price indices, or at least to supervise and monitor whatever agency is entrusted with the responsibility.

3.12 In practice, the government agency that is given the responsibility to compile and publish XMPIs is usually the statistics office or bureau, the central bank, or the customs authority (for goods exports and imports only). The central bank may be entrusted with the task of compiling XMPIs because in open economies the export and import prices are important determinants of domestic inflation, which most central banks want to control using the instruments of monetary policy.

3.13 Price indices for industrial commodities also have a long history. In Canada, a wholesale price index (WPI) of 89 commodities was compiled using an unweighted geometric mean for the period 1867–90. After that, the index was expanded to cover more commodities and to use a Laspeyres index. The first industrial commodities index in the United States was produced in 1902 (covering the period 1890–1901), using an unweighted average of price relatives for about 250 commodities. This index was developed in response to a U.S. Senate Finance Committee request for an investigation into the effects of tariff laws on prices of domestic and foreign agricultural and manufactured products. A system of weighting was first used in 1914. The original index was also referred to as the WPI because it covered commodity prices before they reached retail markets.

3.14 In Europe, the first WPI for the United Kingdom was prepared by the Board of Trade and presented to Parliament in 1903. The price reference year was 1871, and the series covered the years from 1871 to 1902. The prices were mainly derived from the trade accounts, with weights estimated from different commodities used, or consumed, in the country between 1881 and 1890. This index covered of 45 commodities, mainly basic materials and foodstuffs. Following World War II, a number of countries also began collection of data on wholesale prices of commodities in an effort to measure price changes at an earlier level in the production process. Around 1970 Eurostat, the Statistical Office of the European Union (EU), began a systematic program to encourage members to collect industrial output prices in an effort to get information on prices as products left producers' factories. These price indices were thus called producer price indices—PPIs—because they attempted to measure the change in prices producers received at the factory gate. In the past 5–10 years, many national statistical agencies have been progressively extending coverage of their national PPIs to measure changes in service industry prices, which in many countries now account for nearly two-thirds of GDP.

3.15 PPIs usually are compiled monthly, although some countries compile them only quarterly. Countries also try to publish them as soon as possible after the end of the month to which they refer, sometime within two weeks of the reference month. Moreover, most countries prefer not to revise them once they have been published. In contrast to many other kinds of statistics, most of the required data, at least on prices, can be collected at the same time.

3.16 PPIs have two characteristics that users find important. They are published *frequently*, usually every month but sometimes every quarter. They are available *quickly*, usually about two weeks after the end of the month or quarter. PPIs tend to be closely monitored and attract a lot of publicity. In many countries the PPI is not revised once it is published, which is viewed as an advantage by many users.[1]

3.17 The export and import price indices usually are thought of in connection with customs records, because these have traditionally been the main source of information. These records pertain only to goods in almost all countries, as well as the shipping and insurance services involved in the transport of imports. As such, they traditionally have been compiled for goods as indices of the *unit values* of detailed customs classes. As discussed in Chapters 2 and 5, detailed customs goods classes rarely contain only one product. Thus unit values will suffer from composition effects, wherein the product composition of the unit value for a given customs class varies from period to period. This can cause the unit value price relative from period to period to change even if the prices of the component products have not. Thus the unit value price index tends to be biased in unpredictable directions. Of course, bias in one direction of the unit value price implies bias in the opposite direction of the customs volume index. Comparisons with survey-based data illustrate the extremely volatile nature of unit value indices (see Chapter 2). Further, with customs unions, inter-area trade data will generally not be available.

3.18 As a result, a number of countries have discontinued computing the price relatives as unit values for some or all of the detailed classes of their XMPIs and for some countries of origin/destination. They instead directly survey establishments engaged in international trade to record the prices received by exporters and paid by importers to form the detailed price relatives and indices comprising their XMPIs.[2] Customs unit values and these surveys are the subject of Chapters 2 and 6.

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[1]In many countries both the XMPI and consumer price index (CPI) are not subject to revision once published unless an error is discovered in price collection or compilation. In a number of countries, however, it is standard practice to revise the XMPI once more complete information is available. For example, in the United States the XMPI is revised with a three-month lag; that is, the most recent three months are preliminary (subject to revision), whereas the fourth month's data are final.

[2]Among the countries that are exceptions to extensive use of customs unit values in trade price indices are Australia, Czech Republic (partially), Estonia, Latvia (nonhomogeneous goods), Mauritius (partially), Singapore, Sweden, United Kingdom (manufactured goods), and United States. These countries instead make substantial use of price surveys of narrowly defined products. See United Nations (2005).

C. International Standards for Price Indices

3.19 Once a statistic is accorded official status and given some prominence, the establishment of international standards usually follows. International standards are needed for several reasons—and not merely in order to compile internationally comparable statistics. The first international standards for XMPIs were promulgated in 1981 by the United Nations (UN). The UN Statistical Commission at its nineteenth session requested the preparation of manuals on the practical aspects of collecting and compiling price and quantity statistics within the overall framework of the "Guidelines on Principles of a System of Price and Quantity Statistics," which was issued in 1977. The United Nations Statistical Office released the "Manual on Producers' Price Indices for Industrial Goods" in 1979 to provide practical guidance on the preparation of industrial PPIs, and "Strategies for Price and Quantity Measurement in External Trade" was released in 1981 to provide guidance on XMPIs.

3.20 This *Manual* discusses revised and updated methods for XMPI compilation based on current practice and recent developments in price index number theory.

3.21 Some international statistical standards are developed primarily to enable internationally comparable data to be collected and published by international agencies, such as the statistical offices of the United Nations, the International Labor Office (ILO), the International Monetary Fund (IMF), or the Organization for Economic Cooperation and Development (OECD). The publication of such data by an international agency is often seen as a guarantee that the data conform to internationally accepted standards even though this may not always be the case in practice. Although national statistical offices actually supply the data to the international agencies, their publication by the international agencies is often interpreted as a public endorsement of their reliability, which enhances their status and credibility even within their own country.

3.22 However, international standards are not developed simply to enable internationally comparable data to be compiled. Many countries choose to use them as norms or standards for their own statistics. In this way, small national offices with limited resources of their own benefit from the collective views and experience of experts from a wide range of countries on which the international standards are based.

C.1 Current revision

3.23 This *Manual* has been developed in response to several factors. A considerable amount of work on the methodology of price indices, covering both theoretical issues and optimal methods of calculation, was undertaken at an international level during the 1990s as a result of the formation of an international group of price experts. This group, the International Working Group on Price Statistics, established under the auspices of the United Nations Statistical Commission, met for the first time in Ottawa in 1994 (and is therefore called the "Ottawa Group"). It brought together leading experts on price indices from national statistical offices and universities from around the world. During the course of its seven meetings through 2002, well over a hundred papers on the theory and practice of price indices were presented and discussed. This collective activity at the international level has inevitably led to some rethinking about, and elaboration of, the previous international standards on consumer price indices (CPIs), PPIs, and XMPIs as embodied in the *ILO Manual on Consumer Price Indices* (Turvey and others, 1989), the *Manual on Producers' Price Indices for Industrial Goods* (UN, 1979), and *Strategies for Price and Quantity Measurement in External Trade* (UN, 1981). This current *XMPI Manual* also incorporates approaches to the measurement of prices of exports and imports for services.

3.24 Another factor is the high priority accorded to the control of inflation as a policy objective in most countries, after the experience of high, or even hyper, inflation in the past three decades of the 20th century. The slowing of inflation in many parts of the world in the 1990s, compared with the 1970s and 1980s, far from reducing interest in its measurement, has stimulated a demand for more accurate and reliable measures of inflation. Whereas an error or bias of 1, or even 2, percentage points in the annual rate of inflation may not be considered so important when inflation is running at 10, 20, or more percent, it becomes very significant when the rate of inflation itself is estimated to be only 1 or 2 percent. Inflation may slow down to the point at which it is not even clear whether prices are rising or falling, on average.

3.25 The need for a revised manual on XMPI compilation, as was the case of the PPI revised manual, followed from concern over CPI measurement issues. Users of CPIs in some countries have become convinced that the indices are subject to an upward bias, mainly as a result of their failing to make proper allowance for improvements in the quality of many goods and services, especially newer goods, such as computers, that are subject to rapid technological progress. The treatment of changing quality has long been recognized as particularly difficult on both conceptual and practical grounds. This topic has been intensively investigated, with numerous new studies on the subject appearing in the 1990s.

3.26 It has also been realized that, because of the widespread use of price indices for the index linking of social benefits, such as pensions and other government payments, and as an escalator for price adjustments to long-term contracts, the cumulative effects of even small potential biases can have considerable financial consequences for government budgets and private industry purchases over the long term. This has led to governments themselves scrutinizing the accuracy and reliability of price indices more intensively than in the past.

3.27 Within the European Union, the convergence of inflation was deemed to be an important prerequisite for the formation of a monetary union. This requires precisely defined price indices that are comparable among countries. An intensive and prolonged review of all aspects of the compilation of CPIs was undertaken during the 1990s by all the national statistical offices of the member countries of the European Union in collaboration with Eurostat. This work culminated in the elaboration of a new set of international standards for the 29 member and candidate countries of the European Union and led to the development of the EU's Harmonized Indices of Consumer Prices, or HICPs. Work on the HICPs proceeded in parallel with that of the Ottawa Group, many of whose experts also participated in the development of the HICPs.

3.28 The need for revising the ILO manual to incorporate these new developments was one of the major recommendations at the 1997 joint United Nations Economic Commission for Europe (UNECE)/ILO meeting on the CPI. Similarly, the Inter-Secretariat Working Group on Price Statistics (IWPGS) came to the conclusion that a new PPI manual was long overdue as well as a manual on XMPIs.

3.29 Significant developments have taken place in the practice of price index construction that now necessitate a revision of the 1981 UN manual. Among these are emergence of the economies in transition, increased inflation, the reality that XMPIs may overstate inflation even when international standards are followed, the need for constructing and publishing more than one index to meet specific requirements (e.g., XMPIs by destination country of export and source country of import), and so on.

C.2 Responsibilities of the international agencies

3.30 The traditional practice of index linking wages and contracts in many countries has meant that, at both a national and an international level, ministries or departments concerned with economic policies and statistics have taken responsibility for price indices. However, many government departments—especially ministries of finance, economics, industry, and trade, and of course central banks—are concerned about inflation and have acquired an interest in a variety of price indices as key indicators of inflation and its transmission.

3.31 All the international agencies concerned with general economic policy now attach importance to XMPIs and their movements. In addition, Eurostat, the ILO, the IMF, the OECD, the World Bank, the UNECE, and the World Trade Organization (WTO) all have a strong interest in XMPIs. Some of these agencies have provided technical assistance in the compilation of XMPIs to countries in transition as well as to developing countries. The agencies therefore agreed to pool their resources and collaborate in the present revision of the *Export and Import Price Index Manual* (*XMPI Manual*), using the IWGPS to manage the process.

C.3 Links between the new *CPI, PPI,* and *XMPI Manuals*

3.32 One of the first decisions of the IWGPS was to produce a CPI and a PPI manual in parallel. Movements in producer prices are clearly important for the measurement of inflation and the analysis of the process of inflation within an economy. PPIs have been comparatively neglected, however. Whereas there has been an international manual on CPIs for more than 70 years, there has been a manual on the PPI covering industrial output for only about 25 years. Following the publication of those manuals in 2004, the IWGPS turned its attention to a manual on XMPIs to update the previous manual, which was published nearly 30 years ago.

3.33 This new *Export and Import Price Index Manual* was based, wherever appropriate, on the *Consumer Price Index Manual* (ILO and others, 2004a) and

Producer Price Index Manual (ILO and others, 2004b). XMPIs and PPIs especially have a lot of methodology in common and this *Manual* draws heavily from the *PPI Manual*. Both draw on the same theoretical literature pertaining to index numbers. Whereas CPIs also draw on the economic theory of consumer behavior, both PPIs and XMPIs draw on the economic theory of production and the short-term rigidities in the production process.[3] The economic theory of all of these indices leads to the same kinds of conclusions with regard to index number compilation. It was therefore decided that the IWGPS manuals should be similar in form and as consistent as possible, sharing common text when appropriate.

C.4 Inter-Secretariat Working Group and the technical expert groups

3.34 Responsibility for the production of the *CPI*, the *PPI*, and the *XMPI Manuals* rested with the same Inter-Secretariat Group consisting of staff from the statistical offices, departments, or divisions of the ILO, the IMF, the World Bank, the UN, the OECD, and the EU. Expert advice on the contents of the three *Manuals* was provided by three parallel technical expert groups consisting of invited experts on CPIs, PPIs, or XMPIs from national statistical offices and universities together with experts from the international agencies themselves. To ensure consistency, there was overlap of membership among the expert groups.

3.35 Most members of the technical expert groups also participated in meetings of the Ottawa Group, which supported the decision to revise the *CPI Manual* and to produce a new *PPI Manual*. The *Manuals* draw on the contents and conclusions of the papers presented at meetings of the Ottawa Group and the Voorburg Group, thus providing the outlets through which the groups' conclusions can influence the actual compilation of price indices. In addition, the *XMPI Manual* has been drafted with notification to and comment from the *Interagency Task Force on Trade Statistics*, comprising the WTO, UN Statistics Division (UNSD), UNCTAD, the World Customs Organization (WCO), the UN Regional Economic Commissions, the Commission of the EU, and other agencies.

D. Purpose of Export and Import Price Indices

D.1 Background

3.36 The XMPIs are a weighted average of the price changes in groups of products between one time period and another. The average price change over time cannot be directly observed and must be estimated by measuring actual prices at different points in time. Price index numbers are compiled from the collected price observations through time; their significance lies in a series of index numbers referencing the comparison prices between a particular period and a reference base. For an index to provide information on price changes, at least two index numbers from the same series need to be available.

3.37 XMPIs do not attempt to measure the actual level of prices but measure the average change in prices from one period to another. Changes in the value of exports and imports are usefully, for economic analysis, decomposed into their price and volume components, and XMPIs are measures of the price component. XMPIs, as measures of the price change component, when divided into—that is, used to *deflate*—the change in their counterpart value aggregate, yield a measure of the change in volume terms.

3.38 Several different pricing concepts are used in price index measures and in foreign trade statistics. Among these are the basic prices received by producers, the purchasers' price paid, prices reported as free on board (f.o.b.), and prices including cost, insurance, and freight (c.i.f.). Valuation differences in the observed prices will arise depending on which of these pricing concepts are applied. For example, a national statistical office using establishment survey data may produce XMPIs from a resident's point of view and may value exports at basic prices (factory gate) and

[3]This is not strictly true because there usually is at least a small portion of expenditure on imports and receipts from exports that accrue directly to households as final consumption. A good example is "cross-border shopping," which may be important in some countries. In this case, households may import by visiting a neighboring country where they buy goods and services and carry the goods or the result of the services back with them to their home countries. Exports can occur when the item purchased is a transportable durable good, such as an automobile. In this case, the transaction should be recorded as an import by the country of the purchasing household and as an export by the country of the selling household. If the durable is second-hand, because consumer durables are recorded in consumption on a net acquisitions basis, the import adds to the consumption expenditure of the importing country and subtracts equally from the consumption expenditure of the exporting country, leaving expenditure on GDP unchanged in both countries when the export and import effects are taken into account. If the durable is new, the export is subtracted from capital formation (change in inventories) if purchased from stock. Only if the durable is new and delivered directly to the buyer by the nonresident producer in the period it was produced will there be no offsetting negative final expenditure for the exporting country. See Chapter 15 for further discussion of the effects of international trade on final consumption expenditure.

imports at purchasers' prices. Alternatively, using customs data, exports may be valued f.o.b. and imports c.i.f. Differences in valuation are discussed in more detail in Chapter 4.

3.39 Relative prices are changing all the time, with some prices rising and others falling. Because price changes can vary considerably from product to product, the value of the price index will be dependent on the precise set of goods and services it covers. It will also depend on the weights attached to the different kinds of products within the set.

3.40 The goods and services coverage of the XMPI varies across the world, but almost all cover goods in international trade, as well as the shipping and insurance services supplied on imports. Other internationally traded services, such as business, information, and financial services, also are in scope for the XMPI. Compilers are including them in XMPIs as well as in PPIs and CPIs as international consensus forms on how to measure their prices and output.

D.2 Uses

D.2.1 Sources of inflationary pressure

3.41 As outlined in Chapter 15, the import price index (MPI) serves as one component of inflationary pressure, the other being the PPI, which covers supply from domestic production. Inflation can be imported as part of intermediate consumption (inputs) to establishments, capital formation, final demand by households, governments, and other institutions. The economic analysis of such issues as outsourcing requires detailed data on price changes of domestically produced and imported goods and services and switches from one to the other. Export price indices (XPIs) serve as an indicator of price changes of one component of the economy's total output of goods and services, the other being that directed to the domestic market.

D.2.2 Terms of trade

3.42 Changes in the ratio of export to import prices have a direct impact on changes in the real income of a country. If the prices of a country's exports rise faster (or fall more slowly) than the prices of its imports (i.e., if its terms of trade improve) fewer exports are needed to pay for a given volume of imports. Thus, an improvement in the terms of trade, or a trading gain, makes it possible for an increased volume of goods and services to be purchased by residents out of the incomes generated by a given level of domestic production—the real income of a country increases. The measurement of this terms of trade effect is outlined in the *2008 System of National Accounts (2008 SNA)*, Chapter 15 (Section C) and requires price indices of imports and exports. It is worth noting that there is a difference between changes in the volume of domestic production, as measured by GDP in volume terms, and changes in real gross domestic income (GDI), the latter including, unlike the former, trading gains (or losses) owing to terms of trade effects. The difference between movements in the volume of GDP and real GDI are not always small. If imports and exports are large relative to GDP and if the commodity composition of the goods and services that make up imports and exports is very different, the scope for potential trading gains and losses may be large. This may happen, for example, when the exports of a country consist mainly of a small number of primary products, such as cocoa, sugar, or oil, while its imports consist mainly of manufactured products. Chapter 24 of this *Manual* provides an alternative framework that benefits from being derived from the economic theory of price indices, as outlined in Chapter 18. It shows how the product of a superlative export price index and a superlative import price index can account for changes in the country's real net (primary) income.

D.2.3 Government economic policy

3.43 Foreign trade indices are critical for government economic policy for several reasons. They enable the projection of budget figures and the forecasting of economic trends; they are essential for preparing negotiations with countries providing capital; and they help to conduct market surveys of strategic products.

3.44 Governments make regular projections in their budgets to determine the amount of additional revenue needed to finance current spending and investment programs. As a major component of government revenue, customs revenue is forecasted based on past trends in export and import values that make up the tax base. Understanding and forecasting value changes is facilitated by understanding and forecasts of the price and volume components. Forecasts of revenue can be made for individual product groups based on the principal duty rates. Government budget and public spending program forecasts are in principle always part of a macroeconomic framework representing the economic performance projections for different sectors (such as agriculture, industry, and services) to ensure that budget and public spending program forecasts are consistent with the economic activities in question.

3.45 Customs revenue is often central in the negotiations between governments and foreign capital providers. International agencies often extend loans on the condition that national authorities successfully meet macroeconomic targets, such as the size of the government deficit to GDP. Accurate information regarding international targets is thus essential, to the lender as well as to the authorities of the borrowing government. Accordingly, customs duties and foreign trade (prices and volumes) are closely monitored to ensure that the specified conditions on the flow of international finance are effectively met.

3.46 The breakdown of relative change over time of the value of exports and imports into price and volume components is needed to conduct economic analysis that supports the development of macroeconomic and international trade policy, and to disseminate information needed for effective decision making.

D.2.4 Analysis of competitiveness and productivity

3.47 An industry remains competitive only if its prices are kept in line with the market. Exporters have a critical interest in knowing the evolution of prices for products similar to their own from suppliers in other countries. On the other side of the market, importers also need to know about the evolution of the prices of the products they purchase to secure the least-cost source among supplying countries.

3.48 Studies of competitiveness involve analyzing the evolution of prices, not only in the domestic currency but also in the currencies of the country's main trading partners. For example, a country's stable export price index may mask a devaluation leading to higher competitiveness of the products that the country exports.[4]

3.49 Changes in the productivity of a sector can be measured in terms of changes in the ratio of the volume of net output to its primary inputs, or gross output to primary and intermediate inputs. Outputs and inputs need to be measured in volume terms and this in turn requires appropriate price deflators. The deflators for outputs include those for the domestic and export markets, and for inputs those from the domestic market and those imported—that is, the components of XMPIs.

D.2.5 Drawing up trade contracts

3.50 Businesspeople and potential investors may conduct market surveys on a particular product, in which they analyze corresponding supply and demand, so as to identify whether the product could be profitable for production or export purposes. To increase the accuracy of such an analysis, it requires more detailed information about the product, such as existing world sources of supply relative to potential demand, price, and identification of the main exporters and main national trading partners.

3.51 Import and export prices are used for contract escalation to ensure that the payment for some output or input is equal in real terms to what was agreed upon when the contract was signed. Appropriate import price indices can be used as an escalator for a long-term contract signed by a manufacturing firm to purchase production inputs from abroad. Likewise, appropriate price indices can be used as an escalator for a long-term contract signed by an exporter to deliver a product to a foreign buyer.

D.2.6 Measuring inflation, forecasting future price trends

3.52 Foreign trade indices are also a critical factor explaining the evolution of domestic inflation. Import prices affect the level of domestic inflation in several ways. Decreasing import prices of final goods directly affect the overall level of prices in the economy. Import prices can also be a good indicator of future inflation in a country where many inputs to domestic production are imported. Declining prices of imported inputs to manufactured goods have an indirect impact on the level of output prices by lowering the cost of production. More directly, if import prices for a product are falling, national producers of that product will have to lower their prices to preserve their competitiveness. Any seasonal or trend information embedded in the time series of import prices would contribute to the knowledge about future price levels.

D.2.7 Exchange rate analysis

3.53 To anticipate the effects of exchange rate adjustments, it is important to know the fraction of exchange rate movements that is passed on to the

[4]In a country with the schilling as the national currency, exporting coffee to the United States, for example, the export price index in schillings (S) for coffee shows zero inflation because the price of a kilogram is constant at S 100 in successive years, whereas the export price index in dollars ($) shows a decline of 1.5 [(100/10)/(100/15)]. This is because the exchange rate (S/$) increases (devaluation of the schilling) from $1 = S 10 in the first year to $1 = S 15 in the next.

prices of exports and imports. The answer to this question depends on the extent to which the "law of one price" holds as well as on pass-through rates.[5] Pass-through rates measure the percentage of exchange rate changes that are passed through to the prices of imports and exports. If a local currency appreciates against foreign currencies, import prices will no doubt fall while export prices should rise. A depreciation of the local currency would have the opposite effect. Pass-through rates are usually between zero and 100 but can be negative if a change in the exchange rate has the opposite effect on export or import price indices; these rates can exceed 100 if the export or import price index changes by a greater magnitude than the exchange rate.[6] Changes in the ratio of export to import prices are of interest in determining exchange rate changes, especially for tradable as against non-tradable goods and services.

D.2.8 Compilation of national accounts in volume terms

3.54 Import and export data are very important in the compilation of volume estimates of national accounts. This applies, in particular, if this compilation of national accounts is based on expenditure components or on product balances, as outlined in Chapters 4 and 15. Imports are among the sources of supply of products to the economy, whereas exports are among the uses of products The XMPIs used as deflators and the value aggregates for exports and imports must have prices valued on the same basis.

E. Family of XMPIs

3.55 As noted in Chapter 4, this *Manual* takes the position that there is, in principle, only one economic valuation and timing basis for international goods and services trade flows: the change of ownership basis of the *2008 SNA*. By implication, the buyer becomes liable for the cost of the good or service at its transaction value on the date of change of ownership. Any tariffs or other taxes levied in the course of the item's passage from source to destination, or any transport and distribution services, accrue on the date the importer or exporter becomes liable for the cost of them. The accrual dates for these services could differ from the change of ownership date of the good and, if different, generally would be later than that date. This is less important in domestic statistics than for international trade, because some commodities can lie in entrepot inventory for a considerable period before a final destination is set for them. XMPIs constructed on these principles are the definitive set.

3.56 The main recording system for trade in goods is customs documentation, typically tariff schedules, which captures the c.i.f. value of goods when they cross the frontier of the importing country. A customs agency records exports via declaration forms at the time the imports embark toward a foreign destination, or international waters. Customs documents usually also record a quantity relevant to customs administration (and also relevant to determining shipping costs), typically in units, weight, and/or cubic volume displaced in shipment. As such, the customs basis for recording represents an alternative set of value, price, and volume statistics for international trade to the economic basis of the international statistical standards of the *2008 SNA*. For many types of goods, it may be qualitatively and quantitatively close to the standard. Compilers often use the customs basis estimates as a proxy for the economic concepts in the international standards, as outlined in Chapters 4 and 15.

3.57 When there are measured differences between the ownership-based and customs-based statistics, however, the ownership-based ones should prevail. Say there is a difference in the coverage of goods owing to the recording principles used for customs purposes, as against the desired ownership change principle. Then the customs-based price indices will serve as proxies for the ownership-based price indices if the price change of the trade arising from the difference in recording is similar to that of the price change of the measured customs-based trade. This follows from the fact that price indices are unaffected by changes in coverage, even if the excluded products are substantial, as long as the price changes of the excluded products are similar, on aggregate, to those of the included ones.

E.1 Industry aggregation

3.58 Breakdowns of the XMPIs by the industries of the reporting establishments assist us in gauging the impact of world price movements by domestic industry and facilitate constructing price indices for intermediate consumption in the national accounts.

[5]The assumption of one price states that the price of an item, adjusted for tariffs and transport costs, is the same in all countries. Exchange rates account for differing price levels.

[6]The pass-through rate for imports is the ratio of the import price index deflated by the average exchange rate index.

E.2 Macroeconomic aggregations

3.59 High-level aggregations of the XMPIs, such as total exports and imports, contribute to the price index for GDP by accounting for the price change in net exports. Their direction of trade statistics—exports and imports by destination or source country, respectively—may also contribute to our understanding of how exchange rates and bilateral trading conditions affect the rates of change in export and import prices.

E.3 Commodity analysis

3.60 Analysis of MPIs by commodity reveals the sources of inflationary pressure from imports, which may be priced in international markets and thus outside the control of domestic agencies. Particularly important examples are the price change of crude oil and food. Aggregations of commodities can also be constructed to show the total impact of commodity price change on the economy. Analysis of the XPI by commodity contributes to our understanding of price pressures on domestic production capacity arising from changes in export demand. When the terms of trade are viewed as the relative rate of change in the prices of a country's package of exports with its package of imports, commodity breakdowns allow us to see how world commodity price movements contribute to movements in the terms of trade for a given country. The analysis of the effect of outsourcing requires commodity-level data on the price changes of domestic and imported goods and services and, when it occurs, switches from one to the other. MPIs are but one component of the quite demanding data needs for the task.

E.4 Stage of production

3.61 A further method for analysis is to aggregate by stage of production, in which each commodity is allocated to the stage in which it is used. This differs from stage of processing because a product is included in each stage to which it contributes and not assigned solely to one stage. The classification of products to the different stages is usually achieved by reference to input/output tables in order to avoid multiple counting of the stages that are not aggregated. There is a growing interest in this type of analysis—for example, these types of indices are already compiled on a regular basis in Australia (see, for example, Australian Bureau of Statistics, 2001).[7] The XMPIs may be broken down with the same classification by product stage of processing as the PPI.

E.5 Final expenditure price index

3.62 A further variant is the final expenditure price index (FEPI). This measures prices paid by consumers, businesses, and government for final purchases of goods and services. Intermediate purchases are excluded. XMPIs are used as proxies for the final prices paid for investment goods by businesses and government in the FEPI model used in Australia. This is because most XMPIs reflect changes in basic prices or producers' prices (not purchasers' prices). This topic is further considered in Chapter 6.

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[7]This topic is also considered in Chapter 15.

4. Coverage, Valuation, and Classifications

A. Introduction

4.1 Price indices are compiled as weighted averages of changes in the prices of goods and services. The weights correspond to and derive from the value aggregate for which the index is designed: in this case, exports and imports of goods and services. Because changes over time in value aggregates for goods or services comprise changes in the sum of prices *times* quantities across the types of goods and services in the aggregate, the prices forming the price ratios in the associated price index must be representative of the price changes of the value aggregate.

4.2 The properties of the value aggregate that directly translate into properties for the prices and weights associated with the price index are valuation, coverage, and classification of the goods and services making up the aggregate. Coverage, considered in Section B, defines the scope of goods and services contained in the aggregate; valuation, considered in Section C, defines what the prices in the value aggregate of interest include and do not include; and classification, in Section D, divides that aggregate into a standard set of subaggregates.

B. Coverage

B.1 Goods

4.3 In defining the coverage of goods, the concepts followed in this *Manual* are the same as those required in the *2008 System of National Accounts (2008 SNA)*[1] and the *Balance of Payments Manual, Sixth Edition (BPM6)*.[2] Thus, exports and imports cover, in principle, all goods for which a change of ownership takes place between the residents of a given economy and nonresidents. It therefore encompasses sales, purchases, barter, or other transfers of all goods between resident and nonresidents.

4.4 The main source for data on exports and imports is the international merchandise trade statistics. These data are based on administrative documents and include the physical movement of goods across the borders of a country.[3] The nature of resident to nonresident transactions implies that when a change of ownership occurs, the goods usually cross the customs frontier. However, data on merchandise trade as compiled primarily from customs warrants may normally only record goods that are subject to customs controls. The effective coverage of such statistics is therefore determined and limited by the effective coverage of the customs procedures in force for the country.[4] As an extreme example, intra-country trade between member countries of the European Union is excluded from customs statistics. Thus, the process of recording trade in goods may not necessarily coincide with the change of ownership because goods may change ownership without having to cross the customs frontier or may cross the customs frontier without changing ownership. Nonetheless, it is important to establish what should be covered in principle in order that the shortcomings of the practice can be identified.

4.5 Because there is a change of ownership of goods between a resident and a nonresident, *the* BPM6

[1]Commission of the European Communities and others (2008). The reference to the *2008 SNA* used in this *Manual* is to the final draft of Volume 1 (Chapters 1–17) of the updated *System of National Accounts* adopted by the 39th session of the United Nations Statistical Commission, February 26–29, 2008, available at http://unstats.un.org/unsd/sna1993/draftingphase/ChapterList.asp.

[2]IMF, sixth edition of the IMF's *Balance of Payments and International Investment Position Manual (BPM6)*, available at http://www.imf.org/external/pubs/ft/bop/2007/bopman6.htm. This is an update of *BPM5*.

[3]As defined in *International Merchandise Trade Statistics: Concepts and Definitions* (*IMTS*) (United Nations, 1998a).

[4]Compilers of international merchandise trade statistics make some adjustments to the basic customs data that are partly in line with what is sought after by the *2008 SNA* and the *BPM6*. The discussion of coverage here is phrased in terms of the needs of coverage of the *2008 SNA* and the *BPM6* and actuality of data from customs documents—United Nations (2004), for which a draft supplement had been prepared at the time of writing.

includes the following cases in the balance of payments definition of general merchandise:

(a) Banknotes and coins not in current circulation and unissued securities;

(b) Electricity, gas, and water;

(c) Products such as noncustomized packaged software (systems and applications), and video and audio recordings on physical media, such as disks and other devices, with a license for perpetual use (i.e., not through end-user or other licenses) are included in general merchandise if provided on disk, CD-ROM, or other magnetic media. These products are included at their full transaction value (i.e., not at the value of the empty disks, or other storage device);

(d) Goods procured in ports by carriers. Goods such as fuels (bunkering), provisions, stores, ballast, dunnage, and so forth procured by nonresident transport operators in ports from resident providers are included in exports of general merchandise. Similarly, goods procured by resident transport operators from nonresident providers are included in imports;

(e) Goods supplied or acquired by carriers away from the territory of residence of the operator. For example, fish and other marine products caught by ships operated by residents of the compiling economy and sold abroad directly should be included. Similarly, oil and minerals retrieved from the ocean floor by resident operators and sold abroad directly should be included. The goods could be acquired or sold in foreign ports or on the high seas to foreign vessels;

(f) Goods under financial leases acquired by the lessor. Because the lessee is the economic owner, a change of ownership between the seller of the goods and the lessee is recorded at the start of the lease. The lessor acquires legal title, but does not acquire economic ownership;

(g) Goods sent abroad without a change of ownership, but later sold. Goods sent abroad on consignment or for storage, repair, exhibition, processing, and so forth without a change of ownership are not recorded at the time they are sent abroad, but if they are later sold to a resident of a different economy from that of the owner, they should be recorded in general merchandise;

(h) Equipment that is sold while outside the territory of residence of its original owner. For example, equipment originally taken out of the territory for temporary purposes, such as construction, exhibition, or fishing, may be subsequently sold or given away;

(i) Illegal goods;

(j) Smuggled goods that are otherwise legal;

(k) Gifts and parcel post where there is a change of ownership;

(l) Goods lost or destroyed after ownership has been acquired by an importer but before the goods have crossed a frontier (however, goods lost or destroyed before ownership has been acquired by an importer are excluded from merchandise trade);

(m) Livestock that changes ownership;

(n) Government sales of goods to and purchases of goods from nonresidents. Acquisitions of military equipment from nonresidents should be included in general merchandise. Goods supplied by governments to their own embassies, military bases, and so forth involve resident-to-resident transactions and so are not covered in the international accounts. Expenditure by embassies and so forth is included under government goods and services n.i.e;

(o) Goods financed by grants or loans;

(p) Humanitarian aid in the form of goods;

(q) Goods transferred to or from a buffer stock organization;

(r) Goods acquired to be processed without passing through the territory of the owner and goods sold after processing without passing through the territory of the owner; and

(s) Any other goods exempted from customs procedures or otherwise excluded from data sources. (Cases that sometimes arise include shuttle trade; acquisition of ships, aircraft, and satellites; trade between free trade zones of an economy and residents of other economies; goods in bonded warehouses in economies that use the special trade system; and amounts below customs thresholds.)

Goods for resale acquired by travelers while on visits (sometimes called shuttle trade) are included in general merchandise. *Shuttle trade covers transactions involving* *the purchase of goods in an economy by travelers (nonresidents) who then transport these goods back to their economy of residence where they are to be sold; goods purchased by travelers in their home country for resale abroad; and goods purchased by travelers abroad in one economy and sold abroad in a second economy.* It is sometimes also called informal cross-border trade. Because the intent of this travel is not to acquire goods for personal use—recorded under travel—but to engage in a business and to make a profit, the goods acquired and sold are recorded under general merchandise. Goods for own use or to give away acquired by travelers in excess of customs thresholds are also included in general merchandise. For example, durable goods (such as cars and electronics) and valuables (such as jewelry) may be acquired in this way. This treatment is consistent with international merchandise trade statistics, but care is needed to avoid double counting such goods by including them also under travel.

4.6 Because there is no change of ownership of goods between a resident and a nonresident, or because the goods have no value, *the* BPM6 *excludes the following cases from general merchandise:*

(a) Transit trade. These goods are admitted under special customs procedures that allow the goods to pass through the territory. They are excluded from the general merchandise of the territory of transit;

(b) Migrants' personal effects. The personal property that accompanies people changing residence is not classified as a transaction because there is no change in ownership;

(c) Goods consigned to embassies, military bases, and so forth from their home authorities and vice versa;

(d) Goods sent to an enterprise's external operations where those operations are not sufficiently substantial to constitute a branch. A common example is goods sent abroad from the home base for use in a construction project not undertaken by a separate entity; these goods are not included in exports of general merchandise of the territory of the home base;

(e) Goods temporarily exported or imported without a change of ownership. Examples include goods for repair, as part of an operating lease, for storage, and animals or artifacts for participation in exhibitions or competitions (such movements of goods should be tracked, so as to identify cases where the goods are subsequently sold, rather than returned);

(f) Goods for assembly, packing, labeling, or processing by an entity that does not own the goods concerned;

(g) Returned goods. In these cases, the goods were not accepted, or a change of ownership occurred but the parties later agreed to annul the change of ownership. It is recommended that revised entries should be made to exports and imports for the period when the goods were initially recorded, to remove the voided transaction especially for returns of occasional, high-value goods. However, for statistical convenience, deductions from exports and imports may be made in the periods when the goods are returned for minor cases;

(h) Samples of no commercial value;

(i) Trade in goods between free trade zones and residents of the same economy; and

(j) Any other goods that have been included in the data source although there was no change of ownership.

B.1.1 Special cases: Goods sent abroad for processing and goods under merchanting

4.7 The value of goods sent abroad for processing without a change of ownership is excluded from trade in goods. Instead, the value of the work done on the goods by the resident of one economy on behalf of a nonresident would be considered a manufacturing service and classified as *manufacturing services on physical inputs owned by others.* Thus, after processing, the manufacturing service would be recorded as an export of services by the economy undertaking the processing and an import of services by the economy of the unit that owns the goods. Sale of the goods after processing (i.e., including the value of the manufacturing service) would be recorded in the merchandise trade statistics.

4.8 *Merchanting* is defined in the *BPM6* as the purchase of goods by a resident from a nonresident combined with the subsequent resale of the goods to another nonresident without the goods being present in the compiling economy. Thus, the physical possession of the goods by the owner is not necessary for the transaction to take place. The acquisition of the goods by merchants is recorded in the *BPM6* as negative exports of the economy of the merchant and the subsequent sale is shown as positive export of the economy of the merchant.

B.2 Services

4.9 The *BPM6* defines services as the result of a production process that changes the conditions of consuming units or facilitates the exchange of products or financial assets. They are generally not separate entities over which ownership rights can be established and cannot generally be separated from their production. However, this definition is not clear-cut, and in practice the distinction between goods and services may be based on other considerations, such as data sources.

4.10 The *BPM6* identifies 12 broad groups of services as follows:

1. *Manufacturing services on physical inputs owned by others*
2. *Maintenance and repair services n.i.e.*
3. *Transport*
4. *Travel*
5. *Telecommunications, computer, and information services*
6. *Construction*
7. *Insurance and pension services*
8. *Financial services*
9. *Charges for the use of intellectual property*
10. *Other business services*
11. *Personal, cultural, and recreational services*
12. *Government goods and services n.i.e.*

4.11 These broad groupings are based mainly on the type of product and are closely aligned with the Central Product Classification (CPC). However, travel, construction, and government goods and services are based on the type of transactor and are therefore not defined in the CPC. These three groups may cover a wide range of both goods and services, including many of the services classified according to product. For instance, recreational services acquired by a transactor undertaking travel would be classified as travel services. Likewise, goods acquired by these transactors are excluded from trade in goods and included in the service categories. However, not all goods acquired by these transactors would be classified as travel services. It excludes goods for resale (acquired for shuttle trade), the acquisition of consumer durables (motor vehicles and electronic items), and valuables.

4.12 From a resident's perspective, *travel services* covers the acquisition of goods and services in an economy by nonresidents during visits to that economy and the acquisition of goods and services by residents in other economies during visits abroad. It represents a form of consumption expenditure, but unlike most other forms of consumption expenditure, it takes place away from the economy of the resident. It follows, therefore, that for residents undertaking consumption expenditure abroad the expenditure is classified as imports of travel services, whereas expenditure of nonresidents in the domestic economy is classified as exports of travel services. The residency criteria are critical to determining which transactors would be considered as undertaking travel services, and the price statistician would need to work closely with the national accounts and balance of payments compilers to determine the transactors and expenditure that would fit the criteria.

4.13 For practical purposes, the value of imports of goods, as recorded by customs authorities and in merchandise trade statistics, includes the cost of insurance and freight services. The *2008 SNA* and the *BPM6,* however, adopt an f.o.b. (free on board) valuation principle for both imports and exports (see Section C). This requires, therefore, that insurance and freight be deducted from the value of imports and classified separately as services.

C. Valuation

C.1 Valuation methods

4.14 Three methods of valuation are used in the *2008 SNA*. The basic price is the unit price received by the producer from the purchaser, minus any tax payable or subsidy receivable as a result of its production or sale, and excluding any transport charge separately invoiced by the producer. The basic price *plus* taxes on products resulting from production (including value-added tax (VAT) not deductible by the producer) *less* subsidies on products resulting from production *equals* the producer's price. The producer's price *plus* separately invoiced transportation charges, *plus* wholesale and retail distribution margins, *plus* VAT not deductible by the purchaser *equals* the *purchaser's price*. The focus here is on basic and purchasers' prices, and the three factors that need to be considered in converting output and imports from basic prices to purchasers' prices, and *vice versa*, are transport margins, trade margins, and taxes less subsidies on products. Thus the basic price is what the supplier receives per unit of a good or service exchanged, and the purchaser's price is what the purchaser pays. Basic prices thus pertain to the supply of goods and services while purchasers' prices pertain to the use of goods and services.

4.15 United Nations (1998a and 2004) guidelines define f.o.b. values to include the value of the goods and the value of services performed to deliver goods to the border of the exporting country. C.i.f. (cost, insurance, and freight) values include the transaction value of the goods and the value of the services performed to deliver the goods from the border of the exporting country to the border of the importing country.[5] United Nations (1998a and 2004) recommend that the statistical value of *imported* goods be c.i.f. and *exported* goods f.o.b. F.o.b. values can be applied to imports, but they exclude the transport cost of the imported goods to the port or place of importation; the cost of insurance while in international carriage; and the cost of loading, unloading, and handling charges associated with the transport of the imported goods to the port or place of importation.[6]

4.16 Thus exports at basic prices (from a resident supplier) *plus* taxes, subsidies, and other margins to get the good to the border of the exporting country *equal* exports f.o.b. *plus* costs to get the good to the border of the importing country *equal* imports c.i.f. *plus* costs to get the good to the purchaser *equal* imports at purchasers' price (by a nonresident user). It may be that our analytical need is for imports at basic prices as a supply by a nonresident supplier and exports at purchasers' prices as a use by a nonresident user. The national accounts distinguish between accounts for resident institutional units as well as nonresident (rest of the world) ones. In this case, exports at basic prices (from a nonresident supplier) *plus* taxes, subsidies, and other margins to get the good to the border of the exporting nonresident country *equal* exports f.o.b. *plus* costs to get the good to the border of the importing resident country *equal* imports c.i.f. *plus* costs to get the good to the purchaser *equal* imports at purchasers' price (by a resident user). The link between the valuation principle and supplier/user status therefore has direct implications for the valuation basis appropriate for exports and imports.

4.17 When we take the point of view of a unit that has a center of economic interest in the territory of the economy, we will say we are taking the *resident approach to international trade flows,* and we value exports of the total economy at basic prices and imports at purchasers' prices. When we take the point of view of a unit that has a center of economic interest in the rest of the world, we will say we are taking the *nonresident approach to international trade flows,*[7] and we value exports of the total economy at purchasers' prices and imports at basic prices.

4.18 Exports f.o.b. thus are in practice the same as exports valued at the frontier of the exporting economic territory, say country A. These f.o.b. exports from A may be imports to country B, but if valued at c.i.f., will differ from the f.o.b. export value because they will include transportation, insurance, unloading, and handling charges. However, imports to country B valued at f.o.b. of the exporting country A will exclude such costs and be equal to f.o.b. exports from A. The identities between, respectively, f.o.b. exports and f.o.b. imports of the exporting economic territory and, respectively, imports and exports of the rest of the world make f.o.b. valuation a useful accounting tool. Notwithstanding its c.i.f. recommendation for valuing imports, United Nations (2004) acknowledges that f.o.b. values for imports assist reconciling import data with the corresponding f.o.b. export data from the country of origin and allow world trade at a common valuation to, at least in principle, balance.

4.19 If the source data for the weights and unit values (as proxies for prices) are derived from customs declarations there is a need to identify how closely the c.i.f. and f.o.b. valuations meet analytical needs within and outside of the *2008 SNA*. Because trade transactions for balance of payments and national accounts statistics should be recorded on the basis of their value at the time of change in ownership between residents and nonresidents, and because some freight and insurance services are supplied by residents of the importing country, the c.i.f. values need to be separated into their f.o.b. and freight and insurance components. Further, balance of payments and national accounts statistics distinguish between goods and services, and freight and insurance margins have to be shown separately to allow for this.

4.20 Valuation issues relating to the timing of recording, that is, at change of ownership, are considered in Section C.2. There is the need to consider what valuation

[5]Chapter 14 and Annex B of United Nations (2004) outline a number of valuation possibilities and how they differ from a c.i.f. and an f.o.b. valuation.

[6]Under f.o.b. the cost of loading may be divided between buyer and seller and only partially included in the value of the shipment when it should in principle be included in full in the value of exports.

[7]This is discussed further in Dridi and Zieschang (2004).

principles are desirable for analytical purposes. This is undertaken in Section C.3. These valuation principles will be seen to be identified according to basic and purchasers' prices, and although such principles can be applied when obtaining data from surveys of establishments, there remains a problem if the data source is customs declarations. In Section C.4, the correspondence between the valuation principles used in customs declarations and those required to meet these analytical needs is considered. Issues relating to the treatment of services and the unit of account are outlined in Sections C.5 and C.6.

C.2 Valuation and time of recording

4.21 For any given transaction between residents of different economic territories, a single date of recording in concert with a unique valuation for that date ensures international accounting consistency: The value of the seller's export is the same as the value of the buyer's import, and both are recorded or *accrued* on the same date. The *2008 SNA* and the *BPM6* recommend that exports and imports of goods be recorded in the accounts of the transacting parties at the market price prevailing when *change of ownership* occurs. This has implications for international transport charges and goods under merchanting.

C.2.1 International transport charges

4.22 The treatment of transport margins for valuation depends on when the change in ownership occurs. If, for example, the supplier (exporter) delivers the commodity to the importer without an explicit charge for transportation, the transport margin is part of the basic price—change of ownership takes place on delivery. If the purchaser (importer) is responsible for transport, the basic price excludes transportation—change of ownership takes place on leaving the supplier (exporter). However, the issue is complicated if the transportation is subcontracted to a third party, further depending on whether the third party is subcontracted by the exporter or importer and where the third party contractor is resident, especially if it is in a third country. For example, say an enterprise A in one country exports goods to another enterprise B in a different country and employs a subcontracting enterprise C to undertake the transport. If C is resident in A's country and is contracted by A to provide transport, the transport is a cost to A that is included in the export basic price. Were B to contract with C (resident in A's country) there would be an export of transport services by C separate from the export of goods by A. If C was resident in B's country, and contracts with A, there are imports of transport services from B to A, which are then included in exports from A to B. If, however, C (resident in B's country) contracts with B, the transport charges do not figure in the traded values. To further complicate matters, if C is in a third country and contracts with A, then C is exporting services to A and A includes the value of such services as part of the basic price of the good. Were C to contract with B, there is only an export of services from C to B. Note that the purchasers' price includes the value of the goods and transport charges in all cases except when B undertakes the transport directly as a secondary activity and where B subcontracts C to undertake the transport. In such cases the change in ownership takes place on leaving the supplier, and transport charges are excluded from exports and imports. These principles follow those given in *2008 SNA*, Chapter 14, Section B.5. They are important because the difference between basic and purchasers' prices depends in part on the inclusion or otherwise of transport charges, which in turn is dictated, at least in principle, by the concept of change in ownership. Section C.4 will consider how the practical implementation of such principles is constrained by data sources.

C.2.2 Recording practice

4.23 The *BPM6* notes that goods are considered to change ownership when the parties enter the goods in their books as a real asset. United Nations (1998a and 2004) recommend that the time of recording be based on when the goods enter or leave the economy. For customs-based data-collection systems, the date of lodgment of the customs declarations is used as a useful approximation. In practice, data sources for trade statistics for international goods generally deviate from the valuation and time of recording ideals, and there are generally lags between the time of export and the time of import as the goods are in transit to the destination.

4.24 For high-value capital goods that may take several months or years to complete, the *BPM6* notes that when a contract of sale is agreed in advance, progressive change of ownership takes place rather than the change of ownership at completion. Thus, when the contract calls for progress payments, the transaction value may be approximated by the value of stage payments made in each period.

4.25 The World Trade Organization (WTO) Agreement on Valuation[8] establishes the basis for the customs

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[8]Agreement of Implementation of Article VII of the General Agreement on Tariffs and Trade, 1994.

valuation of international trade. The customs value is based on the transaction value of the imported goods and is equivalent to the market price charged to the importer for the goods at the time of export, plus some adjustments. The adjustments permitted under the agreement cover the following:

(a) Charges that may be incurred by the buyer before the goods are exported, but are not included in the price payable;

(b) Royalties and license fees that must be paid by the buyer as a condition of the sale of the goods; and

(c) The value of goods and services supplied by the buyer and used in the production of the goods.

The customs value including the adjustments under the agreement is essentially the f.o.b. value in the *2008 SNA*.

4.26 For cases where the transaction value cannot be determined, the WTO has established a hierarchical list of alternative methods that could be considered. The two most important alternatives are the transaction value of identical goods or the transaction value of similar goods, respectively. United Nations (1998a and 2004) recommend that countries follow the WTO guidelines in the valuation of goods for statistical purposes. The statistical value differs from the customs value in such a way that compilers make adjustments "to promote the comparability of international merchandise trade statistics" (United Nations, 1998a, paragraph 116, Chapter IV). The data compilers are encouraged to use additional sources to make sure that imported goods are correctly valued at c.i.f. and exported goods are correctly valued at f.o.b.

C.3 Analytical needs for valuing imports and exports

4.27 Having outlined different valuation methods and the principle of timing of recording, it is necessary to determine what valuation method is appropriate, and this will depend on the analytical/users' needs. Two approaches will be outlined that meet different analytical needs. The first derives from the need for export and import price indices (XMPIs) to be used to deflate the values of exports and imports within the 2008 integrated system of national accounts. The second approach derives directly from the quite specific analytical needs of productivity analysis, the transmission of inflation, and terms of trade (effects) measurement. It is stressed from the outset of this section that although the valuation needs are considered in terms of basic and purchasers' prices, countries may not have data available valued in these terms, especially if the basis of their data source is customs documentation. Section C.4.2 considers the suitability of customs-based valuation systems to meet the analytical needs outlined below and the nature of their shortcomings.

4.28 The first approach to valuation derives directly from the *2008 SNA* (Chapter 14), from the supply and use tables (SUTs) and goods and services account. Exports and imports are nominal value aggregates within the *2008 SNA*. As such, an appropriate approach to the valuation of the prices and weights used in the compilation of exports and import price indices is to follow the valuation principles of their counterpart value aggregate. Price deflators are applied to value aggregates at a detailed product level. Exports and imports are given for product groups in the SUTs, but they are not given in isolation. As outlined in Section C.3.1, for each product group they are given along with, and reconciled with, corresponding values of output, consumption, and investment to form commodity balances. Because the *2008 SNA* also advises that SUTs be developed in volume terms, the valuation of the price indices for exports and imports to be used as deflators for the detailed product groups of the SUTs to derive the volume estimates should emanate from the valuation required for the SUT in volume terms. In this way the values, price indices, and volume estimates for exports and imports benefit from their reconciliation at detailed product levels with estimates of output, intermediate and final consumption, and capital formation and have a valuation that is designed to enable this. This approach, outlined in Section C.3.1, is appropriate when the primary purpose of the price indices is for use as deflators of export and import value aggregates in the *2008 SNA*. The valuation requirements for nominal value aggregates for exports and imports thus carry over to the price indices used as their deflators. For SUTs valued at purchasers' prices, and the needs of deflators at the product level, Section B.3.1 outlines how the valuation of the supply of imports is at basic prices with taxes and margins added subsequently to raise it to purchasers' prices, and the value of the use of exports is at purchasers' prices (*2008 SNA*, paragraph 14.75). The valuation principles appropriate for export and import price indices used as deflators will be shown to require a perspective from *a nonresident producer* for whom exports are a use and imports a supply.

4.29 The second approach to valuation principles for export and import price indices is dictated by the analytical needs to analyze the transmission of inflation (including effects on outsourcing), terms of trade measurement, and productivity analysis. Such needs require a resident unit's perspective from which exports are a supply and imports a use.

4.30 These two approaches to valuation are outlined in Sections C.3.1 and C.3.2, respectively. Also required for valuation are principles for the time of recording in a manner consistent with the *2008 SNA*, and these are outlined in Section C.4.

C.3.1 Valuation based on the 2008 SNA

4.31 The *2008 SNA* equates, as outlined in Chapter 15, the sum of the value of transactions for goods and services (hereafter, products) supplied to the economy by domestic production (output) and imports with the sum used for intermediate consumption, final consumption, capital formation (including inventories), and exports—that is,

supply: output + imports
+ taxes less subsidies on products
= *uses:* intermediate consumption
+ final consumption + capital formation
+ exports. (4.1)

4.32 Because imports are supplied to the domestic economy by the rest of the world and exports produced by the domestic economy and used by the rest of the world, this perspective is referred to in this *Manual* as the *nonresident's perspective*. Because the uses of products are usually valued at purchasers' prices, but output is valued at basic prices, it is necessary to add taxes less subsidies on products to the supply side so that both sides are at purchasers' prices and the identity given by equation (4.1) balances. XMPIs are used to deflate exports and imports, and in practice deflation is undertaken not for these aggregates of all products but at a detailed product group level. For the identity to balance *at the product group level* it is necessary to add trade and transport margins and taxes less subsidies as separate items to the supply at basic prices on the left-hand supply.[9] Such commodity balances are used by national accountants to validate data and, where necessary, to estimate missing values as residuals. Supply and use tables consist of a set of such balances covering all products in an economy organized in matrix form with product groups in rows.

4.33 Note that, as explained in Chapter 15 and Chapter 6 of the *2008 SNA*, rearranging the above identity yields the goods and services account identity:

output – intermediate consumption
+ taxes – subsidies
= final consumption + capital formation
+ exports – imports, (4.2)

where the left-hand side of equation (4.2) is GDP from the production approach and the right-hand side is GDP from the expenditure approach. Again, trade and transport margins are no longer included because the identity given by equation (4.2) is for the economy as a whole, and the commodity balances for the trade and transport industries that supply the margins are included in the aggregates.

4.34 It is reiterated that the use of XMPIs for deflating exports and imports in national accounts is at the product group level. This is primarily because the value aggregates for a product group will include a wider range of products than those sampled for the price indices. The assumption is that the sampled price changes apply to all products in the group and this is likely to be more feasible if applied at the group level than for the aggregate as a whole. In much the same way that exports and imports are identified as part of SUTs at current prices for individual products, so that their estimates can benefit from being reconciled with other components of equation (4.1), the deflation of imports and exports are identified as part of SUTs in volume terms so they too can be reconciled with deflated values of other components of equation (4.1). SUTs in volume terms require that for each product group *the deflated values* of output, intermediate consumption, final consumption, capital formation, exports, and imports reconcile. The *2008 SNA* advises that SUTs are compiled at current prices and in volume terms at the same time so that the volume estimates are consistent with the value estimates, which in turn are all consistent across the supply and use components.[10] The deflation of exports and imports should not be a piecemeal activity but undertaken as part of a consistent and integrated system.

[9]The inclusion of trade and transport margins was unnecessary in equation (4.1), whose aggregates are summed over all products, because the margins used canceled with the margins supplied by the industries supplying trade and transport services.

[10]A detailed account and illustration of SUTs in volume terms can also be found in Chapter 9 of Statistical Office of the European Communities (Eurostat) (2008).

4.35 Supply and use tables are a powerful tool used to compare and reconcile data from various sources. Their features are outlined in detail in Chapter 15. They can be valued at basic prices but are more commonly valued at purchasers' prices. The use table is a matrix of products in the rows and details of their uses (intermediate consumption, final consumption, capital formation, and *exports*) in the columns. The supply table is a matrix that has as its rows the same products as the use matrix, with columns for *imports*, domestic production at basic prices, and a valuation adjustment necessary to raise their sum, the total supply, to purchasers' prices. Supply (and imports as a component of it) should thus be measured at purchasers' prices and uses (and exports as a component of it) at basic prices. Therefore, to derive SUTs at purchasers' prices it is necessary to raise the supply table from basic to purchasers' prices. (It also follows that to derive SUTs at basic prices it is necessary to lower the use table from purchasers' to basic prices.) Chapter 14 of the *2008 SNA* stresses the need for a common valuation for supply and use so that the product rows can balance and thus be useful for reconciliation. The *2008 SNA* further emphasizes the usefulness of balancing SUTs in volume terms and advises that, for consistency, SUTs in current values and in volume terms be prepared at the same time (paragraphs 14.134 and 15.96). It follows that there is a need for price indices to deflate the components of SUTs at current prices to derive corresponding tables in volume terms. Two of these components are, of course, exports and imports.

4.36 Although it is recognized that the valuation of SUTs in current prices is generally at purchasers' prices, *2008 SNA* notes that there are arguments for (and against) the use of basic prices, as against purchasers' prices, for valuing SUTs. The principle argument for a valuation at basic prices is that it facilitates the compilation of SUTs in volume terms, a matter that is of interest to this *Manual*. The case for deflating the components of an SUT at basic prices[11] is that producer price indices (PPIs), used at a detailed level for deflating both output and intermediate consumption, are compiled at basic prices and thus are well suited for deflating the rows of a basic price SUT. Further, a use—for example, intermediate consumption of a product—valued at purchasers' prices can be decomposed into that part of its value at basic prices derived from domestic production, and that derived from imports,[12] trade margins, transport margins, taxes, and subsidies on products. This decomposition enables separate deflators to be applied to the constituent elements. This is not to say that—after the removal of trade margins, transport margins, taxes, and subsidies on products and separation of uses of intermediate consumption into that sourced domestically and that sourced from imports—the resulting rows will be sufficiently homogeneous to apply deflators without any concern. But much of the heterogeneity owing to varying margins for trade, transport, taxes, and subsidies will at least be removed from an aggregate otherwise valued at purchasers' prices. Further, separate deflators can be more appropriately targeted to that part of intermediate consumption, for this example, that is domestically sourced and that part that is foreign sourced. The analysis of changes in the volume of different uses that is outsourced is of much interest in economic analysis.

4.37 The case against valuing the SUT at basic prices is first, as outlined in *2008 SNA*, Section D.2, the arduous task of the decomposition of the use table (at purchasers' prices) into its six constituent components of basic price of domestic production, imports, trade margins, transport margins, taxes, and subsidies. Six matrices must be set up to enable the decomposition *of each element* of the use table. This is a much more resource-intensive task than setting up the six columns on the supply side to raise supply at basic prices to purchasers' prices. Second, the deflation of margins and taxes is problematic on conceptual as well as practical grounds. Third, while it has been argued that PPIs are usefully available for deflation at basic prices, it is also the case that the consumer price index (CPI) is suitable for deflating detailed components of household consumption expenditure at purchasers' prices.

4.38 XMPI compilers are advised to discuss with national accountants their needs in this respect. If national accountants compile, or intend to compile, SUTs in volume terms, XMPI compilers need to find out which valuation system is used, basic or purchasers' prices. It may, of course, be difficult to tailor XMPIs to such needs. There will be resource considerations, because this will be one of the other analytical needs outlined below. The data available for XMPIs may be

[11]For more details on the case for valuing SUTs in volume terms at basic prices, see *2008 SNA,* Section D.3, and United Nations (1999b).

[12]*2008 SNA,* paragraph 14.132, advises that for the individual product rows of each use at basic prices, imports are separated from domestic production. In some product groups a use may only be domestically produced or only imported, and in other cases, in the absence of hard data, informed judgment may have to be used to allocate the respective shares.

customs data and unit value indices with their attendant problems, outlined in Chapter 2, and valuation (imports are valued c.i.f. at the point of entry into the importing country and exports at f.o.b. at the point of exit from the exporting country) that exactly match neither basic nor purchasers' prices. There are also likely to be problems in valuing goods for processing and merchanted goods, as outlined below, as well as matching at a detailed level products classified in the rows of the SUT by Central Product Classification and those classified for traded goods by the Harmonized Commodity Description and Coding System or Harmonized System (HS) or Standard International Trade Classification (SITC). However, at least there will be some understanding of the extent to which the XMPIs necessary for the deflation of SUTs are appropriate in valuation terms for such use and, if not, the factors governing discrepancies. It may the case that for product groups of major importance to the economy, adjustments are made for that product group for this purpose so that the valuation is rendered more appropriate to the need.

4.39 The compilation of national accounts may in some countries not benefit from SUTs in volume terms, or even at current prices. In such cases the XMPI compilers, along with the compilers of PPIs and CPIs, need to discuss with national accountants their present and planned needs for deflators. Attention should be directed to the valuation system of imports and exports currently used for the national accounts so that the price indices used are effective in decomposing the change in the value aggregate into its price and volume components. Such valuations, at least for goods, will for the large part derive from the customs source data used.

C.3.2 Valuation based on a resident's perspective

4.40 The valuation principles appropriate for export and import price indices used to analyze the transmission of inflation (including outsourcing), terms of trade measurement, and productivity analysis require a resident unit's perspective. For a resident producer, exports are a component of output and imports a component of intermediate consumption, products used by producing units as inputs. Exports may also include existing (transportable) fixed assets, withdrawals from inventories, and valuables. More important, imports also include imported products for final consumption (including that by households and government), as well as imports of transportable assets, additions to inventories, and valuables. What matters for resident suppliers is the value received for supplying products to the export market, and in this respect a valuation of exports is meaningful at basic prices. As will be outlined below, the basic price will include transport and insurance if it is bundled with the price of the good, but not if it is a separately invoiced service, in which case, if supplied by a resident supplier, it will be a separately classified export of a service. What matters for resident purchasers using imports is the price they pay for the imports, and in this respect a valuation of imports is meaningful at purchasers' prices. This requires *a resident's perspective* for which imports are a use and exports a supply.

4.41 Consider the uses of goods at purchasers' prices in the international trade context of this *Manual*. When taking the resident's view of imports as an intermediate or final use of goods and services, it is necessary to distinctly consider the import status of separately invoiced items as well as the distribution margins in the same purchase transaction. These separately invoiced "sub-transactions" for goods, transport services, insurance services, distribution services, and the like, must be classified as imported or domestically supplied to fully account for the contribution imports make to a given goods and services flow for the purchasers' values of intermediate uses by product and industrial activity and of final uses by product. GDP estimated from the *production approach* is based on the *value added* to the value of goods and services used in the production process (intermediate consumption) to generate the value of output. GDP can be thought of as being equal to the sum of the value added produced by all institutional units resident in the domestic economy. Taxes less subsidies on products need to be added back to value added to ensure that the values of supplies are equal to the values of uses at purchasers' prices. GDP is defined from the production approach on the left-hand side of equation (4.2); therefore, it is the sum of value added by resident producers plus the value of taxes less subsidies on products. The valuation from the production side is from the *resident producer's* perspective: the prices at which they sell and buy. The production approach from a resident's perspective, in concert with utility maximization approaches for consumption, and investment function approaches for capital formation, would be appropriate for import and export price and volume series used for the analysis of (the resident country's) productivity change, changes in the terms of trade, and transmission of inflation.

4.42 As outlined in Chapter 15, the resident's perspective can be phrased within the accounts of the

2008 SNA. The affected accounts are the production account for output and intermediate consumption, the use of income account for consumption, and the capital account for capital formation.

C.4 Practice and data sources

4.43 The valuation used in practice depends on the source data available. Two sources are considered, establishment-based survey data and data derived from customs declarations.

C.4.1 Establishment-based survey data

4.44 Establishment surveys are an important and, for commodities not subject to customs controls,[13] necessary data source. Establishment-survey price data have specific advantages above unit value customs-based data, as outlined in Chapter 2, that include enabling the valuation of transactions to be better determined. It is possible in a survey to better define the required price according to the principles underlying the need. For example, for the analysis from a resident producer's perspective, exports should be valued at basic prices and imports at purchasers' prices. For the SUTs, imports are valued at basic prices and exports at purchasers' prices. The prices requested in the survey can be defined according to needs. Information requested will include the transaction price, the transport costs, if separately invoiced, and other salient information on the contracting arrangements for transportation. If transport services are separately invoiced, the basic price excludes them; if not, the basic price includes them. Adjustments for taxes and subsidies can be made depending on whether a basic or purchasers' price valuation is required. This *Manual* recommends that priority be given to the use of establishment surveys for exported and imported commodities that account for a large proportion of traded values undertaken by a relatively small number of establishments, such as oil and some basic commodities. Establishment surveys should be used not only to minimize the unit value bias that arises from customs-based data, but also to better adopt the appropriate valuation principle.

[13]Examples of commodities that may be excluded from customs statistics are services; goods circulating in customs unions; goods delivered to offshore establishments such as oil platforms; certain types of small-volume but high-value goods carried by persons, such as diamonds; and ships and aircraft whose transit from one country to another may not be readily distinguished from those that are part of trade.

C.4.2 Data derived from customs declarations

4.45 For some commodities, information from customs statistics may be used for weights and for unit value changes, as proxies for price changes. Information on imports from customs declarations are valued c.i.f. (i.e., they include the cost of carriage, insurance, and freight) at the point of entry into the importing economy. This valuation is standard, regardless of whether any of the c.i.f. elements are provided by domestic enterprises because import duties are imposed on the c.i.f. valuation. If it is the exporter that contracts the delivery—transport charges are not separately invoiced (whatever the nationality of the carrier)—the inclusion of the cost of transport in the value of the good imported renders the c.i.f. valuation close to a basic price valuation, at least with regard to incorporating transport charges.[14] A c.i.f. valuation based on customs declarations excludes the cost of transport from the border of the importing economy to the premises of the importer. Transport for this may be provided by either a resident or nonresident carrier. If a resident enterprise undertakes this element of the transport it will be excluded from the import value of the good, and the basic price plus taxes less subsidies should equal the purchaser's price.

4.46 If the importer undertakes delivery itself or contracts with a unit resident in the same economy to do so—transport charges are separately invoiced—there is in fact no import of transport services even though it will appear there is when imports of goods are recorded c.i.f. The *2008 SNA* (paragraph 14.71) advises that fictional exports of the same amount of transport services are recorded to correct for this. If the importer contracts the delivery and the carrier is not co-resident with the importer—again transport charges are separately invoiced—an import of services takes place and, while its value will be included in c.i.f., the required valuation for the *2008 SNA* is to exclude it and separate the c.i.f. value into goods only and transport service elements. Because the required basic price is one that excludes transport costs, an f.o.b. valuation based on customs documentation will better approximate a basic price valuation than a c.i.f. one. An f.o.b. valuation based on customs declarations excludes the cost of transport from the producer to the border of the exporting economy and will therefore differ from a basic price valuation.

[14]However, describing this as c.i.f. is not helpful in an *SNA* context because it is a legitimate part of the cost of the imported good and should not be seen as a separate import of transport services.

If the importer undertakes delivery itself or contracts with a unit resident in the same economy to do so, the basic price plus transport costs plus taxes less subsidies payable by the purchaser should equal the purchasers' prices.

4.47 It may be that resources or information are unavailable to determine the party responsible for transport costs. A c.i.f. valuation of imports would have the transport costs and insurance for transit included in the c.i.f. valuation for imported goods, but the value of transport costs and insurance that is separately invoiced will also be included—in fact, double counted—in the imports of transport and insurance services. A global c.i.f./f.o.b. adjustment is entered in the supply table to adjust for double counting such services.

4.48 Exports are valued f.o.b. (free on board) at the point of exit from the economy of the exporter. The *2008 SNA* requires that valuation take place when the change of ownership takes place and the f.o.b. valuation is at the border of the exporting country. As considered above, this will vary with each transaction depending on the nature of the contract for transportation services. The *2008 SNA* (paragraph 14.112) notes that an assumption that the change in ownership takes place at the national border may be the only practical assumption given customs data sources.

C.4.3 Valuation practice

4.49 Compilers need to identify a target conceptual basis for valuation. Compilers would need to consult with national accountants to determine the valuation basis used in the accounts for imports and exports at nominal values so that consistent price indices are derived as deflators. Transport costs can be an important element of the value of imports and exports both at purchasers' prices and, if not separately invoiced, at basic prices. For the weights for XMPIs, compilers need to discuss with national accountants the basis for the valuations used to ensure that the weights are appropriately valued or, at least, they are aware of, and can record in metadata, any shortcomings. It should be noted that for price change measurement if, for example, the transport margin for transporting goods from the border of a country to the establishment is omitted, the price change is in error only if the transport margin, as a percentage, changes over time.

4.50 If the data source is establishment surveys, then the questions on prices can be framed so that the appropriate concept(s) can be used for the measure. If only customs data are available, then the limitations of the valuation system from such data for different analytical purposes can at least be noted in metadata. In the longer run product sectors should be identified that make up significant shares of trade and of which a large part is undertaken by a relatively small number of establishments or, if by a larger number of establishments, ones for which representative samples can be easily surveyed. Such "low-hanging fruit" should be the subject of establishment surveys to be combined with data from customs sources to form, for price measurement, hybrid indices as outlined in Chapter 2.

C.5 Services

4.51 This *Manual* follows the guidelines of the *2008 SNA* and the *BPM6* in the valuation of services. Services should be valued at market prices at the time the services are delivered.[15] Where a market price is not available, then an equivalent market price based on similar market conditions should be adopted. This may not be the case in transactions between affiliated units, which may not reflect market conditions. These units often use transfer prices, which are assumed accounting values that may have only a loose relationship to the prevailing market prices when the transactions are recorded.[16]

4.52 Services generally are consumed at the point of production. Therefore, measuring international trade in services does not present the problems of differences in dating transactions presented by trade in goods, at least in principle.[17] However, it may be difficult to determine the residency of the units undertaking the transaction. As trade in services usually necessitates direct interaction between the units, there may be no official intermediary (like the customs authority for goods) identifying the economic agents, much less valuing and dating their transactions in services.

4.53 The value of insurance services and some financial services are not directly observable and must be

[15]As a clarification regarding international transportation and insurance services, these would be delivered at the time the goods are loaded onto the carrier.

[16]For a more in-depth discussion on the treatment of transfer prices see Chapter 18 of this *Manual*.

[17]However, establishing the date of delivery for the international transportation and insurance of goods may be a problem, depending on transit times relative to the accounting period, because they are recorded in customs administrative systems when crossing the frontier of the destination country rather than the frontier of the source country.

derived from other observable transactions. The total value of insurance and pension services is derived as the margin between the amounts accruing to the insurance company and the amounts accruing to policyholders. The amounts accruing to enterprises would cover premiums or contributions and "premium supplements," which is the investment income on technical reserves, whereas the amounts accruing to policyholders would be claims or benefits. However, non-life insurance claims are usually subject to wide fluctuations owing to sudden major catastrophes, such as earthquakes and hurricanes. Thus, if the margin is estimated using only the claims payable, then the value of insurance services would fluctuate widely and may even be negative. In this regard, the estimation of insurance service charges for non-life insurance includes an adjustment for claims volatility.

4.54 Depending on the nature of the service, the charges for financial services may be explicit, which are directly observable, or implicit. Explicit charges include commissions, loan application fees, and loan fees. For financial intermediaries receiving interest income, the actual interest includes both an income component and a charge for the services. Thus, following the *2008 SNA*, the financial services provided by financial intermediaries in one economic territory to account holders in another, and charged through the interest income, is estimated as the sum of the following:

(1) The difference between a reference rate and the interest rate paid to liability holders times the liabilities outstanding, and

(2) The difference between the rate earned on assets and the reference rate times the assets on account (at market value).

C.6 Unit of account

4.55 The goods and services should be converted from the transactions currency to the unit of account used in the statistics at the prevailing exchange rate at the time of the transaction. If this prevailing rate is not available then the average rate, calculated as the midpoint between the selling and buying rates, should be used. This is consistent with the conversion principles recommended by the *2008 SNA* and the *BPM6*. Note that the accrual principle weighs heavily here, because it may provide a date of conversion, and thus an exchange rate that differs significantly from the dates on customs documents.

D. Classifications

4.56 This section describes some of major systems that are used for classifying goods and services. Classifications for merchandise trade statistics have been long established by the United Nations,[18] in particular, and accepted internationally for the economic analysis of the movement of goods and for customs purposes. Most countries use the Harmonized System to classify trade in goods, and this is the preferred system for producing XMPIs. Because many countries have historically used the Standard International Trade Classification, they may choose to produce XMPIs on both systems to have a longer history of detailed price indices on the same classification system. In any event, there is a correspondence between the two systems.[19] The *2008 SNA* recommends use of the International Standard Industrial Classification of All Economic Activities (ISIC) for activities or industries, the CPC for domestic products, and the closely related Harmonized Commodity Description and Coding System for exported and imported products. Each country may adapt the international standard to its specific circumstances. If the adaptation amounts to adding further detail, the classification is said to be derived from the international standard. The Nomenclature génerale des Activités économiques dans les Communautés Européennes, or the General Industrial Classification of Economic Activities within the European Communities, is an industrial classification derived from the ISIC. If the adaptation reorganizes the way in which detailed categories are grouped compared with the international standard but provides for a cross-classification at some level of detail, it is said to be related. The North American Industrial Classification System being implemented by Canada, Mexico, and the United States is an industrial classification related to the ISIC. The European Union's PRODCOM classification of industrial products is derived from its Classification of Products by Activity, which, in turn, is related to the international standard CPC through a cross-classification defined at a high level of product detail.

4.57 Classifications of trade in services have only recently gained prominence, primarily owing to the increasing interest in international trade in services. In addition to the current trade classifications, other broad classifications have been developed to identify the details

[18]Details of the classification systems are usefully available at http://unstats.un.org/unsd/class/family/default.asp. The site also includes correspondence tables between different systems where relevant.

[19]Available at http://unstats.un.org/unsd/cr/registry/regso.asp?Ci=51&Lg=1.

of goods and services that are produced. They are generally linked to the merchandise trade classifications, which serve as useful starting points for coverage of goods.

D.1 The Harmonized System

4.58 The Harmonized Commodity Description and Coding System, or Harmonized System as it is commonly known,[20] was introduced in 1988 by the World Customs Organization (WCO), as a replacement for the Standard International Trade Classification as the global standard by which imports and exports are classified. The WCO estimates that over 98 percent of the merchandise in international trade is classified using the HS.

4.59 In the HS, goods are classified primarily according to the component material or the type of product, degree of processing, function, and economic activity. Goods are classified under 21 main sections, which are further subdivided into 97 chapters, 1,241 headings, and 5,113 subheadings. Descriptions are common across all countries down to the six-digit level; however, for statistical or tariff purposes, countries are allowed to include additional digits on a country-specific basis.

4.60 The chapters of the HS are grouped as follows:

01–05 Live Animals and Animal Products

06–14 Vegetable Products

16–24 Prepared Foodstuffs

25–27 Mineral Products

28–38 Products of the Chemical or Allied Industries

39–40 Plastics and Rubber and Products Thereof

41–43 Raw Hides and Skins and Products Thereof

44–46 Wood, Cork and Straw and Articles Thereof

47–49 Pulp, Paper and Articles Thereof Including Books and Newspapers

50–63 Textiles and Textile Articles

64–67 Footwear and Headgear Products

68–70 Articles of Stone, Ceramics, and Glass

71 Pearls and Precious and Semi-Precious Stones and Metals

72–83 Base Metals and Base Metal Articles

84–85 Machinery and Electrical Equipment

86–89 Transportation Equipment

90–92 Optical, Clocks, Musical Instruments

93 Arms and Amunition

94–96 Miscellaneous Manufactured Articles

97 Works of Art, Collectors' Pieces and Antiques

D.2 Standard International Trade Classification

4.61 The Standard International Trade Classification (SITC) has been in use since 1961, and before the advent of the HS it was the only trade classification in use. It covers all goods that enter international trade. It groups goods according to the (1) materials, (2) degree of processing that they have undergone, (3) uses of the product, and (4) the importance of the product in merchandise trade. The classification structure of SITC Rev. 4 consists of five levels comprising 10 sections, 67 divisions, 262 groups, 1,023 subgroups, and 2,970 items at the five-digit level.

D.3 Central Product Classification

4.62 The Central Product Classification (Version 1.1.1)[21] is used to classify all goods *and services* and covers broadly the goods and services that fall within the production boundary of the *2008 SNA*. It classifies products according to their physical characteristics, intrinsic nature, and industry of origin.

4.63 The products in the CPC are classified according to the type of production defined in the International Standard Industrial Classification of All Economic Activities (ISIC) in such a way that there are CPC subclasses corresponding to the principal type of product produced in each four-digit activity of the ISIC. The classification system consists of 10 sections, 70 divisions, 305 groups, 1,167 classes, and 2,098 subclasses. Its critical role in the *2008 SNA* is principally to define the product rows of the *SNA's* Supply and Use Table. As discussed in Chapter 15, the SUT is, as well, the conceptual framework for the index product weights of the PPI, CPI, and XMPIs.

[20]The latest version at the time of writing was released in 2007. Details are available at http://www.wcoomd.org/home_wco_topics_hsoverviewboxes.htm.

[21]Version 2 is in draft form at the time of writing available at http://unstats.un.org/unsd/cr/registry/regct.asp?Lg=1.

D.4 International Standard Industrial Classification of All Economic Activity (ISIC)

4.64 The International Standard Industrial Classification of All Economic Activity (ISIC) classifies producer units according to their major kind of activity mainly on the principal class of goods produced or services rendered; that is, ISIC classifies principally by an output-type criterion. The categories of the ISIC at the most detailed level (classes) are delineated according to what is in most countries the customary combination of activities described in statistical units. The groups and divisions, the successively broader levels of classification, combine the statistical units according to the character, technology, organization and financing of production. Wide use has been made of the ISIC, both nationally and internationally, in classifying data according to kind of economic activity. The major categories of the ISIC Revision 4, are shown in Chapter 15, Box 15.2.

D.5 Statistical Classification of Products by Activity in the European Union

4.65 The Statistical Classification of Products by Activity in the European Union (CPA)[22] was developed by the European Union for use in member countries. The classification is based on the industrial origin of the products, and it is structurally linked to the Revised Statistical Classification of Economic Activities in the European Community (NACE). It has six levels made up of 17 sections, 60 divisions, 220 groups, 492 classes, 946 categories, and 2,303 subcategories. The European Union's PRODCOM classification of industrial products is derived from the CPA.

[22] The latest version at the time of writing is the 2002 version available at http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=236&Lg=1.

D.6 Extended Balance of Payments Services Classification

4.66 The Extended Balance of Payments Services Classification (EBOPS) represents an extension to the *BPM6* classification of international trade in services and reflects the main categories of the standard components of balance of payments. It is primarily a product-based classification and is therefore described in terms of the CPC. However, as with the *BPM6*, the EBOPS includes some classes that are not compatible with international product classifications such as the CPC or CPA (see Section B.2).

4.67 The EBOPS also includes various supplementary items and alternative groupings that are intended to provide additional information on the transactions recorded in the system. For example, the EBOPS includes an alternative grouping for *travel,* which presents a reconfiguration of the travel categories in the *BPM6* and presents different travel subgroupings such as goods and accommodation.

5. Data Sources

5.1 The two main approaches (the unit value approach and the survey pricing approach) of compiling the imports and export price indices are associated with different sources of statistical information. We consider administrative and survey sources in the next section. We further subdivide the discussion by considering sources for goods and sources for services as well as sources for index weights and sources for prices. Chapter 6 considers sampling issues for price collection with a particular emphasis on survey-based price indices, and Chapter 7 outlines the principles and practice of the collection of prices for the sampled items in the sampled establishments.

A. Administrative Sources

5.2 The administrative sources of data are (1) customs data, (2) data from the international transaction reporting system (ITRS), and (3) other administrative data.

A.1 Customs data

A.1.1 Customs declaration

5.3 Customs data are the basic data source for the weights and the unit value approach of compiling the import and export price indices for *goods*. Customs data are used to decompose the value flows in foreign trade statistics into price and quantity factors as well as provide value weights for compiling indices as weighted averages of price relatives.

5.4 The regular customs documents (*customs declarations*) are forms filled in by exporters and importers and submitted to the customs. A goods declaration is "a statement made in the manner prescribed by customs, by which the persons concerned indicate the customs procedure to be applied to the goods and furnish the particulars that customs requires for its application."[1]

[1]WCO (2006), General Annex, Chapter 2, E19./F8. Available at http://www.wcoomd.org/kybodycontent.htm.

5.5 In most countries, a customs declaration is required for merchandise imports and exports, whether or not these goods are subject to customs duties (but there are important exceptions to this, which will be further noted). In principle, a customs declaration identifies the importer or exporter, the product code, the value of the shipment, the shipping quantities, the duties paid, the country of origin or destination, the port of entry or exit, the mode of transport, the costs of transport, and the costs of insurance and freight. Customs, the statistical office, or another agency compiles statistics on foreign trade on the basis of electronic copies of the customs declarations.

5.6 The principal *data items* on customs documents used in the calculation of the price indices are as follows:

- *The detailed commodity code:* For almost all countries, the classification structure is based on the Harmonized System (HS), using the first six digits and extended with two to four more digits for national purposes.
- *The country code:* This is a code designating the country of last known destination for exports or the country of origin/consignment for imports.
- *The quantity (or quantities) of exported or imported commodities:* The World Customs Organization (WCO) has recommended the use of standard units of quantity for weight, length, area, volume, electrical power, and number. One of the above standard units of quantity is specified for each HS six-digit subheading. It is also recommended that in cases where the standard unit is other than weight, a weight also be reported. The weight figures must be reported on a net basis (excluding packing).
- *The value of the exported or imported commodities:* The customs value should, to the greatest extent possible, be based on the price actually

paid or payable for the goods being valued.[2] That price, subject to certain adjustments, is called the "transaction value." Where there is no transaction value or when the transaction value cannot be accepted because the price has been influenced by distortions resulting from certain conditions or restrictions, the agreement provides for other methods of determining customs values. The WTO Agreement on Valuation also allows countries to include in or exclude from the customs value, in whole or in part, such components as (1) the cost of transport of the imported goods to the port or place of importation; (2) loading, unloading, and handling charges; and (3) the cost of insurance.

A.1.2 The statistical value in merchandise trade statistics

5.7 As mentioned above, the WTO Agreement on *valuation* should be followed in determining the customs value of the imported and exported goods. Although the agreement allows including or excluding various types of costs to the customs value of goods, United Nations (1998a)—*International Merchandise Trade Statistics: Concepts and Definitions (IMTS),* Rev. 2—provides that the statistical value of imported goods should be a c.i.f.-type value (cost, insurance, and freight, i.e., including transport and insurance costs up to the border of the compiling country) and that of exported goods should be an f.o.b.-type value (free on board, i.e., excluding transport and insurance costs beyond the border of the compiling country).

5.8 The customs value is used for determining the statistical value. The statistical value should not include taxes due on exports or imports such as customs duties, value-added tax, excise duties, levies, export refunds, or other taxes with similar effects.

5.9 *Accuracy* is probably the most important characteristics for evaluating the quality of the basic data. Usually the customs declarations are validated immediately when they are submitted to the customs office in a computerized system,[3] to verify the customs operations data. At the statistical office, the customs declarations are further validated—for codes of nomenclatures, for plausibility of values, etc. The most routine validation procedures are the following:

- Validation for outliers in the customs values;
- Validation for misclassification or missing codes (country code, commodity code, currency code, mode of transport, quantity measures, etc.);
- Validation for customs procedures (imports, exports, temporary admission, temporary exportation, re-importation, reexportation, etc.);
- Validation for time of recording (the date when the goods enter or leave the country);
- Validation for internal consistency (ratios of gross weight to net weight, value in currency multiplied by exchange rate equal to statistical value in domestic currency; improbable unit values; improbable border point/means of transport; improbable quantity/means of transport; improbable seasonal goods/tariff information; etc.); and
- Validation for consistency with other data sources (partner country data, domestic production data, and world commodity prices).

The computerization of customs operations should enable most customs data to be easily accessible to statistical agencies in a *timely* and regular manner.

5.10 The main challenge in using customs unit values for import and export price indices is to ascertain that product categories are sufficiently homogeneous to minimize distortion in price measurement owing to compositional changes (see Chapter 2 and the illustration in Table 6.1 on bias and compositional change). Chapter 6, Section C, outlines some routines for testing for heterogeneity within classes, and Table 6.2 illustrates how hybrid indices are compiled using both unit value indices, when justified, and price relatives, otherwise. As discussed in Chapter 2, the use of unit value indices is considered likely to be warranted in only a limited number of cases. When using the unit value indices from customs data for compiling the price indices, there are several further *practical problems* that cannot be resolved easily: (1) the appearance of new products, (2) quality changes, (3) the unique goods, and (4) the seasonal and other discontinuities in appearance of commodities. The possibilities available under the unit value approach to resolve those problems are very limited. The options available are either to accept the data as sufficiently comparable for practical use or to reject the data as a basis for decomposition of value series. For most price index

[2]The World Trade Organization (WTO) Agreement on Valuation (WTO, 1994).

[3]See www.asycuda.org/ for details of the system.

purposes, unique or one-of-a-kind goods should be excluded along with shipments of personal effects.

5.11 But in spite of these and other exclusions, it is almost impossible to rival the coverage of the customs data for goods. In addition to their coverage, the customs data are updated on a continuous basis. This is why customs data remain an important data source for weights whether price changes are measured as unit values or price relatives. The form of *access to customs data* is of importance for the decomposition of imports and exports values into price and quantity elements. The recommended practice is that the statistical data compiler has an access to individual records. The availability of individual customs declaration data makes it possible to sample individual transactions, to exclude some specific transactions, or to adjust some transaction on the basis of knowledge derived from other sources. Moreover, the availability of individual transactions data makes it possible to calculate statistical measures for each commodity or commodity/country combination.

A.1.3 Customs quantity concept

5.12 On the customs form, information is submitted on gross *weight,* net weight, and—for some special commodities—quantity in units other than weight. Only net weight and quantity in other units are used for compiling price indices.

5.13 As noted above, a customs document does not necessarily identify the *transacted quantity*. For each HS subheading, the term "quantities" refers not to the physical measure but to the measures of the customs tariff heading. These generally are more closely related to a *shipping quantity* such as weight, as noted above. Thus, even if all of the clearances in an HS class are for very similar goods, the customs quantity may not be close to the transaction quantity concept needed for the decomposition of value flows. The problem worsens when the HS class contains a heterogeneous assortment of items. Compilers must decide whether the quantity measure is acceptable—whether the corresponding specification in the customs tariff contains one commodity only or whether the quantity measure should be rejected as a uniform measure. Compilers also must decide whether the customs class contains two or more distinct types of goods. To identify subclasses of customs clearances, compilers usually supplement the commodity classification with additional data fields such as (1) country of origin/destination, (2) size of transaction, (3) mode of transport, and (4) identity of the importer and the exporter. The next chapters deal further with methods to decide when detailed customs classes are acceptable as product specifications for price indices.

A.1.4 Customs price concept: The unit value

5.14 The customs price concept for a given detailed class of goods is the *unit value,* defined as the ratio between the total value of clearances in the class and the total quantity cleared in the class. These unit values may or may not be a good source of price information. The main issue is that the elementary aggregates, which the customs information can define, contain multiple products about which customs data can say little. Consequently, supplementary surveys also may be needed in identifying and measuring the average transaction prices for the elementary items that make up the detailed customs aggregates of transactions. Additional surveys also are needed to measure the prices of the goods and services lying outside the scope of ordinary customs administration such as international trade in services unrelated to shipping imported goods. (See Chapter 2 for a detailed discussion of issues related to unit values.)

A.1.5 Customs coverage

5.15 In international merchandise trade statistics, the objective is to record goods entering and leaving the economic territory of a country. In practice, what is recorded are goods that enter or leave the statistical territory, which is the territory with respect to which data are being collected. Customs declarations record the goods that enter or leave the customs territory of a country, because that is the only territory to which customs law applies. The statistical territory (i.e., the reference territory for which merchandise trade statistics are produced) may coincide with the economic territory of a country or with some part of it. It follows that when the statistical territory of a country and its economic territory differ, international merchandise trade statistics do not provide a complete record of inward and outward flows of good. There are two trade systems in common use by which international merchandise trade statistics are compiled: the general trade system and the special trade system.

5.16 *The special trade system* is in use when the statistical territory comprises only a particular part of the economic territory. The special trade system (strict definition) is in use when the statistical territory comprises only the free circulation area, that is, the part within which goods "may be disposed of without customs restriction." Consequently, in such a case, imports include all goods entering the free circulation area of a compiling country, which means cleared through customs for home use, and exports include all goods leaving the free circulation area of a compiling country. However, under the strict definition, goods imported for inward processing and goods that enter or leave an industrial free zone would not be recorded because they would not have been cleared through customs for home use.

5.17 The *general trade system* is in use when the statistical territory of a country coincides with its economic territory. In addition to the special trade system, the general trade system covers merchandise that enters or leaves premises for inward processing of industrial free zones, and premises for customs warehousing or commercial free zones. The *IMTS*, Rev. 2, advises using the general trade system because it provides a more comprehensive recording of the import and export flows than the special trade system does. It also provides a better approximation of the change of ownership criterion used in the Commission of the European Communities and others (*2008*), *System of National Accounts 2008 (2008 SNA)*.[4]

5.18 Customs data normally cover all transactions in goods flowing across the borders. However, some countries do not record very low value transactions, because the effort to record them outweighs the usefulness of the data for statistical purposes. It is often the case that special transactions (industrial plants, vessels and aircraft, sea products, staggered consignments, military goods, offshore installations, spacecraft, motor vehicle and aircraft parts, postal consignments, petroleum products, and waste products) are not recorded through customs declarations. Not all countries record through their customs declarations the international transactions in imports and exports of electricity, gas, and water. Customs often exclude or do not cover well the trade flows between countries that belong to customs unions, such as the European Union (EU)[5] and the Southern African Customs Union. The same can be said for the free zones that some countries have set up for processing imported materials into manufactured articles. In addition to the gaps in the domain of international transactions customs data cover, there are *underreporting* and *misreporting* problems that include the following:

- Not all of the information required by the form is collected on every declaration, particularly data on insurance and freight;
- Customs administrations collect the declarations mainly for revenue purposes and tend to pay more attention to the accuracy of the details on import declarations than those of export declarations, because the latter usually are not subject to customs duty;
- The quality of data on imported commodities varies from country to country; some commodities are subsidized whereas others are not; and some importers undervalue imports to avoid high import duties; and
- Despite the provisions of the WTO Agreement on Valuation, trade among related enterprises may reflect transfer pricing valuations significantly different from market values in order to affect tax advantages for the group.

A.2 International transactions reporting system

5.19 Many countries use an ITRS to collect data for their balance of payments statistics. The ITRS records transactions between residents and nonresidents whose settlement is carried out through commercial banks. In principle the ITRS covers trade in both goods and services, but in practice it is mostly used for the compilation of data on trade in *services*. However, it could be primarily used as a source for compiling weights for import and export indices mainly for services. The ITRS data are primarily collected from commercial banks. The data items collected by the ITRS forms usually are the direction of a transaction, the purpose of the payment, the currency used, the value of a transaction, the classification of a transaction, and the country of the nonresident party. It should be mentioned that the ITRS data could be a source for compiling weights for imports and exports indices only if the transactions are classified on a very detailed level—for example, a five-digit Central Product Classification (CPC) code. The transactions might be expressed in different currency. In this case, they are converted (by use of the midpoint exchange rate applicable for each transaction) to the common unit of account in which the balance of payments is compiled. ITRS information records

[4]However, the *2008 SNA* identifies trade in goods for processing as relating only to the service component of the processing activity, as was outlined in Chapter 4 and in more detail in *2008 SNA,* Chapter 15.

[5]With the removal of frontier controls between the EU member states, a new system, known as Intrastat, was devised to collect statistics on intra-EU trade. Intrastat records the arrival and the dispatch of goods between the member states. The information is collected directly from enterprises and makes use of value-added tax (VAT) data and the VAT reporting system. Intrastat does not cover private individuals and small enterprises that are exempt from VAT declarations.

transactions on the date of payment, which is generally considered a good approximation for the date of change in ownership. When valuating the transactions, there are potential problems with the bundling of transactions (transactions that should be classified into different CPC groups) and recording on a net basis (foreign exchange payments may cover both credit and debit transactions). ITRS data vary in coverage from country to country, depending in part on variations in the transaction threshold at which financial institutions must report information into the system. There also are variations in the scope of coverage of international transactions in payment for services. These variations depend in part on the nature of the transactions. ITRS bank settlements are often supplemented by collection of information settled outside the domestic banking system (e.g., via accounts held abroad by residents) or by transactions for which only net payments are made, such as those taking place in clearing or netting schemes.

A.3 Other administrative data

5.20 Export and import data for *services* transactions typically are not collected by customs sources. Trade data on services may be collected by several agencies that focus on specific industries. The agencies' survey instruments and databases are specific to the needs of the agency and its data users and may serve as a source for data on weights and a sampling frame to select service establishments for price surveys discussed in the next section.

5.21 The country's Ministry or Department of Transportation database can be a source of information on international transportation exports. For example, these data can be used to select a sample of air carriers that regularly provide data on airfreight. The data may include the origin and destination airports, shipment weight, dimensions of shipment, whether shipment is containerized, type of product shipped, type of buyer of the service, and any special services provided by carriers. The same database is used as the primary sampling source for air passenger fares. The required information in this case is data on passenger counts, revenues, origin and destination airports, fare classes for international trips (business, first, or economy class), and fare type (one-way or round-trip).

5.22 In addition, the national regulatory authorities for telecommunication and postal services might collect information on volume (and permit deriving a form of "unit values") for many communication and postal services.

5.23 The main source of data for exports of travel and tourism goods and services purchased by international visitors during their stay in the country may be the Ministry or Department of Tourism database.[6] This database usually covers expenditure data on the following activities: round-trip international airfare, tour packages, airport expenditures, transportation, lodging, food and beverages, gifts and souvenirs, entertainment and recreation, and other.

5.24 The Ministry of Finance or Treasury can be a significant source of information. International trade within a customs union may be covered, for example, by requiring additional information itemizing purchases of goods and services by source country and sales by destination country on value-added tax returns.

B. Survey Sources

5.25 When customs or other administrative sources are seen to be inadequate for identifying products and tracking their prices, compilers can undertake establishment surveys to fill this gap. Because services are not covered in customs data, prices of internationally traded services generally will be collected via surveys. The surveys may take the form of a collection directed specifically at prices for foreign trade, or they may have been undertaken for another purpose, such as compiling the producer price index (PPI).

B.1 Export and import price surveys

5.26 The export and import price surveys are not much different in concept from any price surveys, for example, the PPI survey. Calculating the foreign trade price indices entails collecting prices from businesses relating to particular products (imported or exported goods and services) and time periods. These businesses can be both importers and exporters of products. The frequency of price collection is either monthly or quarterly.

5.27 In the standard methodology, a set of establishments is selected, preferably with the selection probability of each establishment proportional to the establishment's share in imports or exports. This may be accomplished explicitly by taking probability-based

[6] The UN and World Tourism Organization define an "international visitor" as "any person who travels to a country other than that in which s/he has his/her usual residence but outside his/her usual environment for a period not exceeding 12 months and whose main purpose of visit is other than the exercise of an activity remunerated from within the country visited."

export and import samples of establishments from lists, or *frames*, of establishments engaged in external trade that are assembled from tariff and export declaration documents and from an ITRS if services are involved. The sampling may also be by selecting the set of establishments representing the top, say, 50 to 75 percent of the value of trade during the period referenced by the frame. The first are called *probability samples* and the second *cutoff samples*. Both types of samples require the existence of the described frame, which is often taken from customs records for goods. Such sampling approaches are discussed in more detail in Chapter 6.

5.28 The price surveys require weights for the products, establishments, and transactions. The customs data on shipment values can be used to derive weights at each level of sampling. Each establishment is assigned its own weight. When probability sampling is used, the weight is the sampling fraction (e.g., 1/10) multiplied by the value of shipments for the strata. So if the value of shipments for the strata is 150,000, then the establishment's weight is 15,000 (150,000 × 1/10). Note that each of the selected establishments in the strata has the same weight.

5.29 When cutoff sampling is used, the weight is the establishment's value of shipments plus a pro rata proportion of the value of shipments for establishments not included in the sample. Table 5.1 contains an example of assigning weights to sampled establishments. A cutoff sample for all establishments with a share of, say, 0.020 or more is selected for the sample. The sample has 10 establishments with a total value of imports of 56,000, which covers 70 percent of the total import value. Each establishment's share in the sample is also calculated. For example, establishment 0193 has a value of import shipments of 15,600, representing a 0.195 share of total imports. Its share of the total sample is 0.279. The value of shipments for the establishments not selected in the sample is 24,000, which must be allocated to those selected. The final weight (out of 80,000) for the selected establishment will be its own weight (15,600) plus its pro rata share of the weight for the nonselected establishments (15,600/56,000 × 24,000 = 6,686), that is, 22,286. The final weight for the other establishments is calculated in the same way. Note that each establishment's weight will be different.

5.30 Of course the cutoff sample assumes that the price changes for the larger establishments accounting for the top 70 percent of imports will be the same, on aggregate, as that for the establishments accounting for the bottom 30 percent. If the latter price movements are expected to differ substantially from the former, then a sample of the latter ones may be selected and the weight of 24,000 apportioned to the selected smaller establishments using principles similar to those outlined above.

5.31 Within each establishment, there will be a sample of eligible products (see Chapter 6, Section G.2, for a description of product sample selection). For each sampled product within the establishment, we

Table 5.1. Example of Assigning Weights

Establishment Number	Import Value	Import Share	Sample Share	Final Weight
0193	15,600	0.195	0.279	22,286
0125	10,500	0.131	0.188	15,000
0105	8,600	0.108	0.154	12,286
0090	5,800	0.073	0.104	8,286
0169	3,900	0.049	0.070	5,571
0054	3,100	0.039	0.055	4,429
0132	2,600	0.033	0.046	3,714
0019	2,500	0.031	0.045	3,571
0130	1,800	0.023	0.032	2,571
0011	1,600	0.020	0.029	2,286
Sampled	**56,000**			
Not sampled	**24,000**			
TOTAL	**80,000**	**0.700**	**1.000**	**80,000**

can calculate its share of imports among the other selected products. Assume that for establishment 0193 there are three eligible products—product 1 with an import value of 5,000; product 2 with an import value of 3,000; and product 3 with an import value of 2,000. To derive each product's weight, we take the product's share in the sample multiplied by the establishment's weight. We calculate product 1's weight as 7,800—that is, 5,000/(5,000 + 3,000 + 2,000) × 15,600. The weights for the other products are derived in the same way.

5.32 Sample transactions then are selected from each establishment. A methodology called *disaggregation* may be used to select a sample of transactions with probability proportional to the importance of the product and transaction type in the establishment's total value of exports or imports. Alternatively, an establishment representative may be asked for the items among those exported or imported that collectively account for, say, 50 to 75 percent of the value of export or import business done by the establishment. (The sampling approaches are discussed in detail in Chapter 6.) For each transaction, the price and the quantity transacted are recorded. In addition, a set of transaction and product characteristics is recorded. Among these characteristics would be the date of shipment as a best convention for the desired change of ownership principle.

5.33 Identification of elementary items within the elementary aggregate could then proceed using the price (shipment unit value) and the characteristics information to cluster the transactions. Elementary items would be equated with the identified clusters. If there is little bunching or clustering of transactions, a regression analysis of price on characteristics would be run to see if elementary items are effectively distributed along a price-characteristics locus. If the regression fits well, then the coefficients from the hedonic regression can be used via so-called hedonic quality adjustment methodologies to adjust for changes in the elementary item composition of the elementary aggregate (see Chapter 8).

B.2 Producer price index

5.34 Because establishments directly involved in export and import trade often specialize in international wholesale and retail distribution, these distributive activities are likely to be heavily represented in the target population of international transactions in goods and services. However, producers specialized in other activities also may engage directly in transactions with nonresident buyers to sell their output. Hence, the PPI price surveys, which usually cover the nondistributive activities of mining, manufacturing, and energy production and distribution, also can be sources of price data for the export price index, provided that export transactions in the PPI price sample are identified as such. There is a good a priori reason for integrating price collection between the export price index and the PPI in order to place the minimum response burden on establishments that are contacted to report prices for both the PPI and the export price index.[7] Further, as a PPI is developed for distributive services, PPI coverage of the specialized export-import firms important in the international trade price indices can be employed as part of the calculation of the output price index for the wholesale/retail margin, which is the national accounts measure of output for the distributive services group.

B.3 Consumer price index

5.35 Household purchases of goods and services abroad as a result of recreational tourism are in scope for the consumer price indices (CPIs) of most countries in principle. Those flows of goods usually would be measured via the passenger debarkation documents collected by customs at ports, border crossings, and international airports. Household purchases of goods and services abroad are thought to be an important component of household consumption, particularly for countries too small to have an advanced retail distribution industry, but border on larger countries that do possess such an industry.

5.36 Few countries currently attempt to collect prices for the imports generated by cross-border shopping because it involves collecting information from nonresident retailers or establishing data-sharing agreements with the statistical offices of neighboring countries. In the latter case, the prices of household imports for one

[7]In some countries, export and import price indices are estimated from components of the wholesale price survey. However, the use of wholesale price indices as proxies for import and export price indices is likely to introduce bias in foreign trade indices. Two important reasons for this are, first, that the price representation in terms of firms and commodity items in the domestic market may be significantly different from the situation in the external market and, second, that prices usually move in different ways in the domestic and external market owing to the existence of different competitive conditions and tax structures.

country would be in scope for the export retail distribution price index and the CPI surveys of its neighboring countries as well as the other countries comprising the tourist destinations of its residents. Household expenditure surveys generally do not exclude goods and services purchased abroad, and thus most CPIs include these purchases in their expenditure weights. By implication, statistical offices impute the price index of cross-border shopping for each good or service item by the price index of domestic purchases of the item.

C. Summary

5.37 Table 5.2 summarizes the sources of data for export and import price indices (XMPIs) discussed in this chapter. It shows the types of goods and services from the Central Product Classification (CPC) and the broad types of source information used for goods and for services including customs documentation, the international transaction reporting system (ITRS), and sample surveys.

Table 5.2. Data Sources for Export and Import Price Indices

	CPC Division		Weights	Prices
Goods	0	Agriculture, forestry, and fishery products	Administrative Customs administration ITRS	Administrative Customs administration (for commodity codes containing a single elementary item)
	1	Ores and minerals; electricity, gas, and water		
	2	Food products, beverages and tobacco; textiles, apparel, and leather products	Sample survey Establishment surveys of free trade zones	Sample survey Establishment price surveys (for commodity codes containing multiple elementary items)
	3	Other transportable goods, except metal products, machinery, and equipment		
	4	Metal products, machinery, and equipment		
Services	5	Intangible assets; land; constructions; construction services*	Administrative Customs administration (international transport and insurance) ITRS	
	6	Distributive trade services; lodging; food and beverage serving services; transport services; and utilities distribution services	Sample survey General establishment surveys Customs administration for transport services of imports	Sample survey Establishment price surveys
	7	Financial and related services; real estate services; and rental and leasing services		
	8	Business and production services		
	9	Community, social, and personal services		

*Following United Nations (2002), the *Manual on Statistics for International Trade in Services,* Table A.IV.2, this comprises 51290—Other nonfinancial intangible assets (comprising royalties for the right to use exclusive rights owned by other economic agents) and 54—Construction services. It *excludes* the remainder of CPC division 51—Intangible assets, as well as 52—Land and 53—Constructions.

6. Sampling Issues in Price Collection

A. Introduction

6.1 Price surveys normally collect information on products—their prices and price-determining characteristics—from establishments. For foreign trade in goods, however, administrative sources (customs records) usually are available from which compilers can calculate unit values. The central question of external trade price indices thus arises: Is a unit value a price and a valid data element for constructing an export or import price index? Can we, therefore, use it rather than spending the effort and cost to collect prices from establishments? In Chapter 2, concerns were expressed about the widespread use of unit values as surrogates for price indices, even at the most detailed level of the Harmonized System (HS) classes of goods, and even if subdivided by source (import) or destination (export) country. A strategy was outlined in Chapter 2 for countries whose trade price indices rely on unit values for moving from such a system to a hybrid system that includes actual prices surveyed from establishments, and subsequently, resources permitting, to one in which most of the commodities included have prices based on survey information. A price must be associated with a given and complete description of the product, encompassing the product and transaction characteristics that affect the exchange value or price. Unit values more often than not are averages across a variety of such descriptions, and thus they are subject to composition effects. Unit values will change not only because of change in the price of any given product description within the HS class, but also because an HS class contains a different assortment of priced product descriptions from month to month, quarter to quarter, and year to year.

6.2 More often than not, as outlined in Chapter 2, a given HS class, even crossed with country of origin (import) or destination (export), does not define a homogeneous class of transactions. As a product description, it is insufficient. Unit value indices are used by many countries and a move to price indices has resource consequences. The preferable, though resource-intensive, approach is a one-off switch to an index based on establishment-based price surveys. This may be prompted by a country joining a customs or monetary union. Although the main problem with simply introducing a new program is the resource cost, if a producer price index (PPI) program is already established, there will be natural synergies between the export and import price indices (XMPIs) and the PPI. If resource constraints preclude this, one possibility is to identify whether there are particular products less prone to unit value bias and to utilize unit value indices only for those subaggregates in a hybrid overall index. The compilation technique for such hybrid indices was outlined in Chapter 2, Sections E.1 and E.2, and an example is provided in Table 6.2 of this chapter. The use of hybrid indices is a strategic option outlined in Chapter 2, Section E. *It is stressed that this is a strategy for statistical offices with limited resources.* The intention of this gradualist approach is that it be a staged progression for an eventual move to a system in which the primary data source would be survey-based prices of well-specified representative items. A gradualist approach has major resource benefits. There will be some "low-hanging fruit," that is, establishments responsible for relatively high proportions of exports and imports, some of which may be owned by the state and may already have some reporting obligation. Likely examples of such commodity groups include natural gas, petroleum, electricity, and airlines. There will also be industries in which unit value indices are *prima facie* inadequate measures of price changes, largely because of the churn in highly differentiated products, or the custom-made nature of the products, such as ships and oil platforms. Further, there may be industries that account for a substantial proportion of trade, and the payoff of reliable data far outweighs the survey costs—for example, the use of surveys of fish-processing plants for major exporters of fish products and of agricultural marketing cooperatives for exports of primary products.

6.3 The gradualist approach requires as a first step a rigorous evaluation of each commodity group to determine the relative payoff and cost of abandoning unit value indices. A good starting point would be a listing by commodity group that includes the weight, the perceived reliability of the unit value series, the likely source and reliability of alternative series, and a grade for the relative cost of obtaining such data. The initial aim would be to identify important commodity groups whose current series are deemed unreliable but for which there are readily available alternative sources. The gradualist approach is outlined in Chapter 2, Section G. The evaluation implies that we also look to see if there are commodity classes that are *prima facie* suitable for unit value indices, at least in the short run. Section C of this chapter outlines some simple distributions and summary measures to help identify potentially homogeneous classes for which unit value indices may be used as a starting point.

6.4 Empirically, a product description may be deemed adequate for the use of unit value indices if the variability of the prices of transactions classified under that description is low. However, this empirical justification is not sufficient. The dispersion must be expected to continue to be low, and this is likely to be the case if the customs class is a single product, not differentiated. XMPI compilers cannot be expected to be experts in the details of every commodity market. If the dispersion in prices is particularly low for a detailed HS class, good practice would be for the compiler to contact the importing/exporting establishment to determine whether commodities are available in differentiated forms not only with regard to their characteristics but also to the terms of sale.

6.5 When the detailed customs class is not a single product, it is necessary to identify well-specified representative products within the class and track their prices over time so that the prices of like items are compared with like. The standard approach in this case, which is no different than for other price indices such as the consumer price index (CPI) and PPI, is to design a survey of establishments engaged in foreign trade, select a representative sample of fully described products from them, and follow the prices of this sample of products to construct export and import price indices. The focus of this chapter is with obtaining price information from surveys of importing and exporting establishments.

6.6 In an ideal world, it would always be possible to use statistically sound sampling techniques to produce price indices with a high degree of accuracy and within given resource constraints. There are, however, factors mitigating against achieving an ideal, statistically efficient sample within a given sampling budget: (1) accurate estimates of population variances, required for allocation of sample units to strata, are rarely available; (2) sampling frames are always deficient to some extent, missing some key information, such as births of new establishments, or desired stratification variables; and (3) response rates are unpredictable and may prove to be deficient, which affects the accuracy of the price index levels and measured price changes.

6.7 The aim of the sampling statistician is, therefore, to make the best use of what is available and to apply the principles of sampling theory in a commonsensical and practical way. Arguably the most important steps in sampling are to establish and understand fully what the survey is trying to estimate, the limitations of the sampling frame, and the environment in which the survey will be conducted, that is, likely response rates, data quality, and levels of resources.

6.8 There is a direct relationship among the uses of the export price index (XPI) and import price index (MPI), the scope of the coverage of the external trade price survey, and the requirements for sampling frames. Two of the major uses of the XPI and MPI are as general indicators for inflation in a country's international trade and as deflators for goods and services trade in the national accounts. The broader the trade coverage of the indices in terms of economic activities, particularly services, the more useful they are in inflation analysis and compiling volume GDP measures. But broad coverage requires the ability to develop sampling frames for the full range of economic activities engaged in international trade, including both goods-producing and service-producing activities. These sampling frames also must be kept up to date by recording both the births and deaths of enterprises in each sector as well as new products produced and purchased and nearly obsolete ones.

6.9 Once coverage and uses have been established, a sample design can be drawn up, with decisions made about stratification, sample size, and allocation. Random sampling techniques may be employed in countries where large amounts of data are available and reasonable estimates of variance can be made. In many country situations, only limited details of sampling parameters are available, and the statistician may have to fall back on procedures that use expert knowledge at many stages in the selection process. To the extent possible, acceptable, practicable sampling procedures should be used. Judgmental approaches should be used only as a last resort.

6.10 As with most panel samples collected through time, price surveys suffer from problems associated with a changing population. Any sample of establishments and products will become increasingly unrepresentative over time, and it is likely to be depleted as establishments cease the sale or the production of selected products or cease operations altogether. Some form of panel rotation or supplementation for the samples is advised to minimize any bias caused by sample attrition, noncoverage of new products, new establishments, and new production technologies.

B. Starting Position

6.11 The data to be collected must be identified and understood. Focusing on goods first, if there are customs records available on a timely basis, the problem is to determine which of the detailed customs strata or classes contain a single product. For these classes, use of already collected administrative information is more efficient and much less expensive than developing establishment survey sources from which prices are directly collected.[1] However, such cases, as outlined in Chapter 2, are likely to be the exception rather than the rule, but may be a necessary undertaking because of resource requirements. Regardless of the source of information, whether administrative or survey, the source must be assessed for its compliance with the statistical concepts set out in Chapters 3 and 4.

B.1 Collecting transaction prices

6.12 It can be difficult to define and collect prices for many goods and services. Often the quoted list or book price does not represent the price received or paid by the establishment. Ideally, we want to collect actual prices received for a representative sample of establishment transactions. For goods, this can be achieved quite regularly. This is also the case with most services. However, for some services—for example, banking and insurance services—the volume and price of financial intermediation are not clear-cut and the price may have to be derived from transaction information. (Additional information on prices for these services is provided in Chapter 11.) In addition, the prices collected should be based on transactions rather than estimates, except as a last resort.

[1]An exception would be when establishment price data are already collected for a PPI and the output (input) is mainly for exports (imports), or if the establishment is willing to extend the survey to domestic and foreign markets.

B.2 Valuation

6.13 Valuation principles were outlined in Chapter 4. They were identified as depending on the needs of users and being constrained by data sources. For estimates of GDP by the expenditure approach, the *2008 System of National Accounts (2008 SNA)*, Commission of the European Communities and others (2008), advises that the supply of goods and services should be valued at basic prices, and, as a supply component, so should MPIs. The *2008 SNA* recommends purchasers' prices for the uses of goods and services. As the *2008 SNA* views exports as a final use of domestically produced goods and services by nonresidents, exports should be valued at purchasers' prices. They thus include all relevant taxes on products levied by the government of the source country, and all transport, insurance, and distribution charges to the point of international shipment. However, *2008 SNA* also gives the pros and cons of using basic or purchasers' prices for the detailed deflators required for supply and use tables in volume terms.

6.14 As we noted in Chapter 4, a resident's approach, useful for the analysis of productivity, terms of trade, and transmission of inflation, requires different principles, and if data are from customs, the usual free on board (f.o.b.) prices for exports and cost, insurance, and freight (c.i.f.) prices for imports may not live up to some of these needs.

B.3 Date of recording transactions

6.15 In line with the valuation of international trade in the *2008 SNA*, accrual accounting rules should be followed as far as possible, so that in the XPI and MPI, prices are recorded at the time of shipment (goods) or delivery (services). Although country practices often differ—for example, prices may be recorded at the time of purchase or order—the preferred timing generally is at the time of shipment for goods and of delivery for services. Prices could be an average of several observations during the month or the price on a particular day of the month; both approaches are used and are acceptable.

B.4 Structured product descriptions

6.16 The price-determining characteristics of each product or variety should be identified so that transaction specifications are complete; that is, so that there is little variance in the price of a product with that description at any given point in time. For example, the price per liter of paint will depend on the number of cans to be shipped, brand, type and quality of paint, terms of payment (net

30 days), type of customer, and any special discounts that may apply. The *product specification* or description in XPIs and MPIs, as for other price indices, should be systematic and permit statistical analysis of how product and transaction characteristics contribute to the transaction price at any given point in time. International good practice in product description is exemplified in the *Structured Product Descriptions* of the International Comparisons Program, which are similar to the "checklists" used by certain national statistical offices.

C. Goods: Testing Customs Elementary Aggregates for Multiple Elementary Items

6.17 The first phase in setting up a hybrid compilation system for price indices for export and import goods is to identify the elementary items whose prices will be tracked by the index. Resource constraints may dictate that unit value indices have to play a major role in XMPI measurement, and the logic of this process is to begin with evaluating the data already available from customs sources to form hybrid indices. The objective is to test whether each good's elementary aggregate defined by the detailed customs commodity code and destination or source country comprises a single elementary item. If so, because the unit value can be used as a surrogate price, a price relative can be formed from them directly, and no further collections are required, assuming customs timing errors are not too severe relative to the change of ownership principle. The requirement that a single elementary item comprises a code is a restrictive one—for example, "*Harmonized Code 6402991815: tennis shoes, basketball shoes, gym shoes, training shoes and the like*" contains trade in an item of a single quality under the same terms of sale over time, and will continue to do so. If the good's elementary aggregate, defined by the detailed customs commodity code and destination or source country, cannot be deemed to comprise a single elementary item, additional surveys will be required to identify the underlying elementary items within those commodity codes. Yet there may be cases where unit values have to be used in hybrid indices that cover more than one item in a class. To evaluate the fitness of customs unit values as the basis for elementary aggregate price indices, we consider two suites of testing protocols below—though see also Chapter 10, Section D, for an account of outlier detection routines.[2] These are illustrative and there are many alternative approaches that may be more suitable for a particular country's needs. In adopting a set of tests a country should apply them to past data to decide on not only the appropriate methods but also the appropriate (cutoff) parameters to adopt for a particular method. Contacts with importing and exporting establishments will be useful to decide on what constitutes outliers or alternative groups.

6.18 However, the use of unit values in such circumstances is born out of limited resources to do otherwise and should not be relied upon. This explains the rationale behind the strategy in Chapter 2, Section G, of such hybrid indices being part of a staged progression to XMPIs, primarily based on survey data.

C.1 Price dispersion test

6.19 Our definition of an elementary item is based fundamentally on the price dispersion of all transactions falling within the group defined by the item. We can consider a given domain of export or import transactions defined by a particular set of commodity and transaction characteristics to be an elementary item if there is very little price dispersion within the domain at any given point in time. An elementary aggregate defined by a customs commodity class crossed with destination/source country may satisfy this condition. In this case, there is one elementary item in the elementary aggregate, and the unit value that can be derived from customs information may be considered a reasonable estimator of the desired unit value estimate for the elementary item.

6.20 We would test this for a given month by constructing a unit value for every transaction (customs document) in the domain in that month and measuring the statistical variance of the resulting collection of unit values. A sufficiently low variance measurement would allow us to use the customs unit value of the elementary aggregate as the basis for an index. There are caveats to the accuracy of this empirical test, however.

- First, as noted in the previous section, the quantity measure available on customs documents is a shipping quantity rather than a transaction quantity. The shipping quantity and the transaction quantity must be in a fixed (for all time) proportion to one another across all transactions in the elementary aggregate for the customs (shipping) unit values to be accurate estimators for the desired transaction unit values. They can differ because of recording errors in the shipping quantities on the customs form, as already noted.

[2]Systems for outlier detection tailored to the needs of unit value indices are outlined in Technical Annex B, pages 190–98, of WTO, UNCTAD, and ITC (2007).

- Second, the date of each customs document must be within the same month as the date of the change of ownership of the goods in question (i.e., the date the transaction accrues).

6.21 If the first condition is seriously violated, the shipping unit values are inaccurate estimators for the desired transaction unit values. If the second condition is violated, the unit values will be classified in the wrong month even if they are correctly calculated. Both of these types of error would tend to cause us to conclude that a customs elementary aggregate is not coincident with a single elementary item, even if it is true.

6.22 Consider the following price dispersion testing suite to ascertain whether a customs elementary aggregate contains one or multiple elementary items:

- Compute trade values, total shipping quantities, and unit values for each customs document classified into a set of *cells* of data defined by the *Month or quarter* of observation, Detailed (6- to 10-digit HS) *commodity code,* and Destination (exports) or source (imports) *country.*
- Compute and examine the mean and standard deviation of the unit values in each cell.
- For those cells whose unit value coefficient of variation is *greater than 0.5, test for multiple elementary items* within the cell.
  - Construct a histogram of the unit values within the cell.
  - Examine the unit values in the cell for a few extreme upper or lower outlier unit values—for example, three or more standard errors from the mean unit value—whose removal might significantly reduce the coefficient of variation.
- If there is evidence of outliers, examine the customs document of the outlier for errors (e.g., in order of magnitude) in the value or quantity recorded, or consult the text description for indications of different product or transaction characteristics explaining the unusual unit value from the document. Examine several quarters of data to verify the persistence of any observed patterns.
- Examine the histogram for evidence of separate clusters of unit values. If two or more clusters are visually identified, examine the customs forms in the identified clusters for comments indicating differences in product or transaction characteristics explaining the difference in the unit values represented between the clusters. Examine several quarters of data to verify the persistence of any observed patterns.
- If outliers or clustering identified by differences in product or transaction characteristics are not present or occur only once in several quarters, trim the outliers and smallest clusters from the cell and examine the time series of the resultant estimates of unit values for the cell for erratic temporal behavior inconsistent with any available anecdotal evidence about the prices in the product class and concurrent exchange rate movements.
- If outliers or clustering identified by differences in product or transaction characteristics are persistent (e.g., more than one in five quarters),
  - Recommend adding a coded data item identifying the clusters of items on the customs forms for the product class to permit subclassifications of customs forms within the existing cell; and/or
  - Consider collecting a survey of exporters/importers (discussed below) to measure and test for statistically significant price-determining characteristics within the product cell.
- If there is no conclusive evidence of outliers and clustering but there is an erratic time series of cell unit values at the desired frequency of the export or import price index (e.g., quarterly or monthly), Consider collecting a survey of exporter/importers (discussed below) to measure and test for statistically significant price-determining characteristics within the product cell. (Some commodities do have erratic price variations, which the unit value can truly reflect. This should also be taken into account.)
- If the cell coefficient of variation in unit values is consistently less than or equal to 0.5; or there is no conclusive evidence of outliers and clustering and a reasonable time series of unit values for outlier-adjusted cells at the desired frequency of the export or import price index, can be formed, then consider the commodity/country cell as an elementary item and the cell unit values as the average prices of elementary items.

6.23 In the presence of some price dispersion in the elementary aggregate, a further test would require additional information on the price-determining characteristics of the transactions in the customs elementary aggregate, if available. If there is no variation in any of these characteristics, we would tend to accept that there is a single elementary item in the aggregate. If, however, there is significant variation in the price-determining characteristics, we would conclude that the customs aggregate contains more than one elementary aggregate.

6.24 Detection of variation in price-determining characteristics usually would proceed judgmentally on the basis of any text notes included on the customs documents in the domain of the elementary aggregate. Multiple elementary items also might be suspected if there are dissimilar clusters of shipment unit values within the domain. Credible evidence of multiple elementary items within a customs elementary aggregate would be the basis for augmenting customs data with survey information in order to identify those elementary items by measuring the associated product and transaction characteristics common to the elementary aggregate.

C.2 Quantity proportionality test

6.25 We note for completeness that if the quantities of elementary items transacted within an elementary aggregate are highly correlated from period to period—that is, product quantities remain in roughly fixed proportions—then unit values across elementary items can be used to track price change in the elementary aggregate *even if it fails the price dispersion suite*. Fixed-quantity proportions over time at high levels of product detail are, however, an untenable assumption for most price indices, and this is certainly true of export and import price indices. Table 6.1 illustrates the pitfalls of using unit values to produce a price index for a metals elementary aggregate in a case where there has been no change in prices, but there has been a shift in relative quantities. The challenge to constructing a test of proportional quantities within a customs product-destination/source cell is that it often is not possible to obtain repeated observations of the value and quantity shipped by a given shipper of a given specific product, because this would require a laborious process of going through customs documents from month to month looking for possibilities of matches. In some months for some types of products, there will be few if any matches to be found even if the effort is spent to find them. This tends to bolster the view that if the price dispersion suite fails, the default methodology is to survey importers/exporters in the cell.

6.26 Nevertheless, an approach to testing for proportional shipping quantities would be to examine the average shipping quantity across quantiles (say, quintiles or deciles) of the values of shipments, and compare relatives of the average quantity shipped between months for each quantile. If the quantity relatives are all the same across quantiles or tightly clustered, some support would be lent to the proposition that relative quantities have not changed.

D. Goods and Services: Surveying Enterprises to Identify Elementary Items

6.27 When a customs elementary aggregate is deemed to contain multiple differentiated elementary items, or in order to survey goods and services transactions beyond the scope of customs sources, it is necessary for good measurement to design surveys of enterprises to obtain elementary item prices. In part because statistical surveys can be designed for export and import price indices, whereas customs administrative files are designed principally for tax collection, surveys can capture information on the characteristics of goods and services to the level of specificity needed for a complete product description. The kinds of characteristics of products and transactions on which survey information is to be recorded for each product type are determined as a result of review of trade association literature, comment fields on customs forms, various press sources, and previous survey experience with the elementary aggregate, if available.

6.28 These aggregates generally will be defined for goods by the same detailed HS codes used by the customs administration, crossed with the destination (exports) or source (imports) country. The particular goods aggregates subject to survey will be those that testing has indicated contain multiple elementary items, or for which no customs data are available.

6.29 For services, a starting set of definitions for elementary aggregates would be the five-digit codes under the United Nations' Central Product Classification, Version 1.1, division 59.

6.30 In designing a sample of exporters or importers, the first step is to assemble a comprehensive list or frame of resident trader establishments in the various goods and services to be surveyed. For goods, sample frames for establishments normally are set up using the customs source, as the names of the seller and buyer should be on each customs form. The seller's name would be captured from export declarations to form an establishment sample frame for exports, and the buyer's name would be captured from tariff filings to form the frame for imports.

Table 6.1. Unit Values and Product Mix

This example uses import data for three commodities considered to be three specifications of the same item where the nature of the sales contract (spot versus long term) affects the price.

Commodity		Period t–1			Period t		
2616.10.00.10	SILVER ORES AND CONCENTRATES: COPPER CONTENT						
Source: Orlandia Shipping volume: metric ton		Value (V) (1)	Quantity (q) (2)	Unit value (p) (3) = (1)/(2)	Value (V) (1)	Quantity (q) (2)	Unit value (p) (3) = (1)/(2)
		Observed consignment customs value and shipping volume					
A–	Acme Ores, spot market transaction, Orlandia metals exchange	300	6	50	600	12	50
B–	Acme Ores, contract with Orlandia Metals Corporation	450	10	45	450	10	45
C–	Metals, Inc., contract with Orlandia Mining Company	1200	30	40	1200	30	40
	Total	1950	46	135	2250	49	135
Weighted average unit value (A, B, C)				42.39			45.92

Note: Illustrating the discussion in the text, we have been able to match only three unit value calculations in this customs aggregate between periods t–1 and t. The price dispersion test for these three would be inconclusive, though possible elementary items would be [A B], which combines on the basis of the shipper (here, Acme Ores) or [B C], which combines on the basis of the type of transaction (here, long-term contract). A quantity proportionality test applied to this example suggests that quantities are not proportional for any combination except [B C]. Hence there are at least two elementary items in this matched subset of customs elementary aggregate 2616100010, namely A and [B C]. Accordingly, it will be necessary to survey transactors in silver ores with copper content in order to accurately measure price change.

To demonstrate the importance of identifying and tracking elementary items in customs aggregates observe that in this example prices remain unchanged, but quantities (product mix) change. This leads to change in the unit value of the customs subset from

$$\mathbf{42.39} = \left[\sum_{j=A}^{C} p_j^{t-1} q_j^{t-1} \Big/ \sum_{j=A}^{C} q_j^{t-1}\right] \text{ to } \mathbf{45.92} = \left[\sum_{j=A}^{C} p_j^{t} q_j^{t} \Big/ \sum_{j=A}^{C} q_j^{t}\right].$$

In other words, the unit value index based on data for specific transactions A, B, and C equals **108.33** (or 45.92/42.39 × 100). On the other hand, because the price relative of each of the individual goods is equal to **one** between $t-1$ and t in our example, any price index formula applied to the price, quantity, and value data for varieties A, B, and C would yield the value **100.00** for the commodity group comprising them. The unit value thus has a **composition error** of 108.33 − 100.00 = **8.3 percent.**

6.31 Lists of establishments engaged in international trade in services also may be set up from administrative sources—for example, exporters of services might be assembled from individual and business income tax filings reporting a foreign source of, respectively, earned income and sales, after making a comparison with foreign sales of goods from examination of shippers' export declarations collected by customs. The most efficient source would most probably be available to balance of payments compilers that in many cases come from commercial bank reports of international payments. XMPI compilers should examine these records and ensure good cooperation with balance of payments compilers. A good, if less focused, alternative would be implementing or expanding general establishment survey collections for service activities, such as for the PPI. These collections are based on survey frames compiled from lists of resident establishments and enterprises extracted from business registers, or from frames assembled from tax filings. Elementary items comprising export transactions can be determined and collected from the same sample of establishments used for these general collections, but large increases in sample size may be required to capture an adequate sample of products and transactions in external trade.

6.32 However, for transportation and transportation insurance activities related to imports, if the establishment's information is recorded on properly completed customs documents then customs sources can be used to assemble the survey frame of establishments.

6.33 Other factors to consider in sourcing data for trade price indices from direct price surveys are the following:

- The elementary items should be periodically reselected within the elementary aggregates, at least every five years, to keep the sample representative of current trade flows—usually a costly process for both the statistical agencies and the respondents;
- For customs elementary aggregates containing multiple elementary items, the coverage of commodities and traders from samples may be volatile from month to month or quarter to quarter if infrequent (or casual) traders account for an important share of exports or imports; and
- Through a properly designed questionnaire and good response from sampled establishments, the timing of prices collected through direct surveys can be made to closely approximate the change of ownership accrual principle required by the fifth edition of the *Balance of Payments Manual (BPM5)* and *2008 SNA*.

6.34 Table 6.2 provides a stylized example of how both customs and survey sources of price data could be incorporated into an import price index. In this example customs classes

- 2616100010 SILVER ORES AND CONCENTRATES: COPPER CONTENT;
- 2701200000 BRIQUETTES, OVOIDS, AND SIMILAR SOLID FUELS FROM COAL, and
- 2707100000 BENZENE, WEIGHT OF AROMATIC CONSTITUENTS GREATER THAN NONAROMATIC

are considered to contain only one elementary item, while

- 8419899085 INDUSTRIAL MACHINERY, PLANT OR EQUIPMENT FOR THE TREATMENT OF MATERIALS, INVOLVING A CHANGE IN TEMPERATURE

contains multiple elementary items. Hence, a sample of elementary items is developed for industrial machinery, containing

- Cracking tower, heavy crude oil feedstock; mfr: Petro-Equipment Corporation
- Paint drying booth; 4m × 10m; mfr: Radiant Products, Pte.

E. Common Problems in Price Survey Sampling

6.35 There may be many reasons why price surveys are thought to be unrepresentative and thus liable to lead to inaccurate results. All national price surveys suffer from problems to some extent. The following are some examples:

- Samples are selected purposively rather than using probability sampling methods, increasing the chances of bias. For example, establishments may be selected for their convenient geographical location or because they are known to be good respondents;
- Without probability selection methods, estimates of statistical accuracy cannot be made (but without some initial estimate of variance, a randomly selected sample cannot be optimized—that is, lowest variance given cost constraints—either. This difficult problem is dealt with later.);
- The sample size for an industry or commodity may have become outdated if the industry or commodity has grown or contracted because the base period (period when the sample was selected);
- New products may not be identified or included in the survey. This problem may be relieved to some extent by rotating the sample of establishments;
- The sampling frame may be out of date or may not include certain groups of the target population. For example, a common problem in the PPI is that information on small producers is unreliable because this group often is volatile and difficult for administrative authorities to track, resulting in the weight for small producers possibly being wrong (typically they are underrepresented); and
- Surveys may be voluntary, increasing the chance of nonresponse bias that results when those who do not respond have different price experiences than those who do respond.

6.36 A decision should be made about the **level of accuracy** required:

- *Ideally, a maximum acceptable sampling error should be identified for each published index.*

Table 6.2. Using Price Surveys and Customs Unit Values in the Same "Hybrid" Index

Commodity/Source (Destination)		Period t–1			Period t	
Trade index item group—import example	Value (V) (1)	Quantity (Q) (2)	Unit value or price (P) (3) = (1)/(2)	Value (V) (4)	Quantity (Q) (5)	Unit value or price (P) (6) = (4)/(5)
1 – 2616.10.00.10 SILVER ORES AND CONCENTRATES: COPPER CONTENT; Source: Orlandia; Shipping volume: metric tons	300	6	50	1200	20	60
2 – 2701.20.00.00 BRIQUETTES, OVOIDS, AND SIMILAR SOLID FUELS FROM COAL; Source: Mineland; Shipping volume: metric tons	450	10	45	450	10	45
3 – 2707.10.00.00 BENZENE, WEIGHT OF AROMATIC CONSTITUENTS GREATER THAN NONAROMATIC; Source: Lubovia; Shipping volume: metric tons	1200	30	40	600	22	27
Total value of unit value items	1950			2250		
4 – 8419.89.90.85 INDUSTRIAL MACHINERY, PLANT, OR EQUIPMENT FOR THE TREATMENT OF MATERIALS, INVOLVING A CHANGE IN TEMPERATURE; Source: North Machindia. The following specific transactions collected from a survey sample:	***600***			***700***		
A – Cracking tower, heavy crude oil feedstock; mfr: Petro-Equipment Corporation	***500***		***500***	***500***		***500***
B – Paint drying booth; 4m × 10m; mfr: Radiant Products, Pte.	***100***		***10***	***200***		***30***
Total value of hybrid items (unit value and directly priced)	**2550**			**2950**		

Unit value price index for items 1–3, and also aggregate unit value price index if index for 1–3 is imputed to item 4

Laspeyres: $[300/1950 \times 60/50 + 450/1950 \times 45/45 + 1200/1950 \times 27/40] \times 100 =$ 83.08

Paasche: $([1200/2250 \times (60/50)^{-1} + 450/2250 \times (45/45)^{-1} + 600/2250 \times (27/40)^{-1}])^{-1} \times 100 =$ 96.20

Fisher Ideal: $[(83.08 \times 96.20)^{1/2}] =$ 89.40

Elementary aggregate price index for item 4

Laspeyres: $[500/600 \times 500/500 + 100/600 \times 30/10] \times 100 =$ ***133.33***

Paasche: $((500/700) \times (500/500)^{-1} + (200/700) \times (30/10)^{-1})^{-1} \times 100 =$ ***123.53***

Aggregate hybrid external trade price index

Laspeyres: $1950/2550 \times 83.08 + 600/2550 \times$ ***133.33*** $=$ **94.90**

Paasche: $(2250/2950 \times (96.20)^{-1} + 700/2950 \times (\mathbf{123.53})^{-1})^{-1} =$ **101.53**

Fisher Ideal: $[(94.90 \times 101.53)^{1/2}] =$ **98.16**

Note: This example presumes that the elementary aggregates silver ores with copper content, coal briquettes, and aromatic benzene each comprise a single elementary item and consequently that period-to-period relatives of their unit values are proper estimators for their price indices.

However, sampling error can be assessed only if probability sampling techniques have been used. This often means starting with some estimates of variance for the component index to determine initial sample sizes. Then, once samples have been collected and variances calculated, the sample can be optimized based on the new variance information. However, the calculation of variances and sampling errors is very difficult to accomplish.[3]

- *In practice, there is a trade-off between cost and accuracy.*

A high level of accuracy that would be desirable requires larger sample sizes that may not be affordable. In such cases, costs often determine the sample sizes, and the level of accuracy may suffer somewhat. Many countries find that they can contain costs and maintain acceptable levels of accuracy by using cutoff sampling (discussed later).

6.37 Once the coverage is decided, the **population** to be sampled should be identified and the sampling frame reviewed to determine whether the existing frame needs to be supplemented.

- *Does the frame contain all of the units in the target population? Does it cover all of the industries that are in the scope and all of the establishments in the targeted industries? Will separate frames have to be developed for each industry, group, or division?*

Customs documents contain useful information on enterprises trading. Most business registers have a cutoff (threshold) below a certain size (number of employees or value of sales), and it is usual that some industries are less well covered, for example, construction and retail trade. Also, there is a need to identify establishments separately from parent enterprises.

- *How are units defined in the frame? There are probably borderline units where it is uncertain if they belong in the population.*

Although the PPI survey could in principle serve as the "spine" for a supplemental survey of the prices for internationally traded goods and services, it may not have a sharp enough focus on international trade and will typically cover only exports. For example, ancillary or auxiliary units of an enterprise may be out of scope, and the PPI sample might have to be uneconomically large to detect foreign trade when this trade covers a rather small part of total output. Thus, compilers need to develop a separate establishment sample frame for establishments engaged in international trade to select the sample of establishments that will report the prices of internationally traded products. Customs documents have the required information for this purpose.

- *Are units mutually exclusive?*

There could be double counting, which occurs when an establishment could be included both in its own right and as part of its parent enterprise.

- *Is there information available to allow stratification?*

We need certain data elements that will serve as stratification variables—for example, sales or purchases from abroad by type of good or service, industrial classification, production or sales, number of employees, and location of establishment—in order to select the sample. Again customs documents have information on the value of trade, HS classification, and location.

- *Is there information available to allow weighting for probability proportionate to size (PPS) selection?*

We will need measures of size, such as output, total sales, and value of shipments. Customs documents should be helpful in this respect. If such measures of value are not available, "number of employees" may have to be used as a proxy.

6.38 The level of available **resources** should be decided:

- *This will be a constraint on sample sizes.*

It is generally more expensive to increase the number of establishments sampled, as opposed to increasing the number of prices collected from each establishment. On the other hand, simply increasing the second may add little to accuracy, when intra-establishment (within an establishment) variance is low compared with inter-establishment (between establishments) variance.

- *And this may dictate the methods of measurement.*

The methods of measurement would include, for example, personal visits, telephone collection, or postal or electronic questionnaires.

[3]The United Kingdom, for example, has estimates of sampling variance for its PPI; see Bucknall and others (2005) and Morris and Green (2007).

6.39 Legislative issues may affect the sample design.

- *Will the survey be voluntary or statutory?*

This will affect response rates, which, in turn, have implications for accuracy and sample sizes. Statutory surveys will have higher response rates, although they may result in lower data quality.

- *Are there rules concerning confidentiality?*

This may impose a lower limit on sample sizes—for example, a minimum of four units per stratum may be required. In many small countries, this could be problematic when there are only one or two major exporters for certain goods and services. In such instances, the national statistical office may have to request approval from the companies to publish detailed product-level price indices.

F. Sample Design

6.40 Given information about what the XMPI survey is intended to achieve, the format of the inputs and outputs, desired level of accuracy, and available resources, the process of designing the sample can begin.[4] Again, decisions need to be made, but the main objective of the design process is clear—to maximize efficiency—that is, to minimize sampling and nonsampling errors and to minimize costs.

6.41 Decisions will need to be made about the following:

- Sampling techniques (probability vs. nonprobability),
- Sampling frames,
- Sample structures and stratification,
- Sample allocation between strata, and
- Methods for reducing nonsampling errors.

F.1 Sampling techniques

F.1.1 Probability versus nonprobability sampling

6.42 The statistician, confronted with any measurement problem, must initially consider the possibility of installing a rigorous probability sample. In the context of XMPIs, probability sampling means the selection of a sample panel of exporters, importers, and products (transactions) from a universe of foreign trade activity in which each producer and product has a known chance of selection.

6.43 Nonprobability sampling is known as judgmental or purposive sampling, or expert choice, and samples are chosen by experts to be representative. In practice, however, different experts would rarely agree on what is representative, and the samples are subject to biases of unknown size. Judgmental sampling may be justified when sample sizes are small, but concern about their biases increases with sample size.

6.44 Alternatively, in cutoff sampling, all establishments and/or products over a certain threshold of size are selected for inclusion in the sample. Many countries have found cutoff sampling to be an efficient way to identify the import units for their establishment surveys. Not only does cutoff sampling save resources, it has been shown to approximate probability samples. Cutoff sampling is discussed in more detail below; see also Cope and Freeman, 1998.

6.45 Using a probability sample comes with two well-known advantages. First, it ensures that the items to be priced are selected in an impartial and objective fashion. In the absence of probability sampling, a danger exists that only items that are easy to price will be selected, resulting in biased estimates (indices). In particular, there is likely to be poor coverage of technologically advanced items, like machine tools, electronic equipment, aircraft, or home electronics. These are difficult to price because of rapid changes in specifications. There is also a tendency to place too much emphasis on simpler products, like food items, cement, textiles, or steel bars, for which a comparable series of price quotations can easily be provided.

6.46 The second advantage is that a probability sample permits the measurement of the quality of the survey results through estimates of the variance or sampling error. The quality of results in this context relates to the chance of a difference between the results obtained from the sampled observations and the result that would have been obtained in a complete enumeration of all reporting units in the universe. The use of a probability sample, of course, does not permit the measurement of errors arising from nonresponse, inaccurate reports, obsolete weights, unrepresentativeness of the commodities priced, or any other nonsampling source.

[4]There are many textbooks that can be consulted on the theory and application of sampling. One text used quite often is Cochran (1977), available worldwide.

6.47 Probability sampling conceivably could be used at all stages of the selection process. For example, a random sample of products could be selected from a comprehensive list of all goods produced by all mining and manufacturing firms. For each selected commodity, a random sample of producers could be picked using a comprehensive list of producers; for each selected producer, a random sample of specific brands could then be chosen for regular price reporting from a complete list of each producer's output. A less rigorous approach might involve random choice of producers or retailers, followed by a purposive selection of individual products or items; alternatively, the producers or retailers might be selected on a nonprobability basis using cutoff sampling (described next), while a random sample is picked from all items made by the selected producers. This mixture of nonrandom with random selection procedures and cutoff sampling procedures narrows the interpretation that may be placed on estimated sampling errors but still will retain the advantage that a certain amount of objectivity is imparted to the selection process.

6.48 Optimal sample design requires, for all units in the population, information that will allow effective stratification and increased efficiency owing to selection by PPS. Different variants of probability sampling can be used by statistical agencies:

Simple random sampling—every possible unit has an equal chance of being drawn;

Systematic sampling—every *k*th unit is selected, after a random start. This sampling is affected by any ordering or pattern in the sampling frame. Ordering leads to a form of implicit stratification, and a pattern in the frame can lead to biased samples;

PPS—each unit has a probability of selection in proportion to its size (or some other indicator of importance, but size is commonly used). Once these probabilities of selection are assigned, either simple random or systematic sampling techniques can be used.

6.49 Despite the attractions of probability sampling methods, there will be situations where it is neither necessary nor desirable. Price indices are an area of statistics where the risks in not having a probability sample are relatively low. The potential diversity of the change in prices charged by various producers of a given commodity over many time periods is relatively low. Compare this to the potential diversity for sales or capital expenditures of firms making the same product over the same period of time. The largest firm may become the smallest, and vice versa. Some may even abandon production of the commodity, and new firms may enter. In summary, the measurement of price changes appears to require less rigor with respect to probability sampling than do other areas of statistical measurement. The additional costs that may be incurred using probability sampling can be allocated to other areas in the survey, such as price data collection or improvements to source data on weights.

6.50 That said, without probability sampling, statistical agencies will not be able to produce meaningful measures of sampling error to guide users in distinguishing between real changes in prices and those owing to statistical noise. They also will experience difficulty in statistical decision making to improve the sample design and allocate resources more efficiently. Good measures of sampling error provide statistical offices with data for reallocating the sample to areas with high variance to reduce statistical error.

6.51 In several countries, the range of domestically produced mining and manufacturing goods is so limited and the number of firms producing them so small that there is no point in making a selection; the survey should try to cover all products and all producers.

6.52 In other cases, there may be no practical way of determining the universe in advance. A basic requirement for probability sampling is to define the universe (or population) and to identify all units in the universe. For XMPIs, the universe of goods is usually defined by the customs data for goods and the balance of payments information for services.

6.53 The cost of installing and administering a probability sample may be judged too high. There clearly are high costs involved in the design, selection process, control, and administration of a probability sample for collecting price observations.

6.54 Estimates of variability in price movements also are needed. This information is rarely available for all units in the population, certainly not at a detailed product or item level. One way of dealing with this is to use a two-phase sample, where certain information is collected from a sample of units, and then these units are resampled using this information.

6.55 Probability selection often will be inappropriate because the survey of producers' prices ideally should form part of an integrated program of price statistics. This means that the choice of items to be priced at the intermediate (i.e., producers' prices) stage may depend on the items selected for pricing at an earlier stage (e.g., imports) or at a later stage (e.g., exports, consumption).

6.56 Thus, for most countries a strict probability approach will not be possible or the costs will greatly outweigh the advantages, so a combination of probability and purposive sampling techniques is employed.

F.1.2 Cutoff sampling

6.57 Cutoff sampling is a strategy frequently used by countries to select samples. In this approach, a predetermined threshold is established with all units at or above the threshold included in the sample (selected with certainty) and units below the threshold level not included (zero probability of selections). Cutoff sampling generally results in a high degree of coverage among a small number of prospective units. This occurs because the distribution of the selection variable (e.g., value of imports or exports) is concentrated in a small number of large establishments.[5]

6.58 The problem with such an approach is that the smaller establishments may have different price movements from the larger units and, thus, introduce an element of bias into the price index. The bias would be the difference between the average price change for the noncovered units and the price change for the overall population. If the importance of units excluded is very small or the bias is very small, the effect on the overall error may be very small. Usually the total error is measured by the root mean square error, RMSE, ($\sqrt{\boldsymbol{Variance} + \boldsymbol{Bias}^2}$), and the sample with the lower total error is deemed more efficient. Thus, the approach that produces the lowest total error or RMSE will be preferred. It is possible that a cutoff sample could be more efficient if the bias component of the excluded units is small. For example, if the noncovered units have substantial variation with regard to price change but small bias (i.e., the average price change is not much different), the RMSE could be smaller using the cutoff sample, and the survey costs could be much lower.

6.59 Cutoff sampling has a great deal of practicality for selecting the establishments and products in a multistage sampling scheme. For example, in selecting the importers of goods that will be included in the sample, a threshold can be established that all establishments covering up to, say, 70 percent of goods imports or exports will be chosen. Another aspect of sampling where the cutoff approach can be used is in the selection of the representative products in the sample where all products that represent up to, say, 70 percent of import or exports (at, say, an eight-digit HS classification) are included in the sample.

6.60 Cutoff sampling is not the same as probability sampling. Sampling errors for cutoff samples will not be as accurate because the sample is not necessarily representative of the index population. Statistical offices will need to make special efforts to measure bias among smaller firms in order to calculate the RMSE to get a meaningful measure of error.

F.1.3 Multitiered stratification

6.61 Alternatively, it may be useful to use stratified samples in which various classes of establishments are sampled separately. Often it is helpful to identify three or four strata based on their size, such as large, medium-sized, and small establishments, with each stratum having a different sampling rate. For example, large establishments (based on turnover or number of employees) may be sampled with certainty (i.e., all selected in the sample), medium-sized establishments may be sampled at a rate of 25 percent (one out of every four), and small establishments may be sampled at a rate of 2 percent (one out of every 50).

F.2 Sampling frames

6.62 Whether selecting a sample using probability or nonprobability techniques, we need to define the universe (population) from which we wish to sample, that is, construct a sampling frame. In most countries it is possible to define the population using various lists of enterprises (business registers), compiled for administrative purposes. For the XMPI, these business registers can be constructed from customs documents such as import and export declaration forms. An establishment will be included in the frame if it is named on a customs form as the recipient of an import or the declarer of an export.

6.63 Several attributes would make up the ideal **sampling frame.**

- *The ideal sampling frame would be a complete list of all eligible units (producing and exporting) within the geographic area and industry or product coverage required.*

[5] See de Haan, Opperdoes, and Schut (1999) for an analysis of cutoff sampling in the CPI.

6.64 Registers typically are compiled as the by-product of an administrative system, such as tax collection, social security schemes, and customs records. Alternatively, lists can be compiled using records such as bank accounts. Tax and social insurance lists generally contain, at a minimum, information about geographical location and size (turnover or number of employees), but they may not indicate the principal activity of an enterprise or identify it as an exporter. In trade statistics, the main administrative source for assembling a list of establishments engaged in international trade is customs records, which only cover goods trade and the associated transport and insurance on imports. Supplementary lists from balance of payments sources will be needed for enumerating resident establishments purchasing services from and providing services to nonresidents in transport and insurance services on exports, as well as passenger transport, life and property insurance services, business services, and others.

- *The ideal sampling frame would also be updated instantly with all births and deaths of units and changes in addresses, fax numbers, and so forth.*

6.65 Maintaining an up-to-date register is resource-intensive. It generally is the case that information about the bigger units is more up to date than data on smaller units. This is a particular problem during periods of changing economic structure when some industries or residential areas are expanding and new units may be starting up in large numbers. If units are not removed from the sampling frame when they no longer exist, they may be selected as part of the sample. This needs to be borne in mind when determining sample sizes. Also, a common error with systematic sampling is to substitute the next unit in the list when a dead unit is sampled, but this should be avoided because the probability of selection of that next unit is enhanced. The sampling interval should be repeated as usual, and dead units simply dropped.

- *The ideal sampling frame would hold certain fields for each unit, allowing sorting of the list and stratification as required.*

6.66 For example, industry classification at the International Standard Industrial Classification of All Economic Activities (ISIC) four-digit level and information about value of output would be maintained for XMPI purposes (ideally of each product, at the six-digit Statistical Classification of Products by Activity in the European Union (CPA) level, produced by each unit, and the destination/source country for each transaction). This information would be updated annually. Export and import values would be maintained using both customs and balance of payments records as part of the unique business identification maintained for each establishment.

6.67 Lists maintained primarily for tax collection purposes are likely to hold information on the values on which taxes are levied—for example, imports at c.i.f.

- *An ideal sampling frame would identify each unit uniquely at the correct institutional level.*

6.68 In practice, some units may be listed more than once, and others may be grouped under one listing. Ideally, a structure would identify enterprises and their corresponding establishment structure with separate classification and other stratification information for each establishment. If such information is not immediately available from the business register, additional steps or surveys may be needed to collect this information as part of the process of sample frame refinement.

F.3 Sample structure

6.69 The sample structure for XMPIs is straightforward because priority proceeds from classification by product at the highest level, to destination/source country, to industry supplying exports or purchasing imports.

6.70 Consider the XPI or MPI structure using the following example:

(1) We require XPIs (MPIs) for products (five-digit Central Product Classification), but also want to publish time series by industries (four-digit ISIC) and destination (source) country.

(2) There are establishments trading in a range of products that are characteristic of more than one industry and with more than one destination (source).

(3) The shares of exports (imports) by product are the most stable over time, followed by the shares by industry, while the shares by destination (source) may be stable in some instances and volatile in others, particularly within a product class.

(4) Seasonal availability affects the sample size because it will be predictably lower during off-season periods; this should be taken into consideration in the design of minimum sample strata, so that the several similar products included within the stratum have overlapping and, collectively, year-round availability. Also,

sample sizes for these strata should be increased because of the higher variability in price movements among seasonal products.

6.71 Given the uses of XMPIs and the relative stability of trade by product, the first step in this process involves selecting the products across industries and destination/source countries that will be represented in the XMPIs. In most countries, some products account for an extremely small part of trade—for example, products that comprise less than 0.02 percent of total trade in a major product group for either exports or imports. (If this is not the case, then all products could be included for estimation.) It would be possible to use a cutoff approach where those products below the threshold level (in this example, 0.02 percent of sales) are excluded from the sample of products, but their weight is allocated to another closely related stratum or distributed across a number of other strata. Sampling frames for exports or imports then are built for cells cross-classified by product.

6.72 The statistical office should review the products that fall below the cutoff point and determine if any traditionally important products should be included anyway. Also, newly emerging products that are expected to grow in importance might be included because they will eventually exceed the threshold. Finally, for the products not selected, the statistical office should determine if groupings of these products can be made that reach the threshold level.

6.73 Because most uses of XMPIs focus on the product breakdowns there should be a product orientation to the sample design. To construct *product XMPIs*, for exports we would need export or sales information and import or purchase information for each establishment for each six-digit HS product that it produces, enabling us to form a list of all producers for each six-digit product. From each list we would sample transactions and weight them accordingly to give product XMPIs.

6.74 To bring in industry detail, a compromise is to employ a two-stage[6] sampling scheme—that is, the frame is stratified first by product, then stratified by amount of trade within each product cell. Next, establishment samples are selected to represent each of these cells or strata. The establishment samples are pooled to determine the selected establishments representing more than one product category. The establishments are visited, coded by industry, and transaction samples selected from each product stratum the establishment was selected to represent. Each transaction selected must then be classified under a product heading, and coded as well with a destination (source) country. Product XMPIs can be compiled using all prices for each product, regardless of the industry in which the establishments are classified or the destination (source) of the transaction. With two-stage sampling of this sort, some accuracy of the industry and source and destination XMPIs will be sacrificed.

[6]A distinction is made between *two-stage* sampling, where a sample of establishments is selected and then a sample of transactions is selected from each, and *two-phase* sampling, where a sample of establishments is selected to provide detailed output data, and this sample then is used as a new sampling frame. This new frame can be sorted and stratified much more effectively than the original frame as a result of the information collected in phase one.

F.4 Stratification

6.75 It is a well-known principle of sampling that stratification into segments for which the dispersion of price changes is lower (more homogeneous) than the overall dispersion tends to increase the efficiency of the sample by reducing variance.

6.76 For example, in the two-stage sample described above, the list of price-forming units is first stratified by product, for example, at the five-digit Standard International Trade Classification (SITC) or six-digit HS code. Each product stratum then can be further stratified by variables appropriate for that product group. The ideal variant for stratification is the value to be measured in the survey—that is, price change—but in practice we use proxy variables that we assume to be correlated with price change. For example, the size of the production unit may cause differences in production technologies and, thus, different responses to changes in demand or input costs.

6.77 In the U.S. XMPI, the sample design ensures that all units (i.e., products or producers) above a certain size are included. The remaining units are sampled with probability of selection proportionate to size. The alternate approach of setting broad strata, such as those with value of sales of 1 million to 5 million, 5 million to 10 million, and so forth, will result in units within each stratum having an equal chance of selection and, when selected, an equal weight. In a PPS sample design, a unit with 5 million in sales will have roughly a five times greater chance of selection than a unit with 1 million in sales. Further, the unit falling into the sample on a PPS selection would have a weight proportionate to its size, an additional improvement over broad stratum sampling.

6.78 Ideally, stratification should be optimized to minimize sampling errors. For example, the number of strata (L) can be optimized based on a relationship such as

$$V(\bar{y}_{st}) = \frac{S_y^2}{n}\left[\frac{\rho^2}{L^2} + (1 - \rho^2)\right], \tag{6.1}$$

where S_y^2 is the variance of the variable being estimated (y), in this case price change; n is sample size; and ρ is the correlation between y and the variable used for stratification, in this case a proxy for price change such as output or sales.

F.5 Sample allocation

6.79 Given that there is always an upper limit on the amount of data that can be collected because of resource constraints, decisions must be made about how to allocate the data collection between the strata—that is, we must decide how many establishments to sample in each stratum and how many prices to collect from each. It is generally more expensive to increase the number of establishments sampled as opposed to increasing the number of prices collected from each establishment, although simply increasing the latter may add little to accuracy when intra-establishment (within establishment) variance is low. So, it is generally the case that the number of establishments to be sampled is the constraint, rather than the total number of prices collected.

6.80 Ideally, the sample allocation would be optimized so that accuracy is maximized within the cost constraint, according to some equation linking sample size with accuracy. For example, the simplest form of optimal allocation is to make the sampling fraction (f_h) in a stratum (h) proportional to the standard deviation S_h in the stratum, and inversely proportional to the square root of the cost (c_h) of including a unit from that stratum in the sample—that is,

$$f_h \propto S_h/\sqrt{c_h}. \tag{6.2}$$

Thus more heterogeneous and cheaper strata are sampled at higher rates. Often, costs do not differ between strata, so the optimum allocation reduces to $f_h \propto S_h$, the so-called Neyman allocation.

6.81 If probability sampling techniques have been used, it is possible, in theory, to estimate variances at each level. Typically, the frame first is stratified by six-digit product codes. Two-stage PPS sampling is employed to select establishments within each code and then transactions from each establishment. Establishment industry classification and the destination/source of the product transactions are recorded at initiation of the establishment.

6.82 The variance of each XMPI will depend on the variance between (inter) establishments producing a product, and the variance within (intra) each establishment in the sample. The intra-establishment variance might be because of differences in variety or terms of transaction, but it likely will be relatively small compared to the inter-establishment variance. So, an optimization model will allocate the sample of establishments in proportion to the variance within strata, but it will suggest collecting a fair but low number of prices for each product from each establishment.

6.83 Calculation of the variances of the product XMPIs is complex, and thus the optimization algorithm also is complex. There are variances between establishments in each industry and within each product stratum in each establishment in the sample.

6.84 The above examples assume that probability sampling techniques are used and that variances therefore can be estimated. In sample surveying, however, we usually assume very limited information about the frequency distribution followed by sample measurements. This means that in practice, optimization often is done using a variety of pieces of information, applied to more or less formal optimization models. Information that may be available includes the following:

- The total sample size that resources allow;
- The number of units in each product frame;
- The economics of each market—that is, the value of output, company and product composition, product dispersion, price-setting mechanisms, and so forth;
- Which XMPIs need to be published—it may be necessary to allocate larger sample sizes to some industries or products than simple empirical methods would indicate in order for XMPIs to be published at a detailed level without fear of breaching confidentiality guidelines; and
- Response rates.

6.85 The aim often is simply to produce product indices with comparable accuracy and to publish a reasonable amount of industry and destination/source detail. As for the number of prices collected from each establishment, it may be necessary to use a general rule, such as the average number of prices should be about 4 or 5 with no single establishment providing more than 15 or 20.

G. An Example of Sample Selection and Recruitment of Establishments

6.86 For sample selection to proceed, all of the earlier steps of sample design must have been completed. Decisions have been made on the sampling techniques to use at each stage of the sampling process. Assume for simplicity that manufactured goods have been chosen as the first area to be included in the XMPI. (Subsequently, mined ores, agricultural products, power and other public utility services, transport services, etc., may be added.) For this purpose, information on establishments such as industry, output, sales, name, and location is available from a recent Census of Manufacturing or a Census of Establishments. Products at the six-digit HS level have been selected using a cutoff sampling strategy. All product groups with imports greater than 0.02 percent of total imports, free on board (f.o.b.) have been chosen. (The cutoff value—0.02 percent—is determined by the market share considered significant within the country. If the number of product groups is too large given the resources available, a higher cutoff threshold may need to be used.)

6.87 In addition, the production of quite a few products is concentrated among a few large enterprises, while others have less concentrated production. It would be helpful to stratify the industries by size of firm. In those industries where production is highly concentrated among a few large enterprises (e.g., three firms represent 90 percent of production), the large enterprises are selected. In those industries with a more dispersed concentration, the largest firms could be selected with certainty (i.e., chosen with a probability of 1.0) while a sample of smaller firms could be selected using random sampling techniques (e.g., PPS sampling as described below). In general, the number of sampling units for the smaller firms should increase as the concentration ratio (percentage of industry output by large firms) becomes smaller. For example, for industries where the concentration ratio is 70 percent, a sample of four units among the smaller establishments might be adequate, but if the concentration ratio is less than 50 percent, the number of units might be twice that size. Using such a process also requires that appropriate weights be assigned to each selected unit. For the certainty units, the weight would be the firm output (sales), whereas for other units it would be the sampling interval (see the example below).

6.88 At this point the frame is stratified, allocations of sampling units have been made, and the sampling technique has been decided upon. Usually, three phases are left to sample selection:

(1) Select establishments;

(2) Recruit establishments; and

(3) Select transactions.

G.1 Selection of establishments using probability techniques

6.89 The sampling frame of establishments has been stratified by four-digit industry codes and size for probability sampling (purposive sampling could be used instead, and some of the issues involved in this are discussed in Section G.4). In this situation, either systematic or PPS sampling could be used, or a combination of the two. A common application of PPS is to assign a probability of 100 percent to units in the largest size strata (as discussed above), and then select randomly from each of the other strata, with probability of selection proportionate to size.

6.90 A combination of systematic sampling and PPS is used in the United States, where a stratum frame would be ordered by size and cumulative totals calculated. For example, assume that we know the average cost per establishment for collecting price information, and that the costs will not vary significantly by industry. Based on this information, we determine that the number of establishments in the sample would be 400 (total data collection costs divided by average cost per establishment). If the industry for which we are drawing the sample represents 1.0 percent of the total sector output, then we would allocate four establishments to the industry ($400 \times .01$), and we can proceed to draw the sample from the frame. Assume the information in Table 6.3 is available from the sampling frame.

6.91 The sampling interval is calculated as follows:

$$\text{Sampling interval} = \frac{\text{cumulative grand total}}{\text{number of sample units}} = \frac{580}{4} = 145.$$

All establishments with production values greater than the sampling interval (145) have 100 percent probability of selection and are known as "certainty units" (establishment E). These selected units are removed from the frame, we recalculate the cumulative size, and a new sampling interval is calculated using the reduced

Table 6.3. Step 1 for Establishment Sample Selection

Establishment Identifier	Size (Value of production in millions)	Cumulative Size	Cumulative Percent
E	200	200	34
C	100	300	52
D	80	380	66
B	60	440	76
G	50	490	84
F	40	530	91
H	30	560	97
A	20	580	100

Table 6.4. Step 2 for Establishment Sample Selection

Establishment Identifier	Size (Value of production in millions)	Cumulative Size
C	100	100
D	80	180
B	60	240
G	50	290
F	40	330
H	30	360
A	20	380

frame and the remaining number of sample units to be allocated (as shown in Table 6.4).

$$\text{Sampling interval} = \frac{\text{cumulative grand total}}{\text{sample allocation}} = \frac{380}{3} = 127.$$

6.92 If there are new certainty units in the reduced sample, these are removed (not in this case), and the process is repeated until a sampling interval is calculated for which there are no certainty units. This sampling interval is used for systematic sampling. The remaining sample is sorted (largest to smallest as shown in Table 6.4), a random number between 0 and 1 is generated, and the sampling interval is multiplied by this random number to give the starting point for the sampling pattern.

$$\text{Random number} = 0.34128.$$
$$\text{Starting point:} = 0.34128 \times 127 = 43.$$

Sampling pattern:

43	(43 + 127)	(43 + 127 + 127)
43	170	297.

Thus establishments C, D, and F are selected, giving a total sample of C, D, E, and F.

6.93 The weights assigned to each establishment would be as follows. Establishment E will have a weight of 200. It was selected with certainty, and it will maintain the same weight because it is representing itself in the sample. Establishments C, D, and F will each have a weight of 127 because they are representing all the other establishments not selected in the sample. Thus, the total of their weights must be the total of all the noncertainty establishments, which is 380 in this example. Additional detail on the source of weights was presented in Chapter 5.

6.94 An alternative approach in some countries is to use cutoff samples so that a certain level of output or sales is achieved. For example, there may be a desire to have the sample represent 70 percent of the output in each industry in the sample. In such a case, a cutoff sample is used. Establishments in the product sampling frame are ranked in order of the output (largest to smallest). The percentage of output that each establishment represents to the total for the product group is calculated. The cumulative percentage then is derived. A cutoff of 70 percent is established, so that all establishments below this threshold in the cumulative rankings are dropped and the sample will consist of those remaining. This approach guarantees that the sample consists of large establishments.

6.95 In the previous example if one used the cutoff procedure, establishments E, C, D, and B would have been selected because their cumulative percentage of output is 76.

G.2 Selection of products and establishments in small developing countries

6.96 An alternative sampling strategy that statistical offices in developing countries have applied successfully is to use a cutoff approach for selecting products and establishments. The first step is to tabulate the import (export) data from customs records for one or two recent years to derive the total value of shipments by eight-digit HS code or five-digit SITC. These represent the products for external trade in the country's recent experience. (One-of-a-kind shipments and shipments of personal effects should be deleted.)

(1) Order the products that remain from highest to lowest in terms of their value of shipments and calculate the total value. This represents the value of regular recurring imports (exports).

(2) Compute each product's (again at the eight-digit HS code) relative importance by dividing the value of shipments for the product by the total value of recurring shipments.

(3) Calculate the cumulative relative importance for each product.

(4) Apply a cutoff threshold of 70 percent so that all products with a cumulative total less than or equal to 70 percent are accepted for the product sample.

(5) Tabulate the same information by establishment and select the establishment sample using a cutoff sample with a threshold of 70 percent.

(6) Select those establishments that import (export) the products selected in the product sample. The result is a sample list of important establishments that import (export) the selected products.

6.97 The resulting sample normally would represent approximately 50 percent of imports (exports). The statistical office should recruit the sampled establishments and select representative transactions within the establishment for the products identified.[7]

G.3 Recruiting establishments

6.98 Recruiting an establishment means securing the cooperation of its staff (particularly if the survey is voluntary), so that data will be of a high quality. It is highly recommended that each establishment receive a personal visit during which the purpose and function of the price survey are explained, and the sample of transactions or varieties to be priced is selected. Supplementary data for weighting transactions also can be collected during the visit. All these tasks can be more effectively carried out via personal visits rather than via telephone calls or mailed questionnaires.

G.4 Selecting products and transactions in the establishment

G.4.1 Probability and cutoff sampling procedures

6.99 The probability approach also can be used for selecting products and transactions by soliciting information from establishment records. Once in the establishment, however, the respondent may be reluctant to provide detailed records for selecting products and transactions. One alternative would be to ask the respondent to list the products produced and provide an estimate of the percentage each product represents of total sales. This information can be used to select the sample by ranking the products from highest to lowest and then making the selection using the same techniques discussed above.

6.100 Another alternative, if the respondent is unwilling to provide product percentages, is to ask him or her to rank the products in order of importance. Using the ranking information, estimated percentages can be established. Consider the information in Table 6.5 that is provided by a respondent in an establishment with eight products.

6.101 The respondent was able to rank the products in order of importance. Each product can then be assigned its importance based on the reverse order of its ranking: Product G is assigned 5, Product H is assigned 4, and so forth. Next, an estimated percentage of sales is calculated using each importance as a percentage of the total of the assigned importances. Assume that the sample design indicates that three products are wanted for this establishment. These percentages can then be used to select a sample of products through the probability sampling procedures described above or through cutoff sampling procedures.

6.102 If probability procedures are used, the sampling interval is first calculated:

$$\text{Sampling interval} = 100/3 = 33.$$

[7]In small countries, there may only be a few products that are exported. In such a case, all the large exporters are known to the statistical office and the sample of exporters would be those few key establishments. In order to publish product-level indices in this case, the statistical office may have to seek approval from the exporter because of confidentiality requirements.

Table 6.5. Selection of Products Using the Ranking Method

Product	Ranking	Importance	Estimated Percentage	Cumulative Percent
G	1	5	33	33
H	2	4	27	60
I	3	3	20	80
J	4	2	13	93
K	5	1	7	100
Total		15	100	

A random number is selected to determine the starting point and the sampling pattern:

Random number = 0.45814.

Starting point = 0.45814(33) = 15.

Sampling pattern = 15, 48 (15 + 33), and 81 (48 + 33).

The selected sample will be products G, H, and J. (Note that we do not select product I because it is below the third interval in the sampling pattern.)

6.103 If the cutoff procedure is used, the first two or three products (G, H, and I) will be selected depending on whether we used a 50 or 70 percent threshold. With the cutoff procedure, the most important products are selected.

6.104 In addition, representative transactions for continuous pricing need to be identified. The respondent should be asked to supply information on various transactions that apply to the selected products. Again, the data can be in the form of actual values from company records, estimated percentages, or by ranking. If two transactions per product are required, then the same procedures as those just described would be followed to select the two transactions. (Normally, we would expect to have two to four transactions per product depending on its importance and the number of transactions the establishment normally has for each product. Because respondent burden may be problematic, we would not want to have much more than 20 observations per respondent.)

6.105 In the above examples, if the respondent could not provide any information or if he or she says that they are all equally important, then equal probability would be assumed. In such a case, each product or transaction would be assigned the same importance (i.e., 100 divided by the number of products), and the selection procedure would continue as explained above.

G.4.2 Purposive sampling

6.106 Because the selection will be based largely on the judgment of the members of the establishment's staff present at the recruitment meeting (respondents), it is important that these people are knowledgeable and hold senior positions, probably from the marketing, sales, or accounting departments.

6.107 The first step is to stratify by products produced by the establishment selected for the product group sample. As a general guide, it is reasonable to have between 3 and 10 detailed product strata (depending on the size of the establishment) that are deemed representative of the establishment's output. It should be possible to obtain a sales figure or estimate for each stratum, or at least to order the strata by size. If the establishment is both an importer and exporter, the product strata should be further stratified between imports and exports. Separate prices should be collected for exports and domestic products, as necessary.

6.108 Then for each stratum, one or two specific transactions should be chosen, bearing in mind the general rule that the average number of prices from establishments should be around 4 or 5, with no single establishment providing more than 15 or 20 (strata may have to be combined if the number is too large). The aim is to choose transactions and terms of sales that account for a significant proportion of sales, are broadly representative of other production, and are expected to be available for sale or stay in production at future price collections.

6.109 Weights for each transaction selected could be determined by proportional allocation of the establishment weight to each product and transaction selected. This procedure is discussed in Chapter 5, Section B.1

G.5 Recording product specifications

6.110 After transactions have been selected, the price-determining characteristics must be carefully discussed and recorded on the collection form. (See Chapter 7 for more details on recording product specifications.) Examples of such characteristics are as follows:

Product specifications:

- Type of product,
- Brand name or model number, and
- Main price-determining characteristics—size, weight, power, etc.

Transaction specifications for the XMPI:

- Type of buyer—exporter, wholesaler, retailer, manufacturer, government;
- Type of contract—single or multiple deliveries, orders, one-year, agreed volume;
- Unit of measure—per unit, meter, ton;
- Size of shipment—number of units;
- Delivery basis—free on board, sale with or without delivery to customer;
- Type of price—average, list, f.o.b., net of discount, c.i.f.; and
- Type of discount—seasonal, volume, cash, competitive, trade.

H. Sample Maintenance and Rotation

6.111 Price surveys are panel surveys in that data are collected from the same establishments on more than one occasion. The general problems with such surveys are that the panel becomes depleted as establishments stop producing, the panel becomes increasingly unrepresentative as time passes and the universe changes, and some establishments may resent the burden of responding and leave the panel or provide data of poor quality. All these problems cause bias.

6.112 A widely used method to alleviate some of these problems is to limit the length of time that establishments stay on the panel by using some form of panel rotation.[8] Rotation has two main benefits: (1) it ensures that most producers participate in the survey for a limited time and, therefore, the burden is shared among enterprises, and (2) it helps to alleviate the problems caused by a sample being out of date—that is, sample depletion and not being representative of current trends. Recruiting new establishments helps to ensure that new products are represented in the price surveys.

H.1 Approaches to sample rotation

6.113 Obviously sample rotation has a cost because new panel members need to be recruited. There are several options regarding how rotation might be done. First, a rotation rate should be fixed. For example, if the whole panel is to be rotated every five years, then the annual rate is 20 percent. This could be implemented by dividing the industry headings into five groups and dealing with one group each year. Or 20 percent of all respondents, across all industries, could be dropped each year and replacements recruited. An establishment's rotation cycle could be related to its size, so that larger establishments stay in the sample for more than five years, and small establishments stay in for fewer than five years.

6.114 If sample rotation is done by product group, this provides a good opportunity to review the sample design and reallocate and select new establishments as necessary. Rotation and sample revision fit best within a system of annual chain linking in which the product structure and weights can be updated each year.[9]

6.115 Where cutoff samples are used, the same procedures as those described in Section G.2 could be completed as each year's customs records become available to identify new establishments and those that have become more important in external trade. New establishments that now fall within the cutoff threshold can

[8]In many countries, the rotation is limited to the smaller respondents, for whom it is felt that responding to surveys imposes a significant burden. This need not be the general case, and the use of full-panel sample rotation is encouraged.

[9]Annual weight update is not a requirement for sample rotations; it simply makes the process a bit easier because weights already are being updated at most levels of the index. When there is no system for annual weight updates, sample rotation does require a two-tier system of weights—fixed weights at higher levels of aggregation for aggregating to higher-level indices and separate weights for low-level indices that are updated periodically.

be added and old establishments that fall outside the threshold could be dropped. Thus, the sample would reflect the most current establishments and products important to the country.

H.2 Procedures for introducing a new sample of establishments

6.116 The procedures used to introduce a new sample of establishments are similar to the overlap procedure used for linking replacement price observations or introducing a new product structure in a weight update. Assume the rotation strategy calls for replacing 20 percent of all products. If the XMPI sample consists of 100 six-digit product groups, then each year the statistical office will replace the samples in 20 product groups. For each of the targeted product groups, a sampling frame is needed to select a new sample of establishments. The staff must then recruit the establishments, as discussed in Section G.

6.117 The new establishment sample will have new weights for the selected establishments, products, and transactions. The new sample and weights will be used directly to replace the old sample. During the same month, the data collection staff will have to collect price observations for both the new and the old sample. The old sample prices are used to calculate the index in the usual way, and the new sample will provide new base-period prices to calculate the index in the next period using the new weights. For example, the old sample for a particular product market may consist of five establishments and 20 price observations, whereas the new sample may have eight establishments and 32 price observations. Both samples are collected during the overlap month—that is, 13 establishments with 52 price observations (assuming no establishment in the old sample is also in the new). The 20 observations from the old sample are used for the current-period index calculation. The 32 price observations for the new sample provide basic data for setting new base prices in the new sample.

6.118 The index formula used will influence the relationship between the price reference period for the weights and the reference period for the base prices. If the statistical office compiles a Lowe or Laspeyres index, it will use the first set of prices collected in the new sample to set the base prices for the index. The base price reference period and the weight reference period need to align if the Laspeyres price index is used. If the weight reference period for the establishment and product weights are, for example, annual shipment values for 2005 and the prices collected for the new sample are for June 2007, then the new prices will have to be estimated backward to the annual average for 2005. This is accomplished by applying the price change for the product between June 2007 and the annual average for 2005 to the June 2007 price observations. For example, if the prices in the product (over all establishments) rose by 10 percent between the annual average index for 2005 and the June 2007 index, then each price observation would be deflated by the factor 1.10. This calculation adjusts the new price observations for the average price change in the product market between the weight reference period and the current period.

6.119 Consider a similar example for the Lowe index. Again, assume that the weight reference period is for 2005 and that the base price reference period is December 2006. In this case, the statistical office will need to update the weights for price changes between the 2005 annual average and December 2006. The price index for the product group is used to calculate the price change between 2005 and December 2006 and this price change is applied to all the weights. Next, the June 2007 prices will need to be adjusted backward to December 2006. The product price index is used to measure the price change between December 2006 and June 2007. This price relative then is used to deflate the June 2007 price observations to obtain December 2006 base prices.

6.120 If the estimation method is a Young index, the process is much simpler because the new weights are used directly in the computation of the index using the new prices without any adjustments. (See Chapter 16, Sections D.2 and D.3, for a discussion of the Lowe and Young indices.)

6.121 These procedures ensure that the new prices and weights are consistent with the index number formula within each 6- to 10-digit product group selected for sample rotation. For higher-level indices, the weight reference period may not be the same as for the products going through sample rotation. In practice, the aggregation weights used to combine group-level indices may have a different price reference period than for the sample rotation groups. For example, the product group weights used to produce higher-level indices (three-digit, two-digit, etc.) may have a reference date of 2005 because they come from customs records for 2005. The index reference period might also be 2005 = 100, because of a statistical agency policy to re-reference index numbers once every five years. On the other hand, the weights from the establishment sampling frame used to draw the rotated sample may be for 2006, because the weights for the rotated product groups are taken from the most recently available customs data or services trade survey (perhaps with a special supplement for product groups scheduled for sample rotation). The price index reference period could be December 2006 because the price information is readily available from sample respondents.

6.122 Thus, there can be a difference between the base price reference period for the new sample at the lowest level (elementary aggregate)—December 2006—and the index reference period for higher-level indices—annual average for 2005. In such cases, the price change from the lower-level indices will be used to move the higher-level indices forward to the current period. For example, in product 8411.11.40 (8411 Turbojets, turbopropellers and other gas turbines, and parts thereof: Turbojets: 8411.11 of a thrust not exceeding 25 kN: 8411.11.40.00 Aircraft turbines) the index level in December 2006 was 108.0, and in September 2007 it was 110.2 with an index reference period 2005 = 100. The sample of 10 establishments and 40 price observations for this product was rotated in January 2007 using base prices from December 2006. The elementary indices for the products in this industry have a price reference date of December 2006. To estimate the product index, the statistical office will have to use the price change from the new sample and link it to the level of the higher-level index. This can be done in two ways, depending on whether the statistical office uses a direct or chained price index formula (see Chapter 10, Section B.3). Assume a direct index is used where the current price for October 2007 is compared to the base price in December 2006, resulting in a price index of 102.96 (December 2006 = 100). The long-term price relative (1.0296), times the product 8411.11.40 price index for December 2006 (108.0), gives the October 2007 index level of 111.2. Alternatively, if the monthly chained index form is used, where the October prices are compared to the September prices, then the lower-level index is linked to the September 2007 higher-level index. Assume the one-month price relative was 1.0091 in October 2007. The September 2007 product 8411.11.40 index (110.2, where 2005 = 100) is multiplied by this price relative to derive the October 2007 product index of 111.2. The results of the formulas should be the same. The advantage to using the monthly chained index form is that it facilitates making quality adjustments as discussed in Chapter 8, Section C.3.3.

I. Summary of Sampling Strategies for the XMPI

6.123 The approach to a sampling strategy in the XMPI requires a number of steps to gain enough information to design a survey that will produce reasonable estimates of price change within the level of resources provided. The following points provide a logical sequence to the sampling issue as presented in this chapter.

(1) *Determine the survey objectives, uses, coverage, and resources before determining the data to be collected, the periodicity of collection, and the type of sampling that will be employed.*

It is important to decide at the beginning of the process if price changes for the product and/or destination (origin) will be needed as well as products and the degree of accuracy required. It will also be important to decide whether monthly or quarterly indices will be produced. These, in turn, will determine the level of resources allocated to the program. Alternatively, if there is a fixed level of resources available, it is possible to work with cost controls to determine affordable sample sizes and collection frequency at the expense of accuracy.

(2) *Identify sources to use to develop a sampling frame for selecting the establishments and products for covered sectors and industries.*

The availability of up-to-date business registers with appropriate selection parameters (e.g., industrial codes and measures of size) could serve as a source for developing sampling frames for selected industries. Many of the sources of weight data discussed in Chapter 5 also could be used to develop a sampling frame. For international trade in goods, tariff and export declaration administrative records are an excellent source for a list of establishments engaged in international trade. Others include industrial census, bank surveys, and administrative records.

(3) *Use probability sampling techniques to the extent possible.*

Although probability sampling throughout the selection process is a desirable goal, it may not be entirely affordable. An alternative is to use cutoff sampling at various stages in the process.

(4) *To make the sample more efficient, use multiple levels of stratification within the sample design.*

In most cases, two strata will be identified within the sample—product and establishment. However, the sample could be more efficient and representative if additional strata are used, such as establishment industry code, establishment size (large, medium, and small), region or location (if there are price trend differences by location within a country), and export versus domestic market production (if there are price trend differences for these markets). Additional strata will be helpful to the design wherever there might exist differing price trends or price variability within the chosen strata.

(5) *The price sample should be based on actual transactions with the characteristics of those transactions fully described.*

Often there is a tendency to use average prices or unit values (sales value ÷ quantity sold) as the price reported in the XMPI. These are not true transaction prices, in that they represent the average of a number of transactions for which there could be differences in quality or pricing characteristics. Therefore, it is important to select a sample of individual transactions with a detailed description of all of the characteristics that determine the price. These transaction prices and their characteristics then will be observed through time. So-called transfer prices between related establishments that may have little or no relationship to market values can be especially problematic for internationally traded items, particularly between countries with large trade volumes and integrated economies, such as the United States, Canada, and Mexico, or the member countries of the European Union.

(6) *Initial recruitment of establishments should be completed by personal visits.*

Initial sample recruitment should be conducted through personal interviews with establishment managers in order to accurately select representative products and transactions. The purpose of the survey must be explained, along with the need for the continuous reporting of price data for the selected transactions.

(7) *Samples of establishments and products must be maintained so the reliability of the XMPI remains intact. A program of sample maintenance is needed for this purpose, and sample rotation may also be desirable.*

6.124 Products produced by establishments will frequently change in response to market conditions. Also, establishments will cease operations and new ones will begin production. The XMPI sample sizes must be maintained in order for XMPI estimates of price change to be accurate. Therefore, it is necessary to have a program targeted toward keeping the sample intact and the products representative of current trade in terms of both the goods being produced and the establishments producing them. Fortunately, for international trade in goods, there is a large volume of current administrative information to draw upon for constructing the establishment frame.

7. Price Collection

A. Introduction

7.1 This chapter gives an overview of price collection issues. The focus of the chapter is on price collection for the survey pricing approach to export and import price indices (XMPIs). In Chapter 5, two sources of data for XMPIs were considered: administrative sources and survey sources. The former relies on established data collection procedures, primarily customs data. *The design and implementation of such administrative data collection procedures are, to a large extent, not dictated by the needs of export and import price index number construction and thus are not the concern of this chapter.* In Chapter 2, consideration was given to circumstances under which administrative sources might prove unreliable for tracking prices, and price surveys may be more fruitfully employed. Further sources of data, used for import price indices, are *series of world market prices* and *mirror prices from other countries. Again these series are collected by other organizations for other purposes and are not the direct concern of this chapter.*

7.2 It should be reiterated that the establishments to be surveyed will include those that specialize in the distributive export and import trade, as well as producers in manufacturing, mining, energy, and other nondistributive industries that export their output and import their inputs. There may well be a program of price surveys of resident establishments for the producer price index (PPI), possibly for intermediate inputs and outputs. Price collection for XMPIs should be integrated into the PPI surveys so as to, first, enhance the comparability of the PPI and XMPIs; second, not unnecessarily increase the number of responding establishments; third, take advantage of trained price collectors and computer and administrative systems; and, finally, not waste resources on duplication—resources that might otherwise be utilized in improving the quality of the resulting data. The prices collected for the PPI are unlikely to be sufficient for XMPIs; additional establishments will have to be sampled and prices for some of the responding establishments will have to be collected for both the domestic and foreign markets. Not all issues relating to price collection will be considered in this chapter. See Chapter 13 on the organization and management of the PPI system.

7.3 This chapter describes a range of options for each aspect of price collection, but it is not prescriptive, because different solutions can be used depending on individual country circumstances. Price collection is a vital part of the overall XMPI compilation process. Without good-quality price collection procedures, it is difficult or impossible to produce accurate and reliable results, regardless of how rigorous the subsequent processing is throughout the remaining steps of producing the XMPI.

B. Timing and Frequency of Price Collection

7.4 Calculating the XMPI entails collecting prices from businesses relating to particular commodities and time periods. Businesses can be both exporters and importers of commodities, so that prices may be collected for sales of goods and services for use in an export price index or purchases of goods and services for use in an import price index. Both indices serve useful purposes as macroeconomic indicators, as well as being of use as deflators.

7.5 The frequency of collection is commonly monthly, although a number of countries operate a quarterly collection system. For the purposes of this chapter, it is assumed that price collection is monthly. When collecting prices for a particular period, there are two basic choices of collection period: point-in-time or period averages.

B.1 Point-in-time prices

7.6 Point-in-time prices relate to the price of a commodity on a particular date in the month. For example, first day, first Monday, the nearest trading day to the 15th of the month, and so forth. This approach makes the collection date straightforward, and should be well understood by the business establishment that prices provided relate to transactions on that date.

7.7 The main advantage of point-in-time pricing is that comparisons from month to month will be consistent, which is particularly important when there are step changes in prices taking place during the month, such as a general price increase or duty change. One of the disadvantages of a set point in time for XMPIs is that a transaction may not have taken place on the specified date. If this happens, respondents can be asked to provide details of a transaction that occurred as near as possible to the specified date. Another problem is that point-in-time estimates are more susceptible to short-term external influences (e.g., extreme weather, labor stoppages) that could affect the price on the particular day of price collection. They may also miss short-term price changes (e.g., rise and fall) that occur between pricing dates.

B.2 Period prices

7.8 Period prices are an estimate of the price across the month and so are average prices for the month. A period price should take account of when price changes occurred during the month. For example, if a commodity was priced at 10 for the first 10 days of a month and then increased to 15 for the remaining 20 days, the average price would be 13.33 (i.e., $[10 \times 10 + 20 \times 15]/30$). This averaging is usually done by the statistical office and requires the exact date of the price change to be supplied by the respondent.

7.9 This approach usually yields a smoother time series and is less susceptible to the vagaries of the timing of price increases. The method is also easier for respondents because they can select a transaction and specify the relevant transaction date within the period. A key feature (compared to point-in-time estimation) with this method is that where a price changes partly through the month, the full effect of the price change is not included in the index until the following month. This is appropriate for an index used for deflation purposes as well as for an inflation measure.

7.10 Often a single price quotation is taken to represent the average price over the particular reference period. In theory, a more accurate measure of an average transaction price for a homogeneous item is the *unit value price*, which is total sales divided by the total number of units sold in a period.

7.11 If this method is used, the commodity must be either homogeneous or able to be expressed in terms of some common physical unit. A homogeneous commodity can be distinguished by

- Its point of purchase (outlet effect),
- The various competing brands or commodity lines of the commodity being sold at an outlet (brand effect), or
- The various package sizes at which the commodity is sold (packaging effect), and
- Any other price-determining characteristics (quality effect).

7.12 For strictly homogeneous goods, unit values are appropriate for aggregation over time. If, for example, the price over a month of the very same item is higher in the latter weeks than the former the unit value for the month would be an appropriately quantity-weighted average price—it solves the time aggregation problem (Chapter 21, Section B).

7.13 Unfortunately, the use of changes in unit values over time as a surrogate for price changes is very problematic and is not generally recommended, because any change in commodity quality or commodity mix can seriously distort the average unit price, as outlined and illustrated in Chapters 2 and 6. In limited circumstances—for example, for a highly volatile but narrowly defined and homogeneous commodity like petroleum—this method can be used.

7.14 Care must be taken to ensure that the average prices relate to a narrowly defined commodity of constant quality, rather than a broad commodity group. Furthermore, an index will be less timely using period prices, when compared with point-in-time estimates, because the average cannot be calculated until the end of the period.

7.15 Transportation costs should be excluded from the unit value calculation for export price indices because the pricing basis is the *basic price*—that is, the amount received by the producer or distributor, exclusive of any taxes on commodities and transport and trade margins. In other words, the pricing point is ex-factory, ex-wholesaler, ex-farm, and so on.

B.3 Choice of point-in-time or period prices

7.16 The choice of collection period is influenced by a number of issues, such as the frequency of collection, the practicalities of price collection, and the uses of the index. The choice of collection period becomes less important the more frequent the collection; thus, the choice is more important for quarterly collection than monthly collection, although it is still an important consideration for monthly collection. The use of the index is an important consideration. Because XMPIs are used to deflate trade values, ideally the index should relate to the time period of the trade flows. Most economic statistics relate to a period rather than a point in time, so again, in principle, the price index should do the same. It may be that for particular (homogeneous) commodities unit values are used while point-in-time prices are used in other instances.

B.4 Frequency

7.17 A distinction can be made between the frequency of collection and timing of observations. Monthly prices can be observed, for example, quarterly. The choice of collection frequency is determined by issues such as costs and the periodicity needed for deflation of output or sales data. In the European Union (EU), members are required to provide monthly data to the Statistical Office of the European Communities (Eurostat) under the short-term indicators regulation. Normally, prices will be collected from every business in the sample for each time period.

7.18 Although collecting prices for every period is appropriate for most industries, there may be industries where prices are generally stable, commodities take a long time to produce, or they change at predetermined times—for example, each January. Collecting a price every period in such a situation may be an unnecessary burden on businesses. For respondents in these industries, a price for each period is still required although it may be possible to reduce the periodicity of collection; this is a case where using "carry forward" price imputation is desirable. In some exceptional circumstances, respondents may be allowed to give forward prices, but care must be taken to avoid complacency. In these cases, usually involving long-term contracts, respondents can make a commitment that the price will not change in the defined forward period. If this turns out to be incorrect, the frequency of reporting should be changed. United Kingdom, Office for National Statistics (2002) Annexes G and H provide an example of a form used for price collection that makes provision for the inclusion of future prices.

B.5 Definition of a price observation

7.19 Chapter 6 on sampling explained how a representative sample of commodities should be selected. This section describes how prices for these selected commodities should be determined and collected. A price observation can be defined as the price of a specific commodity at the point in time or for the period of price collection and its terms of sale. To ensure consistency in the final index, the price observation should compare like items with like for each period. The commodity should be defined as tightly as possible so that the returned price is consistent from period to period and changes in quality can be identified (see Chapter 8, Section B). The price should be one that a customer has paid for the specified commodity and include all available discounts and special offers—that is, a real transaction price. (See Section D.4.1 below.)

7.20 If the commodity specification changes from one period to another, the price needs to be adjusted to ensure consistency. For instance, the quantity per order may increase, resulting in a lower unit-selling price. If the new quantity sold was available at the same unit price last period as it is this period, this is *not* a genuine price decrease and should not be reflected in the index. Rather, the comparison should be made between the same quantity purchased in both periods so that the index compares the same specifications (i.e., like with like).

7.21 Care should be taken to ensure that the currency of the returned price is clearly denoted, so that prices over time in different currencies are not compared. This is particularly important when respondents are providing price information for sales to both the domestic and export market. Procedures should be in place to convert all returned prices to home currency values. However, it must be clear what coverage is intended, either production for the home market, for the home and export market, or for the export market, so that the price quotations with currency conversions as applicable, can be used for the appropriate indices. The data supplied by the contributor should be in the currency in which the transaction took place; then the currency conversion, undertaken in the national statistical office, should ideally follow the principles in the Commission of the European Communities and others (2008), *System of National Accounts 2008 (2008 SNA),* and be at the midpoint between the buying and selling rate. The midpoint rate is used to exclude any service charge. The conversion should be at the rate prevailing at the time the transaction takes place, which may differ from the time the payment is made. In practice some rule may be used, such as using mid-month exchange rates.

B.5.1 List prices

7.22 The aim of compilers of XMPIs is to measure *actual* prices paid to or received from producers for goods or services. These are commonly referred to as transaction prices. By definition, these prices include all discounts or rebates given.

7.23 The price of goods or services as quoted in a catalog or advertisement is often referred to as the list price, book price, or recommended retail price. These prices are typically higher than transaction prices, as discounts or rebates apply to transaction prices.

7.24 In most areas of the economy, the prices actually paid or received for goods or services are not the list prices. Typically, negotiations between the producer and purchaser result in some form of discount or rebate, particularly to large purchasers. In most cases, they are substantial reductions off the list price and will vary over time. XMPI compilers should ensure that actual transaction prices are obtained rather than list prices.

7.25 It is usually easier for a respondent to provide a list price rather than a transaction price. For the reasons already stated, this is not appropriate. Because it is difficult to price a transaction, to achieve constant quality, compilers should ensure that the commodity priced is the same as that priced in the previous period.

B.6 Issues for high or hyperinflation

7.26 During periods of high or hyperinflation, the timing of price collection takes on significant importance. Prices may well change substantially during the collection period.

7.27 The frequency of collection also becomes more important, so quarterly collection may be inadequate for policymakers in a hyperinflationary period. Even in times of low inflation, it is important that early signals of upstream inflationary pressure are captured. Validation of the data may also prove to be more difficult because it is likely that every price quote would fail validation checks set during "normal" inflation, and it would be more difficult to spot erroneous returns.

7.28 On a wider scale, there is also a potential problem of feedback or circularity fueling inflation. Some companies may use the export or import price index to fix their prices (as part of a contract with the customer), which could then feed into the calculation of future such indices (there is always a risk of this in detailed indices, but the risk would be higher in periods of very high inflation).

C. Commodity Specification

7.29 The XMPI price collection surveys are unusual compared with most business surveys because, as with the PPI survey, there is a requirement to get a detailed commodity specification from respondents before the routine monthly collection can begin; this process is often called initialization or recruitment.

7.30 A separate set of processes and survey forms are required for the initialization procedure. The collection method used can also be different for the initialization period—for example, it may be possible and desirable to make a personal visit to each new respondent at initialization, but subsequent routine price collection would be done via postal collection. The initialization form should put more emphasis on explaining the purpose of the survey and contain more details about the commodity specification requirements. This form should also contain a commodity list for the respondent to identify which commodities they produce from the list. The initialization process may also be conducted by specialist staff such as field officers (see Section D.6). United Kingdom, Office for National Statistics (ONS) (2003) Annexes D and E provides, as an example, a form used by the ONS for the recruitment of items from an establishment for price collection. The following section on commodity specification can apply to both the initialization process and routine collection procedures.

C.1 Purpose of commodity specification

7.31 For each commodity group or service, prices for a set of specific representative commodities need to be fully specified for pricing. These commodities should be typical of the price movements of the range of individual commodities within the commodity group or service under consideration. The selection of commodities from within the commodity range of each establishment would ideally be undertaken from a complete census of the relevant transactions. Obviously in most cases this information is not available. In some cases there can be a trade-off between having infrequent data on a more complete and detailed basis, and more frequent product-updating procedures that rely on the respondents to the price collection to self-select commodities that are representative of their transactions and, hence, the commodity group. The sample selection aspects of commodity selection were covered in Chapter 6.

C.2 Aspects of commodity specification

7.32 There are a number of different aspects of commodity specification. For example, simply giving a commodity name will not be sufficient if the size of the package changes, which would, in turn, affect the price received. The essential purpose of a good commodity specification is to ensure that a consistent price is collected from period to period, relating to a consistent commodity with the same terms of sale in each period. Table 7.1 lists the main criteria that could affect the price of a commodity and could form part of a specification.

7.33 It is useful to supply notes on the completion of the form; an example of such notes on item specification from the U.K. Office for National Statistics' survey is available in United Kingdom, Office for National Statistics (2003) Annexes C and D. The above details combine to give a tighter specification for the commodity than just the description alone. Specifying a commodity in this way also supports the adjustment of the price associated with any changes in the commodity quality or the terms and conditions of sale. Some respondents object to providing full specification details because of concerns about confidentiality, and in these cases the specification details can be held by the statistical office, but a shorter encoded specification can be used on printed material such as forms. If this is done, it is essential to review the specification regularly.

C.3 Other forms of description

7.34 For some industries, a specification for a particular commodity may not be appropriate. For example, some industries produce goods or services on a made-to-order basis, and the same commodity is not produced in successive periods. Examples of this could be furniture manufacturers, shipbuilders, and accounting services. In these instances, a generic specification may be more appropriate. This would be a specification, as described previously, but for a standard product, rather than for a specific product. This could be a commodity that the company has made at some point in the past, or a basic model that it customizes individually for each customer. See Section D.5.2 for more details on this type of pricing.

Table 7.1. Price-Determining Characteristics

Item	Criteria/Reason
Commodity name	Company's name for the commodity within the specified commodity group. This should ideally contain information on the model/variety of the product.
Serial number	For the company's reference. This allows for changes in commodity name.
Description	In addition to the commodity name, this gives an opportunity for the company to specify what (if any) enhancements or add-ons are included in the product. For example, with cars, there are usually a number of options available (metallic paint, sunroof), all of which could affect the price of the product. All important price-determining characteristics should be included not only to ensure that like is compared with like in each month, but also to detect the precise nature of any quality changes so that price adjustments can be undertaken (Chapter 8).
Size of transaction	The amount of the commodity sold in the transaction and whether volume discounts apply.
Units of sale	Units used in describing the product.
Class of customer	Some companies may have different pricing structures for different customers (e.g., retail and trade). A reference number can be used to maintain customer confidentiality.
Discounts	Many companies offer trade, volume, competitive, or preferred customer discounts. All applicable discounts should be described.
Payment terms	Companies may have different prices for different payment or credit terms.
Carriage terms	Whether transport costs are included and what type of transport.
Currency	Currency the price will be provided in.
Destination	Companies often charge different prices for different markets.

D. Collection Procedures

D.1 Survey collection techniques

7.35 The aim of all survey collection techniques is to facilitate the transmission of price data from businesses to the statistical office in a secure and cost-effective manner, while minimizing the administrative burden on the respondent. A range of approaches to XMPI data collection are discussed below—postal survey, automated telephone response, personal interview, telephone interview, and Internet data provision. All of these methods rely on good questionnaire design, good respondent relations, and/or good interviewing techniques. Also, the highly sensitive and confidential nature of the price data provided by businesses may necessitate extra security requirements in data collection and processing.

D.2 Questionnaire design

7.36 Regardless of which data collection method is used, good questionnaire design is essential for the successful collection of prices. The questionnaire should be designed to make it easy for the producer to use and understand what is required.

7.37 The layout should easily facilitate the extraction of data and should contain detailed descriptions of the commodities to be priced. Detailed descriptions not only help the producer but also help in validation and the identification of quality changes. Quality adjustments cannot be made in the absence of detailed commodity specifications (see Chapter 8 for more details on quality adjustment techniques). Detailed commodity descriptions also ensure that the same commodities are priced each month, which gives important continuity and enables the statistical office to validate the data.

7.38 The questionnaire should be designed to help the respondent to extract information quickly and to enable speedy and accurate processing in the office. To meet these objectives the questionnaire should do the following:

- Provide clear instructions on what the respondent is required to do;
- Define why the establishment has been chosen, what the survey is, and how the data are collated and published;
- Enable respondents to complete the form quickly and accurately;
- Ensure supporting notes are available for each item of data to be collected;
- Use plain and clear language;
- Clearly identify the organization from which the survey has been sent and give a contact point and telephone number so respondents can get in touch to resolve any problems;
- Request reasons for price changes; and
- Ask whether the commodities are still representative and sold in volume.

7.39 Different designs can be used to make the questionnaire easier to complete for certain classes of respondents. For example, different designs could be used for different industry questionnaires. Also, a questionnaire with a checklist design that provides all the important specifications and price-determining characteristics will help respondents and data collectors. They will be able to verify the transaction and provide any new specifications or changes to the price basis that may apply when a previous transaction is no longer available and a replacement is selected.

7.40 One way to make the form easier to complete for businesses is to put the last recorded price on the questionnaire using a "tailored form" with unique commodity descriptions for each respondent. This will require the statistical agency to have much better form design and printing capabilities. It is, however, controversial in terms of the impact on the results. Although it is easier for the producer to complete, there is a greater risk that less care will be taken in the completion of the survey, and the producer will be more likely to repeat the last period's price even if a price change has taken place. There is also the risk that confidentiality will be breached if the form goes astray or even to the wrong part of the organization.

7.41 For help in validation and to reduce re-contact with the producer, it is useful to provide a space for "comments" to allow the respondent to explain any unusual movements in their prices. It is also important to emphasize to the respondent that any change in specification must be reported. An example of an XMPI postal survey price collection form that takes many of these issues into consideration is provided in United Kingdom, Office for National Statistics (ONS) (2003) Annexes G and H, though such questionnaires should, where necessary, be tailored to individual country circumstances. Note that the forms in Annexes C and D are used for initialization (recruitment), after discussion with the respondent to provide information on the selected, representative product(s). The forms in Annexes G and H are used for the regular monthly price collection for the initialized commodities.

D.3 Medium of collection

7.42 The following section outlines a range of survey collection methods. The principles of questionnaire design outlined above apply to each of these methods.

D.3.1 Self-completion—Return of data by postal survey

7.43 Key points of good practice that should be followed in questionnaire design are outlined below.

- The form should be clearly addressed to the company in question. It should display on the front page the name of the institution from which it has been dispatched;
- It should explain why it has been sent, how the results will be used, and from whom the end commodity can be obtained;
- It should include the name and number of the direct contact in the office, should respondents require assistance in completing the return; and
- It should include any statutory obligations that respondents are under to complete the form and the penalties for not doing so.

7.44 Within the form, sufficient descriptions and explanations should be included for the respondent to follow, including

- Guidance notes for each section requiring data,
- A clear definition of the commodity requiring data,
- The time period or point in time to be covered by the return,
- Instructions on how to change the description of the product,
- Information on linking the commodity description to industry tariff codes, and
- A period of back prices for amendment if necessary.

7.45 Allow for changes to administrative information on the form, including

- Space to record comments,
- The name and contact number of the person completing the form,
- Changes to the mailing address of the company, and
- Notes on how to return the form (prepaid envelope).

7.46 The main advantage of the postal survey approach is that it is inexpensive, particularly when coupled with modern data-handling technology, which reduces the need for operators to physically type data into systems. Large and dispersed geographical areas can be covered for minimal extra cost. This requires, of course, that the postal system in the country be accurate and dependable with deliveries.

7.47 Disadvantages of the postal survey approach include the difficulty in achieving a high level of response from respondents because the collection mode is not interactive. This can be mitigated if there is legislation in place to penalize for nonresponse. Potential quality problems can arise when respondents do not pay adequate attention to the notes and complete the form incorrectly. For this reason, it is wise to explain the requirement through a meeting or telephone conversation when the respondent is originally selected to participate in the survey; personal contact with the respondent, even by phone, should be encouraged as a general method for improving the quality of data returns. An automated system of reminder letters may be set up that includes letters explaining the usefulness of the data and the importance of maintaining responses from the current randomly selected sample. The reminder letters might be supplemented by telephone calls. Follow-up contacts with those that don't respond and resolving queries about reported data can add a significant cost to the postal approach.

D.3.2 Automated telephone data submission

7.48 Usually the XMPI price surveys collect price details for a small number of commodities from each respondent. The brevity of the questionnaire makes the XMPI surveys ideal for telephone-based data entry systems, in which the respondent reports the information directly over the telephone by following voice prompts and entering data using a touch-tone telephone. The prerecorded dialogues in such systems enable the respondents to report their monthly data quickly and accurately. Usually a letter is dispatched asking the respondent to make a telephone call. This approach has the advantage that it is possible to program the dialogue to allow validation of the data to take place during the telephone call. This can be done by asking the respondent to leave a voice message or by switching the call to a data collection analyst. Generally, this method is beneficial mainly to the statistical office because it reduces desk processing and, hence, reduces costs. Some online validation can take place that may benefit respondents by saving them from being recontacted by the statistical office.

7.49 The possible disadvantages of this system are confusion to some users caused by the technology and, because respondents can leave a voice message without discussion, some further clarification contact may be needed. This method is also less useful when there are complex commodity specifications, which need to be updated frequently.

D.3.3 Personal interview

7.50 A personal interview involves a face-to-face meeting with each respondent on a regular basis (e.g., monthly, quarterly) by a trained interviewer to obtain the data necessary for the survey. The main advantage of this approach is that the data can be validated at the source, and problems and differences of understanding can be resolved during the discussion.

7.51 However, the big disadvantage is the cost of employing interviewers and the large travel costs, particularly where large distances are involved. There is also a disadvantage to the respondent, who would have to spend more time in face-to-face meetings with statistical office representatives. Field collection for the XMPI is not as viable as for the CPI because

- Outlets are not clustered in population centers—they are often in decentralized industrial areas; and
- Inspection of commodities cannot be carried out, leading to less quality control of specifications.

7.52 A further variation here is to use another collection method (e.g., postal questionnaire) on a regular basis and have a less frequent personal interview to clarify details such as the commodity range and representativity. For example, some statistical offices visit each respondent on a rotating basis over a five-year period. This also gives the statistical office the chance to "train" the respondent to provide good-quality data. This approach can be particularly beneficial if used at the point when the business is initially brought into the sample, because many of the problems can be dealt with in a face-to-face meeting.

D.3.4 Telephone interview

7.53 Each respondent would be called during the collection period and asked for the data required for the survey, with the interviewer validating the form when speaking to the respondent. The data collection staff can be assisted with a set dialogue or through computer-aided telephone interviewing. It is important that adequate training be provided to deal with questions that arise during the call. The main advantage of this approach is the data validation during the telephone call, but again, this is costly in terms of staffing and there can be difficulty getting through to the respondent to get the information. Telephone interviewing is becoming more difficult because of technical developments that allow respondents to answer telephone calls only from selected people (via voicemail). Also, the respondent may not have the data immediately at hand, which could lead to guesses being provided rather than the correct data. The main concern with this method is that it is likely to lead to bias caused by respondents repeating previous observations, that is, stating that there has been no change.

D.3.5 Internet data provision

7.54 Using the Internet as a method of data collection offers great potential in terms of efficiency and economy. Respondents can be given the URL and password-protected access to a website that provides a custom-designed questionnaire to be completed online. Automatic e-mails may be generated to periodically remind the respondent to complete the questionnaire. Systems to validate the data in real time as the data are being entered by the respondent are also possible. This is a benefit to the business respondent because it reduces recontact time (although the benefits of a one-to-one dialogue are lost). Because the returned data are in electronic format, it is efficient for further processing by the statistical agency, and the response times are quicker than postal-based collection.

7.55 There are, however, a number of issues related to Internet collection. To be effective a very large proportion of the businesses in the country must have access to the Internet. Also, Internet security is vital, given the commercially sensitive nature of the data.

D.3.6 Electronic capture of data from discs

7.56 Another method to capture data involves the supply of a compact disc containing an "electronic questionnaire." The respondent loads the disc and completes the information before returning the disc to the statistical office. Data are then transferred to the statistical office database. This method allows online validation techniques to be built into the questionnaire to save recontact time. However, the procedures for dealing with compact discs are onerous, and for short surveys such as the XMPI (with few data items to be collected) the benefits are limited.

D.3.7 Electronic data transfer

7.57 An electronic data transfer method of collection involves the transfer of data files directly from the establishment's systems and allows a large volume of data to be collected with a minimum ongoing collection burden to the respondent. The initial setup procedures can be quite burdensome, but the regular collection costs are reduced. The statistical office has to clearly define the data format and information system protocols. It is possible that this type of collection could allow full unit value data to be collected, which can be beneficial for tightly defined and homogeneous commodities.

D.3.8 E-mail collection

7.58 The use of e-mail is another collection method that allows the survey form to be delivered and returned electronically. This approach is less efficient than some of the other electronic methods outlined above; however, it could be useful where postal services are less reliable. It is also useful as a reminder technique, because it offers speedy contact with respondents. Again, security is a key issue, and because e-mail can be less secure than some of the other forms of electronic collection, the legal issues should be examined carefully.

D.3.9 Alternative sources

D.3.9.1 Published sources

7.59 Some data items are available from published sources such as trade publications. Examples include metal prices for metals that are traded on financial markets, which are reported in the financial press and international journals such as *Metal Bulletin*. Published sources provide a high-quality source of price data for these commodities. Their advantage is that they are readily available and relatively inexpensive; they also reduce the respondent burden. Before using published source data the statistical office must be sure that the source is reliable and that the prices reported are genuinely independent market prices. It is important to verify that the prices are actually based on business transactions of these commodities. It is also a good idea to become as familiar as possible about the methodology used by the compiling organization.

7.60 A further subset of published sources that is becoming increasingly important is data collection from company websites. Many companies create extensive websites allowing customers to search by commodity specification and in some cases allowing customers to set their own commodity configuration. It is then possible to buy the commodity directly from the site. This type of website offers tremendous potential for XMPI price collection and also for independent validation of prices received through the more conventional channels. There are a few issues to consider, such as the extent to which the Internet prices are list prices instead of transaction prices and whether large buyers attract lower negotiated prices under long-term contracts, for example.

7.61 Another important issue is that the prices advertised on the Internet may be retail prices. But in some circumstances retail and producer prices are the same. For example, one of the best sectors for which to use Internet advertising for price collection/validation is personal computers (PCs). In this case, major manufacturers have set up websites that enable the public and businesses to buy directly from the manufacturer (again it is important to be aware that bulk discounts could be offered).

7.62 It is worth noting that even if published sources are not used as direct inputs to the compilation of the index, they can provide valuable information for editing, external verification, and the preparation of analyses of the main index movements.

D.3.9.2 Regulatory data sources

7.63 For some commodities or services it is possible to get data from government regulators. It can be difficult to get access to this information if confidentiality constraints apply, but where it is possible to get this type of data, they can be of very high quality. Telecommunication and rail fare data are two examples of prices for services collected in this way in several countries. The potential overlap between retail or producer price can also be an issue when using this type of data, but in many cases there are different tariffs for business and consumer use.

D.4 Field procedures

7.64 The following section outlines practical field practices adopted by many statistical offices.

D.4.1 Price discounts

7.65 Prices collected should be transaction prices, not list prices. This means that all discounts should be taken into account. Discounts can be given for a variety of reasons, such as prompt payment, volume of the purchase, competitive price cutting, and so on. It is important that this is made clear during the survey collection process. Discounts arising from high-volume transactions can cause particular difficulty. Problems occur if the volume sold to the representative customer changes from period to period, which could lead to changes in the discount rate in each period. In such a case the price index would move simply because of change in the volume mix, rather than as a result of a pure price change. This type of problem commonly occurs in quarry commodities, such as road stone and railway ballast. A possible approach in such circumstances is to seek prices for the same specific, typical transaction volume each month.

7.66 A common form of discounting is to provide a larger quantity of the commodity for the same price, sometimes for a limited period. The commodity specification should include details on quantity to enable an adjustment to be made to include this type of discount. Retrospective discounts based on sales volumes are an important feature of the manufacturing sector, but they are difficult to collect with normal survey techniques, so these tend not to be included. An example is the bonus paid to car dealers by manufacturers based on sales volumes, which is separate from the original sales transaction.

D.4.1.1 Rebates

7.67 Rebates are a form of discount where the discount is (generally) paid after the purchase. They are normally based on the cumulative value of purchases over a specified time—for example, a rebate is given at the end of the year based on the customer's total purchases over that year.

7.68 The collection of discounted prices and the identification of discounts are complicated in practice by a number of factors. First, the pricing structure used by the company may be complex and the conditions under which discounts apply may be described in nonstandard terms. Second, differences in pricing and discounting procedures among companies require that data collection be tailored to each company. Third, in a number of areas, the level of discounts and rebates is commercially very sensitive information, and only senior company officials may know the full level of discounts offered to major customers. Taken together, these three factors mean that identifying and keeping track of discounts constitute major tasks facing XMPI compilers.

7.69 Rebates in XMPIs pose major practical problems in that they are often determined by future events—for example, the buyer receives a rebate at the end of the year on the basis of how much he or she purchased during the year. Thus, at the start of the year, although it is known that the buyer will receive a rebate, the precise amount is unknown. The special problem posed by rebates of this sort is that the final price to be paid may not be known until the end of the period concerned, when the total value of purchases will be known and hence the level of rebate can be calculated. This type is often referred to as a retrospective price fall.

7.70 Often a rebate paid to the buyer in the form of a reduction in the cost of the purchase over a year occurs in a particular month. This can lead to reported prices showing dramatic price falls for that particular period. XMPI compilers should take care to ensure this does not occur.

D.4.1.2 Treatment of rebates in XMPIs

7.71 The question arises as to how such rebates should be treated. Should the price paid each month be shown in the index as the price for the item? If so, how should the rebate be treated—as a retrospective price reduction? If so, should the previous prices be revised?

7.72 On balance it is considered that where the rebate is already in existence, the rebate should be treated as a discount and deducted from the monthly price and not treated as a retrospective price reduction. The basis for calculating the rebate should be the buyer's normal volume of purchases (if the buyer is a new customer, then the basis for calculating the rebate should be the average quantity purchased by that category of buyer).

7.73 Changes in the level of rebates should be reflected only when the actual rebate for the same quantity purchased or sold changes. Changes in the rebates paid to a particular customer for changing the volume of purchases should not be reflected as a price change.

7.74 As price indices are designed to measure price changes for a constant quantum of purchases or sales, the rebate collected should be the rebate applicable to that constant quantity and clearly specified in the pricing basis.

7.75 Where rebates are specified in terms of a monetary value of purchases or sales, it is important to realize that because of inflation a monetary value does not represent a constant real quantum. As a consequence, the monetary value should, if possible, be converted to a quantity. If this is not possible, then the dollar value should be updated each year according to the change in the price of the item concerned.

7.76 If the quantity or value of a respondent's purchases or sales changes significantly, the pricing basis should be changed to reflect this. The change in rebate associated with this should not be allowed to affect the index.

7.77 When a number of levels of rebates are offered, it is necessary to ascertain the importance of each level of rebate and to price those that are significant.

7.78 Caution must be adopted when dealing with retrospective price falls. Revisions to previously published indices can create major problems for users, who use them to negotiate contracts.

D.5 Other variables

D.5.1 Quality changes/specification changes

7.79 If any variable in the commodity specification changes, the respondent should be questioned about the change and whether or not new features have been added. If it is a simple change that bears no effect on the price, then the specification should be updated and a marker placed on the commodity description to indicate that it has been changed. If the quality increases (in terms of producer's cost) then this should be reflected in the index as a price fall. If the quality decreases, then this should be reflected as a price rise.

7.80 If the price of the commodity changes and it is solely or partly due to the change in the specification of the product, then the data collection form returned to the respondent should contain the new price, but for index purposes a quality adjustment should be made to record no price changes for the commodity on the price database (because it is not an inflationary pressure causing the price to move). If, for example, the price of the commodity increases by 10 percent, and the respondent's assessment is that only 5 percentage points of the increase were related to the pure price change, with the remaining 5 percentage points being due to a different commodity with a changed specification being quoted (a quality improvement), then the price relative should increase by only 5 percent. Methods for assessing the effects of quality changes on price changes are outlined and discussed in Chapter 8.

D.5.2 Unique commodities

7.81 A unique commodity is a commodity that is only manufactured once to the specification of an individual customer. Within a group of commodities, each commodity will be different from the others—for example, industrial furnaces, ships, or an audit contract. In these cases the price cannot be observed over multiple periods. There are a number of approaches to solving this problem, as follows:

(1) *Model pricing:* Ask the respondent to provide a notional commodity with a basic range of characteristics, based on recent orders. For each period the respondent is asked to supply a hypothetical price quote based on this hypothetical product. It is important to update the commodity specification at regular intervals. The model used can be

  a. An actual commodity sold in some recent period, which is representative of the respondent's output, can be selected and specified in detail as the model to be priced;

  b. A hypothetical model that is representative of the types of commodities produced by the respondent can be established. Although this model may never have been (or never will be) produced, it must represent an item that could be readily produced; and

  c. A component model can be established. These are used in those cases where no single model can represent the output of the respondent. In such cases, a number of models can be selected or a notional model incorporating the key components from the various items produced can be established—that is, incorporating the different types of materials used and different production techniques. In the latter case, the model would be purely hypothetical in that it might never be built, but it would nevertheless be representative for measuring price changes.

(2) *Repeat recent real s ale:* Ask the respondent to provide a price quote for a recent real sale and to provide a hypothetical price for this exact commodity design for the subsequent months. If the order is not repeated again after a reasonable interval—for example, six to eight months—then a replacement commodity is sought.

(3) *Specification pricing:* A base model of the commodity or service is agreed with the company and then in each subsequent month the company supplies the price for each individual part of the model—for example, one hour of an accountant's time or a ton of steel, and so on. When the data are returned to the office they are collated using a formula agreed with the company to arrive at a price each month.

(4) *Component pricing:* This approach entails collecting prices for a selection of component parts and using them as inputs to produce a final output price. It is important to include the prevailing margin achieved by the producer and it is also important to have a dialogue with the producer to ensure that the components remain representative and constitute a very high proportion of the total inputs. For example, a recent EU Task Force report on large capital equipment suggested that this approach would be sufficient if the value added for the assembly of the components did not exceed 10 percent of the total commodity value.

7.82 These approaches are much more burdensome on respondents, because they cannot simply look at recent sales data to provide price quotes. To accurately supply price quotes using these approaches would lead to the respondent incurring substantial costs, so in practice there is an element of estimation in this process.

7.83 In all of the above unique commodity cases, the main difficulty is persuading the respondents of the value of this approach because they do not produce this specific commodity as described. To resolve these issues, a field officer visit may be necessary. It is important to include such base-level commodities as these in XMPIs because they are often associated with high-value goods that would not be included otherwise. It is all too easy to select a related but simple product—for example, in shipbuilding to ignore large, unique ships and to concentrate only on small, regularly produced commodities such as dinghies. Unique ships and dinghies may have quite different price changes.

7.84 A further approach to solving the unique commodity problem could be to build a hedonic model. This would enable the unique model to be valued from its characteristic set. This method is more normally associated with quality adjustment but could be extended where sufficient data are available. See Chapters 8 and 22 on quality adjustment for more details on this approach.

D.5.3 Unit values

7.85 In some circumstances it is possible to use unit value indices to overcome the problems of unique commodities, but this is recommended only for commodities with a very narrowly defined commodity group—for example, for road stone. See Chapter 2 and Section B.2 above for a discussion of the limitations of unit value indices.

D.5.4 Transfer prices

7.86 Affiliated enterprises may set the prices of transactions among themselves artificially high or low, the so-called "transfer prices." Transfer prices should be used with caution because they often do not fully reflect the true value of the goods or services being transacted. It is important to aim to collect market prices, or real transaction prices. The best way to get real transaction prices is to ensure that the price recorded is to a third party, not to another part of the same business. In some circumstances there may be insufficient real market prices for a representative index. Indeed, it is possible that the only price available is to another part of the same business, which is potentially, but not necessarily, a transfer price. Careful attention then has to be paid to market price movements for similar commodities to ensure that the inter-enterprise sales reflect market conditions. It is important to avoid recording stable prices for these transactions over the long term if market prices are changing. In addition, obtaining good weight information on the value of output or sales will also be difficult because the traded values used as weights should reflect market prices. A further, more detailed, discussion of such issues is provided in Chapter 19.

D.5.5 Sampling issues

7.87 The sampling of representative establishments and items was considered in Chapter 6. The objective of price indices is to measure pure price change over time—that is, to measure the extent to which the cost of an identical basket of commodities changes over time, not affected by changes in quality or quantity or the terms of sale. This is often referred to as pricing to constant quality; it is not a simple objective to achieve because the characteristics of commodities being sold in the marketplace, including their terms of sale, change over time. Frequently, the precise commodity priced in one period is no longer available in the next period, either because there has been some change in the characteristics of the commodity or because something new has taken its place. Thus sampling issues are not solely concerned with the initial representativity of the sample, but also with its maintenance.

7.88 Sampling issues, such as sample loss relevance, avoidance of making quality adjustments, selecting noncomparable replacements, and inadequate matching procedures, greatly affect the representativeness of the sample over time. Ignoring such issues could bias the index (see Chapter 9 for further details). XMPI compilers must devise techniques to minimize differences and eliminate their effect on the index.

D.5.6 Price discrimination

7.89 Price discrimination refers to the situation where the same commodity sells at different prices in different markets. An example would be the same grade of wheat sold at different prices in different markets, or a different volume of wheat sold at the same price to different markets. When such price discrimination occurs, the average price of wheat can change over time because of changes in the proportions of wheat sold to each market. The article by Ruffles and Williamson (1997) shows that in the United Kingdom more than three-quarters of exporters in a survey charged different prices for different countries.

7.90 The same commodity of the same quality may be imported at different prices from different suppliers, possibly in different countries, or exported to different foreign purchasers at different prices. How should such price changes be reflected in XMPIs? In general terms, if exactly the same item is sold in different markets at the same terms of sale, then a unit value is appropriate because it would reflect a, say, fall in average price if quantities switched in favor of countries of origin/destination with lower price levels. If the products were very similar, unit values may again be used as long as some adjustment for the quality component of the price differential could be stripped out (see Chapter 8). If the products differed by country of origin, price indices should be used that treat the product to/from a different destination as a different product. While the above general principles hold, the answer depends on the reason for the price discrimination and requires an examination of the various forms of price discrimination. There are four main forms of price discrimination (listed below). All four factors (or any combination) could apply to a particular specification.

D.5.6.1 Differences in selling terms and provision of credit

7.91 The conditions under which goods are sold often vary between markets (buyers). For instance, prices may be lower in one market because the goods are paid for on delivery, whereas prices in another market may be higher, reflecting the fact that goods are sold on credit. In these situations it seems reasonable to argue that identical goods are not being sold in each market. What is in fact being sold in the second market is a mixture of the good and credit. It follows that, in these cases where the credit arrangements are a significant element, shifts in destination should not be reflected as price changes but XMPIs should be compiled as aggregates of price changes across destinations.

D.5.6.2 Differences owing to timing of contracts

7.92 Where goods are sold on a long-term contract basis, price differences may arise between different markets simply because of differences in the period when the contracts in respect of these markets were signed. In these cases it seems clear that changes in prices owing to changes in destination should be reflected in the index. Failure to do so would run the risk of missing out on long-term price changes for commodities where the destination is changing over time.

D.5.6.3 Competitive pressures

7.93 In some markets, goods may have to be sold at lower prices because of competition from other countries (e.g., dumping of EU agricultural commodities), whereas in other markets producers may be able to achieve higher prices because of the absence of such competition. Shifts between markets represent pure price changes and should be reflected as such only if respondents confirm that there is genuinely no change in the specification of the commodity in the two markets. The *2008 SNA* argues that if the price dispersion in a period was not due to quality differences, a unit value index should be used. Yet it notes an important exception regarding the case of institutionalized price discrimination. If different purchasers of the same good or service, say electricity, face different prices and the individual purchasers, say commercial customers and private households, are unable to change from one price to another, then price indexes should be used. The constraint on the availability to the purchaser of different prices must be institutional and not simply an income constraint.

D.5.6.4 Hidden quality differences

7.94 For some items, such as tinplate, respondents supply prices only for broadly specified commodities. In these cases, destination may serve as a de facto quality specification—for example, the quality of tinplate shipped to destination A is different from that shipped to B. In this situation, price changes should not be averaged over destinations in the form of a unit value; changes in destination should not be reflected as price changes.

D.6 Field officer visits

7.95 Field officer visits serve two broad purposes. First, field officers are often used in the initialization/recruitment process to identify representative commodities from within the respondents' commodity range and to discuss the exact reporting requirements for the XMPI. (This approach is used by the United States, Australia, and France; the French use qualified engineering staff to visit companies.) In some regions it is known that a group of commodities are manufactured by a company in a particular sector (e.g., in Europe a detailed survey on commodities by industry of origin (PRODCOM) is conducted by member states). In such a case, there are two options when selecting commodities: either let the business pick the most representative commodity (i.e., the one that accounts for the largest percentage of the respondent's turnover for the class of product) or let the field officer select the commodity with the respondent. Each approach has advantages and disadvantages. For example, only the respondent knows the best or easiest commodity for them to supply data for; however, it is important to get a price quote for several commodities that are representative of the respondent's commodity range. This may require collaboration between the respondent and the field officer. For more information on commodity selection see Chapter 6.

7.96 The second main purpose of the field officer is to assist respondents with problems in completing returns—for example, in the case of unique commodities or late responses. The field officer contacts and visits the company to understand its specific concerns and problems in completing the form and works with the company to overcome them. This is largely a reactive activity used to solve problems, but another alternative is to have a rolling program of visits so that each respondent (or key respondent) is visited over a set period. This helps to keep the respondent educated and ensures that problems do not linger unnoticed for long periods.

D.7 Industry specialists

7.97 The role of the industry specialist is similar to that of the field officer, but the industry specialist concentrates on a narrow range of industries. Because prices are required from very specialized industries, such as chemicals and semiconductors, it is difficult for the statistical office to assure the quality of data returns and have a meaningful dialogue with respondents unless the organization includes analysts with a more detailed knowledge of these complex industries. Small teams become "experts" in certain fields—for example, computers. The teams are fully up to speed on changes in the market, their respondents' activities within the market, and specific problems relating to completion of survey data. These experts then analyze returns in line with the industry intelligence and support respondents when they provide data.

D.8 Delinquency follow-up

7.98 It is important to achieve high response rates and, in order to achieve this, procedures should be established to follow up nonresponders. Problems with maintaining adequate response rates will occur in price surveys that do not have delinquency follow-up procedures, even when the surveys have statutory penalties.

7.99 Delinquency follow-up can be done using any of the data collection methods outlined above. A reminder telephone call is an effective technique because this enables respondents to discuss any difficulties they have with the survey at the same time, and it is often possible to take data over the telephone (although a price taken over the telephone should be marked for later verification). This technique has the advantage of generating quick results, but it does require the statistical agency to maintain an up-to-date list of contacts and their telephone number. Given the labor-intensive nature of reminder telephone calls, it is possible to target these calls on key responders, which are usually the responders with the largest weights.

7.100 A follow-up letter is often effective, particularly if the country has a legal penalty for nonresponse. In such a case it is possible to make the wording of the follow-up letter stronger, with more emphasis on the legal penalty. It is usual to follow a set procedure, including the use of a recorded delivery letter, if formal legal proceedings are to be conducted for nonresponse.

E. Respondent Relations

7.101 Respondents are very important to statistical offices, because without them there are no data. Therefore, developing good relations with and gaining the trust of data providers is an integral part of producing good estimates.

E.1 Dealing with refusals

7.102 On occasion, you will come across a respondent who states, "I've had enough" or "I'm not doing it anymore." In general, providing respondents with relevant information related to their concerns will ensure that they will continue to provide data. In the examples above, refusals could reflect issues with confidentiality, lack of importance, and too many forms.

7.103 Possible ways of dealing with these concerns are as follows:

(1) *Confidentiality:* When dealing with issues of confidentiality, a statistical agency that is independent of other government agencies has an advantage over those that are not. Independence and confidentiality are important to ensure trust of respondents. Reassurance that the data will not be released to any other agency or person is much easier with legislation backing the statistical office.

(2) *Objective of the collection:* Reassure the provider that the prices are aggregated into an index that is published monthly or quarterly, and inform the provider of the importance of contributing and the use of the statistics produced.

(3) *Respondent workload:* Talk with the respondent to ensure that the data are easy to obtain and that the current specifications are still relevant; see if the provider can get rotated out of the sample if the company has been contributing data for many years.

E.2 Reducing respondent workload

7.104 Apart from the ongoing use of tailored forms, which significantly simplify the collection process for respondents, XMPI compilers can actively reduce provider workload by

(1) Identifying commercially available data that meet the methodological requirements of price indices and using these as a substitute for data collected from respondents. The cost incurred in purchasing these data is compared with expected collection costs, with the benefit of reducing provider load taken into account; and

(2) Identifying administrative data sources of prices that meet the methodological requirements of price indices, and using these as a substitute for data collected from respondents.

E.3 Surveys of respondents' views

7.105 It is important to be aware of deficiencies in the data collection process. Some of this may be gleaned from the field officer visits discussed in Section D.6. However, user surveys may be carried out on a sample of respondents to identify whether issues raised by individual respondents can be generalized and to consider further possible areas for improvement to the data collection process. Such surveys should not be carried out too frequently to avoid criticisms of an undue burden on the respondents. The user survey should clearly state that its aim is to make the task of responding easier, as well as to improve the accuracy of responses. Such surveys can examine issues such as the adequacy of the commodity specifications, the representativity of the commodities used, the respondent's compliance with requested data definitions (e.g., using transaction rather than list prices), and the respondent's likely use of proposed alternative electronic/telephone data response systems. An example of a user survey for the PPI and XMPIs is shown in the United Kingdom, Office for National Statistics (2003).

F. Verification

F.1 Verification and validation of prices

7.106 Verification aims to identify potentially incorrect prices as early in the process as possible, consult with the respondent, and amend the data if necessary. Three key checks are required:

- Data reported were accurately entered into the processing system,
- All requested data were provided, and
- Data reported were valid (outlier detection).

7.107 This section covers only the first two bullet points above, that is, simple data checks at the point that data enter into the XMPI system. Validation checks to assess whether the data returned by respondents are credible in relation to other data for the same industry or commodity and the treatment of data that are not credible are covered in Chapter 10.

F.2 Verification tolerance

7.108 The first stage in the verification process is the confirmation that the data entered into the system for further processing is an accurate reflection of the data returned. This can be done either by a manual audit or through an automated system. These checks should include the following:

- All data fields required have been completed,
- The data entered in the database agree with those reported, and
- All data fields are completed within an expected parameter range.

When the data have been accurately recorded by the statistical office but basic data checks are not passed, the analyst will need to contact the respondent in order to verify the information or to get the correct data. Returned prices may be compared to those received for the previous period. If the price change is outside a specified range, then the price should be marked for further investigation. Respondents providing dubious prices can then be contacted to check that the large change is correct and to provide a reason for the large change. Large price changes fall into two main categories: those that are erroneous and those that are correct but genuinely unusual. The second category is more difficult to deal with because they could be outliers, which might result in the need for special treatment within the estimation procedure. Outlier treatment is discussed in Chapter 10.

F.3 Setting tolerances

7.109 The tolerances for data verification checks should be set so that any changes outside the boundaries of expectation are flagged for the data reviewer.

7.110 Tolerances may need to be set independently for each commodity group. For commodities that have volatile prices, such as oil or seasonal items, it may be appropriate to have quite wide verification tolerances. Other commodities may have more stable prices and thus narrower tolerances would be more appropriate. To set verification tolerances for a particular product, price changes over a period of time, say two or more years, need to be analyzed. The range of price changes can then be considered, and the top and bottom 10 percent, for example, can be used to set the tolerances for verification.

7.111 In addition to checking for large price movements, another check is for prices that have not moved for a considerable period. Most companies will review their prices on a regular basis, often annually. If a company's price has not moved for 15 months, for example, they may be returning the same price out of habit. In these cases the company should be contacted to see if the true price is being returned. In periods of low inflation it is likely that the periods of stability between price changes could get longer and also that the number of companies not reporting price changes for considerable periods could increase.

G. Related Price Issues

G.1 Lagged prices

7.112 In some cases it is not possible to get prices in time for the current period's compilation. In order to ensure that the XMPI is published in a timely manner, the previous available price or lagged price can be used. The need for these lagged prices is often when the required data come from administrative sources, and the delays result from the time required by the external supplier to collate data. Examples include financial intermediation service charges and insurance premiums.

G.2 Seasonal commodities

7.113 Some commodities are only available for part of the year—for example, items associated with religious festivals and certain fresh fruits or vegetables. A practice commonly adopted in such cases is to carry the last reported price forward, until the next season's trading starts and a new price can be collected. This procedure tends to dampen the index movements when the commodity is out of season, and it causes upward and downward movements in the commodity index when the commodity is back in season. One solution is to impute the missing prices based on the short-term movements of prices for similar commodities. An alternative solution is to have variable weights for each period, so that the commodity has a zero weight when not in season. The disadvantage of this is that it makes analysis of index point effects less straightforward, because care has to be taken to ensure that the correct weights are applied in each month. See Chapters 11 and 23 on seasonal commodities and Chapter 8 on imputation techniques for more details.

8. Treatment of Quality Change

A. Introduction

A.1 Why quality change is an issue

8.1 When routinely compiling an export or import price index (XMPI), specific varieties of commodities in the index regularly appear and disappear. New goods and services can appear because technical progress makes production of new varieties possible. Even without technical progress in the supplying activity, however, commodities previously feasible, but not produced, may emerge because the technology of the using activity or the tastes of the final consumer have shifted. Existing varieties often decrease in importance or disappear from the market altogether as new varieties appear. For price collection using establishment surveys, the priced set of commodities often is a small sample of the full range of commodities that exist at any given time. Commodities in the sample may appear and disappear not because they are truly new to, or no longer produced or used by, all establishments, but because they may be only new to, or no longer produced by, the establishments in the sample. New goods and services may be new only in the sense that they were not traded in the previous or reference month but now reappear, while old goods and services may be old only in the sense that they are not traded in the current month, but once were.

8.2 This chapter covers how to deal with the problem of continuous change in the assortment of transactions whose prices make up XMPIs. The overarching principle for designing methods to deal with variety turnover is that, at the most detailed level, the prices of items between any two periods may be directly compared only if the items are *essentially the same*. Violating this principle would mean that a given monthly price ratio measures not only the change in price, but also the value of the qualitative difference between two items. This contaminates the estimate of relative price change with an element, quality, that measures relative volume rather than price. It degrades the accuracy of the price index formed with the price ratios or relatives for the measured transactions.

8.3 What does "essentially the same" mean in practical terms? For measurement purposes, a commodity equates to a *complete description* of price-determining characteristics. It may, for example, be the case that an imported good can be purchased with a warranty or an extra option for the same price. But then the product would not be the same good, and the price of the option may decrease over time while that of the warranty increases. All items have to have the same commodity description. The form of this description often is simply text. It also can be highly structured. In *structured commodity descriptions*, the commodity's characteristics are specific levels of indicators for several dimensions that are known to affect the average transaction price.[1] Each set of these indicators' levels frames a specific commodity. Examples of these dimensions are the horsepower of an automobile, the speed of a computer, or the species of a piece of fruit. Examples of commodity-determining levels or specific settings of these respective dimensions are 325 horsepower, 2 gigahertz, or flame red grape. *The comparison of prices of varieties with like characteristics over time requires that detailed information is available to the price statistician each month on price and quality specifications, something that can be collected from establishment surveys but not from customs documentation.*

8.4 For price measurement purposes, the comparative quality of a commodity comprises its description and price. Distinct descriptions represent different qualities of commodities, to the extent that they contain different levels of characteristics that affect the average price of transactions of things with that description in a given month. If commodities with two distinct descriptions

[1]See Chapter 6 on structured commodity descriptions, also termed checklists by some statistical agencies.

are transacted at the same time, the description with the higher price must be the higher quality. This corresponds to what is called a revealed preference or value in use of the commodity (demand side), as well as higher content in the input needed to make the commodity (supply side). For index compilers, then, quality is an ordinal concept, comprising the set of complete commodity descriptions ordered by price for a given month.

8.5 When an existing variety of a commodity disappears and a new one appears, a new description manifests itself as well. The new description is different from the descriptions of existing commodities because the level of at least one characteristic in the description has changed. The difference in the characteristic explains the difference in price compared with varieties already available. For example, a new variety of computer emerges with a processor speed of 3 gigahertz instead of 2, and it has, say, a $325 premium over 2 gigahertz computers already available. Thus, the value of the additional gigahertz of speed is $325 and the new computer is, by implication, of higher quality than the old one. Some commodities may be quite distinct in their newness in a manner that cannot be described by the existing characteristic set, and the treatment of such completely new commodities is the subject of Chapter 9.

8.6 When a variety of a commodity is no longer sold, and *price collection is based on establishment surveys,* there are a number of options open to the price statistician. As will be outlined below, these include the use of a comparable replacement; a noncomparable replacement, with an explicit quality adjustment to one of the prices to make the items comparable, or an adjustment based on their relative prices in some overlap period; or more simply, the dropping of the variety from the sample until a new set of commodities is selected on rebasing or sample rotation. In each case, a quality adjustment is being undertaken, be it implicit or explicit. It is sometimes said by statistical offices that they do not undertake quality adjustments in their XMPI compilation. Some adjustments are always made; they are implicit in the practice of treating missing varieties. A number of empirical studies for consumer and producer price indices (CPIs and PPIs) have found that the choice of method for dealing with such missing values can matter substantially. This chapter outlines alternative methods and is also a guide to selecting methods based on the measurement circumstances so that the methods chosen can be the appropriate ones.

8.7 For *price collection based on unit values from customs returns*, there is no careful matching of the prices of like items with like. The implicit method of dealing with new and old varieties is to bundle their value with the value of existing varieties, for the commodity group or shipment in question, and calculate unit values. This method of ignoring the quality change and the mix of quality varieties within the commodity group makes, as discussed in Chapter 2, the resulting index subject to unit value bias. Indeed the information on customs documentation is insufficiently detailed for the price statistician to even be aware that changes in varieties are taking place, let alone make the changes required for proper price measurement. As such this chapter will be largely devoted to issues that arise from price measurement using establishment surveys.

8.8 The issue of missing prices is particularly severe for XMPIs as compared with CPIs and PPIs. For CPIs an item's price is missing if it is not in the sampled outlet when the price collector visits the outlet. It may be out of stock or no longer sold. For a PPI, an item's price is missing if the reporting establishment has not sold/purchased or priced the item in the period that constitutes the inquiry. It may have had no sales in that period and not be priced, or be no longer produced. However, for XMPIs, an item's price is missing if the reporting establishment does not export/import (or price) the item in the period that constitutes the inquiry. It may still be produced/purchased for/from the domestic market, but it is still missing. Feenstra and Diewert (2000) investigated various imputation techniques for missing prices for XMPIs making use of a data set from the International Price Program of the United States Bureau of Labor Statistics (BLS) that consisted of all price quotes received from January 1997 to December 1999 at the most elementary "item" level. Included in this data set was an indicator variable for whether each price quote is imputed or not owing to being unavailable at the time of reporting. Of the 893,935 monthly observations at the elementary "item" level, over the three years of data more than one-third was imputed.

A.2 Why the matched-models method may fail

8.9 The matched-models approach to variety turnover described in Section A.1 is subject to three broad sources of error: (1) missing commodities (hereafter also including varieties of a commodity), (2) sample space change (sampling issues), and (3) new commodities. *Missing commodities* are concerned with the solution of the problem of "what to do when a commodity is no longer imported or exported in the month in question" by means of replacement varieties or imputations.

In solving the problem the methods attempt to preserve the original matched sample with one-on-one replacements. For *sample space changes,* the issue of concern is with incorporating the price changes of unmatched old and new models that are no longer traded or are introduced outside of the matched sample. However, when a very *new commodity* arrives, which cannot by definition be easily linked to existing ones, a different problem arises.

A.2.1 Missing commodities

8.10 Compilers measure the long-run price change for a commodity by comparing the price of the commodity in the current period, usually month, with its corresponding price in the price reference period. This reference period would be the month when the commodities in the index entered the sample. When a commodity is missing, it may be because it has been discontinued, or it may not be available to the same specification—its quality has changed. We thus encounter the first potential source of error in the matched-models method. There are several specific contexts for this. It may be a seasonal commodity, or the commodity may be a custom-made good or service supplied each time to a customer's specification, or it may be that it is in short supply, or simply no longer demanded and supplied. There are four main approaches for dealing with missing commodities:

Approach 1: The price change of the discontinued commodity may be *imputed by the aggregate price change of a group of other commodities* whose price evolution compilers judge to be similar to that of the missing commodity. Such imputations should only be undertaken for short periods.

Approach 2: A *replacement commodity may be selected, comparable in quality* to the missing commodity, and its price used directly to form a price relative.

Approach 3: The replacement may be deemed noncomparable with the missing commodity, but prices of both the missing and replacement commodities may have been available in an *overlap period* before the commodity was missing. Compilers use the price difference in this overlap period to adjust the quality of the replacement commodity's price until there are at least two observations on the replacement commodity.

Approach 4: The price of a noncomparable replacement may be used with an *explicit adjustment for the quality difference* to extract the pure price change.

8.11 This chapter discusses these four approaches to quality adjustment in some detail along with the assumptions they imply. Because the prices of the unavailable commodities are not measured by definition, the veracity of some of the maintained assumptions about their price changes, had they been available, is difficult to establish. Nevertheless, the objective of each of the methods is to produce matched comparisons of the prices of commodities: to compare like with like from month to month. When commodities are replaced with new ones of a different quality, then a quality-adjusted price is required to produce a match. If the adjustment is inappropriate, there is an error, and, if it is inappropriate in a systematic direction, there is a bias. Careful quality-adjustment practices are required to avoid error and bias.

A.2.2 Sampling issues

8.12 Sampling issues comprise four main areas of concern. First, samples lose relevance. A given set of matched models or commodities is likely to become increasingly unrepresentative of the population of transactions over time. There is evidence that the prices of old commodities when dropped are relatively low and the prices of new ones relatively high, and their prices are different even after quality adjustment. For strategic reasons, firms may wish to dump old models, to make way for the introduction of new models priced relatively high. Ignoring such unmatched old and new models in XMPI measurement will bias the indexes downward (see Section G.2.3 in this chapter). Ironically, the matched-models method that compilers employ to ensure constant quality may itself lead to bias, by ignoring such unmatched models, especially if used with an infrequently updated commodity sample.

8.13 Second, because of the additional resources required to make quality adjustments to prices, it may be in the interests of respondents and their counterparts in statistical offices to avoid making noncomparable replacements and quality adjustments. They keep with their commodities until they are no longer produced—that is, they continue to monitor old commodities with limited sales. Such commodities may exhibit unusual price changes as they near the end of their life cycle. These unusual price changes arise because marketing strategies typically identify gains to be made from different pricing strategies at different times in the life cycle of commodities, particularly at the introduction and end of the cycle. Yet their weight in the index, which is based on their export revenue/cost share when they were sampled, would remain constant in the index and probably would be too high at the end of the life cycle. Further, new, and therefore unmatched commodities with possibly large sales would be ignored. Undue weight would be given to the unusual price changes of matched commodities at the end of their life cycle. This issue again is resolved by more frequent sample reselection of commodities within a given sample of establishments.

8.14 Third, the methodology for selecting replacement commodities may be such that respondents are advised to choose comparable replacements to avoid the need for explicit quality adjustments to price. Obsolete commodities are by their nature at the end of their cycle, and replacements, to be comparable, must also be near or at the end of their cycles. Obsolete commodities with unusual price changes at the end of their cycle may be replaced by other obsolete commodities with unusual price changes. This compounds the problem of unrepresentative samples and continues to bias the index against technically superior commodities delivering cheaper service flows.

8.15 Finally, the sampling problem with the matching procedure occurs when the respondent continues to report prices of commodities until replacements are forced, that is, until the commodities are no longer available, but the respondent has instructions to replace them with popular commodities. This improves the coverage and representativity of the sample. But the wide disparity between the characteristics of the old, obsolete commodities and new, popular ones makes accurate quality adjustment more difficult. The (quality-adjusted) price changes of very old and very new commodities may not be similar as required by the imputation methods under approach 1. The differences in quality are likely beyond what can be attributed to price differences in some overlap period under approach 3, because one commodity is in the last stages of its life cycle and the other in its first. Further, the technical differences between the commodities are likely to be of an order that makes it more difficult to provide reliable, explicit estimates of the effect of quality differences on prices under approach 4. By implication, many of the methods of dealing with quality adjustment for unavailable commodities will work better if the switch to a replacement commodity is made sooner rather than later. Sampling issues thus are closely linked to quality-adjustment methods. This will be taken up in Chapter 9, in the section on commodity selection and the need for an integrated approach to dealing with both representativity and quality-adjusted prices.

A.2.3 New commodities

8.16 The third potential source of error is distinguishing between new commodities and quality changes in old ones, also covered in this chapter. When a truly new commodity is introduced, there are at least two reasons why early sales are at high prices that later fall, often precipitously: capacity limitations and market imperfections. Both of these may be present shortly after introduction of a new commodity because there are few suppliers for it.

8.17 Early in the commodity life cycle, production processes may have limited capacity; therefore, producers find themselves operating at relatively high and increasing marginal costs of production. Marginal costs of operation tend to decline as more producers enter the market or as existing producers redesign and upgrade production facilities for higher volume. Both of these bring operating levels back from high marginal cost, near full capacity levels.

8.18 With or without early capacity constraints, the small number of suppliers early in the life cycle allows what economists call *market imperfections* to arise. In an imperfectly competitive market, the producer can charge a monopoly price higher than the marginal cost of production. As more competitors enter the market for the new good or service, the monopoly power of early sellers decreases and the price tends to drop toward marginal cost. Further, the market may be one in which subgroups are willing to pay a premium for a new variety/commodity and suppliers will accordingly practice price discrimination.

8.19 The initially high price at introduction and its full subsequent decline would not be brought into the index fully by the usual methods. Compilers commonly either wait until the index is rebased or until a commodity in the sample becomes unavailable to seek a replacement commodity and admit the possibility of detecting a new good. After capacity constraints or monopoly profits diminish, subsequent price changes may show little difference from other broadly similar commodities. Standard approaches thus wait too long to pick up these early downward trends in the prices of new goods.

8.20 At the extreme, capturing the initial price decline requires a comparison between the first observed price and a hypothetical price for the period before its introduction. The hypothetical price would be the price below which there would be no positive market equilibrium quantity bought and sold.[2] Again, frequent resampling offers the possibility of catching new goods early in the commodity cycle when their prices are high and market share relatively low, thereby capturing early price declines as producers relieve capacity constraints and new entrants compete market imperfections away.

8.21 Finally, it is important to emphasize that there is not only a price decline but also a market share increase in the stylized commodity life cycle. Frequent resampling and focused scanning for new commodities should be at least somewhat effective in capturing the price declines in early commodity cycles. Compilers face a potentially serious problem, however, if they have no market share information to go with the prices. The stylized facts of the commodity cycle are that a new commodity comes in at a high price and a low market share. The price then declines and market share increases. Both prices and market share then stabilize for a period, until a successor commodity emerges at a high price and low market share and then begins to take market share from the now mature existing commodity. Early and normally large price declines for new commodities thus should figure into the elementary aggregate price index at a relatively low weight, while later and normally smaller price declines figure in at a successively higher weight. Without current market share data, early price declines may well be overemphasized and the growth in the price index for the elementary aggregate underestimated.

A.3 Temporarily missing commodities

8.22 Commodities that are *temporarily* missing are not available and thus not priced in the month in question, but are expected to be priced in subsequent months. The lack of availability may be because, for example, inventories are insufficient to meet demand, or material inputs are seasonal, as is the case with some fruits and vegetables for food canning. There may also be shortages. The treatment of seasonal goods is the subject of Chapter 23. Such goods may go missing in some months, but are distinguished by the fact that they are expected to reappear, at a similar level of quality, in the next season. It may be that commodities are only temporarily missing because they are switched in a particular month to be sold to or purchased from the domestic market, thus not figuring in export and imports.

[2]This hypothetical price is the *reservation price* for the import price index. It is the highest notional price at which the quantity demanded by the purchasing establishment would have been zero. The user's reservation price thus will be *higher* than the first observed price (Hicks, 1940).

8.23 Standard practice for good survey management requires that seasonal commodities be separately identified by the respondents as "temporarily missing" or "seasonal," so compilers can remain alert to the commodity's reappearance later in the year. Principles and methods for dealing with such commodities are outlined in Armknecht and Maitland-Smith (1999), Feenstra and Diewert (2000), and Chapter 23.

A.4 Outline for the remainder of the chapter

8.24 Section B considers further what is meant by quality change and then considers conceptual issues for the valuation of quality differences. The meaning of quality change requires a conceptual and theoretical platform so that adjustments to prices for quality differences are made against a well-considered framework. Readers interested only in methods of quality adjustment will find them in Sections C through G. Section C provides an overview of the methods available for dealing with unavailable price observations. Methods for quality-adjusting prices are classified into two types: *implicit* and *explicit* adjustments, covered in greater depth in Sections D and E, respectively. Section F considers how to choose among methods of quality adjustment.

8.25 The implicit and explicit adjustment methods to be used when matching fails—when matched models are missing—are first outlined. However, where commodities are experiencing rapid technological change, these methods may be unsuitable. The use of imputations and patching in of quality-adjusted replacement prices when the matching fails is appropriate when failed matches are the exception. But in high-technology commodity markets likely to experience rapid turnover of models, they are the rule. Section G considers alternative methods using chained or hedonic frameworks to meet the needs of rapidly changing production and purchasing portfolios. Section H examines frequent resampling as an intermediary, and for imputation a more appropriate, approach. Chapter 23 discusses issues relating to seasonal commodities in more detail.

B. What Is Meant by Quality Change

8.26 Before turning to the methods of quality adjustment, the nature of quality change is first discussed, along with a brief outline of the conceptual basis for the indices. Sections C to G include material on the methods price statisticians might employ in dealing with the problem of quality adjustment. In choosing between, and applying, some of these methods, arguments may be made concerning the conceptual basis as to how quality should be valued.

B.1 Nature of quality change

8.27 Bodé and van Dalen (2001) undertook an extensive study of the prices of new automobiles in the Netherlands between 1990 and 1999. The average price increase per car over this period was about 20 percent, but the mix of average quality characteristics changed at the same time. For example, the horsepower (HP) of new cars increased on average from 79 to 92 HP; the average efficiency of fuel consumption improved from 9.3 to 8.4 litres/100 km; the share of cars with fuel injection went from 51 percent to 91 percent; the share of cars with power steering went from 27 percent to 94 percent; and the share of cars with airbags went from 6 percent to 91 percent. There were similar increases for central locking, tinted glass, and many more features.

8.28 Standard price index practice matches the prices of a sample of models in, for example, January with the same models in subsequent months. This holds the characteristics mix constant to keep quality differences from contaminating the estimate of price change. However, as considered later in this chapter, the resulting sample of matched models (commodities) is one that gives less weight (if any) to models subsequently introduced. Yet the later models benefit from more recent technological developments and may have different price changes given the quality of services they provide. One approach to correct for such quality changes using the whole sample of both new and existing models is a dummy variable hedonic regression (see Section G.2.1). Bodé and van Dalen (2001), using a variety of formulations of hedonic regressions, found the quality-corrected prices of these new automobiles to be about constant over this period. In this case, the value of the quality improvements explained the entire nominal price increase.

8.29 Recorded changes in prices are the outcome of shifts in both demand and supply. Chapter 22 explains that these shifts arise from a number of sources, including environmental changes; changes in users' technology, tastes, and preferences; and changes in producers' technology. More formally, the observed data on prices are the loci of the intersections of the demand curves of different final users with varying tastes or intermediate users with possibly varying technologies, and the supply curves of different producers with possibly varying technologies. Separately identifying the effects of changes in environment, technology, and tastes and preferences on the spectrum of commodity characteristics present in markets at any given time is conceptually and empirically difficult. Fortunately, compilers do not have to separately identify these effects to produce a good price index in the face of quality change. They need only to identify their overall impact.

8.30 Our concern is not just with the changing mix of the observed characteristics of commodities. There is the practical problem of not always being able to observe or quantify characteristics, such as style, reliability, ease of use, and safety. There are other, less obvious, differences in quality, including different times of the day or periods of the year. For example, electricity[3] or transport provided at peak times must be treated as being of higher quality than the same amount of electricity or transport provided at off-peak times. The fact that pricing peaks exist shows that purchasers or users attach greater utility to the services at these times.

8.31 It may be that the commodities, although essentially the same, differ in other respects, say, the warranties attached, reliability or speed of delivery, or versatility of batch size. The matter is of some importance. If the item is the same in all respects and a higher price is obtained by, say, exporting to country A rather than B, then a switch of some output from A to B is essentially a price and revenue increase from the same output. The measurement approach would be to add values of exports of A and B together and divide by the total quantity. However, if they are considered to be different commodities, then the aggregate price change is a weighted average of the price changes of the two countries.

8.32 There is a very strong likelihood some price-determining characteristics will be unmeasured in any quality-adjustment situation. Compilers cannot produce timely statistics if they are perpetually seeking more data on characteristics to produce a still better quality adjustment. How many characteristics data are enough? Characteristics data are sufficient when commodities are described completely enough. Commodities are described completely enough when there is low

[3]International sales and purchases of electricity, gas, and water, although not always recorded by the customs authorities of some countries, constitute international transactions. United Nations (2004, paragraph 255, pp. 44–45) advises that enterprise surveys be used to collect such data.

variability of prices over transactions with that description in any given month. If we use characteristics from a structured commodity description to estimate a hedonic regression model, the model will fit well only if the structured descriptions are reasonably complete. The first criterion for sufficiency of structured characteristics data, then, is a good fit to a hedonic model. If there is a good fit using a set of objective characteristics, there may be still other characteristics, such as style and reliability, not yet included in the structured description and thus unmeasured, but they cannot contribute much more to the fit of the model. A second, qualitative criterion is that the included characteristics be meaningful to the participants in the market for the commodity.

B.2 Conceptual issues

8.33 An export price index (XPI) from a resident producing unit's perspective is an index designed to measure the average change in the price of goods and services exported. Section B.2.1 focuses on a conceptual framework for the quality-adjustment problem for XPIs and the restrictive assumptions that have to be maintained to use the *resource cost approach*. The principles relating to an import price index (MPI) follow in Section B.2.2. It outlines the quality-adjustment framework for MPIs and the restrictive assumptions that have to be maintained to use the *user value* approach to quality adjustment. The discussion continues in Section B.2.3 with a brief introduction to two problems associated with resource cost and user value approaches. The first, in Section B.2.4, is when technology substantially changes and fixed-input *output export indices* make little sense for valuing higher-quality commodities produced at much lower unit cost. The second is the reconciliation problem in national accounts at constant prices, a problem that leads this *Manual* to recommend a unified valuation system in Section B.2.5.

B.2.1 Fixed-input output export price index

8.34 In this *Manual*, the principal conceptual basis for the XPI is the *fixed-input output export price index* (FIOXPI). It is an XPI that is based on the resident producers' perspective—that is, an *output* one as opposed to a nonresident-producing establishment that treats exports as uses or inputs. The FIOXPI thus aims to measure an output export price index constructed on the assumption that inputs and technology are fixed.[4] Chapter 18 defines the economic theoretical XPI as a ratio of revenue functions. The revenue function of an establishment expresses the value of its output for export as a function of the prices it receives and the quantities of inputs required to produce the output. It recognizes that only a finite number of varieties or commodities are producible for export at any given time, but also grants that, for given inputs and technology, there may be a continuum of designs from which producers select this finite number of commodities. Hence, in response to changes in preferences or the technologies of producers using a given establishment's output, there may be different sets of commodities produced from period to period from a given set of inputs and technology.

8.35 Compilers and price index theorists are used to thinking in the narrower framework comparing the prices of exactly the same things from period to period. For example, they would (want to) measure the price change of shirts exported on the assumption that the cutting, sewing, folding, packaging, and so forth were all undertaken in the same way from the same labor, capital, and material inputs in the two periods being compared. If the revenue from exports increased by 5 percent, given that everything else remained the same, then this is the change in the output export price. If everything else does not change, then a measure of a "pure" price change results.

8.36 Even if technology and inputs remain the same, however, the way things are produced and sold may change. For example, the shirtmaker may start improving the quality of its shirts by using extra cloth and more stitching using the same machinery. The "price basis" or commodity description underlying this comparison has changed within a given technological framework. A direct comparison of shirt prices in successive months includes in this case not only the effects on revenue from price changes from exports, but also changes in commodity characteristics and thus quality. To include the increase in revenue resulting from improved quality would be to misrepresent price change—to bias the index upward. Prices would not, in fact, be rising as fast as indicated by such an unadjusted index.

8.37 A pure price relative for a commodity fixes the commodity description or price basis by definition. For the price basis not to change, the commodity's observable characteristics and the manner in which the commodity is sold for export must remain fixed. The FIOXPI for an elementary aggregate may evolve because producers adjust quantities and revenue shares in response to changes in the relative prices of commodities. Further, "new"

[4]See Chapter 18 for more on this conceptual framework.

commodities that are feasible with the same inputs and technology, but were not previously produced, may appear and supplant existing commodities.

8.38 A variant of the FIOXPI framework underlies the *resource cost* approach to explicit quality adjustment for export prices. In the resource cost approach, when quality changes, the compiler asks the establishment representative how much it cost to produce for export the new commodity and how much it would have cost to produce the old commodity for export in the current period. She then divides the price relative between the new and the old commodities by their relative cost. Resource cost adjustment relies on holding input prices relative to total cost fixed, rather than holding input quantities fixed, when comparing the prices for a given set of commodities between two periods. This variant of the FIOXPI is based on the concept of a ratio of *indirect* revenue functions, so named because they maximize revenue subject to a cost function constraint rather than a production function constraint.[5] Although the direct revenue function of the FIOXPI increases with inputs, the indirect revenue function increases with total cost. If commodity characteristics change along with prices, the resource cost adjustment for the change in quality is the factor that, when used as a multiplier for observed total cost, would produce the same revenue (given the initial set of commodity characteristics) as the revenue realized through producing the new commodities in the current period. Thus, if the new good is of a higher quality, we would expect this cost multiplier to be positive, and the cost of producing the old commodity in the current period to be less than the cost of producing the new commodity. The cost relative between the two commodities, thus, is greater than one, and, when divided into the price relative between them, lowers the estimate of price change by the percentage value of the quality increase.

B.2.2 Utility maximization and cost minimization—fixed-output input import price index and other indices

8.39 This *Manual* has two principal conceptual bases for import price indices. The first is from the perspective of the resident householder directly importing goods and services for final demand. Approaching the consumer price index from the standpoint of economic theory has led to the development of the concept of a cost of living index (COLI). The theory of the COLI was first developed by Konüs (1924). It rests on the assumption of optimizing behavior on the part of a rational consumer. The COLI for such a consumer has been defined succinctly as the ratio of the minimum expenditures needed to attain the given level of utility, or welfare, under two different price regimes. Details of the theory are provided in Chapters 17 and 21 of the *CPI Manual* and will not be duplicated here. Given that the theoretical COLI measures price changes in terms of expenditure changes necessary to maintain a constant level of utility, it follows that relating, at a conceptual level, issues of quality to the utility derived from the quality change is theoretically appropriate for price measurement.

8.40 The second approach to MPIs is taken from the perspective of the resident producer and is the *fixed-output input import price index* (FOIMPI). It is an MPI that is based on the resident producer's perspective of purchasing imports for intermediate consumption—that is, an *input* one—as opposed to a nonresident producing establishment that treats imports as outputs.[6] It is the relative change in the cost of imports—the market value of inputs—required to produce a fixed level of output, when input prices change between the current period and a base period. Assuming producers minimize the cost of producing output, the input import price index thus is a ratio of cost functions that relate establishment total production cost to its outputs and the input import prices it pays.[7] The prices of imported inputs should include all of the amounts purchasers pay per unit of the commodities they use, including transportation, insurance, wholesale/retail margins, and indirect taxes. Chapter 15 calls these purchasers' prices, following the Commission of the European Communities and others (2008), *System of National Accounts 2008 (2008 SNA)*.

8.41 A variant of the FOIMPI framework underlies the *user value* approach to explicit quality adjustment for input prices. User value adjustment relies conceptually on a variant of the FOIMPI. It holds output prices fixed relative to total revenue, rather than holding output quantities fixed, when comparing the prices for a given set of imported input commodities between two periods. The variant is based on the concept of a ratio of *indirect* cost functions, so named because they minimize cost subject to a revenue function constraint rather

[5]The cost function is itself a derivative of the production function. The indirect revenue function reflects the production function, and thus technology, *indirectly* through the cost function.

[6]It should be apparent that a nonresident's perspective can be taken by simply treating the XPI from an input perspective: as a fixed-output input price index; and an MPI from an output perspective: as a fixed-input output price index.

[7]See Chapter 18, Section C, for more on this conceptual framework.

than a production function constraint.[8] Whereas the direct cost function of the FIOMPI increases with outputs, the indirect cost function increases with total revenue. If commodity characteristics change along with prices, the user value adjustment for the change in quality is the factor that, when multiplied by observed total revenue, would produce the same cost in the current period (given the initial set of commodity characteristics) as the cost realized using the new commodities as imported inputs. Thus, if the new imported input is of a higher quality, we would expect this revenue multiplier to be positive, and the revenue possible from using the old commodity in the current period to be less than the revenue realized from using the new commodity. The revenue relative between the two commodities thus is greater than one, and, when divided into the input price relative between the two input commodities, lowers the estimate of their price change by the percentage value of the quality increase.

8.42 Triplett (1990, pp. 222–23) summarized the history of thought on the resource cost and user value methods of quality adjustment:

> Fisher and Shell (1972) were the first to show that different index number measurements (they considered output price indexes and consumer price indexes) imply alternative treatments of quality change, and that the theoretically appropriate treatments of quality change for these two indexes correspond respectively, to "resource-cost" and "user-value" measures. Triplett (1983) derives this same result for cases where "quality change" is identified with characteristics of goods—and therefore with empirical hedonic methods [discussed later]; the conclusions are that the resource-cost of a characteristic is the appropriate quality adjustment for the output price index, and its user-value is the quality adjustment for the COL index or input index.
>
> Intuitively, these conclusions are appealing. The output index is defined on a fixed value of a transformation function. The position of a transformation function, technology constant, depends on resources employed in production; accordingly, "constant quality" for this index implies holding resources constant, or a resource-cost criterion.
>
> On the other hand, the COL index is defined on a fixed indifference curve, and the analogous input-cost index is defined on a fixed (user) production isoquant. For these two "input" price indexes, "constant-quality" implies holding utility or output constant, or a user-value criterion. . . .

[8]The revenue function is itself a derivative of the production function. The indirect cost function reflects the production function, and thus technology, *indirectly* through the revenue function.

B.2.3 A problem with these concepts and their use

8.43 Chapter 18 recognizes the FIOXPI as the appropriate basis for an XPI and the FOIMPI for the MPI if a resident's perspective is utilized.

8.44 As shown in Section B.2.1, the resource cost method has a microeconomic rationale within the indirect revenue framework for quality-adjusted output export price measurement. However, the correctness of dividing a price relative by a resource cost ratio for a given commodity requires two potentially restrictive assumptions. The production process for the export commodity whose price is adjusted must be *separable* from the process for the rest of the outputs of an establishment, and the *returns to scale* of that process must be constant and equal to one.[9] These assumptions would be unlikely to be confirmed were the data available to empirically test them (and these data usually are not available to compilers).

8.45 As shown in Section B.2.2, the user value method also has a microeconomic rationale within the indirect cost framework for quality-adjusted input import price measurement. However, the correctness of dividing a price relative by a user value ratio for a given commodity requires two potentially restrictive assumptions. The input requirements for the item whose price is adjusted must be separable from the requirements for the rest of the inputs an establishment uses, and the returns to scale of that process must be constant and equal to one. These assumptions would be unlikely to be confirmed were the data available to empirically test them (and these data usually are not available to compilers).

B.2.4 When technology changes

8.46 The problems with traditional resource cost and user value approaches to explicit quality adjustment compound in the presence of technical (and taste) change. Throughout the earlier sections, this chapter has noted the similarity of effects on XMPIs between

[9]See Chapter 21, Section B.6, on the "resource cost" decomposition of the relative change in revenue when both prices and commodity characteristics change. Separability implies, for practical purposes, that any particular commodity whose quality has changed must have its own production process unaffected by the production of other, more or less similar commodity varieties. Constant returns to scale reinforce this restriction by implying that the output of a commodity may be increased by any given proportion by increasing inputs by the same proportion, without regard to the production of other distinct, more or less similar commodity varieties.

relative price change, preference change, change in the using technology, and change in the supplying technology. Broadly, all affect the assortment of commodities available at any given time and the relative importance of the commodities in the subset of that assortment persisting from period to period. As noted in Chapter 18, however, changes in weights arising from suppliers' and users' responses to relative price changes given fixed technology and preferences have predictable outcomes. They are the foundation for well-known theorems on the downward (upward) bias of Laspeyres price indices and the upward (downward) bias of Paasche price indices for output (input) price indices. Normally, considering substitution effects alone leads to the standard expectation that the Laspeyres output (input) price index will lie below (above) the Paasche output (input) price index.

8.47 The export and import value shares compilers observe and use as weights reflect changes in relative prices, technology, and tastes simultaneously. Changes in the relative importance of commodities, including their emergence and disappearance, can be unpredictable. Technology change can augment the substitution effects from relative price change, or it can more than offset substitution effects. As a result, the Laspeyres output price index may lie above the Paasche output price index, and the Laspeyres input price index may lie below the Paasche input price index in any given period-to-period comparison.

8.48 Regarding the resource cost method, an establishment representative can find it problematic to assess the cost of changes in the price basis of an output good or service arising partly or wholly from a change in production technology. Much of the cost of the improved reliability, efficiency, design, flexibility, durability, and other characteristics are difficult to measure. Moreover, the changes in technology that generate them include changes in plant and machinery, quality monitoring, inventory control, labor requirements, the manner in which work is organized, types of materials used, packaging, and selling techniques, all of which are difficult to measure in terms of the simple costing referred to above. The new technologies in high-technology commodities require new methods of production. These production technologies may change, possibly more than once during a year. Asking the cost of a previous variety produced under the current production process or the cost of the current variety under the previous generation process may be conceptually appropriate, but practically impossible. Yet not answering the cost question under the condition that technology is fixed in the current or previous generation can produce wildly inaccurate results. Consider the market for personal computers, where price declines have been accompanied by rapid quality improvements.

8.49 Holdway (1999) illustrated the problem of using a fixed-input output price index (FIOPI) for computer microprocessing units (cpus) such as an Intel Pentium III. He considered changes in the speed of new generations of microprocessors and used the example of the transition from 66 megahertz (MHz), costing \$230 when it was discontinued, to be replaced by the 90 MHz model valued at \$247 in the same month. The additional cost of the 24 MHz at that month's technology's resource costs has to be estimated. Say the cost of a single unit MHz was estimated to be \$2.0833, which when multiplied by 24 equals \$50. So what is the pure price difference between these two cpus? To make the new 90 Mhz cpu equivalent to the old 66 MHz one, the \$50 has to be subtracted from its price and compared with the price of the old one, that is, $[(247 - 50)/230] - 1 = -0.143$, a 14.3 percent fall. This is instead of a nominal price increase of $[(247/230) - 1] = 0.074$ or 7.4 percent.

8.50 Suppose, however, the establishment reports the unit cost of the 66 MHz unit at the technology prevailing when the older, slower unit was designed rather than the unit cost of a 66 MHz unit from the newer technology underlying the 90 MHz chip. In this case it is very easy to misapply the resource cost method by not comparing costs within a given generation of production technology. The new 90 Mhz cpus were built using a better technology. They used 0.50 as opposed to 0.80 micron technology, allowing more features to be packed into a smaller section of a silicon wafer, which improved performance. Also, the technology used to produce them, including an amortization factor for plant and capital equipment, lowered unit costs (see Holdway, 1999, for details). Say an estimate was requested as to how much extra it would cost to produce a 90 Mhz cpu than a 66 Mhz one, while maintaining that the cost assessment should assume the 66 Mhz wafer technology. Suppose unit costs for the higher-performance cpu were \$100 more because the old technology was less efficient than the new technology, a common occurrence in high-technology industries. Application of the resource cost method now provides an estimate of $(247)/(230 + 100) - 1 = -.252$, a 25.2 percent decrease.

8.51 In the latter cases the method breaks down. The unadjusted price increase was 7.4 percent. With a resource cost adjustment using estimates based on the new technology there was a decline of 14.3 percent. Adjusting the prices based on estimates using the old technology to produce the new, higher-performing chip results in a decrease of 25.2 percent. In both cases the cost declines represent different levels of technology, and the resource cost approach can give widely different answers. In the computer and electronics and other industries where unit prices are falling and technology rapidly changing, resource cost quality-adjustment procedures can be very misleading as major technology shifts occur.

8.52 PPIs, like XMPIs, cannot, of course, hold the price basis constant over very long periods. For example, in the 45 years since the introduction of the commercial computer the price of computing power has been estimated to be less than one-half of one-tenth of 1 percent (0.0005) of what it was at its introduction. It has decreased by more than two thousand fold (Triplett, 1999). Yet if these price changes reflected overall changes in producer prices, absurd estimates of output growth at constant prices would result. The tastes and expectations of consumers along with the technology of the producers change over time, and these changes will be shown in Chapter 22, Section H, to affect the implicit prices attributed to the quality characteristics of what is bought and sold.

8.53 Because of the effects of relative price change, technology change, and taste change, we again would prefer to use the (observed) overlap price and hedonic methods, if it is feasible to use them, rather than the resource cost and user value approaches. Further, rapid technical and taste changes must also be met by more frequent sample updates to avoid rapid loss of sample relevance.

B.2.5 Consistency between supply and use price statistics: Assessing commodity quality at supply and use values

8.54 Quality assessments must be consistent throughout the supply and use accounts for goods and services. As discussed in Chapter 15, the XMPIs cover aggregates in the supply and use tables of the *2008 SNA,* balancing the sources of goods and services supply in the current period with the uses of those goods and services. The sources of supply are domestic production and imports, plus adjustments for transport and distribution services to get goods to their users and taxes and subsidies on commodities. The uses of goods and services are intermediate consumption, final consumption, capital formation, and exports. Each good or service commodity thus has its own row in the matrix of supply and use, whose columns are the aforementioned components of supply and use. Even at this highest level of detail, the supply of every distinct good or service, adjusted for transport and distribution margins and taxes, must balance its uses. This will be identically true in both value and volume terms.

8.55 Because every transaction cannot be tracked, however, supply and use tables cannot be produced at the level of elementary items. It is feasible to track supply and use only at the level of elementary aggregates, basic headings, or even higher-level aggregates of goods and services. Thus, each row of such a supply and use table necessarily contains some quality heterogeneity and we can speak of it only in average terms. Changes in the total supply and total uses of these detailed aggregates of goods and services thus comprise four parts. There are average quality changes, changes in basic prices, changes in taxes and subsidies on commodities, and average quantity changes of the elementary commodities comprising the aggregate. Volume change for an aggregate is an amalgam of quality and quantity changes. Clearly, adjusting price change to eliminate the effects of changes in quality is important here, lest volume be understated or overstated by the amount of quality change erroneously ascribed to price change. The context also highlights the need to have a single valuation of quality change, not one from the supply side—for example, imports of a good—and one from the uses side—intermediate consumption of that same good. Thus, as outlined in the *2008 SNA*, Chapter 15, similar estimates should be used for supply and use quality adjustments if the supply and use accounts are to balance in both value and volume terms.

C. An Introduction to Methods of Quality Adjustment When Matched Items Are Unavailable

C.1 Introduction

8.56 It may be apparent from the preceding text that quality adjustments to prices are not going to be a simple issue or involve routine mechanical methods whereby a single methodology will be applied to prices in all commodity groups to yield adjustments. A number of alternative approaches will be suggested, and some will be more appropriate than others for specific items regardless of their commodity group. An understanding of the technological features of the exporting/importing industry, the commodity market, and alternative data sources will be required for the successful implementation of a quality-adjustment program. Specific attention must be devoted to commodity areas with relatively high weights and where large proportions of commodities are turned over. Some of the methods are not straightforward and require some expertise, although methods learned and used on some commodities may be applicable elsewhere. The issue of quality adjustment is met by developing a gradual approach on a commodity-by-commodity basis. It is emphasized that such concerns should not be used as reasons to obviate the estimation of quality-adjusted prices. The practice of statistical agencies in dealing with missing commodities, even if it is to ignore them, implicitly involves a quality adjustment, and the form of the implicit one undertaken may not be the most appropriate one and may even be misleading. The extent of quality changes and the pace of technological change require that appropriate methods be used.

8.57 To measure aggregate price changes, a representative sample of commodities is selected along with a sample of exporting/importing firms and a host of details that define each *price*, including details on the conditions of the sale where relevant. This is to establish an insight into the price basis of the commodity. This is then followed by a periodic survey for which the firms report prices (reprice the commodity) each month for these selected commodities. They do so to the same specifications, that is, on the same price basis. The detailed specifications are included on the repricing form each month as a prompt to ensure that the price basis has remained the same. Respondents must be aware of the need to report the details of any change in the price basis; confusion may lead to biased results. It must be borne in mind that firms have no incentive to report such changes because this will invariably involve additional work in costing the change. Attention should also be devoted to ensuring that the description of the price basis contains all pertinent, price-determining elements. If an element is excluded, any change is much less likely to be reported. In both of these cases, the quality change would be invisible to the price-measurement process.

C.2 Methods for making quality adjustments

8.58 When a commodity is missing in a month for reasons other than being off-season or off-cycle, the replacement may be of a different quality—the price basis may have changed, and like may be no longer compared with like. A number of approaches exist for dealing with such situations. Though the terms differ among authors and statistical agencies, they include the following:

- *Imputation*—Used when no information is available to allow reasonable estimates to be made of the effect on price of a quality change. The price change of all commodities—or of more or less similar commodities—is assumed to be the same as that for the missing commodity.
- *Overlap*—Used when no information is available to allow reasonable estimates to be made of the effect on price of a quality change, but a replacement commodity exists in the same period as the old commodity. The price difference between the old commodity and its replacement in the same overlap period is then used as a measure of the quality difference.
- *Direct comparison*—Used if another commodity is directly comparable, that is, so similar it has more or less the same quality characteristics as the missing one. Its price then replaces the unavailable price. Any difference in price level between the new and old is assumed to be because of price changes and not quality differences.
- *Explicit quality adjustment*—Used when there is a substantial difference in the quality of the old and replacement commodities. Estimates of the effect of quality differences on prices are made to enable quality-adjusted price comparisons.

8.59 Before these methods are outlined and evaluated, the extent of the problem should be addressed. This situation arises when the commodity is unavailable. It is not just a problem when *comparable* commodities are unavailable, for the judgment as to what is and what is not comparable itself requires an estimate of quality differences. Part of the purpose of a statistical meta-information system for statistical offices (outlined in Chapter 9) is to identify and monitor the sectors that are prone to such replacements and determine whether the replacements used really are comparable.

8.60 Quality-adjustment methods for prices are generally classified into the implicit or imputed (indirect) methods explained in Section D (the differences in terminology are notorious in this area) and explicit (direct) methods explained in Section E. Both decompose the price change between the old commodity and its replacement into quality and pure price changes. However, in the latter, an explicit estimate is made of the quality difference, usually on the basis of external information. The pure price effect is identified as a remainder. For implicit adjustments, a measurement technique is used to compare the old commodity with the replacement, so that the extent of the quality and pure price change is implicitly determined by the assumptions of the method. The accuracy of the method relies on the veracity of the assumptions, not the quality of the explicit estimate. In Sections D and E, the following methods are considered in detail:

Implicit methods:

- Overlap,
- Overall mean/targeted mean imputation,
- Class mean imputation,
- Comparable replacement,
- Linked to show no price change, and
- Carryforward.

Explicit methods:

- Expert judgment,
- Quantity adjustment,
- Differences in production/option costs, and
- Hedonic approach.

C.3 Some points

C.3.1 Additive versus multiplicative

8.61 The quality adjustments to prices may be undertaken by either adding a fixed amount or multiplying by a ratio. For example, where *m* is the old commodity and *n* its replacement for a comparison over periods t, $t + 1$, $t + 2$, the use of the overlap method in period $t + 1$ required the ratio p_n^{t+1}/p_m^{t+1} to be used as a measure of the relative quality difference between the old item and its replacement. This ratio could then be *multiplied* by the price of the old item in period t, p_m^t to obtain the quality-adjusted prices p_m^{*t} shown in Table 8.1. Such multiplicative formulations are generally advised because the adjustment is invariant to the absolute value of the price. It would be otherwise possible for the absolute value of the change in specifications to exceed the value of the commodity in some earlier or—with technological advances—later period. Yet for some commodities, the worth of the constituent parts is not in proportion to their price. Instead, they have their own intrinsic, absolute, additive worth, which remains constant over time. Producers selling over the Internet may, for example, include postage, which in some instances may remain the same irrespective of what is happening to price. If postage is subsequently excluded from the price, the fall in quality should be valued as a fixed sum.

Table 8.1. Estimating a Quality-Adjusted Price

	t	$t+1$	$t+2$
Old item *m*	p_m^t	p_m^{t+1}	
Replacement *n*	p_m^{*t}	p_n^{t+1}	p_n^{t+2}

C.3.2 Base- versus current-period adjustment

8.62 Two variants of the approaches to quality adjustment outlined in Section C.2 are to either make the adjustment to the price in the base period or make the adjustment to the price in the current period. For example, in the overlap method described above, the implicit quality-adjustment coefficient was used to adjust p_m^t. An alternative procedure would have been to multiply the ratio p_m^{t+1}/p_n^{t+1} by the prices of the replacement commodity p_n^{t+2} to obtain the quality-adjusted prices p_n^{*t+2}, and so forth. The first approach is easier because once the base-period price has been adjusted, no subsequent adjustments are required. Each new replacement price can be compared with that of the adjusted base period. For multiplicative adjustments, the end result is the same whichever approach is used. For additive adjustments, the results differ. It is more appropriate to make the adjustment to prices near the overlap period.

C.3.3 Long-run versus short-run comparisons

8.63 Much of the analysis of quality adjustments in this *Manual* has been undertaken by comparing prices between two periods (e.g., periods 0 and 1). For long-run comparisons, suppose the base period is taken as period t and the index is compiled by comparing prices in period t first with $t + 1$, then with $t + 2$, then with $t + 3$, and so forth. The short-run framework allows long-run comparisons—say, between periods t and $t + 3$—to be built as a sequence of links joined by successive multiplication—say, period t with $t + 2$ and period $t + 2$ with $t + 3$. This can also be done by chaining period t with $t + 1$, $t + 1$ with $t + 2$, and $t + 2$ with $t + 3$. In Section H, the advantages of the short-run framework for imputations and to facilitate the introduction of new commodities are outlined. In Section G.3, chained indices and a hedonic approach are considered for industries experiencing a rapid turnover in commodities. These quality-adjustment methods are now examined in turn, and in Section F, the choice of method is discussed.

C.3.4 Statistical metadata

8.64 In Sections D and E, implicit and explicit methods of quality adjustments to prices are discussed. In Section F, the choice between these methods is examined. Any consideration of the veracity of these methods, resource implications, and the choice between them needs to be informed by appropriate information on an commodity-by-commodity basis. Section C of Chapter 9 considers information requirements for a strategy for such quality adjustment that makes use of a statistical metadata system.

D. Implicit Methods

D.1 Overlap method

8.65 Consider an example where the items are sampled in January and prices are compared over the remaining months of the year. Matched comparisons are undertaken between the January prices and their counterparts in successive months. Five commodities are assumed to be sold in January with prices p_1^1, p_2^1, p_5^1, p_6^1, and p_8^1 (Table 8.2, panel a). Two types of similar commodities are produced in the commodity group concerned, A and B. An index at the elementary level is required for the overall price change of these two commodity types. At this level of aggregation, the weights can be ignored. A price index for February compared with January = 100.0 is straightforward in that prices

Table 8.2. Example of Overlap Method of Quality Adjustment

(a) General Illustration

Commodity Type	Item	January	February	March	April
A	1	p_1^1	p_1^2	p_1^3	p_1^4
	2	p_2^1	p_2^2		
	3			p_3^3	p_3^4
	4		p_4^2	p_4^3	p_4^4
B	5	p_5^1	p_5^2	p_5^3	p_5^4
	6	p_6^1	p_6^2		
	7			p_7^3	p_7^4
	8	p_8^1	p_8^2	p_8^3	p_8^4

(b) Numerical Illustration

Commodity Type	Item	January	February	March
A	1	4.00	5.00	6.00
	2	5.00	6.00	
	2. Overlap			**6.90**
	2. Imputation			**6.56**
	2. Targeted imputation			**7.20**
	2. Comparable replacement			**6.50**
	3			6.50
	4		7.50	8.00
B	5	10.00	11.00	12.00
	6	12.00	12.00	
	6. Imputation			**13.13**
	6. Targeted imputation			**12.53**
	7			14.00
	8	10.00	10.00	10.00

Note: Figures in bold are imputations/estimates that may be made by the statistical authorities, as explained in the text.

of commodities 1, 2, 5, 6, and 8 are used and compared only by way of the geometric mean of price ratios, known as the Jevons index (which is equivalent to the ratio of the geometric mean in February over the geometric mean in January—see Chapter 21). In March, the prices for commodities 2 and 6—one of type A and one of type B—are missing.

8.66 In Table 8.2, the lower panel (b) is a numerical counterpart of the upper panel (a), further illustrating the calculations. The overlap method requires prices of the old and replacement commodities to be available in the same period. In Table 8.2(a), commodity 2 has no price quote for March. Its new replacement is, for example, commodity 4. The overlap method simply measures the ratio of the prices of the old and replacement commodity in an overlap period. In this example, the period is February, and the old and replacement commodities are commodities 2 and 4, respectively. The ratio of their prices is taken to be an indicator of their quality differences. The two approaches outlined in Section C.3.2 are apparent: either to insert a quality-adjusted price in January for commodity 4 and continue to use the replacement commodity 4 series, or to continue the commodity 2 series by patching in quality-adjusted commodity 4 prices. Both yield the same answer. Consider the former. For a Jevons geometric mean from January to March *for commodity type A only,* assuming equal weights of unity,

$$P_J(p^1, p^3) = \left[p_1^3/p_1^1 \times p_4^3/\left(\left(p_4^2/p_2^2\right) \times p_2^1\right)\right]^{1/2}$$
$$= \left[6/4 \times 8/((7.5/6) \times 5)\right]^{1/2}$$
$$= 1.386. \tag{8.1}$$

8.67 Note that the comparisons are long-run ones, that is, they are between January and the month in question, March. The short-run (modified) Laspeyres framework provides a basis for short-run changes based on data in each current month and the immediately preceding one. For example, January to October prices for each commodity are compared in terms of January to September and September to October. In Table 8.2(a) and (b), the comparison for commodity type A would first be undertaken between January and February using commodity 1, and then multiplied again by the price change between March and February for commodity 1. Then turn to commodity 2. The price change would first be undertaken between January and February. The result would be multiplied by the comparison between February and March using item 4, commodity 2's replacement. Still, this implicitly uses the differences in prices in the overlap in February between items 2 and 4 as a measure of this quality difference. It yields the same result as before:

$$\left[\frac{5}{4} \times \frac{6}{5}\right]^{\frac{1}{2}} \times \left[\frac{6}{5} \times \frac{8}{7.5}\right]^{\frac{1}{2}} = 1.386.$$

8.68 The advantage of recording price changes for, say, January to October in terms of January to September and September to October is that it allows the compiler to compare immediate month-on-month price changes for data editing purposes. Moreover, it has quite specific advantages for the use of imputations as discussed in Sections D.2 and D.3 for which different results arise for the long- and short-run methods. It further facilitates the introduction of replacement commodities. A fuller discussion of the long-run and short-run frameworks is undertaken in Section H.

8.69 The method is only as good as the validity of its underlying assumptions. Consider $i = 1 \ldots m$ commodities, where p_m^t is the price of commodity m in period t, p_n^{t+1} is the price of a replacement commodity n in period $t + 1$, and there are overlap prices for both commodities in period t. Now item n replaces m but is of a different quality. So let $A(z)$ be the quality adjustment to p_n^{t+1}, which equates its quality to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A(z^{t+1})\, p_n^{t+1}$. Put simply, the index for the commodity in question over the period $t - 1$ to $t + 1$ is

$$I^{t-1,t+1} = \left(p_m^t/p_m^{t-1}\right) \times \left(p_n^{t+1}/p_n^t\right)$$
$$= \frac{p_n^{t+1}}{p_m^{t-1}} \times \frac{p_m^t}{p_n^t}. \tag{8.2}$$

8.70 The quality adjustment to prices in period $t + 1$ is defined as before, $p_m^{*t+1} = A(z^{t+1})\, p_n^{t+1}$, which is the adjustment to p_n in period $t + 1$, which equates its value to p_m in period $t + 1$ (had it existed then). A desired measure of price changes between periods $t - 1$ and $t + 1$ is thus

$$\left(p_m^{*t+1}/p_m^{t-1}\right). \tag{8.3}$$

The overlap formulation equals this when

$$\frac{p_m^{*t+1}}{p_m^{t-1}} = A(z^{t+1})\frac{p_n^{t+1}}{p_m^{t-1}} = \frac{p_n^{t+1}}{p_n^t} \times \frac{p_m^t}{p_m^{t-1}}$$

$A(z^{t+1}) = \dfrac{p_m^t}{p_n^t}$ and similarly for future periods of the series

$$A(z^{t+i}) = \frac{p_m^t}{p_n^t} \text{ for } \frac{p_m^{*t+i}}{p_m^{t-1}} \text{ for } i = 2, \ldots T. \tag{8.4}$$

8.71 The assumption is that the quality difference in any period equates to the price difference at the *time of the splice*. The *timing* of the switch from *m* to *n* is thus crucial. Unfortunately, responding establishments usually continue to report prices on a commodity until it is no longer imported/exported so that the switch may take place at an unusual period of pricing, near the end of item *m*'s life cycle and the start of item *n*'s life cycle.

8.72 But what if the assumption does not hold? What if the relative prices in period *t*, $R^t = p_m^t / p_n^t$, do not equal *A(z)* in some future period, say $A(z^{t+i}) = \alpha_i R^t$? If $\alpha_i = \alpha$, the comparisons of prices between future successive periods—between $t + 3$ and $t + 4$—are unaffected, as would be expected, because commodity *n* is effectively being compared with itself.

$$\frac{p_m^{*t+4}}{p_m^{t-1}} \Bigg/ \frac{p_m^{*t+3}}{p_m^{t-1}} = \frac{\alpha R^t}{\alpha R^t} \frac{p_n^{t+4}}{p_n^{t+3}} = \frac{p_n^{t+4}}{p_n^{t+3}}. \qquad (8.5)$$

8.73 However, if differences in the relative prices of the old and replacement commodities vary over time, then

$$\frac{p_m^{*t+4}}{p_m^{t-1}} \Bigg/ \frac{p_m^{*t+3}}{p_m^{t-1}} = \frac{\alpha_4 p_n^{t+4}}{\alpha_3 p_n^{t+3}}. \qquad (8.6)$$

Note that the quality difference here is not related to the technical specifications or resource costs but to the relative price that purchasers pay.

8.74 Relative prices may also reflect unusual pricing policies aimed at minority segments of the market. In the example of pharmaceutical drugs (Berndt, Ling, and Kyle, 2003), the overlapping prices of a generic and a name brand commodity were argued to be reflective of the needs of two different market segments. The overlap method can be used with a judicious choice of the overlap period. It should be a period before the use of the replacement, because in such periods the pricing may reflect a strategy to dump the old model to make way for the new one.

8.75 The overlap method is implicitly employed when samples of commodities are rotated, meaning that the old sample of commodities is used to compute the category index price change between periods $t - 1$ and *t*, and the new sample is used between *t* and $t + 1$. The splicing together of these index movements is justified by the assumption that—on a group-to-group rather than commodity-to-item level—differences in price levels at a common point in time accurately reflect differences in qualities.

8.76 The overlap method has at its roots a basis in the law of one price. The law states that when a price difference is observed, it must be the result of some difference in physical quality or some such factor for which consumers are willing to pay a premium, such as the timing of the sale, location, convenience, or conditions. Economic theory would dictate that such price difference would not persist given markets made up of rational producers and consumers. However, empirically, different prices are observed in markets for identical products sold with the same conditions of sale. There are a number of possible reasons for this. First, purchasers may not be well informed about the range of prices on offer and in general will not search out for the lowest prices because costs are incurred in the process. Second, prices do not change smoothly, there being costs associated with making price changes. When they do occur they may be lumpy to catch up on previous changes and not all suppliers may change their prices at the same time. Third, purchasers may not be free to choose the price at which they purchase because the seller may be in a position to charge different prices to different categories of purchasers for identical goods and services sold under exactly the same circumstances—in other words, to practice price discrimination. Finally, buyers may be unable to buy as much as they would like at a lower price because there is insufficient supply available at that price. This situation typically occurs when there are two parallel markets. There may be a primary, or official, market in which the quantities sold, and the prices at which they are sold, are subject to government or official control, while there may be a secondary market—a free market or unofficial market—whose existence may or may not be recognized officially.

8.77 There is extensive literature in economics dealing with theory and evidence of price dispersion and its persistence, even when quality differences have been accounted for. Similar issues arise for goods and services exported and imported with the added complication that volatile exchange rate changes may increase price dispersion, especially if there is a catch-up on the changes not passed through immediately.

D.2 Overall mean/targeted mean imputation

8.78 This method uses the price changes of other commodities as estimates of the price changes of the missing commodities. Consider a Jevons elementary price index, that is, a geometric mean of price relatives (Chapter 21). The prices of the missing items in the current period, say, $t + 1$, are imputed by multiplying their prices in the immediately preceding period t by the geometric mean of the price relatives of the remaining matched items between these two periods. The comparison is then linked by multiplication to the price changes for previous periods. It is the computationally most straightforward of methods, because the estimate can be undertaken by simply dropping the items that are missing from both periods from the calculation. In practice, the series is continued by including in the database the imputed prices. It is based on the assumption of similar price movements for the omitted commodities and the class of commodities used for the imputation. A targeted form of the method would use similar price movements of a cell or elementary aggregate of similar commodities, or be based on price changes at a higher level of aggregation if either the lower level had an insufficient sample size or price changes at the higher level were judged to be more representative of the price changes of the missing commodity.

8.79 In the example in Table 8.2(b), the January to February comparison for both commodity types is based on commodities 1, 2, 5, 6, and 8. For March compared with January—weights all equal to unity—the commodity 2 and commodity 6 prices are imputed using the short-run price change for February (p^2) compared with March (p^3) based on commodities 1, 5, and 8. Because different formulas are used for elementary aggregation, the calculation for the three main formulas are illustrated here (see Chapter 21 for choice of formulas). The geometric mean of the price ratios—the Jevons index—is

$$P_J(p^2, p^3) = \prod_{i=1}^{N}\left[p_i^3/p_i^2\right]^{1/N}$$
$$= \left[\left(p_1^3/p_1^2\right) \times \left(p_5^3/p_5^2\right) \times \left(p_8^3/p_8^2\right)\right]^{1/3}$$
$$= ((6/5) \times (12/11) \times (10/10))^{1/3}$$
$$= 1.0939, \qquad (8.7)$$

or a 9.39 percent increase. The ratio of average (mean) prices—the Dutot index—is

$$P_D(p^2, p^3) = \sum_{i=1}^{N} p_i^3/N \Big/ \sum_{i=1}^{N} p_i^2/N$$
$$= \left[\left(p_1^3 + p_5^3 + p_8^3\right)/3\right] \div \left[\left(p_1^2 + p_5^2 + p_8^2\right)/3\right]$$
$$= (6 + 12 + 10)/(5 + 11 + 10) = 1.0769, \qquad (8.8)$$

or a 7.69 percent increase. The average (mean) of price ratios—the Carli index—is

$$P_C(p^2, p^3) = \sum_{i=1}^{N}\left(p_i^3/p_i^2\right)/N$$
$$= \left[\left(p_1^3/p_1^2\right) + \left(p_5^3/p_5^2\right) + \left(p_8^3/p_8^2\right)\right]/3$$
$$= \left[(6/5 + 12/11 + 10/10)\right]/3 = 1.09697, \qquad (8.9)$$

or a 9.697 percent increase.

8.80 In practice the imputed figure would be entered onto the data sheet. Table 8.2(b) has the overall mean imputation in March for commodities 2 and 6, using the Jevons index, as 1.0939 × 6 = 6.563 and 1.0939 × 12 = 13.127, respectively (bold type). It should be noted that the Dutot index is in this instance lower than the Jevons index, a result not expected from the relationships established in Chapter 21. The relationship in Chapter 21 assumed the variance in prices would increase over time, whereas in Table 8.2(b) it decreases for the three commodities. The arithmetic mean of price relatives—the Carli index—equally weights each price change, but the ratio of arithmetic means—the Dutot index—weights price changes according to the prices of the commodity in the base period relative to the sum of the base-period prices. Item 1 has a relatively low price, and thus weight, in the base period 1 of 4, but this commodity has the highest price increase, one of 6/5. Therefore, the Dutot index is lower than the Carli index. However, in Chapter 21 we establish that the Carli index has an upward bias.

8.81 As noted above, it is also possible to refine the imputation method by targeting the imputation: including the weight for the unavailable commodities in groupings likely to experience similar price changes—say, by commodity type. Any stratification system used in the selection of establishments and commodity varieties would facilitate this. For example, in Table 8.2(b) assume that the price change of the missing commodity 2 in March is more likely to follow price changes of commodity 1, and commodity 6 is more likely to experience price changes similar to commodities 5 and 8. For March compared with February, with weights all equal to unity, the geometric mean of price ratios (Jevons) is

$$P_J(p^2, p^3) = \prod_{i=1}^{N}\left[p_i^3/p_i^2\right]^{1/N}$$
$$= \left[\left(p_1^3/p_1^2\right)^2 \times \left(p_5^3/p_5^2 \times p_8^3/p_8^2\right)^{3/2}\right]^{1/5}$$
$$= \left[(6/4)^2 \times (12/11 \times 10/10)^{3/2}\right]^{1/5}$$
$$= 1.1041. \qquad (8.10)$$

Note the weights used: for commodity type A, the single price represents two prices; for commodity type B, the prices represent three, or 3/2 = 1.5 each.

8.82 The ratio of average (mean) prices—the Dutot index—is

$$P_D(p^2, p^3) = \sum_{i=1}^{N} p_i^3/N \Big/ \sum_{i=1}^{N} p_i^2/N$$
$$= \left[\left(2p_1^3 + 1.5p_5^3 + 1.5p_8^3\right)/5\right] \div \left[\left(2p_1^2 + 1.5p_5^2 + 1.5p_8^2\right)/5\right]$$
$$= [(2 \times 6 + 1.5(12 + 10))] \div [2 \times 5 + 1.5(11 + 10)]$$
$$= 1.0843. \tag{8.11}$$

8.83 The average (mean) of price ratios—the Carli index—is

$$P_C(p^2, p^3) = \sum_{i=1}^{N} \left(p_i^3/p_i^2\right)/N$$
$$= \left[\frac{2}{5}\left(p_1^3/p_1^2\right) + \frac{3}{5}\left(p_5^3/p_5^2 + p_8^3/p_8^2\right)/2\right]$$
$$= 2/5 \times 6/5 + 3/5[(12/11 + 10/10)/2]$$
$$= 1.1073. \tag{8.12}$$

8.84 Alternatively, and more simply, imputed figures could be entered in Table 8.2(b) for commodities 2 and 6 in March using just the price movements of A and B for commodities 2 and 6, respectively, and indices calculated accordingly. Using a Jevons index for commodity 2, the imputed value in March would be 6/5 × 6 = 7.2, and for commodity 6 it would be $[(12/11) \times (10/10)]^{1/2} \times 12 = 12.533$. It is thus apparent that not only does the choice of formula matter, as discussed in Chapter 21, but so too may the targeting of the imputation. In practice, the sample of commodities in a targeted subgroup may be too small. An appropriate stratum is required with a sufficiently large sample size, but there may be a trade-off between the efficiency gains from the larger sample and the representativity of price changes achieved by that sample. Stratification by commodity group and source/destination country may be preferred to stratification just by commodity group if differences in source/destination country price changes are expected, but the resulting sample size may be too small to allow this to be undertaken. In general, the stratum used for the target should be based on the analyst's knowledge of the commodity group and an understanding of similarities of price changes between and within strata. It also should be based on the reliability of the available sample to be representative of price changes.

8.85 The underlying assumptions of these methods require some analysis because—as discussed by Triplett (1999 and 2004)—they are often misunderstood. Consider $i = 1 \ldots m$ commodities where, as before, p_m^t is the price of commodity m in period t, and p_n^{t+1} is the price of a replacement commodity n in period $t + 1$. Now n replaces m but is of a different quality. As before, let $A(z)$ be the quality adjustment to p_n^{t+1}, which equates its quality services or utility to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A(z)\,p_n^{t+1}$. For the imputation method to work, the average price changes of the $i = 1 \ldots m$ commodities, including the quality-adjusted price p_m^{*t+1} given on the left-hand side of equation (8.13), must equal the average price change from just using the overall mean of the rest of the $i = 1 \ldots m - 1$ commodities on the right-hand side of equation (8.13). The discrepancy or bias from the method is the balancing term Q. It is the implicit adjustment that allows the method to work. The arithmetic formulation is given here, although a similar geometric one can be readily formulated. The equation for one unavailable commodity is given by

$$\frac{1}{m}\left[\frac{p_m^{*t+1}}{p_m^t} + \sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] = \left[\frac{1}{(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] + Q, \tag{8.13}$$

$$Q = \frac{1}{m}\frac{p_m^{*t+1}}{p_m^t} - \frac{1}{m(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}, \tag{8.14}$$

and for x unavailable commodities by

$$Q = \frac{1}{m}\sum_{i=m-x+1}^{m}\frac{p_m^{*t+1}}{p_m^t} - \frac{x}{m(m-x)}\sum_{i=1}^{m-x}\frac{p_i^{t+1}}{p_i^t}. \tag{8.15}$$

8.86 The relationships are readily visualized if r_1 is defined as the arithmetic mean of price changes of commodities that continue to be recorded and r_2 is defined as the mean of quality-adjusted unavailable commodities—that is, for the arithmetic case where

$$r_1 = \left[\sum_{i=1}^{m-x} p_i^{t+1}/p_i^t\right] \div (m - x)$$
$$r_2 = \left[\sum_{i=m-x+1}^{m} p_i^{*t+1}/p_i^t\right] \div x, \tag{8.16}$$

then the bias of the arithmetic mean of ratios from substituting equation (8.16) into equation (8.15) is

$$Q = \frac{x}{m}(r_2 - r_1), \tag{8.17}$$

which equals zero when $r_1 = r_2$. The bias depends on the ratio of unavailable values and the difference between the mean of price changes for existing commodities and the mean of quality-adjusted replacement price changes. The bias decreases as *either* (x/m) *or* the difference between r_1 and r_2 decreases. Furthermore, the method relies on a comparison between price changes for existing commodities and *quality-adjusted* price changes for the replacement/unavailable comparison. This is more likely to be justified than a comparison without the quality adjustment to prices. For example, let us say there were $m = 3$ commodities, each with a price of 100 in period t. Let the $t + 1$ prices be 120 for two commodities, but assume the third is unavailable, that is, $x = 1$, and is replaced by a commodity with a price of 140, of which 20 is the result of quality differences. Then the arithmetic bias as given in equations (8.16) and (8.17) where $x = 1$ and $m = 3$ is

$$1/3\left[(-20 + 140)/100 - \left[\left(\frac{120}{100} + \frac{120}{100}\right)/2\right]\right] = 0.$$

8.87 Had the bias depended on the *unadjusted price* of 140 compared with 100, the imputation would be prone to serious error. In this calculation, the direction of the bias is given by $(r_2 - r_1)$. and does not depend on whether quality is improving or deteriorating, that is, whether $A(z) < 1$ or $A(z) > 1$. If $A(z) < 1$, a quality improvement, it is still possible that $r_2 < r_1$ and for the bias to be negative, a point stressed by Triplett (2004).

8.88 It is noted that the analysis here is framed in terms of a short-run price change framework. This means that the short-run price changes between two consecutive periods are used for the imputation. This is different from the long-run imputation, where a base-period price is compared with prices in subsequent months and where the implicit assumptions are more restrictive.

8.89 Table 8.3 provides an illustration whereby the (mean) price change of commodities that continue to exist, r_1, is allowed to vary for values between 1.00 and 1.50: no price change and a 50 percent increase. The (mean) price change of the *quality-adjusted* new commodities compared with the commodities they are replacing is assumed to not change, that is, $r_2 = 1.00$. The bias is given for ratios of missing values of 0.01, 0.05, 0.10, 0.25, and 0.50, arithmetic means and geometric means. For example, if 50 percent of price quotes are missing and the missing quality-adjusted prices do not change, but the prices of existing commodities increase by 5 percent ($r_1 = 1.05$), then the bias for the geometric mean is represented by the proportional factor 0.9759; that is, instead of 1.05, the index should be $0.9759 \times 1.05 = 1.0247$. For an arithmetic mean, the bias is -0.025; instead of 1.05, it should be 1.025.

8.90 Equation (8.17) shows that the ratio x/m and the difference between r_1 and r_2 determine the bias. Table 8.3 shows that the bias can be quite substantial when x/m is relatively large. For example, when $x/m = 0.25$, an inflation rate of 5 percent for existing commodities translates to an index change of 3.73 percent and 3.75 percent for the geometric and arithmetic formulations, respectively, when $r_2 = 1.00$, that is, when quality-adjusted prices of unavailable commodities are constant. Instead of being 1.0373 or 1.0375, ignoring the unavailable commodities would give a result of 1.05. Even with 10 percent missing

Table 8.3. Example of the Bias from Implicit Quality Adjustment for $r_2 = 1.00$

	Geometric Mean Ratio of missing commodities, *x/m*					Arithmetic Mean Ratio of missing commodities, *x/m*				
r_1	0.01	0.05	0.10	0.25	0.50	0.01	0.05	0.10	0.25	0.50
1.00	1	1	1	1	1	0	0	0	0	0
1.01	0.999901	0.999503	0.999005	0.997516	0.995037	−0.0001	−0.0005	−0.001	−0.0025	−0.005
1.02	0.999802	0.999010	0.998022	0.995062	0.990148	−0.0002	−0.0010	−0.002	−0.0050	−0.010
1.03	0.999704	0.998523	0.997048	0.992638	0.985329	−0.0003	−0.0015	−0.003	−0.0075	−0.015
1.04	0.999608	0.998041	0.996086	0.990243	0.980581	−0.0004	−0.0020	−0.004	−0.0100	−0.020
1.05	0.999512	0.997563	0.995133	0.987877	0.975900	−0.0005	−0.0025	−0.005	−0.0125	−0.025
1.10	0.999047	0.995246	0.990514	0.976454	0.953463	−0.0010	−0.0050	−0.010	−0.0250	−0.050
1.15	0.998603	0.993036	0.986121	0.965663	0.932505	−0.0015	−0.0075	−0.015	−0.0375	−0.075
1.20	0.998178	0.990925	0.981933	0.955443	0.912871	−0.0020	−0.0100	−0.020	−0.0500	−0.100
1.30	0.997380	0.986967	0.974105	0.936514	0.877058	−0.0030	−0.0150	−0.030	−0.0750	−0.150
1.50	0.995954	0.979931	0.960265	0.903602	0.816497	−0.0050	−0.0250	−0.050	−0.1250	−0.250

($x/m = 0.1$), an inflation rate of 5 percent for existing commodities translates to 4.45 percent and 4.5 percent for the respective geometric and arithmetic formulations when $r_2 = 1.00$. However, consider a fairly low ratio of x/m, say, 0.05; then even when $r_2 = 1.00$ and $r_1 = 1.20$, Table 8.3 finds 18.9 percent and 19 percent corrected rates of inflation for the respective geometric and arithmetic formulations. In competitive markets, r_1 and r_2 are unlikely to differ by substantial amounts because r_2 is a price comparison between the new commodity and the old commodity *after adjusting for quality differences*. If r_1 and r_2 are the same, then there would be no bias from the method even if $x/m = 0.9$. There may, however, be more sampling error. It should be borne in mind that it is not appropriate to compare bias between the arithmetic and geometric means, at least in the form they take in Table 8.3. The latter would have a lower mean, rendering comparisons of bias meaningless.

8.91 An awareness of the market conditions relating to the commodities is instructive to any understanding of likely differences between r_1 and r_2. The concern here is when prices vary over the life cycle of the commodities. Thus, at the introduction of a new model, the price change may be quite different from price changes of other existing commodities. Assumptions of similar price changes, even when quality adjusted, might be inappropriate. Consider the example of personal computers: new computers enter the market at prices equal to or lower than prices of previous models but with greater speed and capability. An assumption that $r_1 = r_2$ could not be justified.

8.92 Some of this bias relates to the fact that markets are composed of different market segments and producers tailor their output to meet such needs. Indeed, the very training of marketers involves consideration of developing different market segments and ascribing to each segment appropriate *pricing, commodity quality, promotion,* and *place* (methods of distribution). This is known as the "4 Ps" of the marketing mix. In addition, marketers are taught to plan the marketing mix over the commodity's life cycle. Such planning would allow for different inputs of each of these marketing mix variables at different points in the life cycle. This includes *price skimming* during the period of introduction, whereby higher prices are charged to skim off the surplus from segment(s) of purchasers willing to pay more. The economic theory of price discrimination would also predict such behavior. Thus, the quality-adjusted price change of an old commodity compared with a new replacement commodity may be higher than price changes of other commodities in the commodity group. After the introduction of the new commodity, its prices may fall relative to others in the group. There may be no law of one price *change* for differentiated commodities within a market. Berndt, Ling, and Kyle (2003) clearly showed how, after patent expiration, the price of brand name prescription pharmaceuticals can increase with the entry of new generic pharmaceuticals at a lower price, particularly as loyal, less-price-sensitive customers maintain their allegiance to the brand name pharmaceuticals.

8.93 There is little in economic or marketing theory to support any expectation of similar (quality-adjusted) price changes for new and replacement commodities and other commodities in the commodity group. Some knowledge of the realities of the particular market under study would be helpful when considering the suitability of this approach. *Two things matter in any decision to use the imputation approach. The first is the proportion of replacements, and Table 8.3 provides guidance here. The second is the expected difference between* r_1 *and* r_2*, and it is clear from the above discussion that there are markets in which they are unlikely to be similar.* This is not to say the method should not be used. It is a simple and expedient approach. Arguably what should not happen is that the method is used as a default process without any prior evaluation of expected price changes and the timing of the switch. Furthermore, attention should be directed to its targeted use, using commodities expected to have similar price changes. However, the selection of such commodities should also be based on the need to include a sufficiently large sample so that the estimate is not subject to undue sampling error.

8.94 Some mention should be made of the way these calculations are undertaken. A pro forma setting for the calculations—say, on a spreadsheet—would have each commodity description and its prices recorded on a (usually) monthly basis. The imputed prices of the missing commodities are inserted into the spreadsheet being highlighted as imputed. The reasons for highlighting such prices are (1) because they should not be used in subsequent imputations as if they were actual prices and (2) the inclusion of imputed values may give the false impression of a larger sample size than actually exists. Care should be taken in any audit of the number of prices used in the compilation of the index to code such observations as imputed. It is stressed that this is an illustration of a *short-run* imputation, and, as will be discussed in Section H, there is a strong case for using *short-run* imputations against *long-run* ones.

D.3 Class mean imputation

8.95 The *class mean* (or *substitution relative*) method of implicit quality adjustment to prices as used in the U.S. CPI is discussed in Armknecht, Lane, and Stewart (1997). It arose from concerns similar to those considered in Section D.2, namely that unusual price changes were found in the early introductory period when new models were being introduced, particularly for consumer durables.

8.96 The class mean method was adopted in the U.S. CPI for automobiles in 1989 and was phased in for most other nonfood commodities beginning in 1992. It differed from the imputation method only in the source for the imputed rate of price change for the old commodity in period $t + 1$. Rather than using the category index change obtained using all the nonmissing commodities in the category, compilers based the imputed rate of price change on constant quality replacement commodities—those commodities that were judged comparable or that were quality adjusted directly. The class mean approach was seen as an improvement on the overall mean imputation approach because the imputed price changes were based on items that had not just had a replacement. Instead, these items' replacement prices benefited from a quality adjustment, or the new replacement commodity had been judged to be directly comparable. However, it may be the case that sufficiently large samples of comparable substitutes or directly quality-adjusted commodities are unavailable. Or it may be that the quality adjustments and selection of comparable commodities are not deemed sufficiently reliable. In this case, a targeted imputation might be considered. The targeted mean is less ambitious in that it seeks only to capture price changes of similar commodities, irrespective of their point in the life cycle. Yet it is an improvement on the overall mean imputation as long as sufficiently large sample sizes are used.

D.4 Comparable replacement

8.97 In comparable replacement, the respondent makes a judgment that the replacement is of a similar quality to the old commodity and any price changes are untainted by quality changes. For commodity type A in Table 8.2(b), commodity 3 might be judged to be comparable to commodity 2 and its prices in subsequent months used to continue the series. In March, the price of 6.5 would be used as the price in March for commodity 2, whose January to March price change would be $6.5/6 \times 100 = 108.33$ or 8.33 percent. The method of comparable replacement relies on the efficacy of the respondents and, in turn, on the adequacy of the specifications used as a description of the price basis. Statistical agencies may be rightly wary of sample sizes being worn down by dropping commodities using imputation and also of the resource-intensive explicit estimates outlined below. The use of repriced commodities of a comparable specification has much to commend it. If, however, the quality of commodities is improving, the preceding commodity will be inferior to the current ones. In addition, continually ignoring the small changes in the quality of replacements can lead to an upward bias in the index. The extent of the problem will depend on the proportion of such occurrences, the extent to which comparable commodities are accepted in spite of quality differences, and the weight attached to them. Proposals in Chapter 9 to monitor types of quality-adjustment methods by commodity area will provide a basis for a strategy for applying explicit adjustments where they are most needed.

D.5 Linked to show no price change

8.98 Linking attributes any price change between the replacement commodity in the *current* period and the old commodity in the preceding period to the change in quality. A replacement commodity 7 is selected, for example, in Table 8.2(b) from commodity type B for the missing March commodity 6. The replacement commodity 7 may be of a very different quality compared with commodity 6, with the price difference being quite large. The change in price is assumed to be due to a change in quality. An estimate is made for p_7^2 by equating it to p_7^3 to show no change—that is, the assumed price of commodity 7 in February is 14 in Table 8.2(b). There is, therefore, assumed to be no price change over the period February to March for commodity 7. The January to March result for commodity 6 is $(12/12) \times (14/14) = 1.00$, or no change. However, for the period March to April, the price of item 7 in March can be compared with the imputed p_7^2 for February and linked to the preceding results. So the January to April comparison is composed of the January to February comparison for commodity 6 and linked to (multiplied by) the February to April comparison for item 7. This linking is analogous to the procedures used for the chained and short-run framework discussed in Sections G.3 and H.3. The method is born out of circumstances where comparable replacement commodities are not available, and there are relatively large price differences between the old and replacement commodities, having significant differences in price base and quality. It is not possible to separate out how much of this difference is due to price changes and how much to quality changes, so the method attributes it all to quality and holds price constant. The method introduces a degree of undue price stability into the index. It may well be the case that the period of replacement is when substantial price changes are taking place, these changes being wrongly assigned to quality changes by this method. Article 5 of the European Commission (EC) Regulation No. 1749/96 requires member states to avoid such automatic linking. Such linking is equivalent to the assumption that the difference in price between two successive models is wholly attributed to a difference in quality (Eurostat, 2001, p. 125). It should not be used.

D.6 Carryforward

8.99 With this method, when a commodity becomes unavailable—say, in period $t + 2$—the price change calculation uses the old t price, carried forward as if there was no change. Thus, from Table 8.2(a) for commodity type A for the January to March Jevons and Dutot indices (Chapter 21, Section B),

$$P_J(p^1, p^3) = \left[\left(p_1^3/p_1^1 \times p_2^2/p_2^1\right)\right]^{1/2}, \text{ and}$$

$$P_D(p^1, p^3) = \left[\left(p_1^3 + p_2^2\right)/\left(p_1^1 + p_2^1\right)\right], \tag{8.18}$$

with p_2^2 filling in for the missing p_2^3. This introduces undue stability into the index, which is aggravated if the old price p_2^2 continues to be used to fill in the unobserved prices in subsequent periods. It introduces an inappropriate amount of stability into the index and may give a misleading impression of the active sample size. The practice of the carryforward method is banned for harmonized CPIs under Article 6 of the EC Regulation No. 1749/96 for Harmonized Indices of Consumer Prices (Eurostat, 2001, p. 126). To use this method, an assumption is made that the price from this commodity type would not change. This method should be used only if it is fairly certain that there would be no price change. It should otherwise not be used.

E. Explicit Methods

8.100 All of the aforementioned methods do not rely on explicit information on the value of the change in quality, $A(z)$. Now methods that rely on obtaining an explicit valuation of the quality difference are discussed.

E.1 Expert judgment

8.101 Comparable replacements can be considered to be a special case of "subjective quality adjustment," because the determination of commodity equivalence is based on the judgment of the respondent. It is important to mention this because an objection to subjective methods is the inability to provide results that can be independently replicated. Yet in comparable replacement, and for the selection of representative commodities, a subjective element is part of normal procedure. This is not, of course, a case for its proliferation.

8.102 Respondents may be asked to quantify the "production cost" or relative price change that can be attributed to the quality change. Alternatively, the use of experts' views may be appropriate for highly complex commodities where alternative methods are not feasible. Experts, as noted above, should be directed to the nature of the estimate required as discussed in the conceptual section. More than one expert should be chosen, and, where possible, they should be from different backgrounds. Some indication of the interval in which their estimate should lie is also advisable. The well-used Delphi method may be applicable. In this approach, a panel of experts work separately to avoid any bandwagon effect regarding their estimates. They are asked to provide an estimate of the average and range of likely responses. The median is taken of these estimates, and any estimate that is considered extreme is sent back to the expert concerned. The expert is asked to identify reasons for the difference. It may be that the particular expert has a useful perspective on the problem that the other experts had not considered. If the expert argues a case, the response is fed back to the panel members, who are asked if they wish to change their views. A new median is taken, and there are possible further iterations. It is time consuming and expensive but illustrates the care needed in such matters. However, if the adjustment is needed for a commodity area with a large weighting in a trade price index, and no other techniques are available, it is a possible alternative. In all of this guidelines are required as to the conceptual base for the valuation, as discussed in Section B above and Section E.3 below.

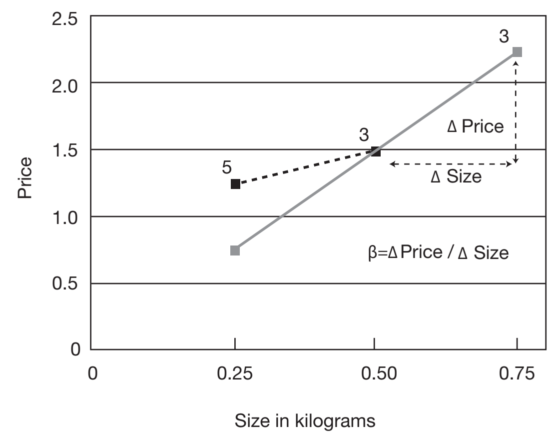
E.2 Quantity adjustment

8.103 This is one of the most straightforward explicit adjustments to undertake and is applicable to commodities for which the replacement is of a different size than the available one. In some situations, there is a readily available quantity metric that can be used to compare the commodities. Examples are the number of units in a package (e.g., paper plates or vitamin pills), the size or weight of a container, or the size of sheets or towels. Quantity adjustment to prices can be accomplished by scaling the price of the old or new commodity by the ratio of quantities. The index production system may do this scaling adjustment automatically by converting all prices in the category to a price per unit of size, weight, or number. Such scaling is most important. For example, it should not be the case that because a respondent reports that a soft drink is now only available in 1-liter containers instead of the previously recorded 0.5-liter ones, its price has doubled.

8.104 There is, however, a second issue. It should be kept in mind that a pure price change is concerned with changes in the revenue received from the sale of the exact same commodities, produced under the exact same circumstances, and sold under the exact same terms. In the pharmaceutical context, for example, prices of bottles of pills of different sizes differ. A bottle of 100 pills, each pill having 50 milligrams of a drug, is not the same as a bottle of 50 pills of 100 milligrams each, even though both bottles contain 5,000 milligrams of the same drug. It may also be reasonable to decide that a bottle of aspirin, for example, containing 500 tablets may not have 10 times the quality of a 50-tablet bottle. If the smaller size is no longer available and there is a change, for example, to a larger size container, and a *unit* price decrease of 2 percent accompanies this change, then it should not be regarded as a price fall if there is a differential in the cost of producing and margin on selling the larger size of 2 percent or more. If, however, the respondent acknowledged that the change in packaging size for this commodity led to a 1 percent saving in resource costs (and margin), and prices of other such commodities without any quantity changes were also falling by 1 percent, then the pure price change would be a fall of 1 percent. In practice, the respondent may be able to make some rough estimates of the effect on the unit cost of the change in packaging size. However, it may well be that no such information is available, and the general policy is to not automatically interpret unit price changes arising from packaging size changes as pure price changes if contrary information exists.

8.105 Consider another example: a brand name bag of fertilizer of a specific type, previously available in a 0.5 kg. bag priced at 1.5, is replaced with a 0.75 kg. bag at 2.25. The main concern here is with rescaling the quantities as opposed to differential cost or margin adjustments. The method would use the relative quantities of fertilizer in each bag for the adjustment. The prices may have increased by [(2.25/1.5) × 100 = 150] 50 percent, but the quality (size)-adjusted prices have remained constant [(2.25/1.5) × (0.5/0.75) × 100 = 100].

Figure 8.1. Quality Adjustment for Different-Sized Items



8.106 The approach can be outlined in a more elaborate manner by referring to Figure 8.1. The concern here is with the part of the unbroken line between the price and quantity coordinates (1.5, 0.5) and (2.25, 0.75), both of which have *unit* prices of 3 (price = 1.5/0.5 and 2.25/0.75). There should be no change in quality-adjusted prices. The delta symbol (Δ) denotes a change. The slope of the line is β, which is ΔPrice/ΔSize = (2.25 − 1.5)/(0.75 − 0.50) = 3, that is, the change in price arising from a unit (kg.) change in size. The quality (size)-adjusted price in period $t - 1$ of the old m bag is

$$\hat{p}_m^{t-1} = p_m^{t-1} + \beta \Delta size$$

$$= 1.5 + 3\,(0.75 - 0.5) = 2.25. \qquad (8.19)$$

The quality-adjusted price change shows no change as before:

$$p_n^t / \hat{p}_m^{t-1} = 2.25/2.25 = 1.00.$$

The approach is outlined in this form so that it can be seen as a special case of the hedonic approach discussed later, where price is related to a number of quality characteristics of which size may be one.

8.107 The method can be seen to be successful on intuitive grounds as long as the unit price of different-sized bags remains constant. If the switch was from a 0.5 kg. bag to a 0.25 kg. one priced at 0.75, as shown by the continuation of the unbroken line in

Figure 8.1, to coordinate (0.75, 0.25) quality-adjusted prices would again not change. However, assume the *unit* (kg.) prices were 5, 3, and 3 for the 0.25, 0.5, and 0.75 kg. bags, respectively, as shown in the example below and in Figure 8.1 by the *broken* line. Then the measure of quality-adjusted price change would depend on whether the 0.5 kg. bag was replaced by the 0.25 kg. one (a 67 percent increase) or the 0.75 kg. one (no change). This is not satisfactory because the choice of replacement size is arbitrary. The rationale behind the quality-adjustment process is to ask: does the difference in unit price in each case arise from differences in unit costs of producing and margins on selling? If so, adjustments should be made to the unit prices to bring them in line; if not, adjustments should be made to the unit price for that proportion due to changes in costs or margins from economies or diseconomies of package size production. It may be obvious from the nature of the commodity that a commodity packaged in a very small size with a disproportionately high unit price has an unusually high profit margin or will have quite different unit production costs, and an appropriate replacement for a large-sized commodity would not be this very small one.

Example of Quantity Adjustments

Size	First Price	First Unit Price	Second Price	Second Unit Price
0.25	0.75	3.00	1.25	5.00
0.50	1.50	3.00	1.50	3.00
0.75	2.25	3.00	2.25	3.00

E.3 Differences in production and option costs

8.108 A natural approach is to adjust the price of the old commodity by an amount equal to the costs of the additional features. This approach is associated with resource cost valuations discussed in Section B.2. Yet Section B.2 advocated a user value approach, the appropriate valuation being the change in production costs associated with a quality change plus any price-cost margin. This amounts to a comparison of relative prices using

$$p_n^t/\hat{p}_m^{t-1}, \text{ where } \hat{p}_m^{t-1} = p_m^{t-1} + x \tag{8.20}$$

and x is the cost or contribution to revenue of the additional features in period $t-1$. The respondent is a natural expert source of such information. Greenlees (2000) provided an example for new trucks and motor vehicles in the United States in 1999. Just before the annual model year introductions, BLS staff visit selected manufacturers to collect cost information. The data are used in the PPI and International Price Comparison programs, as well as in the CPI, and the information-gathering activity is a joint operation of the three programs. Allowable commodity changes for the purpose of quality adjustments include occupant safety enhancements, mechanical and electrical improvements to overall vehicle operation or efficiency, changes that affect length of service or need for repair, and changes affecting comfort or convenience.

8.109 As an example of option cost adjustments, assume the producer prices for a commodity in periods t and $t+2$ were 10,000 and 10,500, respectively, but assume the price in period $t+2$ is for the item with a new feature or option. Also, let the price of the additional feature in period $t+2$ be 300. Then the price change would be 10,200/10,000 = 1.02, or 2 percent. The adjustment may take a multiplicative form (see Section A); the additional options are worth 300/10,500 = 0.028571 of the period $t+2$ price. The adjusted price in period t is, therefore, 10,285.71 and the price change 10,500/10,285.71 = 1.020833, or about 2 percent. If in subsequent periods either of these elements changes, then so too must $\hat{p}_n^{t-1}$ for those comparisons. Option cost is thus a method for use in stable markets with stable technologies. Alternatively, it may be preferable to estimate a one-off adjustment to the preceding base-period price and then compare all subsequent commodities with the new option to this estimate; that is, 10,500/10,300 = 1.019417, or approximately 2 percent.

8.110 In the example above, the prices available for the options were sales prices. For resource cost estimates, the sales prices as estimates of user values must be adjusted to cost estimates by removing markups and indirect taxes. Similarly, and more appropriate to the context of Section B.2, production costs of options need to be upgraded to user values by adding price-cost markups and indirect taxes. Often such data are available for only one period. If the markups are considered to be in the same proportion in subsequent periods, then there is no problem because the retail price changes would proxy the producer ones after adjustment for proportionate margins. However, if the average age or vintage of the commodities has changed, then they will be at different stages in their life cycles and may have different margins.

8.111 Consider the addition of a feature to a commodity. Chairs, for example, can be produced and sold as standard or with a lever mechanism for height adjustment. The specification may always have been the standard model, but this may no longer be in production. The new spec may be a model with height adjustment. The cost of the option is, therefore, known from before, and a continuing series can be developed by using equation (8.20) and simply adding the option cost back into the base-period, old price. Even this process may have its problems. First, the cost of producing something as standard, because all new chairs now have the height adjuster, may be lower than when it was an option. The option cost method would thus understate a price increase. It may be that the manufacturer has an estimate of the effects of such economies of scale to allow for further adjustments. Triplett (2004) cited a study by Levy and others (1999) in which an automobile antitheft system was installed as standard, but disabled when the option was not required. It was seemingly cheaper to produce this way. Second, by including something as standard, the revenue received may be less for some sales than the marginal cost of producing it. The decision to include it as standard precludes buyers from refusing it. It may be that they will turn to other manufacturers who allow them to exclude the option, although it is unlikely that this will be the sole criterion for the purchase. The overall effect would be that the estimate of the option cost, priced for those who choose it, is likely to be higher than the implicit revenue purchasers accord it as standard. Third, the height adjuster may be valued at an additional amount x when sold separately. There is likely to be a segment of the market that particularly values price adjusters and is willing to spend the additional amount. However, when it is sold as standard, many of the purchasers will not value it so highly because these were the very ones who chose the standard chair. The overall user value would be less than x, although it is not immediately apparent how much less. Some statistical offices take one-half x as the adjustment. Some insight into the proportion of the market purchasing the standard commodities would help generate more precise estimates.

8.112 Option cost adjustments are similar to the quantity adjustments, with the exception that the additional quality feature of the replacement is not limited to size. The comparison is $p_n^t/\hat{p}_m^{t-1}$, where $\hat{p}_m^{t-1} = p_m^{t-1} + \beta\Delta z$ for an individual z characteristic where $\Delta z = \left(z_n^t - z_m^{t-1}\right)$. The characteristic may be that the amount of random access memory (RAM) on a personal computer (PC) as a specific model is replaced by one that is identical except for the amount of RAM. If the relationship between price and RAM is linear, this formulation is appropriate. On the web pages of many computer manufacturers, the price of additional RAM is independent of other features, and a linear adjustment is appropriate. Bear in mind that a linear formulation values the worth of a fixed additional amount of RAM as the same, irrespective of the machine's total amount of RAM.

8.113 The relationship may be nonlinear. For example, for every additional one unit of x, y increases by 1.5 percent ($\beta = 1.015$), in this case

$$\hat{p}_m^{t-1} = p_m^{t-1}\beta^z \tag{8.21}$$

for $p_n^t/\hat{p}_m^{t-1}$ as a measure of quality-adjusted price changes. Again, the z change may reflect the service flow, but the nonlinearity in the price$-z$ relationship may reflect the increasing or decreasing utility to the scale of the provision. The characteristic may be priced at a higher rate in up-market models of the commodity versus down-market ones, that is, $\beta \geq 1$ in equation (8.21).

8.114 The similarity between the quantity adjustment and the option cost approach can be identified by simply considering Figure 8.1 with the z characteristic being the option horizontal axis. The similarity between the quantity adjustment and the option cost approach is apparent because both relate price to some dimension of quality: the size or the option. The option cost approach can be extended to more than one quality dimension. Both approaches rely on the acquisition of estimates of the change in price resulting from a change in the options or size: the β slope estimates. In the case of the quantity adjustment, this is taken from a commodity identical to the one being replaced except for the size. The β slope estimate in this case would be perfectly identified from the two pieces of data. It is as if changes in the other factors' quality were accounted for by the nature of the experiment; this is done by comparing prices of what is essentially the same thing except for change in quantity. There may be, for example, two items that are identical except for a single feature. This allows the value of the feature to be determined. Yet sometimes the worth of a feature or option has to be extracted from a much larger data set. This may be because the quality dimension takes a relatively large range of possible numerical values without an immediately obvious consistent valuation. Consider the simple example of one feature varying in a commodity: processor speed in a PC. It is not a straightforward matter to determine the value of an additional unit of speed. To complicate matters, there may be several quality dimensions to the items, and not all combinations of these may exist as items in the market in any one period. Furthermore, the combinations existing in the second period being compared may be quite different from those in the first. All of this leads to a more general framework.

E.4 Hedonic approach

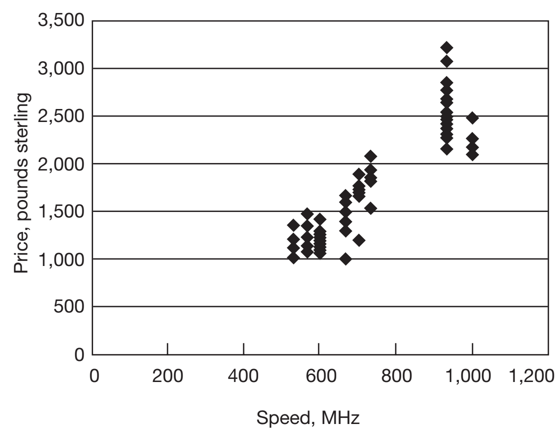
E.4.1 Principles and method

8.115 The hedonic approach is an extension of the two preceding approaches. First, the change in price arising from a unit change in quality—the slope of the line in Figure 8.1—is now estimated from a data set comprising prices and quality characteristic values of a larger number of varieties. Second, the quality characteristic set is extended to cover, in principle, all major characteristics that might influence price, rather than just the quantity or option adjustment. The theoretical basis for hedonic regressions will be covered in Chapter 22 and is briefly reviewed after the following example.

8.116 First, it should be noted that the method requires an extension of the data set to include values for each commodity of price-determining quality characteristics. Under the matched-models method, each respondent needed to supply sufficient data on each item to allow it to be identified for subsequent repricing. The extension required is that all price-determining characteristics should be available for each item. Checklists for the characteristics of a commodity have been found by Merkel (2000) to improve the quality of data collected, as well as to serve the needs of hedonic adjustments (see also Chapter 7 on price collection). If a commodity is missing, any difference in the characteristics of its replacement can be identified, and, as will be shown, a valuation can be ascribed to such differences using the hedonic approach.

8.117 Appendix 8.1 provides data taken from the U.K. Compaq and Dell websites in July 2000 on the prices and characteristics of 64 desktop PCs. Figure 8.2 is a scatter diagram constructed from these data relating the price (£) to the processing speed (MHz). It is apparent that PCs with higher speeds command higher prices—a positive relationship. Under the option cost framework described above, a switch from a 733 MHz PC to a 933 MHz one would involve a measure of the slope of the line between two unique points. The approach requires that there are 733 MHz and 933 MHz PCs that are otherwise identical. From Figure 8.2 and Appendix 8.1, it is apparent that in each instance there are several PCs with the same speed but different prices, owing to differences in other things. To estimate the required value given to additional units of speed, an estimate of the slope of the line that best fits the data is required.

Figure 8.2. Scatter Diagram of PC Prices



In Figure 8.1, the actual slope was used; for the data in Figure 8.2, an estimate of the slope needs to be derived from an estimate of the equation of the line that best fits the data, using ordinary least squares (OLS) regression. Facilities for regression are available on standard statistical and econometric software, as well as spreadsheets. The estimated (linear) equation in this instance is

$$\hat{\text{Price}} = -658.436 + 3.261\ \text{Speed} \qquad \overline{R}^2 = 0.820. \tag{8.22}$$

8.118 The coefficient on speed is the estimated slope of the line: the change in price (£3.261) resulting from a 1 MHz change in speed. This can be used to estimate quality-adjusted price changes for PCs of different speeds. The $\overline{R}^2$ finds that 82 percent of price variation is explained by variation in processing speed. A t-statistic to test the null hypothesis of the coefficient being zero was found to be 18.83; recourse to standard tables on t-statistics found the null hypothesis was rejected at a 1 percent level. The fact that the estimated coefficient differs from zero cannot be attributed to sampling errors at this level of significance. There is a probability of 1 percent that the test has wrongly rejected the null hypothesis. However, the range of prices for a given speed—933 MHz, for example—can be seen from Appendix 8.1 to be substantial. There is a price range of about £1,000, which suggests other quality characteristics may be involved. Table 8.4 provides the results of a regression equation that relates price to a number of quality characteristics using the data in Appendix 8.1. Such estimates can be provided by standard statistical and econometric software, as well as by spreadsheets.

Table 8.4. Hedonic Regression Results for Dell and Compaq PCs

Dependent Variable	Price	Natural Log of Price
Constant	−725.996 (2.71)**	6.213 (41.95)***
Speed (processor, MHz)	2.731 (9.98)***	0.001364 (9.02)***
RAM (random access memory, megabytes)	1.213 (5.61)***	0.000598 (5.00)***
HD (hard drive capacity, megabytes)	4.517 (1.96)*	0.003524 (2.76)**
Brand (benchmark: Compaq Deskpro)		
Compaq Presario	−199.506 (1.89)*	−0.152 (2.60)**
Compaq Prosignia	−180.512 (1.38)*	−0.167 (2.32)*
Dell	−1,330.784 (3.74)***	−0.691 (3.52)***
Processor (benchmark: AMD Athlon)		
Intel Celeron	393.325 (4.38)***	0.121 (2.43)**
Intel Pentium III	282.783 (4.28)***	0.134 (3.66)***
ROM-drive (benchmark: CD-ROM)†		
CD-RW (compact disk-rewritable)	122.478 (56.07)***	0.08916 (2.88)**
DVD drive (digital video disk)	85.539 (1.54)	0.06092 (1.99)*
Dell × Speed (MHz)	1.714 (4.038)***	0.000820 (3.49)***
N	63	63
$\bar{R}^2$	0.934	0.934

† Read-only memory.
Note: Figures in parentheses are *t*-statistics testing a null hypothesis of the coefficient being zero.
***, **, and * denote statistically significant at a 0.1 percent, 1 percent, and 5 percent level, respectively, tests being one-tailed.

8.119 The first column provides the results from a linear regression model, the dependent variable being price. The first variable is processor speed with a coefficient of 2.731; a unit MHz *increase* in processing speed leads to an estimated £2.731 *increase* (positive sign) in price. A change from 733 MHz to 933 MHz would be valued at an estimated 200 (2.731) = £546.20. The coefficient is statistically significant, its difference from zero (no effect) not being due to sampling errors at a 0.1 percent level of significance. This estimated coefficient is based on a multivariate model; the coefficient measures the effect of a unit change in processing speed on price *having controlled for the effect of other variables* in the equation. The result of 3.261 in equation (8.22) was based on just one variable and did not benefit from this. That number is different from this improved result.

8.120 The brand variables are dummy intercepts taking values of 1 if, for example, it is a Dell computer and zero otherwise. Although brands are not in themselves quality characteristics, they may be proxy variables for other factors such as after-service reliability. The inclusion of such brand dummies also reflects segmented markets as communities of buyers as discussed in Chapter 22, Appendix 22.1. Similar dummy variables were formed for other makes and models, including the Compaq Presario and Compaq Prosignia. The Compaq Deskpro, however, was omitted to form the benchmark against which other models are compared. The coefficient on Dell is an estimate of the difference between the worth of a Dell and a Compaq Deskpro, other variables being constant (i.e., £1,330.78 cheaper). Similarly, an Intel Pentium III commands a premium estimated at £282.78 over an AMD Athlon.

8.121 The estimate for processor speed was based on data for Dell and Compaq PCs. If the adjustment for quality is between two Dell PCs, it might be argued that data on Compaq PCs should be ignored. Separate regressions could be estimated for each make, but this would severely restrict the sample size. Alternatively, an interaction term or slope dummy can be used for variables that are believed to have a distinctive brand-interaction effect. Take Dell × Speed, which takes the value of speed when the PC is a Dell and zero otherwise. The coefficient on this variable is 1.714 (see Table 8.4); it is an estimate of the additional (positive sign) price arising for a Dell PC over and above that already arising from the standard valuation of a 1 MHz increase in speed. For Dell PCs, it is 2.731 + 1.714 = £4.445. Therefore, if the replacement Dell

PC is 200 MHz faster than the unavailable PC, the price adjustment to the unavailable PC is to add 200 × £4.445 = £889. Interactive terms for other variables can similarly be defined and used. The estimation of regression equations is easily undertaken using econometric or statistical software or data analysis functions in spreadsheets. An understanding of the techniques is given in many texts, including Kennedy (2003). In Chapter 22, Appendix 22.1, econometric concerns particular to the estimation of hedonic regressions are discussed.

8.122 The $\overline{R}^2$ is the proportion of variation in price explained by the estimated equation. More formally, it is 1 minus the ratio of the variance of the residuals $\sum_{i=1}^{n}\left(p_i^t - \hat{p}_i^t\right)^2/n$, of the equation to the variance of prices $\sum_{i=1}^{n}\left(p_i^t - \overline{p}_i^t\right)^2/n$. The bar on the R^2 denotes that an appropriate adjustment for degrees of freedom is made to this expression, which is necessary when comparing equations with different numbers of explanatory variables. At 0.934, $\overline{R}^2$ is high. However, high $\overline{R}^2$ can be misleading for the purpose of quality adjustment. First, such values inform us that the explanatory variables account for much of price variation. This may be over a relatively large number of varieties of goods in the period concerned. This is not the same as implying a high degree of prediction for an adjustment to a replacement commodity of a single brand in a subsequent time period. For their accuracy, predicted values depend not just on the fit of the equation but also on how far the characteristics of the commodity whose price is to be predicted are from the means of the sample. The more unusual the commodity, the higher the prediction probability interval. Second, $\overline{R}^2$ informs us as to the *proportion* of variation in prices explained by the estimated equation. It may be that 0.90 is explained, while 0.10 is not. If the dispersion in prices is large, this still leaves a large absolute margin of prices unexplained. Nonetheless, a high $\overline{R}^2$ is a necessary condition for the use of hedonic adjustments.

8.123 Hedonic regressions should generally be conducted using a semi-logarithmic formulation (Chapter 22). The dependent variable is the (natural) logarithm of the price. However, the variables on the right-hand side of the equation are taken in their normal units, thus the semi-logarithmic formulation. A double-logarithmic formulation also takes logarithms of the right-hand side z variables. However, if any of these z variables are dummy variables—taking the value of zero in some instances—the double-logarithmic formulation breaks down. Logarithms of zero cannot be taken (thus the focus on the semi-logarithmic form). This concern with linear and semi-log formulations is equivalent to the consideration of additive and multiplicative formulations discussed in Section A. A linear model would, for example, ascribe an extra £282.78 to a PC with an Intel Pentium III as opposed to an AMD Athlon, irrespective of the price of the PC. This is common in pricing strategies using the World Wide Web. However, more often than not, the same options are valued at a higher price for up-market goods and services. In this case our equation (8.22) above, for a multivariate model, is

$$\text{Price} = \beta_0 \times \beta_1^{z_1} \times \beta_2^{z_2} \times \beta_3^{z_3} \times \ldots\ldots \times \beta_n^{z_n} \times \varepsilon \text{ or}$$
$$\ln \text{Price} = \ln \beta_0 + z_1 \ln \beta_1 + z_2 \ln \beta_2 + z_3 \ln \beta_3 + \ldots\ldots z_n \ln \beta_n + \ln \varepsilon. \quad (8.23)$$

8.124 Note that this is a semi-logarithmic form; logarithms are taken of only the left-hand-side variable, that is, price. Each of the z characteristics enter the regression without having logarithms taken. This has the advantage of allowing dummy variables for the possession or otherwise of a feature to be included on the right-hand side. Such dummy variables take the value of 1 if the commodity possesses the feature and zero otherwise, it not being possible to take a logarithm of the value zero. Issues on choice of functional form are discussed in more detail in Chapter 22.

8.125 The taking of logarithms in the first equation (8.23) allows it to be transformed in the second equation to a linear form. This allows the use of a conventional OLS estimator to yield estimates of the logarithm of the coefficients. These are given in column 3 of Table 8.4 and have a useful direct interpretation: if these coefficients are multiplied by 100, they are the percentage change in price arising from a one-unit change in the explanatory variable. For processor speed, there is an estimated 0.1364 percent change in price for each additional MHz the replacement commodity has over and above the unavailable one. When dummy variables are used, the coefficients—when multiplied by 100—are estimates of the percentage change in price given by $(e^{\beta} - 1)\,100$; for example, for a rewritable compact disc drive (CD-RW) compared with a read-only CD drive (CD-ROM), it is $(e^{0.08916} - 1)100 = 9.326$ percent. There is some bias in these estimated coefficients on dummy variables for the (semi-) logarithmic equation; one-half of the variance of the regression equation should be added to the coefficient before using it (Teekens and Koerts, 1972). For CD-ROM, the t-statistic is 2.88; this is equal to the coefficient divided by its standard error. The standard error is 0.08916/2.88 = 0.03096, and the variance is $0.03096^2 = 0.000958$. To adjust to variance of the regression equation, add 0.000958/2 to 0.08916 = 0.089639 or 8.9639 percent.

8.126 The approach is particularly useful when the market does not reveal the price of the quality characteristics required for the adjustment. Markets reveal prices of commodities, not quality characteristics, so it is useful to consider commodities as tied bundles of characteristics. A sufficiently large data set of commodities with their characteristics, and sufficient variability in the mix of characteristics between the commodities, allows the hedonic regression to provide estimates of the implicit prices of the characteristics. The formal theory is provided in Chapter 22. There are a number of ways of implementing the method, which are outlined below. Before doing so, it is useful to note how these coefficients should be interpreted in light of theoretical needs.

E.4.2 On theory

8.127 Some mention should be made of the interpretation of the coefficients from hedonic regressions. The matter will be discussed in further detail in Chapter 22, Section B.5. This section summarizes the conclusion. There used to be an erroneous perception that the coefficients from hedonic methods represented estimates of user value as opposed to resource cost. Rosen (1974) showed that hedonic coefficients may be reflective of both user value and resource cost, both supply and demand influences. There is, in econometric terms, an identification problem, in which the observed data do not permit the estimation of the underlying demand and supply parameters. However, suppose the *production technology of sellers is the same* but buyers differ. Then the hedonic function describes the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. There are different tastes on the user side, so what appears in the market is the result of firms trying to satisfy purchasers' preferences all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. Now suppose sellers differ but *buyers' tastes are the same*. Here the hedonic function $p(z)$ identifies the structure of demand. Of these possibilities, uniformity of tastes is unlikely while uniformity of technologies is more likely, especially when access to technology is unrestricted in the long run. Griliches (1989, p. 120) has argued in the context of a CPI,

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions *per se*, though it can also be useful for these purposes . . . . what is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

8.128 It is thus necessary to take a pragmatic stance. In many cases, the implicit quality adjustment to prices outlined in Section C may be inappropriate because their implicit assumptions are unlikely to be valid. The practical needs of economic statistics require in such instances explicit quality adjustments. To not do anything on the grounds that the measures are not conceptually appropriate would be to ignore the quality change and provide wrong results. Hedonic techniques provide an important tool, making effective use of data on the price-quality relationship derived from other commodities in the market to adjustment for changes in one or more characteristics.

8.129 The proper use of hedonic regression requires an examination of the coefficients of the estimated equations to see if they make sense. It might be argued that the very multitude of distributions of tastes and technologies and interplay of supply and demand make it unlikely that *reasonable* estimates will arise from such regressions. A firm may apply and cut a profit margin and prices for reasons related to long-run strategic plans, for example, yielding coefficients that *prima facie* do not look reasonable. This does not negate the usefulness of examining hedonic coefficients as part of a strategy for evaluating estimated hedonic equations. First, there has been extensive empirical work in this field, and the results for individual coefficients are, for the most part, quite reasonable. Even over time, the empirical evidence is that there are quite sensible patterns of decline in individual coefficients. Second, as shall be seen, it might be argued that the prediction and its error should be our concern and not the values of individual coefficients.

E.4.3 Implementation

8.130 The implementation of hedonic methods to estimate quality adjustments to noncomparable replacements can take a number of forms. The first form is when the repricing is for a commodity with different characteristics. What is required is to adjust either the price of the old or replacement (new) commodity for some valuation of the difference in quality between the two commodities. This patching of missing prices is quite different from the use of hedonic price indices to be discussed in Section G.2 in this chapter and in Chapter 22. These use hedonic regressions to provide hedonic price indices of overall quality-adjusted prices. The former is a partial application, used on noncomparable replacements when commodities are no longer produced. The latter, as will be seen in Section G.2, is a general application to a sample from the whole data set. The partial patching is considered here.

8.131 Hedonic imputation: *predicted vs. actual*—In this approach, a hedonic regression of the (natural logarithm of the) price of model i in period t on its characteristics set z^t_{ki} is estimated *for each month,* as given by

$$\ln p^t_i = \beta^t_0 + \sum_{k=1}^{k} \beta^t_k z^t_{ki} + \varepsilon^t_k. \tag{8.24}$$

8.132 Let us say the price of a commodity m available in January (period t) is unavailable in March (period $t + 2$). The price of commodity m can be predicted for March by inserting the characteristics of the old unavailable commodity m into the estimated regression equation for March; this process is repeated for successive months. The predicted price for the old commodity in March and the price comparison with January (period t) are given, respectively, by

$$\hat{p}^{t+2}_m = \exp\left[\hat{\beta}^{t+2}_0 + \sum \hat{\beta}^{t+2}_k z^t_{k,m}\right], \tag{8.25a}$$

and $\hat{p}^{t+2}_m / p^t_m$, that is, the *old* model's price is adjusted. In the example in Table 8.2(a), $\hat{p}^3_2$, $\hat{p}^4_2$, and so forth, and $\hat{p}^3_6$, $\hat{p}^4_6$, and so forth would be estimated and compared with p^1_2 and p^1_6, respectively. The blanks for commodities 2 and 6 in Table 8.2(a) would be effectively filled in by the estimated prices from the regression equation.

8.133 An alternative procedure is to select for each unavailable m commodity a replacement commodity n. In this case, the price of n in period $t + 2$ is known, and a predicted price for n in period t is required. The predicted price for the new commodity n and required price comparison are

$$\hat{p}^t_n = \exp\left[\hat{\beta}^t_0 + \sum \hat{\beta}^t_k z^{t+2}_{k,n}\right], \tag{8.25b}$$

and $p^{t+2}_n / \hat{p}^t_n$, that is, the *new* model's price is adjusted. In this case, the characteristics of commodity n are inserted into the right-hand side of an estimated regression for period t. The price comparisons of equation (8.25a) may be weighted by w^t_m, as would those of its replaced price comparison in equation (8.25b).

8.134 A final alternative is to take the geometric mean of the formulations in equations (8.25a) and (8.25b) on grounds analogous to those discussed in Chapter 16 and by Diewert (1997) for similar index number issues.

8.135 Hedonic imputation: *predicted vs. predicted*—A further approach is the use of predicted values for the commodity in *both* periods, for example, $\hat{p}^{t+2}_n / \hat{p}^t_n$, where n represents the commodity. Consider a misspecification problem in the hedonic equation. For example, there may be an interaction effect between a brand dummy and a characteristic, say, between Dell and speed in the example in Table 8.4. Having both characteristics may be worth more on price (from a semi-logarithmic form) than their separate individual components. The use of $p^{t+2}_n / \hat{p}^t_n$ would be misleading because the actual price in the numerator would incorporate the 5 percent premium while the one predicted from a straightforward semi-logarithmic form would not. It is stressed that in adopting this approach, a recorded, actual price is being replaced by an imputation. Neither this nor the form of bias discussed above is desirable.

8.136 The comparisons using predicted values in both periods are given as

$\hat{p}^{t+2}_n / \hat{p}^t_n$ for the *new* commodity,

$\hat{p}^{t+2}_m / \hat{p}^t_m$ for the disappearing or *old* commodity, or

$$\left[\left(\hat{p}^{t+2}_n / \hat{p}^t_n\right)\left(\hat{p}^{t+2}_m / \hat{p}^t_m\right)\right]^{1/2} \tag{8.26}$$

as a (geometric) mean of the two.

8.137 Hedonic adjustments using *coefficients*—In this approach, a replacement commodity is used and any differences between the characteristics of the replacement n in period $t + 2$ and m in period t are ascertained. A predicted price for n in period t, that is, $\hat{p}^t_n$, is compared with the actual price p^{t+2}_n. However, unlike the formulation in equation (8.25b) for example, $\hat{p}^t_n$ may be estimated by applying the subset of the k characteristics that distinguished m from n, to their respective implicit prices in period t estimated from the hedonic regression, and adjusting the price of p^t_m. For example, if the nearest replacement for commodity 2 was commodity 3, then the characteristics that differentiated commodity 3 from commodity 2 are identified and the price in the base period p^1_3 is estimated by adjusting p^1_2 using the appropriate coefficients from the hedonic regression in that month. For example, for washing machines, if commodity 2 had a spin speed of 800 revolutions per minute (rpm) and commodity 3 had a spin speed of 1,100 rpm, other things being equal, the shadow price of the 300 rpm differential would be estimated from the hedonic regression, and p_2^1 would be adjusted for comparison with p_3^3. Note that if the z variables in the characteristic set are perfectly independent of each other, the results from this approach will be similar to those from equation (8.25b). This is because interdependence among the variables on the right-hand side of the hedonic equation—multicollinearity—leads to imprecise estimates of the coefficients (see Chapter 22, Appendix 22.1).

8.138 Hedonic *indirect adjustment*—An indirect current-period hedonic adjustment may be used, which only requires the hedonic regression to be estimated in the base period t:

$$\frac{p_n^{t+2}}{p_m^t} \div \frac{\hat{p}_n^t}{\hat{p}_m^t}. \tag{8.27}$$

8.139 The first term is the change in price between the old and replacement items in periods t and $t + 2$, respectively. But the quality of the commodity has changed, so this price change needs to be divided by a measure of the change in quality. The second term uses the hedonic regression in period t in both the numerator and denominator. So the coefficients—the shadow prices of each characteristic—remain the same. It is not prices that change. The predicted prices differ because different *quantities* of the characteristics are being inserted into the numerator and denominator; the replacement n characteristics in the former and old commodity m characteristics in the latter. The measure is the change in price after removing (by division) the change in the quantity of characteristics each valued at a constant period t price. Conceptually, the constant valuation by a period $t + 2$ regression would be equally valid and a geometric mean of the two ideal. However, if hedonic regressions cannot be run in real time, equation (8.27) is a compromise. As the spread between the current- and base-period results increases, its validity decreases. As such, the regression estimates should be updated regularly using old- and current-period estimates, and results compared retrospectively as a check on the validity of the results.

E.4.4 Need for caution

8.140 The limitations of the hedonic approach should be kept in mind. Some points are summarized below though readers are referred to the Bibliography and to Chapter 22, Appendix 22.1. First, the approach requires statistical expertise for the estimation of the equations. The prevalence of user-friendly software with regression capabilities makes this less problematic. Statistical and econometric software carry a range of diagnostic tests to help judge if the final formulation of the model is satisfactory. These include $\overline{R}^2$ as a measure of the overall explanatory power of the equation and F-test and t-test statistics to enable tests to be conducted to determine whether the differences between the coefficients on the explanatory variables are jointly and individually different from zero at specified levels of statistical significance. Most of these statistics make use of the errors from the estimated equation. The regression equation can be used to predict prices for each commodity by inserting the values of the characteristics of the commodities into the explanatory variables. The differences between the actual prices and these predicted results are the residual errors. Biased or imprecise results may arise from a range of factors, including heteroscedasticity (nonconstant variances in the residuals suggesting nonlinearities or omission of relevant explanatory variables), a nonnormal distribution for the errors, and multicollinearity, where two or more explanatory variables are related. The latter, in particular, has been described as the "bane of hedonic regressions . . ." (Triplett, 1990). Such econometric issues are well discussed in the context of hedonic regressions (Triplett, 1990; Gordon, 1990; Berndt, 1991; Berndt, Griliches, and Rappaport, 1995; Silver, 1999; Triplett, 2004; and Appendix 22.1) and more generally in introductory econometric texts such as Kennedy (2003). The use of predicted values when multicollinearity is suspected is advised, rather than individual coefficients, for reasons discussed above.

8.141 Second, the estimated coefficients should be updated regularly. However, if the adjustment is to the old model, then the price comparison is between the price of the new model and the quality-adjusted price of the old model. The quality difference between the old and new models is derived using coefficients from a hedonic regression from a previous period as estimates of the value of such differences. There is, at first glance, no need to update the hedonic regression each month. The valuation of a characteristic in the price reference period may, however, be quite out of line with its valuation in the new period. For example, a feature may be worth an additional 5 percent in the reference period instead of 10 percent in the current period because it might have been introduced at a discount at that point in its life cycle to encourage usage. Continuing to use the coefficients from some far-off period to make price adjustments in the current period is similar to using out-of-date base-period weights. The comparison may be well defined but have little meaning. If price adjustments for quality differences are being made to the old item in the price-reference period using hedonic estimates from that period, then there is a need to update the estimates if they are considered out of date—for example, due to changing tastes or technology—and splice the new estimated comparisons onto the old. Therefore, regular updating of hedonic estimates when using the adjustments to the old price is recommended, especially when there is evidence of parameter instability over time.

8.142 Third, the sample of prices and characteristics used for the hedonic adjustments should be suitable for the purpose. If they are taken from a particular industry, trade source, or web page and then used to adjust noncomparable prices for commodities sold by quite different industries, there must at least be an intuition that the marginal returns for characteristics are similar among the industries. A similar principle applies for the brands of commodities used in the sample for the hedonic regression. It should be kept in mind that high $\overline{R}^2$ statistics alone do not ensure reliable results. Such high values arise from regressions in periods before their application and inform us of the proportion of variation in prices across many commodities and brands. They are not in themselves a measure of the prediction error for a particular commodity, sold by a specific establishment of a given brand in a subsequent period, although they can be an important constituent of this.

8.143 Fourth, there is the issue of functional form and the choice of variables to include in the model. Simple functional forms generally work well. These include linear, semi-logarithmic (logarithm of the left-hand side), and double-log (logarithms of both sides) forms. Such issues are discussed in Chapter 22, Appendix 22.1. The specification of a model should include all price-determining characteristics, and simpler specifications should be used only when they are adequate parsimonious representations of the more general form. Typically, a study would start with a larger number of explanatory variables and a general econometric model of the relationship; the final model is a more specific, parsimonious one because it has dropped a number of variables. The dropping of variables occurs after experimenting with different formulations and seeing their effects on diagnostic test statistics, including the overall fit of the model and the accordance of signs and magnitudes of coefficients with prior expectations.

8.144 Finally, the successful design and use of hedonic quality adjustment requires heavy investments over a long period. Required are (1) intellectual competencies and sufficient time to develop and reestimate the model and employ it when commodities are replaced; (2) access to detailed, reliable information on commodity characteristics; and (3) a suitable organization of the infrastructure for collecting, checking, and processing information.

8.145 It should be noted that hedonic methods may also improve quality adjustment in CPIs by indicating which commodity attributes do *not* appear to have material impacts on price. That is, if a replacement commodity differs from the old commodity only in characteristics that have been rejected as price-determining variables in a hedonic study, this would support a decision to treat the commodities as comparable or equivalent and include the entire price difference (if any) as pure price change. Care has to be exercised in such analysis because a feature of multicollinearity in regression estimates is that the imprecision of the parameter estimates may give rise to statistical tests that do not reject null hypotheses that are false, that is, they do not find significant parameter estimates. However, the results from such regressions can nonetheless provide valuable information on the extent to which different characteristics influence price variation. This in turn can help in the selection of replacement commodities. The enhanced confidence in commodity substitution and the quality adjustment of prices from the hedonic approach with its parallel reduction in reliance on linking have been cited as significant benefits in the reliability of the measurement of price changes. The results from hedonic regressions have a role to play in identifying price-determining characteristics and may be useful in the design of quality checklists in price collection (Chapter 7).

F. Choosing a Quality-Adjustment Method

8.146 Choosing a method for quality-adjusting prices is not straightforward. The analyst must consider the technology and market for each commodity and devise appropriate methods. This is not to say the methods selected for one industry will be independent of those selected for other industries. Expertise built up using one method may encourage its use elsewhere, and intensive use of resources for one commodity may lead to less resource-intensive methods in others. The methods adopted for individual industries may vary among countries as access to data, relationships with the respondents, resources, expertise and features of the production, and market for the commodity vary. Guidelines on choosing a method arise directly from the features of the methods outlined above. A good understanding of the methods and their implicit and explicit assumptions is essential when choosing a method.

8.147 Consider Figure 8.3, which provides a useful guide to the decision-making process. Assume the matched-models method is being used. If the commodity is matched for repricing—without a change in the specification—no quality adjustment is required. This is the simplest of procedures. However, a caveat applies. If the commodity belongs to a high-technology industry where model replacement is rapid, the matched sample may become unrepresentative of the universe of transactions. Alternatively, matching may be under a chained framework, where prices of commodities in a period are matched to those in the preceding period to form a link. A series of successive links of matched comparisons combined by successive multiplication makes up the chained matched index. Alternatively, hedonic indices may be used, which require no matching. The use of such methods is discussed in Section G. At the very least, attention should be directed to more regular commodity resampling. Continued long-run matching would deplete the sample, and an alternative framework to long-run matching would be required.

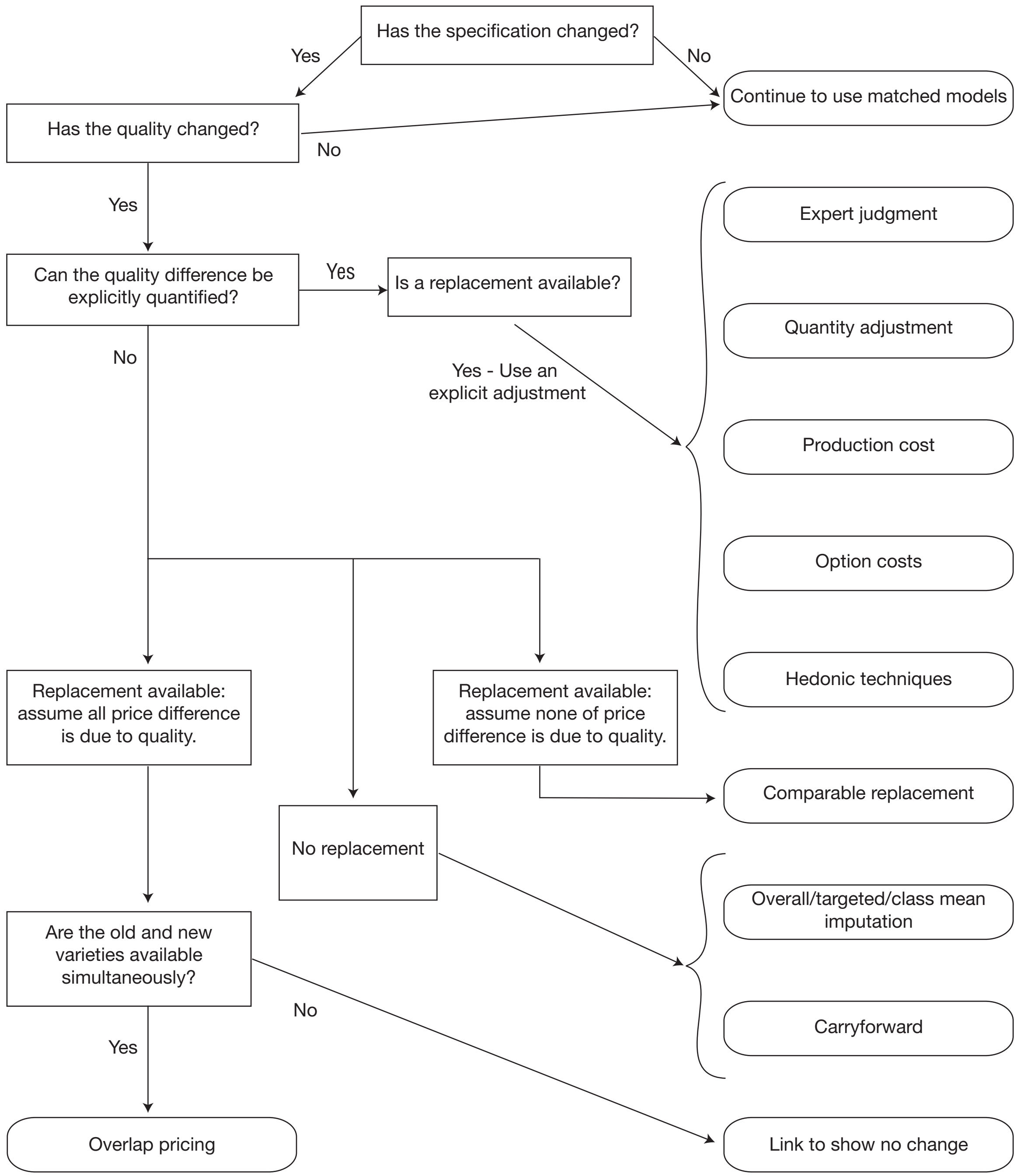
8.148 Consider a change in the quality of a commodity, and assume a replacement commodity is available. The selection of a comparable commodity to the same specification and the use of its price as a *comparable replacement* require that none of the price difference is due to quality. They also require confidence that all price-determining factors are included on the specification. The replacement commodity should also be representative and account for a reasonable proportion of sales. Caution is required when nearly obsolete commodities at the end of their life cycles are replaced with unusual pricing by similar commodities that account for relatively low sales, or with commodities that have substantial sales but are at different points in their cycle. Strategies for ameliorating such effects are discussed below and in Chapter 9, including early substitutions before pricing strategies become dissimilar.

8.149 Figure 8.3 shows where quality differences can be quantified. *Explicit estimates* are generally considered to be more reliable, but they are also more resource intensive (at least initially). Once an appropriate methodology has been developed, explicit estimates can often be easily replicated. General guidelines are more difficult here because the choice depends on the host of factors discussed above, which are likely to make the estimates more reliable in each situation. Central to all of this is the quality of the data on which the estimates are based. If reliable data are unavailable, subjective judgments may be used. Commodity differences are often quite technical and very difficult to specify and quantify. The reliability of the method depends on the knowledge of the experts and the variance in opinions. Estimates based on objective data are, as a result, preferred. Good *production cost* estimates, along with good data on markups and indirect taxes, where applicable, in industries with stable technologies where differences between the old and replacement commodities are well specified and exhaustive, are reliable by definition. The *option cost* approach is generally preferable when old and new commodities differ by easily identifiable characteristics that have once been separately priced as options, but the price of an option will overstate its value when it becomes standard so care must be taken when using this method. The use of *hedonic regressions* for partial patching is most appropriate where data on price and characteristics are available for a range of models and where the characteristics are found to predict and explain price variability well in terms of a priori reasoning and econometrics. Use of hedonic regressions is appropriate where the price of an option or change in characteristics cannot be separately identified and has to be gleaned from the prices of commodities sold with different specifications in the market. The estimated regression coefficients are the estimate of the contribution to price of a unit change in a characteristic, having controlled for the effects of variations in the quantities of other characteristics.

8.150 The estimates are particularly useful for valuing changes in the quality of a commodity when only a given set of characteristics change, and the valuation is required for changes in these characteristics only. The results from hedonic regressions may be used to target the salient characteristics for commodity selection. The synergy between the selection of prices according to characteristics defined as price determining by the hedonic regression and the subsequent use of hedonics for quality adjustment should reap rewards. The method should be applied where there are high ratios of noncomparable replacements and where the differences between the old and new commodities can be well defined by a large number of characteristics.

8.151 If explicit estimates of quality are unavailable and no replacement commodities are deemed

Figure 8.3 Flow Chart for Making Decisions on Quality Change



Source: Chart is extended version of work of Fenella Maitland-Smith and Rachel Bevan given in Triplett (2004).

appropriate, then *imputations* may be used. The use of imputations has much to commend it in terms of resources. It is relatively easy to employ, although some verification of the validity of the implicit assumptions might be appropriate. It requires no judgment (unless targeted) and is therefore objective. Targeted mean imputation is preferred to overall mean imputation as long as the sample size on which the target is based is adequate. Class mean imputation is preferred when models at the start of their life cycles are replacing those near the end of their life cycles, although the approach requires faith in the adequacy of the explicit and comparable replacements being made.

8.152 Bias from using imputation is directly related to the proportion of missing commodities and the difference between quality-adjusted prices of available matched commodities and the quality-adjusted prices of unavailable ones (see Table 8.3). The nature and extent of the bias depends on whether short-run or long-run imputations are being used (the former being preferred) and on market conditions (see Section H). Imputation in practical terms produces the same result as deletion of the commodity, and the inclusion of imputed prices may give the illusion of larger sample sizes. Imputation is less likely to give bias for commodities when the proportion of missing prices is low. Table 8.2 can be used to estimate likely error margins arising from its use, and a judgment can be made as to whether the error margins are acceptable. Its use across many industries need not compound the errors because, as noted in the discussion of this method, the direction of bias need not be systematic. It is cost-effective for industries with large numbers of missing commodities because of its ease of use. But the underlying assumptions required must be carefully considered if widely used. Imputation should by no means be the overall, catchall strategy, and statistical agencies are advised against its use as a default device without due consideration of the nature of the markets, possibility of targeting the imputation, and the viability of estimates from the sample sizes involved if such targeting is employed.

8.153 If the old and replacement commodities are available simultaneously and the quality difference cannot be quantified, an implicit approach can be used whereby the price difference between the old and replacement commodity in a period in which they both exist is assumed to be due to quality. This *overlap* method, by replacing the old commodity with a new one, takes the ratio of prices in a period to be a measure of their quality difference. It is implicitly used when new samples of commodities are taken. The assumption of relative prices equating to quality differences at the time of the splice is unlikely to hold true if the old and replacement commodities are at different stages in their life cycles and different pricing strategies are used at these stages. For example, there may be deep discounting of the old commodity to clear inventories and price skimming of market segments that will purchase new models at relatively high prices. As with comparable replacements, early substitutions are advised so that the overlap is at a time when commodities are at similar stages in their life cycles.

8.154 The use of the *linked to show no change* method and the *carryforward* method is not generally advised for making quality-adjustment imputations for the reasons discussed unless there is deemed to be some validity to the implicit assumptions.

G. High-Technology and Other Sectors with Rapid Turnover of Models

8.155 The measurement of price changes of commodities unaffected by quality changes is primarily achieved by matching models, the aforementioned techniques being applicable when the matching breaks down. But what about industries where the matching breaks down on a regular basis because of the high turnover in new models of different qualities than the old ones? The matching of prices of identical models over time, by its nature, is likely to lead to a depleted sample. There is both a dynamic universe of all commodities produced and a static universe of the commodities selected for repricing. For example, if the sample is initiated in December, by the subsequent May the static universe will be matching prices of those commodities available in the static universe in both December and May but will omit the unmatched new commodities introduced in January, February, March, April, and May, and the unmatched old ones available in December but unavailable in May. There are two empirical questions to answer for any significant bias to be detected. The first question asks whether the sample depletion is substantial; such depletion is a necessary condition for bias. The second asks whether the unmatched new and unmatched old commodities are likely to have different quality-adjusted prices versus the matched ones in the current and base period.

8.156 Thus, the matching of prices of identical models over time may lead to the monitoring of a sample of models increasingly unrepresentative of the population of transactions. There are old models that existed when the sample was drawn but are not available in the current period, and there are new ones coming into the current period that are not available in the base period. It may be that the departures have relatively low prices and the entrants relatively high ones and that by ignoring these prices a bias is being introduced. Using old low-priced commodities and ignoring new high-priced ones has the effect of biasing the index downward. In some industries, the new commodity may be introduced at a relatively low price and the old one may become obsolete at a relatively high one, serving a minority segment of the market. In this case, the bias would take the opposite direction; the nature of the bias depends on the pricing strategies of firms for new and old commodities.

8.157 This sampling bias exists for most commodities. However, our concern is with commodity markets where the statistical agencies are finding the frequency of new commodity introductions and old commodity obsolescence sufficiently high that they may have little confidence in their results. First, some examples of such commodity markets will be given. Then, two procedures will be considered: the use of hedonic price indices instead of partial hedonic patching and chaining.

G.1 Some examples

8.158 Koskimäki and Vartia (2001) attempted to match prices of personal computers over three two-month periods (spring, summer, and fall) using a sample of prices collected as part of the standard price collection for the Finnish CPI, which has some similarities to trade price indexes. Of the 83 spring prices, only 55 matched comparisons could be made with the summer prices, and of those, only 16 continued through to the fall. They noted that the sample of matched pairs became increasingly biased: of the 79 models in the fall, the 16 matched ones had a mean processor speed of 518 MHz compared with 628 MHz for the remaining 63 unmatched ones; the respective hard disk sizes were 10.2 gigabytes (GB) and 15.0 GB; and the percentages of high-end processors (Pentium III and AMD Athlon) were 25 percent and 49.2 percent, respectively. Hardly any change in *matched* prices was found over this six-month period, whereas a hedonic regression analysis using all of the data found quality-adjusted price falls of about 10 percent. Instructions to respondents to hold on to models until forced replacements are required may lead to a sample increasingly unrepresentative of the population and be biased toward technically inferior variants. In this instance, the hedonic price changes fell faster because the newer models became cheaper for the services supplied.

8.159 Silver and Heravi (2005) found evidence of sample degradation when matching prices of U.K. washing machines over a year. By December, only 53 percent of the January basket of model varieties was used for the December/January index, although this accounted for 81.6 percent of January expenditure. Models of washing machines with lower sales values dropped out quicker. However, the remaining models in December accounted for only 48.2 percent of the value of transactions *in December*. The active sample relating to the universe of transactions in December had substantially deteriorated. The prices of unmatched and matched models differed, as did their vintage and quality. Even when prices were adjusted for quality using hedonic regressions, prices of unmatched old models were found to be lower than matched ones; there was also evidence of higher prices for unmatched new models. Quality-adjusted prices fell faster for the matched sample than the full sample: about 10 percent for the former compared with about 7 percent for the latter. Residuals from a common hedonic surface and their leverage were also examined. The residuals from unmatched new models were higher than matched ones, while residuals from unmatched old models were much lower. Unmatched observations had nearly twice the (unweighted) leverage than matched ones; their influence in the estimation of the parameters of the regression equation was much greater and their exclusion more serious.

8.160 These studies, although based on consumer price data, demonstrate how serious sample degradation can occur and how unmatched excluded commodities may be quite different from included ones. Two procedures for dealing with such situations will be considered: the use of hedonic price indices instead of the partial hedonic patching discussed above and chaining. Both rely on a data set of a representative sample of commodities and their characteristics *in each period*. A checklist of structured commodity characteristics to be completed each reporting period is one way changes in quality characteristics can be prompted and monitored: this is especially useful in high-technology industries. If a new commodity is introduced and has or is likely to have substantial sales, then it is included as a replacement or even an addition. Its characteristics are marked off against a checklist of salient characteristics. The list will be developed when the sample is initiated and updated as required. Alternatively, web pages and trade associations may be able to provide lists of models and their prices; however, the need for transaction prices as opposed to list prices is stressed.

G.2 Hedonic price indices

8.161 It is important to distinguish between the use of hedonic regressions to make adjustments for quality differences when a noncomparable substitute is used, as in Section E, and their use in their own right as *hedonic price indices,* which are measures of quality-adjusted price changes. Hedonic price indices are suitable when the pace and scale of replacements of commodities are substantial. There are two reasons for this. First, an extensive use of quality adjustments may lead to errors. Second, the sampling will be from a matched or replacement universe likely to be biased. With new models being continually introduced and old ones dying, the coverage of a matched sample may deteriorate and bias may be introduced as the price changes of the new or old models differ from those of the matched ones. A sample must be drawn in each month, and price indices must be constructed, but, instead of being controlled for quality differences by matching, they will be controlled for, or partialed out, in the hedonic regression. Note that all the indices described below use a fresh sample of the data available in each period. If there is a new commodity in a period, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old commodities drop out, they are still included in the data for the indices in the periods in which they exist. In Section E.4.4 of this chapter, the need for caution was stressed in the use of hedonic regressions for quality adjustments owing to theoretical and econometric issues, some of which will be considered in the appendix to Chapter 22. This need for caution extends to the use of the results from hedonic indices and is not repeated here for the sake of brevity.

8.162 In Chapter 18, theoretical price indices will be defined and practical index number formulas considered as bounds or estimates of these indices. Theoretical index numbers will also be defined in Chapter 22 to include goods made up of tied characteristics, so that something can be said about how such theoretical indices relate to different forms of hedonic indices. A number of forms will be considered in Chapter 22, and the account is outlined here.

G.2.1 Hedonic functions with dummy variables on time

8.163 The sample covers the two time periods being compared—for example, t and $t + 2$—and does not have to be matched. The hedonic formulation regresses the price of commodity i, p_i, on the $k = 2 \ldots. K$ characteristics of the commodities z_{ki}. A single regression is estimated on the data in the two time periods compared, the equation also including a dummy variable D^{t+2} being 1 in period $t + 2$, zero otherwise:

$$\ln p_i = \beta_0 + \beta_1 D^{t+2} + \sum_{k=2}^{K} \beta_k z_{ki} + \varepsilon_i. \qquad (8.28)$$

8.164 The coefficient β_1 is an estimate of the quality-adjusted price change between period t and period $t + 2$. It is an estimate of the change in (the logarithm of) price, having controlled for the effects of variation in quality via $\sum_{k=2}^{K} \beta_k z_{ki}$. Note that an adjustment is required for β_1: the addition of one-half (standard error)^2 of the estimate as discussed in Teekens and Koerts (1972). Two variants of equation (8.28) are considered. The first, the direct *fixed-base version*, compares period t with $t + 2$ as outlined: January–February, January–March, and so forth. The second is a rolling *chained version* evaluated for period t with $t + 1$; then again for $t + 1$ with $t + 2$ and so on, the links in the chain being combined by successive multiplication. A January–March comparison, for example, would be the January–February index multiplied by the February–March one. There is also a *fully constrained version.* This entails a single constrained regression for a period of time—January to December, for example—with dummy variables for each month. However, this is impractical in real time because it requires data on future observations.

8.165 The approach just described uses the estimated coefficient on the dummy variables on time to compare prices in period t with prices in each subsequent period. In doing so, the β parameters on the quality variables are constrained to be constant over the period being compared. A fixed-base, bilateral comparison using equation (8.28) makes use of the constrained parameter estimates over the two periods compared and, given an equal number of observations in each period, is a form of a symmetric average. A *chained* formulation would estimate an index between periods 1 and 4—represented here as $I^{1,4}$—as $I^{1,4} = I^{1,2} \times I^{2,3} \times I^{3,4}$.

8.166 There is no explicit weighting in these formulations; this is a serious disadvantage. In practice. cutoff sampling might be employed to include only the most important commodities. If sales data are available, a weighted least squares (WLS) estimator should be used, as opposed to an OLS estimator. It is axiomatic in normal index number construction that the same weight should not be given to each price comparison because some commodities may account for much larger sales revenues than others. The same consideration applies to these hedonic indices. Diewert (2005b) has argued that sales *values* should form the basis of the weights over quantities. Two commodities may have sales equal to the same quantity, but, if one is priced higher than another, its price changes should be weighted higher accordingly for the result to be meaningful in an economic sense. In addition, Diewert (2005b) has shown that it is value *shares* that should form the weights, because values will increase—over period $t + 2$, for example—with prices, the residuals, and their variance thus being higher in period $t + 2$ than in t. This heteroscedasticity is an undesirable feature of a regression model resulting in increased standard errors. Silver (2002) has further shown that a WLS estimator does not purely weight the observations by their designated weights. The actual influence given is also due to a combination of the residuals and the leverage effect. The latter is higher because the characteristics of the observations diverge from the average characteristics of the data. He suggests that observations with relatively high leverage and low weights be deleted and the regression repeated.

G.2.2 Period-on-period hedonic indices

8.167 An alternative approach for a comparison between periods t and $t + 2$ is to estimate a hedonic regression for period $t + 2$ and insert the values of the characteristics of each model existing in period t into the period $t + 2$ regression to predict, for each item, its price. This would generate predictions of the prices of items existing in period t based on their z_i^t characteristics, at period $t + 2$ shadow prices, $\hat{p}_i^{t+2}(z_i^t)$. These prices (or an average) can be compared with the actual prices (or the average of prices) of models in period t, $p_i^t(z_i^t)$ as a, for example, Jevons hedonic base-period index:

$$P_{JHB} = \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} p_i^t(z_i^t)\right]^{1/N^t}} = \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} p_i^t\right]^{1/N^t}} \approx \frac{\left[\prod_{i=1}^{N^t} \hat{p}_i^{t+2}(z_i^t)\right]^{1/N^t}}{\left[\prod_{i=1}^{N^t} \hat{p}_i^t\right]^{1/N^t}}. \qquad (8.29a)$$

8.168 Alternatively, the characteristics of models existing in period $t + 2$ can be inserted into a regression for period t. Predicted prices of period $t + 2$ items generated at period t shadow prices, $p_i^t(z_i^{t+2})$, are the prices of items existing in period $t + 2$ estimated at period t prices, and these prices (or an average) can be compared with the actual prices (or the average of prices) in period $t + 2$, $p_i^{t+2}(z_i^{t+2})$; a Jevons hedonic current-period index is

$$P_{JHC} = \frac{\left[\prod_{i=1}^{N^{t+2}} p_i^{t+2}(z_i^{t+2})\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} \hat{p}_i^t(z_i^{t+2})\right]^{1/N^{t+2}}} = \frac{\left[\prod_{i=1}^{N^{t+2}} p_i^{t+2}\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} \hat{p}_i^t(z_i^{t+2})\right]^{1/N^{t+2}}} \approx \frac{\left[\prod_{i=1}^{N^{t+2t}} \hat{p}_i^{t+2}\right]^{1/N^{t+2}}}{\left[\prod_{i=1}^{N^{t+2}} \hat{p}_i^t(z_i^{t+2})\right]^{1/N^{t+2}}}. \qquad (8.29b)$$

8.169 For a fixed-base, bilateral comparison using either equation (8.29a) or (8.29b), the hedonic equation is estimated only for one period, the current period $t + 2$ in equation (8.29a) and the base period t in equation (8.29b). For reasons analogous to those explained in Chapters 16, 17, and 18, a symmetric average of these indices would have some theoretical support. It would be useful as a retrospective study to compare the results from both approaches (8.29a) and (8.29b). If the discrepancy is large, the results from either should be treated with caution, similar to the way a large Laspeyres and Paasche spread would cast doubt on the use of either of these indices individually. It would be evidence for the need to update the regressions more often.

8.170 Note that a geometric mean of equations (8.29a) and (8.29b) uses all of the data available in each period, as does the hedonic index using a time dummy variable in (8.28). If in (8.28) there is a new commodity in period $t + 2$, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old commodities drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure, unlike using matched data and hedonic adjustments on noncomparable replacements when commodities are no longer available.

8.171 With the dummy variable approach, there is no explicit weighting in its formulation in equations (8.29a) and (8.29b), and this is a serious disadvantage. In practice, cutoff sampling might be employed to include only the most important commodities or, if value of output data are available, a WLS—as opposed to OLS—estimator used with value of output shares as weights, as discussed in Appendix 22.1.

8.172 The indices ask counterfactual questions. Asking what the price of a model with characteristics z would have been if it had been on the market in a period ignores the likelihood that the appearance of that model would in turn alter the demand for other computers, thus altering the coefficients of the hedonic regression as well. The matter is particularly problematic when *backcasting*, that is, using a current period's specification in some previous period's regression as in equations (8.29a) and (8.29b). If the specifications increase rapidly, it may not be sensible to ask the value of some high-tech model when such technology was in an earlier stage of development. It should be kept in mind that hedonic coefficients may as much reflect production technology as demand (see Chapter 22), and old technologies simply may not have been able to produce goods to the standards of later ones. The question reversed—what would be the value of a previous period's specification in a subsequent period's regression—while subject to similar problems, may be more meaningful. In general, the solution lies in estimating regressions as often as possible, especially in markets subject to rapidly changing technologies.

8.173 There are alternative formulations to those in equations (8.29a and b). Some of the sampled varieties in period $t + 2$ and t will be matched, and there is a case for using the actual prices for these comparisons and predicted prices when unmatched and unavailable, as outlined by de Haan (2007).

8.174 There is also a formal analysis as to why the results from time dummy and the period-on-period (imputation) indices described above differ in Diewert, Heravi, and Silver (2009).

G.2.3 Superlative and exact hedonic indices (SEHI)

8.175 In Chapter 18 Laspeyres and Paasche bounds will be defined on a theoretical basis, as will superlative indices, which treat both periods symmetrically. These superlative formulas, in particular the Fisher index, are also seen in Chapter 17 to have desirable axiomatic properties. The Fisher index is supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds, and it was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index will be shown to be best from the stochastic viewpoint and also not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices are found to correspond to (be *exact* for) underlying Leontief aggregator functions with no substitution possibilities, whereas superlative indices are exact for flexible functional forms including the quadratic and translog forms for the Fisher and Törnqvist indices, respectively. If data on prices, characteristics, *and quantities* are available, analogous approaches and findings arise for hedonic indices (Fixler and Zieschang, 1992 and Feenstra, 1995). Consider a theoretical index for an export price index from the resident producer's perspective, but now only defined over products defined in terms of their characteristics. The prices are still of products, but they are wholly defined through *p(z)*. An arithmetic aggregation for a linear hedonic equation finds a Laspeyres lower bound (as quantities supplied are *increased* with increasing relative prices) is given by

$$\frac{R(p(z)^t, S(v)^{t-1})}{R(p(z)^{t-1}, S(v)^{t-1})} \geq \frac{\sum_{i=1}^{N} x_i^{t-1} \hat{p}_i^t}{\sum_{i=1}^{N} x_i^{t-1} p_i^{t-1}} = \sum_{i=1}^{N} s_i^{t-1} \left(\frac{\hat{p}_i^t}{p_i^{t-1}} \right) \tag{8.30a}$$

where $R(.)$ denote the revenue at a set of output prices; input quantities, v; and technology, S, following the fixed-input output price index model. The price comparison is evaluated at a fixed level of period $t - 1$ technology and inputs. s_i^{t-1} are the shares in total value of output of product i in period $t - 1$, $s_i^{t-1} = x_i^{t-1} p_i^{t-1} / \sum_{i=1}^{N} x_i^{t-1} p_i^{t-1}$ and

$$\hat{p}_i^t \equiv p_i^t - \sum_{k=1}^{K} \beta_k^t \left(z_{ik}^t - z_{ik}^{t-1} \right) \tag{8.30b}$$

are prices in period t adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. Note that the summation is over the same i in both periods because replacements are included when a product is missing and equation (8.30b) adjusts their prices for quality differences.

8.176 A Paasche upper bound is estimated as

$$\frac{R(p(z)^t, S(v)^t)}{R(p(z)^{t-1}, S(v)^t)} \leq \frac{\sum_{i=1}^{N} x_i^t \hat{p}_i^t}{\sum_{i=1}^{N} x_i^t p_i^{t-1}} = \left[\sum_{i=1}^{N} s_i^{*t} \left(\frac{p_i^{t-1}}{\hat{p}_i^t} \right) \right]^{-1} \tag{8.31a}$$

where $s_i^{*t} = x_i^t \hat{p}_i^t / \sum_{i=1}^{N} x_i^t \hat{p}_i^t$ and

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} + \sum_{k=1}^{K} \beta_k^{t-1} \left(z_{ik}^t - z_{ik}^{t-1} \right), \tag{8.31b}$$

which are prices in period $t-1$ adjusted for the sum of the changes in each quality characteristic weighted by its respective coefficients derived from a linear hedonic regression.

8.177 In Chapter 18 an economic theory of index numbers will be outlined and a theoretical output (export) defined for which the Laspeyres P_L and Paasche P_P form bounds on their "true" economic theoretic indices,

$$P_L \leq P(p^0, p^1, \alpha) \leq P_P. \tag{8.32}$$

8.178 The SEHI approach, thus, first utilizes the coefficients from hedonic regressions on changes in the characteristics to adjust observed prices for quality changes. Second, it incorporates a weighting system using data on the value of output of each model and their characteristics, rather than treating each model as equally important. Finally, it has a direct correspondence to formulation defined from economic theory.

8.179 Semi-logarithmic hedonic regressions would supply a set of β coefficients suitable for use with these base- and current-period geometric bounds:

$$\prod_{i=1}^{N} \left(\frac{p_i^t}{\hat{p}_i^{t-1}} \right)^{S_i^t} \geq \frac{R(p(z)^t, q, T)}{R(p(z)^{t-1}, q, T)} \geq \prod_{i=1}^{N} \left(\frac{\hat{p}_i^t}{p_i^{t-1}} \right)^{S_i^{t-1}}, \tag{8.33a}$$

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} \exp\left[\sum_{k=1}^{K} \beta_k^{t-1} \left(z_{ik}^t - z_{ik}^{t-1} \right) \right]$$

$$\hat{p}_i^t \equiv p_i^t \exp\left[-\sum_{k=1}^{K} \beta_k^{t-1} \left(z_{ik}^t - z_{ik}^{t-1} \right) \right]. \tag{8.33b}$$

8.180 In equation (8.33a) the two bounds on their respective theoretical indices have been shown to be brought together. The calculation of such indices is no small task. For examples of its application see Silver and Heravi (2001a and 2003) for comparisons over time and Kokoski, Moulton, and Zieschang (1999) for price comparisons across areas of a country.

8.181 Note that unlike the hedonic indices in Sections G.2.1 and G.2.2, the indices in both parts of equations (8.30), (8.31), and (8.33) need not be based on matched data. Silver and Heravi (2001a and 2003) used scanner data for the universe of transactions via a two-stage procedure whereby, first, cells were defined according to major price-determining features, such as all combinations of brand, outlet type, and (for television sets) screen size—much like strata. There may be a gain in the efficiency of the final estimate because the adjustment is for variation within strata, much in the way that stratified random sampling improves on simple random sampling. The average price in each matched cell could then be used for the price comparisons using (8.30a), (8.31a), or (8.33a), except that—to ensure that each cell's quality differences from characteristics other than these major ones did not influence the price comparison—adjustments would be made for quality changes using (8.30b), (8.31b), or (8.33b). This allowed all matched, old unmatched, and new unmatched data to be included because, if the average price in, say, a cell of (8.30a) was increased because of the inclusion of a new improved product, (8.30b) would be used to remove such improvements, on average. Consider, for example, a brand X, 14-inch television set without stereo sound, assembled by establishments in a given elementary aggregate industrial group. In the next period there may be matched cells: a 14-inch television set for brand X, but which also includes stereo. The new model may have to be grouped in the same cell with the brand X, 14-inch television sets with and without stereo and the average price of the cells compared in (8.30a), (8.31a), or (8.33a), with a quality adjustment for the stereo of the form (8.30b), (8.31b), or (8.33b) undertaken. There may be a gain in the efficiency of the final estimate because the adjustment is for within-strata variation, much in the way that stratified random sampling improves on simple random sampling. The estimated coefficient for stereo would be derived from a hedonic equation estimated from data of other television sets, some of which possess stereo.

8.182 The above has illustrated how weighted index number formulas might be constructed using data on prices, quantities, and characteristics for a product when the data are not matched. This is because continuing with matched data may lead to errors from (1) multiple quality adjustments from products no longer produced and their noncomparable replacements and (2) sample selectivity bias from sampling from a replacement universe as opposed to a double universe.

G.2.4 Difference between hedonic indices and matched indices

8.183 In previous sections, the advantages of hedonic indices over matched comparisons were referred to in terms of the inclusion by the former of unmatched data. This relationship is developed more formally here. Triplett (2004) argued and Diewert (2003) showed that an unweighted geometric mean (Jevons) index for matched data gives the same result as a logarithmic hedonic index run on the same data. Consider two periods so $T = 2$ and assume that the models are matched in each of the two periods so that the set of models in period 1 is equal to that in period 2, $S(1) = S(2)$, and the number of models $N(1) = N(2) \equiv M$ so that the same M models are available in each period. Hence the model characteristics are the same in each, that is, we have $z_{mtk} = z_{mk}$ say, for $t = 1,2$; $m = 1, \ldots, M$; and $k = 1, \ldots K$. Silver and Heravi (2005) have shown the following formula for the log of the *hedonic price index:*

$$\ln P_2/P_1 = \frac{1}{N(1\cap 2)}\Sigma_{m\in S(1\cap 2)} \ln\left(p_{m2}/p_{m1}\right) + \frac{1}{N(1\cap 2)}\sum_{m\in S(2\neg 1)}\left[\ln p_{m2} - \sum_{k=1}^{K} z_{m2k}\beta_k^* - \alpha_2^*\right] - \frac{1}{N(1\cap 2)}\sum_{m\in S(1\neg 2)}\left[\ln p_{m1} - \sum_{k=1}^{K} z_{m1k}\beta_k^* - \alpha_1^*\right]. \quad (8.34)$$

8.184 The first set of terms on the right-hand side of equation (8.34) is the *matched-model contribution* to the overall index, $\ln P_2/P_1$. The next two sets of terms are, respectively, the change in price due to unmatched models existing in period 2 but not in 1, and unmatched models existing in period 1 but not in 2. These expressions are not captured in a matched-models index. If the second set of terms, $[1/N(1\cap 2)] \times \sum_{m\in S(2\neg 1)}\left[\ln(p_{m2} - \sum_{k=1}^{K} z_{m2k}\beta_k^* - \alpha_2^*\right]$, is positive, then the matched-model price index is too low and must be adjusted upward. Consider a new model m introduced in period 2. If (the logarithm of) its price ($\ln p_{m2}$) is above that predicted from a period 2 hedonic regression $\left(\sum_{k=1}^{K} z_{m2k}\beta_k^* - \alpha_2^*\right)$, then this will raise the overall price index, and a matched-model index would be too low if it ignored such new models. Similarly, consider the last set of terms in equation (8.34) and an unmatched old model, introduced in period 1 but no longer available in period 2. If it was priced in period 1 above its period 1 predicted price then the matched-model price index would be too high (note the negative sign). The extent and nature of the bias depend on the pricing strategy of new and old models. The hedonic dummy variable approach, in its inclusion of unmatched old and new observations, can be seen from equation (8.34) to possibly differ from a geometric mean of matched price change. The extent of any difference depends, in this unweighted formulation, on the proportions of old and new commodities leaving and entering the sample and on the price changes of old and new ones relative to those of matched ones. If the market for commodities is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes. Different market behavior and changes in technology will lead to different forms of bias.

8.185 If sales weights replace the number of observations in equation (8.34), then different forms of weighted hedonic indices can be derived as explained in Chapter 22, Section A.5, and formally derived in Silver and Heravi (2005).

G.3 Chaining

8.186 An alternative approach for dealing with commodities with a high turnover is to use a chained index instead of the long-term fixed-base comparison. A chained index compares prices of items in period t with period $t + 1$ ($\text{Index}_{t,t+1}$) and then as a new exercise, studies the universe of commodities in period $t + 1$, and matches them with items in period $t + 2$. These links, $\text{Index}_{t,t+1}$ and $\text{Index}_{t+1,t+2}$, are combined by successive multiplication continuing to, say, $\text{Index}_{t+5,t+6}$ to form $\text{Index}_{t,t+6}$. Only items available in both period t and period $t + 6$ would be used in a fixed-base trade price index. Consider the five commodities 1, 2, 5, 6, and 8 over the four months of January to April as shown in Table 8.2. The price index for January compared with February (J:F) involves price comparisons for all five commodities. For (F:M), it involves commodities 1, 4, 5, and 8; for (M:A), it involves commodities 1, 3, 4, 5, 7, and 8. The sample composition changes for each comparison as commodities die and are born. Price indices can be calculated for each of these successive price comparisons using any of the unweighted formulas described in Chapter 22. The sample will grow when new commodities appear and shrink when old commodities disappear, changing in composition through time.

8.187 Sample depletion may be reduced in long-run comparisons by the judicious use of replacement items. However, as discussed in the next chapter, the replacement sample would include a new commodity only when a replacement was needed, irrespective of the number of new commodities entering the market. Furthermore, the replacement commodity is likely to be either of a similar quality, to facilitate quality adjustment and thus have relatively low sales, or be of a different quality with relatively high sales but requiring an extensive quality adjustment. In either case, this is unsatisfactory.

8.188 Chaining, unlike hedonic indices, does not use all the price information in the comparison for each link. Commodities 2 and 6, for example, may be missing in March. The index makes use of the price information on commodities 2 and 6, when they exist, for the January–February comparison but does not allow their absence to disrupt the index for the February–March comparison. It may be that commodity 4 is a replacement for commodity 2. Note how easily it is included as soon as two price quotes become available. There is no need to wait for rebasing or sample rotation. It may be that commodity 7 is a replacement for commodity 6. A quality adjustment to prices may be required for the February–March comparison between commodities 6 and 7, but this is a short-run, one-off adjustment, The compilation of the index continues for March–April using commodity 7 instead of commodity 6.

8.189 The chained approach has been justified as the natural discrete approximation to a theoretical Divisia index (Forsyth and Fowler, 1981; and Chapter 17). Reinsdorf (1998) has formally determined the theoretical underpinnings of the index, concluding that in general, chained indices will be good approximations of the theoretical ideal. However, they are prone to bias when price changes "swerve and loop," as Szulc (1983) has demonstrated (see also Forsyth and Fowler, 1981; and de Haan and Opperdoes, 1997).

8.190 The dummy variable hedonic index uses all of the data in January and March for a price comparison between the two months. Yet the chained index ignores unmatched successive pairs as outlined above; nevertheless, this is preferable to its fixed-base equivalent. The hedonic approach, by predicting from a regression equation, naturally has a confidence interval attached to such predictions. The width of the interval is dictated by the fit of the equation, the distance of the characteristics from their mean, and the number of observations. Matching, chained or otherwise, does not suffer from any prediction error. Aizcorbe, Corrado, and Doms (2001) undertook an extensive and meticulous study of high-technology goods (personal computers and semiconductors) using quarterly data for the period 1993–99. The results from comparable hedonic and chained indices were remarkably similar over the seven years of the study. For example, for desktop central processing units, the index between the seven years of 1993: Q1 and 1999: Q4 fell by 60.0 percent (dummy variable hedonic), 59.9 percent (chained Fisher), and 57.8 percent (chained geometric mean). The results differed only in quarters when there was a high turnover of commodities, and, in these cases, such differences could be substantial. For example, for desktop central processing units in 1996: Q4, the 38.2 percent annual fall measured by the dummy variable hedonic method differed from the chained geometric mean index by 17 percentage points. Thus, with little model turnover, there is little discrepancy between hedonic and chained matched-models methods and, for that matter, fixed-base matched indices. It is only when binary comparisons or links have a high model turnover that differences arise (see also Silver and Heravi, 2003 and 2005).

8.191 There is a possibility that the introduction of new models and exits of old ones instantaneously affects the prices of all existing models. In such a case, the price changes of existing models will suffice. They will reflect the price changes of new entrants and old departures not part of the sample. This argument is used for the case that direct matched-models comparisons, chained matched-models comparisons, and hedonic indices should give the same results. It is an empirical matter, and its plausibility will vary among industries. It is more likely to apply to fast-moving goods with few to no development costs or barriers to entry.

8.192 It is possible to make up for missing prices by using a partial, patched hedonic estimate as discussed above. Dulberger (1989) computed hedonic indices for computer processors and compared the results with those from a matched-models approach. The hedonic dummy variable index fell by about 90 percent from 1972–84, about the same as for the matched-models approach where missing prices for new or discontinued commodities were derived from a hedonic regression. However, when using a chained matched-models approach with no estimates or imputations for missing prices, the index fell by 67 percent. It is also possible to combine methods; de Haan (2007) used matched data when available and the time dummy only for unmatched data—his double-imputation method.

H. Long-Run and Short-Run Comparisons

8.193 This section outlines a formula to help quality adjustment. The procedure can be used with all of the methods outlined in Sections D and E. Its innovation arises from a possible concern with the long-run nature of the quality-adjusted price comparisons being undertaken. In the example in Table 8.2, prices in March were compared with those in January. Assumptions of similar price changes are required by the imputation method to hold over this period for long-run imputations. This gives rise to increasing concern when price comparisons continue over longer periods, such as between January and October, January and November, and January and December, and even subsequently. In this section, a *short-run* formulation outlined in Sections C.3.3 and D.2 is more formally considered to help alleviate such concerns. Consider Table 8.5, which, for simplicity, has a single commodity A that exists throughout the period, a commodity B that is permanently missing in April, and a possible replacement C in April.

H.1 Short-run comparisons: Illustration of some quality-adjustment methods

8.194 A *comparable replacement C* may be found. In the previous example, the focus was on the use of the Jevons index at the elementary level because it is shown in Chapter 21 that this has much to commend it. The example here uses the Dutot index, the ratio of arithmetic means. This is not to advocate it but only to provide an example using a different formulation. The Dutot index also has much to commend it on axiomatic grounds but fails the commensurability (units of measurement) test and should be used only for homogeneous items. The long-run Dutot index for April compared with January is

$$P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{\text{Apr}}/N}{\sum_{i=1}^{N} p_i^{\text{Jan}}/N}\right],$$

which is 8/5 = 1.30, a 30 percent increase. The *short-run* equivalent is the commodity of a long-run index up to the immediately preceding period and an index for the preceding to the current period, that is, for period $t + 4$ compared with period t:

$$P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{t+3}/N}{\sum_{i=1}^{N} p_i^{t}/N}\right] \times \left[\frac{\sum_{i=1}^{N} p_i^{t+4}/N}{\sum_{i=1}^{N} p_i^{t+3}/N}\right], \tag{8.35}$$

Table 8.5. Example of Long-Run and Short-Run Comparisons

Item	January	February	March	April	May	June
Comparable replacement						
A	2	2	2	2	2	2
B	3	3	4	n/a	n/a	n/a
C	n/a	n/a	n/a	6	7	8
Total	5	5	6	8	9	10
Explicit adjustment						
A	2	2	2	2	2	2
B	3	3	4	**(5/6)6 = 5**	**(5/6)7 = 5.8**	**(5/6)8 = 6.67**
C	**(6/5)3 = 3.60**	n/a	n/a	6	7	8
Total	5	5	6	8	9	10
Overlap						
A	2	2	2	2	2	2
B	3	3	4	**6(4/5) = 4.8**	n/a	n/a
C	n/a	n/a	5	6	7	8
Total	5	5	6	6.8	9	10
Imputation						
A	2	2	2.5	3.5	4	5
B	3	3	4	**(3.5/2.5)4 = 5.6**	**(4/3.5)5.6 = 6.4**	**(5/4)6.4 = 8**
Total	5	5	6.5	9.1	8.4	13

Note: n/a = not available. Figures in bold are estimated quality-adjusted prices described in the text.

or, for example, using a comparison of January with April:

$$P_D \equiv \left[\frac{\sum_{i=1}^{N} p_i^{\text{Mar}}/N}{\sum_{i=1}^{N} p_i^{\text{Jan}}/N}\right] \times \left[\frac{\sum_{i=1}^{N} p_i^{\text{Apr}}/N}{\sum_{i=1}^{N} p_i^{\text{Mar}}/N}\right],$$

which is of course $\frac{6}{5} \times \frac{8}{6} = 1.30$ as before.

8.195 Consider a *noncomparable replacement with an explicit quality adjustment:* say *C*'s value of 6 in April is quality-adjusted to be considered to be worth only 5 when compared with the quality of *B*. The quality adjustment to prices may have arisen from an option cost estimate, a quantity adjustment, a subjective estimate, or a hedonic coefficient as outlined above. Suppose the long-run comparison uses an adjusted January price for *C*, which is *B*'s price of 3 multiplied by 6/5 to upgrade it to the quality of *C*, that is, $6/5 \times 3 = 3.6$. From April onward, the prices of the replacement commodity *C* can be readily compared with its January reference period price. Alternatively, the prices of *C* in April onward might have been adjusted by multiplying them by 5/6 to downgrade them to the quality of *B* and enable comparisons to take place with commodity *B*'s price in January: for April the adjusted price is $5/6 \times 6 = 5$; for May, the adjusted price is 5.8; and for June, it is 6.67 (see Table 8.5). Both procedures yield the same results for long-run price comparisons. The results from both methods (rounding errors aside) are the same for commodity *B*.

8.196 However, for the overall Dutot index, the results will differ because the Dutot index weights price changes by their price in the initial period as a proportion of total price (Chapter 21, equation (21.1)). The two quality-adjustment methods will have the same price changes but different implicit weights. The Dutot index in May is $9/5.6 = 1.607$ using an adjustment to the initial period, January's price, and $7.8/5 = 1.56$ using an adjustment to the current period, May's price. The short-run indices give the same results for each adjustment:

$\frac{8}{5.6} \times \frac{9}{8} = 1.607$ using an adjustment to the initial period, January's price, and

$\frac{7}{5} \times \frac{7.8}{7} = 1.56$ using an adjustment to the current period, May's price.

8.197 The *overlap method* may also take the short-run form. In Table 8.5, there is a price for *C* in March of 5 that overlaps with *B* in March. The ratio of these prices is an estimate of their quality difference. A long-run comparison between January and April would be $\left(6 \times \frac{4}{5} + 2\right)/5 = 1.36$. The short-run comparison would be based on the commodity of the January to March and March to April link: $\left(\frac{6.8}{6} \times \frac{6}{5}\right) = 1.36$.

8.198 At this unweighted level of aggregation, it can be seen that there is no difference between the long-run and short-run results when commodities are not missing, comparable replacements are available, explicit adjustments are made for quality, or the overlap method is used. The separation of short-run (most recent month-on-month) and long-run changes may have advantages for quality assurance to help spot unusual short-run price changes. But this is not the concern of this chapter. The short-run approach does, however, have advantages when imputations are made.

H.2 Implicit short-run comparisons using imputations

8.199 The use of the short-run framework has been considered mainly for temporarily missing values, as outlined by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2000). However, similar issues arise in the context of quality adjustment. Consider again Table 8.5, but this time there is no replacement commodity *C,* and commodity *A*'s prices have been changed to trend upward. Commodity *B* is again missing in April. A long-run imputation for commodity *B* in April is given by $\frac{3.5}{2} \times 3 = 5.25$. The price change is thus $(5.25 + 3.5)/5 = 1.75$, or 75 percent. One gets the same result by simply using commodity *A* $(3.5/2 = 1.75)$, because the implicit assumption is that price movements of commodity *B*, had it continued to exist, would have followed those of *A*. However, the assumption of similar long-run price movements may in some instances be difficult to support over long periods. An alternative approach would be to use a short-run framework whereby the imputed price for April is based on the (overall) mean price change between the preceding and current period, that is, $\frac{3.5}{2.5} \times 4 = 5.6$ in the above example. In this case, the price change between March and April is $(5.6 + 3.5)/(2.5 + 4) = 1.40$. This is combined with the price change between January and March: $(6.5/5) = 1.30$, making the price change between January and April $1.30 \times 1.40 = 1.82$, an 82 percent increase.

8.200 Consider why the short-run result of 82 percent is larger than the long-run result of 75 percent.

The price change for *A* between March and April of 40 percent, on which the short-run imputation is based, is larger than the average *annual* change of *A*, which is just over 20 percent. The extent of any bias from this approach was found in the previous section to depend on the ratio of missing values and the difference between the average price changes of the matched sample and the quality-adjusted price change of the commodity that was missing, had it continued to exist. The short-run comparison is to be favored if, as is likely, the assumption of similar price changes is considered more likely to hold than the long-run one.

8.201 There are data on price changes of the commodity that is no longer available—commodity *B* in Table 8.5—up to the period preceding the period in which it is missing. In Table 8.5, commodity *B* has price data for January, February, and March. The long-run imputation makes no use of such data by simply assuming that price changes from January to April are the same for *B* as for *A*. Let the data for *B*'s prices in Table 8.5 (second to last row) now be 3, 4, and 6 in January, February, and March, respectively, instead of 3, 3, and 4. The long-run estimate for *B* in April is 5.25 as before. The estimated price change between March and April for *B* is now a *fall* from 6 to 5.25. A short-run imputation based on the price movements of *A* between March and April would more correctly show an increase from 6 to $(3.5/2.5) \times 6 = 8.4$.

8.202 There may, however, be a problem with the continued use of short-run imputations. Returning to the data for *A* and *B* in Table 8.5, consider what happens in May. Adopting the same short-run procedure, the imputed price change is given in Table 8.5 as $4/3.5 \times 5.6 = 6.4$ and for June as $(5/4) \times 6.4 = 8$. In the former case, the price change from January to May is

$$\left[\frac{(6.4 + 4)}{(5.6 + 3.5)}\right] \times \left[\frac{(5.6 + 3.5)}{(3 + 2)}\right] = 2.08$$

and in the case of June

$$\left[\frac{(8 + 5)}{(6.4 + 4)}\right] \times \left[\frac{(6.4 + 4)}{(3 + 2)}\right] = 2.60$$

against long-run comparisons for May:

$$\left[\frac{((4/2) \times 3 + 4)}{(3 + 2)}\right] = 2.00$$

and long-run comparisons for June:

$$\left[\frac{((5/2) \times 3 + 5)}{(3 + 2)}\right] = 2.50.$$

8.203 A note of caution is required here. The comparisons use an imputed value for commodity *B* in April and also an imputed one for May. The price comparison for the second term in equation (8.35), for the current versus immediately preceding period, uses imputed values for commodity *B*. Similarly, for the January to June results, the May to June comparison uses imputed values for commodity *B* for both May and June. The pragmatic needs of quality adjustment may demand this. If comparable replacements, overlap links, and resources for explicit quality adjustment are unavailable, an imputation must be considered. However, using imputed values as lagged values in short-run comparisons introduces a level of error into the index that will be compounded with their continued use. Long-run imputations are likely to be preferable to short-run changes based on lagged imputed values unless there is something in the nature of the industry that cautions against such long-run imputations. There are circumstances when the respondent may believe the missing commodity is missing temporarily, and the imputation is conducted under the expectation that production will subsequently continue. A wait-and-see policy is adopted under some rules—three months, for example—after which it is deemed to be permanently missing. These are the pragmatic situations that require imputations to extend over consecutive periods. These circumstances promote lagged imputed values to compare against current imputed values. This is cautioned against, especially over a period of several months. There is an intuition that the period in question should not be extensive. First, the effective sample size is being eaten up as the use of imputation increases. Second, the implicit assumptions of similar price movements inherent in imputations are less likely to hold over the longer run. Finally, there is some empirical evidence, albeit from a different context, against using imputed values as lagged actual values. (See Feenstra and Diewert's 2000 study using data from the U.S. Bureau of Labor Statistics for its International Price Program.)

8.204 The short-run approach described above will be developed in the next section, where weighted indices are considered. The practice of estimating quality-adjusted prices is usually at the elementary commodity level. At this lower level, the prices of commodities may subsequently be missing, and replacements with or without adjustments and imputations are used to allow the series to continue. New commodities and varieties are also being introduced; the switching of sales between sections of the index becomes prevalent. The turmoil of changing quality is not just about the maintaining of similar price comparisons but also about the accurate reweighting of the mix of what is produced. Under a Laspeyres framework, the bundle is held constant in the base period, so any change in the relative importance of commodities produced is held to be of no concern until the next rebasing of the index. Yet capturing some of the very real changes in the mix of what is produced requires procedures for updating the weights. This was considered in Chapter 6. The concern here is with a higher-level procedure equivalent to the short-run adjustments discussed above. It is one particularly suited to countries where resource constraints prohibit the regular updating of weights through regular household surveys.

H.3 Single-stage and two-stage indices

8.205 Consider aggregation at the elementary level. This is the level at which prices are collected from a representative selection of establishments across regions in a period and compared with the matched prices of the same commodities in a subsequent period to form an index for a good. Lamb is an example of a good in an index. Each price comparison is equally weighted unless the sample design gave proportionately more chance of selection to commodities with more sales. The elementary price index for lamb is then weighted and combined with the weighted elementary indices for other commodities to form the XMPI. A Jevons elementary aggregate index, for example, for period $t + 6$ compared with period t is given as

$$P_J = \sum_{i \in N(t+6) \cap N(t)} \left(p_i^{t+6}/p_i^t\right). \tag{8.36}$$

Compare this with a two-stage procedure:

$$P_J = \sum_{i \in N(t+5) \cap N(t)} \left(p_i^{t+5}/p_i^t\right) \times \sum_{i \in N(t+5) \cap N(t+6)} \left(p_i^{t+6}/p_i^{t+5}\right). \tag{8.37}$$

8.206 If a commodity is missing in period $t + 6$, an imputation may be undertaken. If equation (8.36) is used, the requisite assumption is that the price change of the missing commodity, had it continued, is equal to that of the average of the remaining commodities *over the period* t *to* t + *6*. In equation (8.37), the missing commodity in period $t + 6$ may be included in the first stage of the calculation, between periods t and $t + 5$, but excluded in the second stage, between periods $t + 5$ and $t + 6$. The requisite assumption is that price changes *between* t − *1 and* t are similar. Assumptions of short-run price changes are generally considered to be more valid than their long-run counterparts. The two-stage framework also has the advantage of including in the worksheet prices for the current period and the immediately preceding one, which, as will be shown in Chapter 10, promotes good data validity checks.

8.207 Feenstra and Diewert (2000) applied a number of mainly short-run imputation procedures to price comparisons for the U.S. Bureau of Labor Statistics International Price Program. Although such price indices are not the direct interest of this *Manual*, the fact that about one-quarter of the individual commodities tracked did not have price quotations in any given month makes it an interesting area to explore the results from different imputation procedures. When using the two-stage procedure, they advise against carrying forward imputed prices as if they were actual values for the subsequent price comparison. The resulting price relatives for the subsequent period based on prior imputations had a standard deviation about twice that of price relatives where no imputation was required, leading them to conclude that such a practice introduced a significant amount of "noise" into the calculation. Feenstra and Diewert (2000) found more variance in price changes in the long-run imputation method than the short-run method. They also found from both theory and empirical work that when actual prices are available in a future data set and they are used to interpolate back on a linear basis the missing prices, such estimates lead to much lower variances than the short-run imputation approach. However, such linear interpolations require the statistical agency to store past information until a price quote becomes available, interpolate back the missing price, and then publish a revised XMPI.

Appendix 8.1 Data for Hedonic Regression Illustration

Price (£)	Speed (MHz)	RAM	HD	Dell	Presario	Prosignia	Celeron	Pentium III	CD-RW	DVD	Dell × Speed
2,123	1,000	128	40	0	1	0	0	0	0	0	0
1,642	700	128	40	0	1	0	0	0	0	0	0
2,473	1,000	384	40	0	1	0	0	0	0	0	0
2,170	1,000	128	60	0	1	0	0	0	0	0	0
2,182	1,000	128	40	0	1	0	0	0	0	1	0
2,232	1,000	128	40	0	1	0	0	0	1	0	0
2,232	1,000	128	40	0	1	0	0	0	0	0	0
1,192	700	384	40	0	1	0	0	0	0	0	0
1,689	700	384	60	0	1	0	0	0	0	0	0
1,701	700	384	40	0	1	0	0	0	0	1	0
1,751	700	384	40	0	1	0	0	0	1	0	0
1,851	700	384	40	0	1	0	0	0	0	0	0
2,319	933	128	15	0	0	0	0	1	0	0	0
2,512	933	256	15	0	0	0	0	1	0	0	0
2,451	933	128	30	0	0	0	0	1	0	0	0
2,270	933	128	10	0	0	0	0	1	0	0	0
2,463	933	256	10	0	0	0	0	1	0	0	0
2,183	933	64	10	0	0	0	0	1	0	0	0
1,039	533	64	8	0	0	1	1	0	0	0	0
1,139	533	128	8	0	0	1	1	0	0	0	0
1,109	533	64	17	0	0	1	1	0	0	0	0
1,180	533	64	8	0	0	1	1	0	1	0	0
1,350	533	128	17	0	0	1	1	0	1	0	0
1,089	600	64	8	0	0	1	0	1	0	0	0
1,189	600	128	8	0	0	1	0	1	0	0	0
1,159	600	64	17	0	0	1	0	1	0	0	0
1,230	600	64	8	0	0	1	0	1	1	0	0
1,259	600	128	17	0	0	1	0	1	0	0	0
1,400	600	128	17	0	0	1	0	1	1	0	0
2,389	933	256	40	0	1	0	0	1	0	0	0
1,833	733	256	40	0	1	0	0	1	0	0	0
2,189	933	128	40	0	1	0	0	1	0	0	0
2,436	933	256	60	0	1	0	0	1	0	0	0
2,397	933	256	40	0	1	0	0	1	0	1	0
2,447	933	256	40	0	1	0	0	1	1	0	0
2,547	933	256	40	0	1	0	0	1	0	0	0
2,845	933	384	60	0	1	0	0	1	0	0	0
2,636	933	384	60	0	1	0	0	1	0	0	0
1,507	733	64	30	0	1	0	0	1	0	0	0
1,279	667	64	10	1	0	0	0	1	0	0	667

Appendix 8.1 Data for Hedonic Regression Illustration (*concluded*)

Price (£)	Speed (MHz)	RAM	HD	Dell	Presario	Prosignia	Celeron	Pentium III	CD-RW	DVD	Dell × Speed
1,379	667	128	10	1	0	0	0	1	0	0	667
1,399	667	64	30	1	0	0	0	1	0	0	667
1,499	667	128	30	1	0	0	0	1	0	0	667
1,598	667	128	30	1	0	0	0	1	1	0	667
1,609	667	128	30	1	0	0	0	1	0	1	667
1,389	667	64	10	1	0	0	0	1	0	1	667
999	667	64	10	1	0	0	1	0	0	0	667
1,119	566	64	30	1	0	0	1	0	0	0	566
1,099	566	128	10	1	0	0	1	0	0	0	566
1,097	566	64	10	1	0	0	1	0	1	0	566
1,108	566	64	10	1	0	0	1	0	0	1	566
1,219	566	128	30	1	0	0	1	0	0	0	566
1,318	566	128	30	1	0	0	1	0	1	0	566
1,328	566	128	30	1	0	0	1	0	0	1	566
1,409	566	128	10	1	0	0	0	1	0	0	566
1,809	733	384	10	1	0	0	0	1	0	0	733
1,529	733	128	30	1	0	0	0	1	0	0	733
1,519	733	128	10	1	0	0	0	1	0	1	733
1,929	733	384	30	1	0	0	0	1	0	0	733
2,039	733	384	30	1	0	0	0	1	0	1	733
2,679	933	128	30	1	0	0	0	1	0	0	933
3,079	933	384	10	1	0	0	0	1	0	0	933
2,789	933	128	10	1	0	0	0	1	0	1	933
3,189	933	384	10	1	0	0	0	1	0	1	933

9. Commodity Substitution, Sample Space, and New Goods

A. Introduction

9.1 In this chapter, as with Chapter 8, the focus is on new goods (and services) for export and import price indices (XMPIs) based on establishment surveys. This is because when new goods appear in a commodity classification for unit values based on customs data, the unit value index is no longer comparing the prices of like items with like and is affected by the change in compositional mix. The problem of including new goods, and the related problem of the treatment of obsolete ones, has, for unit values based on customs documents, no immediate solution. The problem as outlined in Chapter 2, and Chapter 6, Section C, considers issues of detection of unit value aggregates composed of more than one item that might signal the presence of a new good, though the treatment of such groups is problematic. For XMPIs based on establishment surveys the problem is tractable. In the introduction to Chapter 8, the use of the matched-models method was recognized as the accepted approach to ensure that the measurement of price changes was untainted by changes in the quality of the commodities whose prices are compared. However, it was noted that the approach may fail in three respects: missing commodities, sampling issues, and new goods and services (hereafter "goods" includes services). Missing commodities were the subject of Chapter 8, in which several implicit and explicit methods of quality adjustment to prices, and the choice between them, were discussed. In this chapter, attention is turned to the two other reasons why the matched-models method may fail: sampling issues and new goods. The three sources of potential error are briefly outlined.

- **Missing commodities.** A problem arises when a commodity is no longer produced for export or purchased as an import. An imputation may be made of the price the commodity would have had, had it been available. Alternatively, the responding exporter or importer may choose a replacement commodity of a comparable quality, and its price would be directly compared with the missing commodity's price. If the replacement is of a noncomparable quality, the overlap method might be used to "link in" the price change of the replacement, or an explicit price adjustment might be undertaken. This was the subject of Chapter 8, Sections C through F. In Section G of Chapter 8, a caveat was added. For commodities in industries where model replacements were rapid, continued long-run matching would deplete the sample and quality adjustment becomes unfeasible on the scale required. Chained matching or hedonic indices were deemed preferable.
- **Sampling issues.** The matching of prices of identical commodities over time is, by its nature, likely to lead to the monitoring of a sample of commodities increasingly unrepresentative of the population of transactions. Many new commodities may be traded, but the sample will be constrained to the original matched commodities, and new commodities will be introduced only on a one-for-one commodity replacement basis. Respondents may keep with their selected commodities until they are no longer produced—that is, continue to monitor commodities with unusual price changes and limited sales. Yet on commodity replacement, respondents may select unpopular comparable commodities to avoid explicit quality adjustments; obsolete commodities with unusual price changes are replaced by near-obsolete commodities that also have unusual price changes, compounding the problem of unrepresentative samples. The substitution of a commodity with relatively high sales for an obsolete one has its own problems, because the difference in quality is likely to be substantial and substantive, beyond what can be attributed to, say, the price difference in some overlap period. One would be in the last stage of its life cycle and the other in its first. The issue has implications for sample rotation and commodity substitution.

- **New goods.** A third potential difficulty arises when something "new" is traded. When a new good is produced, it needs to be included in the index as soon as possible, especially if the good is expected to be responsible for relatively high sales. New goods might have price changes quite different from existing ones, especially at the start of their life cycle. In the initial period of introduction of a new commodity, or a variety of an existing one, producers often set higher prices than what might be attainable once the market settles into a competitive equilibrium. But by definition, there is no price in the period preceding the introduction of the new good. So even if prices of new goods were obtained and included in the index as from the initial introduction date, something would still be missing: the initial high price producers can reap by exploiting any monopoly power compared with its hypothetical price in the period prior to its introduction. There is a related problem of "old" goods. Again, the price changes of such goods may be unusual. The goods will be at the end of their life cycle and may be priced at unusually low prices to clear the way for new models. However, there is a price change that is missed. It is the hypothetical price the good would have had, had it existed in the period after its demise, compared with its price in its last period. Such issues are considered in Section D below.

9.2 The problem of missing commodities was the subject of Chapter 8. In this chapter, sampling issues arising out of the matched-models approach and the problem of introducing new goods into the index are considered. As with missing commodities, the sampling issues and new goods problem can be quite severe for trade price indices. Whereas producers have set up costs that usually result in a stream of output over a relatively long period, importers can sporadically purchase and drop new goods and new varieties of existing goods. The industrial concentration of exports and imports is generally higher than it is for production for and consumption in the domestic market. New goods and new varieties of existing goods thus may have a greater impact on trade price indices than on producer or consumer price indices.

B. Sampling Issues and Matching

B.1 Introduction

9.3 The matching procedure has at its roots a conundrum. Matching is designed to avoid price changes being contaminated by quality changes. Yet its adoption constrains the sampling to a static universe of commodities that exist in *both* the reference and base periods. Outside of this there is of course something more: commodities that exist in the reference period but not in the current period, and are therefore not matched; and similarly those new commodities existing in the current period but not in the reference one—the dynamic universe. The conundrum is that the commodities not in the matched universe, the new commodities appearing after the reference period, and the old commodities that disappeared from the current period may be the ones whose price changes differ substantially from existing matched ones. They will embody different technologies and be subject to different (quality-adjusted) strategic price changes. The very device used to maintain a constant quality sample may itself give rise to a sample biased away from technological developments. Furthermore, when this sample is used to make imputations (Chapter 8, Sections D.1 and D.2) as to the price changes of replacement commodities, it reflects the technology of a sample not representative of current technological changes.

9.4 The above problem has been outlined in terms of a commodity having to "exist" in both of the periods being compared. The concern in this respect is for the respondent being able to return a price quote for the month in question for the comparable, matched commodity selected and priced in the price reference period. Of course a commodity may not be exported or imported in a given month and thus not "exist" in the above sense but still be domestically produced for the domestic market of the importing or exporting country. For an export price index such prices for the domestic market may be carried across to the export market only if there is clear evidence that there is not, and will not be, any price discrimination between the two markets. For the discussion below, it is assumed that missing commodities do not have an equivalent, comparable price for the domestic market.

9.5 A formal consideration of matching and the dynamic universe is provided in Appendix 9.1. Three universes are considered:

- An *intersection* universe, which includes only matched commodities;
- A dynamic *double universe,* which includes all commodities in the base comparison period and all in the current period, although they may be of different qualities; and

- A *replacement* universe, which starts with the base period universe but also includes one-to-one replacements when a commodity from the sample in the base period is missing in the current period.

9.6 It is, of course, difficult to ascertain the extent to which matching from the intersection universe constrains the penetration of the sample into the dynamic double universe, because statistical agencies generally do not collect data for the latter. Its extent will, in any event, vary between commodities. A commodity replacement, when a commodity is missing, is an opportunity to bring in a commodity with a relatively large traded value to increase the coverage of the index. However, the selection of commodity substitutes (replacements) by respondents puts coverage of the sample to some extent under the control of the respondents. Guidelines on directed replacements in particular industries have some merit. Second, chaining, hedonic indices (as considered in Chapter 8, Section G) and regular sample rotation also have merit in some industries as devices that help refresh the sample.

B.2 Sample space and commodity replacement or substitution

9.7 The respondents often are best placed to select replacement commodities for repricing. They are aware of not only the technological basis of the commodities being produced or purchased but also their terms of sale. The selection of the replacement for repricing might be quite obvious to the respondent. There may be only a slight, nominal improvement to the commodity. For example, the "improved" commodity is simply a replacement variety sold instead of the previous one. The replacement could have a different code or model number and will be known to the respondent as simply a different color or packaging. The specification list given to the respondent is a critical prompt as to when a repriced commodity is different, and it is important that this list include all price-determining factors.

9.8 The respondent, prompted by the specification list, takes on the role of identifying whether a commodity is of comparable quality or otherwise. If it is judged to be comparable when it is not, the quality difference is taken to be a price difference, and a bias will result if the unrecognized quality changes are in a consistent direction. Informed comparable substitution requires general guidelines on what makes a good substitute as well as commodity-specific information on likely price-determining characteristics. It also requires timely substitution to maximize the probability of an appropriate substitute being available.

9.9 On repricing, respondents traditionally are required to find substitute commodities that are as similar as possible to the commodities being replaced. This maximizes the likelihood that the old and replacement commodity will be judged equivalent and so minimizes the need to employ some method of quality adjustment. Yet, replacement commodities should be chosen so that they intrude into the sampled commodities in a substantial and representative manner so as to make the sampled commodities more representative of the dynamic universe. The inclusion of a popular replacement commodity to refresh the sample—one at the same point in its life cycle as the original popular one selected in the base period—allows for a useful and accurate price comparison and increases the chance of an appropriate quality adjustment being undertaken. It is of little merit to substitute a new commodity with limited sales for a missing commodity with limited sales just because they both have similar features of being "old." The index would become more unrepresentative. Yet if replacements are made for commodities at the end of their life with popular replacement commodities at the start of their life, the quality adjustment will be substantial and substantive. More frequent sample rotation or directed replacements will be warranted for some commodity areas.

- Replacements offer an opportunity to cut back on and possibly remove sample bias in the period of replacement, though not prior to it;
- The more frequent the replacement, the less the bias;
- If there is more than one new (replacement) commodity in the market, there may still be bias because only the most popular one will be selected, and it may be at a different stage in its life cycle than others and priced differently;
- The analysis assumes that perfect quality adjustments are undertaken on replacements. The less frequent the replacement, the more difficult this might be, because the very latest replacement commodity on the market may have more substantial differences in quality than earlier ones;
- If the replacement commodity has relatively high sales, is of comparable quality, and is at the same stage in its life cycle as the existing one, then its selection will minimize bias;
- If there is more than one new (replacement) commodity and the most comparable one is selected at

the old technology, it will have low market share and unusual price changes; and

- Given advance market or production information, replacements undertaken before obsolescence are likely to increase the sample's share of the market, include commodities more representative of the market, and facilitate quality adjustment.

9.10 The problem of commodity substitution is analogous to the problem that arises when an establishment closes. It may be possible to find a comparable establishment not already in the sample, or a noncomparable one for which, in principle, an adjustment can be made for the better quality of service of the new one. It is not unusual for an establishment to close following the introduction of a new factory. Thus, there is an obvious replacement factory. However, if the new establishment has comparable prices but a better range of commodities, delivery, and service quality, there is a gain to purchasers from substituting one factory's output for the other. Yet, because such facilities have no direct price, it is difficult to provide estimates of the value of such services in order for an adjustment to be made for the better quality of service of the new one. The index thus would have an upward bias, which would be lost on rebasing. In such cases, substituting the old establishment, where possible, for a new one that provides a similar standard of service would be preferable.

B.3 Sample rotation, chaining, and hedonic indices

9.11 In the previous section the replacement universe was considered with replacements as substitutes for missing or "obsolete" commodities. The double universe is preferable because it includes information on all commodities in each period. At an elementary aggregate level, customs unit value data might in principle include all such information. However, the unit value change will include changes in quantities as well as prices for what may not be homogeneous commodities. If the commodities are heterogeneous, then survey prices of a narrower yet representative range of commodities should be used for the elementary aggregate in question. Yet following the prices of such representative commodities over time runs the risk of their becoming unrepresentative.

9.12 For some industries, the samples of commodities used will become quite out of date if the sample is not reinitiated until the next rebasing. This is especially the case if the rebasing is infrequent. Sample rotation is equivalent to initiating a new sample, but it is only done for an industry group(s) that maintain the same weights until the next rebasing. Sample rotation is undertaken for specific industries at different points in time to save on the resources required if all the industries were to have their commodities rotated at the same time. The criteria for choice of industries to benefit from sample rotation, and the timing of the rotation, should be clearly and openly scheduled in advance according to objective criteria.

9.13 It is important also to recognize the interrelationships among the methods for handling commodity rotation, commodity replacement, and quality adjustment. When XMPI commodity samples are rotated, this is a form of commodity substitution, except that it is not "forced" by a missing commodity but is undertaken for a general group of commodities to update the sample. Rotation has the effect of making future forced replacements less likely. Yet the assumptions implicit in its use are equivalent to those for the overlap adjustment technique: Price differences are an adequate proxy for the change in price per unit of quality between commodities disappearing from the sample and replacement commodities. Consider the initiation of a new sample of commodities. Prices for the old and new sample are returned in the same month and the new index is compiled on the basis of the new sample, with the results being linked to the old. This is an implicit use of the overlap method, in which all price differences between the new and old commodities are taken to be estimates of the price differential owing to quality differences. Assume the initiation is in January. The prices of an old commodity in December and January are 10 and 11, respectively, a 10 percent increase, and those for the replacement commodity in January and February are 16 and 18, respectively, an increase of 12.5 percent. The new commodity in January is of a better quality than the old, and this difference in quality may be worth $16 - 11 = 5$; that is, the price difference is assumed to be equal to the value of the quality difference, which is the assumption implicit in the overlap method. Had the price of the old commodity in December been compared with the quality-adjusted price of the new commodity in January under this assumption, the price change would be the same: 10 percent (i.e., $(16 - 5)/10 = 1.10$). If, however, the price difference in January was an inappropriate reflection of the quality difference, say the old commodity was being dumped at an unrealistically low price to clear the market for the new one, then the implicit assumption underlying the overlap method does not hold. In practice, the need to simultaneously replace and update a large number of commodities requires the assumptions of the overlap method. This process should not be regarded as error-free, and in cases where the assumptions are likely to be particularly untenable (discussed in Chapter 8, Section D.2), explicit adjustments of the form discussed in Chapter 8, Section E, should, resources permitting, be used.

9.14 Sample rotations to freshen the sample between rebasing are expensive exercises. However, if rebasing is infrequent and there is a substantial loss of commodities in particular industries, then this might be appropriate for those industries. In the next section the need for a metadata system to facilitate such decisions is outlined. The use of more frequent sample rotation aids the process of quality adjustment in two ways. First, the updated sample will include newer varieties, comparable replacements with substantial sales will be more likely to be available, and noncomparable ones will be of a more similar quality, which will aid good explicit quality adjustments. Second, because the sample has been rotated, there will be fewer missing commodities than otherwise and thus less need for quality adjustments.

9.15 A natural extension of more frequent sample rotation is to use a type of chained formulation at the elementary (unweighted) level in which the sample is reselected each period. The prices of all commodities available in each successive linked comparison are compared: Those available, for example, in both January and February are compared for the January to February link, whereas those available in both February and March are compared for the February to March link. The index for January to March is derived by successive multiplication of the two binary links. In Chapter 8, Section G.3, the principles and methods of this chained formulation were outlined in the context of sectors in which there is a rapid turnover of commodities, and such principles are echoed here. Similarly, the use of hedonic indices as outlined in Chapter 8, Section G.2, and short-run comparisons discussed in Chapter 8, Section H, might be useful in this context.

9.16 The above chained formulation allows the price changes of a new good to be included in the index as soon as the good can be priced for two successive periods. For example, a new good that appears in March can be introduced into the index in the March to April link. However, the new good's effect on the price index in the initial period of introduction, March, for the February to March link, is ignored. Similar concerns arise for disappearing commodities. If the last period a price is observed for a commodity is January, its effect on the price index is lost for the January to February link. The incorporation of such price effects into an index is considered in Section D.3.2 and Appendix 9.2.

9.17 If the new good is not entirely new, in the sense that it is providing more services than those of the old good, a hedonic estimate of the reservation price can be used to estimate the cost of the base situation characteristics for the missing price of the disappearing good or the cost of the current situation characteristics for the missing reference price of the new variety. However, this applies only when the good is not entirely new, so that the price can be determined in terms of a different combination of the existing character set. Most likely the (not entirely) new good would have a more of a quality characteristic and the hedonic function can impute its price. However, this would be an out-of-sample prediction and would rely on the assumption that the parameter estimates hold over this extended range of values.

C. Information Requirements for a Strategy for Quality Adjustment

9.18 It should be apparent from the above that a strategy for quality adjustment not only must be linked to sample representativity but also requires building a statistical metadata system. The approach for the index as a whole cannot be described simply. It requires the continual development of market information and the recording and evaluation of methods on a commodity-by-commodity basis. The rationale for such a metadata system relates to the variety of procedures for quality adjustments to prices discussed in Chapter 8, Section C.3.4, and how their suitability might vary on a case-by-case basis, all of which require documentation.

C.1 Statistical metadata system

9.19 The methods used for estimating quality-adjusted prices should be well documented as part of a statistical metadata system. Metadata are pieces of systematic, descriptive information about data content and organization that help those who operate the statistics production systems to remember what tasks they should perform and how they should perform them. A related purpose is to introduce new staff to, and train them in, the production routines. Such data also serve to encourage transparency in the methods used and help ensure that they are understood and continued as staff members leave and others join. The metadata, as proposed in this context, also help identify where current methods of quality adjustment require reconsideration and prompt the use of alternative methods. Indices for the export/import of specific goods, such as personal computers, may be derived using specific compilation/estimation routines and metadata are required to document such procedures. Because so much of the rationale for the employment of different methods is specific to the features of the industries concerned, data should be kept on such features. This would extend to maintaining data on market features, such as the dates for the introduction of new goods and the nature of their technological change. The metadata system should help in the following ways:

- Statistical agencies should monitor the incidence of missing commodities against, say, two-digit chapter or four-digit section of the Harmonized Commodity Description and Coding System as appropriate, and if the incidence is high for particular commodities, at the six-digit level. Where the incidence is high, the ratios of temporary missing prices, comparable replacements, and noncomparable replacements to the overall number of prices, and the methods for dealing with each of these three circumstances, also should be monitored to provide the basis of a statistical metadata system. The advantage of a top-down approach is that resources are saved by monitoring at the detailed level only the commodity groups that are problematic. The metadata might include the following:
  - Commodity-specific information, such as the timing of the introduction of new models; pricing policies, especially in months when no changes were made; and popularity of models and brands according to different data sources.
  - An estimate, if available, of the weight of the commodity concerned so that a disproportionate effort is not given to relatively low-weighted commodities.
  - Information arising from contacts with market research organizations, retailers, manufacturers, and trade associations for commodities for which replacement levels are high. The development of such contacts may lead, for example, to option cost estimates, which can be easily introduced. Where possible, staff should be encouraged to learn more about specific industries whose weights are relatively high and where commodity replacement is common. Contacts with organizations in such industries will allow staff to better judge the validity of the assumptions underlying implicit quality adjustments.
- Industries likely to be undergoing regular technological change should be identified. The system should attempt to ascertain the pace at which models change and, where possible, the timing.
- Price-determining characteristics for commodities undergoing technological change, especially if quality adjustment procedures make use of hedonic regressions. Information may be included from market research organizations, responding businesses, wholesalers, trade associations, and other such bodies. This information should contribute to the statistical metadata system and be particularly useful in providing subsequent guidelines on commodity selection.
- The system should undertake an analysis of what have in the past been judged to be "comparable" replacements in terms of the factors that distinguish the replacement and old commodity. The analysis should identify whether different respondents are making similar judgments and whether such judgments are reasonable.
- When hedonic regressions are used either for partial patching of missing prices or as indices in their own right, information on the specification, estimated parameters, and diagnostic tests of the regression equations should be kept along with notes as to why the final formulation was chosen and used along with the data. This will allow the methodology for subsequent updated equations to be benchmarked and tested against the previous versions.
- Price statisticians may have more faith in some quality-adjustment procedures than others. When such procedures are used extensively, it might be useful to note, as part of the metadata system, the degree of faith the statistician has in the procedures. This may be envisaged as a simple subjective coding on a scale of one to five.

D. The Incorporation of New Goods

D.1 What are new goods and how do they differ from quality changes?

9.20 A new model of a good may provide more of a currently available set of service flows. For example, a new model of an automobile is different from an existing one in that it may have a bigger engine. There is a continuation of a service and production flow, and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of a new good's quality changes against an updated existing model is that, first, the former cannot be easily linked to an existing commodity as a continuation of an existing resource base and service flow because of the very nature of its "newness." Some forms of genetically modified seeds, frozen foods, microwave ovens, and mobile phones, while extensions of existing services, have a dimension of service that is quite new. Second, new goods can generate a welfare gain to purchasers and surplus to producers when purchased/sold at the very time of introduction, and the simple introduction of the new good into the index, once two successive price quotes are available, misses this gain.

9.21 The problem of defining new goods can be considered in terms of defining a monopoly. If there is no close substitute, the good is new. For example, some individual new videos may have quite small cross-elasticities with other videos; their shared service is to provide movie entertainment and they are similar only in this respect. The same argument may apply to some new books and new breakfast cereals. However, Hausman (1997) found cross-elasticities for substitution to be quite substantial for new breakfast cereals. There are many new forms of existing commodities, such as fashionable toys, that are not easily substitutable for similar commodities, and thus manufacturers could generate a substantial surplus over and above what might be expected from their production costs. The ability of manufacturers to generate monopoly surpluses is one consideration when determining whether commodities are new.

9.22 However, the sheer scale of new commodities and new varieties of existing commodities exported and imported makes it impractical to separately monitor and fully incorporate their effect on prices into an index, especially because the techniques for their inclusion are not readily applicable.

9.23 Merkel (2000, p. 6) was more practical in devising a classification scheme that will meet the needs of trade price index number compilation: There are *evolutionary* and *revolutionary* goods. The former are defined as

> extensions of existing goods. From a production inputs standpoint, evolutionary goods are similar to pre-existing goods. They are typically produced on the same production line and/or use largely the same production inputs and processes as pre-existing goods. Consequently, in theory at least, it should be possible to quality adjust for any differences between a pre-existing good and an evolutionary good.
>
> In contrast, revolutionary goods are goods that are substantially different from pre-existing goods. They are generally produced on entirely new production lines and/or with substantially new production inputs and processes than those used to produce pre-existing goods. These differences make it virtually impossible, both from a theoretical and practical standpoint, to quality adjust between a revolutionary good and any pre-existing good.

9.24 Quality adjustments to prices are therefore suitable for evolutionary goods under the fixed-input output export price index framework (discussed in Chapter 8), but unsuitable for revolutionary goods. The definitions are designed to distinguish between the two types of goods not in terms of what is analytically appropriate, but by what is practically meaningful for the needs of trade price index number construction. It is quite possible for a new commodity made from the same inputs and processes as the old one to have a high cross-elasticity of substitution and thus command revenue for each commodity beyond what might be expected from a normal markup. Yet practical needs are important in this context, especially because the methods for estimating the producers' surplus are not practically possible given their substantial resource needs of data and econometric expertise.

D.2 The issues

9.25 There are two major concerns regarding the incorporation of new goods into a trade price index number. First is their identification and detection; second is the related decision on the need and timing for their inclusion. These concerns refers to both the weight and price changes of the new goods. Consider some examples.

9.26 Exports/imports of cellular phones, for example, were in some countries at such a significant level that their early inclusion in trade price indices became a matter of priority. They simply rose from nothing to be a quite large proportion of imports/exports in their industry. Furthermore, their price changes were atypical of other goods in their industry.

9.27 Many new goods can command substantial sales and be the subject of distinct pricing strategies at introduction because of substantial marketing campaigns. Dulberger (1993) provided some estimates for the U.S. producer price index for dynamic random access memory (DRAM) computer memory chips. She calculated price indices for the period from 1982 to 1988 with varying amounts of delay in introducing new chips into the index. The indices were chained so that new chips could be introduced, or not, as soon as they were available for two successive years. Using a Laspeyres chained index, the fall of 27 percent, if there is no delay in introducing new goods, was compared with falls of 26.2 percent, 24.7 percent, 19.9 percent, 7.1 percent, and 1.8 percent, if the introductions were delayed by one year, two years, three years, four years, and five years, respectively. In all cases, the index is biased downwards because of the delay. The longer the delay, the more the price changes of new goods are estimated by goods whose market shares may be quite small. Berndt and others (1997) provided a detailed study of the new anti-ulcer drug Tagamet and found the effects of pre-introduction marketing on its price and market share at introduction to be quite substantial. Not unexpectedly, price falls were found for the generic form of a pharmaceutical on the expiration of the patent, but increases were found for the branded form as loyal customers were willing to pay a premium over the price prior to the patent expiration (Berndt, Ling, and Kyle, 2003).

9.28 Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new goods may lead to errors in the measurement of price changes if the unusual price movement cycles are ignored at critical stages in the good life. Strategies are required for the early identification of new goods and mechanisms for their incorporation either at launch, if preceded by major marketing strategies, or soon after, if there is evidence of market acceptance. This should form part of the metadata system. Waiting for a new good to achieve market maturity may result in an implicit policy of ignoring the quite disparate price movements that accompany their introduction. This is not to say that new goods will always have different price changes. Consider the example of "lite" varieties of foods and beverages, similar to the original ones but with less fat. They had prices very close to the original ones and served to expand the market. Although there was a need to capture such expansion when the weights were revised, the price changes for the existing commodities could be used to capture those of the lite ones.

D.3 Methods

9.29 The methods outlined here include those that fall under what should be normal XMPI procedures and those that would require exceptional treatment. In the former case, consideration is given in Section D.3.1 to the rebasing of the index, rotating of commodities, introduction of new goods as replacements for discontinued ones, and a strategy for dealing with new commodity bias. In the latter, techniques that require different sets of data are outlined. The use of chained matched models and hedonic indices were outlined and discussed in Chapter 8, Section G, "High-Technology and Other Sectors with Rapid Turnover of Models."

D.3.1 Sample rebasing, rotation, directed replacements, and sample augmentation

D.3.1.1 Sample rebasing and rotation

9.30 The concern here is mainly with *evolutionary goods*. A new good may be readily incorporated in the index at the time of rebasing the index or when the sample is rotated. If the new good has, or is likely to have, substantial sales, and is not a replacement for a preexisting one, or is likely to command a much higher or lower market share than the preexisting one it is replacing, then new weights are necessary to reflect this. New weights are fully available only at rebasing, not on sample rotation. There will be a delay in the new commodity's full inclusion, and the extent of the delay will depend on how close its introduction is to the next rebasing and, more generally, the frequency with which the index is rebased. The term "rebasing" here is effectively concerned with the use of new weights for the index. Even if the index were rebased annually and chained, it would take until the annual rebasing before weights could be assigned, and even then there might be a further six-month delay in the sampling and collating of the survey results for the weights. More frequent rebasing allows for an earlier introduction of the new good and is advised when the weights are not keeping pace with innovations in the product market.

9.31 It is quite straightforward to include a new variety into an elementary aggregate, once prices are available in two successive periods. As a replacement for an existing variety, the overlap method may be used (Chapter 8, Section D.1). If only the price in the current period is available, it may still be linked directly to the price of the variety it is replacing, but with an adjustment to the price for any change in quality. This adjustment should follow the principles outlined in Chapter 8. New varieties need not just be introduced on a one-for-one basis. A comparison at the elementary aggregate level between, say, prices in 2005 and prices in June 2006 may be undertaken in two stages: first, by comparing average prices for several varieties in 2000 with average prices of comparable varieties in May 2006; second, by multiplying by a comparison of average prices in May 2006 compared with June 2006. However, the basket of varieties in the May to June 2006 stage may include new varieties in addition to, or as replacements for, the ones used in the 2005 to May 2006 stage. In introducing such varieties there is an implicit weighting, and care has to be exercised to ensure it is meaningful. At the elementary level of aggregation, the Jevons index is the ratio of geometric means, which is equal to the geometric mean of price relatives (Chapter 21, Section B). Equal (implicit) weight is given by the Jevons index to each variety's price relative. The Dutot index is the ratio of arithmetic means. The Dutot index gives each variety's price relative to the weight of its base period price as a ratio of the sum of the prices in the base period of the comparison (Chapter 21, Section B). Chapter 21 explains why Jevons should be generally favored over Dutot as a price index formula at the elementary level.

9.32 Some statistical agencies rotate (resample) commodities within industry groups. Opportunities exist to introduce new commodities within a weighted group under such circumstances. The resource practicalities of such schemes require commodities to be rotated on a staggered basis for different industries, with industries experiencing rapid change being rotated more frequently. For example, DVDs could replace VCR tapes using the overlap method, with the difference in prices in the overlap period assumed to be equal to their quality difference. The assumptions implicit in such procedures have been outlined above, and their likely veracity needs to be considered. Because evolutionary commodities are defined as continuations of the service flow of exiting ones, the hedonic framework may be more suitable; further methods and their choice were discussed in Chapter 8, Sections D through F. However, the principle remains for including new varieties of goods in an index within a weighting system: They must act as a substitute for old varieties of goods.

9.33 Yet in many countries rebasing is infrequent and sample rotation not undertaken. Furthermore, rotating samples on a frequent basis should not be considered as a panacea. Sample rotation is an arduous task, especially when performed over a range of industries experiencing rapid change. Even frequent rotation, say every four years, may miss many new goods. Experience in the United States has found that frequent rotation (resampling) has had a negative impact on participation rates, because respondents shy away from incurring the indirect costs associated with being interviewed about their good range and technology (Merkel, 2000). Yet it is not necessary for statistical agencies to wait until a commodity is obsolete before the new one is introduced. It is quite feasible for statistical agencies to preempt the obsolescence of the old commodity and direct an early substitution of the new. In some industries, the arrival of a new good is well advertised in advance of the launch, whereas in others it is feasible for a statistical agency to have more general procedures for directed substitutions, as is outlined below. Without such a strategy and infrequent rotation and rebasing, a country would be open to serious new good bias. In summary,

- The treatment of a new good as a replacement for an existing one can be undertaken if the old commodity's weights suitably reflect the new good's sales, and if suitable quality adjustments can be made to its price to link it to the existing old price series.
- If the new good does not fit into the preexisting weighting structure, it can be included on rebasing, though this may be infrequent in some countries.
- Regular sample rotation provides a means by which the inclusion of such commodities can be formally reconsidered. Because this is undertaken on a staggered basis, only the weights within the industry, not those between industries, are reallocated.
- Directed sample substitution, as opposed to waiting for sample rotation, may be used to preempt the arrival of new goods.
- Revolutionary commodities, tectonic shifts, and entirely new goods will not fit into existing weighting structures and alternative means are required.
- Directed replacements for evolutionary goods as replacement commodities and for revolutionary goods to augment the sample are considered below.
- The chained framework outlined in Chapter 16, Section F, may be more appropriate for good areas with high turnovers of commodities.

D.3.1.2 Directed replacements and sample augmentation

9.34 For *evolutionary goods* in industries with a rapid replacement and introduction of such goods, a policy of directed substitution might be adopted. Judgment, experience, and a statistical metadata system should help identify such industries. The existing commodities should be coded into well-defined commodity lines. The respondents then are contacted on a regular (say, annual) basis to establish whether a new version has been introduced and, if so, what percentage of the commodity line's value is represented by the new version. Replacement could be decided by a number of criteria. If the new version is designed as a replacement for an existing one, then substitution might be automatic. Once a substitute has been made, the prices require adjustment for the quality differences using the overlap method, imputation, or an explicit estimate based on production or option costs or a hedonic regression.

9.35 It is important to emphasize that, on the introduction of new versions of these evolutionary goods, a price may be charged over and above that which can be ascribed to the resource costs behind its difference from the old one. A new version of, for example, electrical cable may have stronger and more flexible plastic coating and the resource cost behind its production may be quite small. Yet it may be sold at a much higher price than the old version because it's seen to be superior to other such goods in the market. This price increase is a real one that should, for an import price index (MPI), after adjustment for quality, be captured. After a while prices may be reduced as the novelty of the commodity wears off or as competitors bring out a competitive or improved cable. The directed substitution becomes important so that the unusual price increases at the introduction are captured by the XMPI. It is also necessary so that the coverage of commodities becomes more representative. Directed substitution allows both.

9.36 However, for *revolutionary goods* substitution may not be appropriate. First, they may not be able to be defined within the existing classification/weighting systems. Second, they may be primarily produced by a new establishment, or imported by a new wholesaler, which will require extending the sample to include such establishments. Third, there will be no previous commodities to match them against and make a quality adjustment to prices because, by definition, they are substantially different from preexisting goods. Finally, there is no weight to attach to the new establishment and/or commodity.

9.37 The first need is to identify new goods, and the need for contacts with market research companies, trade associations, outlet managers, and manufacturers was discussed in Section C.1 on producing a supporting metadata system. Once the new goods are identified, *sample augmentation* is appropriate for the introduction of revolutionary goods, as opposed to sample substitution for evolutionary goods. It is necessary to bring the new revolutionary good into the sample in addition to what exists. This may involve extending the classification, the sample of establishments/wholesalers, and the commodity list. The means by which the new goods are introduced is more problematic.

9.38 Once two price quotes are available, it should be possible to splice the new commodity onto an existing or obsolete one. This of course misses the impact of the new commodity in its initial period, but as discussed below, including such effects is not a trivial exercise. Consider the linking of a good that is likely to be replaced in the market by the new good. For example, a quite new electrical kitchen appliance may use the price index for existing kitchen appliance up to the period of the link, and then the price changes for the new good in subsequent periods. This would create a separate and additional price series for the new good, which augments the sample, as illustrated in Table 9.1. Commodity C is new in period 2 and has no base period weight. Its price change between periods 1 and 2, had it existed, is assumed to follow the overall index for commodities A and B. For period 3 onward a new, linked price series is formed for commodity C, which for period 3 is $101.40 \times 0.985 = 99.88$, and for period 4 is $101.40 \times 0.98 = 99.37$. New revised

Table 9.1. Sample Augmentation Example

Commodities	Base Weight	Revised Weight	Period 1	Period 2	Period 3	Period 4
A	0.6	0.5	100.00	101.00	101.50	102.50
B	0.4	0.3	100.00	102.00	102.50	103.00
All commodities		*0.8*	*100.00*	*101.40*	*101.90*	*102.70*
C				100.00	98.50	98.00
Spliced C		0.2	100.00	101.40	99.88	99.37
Revised all commodities			*100.00*	*101.40*	*101.50*	*102.05*

weights in period 2 show commodity *C*'s weight to be 20 percent of all of the commodities. The new index for period 3 is

$$101.40\ [(0.8\ (101.9/101.4) + 0.2\ (99.88/101.4))]$$
$$= 0.8\ (101.9) + 0.2\ (99.88) = 101.50$$

and for period 4,

$$101.40\ [(0.8\ (102.7/101.4) + 0.2\ (99.37/101.4))]$$
$$= 0.8\ (102.7) + 0.2\ (99.37) = 102.05.$$

9.39 If commodity *C* were an evolutionary good replacing commodity *B*, there would be no need to introduce new weights and no need to augment the sample, as undertaken above. However, because the revolutionary commodity *C* has no weight in the base period, the splicing requires a revision of the weights at the same time. The selection of the series onto which the new commodity is spliced and, in turn, the selection of the commodity groups for the weight revision requires some judgment. Commodities whose market share is likely to be affected by the introduction of the new good should be selected. If the new good is likely to be responsible for a significant share of traded value, such that it will affect the weights of a broad class of commodity groups, then there may be a case for a realignment of the overall weighting procedure. Such seismic shifts can of course occur, especially in the communications industries, and for a wide range of industries when regulations are removed or trade barriers are relaxed in less developed economies. In some countries, a new industry or plant can, in itself, amount to sizable proportions of a sector's weights. The change in weights also may be required for *disappearing* goods no longer produced in an economy. As noted in Chapter 16, Section F, a chained (unweighted) formulation and hedonic indices may well be appropriate when there is a rapid turnover in such new and obsolete goods. Chaining is an extension of the above procedure and can be used to introduce a new good as soon as it is available for two successive periods.

9.40 Commodity augmentation also may be used for evolutionary goods that are likely to be responsible for a substantial share of the market while not displacing the existing goods. For example, say a local brewery establishes a licensing agreement with a foreign brewery to produce for export foreign-branded as well as their domestic-branded beers. Assume the export revenue for beer from the brewery remains roughly the same, but one segment of the market now drinks foreign-branded as opposed to domestic-branded beer. Respondents may be directed to a forced substitution of some of the sample of domestic-branded beers for foreign ones, with the weight remaining the same. This would be similar to a quality adjustment using a noncomparable replacement as discussed in Chapter 8, Section E. Alternatively, the sample may be augmented because there is concern that a smaller sample of domestic-branded beers may not be sufficiently representative. The augmentation process may be similar to that outlined in Table 9.1, with the new foreign beer *C* accounting for 20 percent of the market. Had the advent of foreign beers displaced some of the alcoholic spirits market, then the revision of weights would extend into this commodity group. As noted in Chapter 8, Section G, chaining and hedonic indices may be appropriate when there is a rapid turnover in new and obsolete goods. With chaining, the good needs to be available only for two successive periods to allow for its introduction.

9.41 There remains the problem of identifying the appropriate effect on a price index of a new good in its first period of introduction. A more serious form of the problem is a new good that is imported for one period only. In Section B.3, mention was made of the use of hypothetical reservation prices for the period prior to the good's introduction. These provide a sound analytical answer to the problem, though econometric estimation problems with the practical estimation of the required parameters and predictions on the scale required were deemed to be a serious constraint.

D.3.2 New goods and disappearing goods at the time of introduction and loss

9.42 In Section B.3, mention was made of the problem of incorporating price information into an index at the time of the introduction of a good, and at the time of the loss of a good. A chained formulation would allow such prices to be incorporated once information was available for two successive periods. For example, a new good that appears in March can be introduced into the index in the March to April link. But the concern here, as noted in Section B.3, is that the new good's effect on the price index in the initial period of introduction, March, for the February to March link, is ignored. Similar concerns arose for disappearing commodities. If, for example, many new goods were being imported, and there was a major shift in expenditure toward them, there would be an increase in the welfare of those purchasing the new goods and such welfare increases should be incorporated into the index at the time of the shift.

9.43 Consider the case of a new good to be introduced into an MPI, say in period 3. A conceptually sound approach to its incorporation into the index is to impute its price for period 2, that is, to estimate its reservation (or choke) price. This is the price that would drive the demand for the good down to zero in the period prior to its introduction (Hicks, 1940; and Hausman, 1997). An analogous approach applies to disappearing goods, where the reservation price for a good last appearing in period 1 is estimated for period 2. Note that Hicks (1940) and Hausman (1997) considered the problem in the context of a consumer price index (CPI). However, such principles carry over to an MPI. For an export price index Fisher and Shell (1972) suggested that the preceding price be imputed as the reservation price given the current period technology, where the reservation price is defined as the maximum price at which zero production of the good is forthcoming, given current period inputs and prices of other outputs in the preceding period. A disappearing good's price has to be imputed in the current period—as the reservation price given the preceding period technology—defined as the maximum price in the current period at which no production of the good is forthcoming, given inputs in the preceding period and prices of other outputs in the current period.

9.44 The econometric estimation of such reservation prices is not practical for general index number compilation. Hausman (1997) provided an example in the context of the CPI, the complexities of which are apparent from the paper and the response to the paper by Bresnahan (1997). Hausman (2003), however, developed a simplified approach requiring an estimate of the price elasticity of demand. Balk (2000b) provided an alternative approach based on changes in expenditure shares of the "old" and "new" commodity and a numerical routine for estimating the elasticity of substitution, as detailed in Appendix 9.2. In order to incorporate these price effects into an index a functional form for the aggregator needs to be assumed, and because the elasticity of substitution is fixed, the form adopts a constant elasticity of substitution. The incorporation of such price effects is a new challenge to statistical offices. Preliminary research studies may be undertaken for goods and services where new (and disappearing) commodities account for a relatively high proportion of expenditure/revenue at the time of introduction (and loss), as a first step to providing estimates of their effects. The inclusion of such effects in the index, at least in the medium term, should be such that they can be separately identified.

E. Summary

9.45 The concern with sample space and new goods in this chapter arises out of a very real concern with the dynamic nature of modern markets. New goods and quality changes are far from new issues and as Triplett (1999) has argued, it has not been demonstrated that the *rate* of new good developments and introductions is much higher now than in the past. However, it is certainly accepted that the *number* of new goods and varieties is substantially greater than before. Computer technology provides cost-effective means for collecting and analyzing much larger sets of data. In Chapter 7, the use of handheld computers for data capture was considered, as was the availability of bar-code scanner data. Yet the proper handling of such data requires consideration of issues and methods that go beyond those normally considered for the static intersection universe, which underscores matched samples. In Appendix 9.1, a formal outline of such sampling issues is provided. In this section some of the more important issues are reiterated.

- Where nothing much in the quality and range of available goods changes, using the matched-models method offers many advantages. It compares like items with like, from like establishments.
- Statistical metadata systems are needed for quality-adjustment issues to help identify the industries in which matching provides few problems. Metadata on quality-adjustment focuses attention on commodity groups that are problematic by collecting and providing information that will facilitate quality adjustment. It also allows for transparency in methods and facilitates retraining.
- Where there is a very rapid turnover in commodities, such that serious sample depletion takes place quickly, replacements cannot be relied upon to replete the sample. Alternative mechanisms, which sample from or use the double universe of commodities in each period, are required. These include chained formulations and hedonic indices as discussed in Chapter 8, Section G.
- Some new goods can be treated as evolutionary and incorporated using noncomparable replacements with an associated quality adjustment. The timing of the replacement is critical for both the efficacy of the quality adjustment and the representativity of the index.
- Instructions to respondents on the selection of replacement commodities are important because

they also have a bearing on the representativity of the index. The replacement of obsolete commodities with newly introduced commodities leads to difficulties in undertaking quality adjustments, whereas their replacement with similar commodities leads to problems of representativity.

- Sample rotation is an extreme form of the use of replacements and is one mechanism for refreshing the sample and increasing its representativity. However, a disadvantage is the possible bias arising from the implicit assumptions underlying the quality-adjustment overlap procedure not being met.
- Revolutionary goods may require the augmentation of the sample to make room for new price series and new weighting procedures. The classification of new goods into evolutionary goods and revolutionary goods has a bearing on the strategy for their introduction, directed replacement (substitution), and sample augmentation.
- The incorporation of the (welfare) effects of new goods at the time of their introduction, and of disappearing goods at the time of their loss, is conceptually sound. Resources permitting, as a first step, research studies should be undertaken for goods and services where new (and disappearing) commodities account for a relatively high proportion of expenditure/revenue at the time of introduction (and loss).

Appendix 9.1 Appearance and Disappearance of Goods and Establishments

9.46 In earlier chapters, especially Chapter 6 on sampling, it was generally assumed that the target quantity for estimation could be defined for a fixed set of goods. In this appendix the important complications arising from the commodities and establishments continually changing are considered. The rate of change is rapid in many industries. With this in mind, sampling for price change estimation is a dynamic rather than static problem. Somehow, the prices of new commodities and in new establishments have to be compared to old ones. *It is important to realize that whatever methods and procedures are used in a price index to handle these dynamic changes, the effects of these procedures will always amount to an explicit or implicit estimation approach for this* ***dynamic universe.***

Representation of change in a price index[1]

9.47 From a sample selection perspective, there are three ways of handling dynamic changes in an elementary aggregate universe, where varieties and establishments move in and out: (1) by *resampling* the whole elementary aggregate at certain points in time, (2) by a one-to-one *replacement* of one variety or establishment for another one, and (3) by *adding and deleting* single observation points (commodities in establishments) within an index link.

Resampling

9.48 In resampling, the old sample is reconsidered as a whole to make it representative of the universe in a later period. This does not necessarily mean that all or even most sampling units have to be changed, only that a fresh look is taken at the representativity of the whole sample and changes undertaken as appropriate. The methods used for resampling could be any of those used for the initial sampling. In the case of probability sampling, it means that every unit belonging to the universe in the later period needs to have a nonzero probability equal to its relative market share of being included in the sample.

9.49 Resampling or *sample rotation* is traditionally combined with the overlap method outlined in Chapter 8, Section D. It is similar to the procedure used when combining two links in a chained index. The first period for which the new sample is used is also the last period for which the old sample is used. Thereby, price change estimation may be based on the old sample up to the overlap period and the new sample from the overlap period onward. Resampling is the only method that is fully able to maintain the representativity of the sample and, resources permitting, should be undertaken frequently. The necessary frequency depends on the rate of change in a particular group of commodities. It relies, however, on the assumption that the price differences between the old and new commodities, at the time of the overlap, are appropriate estimates of quality differences (Chapter 8, Section D). At its extreme, resampling amounts to drawing a new sample in each period and comparing the average price between the samples, instead of the usual procedure of averaging price changes for matched samples. Although being the logical end-point from a representativity point of view,

[1]A fuller version of this appendix can be found in Dalén (1998).

resampling each period would aggravate the quality-adjustment problem by its implicit quality-adjustment procedure and, thus, is not recommended.

Replacement

9.50 A replacement can be defined as an individual successor to a sampled commodity (or a specific establishment) that either disappeared completely from the market or lost market share in the market as a whole. Criteria for selecting replacements may differ considerably. There is first the question of *when* to replace a commodity. Usual practices are to replace either when a commodity disappears completely or when its share of the sales is reduced significantly. Another possible, but less used, rule would be to replace an commodity when another variety within the same group, or representative commodity definition, has become larger with regard to sales, even if the old variety still is sold in significant quantities.

9.51 Second is the question of how to select the replacement commodity. If the rule for initial selection was “the most sold” or “with probability proportionate to (sales) size,” then the replacement rule could follow the same selection rule. Alternatively, the replacement could be that commodity that is “most like” the old one. The advantage of the “most sold” rule is better representativity. The advantage of the “most like” rule is, at least superficially, that it might result in a smaller quality-adjustment problem.

9.52 It is important to realize that, at least with today’s practices, replacements cannot adequately represent new commodities coming into the market. This is because what often triggers a replacement is not the appearance of something new, but the disappearance or reduced importance of something old. If the range of varieties in a certain group is increasing, sampling can represent this increase directly only from the set of new varieties, such as in the case of resampling.

Adding and deleting

9.53 It is possible to add a new observation point into an elementary aggregate within an index link. If, for example, a new brand or model of a durable was introduced without replacing any particular old model, it would be desirable to add it to the sample starting from the time of its introduction. In order to accommodate this new observation into the index system, its reference price needs to be imputed. A practical way to do this is to divide its price in the month of introduction by the price index of all other commodities in the elementary aggregate from the reference period to the month of introduction. In this way, its effect on the index for months up to the introduction month will be neutral.

9.54 Similarly, a commodity that disappears could just be deleted from the sample without replacement. Price change can then be computed over the remaining commodities. If no further action is taken, this means that the price change for the deleted commodity that was measured up to the month prior to deletion will be disregarded from the month of deletion. This may or may not be desirable, depending on the veracity of the implicit assumption as to what its price change would have been had it not disappeared, for the particular commodity group in question.

Formulating an operational target in a dynamic universe

9.55 A rigorous approach to the problem of statistical estimation requires an *index estimation strategy* that includes both the *operational target of measurement* and the *sampling strategy* (design and estimator) needed for estimating this target. This strategy would have to consist of the following components:

(1) A definition of the universe of *transactions* or *observation points* (usually a variety of a commodity in an establishment) in each of the two time periods between which we want to estimate price change;

(2) A list of all *variables* defined on these units. These variables should include prices and quantities (number of units/relative values sold at each price), but also all relevant price-determining characteristics and terms of sale of the commodities (and possibly also of the establishments)—the price basis;

(3) The *target algorithm* (index formula) that combines the variable values defined in (2) for the observation points in the universe defined in (1) into a single value;

(4) Procedures used for *initial sampling* of commodities and establishments from the universe defined in (1);

(5) Procedures within the time span for *replacing, resampling, and/or adding or deleting* observations; and

(6) The *estimation algorithm* (index formula) applied to the sample with the purpose of minimizing the expected error of the sample estimate compared with the target algorithm under (3). This algorithm, in principle, needs to consider all the procedures taken in replacement and resampling situations, including procedures for quality adjustment.

9.56 The kind of rigorous strategy outlined above is generally not used in practical index construction because of its complexity, though its required information system was discussed in Section C.1. A few comments on such possible strategies are made below.

A two-level aggregation system

9.57 A starting point for discussing this objective is a two-level structuring of the universe of commodities and establishments considered in the scope of a price index. These levels are

- The *aggregate* level. At this level there is a fixed structure of commodity groups $h = 1,\ldots,H$ (or perhaps a fixed cross-structure of commodity groups by regions or establishment types) within an index link. New goods and services for updating the universe of commodities would be defined in terms of new groups at this level and moved into the index only in connection with a new index link.
- The *elementary* level. Within this level the aim is to capture the properties of a changing universe in the index by comparing new and old commodities. The micro comparison from s to t must be defined so that it includes new commodities and establishments as they enter into the market and old goods and establishments as they disappear from the market.

The common starting point for three alternative approaches at the elementary level is a pure price formulation of price change from period s to period t at the aggregate level:

$$I^{st} = \frac{\sum_h Q_h P_h^t}{\sum_h Q_h P_h^s} = \sum_h W_h^s I_h^{st}, \qquad \text{(A9.1)}$$

where $W_h^s = \dfrac{Q_h P_h^s}{\sum_h Q_h P_h^s}$ and $I_h^{st} = \dfrac{P_h^t}{P_h^s}$.

The quantities, Q_h, are for $h = 1,\ldots,H$ commodity groups from any period or functions of quantities from several periods, such as a symmetric average of the base and current periods s and t. Special cases of such a pure price index are the Laspeyres ($Q_h = Q_h^s$), Paasche ($Q_h = Q_h^t$), Edgeworth ($Q_h = (Q_h^s + Q_h^t)/2$), and Walsh ($Q_h = [Q_h^s Q_h^t]^{½}$) price indices outlined in Chapters 16 through 18. Alternative formulations for an elementary-level estimation strategy now enter in the definition of I_h^{st}. As a further common starting point, the set of commodities or establishments belonging to h in period u (= s or t) is defined as Ω_h^u. The concept of an *observation point* is introduced, usually a tightly specified commodity in a specific establishment, such that, say, $\Omega_h^u = \{1,\ldots,j,\ldots,N_h^u\}$. For each observation point $j \in \Omega_h^u$, there is a price p_j^u and a quantity sold q_j^u. There are now three possibilities for defining the operational target.

The intersection universe

9.58 The elementary index is defined over the intersection universe, that is, only over observation points existing in both s and t. This index may also be called the *identical units index*. It is equivalent to starting out with the observation points existing in s and then dropping (deleting) missing or disappearing points. An example of such an index is:

$$I_h^{st} = \frac{\sum_{j \in \Omega_h^s \cap \Omega_h^t} q_j p_j^t}{\sum_{j \in \Omega_h^s \cap \Omega_h^t} q_j p_j^s}. \qquad \text{(A9.2)}$$

The intersection universe decreases successively over time as fewer matches are found for each long-run comparison between s and t, s and $t + 1$, s and $t + 2$, and so on, until it eventually becomes empty. An attraction of the intersection universe is that there are, by definition, no replacements involved in this target and, thus, normally no quality adjustments. If the identical units index is combined with a short index link, followed by resampling from the universe in a later period, sampling from this universe is a perfectly reasonable strategy, as long as the assumption implicit in the overlap procedure—that the price differences at that point in time reflect the quality differences—is valid.

The double universe

9.59 The polar opposite approach to the intersection universe is to consider P_h^s and P_h^t as average prices defined over two separately defined universes in the two periods. A double universe target could then be considered; one universe in period s and another in period t. This seems to be a natural way of defining the target, because both time periods should be of equal status and all commodities existing in any of these should be taken into account. The difficulty with this approach is that the two universes are rarely comparable in terms of quality. Some kind of adjustment for average quality change would need to be brought into the index. The natural definition of the average prices involved in this approach is as unit values. This would lead to the following definition of a *quality-adjusted unit value index*:

$$I_h^{st} = \frac{\overline{P}_h^t}{\overline{P}_h^s g_h^{st}}, \text{ where } \overline{P}_h^t = \frac{\sum_{j \in \Omega_h^t} q_j^t p_j^t}{\sum_{j \in \Omega_h^t} q_j^t} \qquad \text{(A9.3)}$$

and $\overline{P}_h^s = \dfrac{\sum_{j \in \Omega_h^s} q_j^s p_j^s}{\sum_{j \in \Omega_h^s} q_j^s}.$

In equation (A9.3), g_h^{st} is the average quality change in h (also interpretable as a *quality index*), which of course needs further definition. For example, g_h^{st} could be thought of as a hedonic adjustment procedure, where characteristics are held constant. Equation (A9.3) was outlined in Chapter 8, Section E, in forms that include explicit hedonic quality adjustments, g_h^{st}, but as part of Laspeyres, Paasche, Fisher, and Törnqvist indices. This operational target is attractive for commodities where the rate of turnover of varieties is very fast, but average quality changes slowly, or reliable estimates of quality changes can be made. The commonly used representative commodity method is not really compatible with a double universe target. It implicitly focuses on preselected primary sampling units that are used for both periods s and t.

The replacement universe

9.60 Neither sampling from the intersection nor from the double universe bears a close resemblance to usual practices for constructing price indices. In particular, the representative commodity method combined with one-to-one replacements, which is the most common sampling method used in practice, needs a rationalization in terms of operational targets that differs from these alternatives. Such a rationalization of sampling from a *replacement universe* is considered below.

Definition 1a: For each $j \in \Omega_h^s$ and $j \notin \Omega_h^t$, we define replacement commodities $a_j \in \Omega_h^t$ whose price enters into j's place in the formula. (For $j \in \Omega_h^s$ and $j \in \Omega_h^t$, $a_j = j$.) In addition to a replacement, a quality change from j to a_j is included, which gives rise to a quality-adjustment factor g_j, interpreted as the factor with which p_j^s must be multiplied for the producer to be indifferent between producing commodities j and a_j at prices p_j^s and $p_{a_j}^t$.

$$I_h^{st} = \frac{\sum_{j \in \Omega_h^s} q_j p_{a_j}^t}{\sum_{j \in \Omega_h^s} q_j p_j^s g_j}. \qquad \text{(A9.4)}$$

However, this first step toward an operational use of the formula requires, first, a need to define g_j, possibly arising from a hedonic regression as described in Chapter 8, Section G.2. Second, there is a need to define a_j. A natural procedure is to use a *dissimilarity function* from j to a_j. The notation $d(j, a_j)$ is introduced for this function. The common procedure of choosing the most similar commodity in cases of replacement now corresponds to minimizing the dissimilarity function. However, some further specifications need to be made. When is the replacement defined to take place? In practice, this ought to be done when the first chosen variety is no longer representative. Mathematically, this could be defined as

Definition 1b: Observation point j should be replaced in the first period in which $q_j^t < cq_j^s$, where c is a suitably chosen constant between 0 and 1 (a modification would be required for seasonal commodities).

The *choice* of replacement point would then be governed by a rule such as Definition 1c.

Definition 1c: a_j should be chosen so that $d(j, a_j)$ is minimized for j.

However, because some priority should be given to observation points that are "important" in terms of quantities or values, Definition 1c can then be modified to become Definition 1d.

Definition 1d: a_j should be chosen so that $d(j, a_j)/q_{a_j}^t$ is minimized for j. (Some other function of $d(.)$ and $q_{a_j}^t$ could be chosen in its place.)

9.61 The dissimilarity function needs to be specified; it may depend on the commodity group h. In general this must be some kind of metric defined on the set of characteristics of the commodity and establishment in question. For example, priority could be given to its dissimilarity to either "same establishment" or "same good," which could easily be worked into such a metric. A more troublesome concern is the inclusion of as many new points in Ω_h^t as possible into the index definition, to make the sample representative. As Definitions 1a–d now stand, the same new point could replace many predecessors, whereas there may be many new points that will not be sampled unless there was a need for a replacement. This shortcoming of the replacement universe is an inherent trait in the replacement method as such. The replacement method is designed only to maintain the representativity of the old sample, not that of the new sample.

Appendix 9.2 New Goods and Substitution

9.62 The case here is concerned with estimating the effect of introducing new goods for a consumer price index (CPI), though there is a direct parallel to purchases for an import price index (MPI). The principles follow on for disappearing goods for an MPI, and to new and disappearing goods for an export price index. The approach identifies new goods as a special case of substitution. In each period, a consumer, faced with a set of prices, decides what to consume. The relative sales of the different commodities may change over time. Consumers may decide to consume less of one existing commodity and more of another existing one, or substitute consumption of an existing old commodity by a new one not previously available, or discontinue consumption of an existing commodity and substitute it by consumption of an existing or new one. Such changes are generally prompted by changes in relative prices. In many cases the "decision" of the consumer is tied to that of the producer or retailer, as commodities are no longer produced or sold so as to make way for new ones. Such substitutions between commodities apply as much to radically new goods as to new models of existing goods. In economic theory, the *elasticity of substitution*, denoted as σ, is a measure of the change in the quantity of, say, commodity i relative to commodity j, that would arise from a unit change in the price of commodity i relative to commodity j. A value of zero would imply that a change in price would lead to no substitution between the consumption of commodities and $\sigma > 1$ would imply that the change in expenditure arising as a result of substituting commodities is positive: It is worth switching.

9.63 There is an intuition here that, if σ is known, and the extent to which substitutions occur in terms of their expenditure shares is also known, then estimates of the underlying price change that prompted the substitution can be derived. This applies as much to substitution between existing commodities as to substitution between existing, discontinued, and new ones. The framework for operationalizing the inclusion of the effects of substitutions for the CPI use was proposed by Shapiro and Wilcox (1997)—see also Lloyd (1975) and Moulton (1996)—whereby the usual Laspeyres formulation was generalized to include the (demand) elasticity of substitution:

$$\left[\sum_{n\in 0,\, t} w_0 \left(\frac{p_{it}}{p_{i\,0}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}, \qquad \text{(A9.5)}$$

where w_0 are expenditure shares in the base period and the summation is over matched commodities available in both periods. The correction, using σ, incorporates a substitution effect into the basic Laspeyres formula. If $\sigma = 0$, the formula is the traditional Laspeyres one. As $\sigma \rightarrow 1$, the formula tends toward a base-period weighted geometric mean. To use this formulation to generalize across the commodities in the summation, the restriction must apply that for any pair of commodities, the elasticity of substitution must be the same. The elasticity of substitution must also be the same over time. Such forms are referred to as constant elasticity of substitution functional relationships.

9.64 Feenstra (1994), Feenstra and Shiells (1997), and Balk (2000b) have extended the substitution to discontinued and new commodities. The advantage of equation (A9.5) is that, given an estimate of σ, a cost-of-living index that includes an estimate of substitution effects can be measured in real time. The incorporation of the effects of new and discontinued commodities follows directly from this. Alternative frameworks for including substitution effects (discussed in Chapter 18) require expenditure data for the base and current periods.

9.65 To extend the framework to new commodities, one must know how expenditures shift between new, existing, and discontinued commodities. Let λ^t be the expenditure share of matched existing commodities out of the total in period t. The total includes existing and new commodities, so $1 - \lambda^t$ is the share of new commodities in period t. Similarly, $1 - \lambda^0$ is the expenditure share of old, discontinued commodities in period 0. The generalized Laspeyres index, which includes substitution between existing and old and new commodities, is given by

$$\left[\frac{\lambda^t}{\lambda^0}\right]^{1/(\sigma-1)} \left[\sum_{n\in 0,\, t} w_0 \left(\frac{p_{it}}{p_{i0}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}. \qquad \text{(A9.6)}$$

Equation (A9.6) requires only the price relatives, the base-period weights, the ratio of expenditure shares, and an estimate of the elasticity of substitution. It can be derived in a number of alternative forms, including generalized, Paasche, Fisher, or Sato-Vartia indices.

9.66 Although there is an intuition behind the above formula, its formal correspondence to an index of consumer prices defined in economic theory is given by Balk (2000b). De Haan (2001) shows how the Fisher equivalent could be derived from a decomposition of a Fisher index when there are new and disappearing goods. The derivations show how the framework requires that $\sigma > 1$, a factor prompting Balk (2000b) to argue for its use for lower-level index aggregation where this is more likely. The remaining problems are the estimation of σ, the availability of data on current expenditure shares, and the validity of the implied constant σ. There are also some conceptual issues. Increases in utility are regarded as having resulted from increases in the desirability of the commodities included in the above aggregation. If such commodities improve, then utility increases. Yet there are other goods outside of the aggregation or system of demand equations. Deterioration in such goods will lead to increases in the desirability of the included commodities and decreases in utility. For example, if a consumer switches to private transport as a result of a deterioration in public transport, this should not be measured as a welfare gain resulting from better private transport, even though the expenditure flows in equation (A9.6) shift that way (Nevo, 2001).

9.67 The direct estimation of σ requires considerable econometric expertise. This puts it outside the routine construction of index numbers (see Hausman, 1997 and 2003). Balk (2000b) showed how an alternative numerical routine might work. De Haan (2001) used scanner data to apply the methodology to a generalized Fisher index. He applied Balk's routine to nine product groups, using data from the Dutch CPI, and found values of σ that exceeded unity. He advised use of chained indices to maximize the matching of ongoing commodities, a principle discussed in Chapter 8, paragraphs 8.153 to 8.158. De Haan (2001) found major discrepancies between a generalized and ordinary Fisher for at least six of the products, and argued for the need to incorporate the effects of new goods (see also Opperdoes, 2001). He also demonstrates how sensitive the procedure is to the selection of σ: For a share in current expenditure for new commodities of 4.8 percent, and $\sigma = 1.2$, a Paasche-type index that includes new goods would be 93 percent below the Paasche price change for ongoing goods only. For $\sigma = 5.0$ and the same expenditure share, the discrepancy falls to 34.1 percent. For very large values, say $\sigma > 100$, the two indices would be relatively close. In such cases, the goods are almost identical, being near-perfectly substitutable; a switch to a new good would have little effect, the new and existing goods having similar prices.

10. XMPI Calculation in Practice

A. Introduction

10.1 This chapter provides a general description with examples of the ways in which export and import price indices (XMPIs) are calculated in practice. The methods used in different countries are not exactly the same, but they have much in common.

10.2 As a result of the greater insights into the properties and behavior of price indices that have been achieved in recent years, it is now recognized that some traditional methods may not necessarily be optimal from a conceptual and theoretical viewpoint. Concerns have also been voiced in a number of countries about possible biases that may be affecting XMPIs. These issues and concerns need to be addressed in this *Manual*. Of course, the methods used to compile XMPIs are inevitably constrained by the resources available, mainly for collecting and processing prices. In some countries, the methods used may be severely constrained by a lack of resources.

10.3 The calculation of XMPIs usually proceeds in two stages. First, price indices are estimated for the unweighted *elementary aggregates,* and then these *elementary price indices* are averaged to obtain *higher-level indices* using the relative traded values for the elementary aggregates as weights. Section B starts by explaining how the elementary aggregates are constructed and which economic and statistical criteria need to be taken into consideration in defining the aggregates. The index number formulas most commonly used to calculate the elementary indices are then presented and their properties and behavior illustrated using numerical examples. The pros and cons of the various formulas are considered together with some alternative formulas that might be used. Much more detail on the properties of elementary aggregate index number formulas is provided in Chapter 21. The problems created by disappearing and new commodities are also explained as are the different ways of imputing for missing prices, considered in further detail respectively in Chapters 8 and 9.

10.4 The data for the measurement of price changes at the elementary aggregate level may be compiled using unit value indices from customs documentation as opposed to price relatives of detailed descriptions of commodities from establishment surveys. Unit value indices—ratios of the value of exports/imports divided by the quantity in one period compared with that in a reference period—may be used to represent the price changes of a commodity classification at the elementary level, prior to aggregation by weights, only if the items within the classification are homogeneous. This is unlikely for unit values from customs data. See Chapter 2 for issues concerning such a use of unit value indices as "plug ins" or proxies for price relatives at the elementary level. If more than one unit value index from customs data or price relative from establishment surveys is to be aggregated as unweighted averages at the elementary level, then the discussion in Section B of this chapter applies. Section B considers and illustrates the formulas that may be used to aggregate the price relatives when no information on weights is available, that is, for elementary price index number formulas. The first order solution to the problem is of course to attempt to obtain weighting information on the relative shares of imports/exports the price relatives represent. Section B and Chapter 21 address the index number problem when no such information is available.

10.5 Section C of the chapter is concerned with the calculation of higher-level indices, when information on weights is available. If elementary aggregate indices, be they based on unit value indices from customs data or price relatives from establishment surveys, are to be aggregated using weights—that is, be aggregated at the higher level—then the concern is with the choice of weighted formulas as discussed in Section C of this chapter. The focus is on the ongoing production of a monthly price index in which the elementary price indices are averaged, or aggregated, to obtain higher-level indices. Price updating of weights, chain linking, and reweighting are discussed, with examples provided. The problems associated with introduction of new elementary price indices and new higher-level indices into the XMPI are also covered. The section explains how it is possible to decompose the change in the overall index into its component parts. Finally, the possibility of using some alternative and rather more complex index formulas is considered.

10.6 Section D concludes with data editing procedures, because these are an integral part of the process of compiling XMPIs. It is essential to ensure that the right data are entered into the various formulas. There may be errors resulting from the inclusion of incorrect data or from entering correct data inappropriately, and errors resulting from the exclusion of correct data that are mistakenly believed to be wrong. The section examines data editing procedures that try to minimize both types of errors.

B. Calculation of Price Indices for Elementary Aggregates

10.7 XMPIs typically are calculated in two steps. In the first step, the elementary price indices for the elementary aggregates are calculated. In the second step, higher-level indices are calculated by averaging the elementary price indices. The elementary aggregates and their price indices are the basic building blocks of the XMPI.

B.1 Construction of elementary aggregates

10.8 Elementary aggregates are constructed by grouping individual commodities and individual services into groups of relatively homogeneous commodities or services. They may be formed for groups of commodities or services irrespective of the country of destination (export) or the country of origin (import), but it is also possible to form elementary aggregates according to country of destination or origin, or for different types of establishments, or even individual establishments. The actual formation of elementary aggregates thus depends on the circumstances and the availability of information, and they may therefore be defined differently in different countries. However, some key points should be observed:

- Elementary aggregates should consist of groups of commodities or services that are as similar as possible, and preferably fairly homogeneous.
- They should also consist of commodities that may be expected to have similar price movements. The objective should be to try to minimize the dispersion of price movements within the aggregate.
- The elementary aggregates should be appropriate to serve as strata for sampling purposes in light of the sampling regime planned for the data collection.

10.9 Each elementary aggregate, whether relating to the whole export or import, the country of destination or origin, or a group of establishments, will typically contain a very large number of individual commodities or services. Unit value indices from customs data benefit from covering the vast majority of transactions for merchandise goods.[1] However, for establishment surveys, in practice only a small number can be selected for pricing. When selecting the commodities, one must take into account the following considerations:

- The transactions selected should be ones with price movements that are believed to be representative of all the commodities within the elementary aggregate.
- The number of transactions within each elementary aggregate for which prices are collected should be large enough for the estimated price index to be statistically reliable. The minimum number required will vary between elementary aggregates, depending on the nature of the commodities and their price behavior.
- The object is to try to track the price of the same product over time for as long as the product continues to be representative. The commodities selected should therefore be ones that are expected to remain on the market for some time so that like items can be compared with like and problems associated with disappearing commodities and selection of replacements can be reduced. Prices of commodities of matched quality need to be monitored because the aim of the measure is to be one of pure price changes unaffected by changes in the quality composition over time, as may be the case with unit values.

10.10 The individual commodities should be grouped into elementary aggregates by use of a product (commodity) or activity (industry) classification, such as the Harmonized Commodity Description and Coding System (HS) or the International Standard Industrial

[1]In practice the coverage is less, so because large proportions of trade may be deleted if no quantity information is available, the unit value changes are considered to be outliers, and customs documentation does not cover the commodities concerned.

Classification of Economic Activities (ISIC). It is useful to assign a detailed product or activity code to each sampled commodity in order to facilitate the grouping of individual observations into elementary aggregates and the calculation of elementary indices. Similarly, the elementary aggregates should be appropriately coded to allow further aggregation into higher-level indices. This is dealt with below in Section C.1.1. The classifications were presented in more detail in Chapter 4.

B.2 Calculation of elementary price indices

10.11 An elementary price index is the price index for an elementary aggregate. Various methods and formulas may be used to calculate elementary price indices. This section provides a summary of pros and cons that statistical offices must evaluate when choosing a formula at the elementary level; Chapter 21 provides a more detailed discussion.

10.12 Often it is not possible to obtain information about the relative importance of the individual commodities that enter into the elementary aggregates. Or it may be considered too time consuming and resource demanding to obtain and maintain individual weights, compared with the possible improvements the use of such weights would add to the index. If such information has to be collected from the respondents it will also add to the establishment's response burden. In many countries much aggregation is thus done without the use of weighting data. This section, therefore, focuses on the calculation of unweighted elementary price indices. The calculation of weighted elementary indices is dealt with in Section C.

10.13 The methods statistical offices most commonly use are illustrated by means of a numerical example in Table 10.1. It is assumed that prices are collected for four representative commodities within an elementary aggregate. The quality of each commodity remains unchanged over time so that the month-to-month changes compare like items with like. No information on weights is available. Assume initially that prices are collected for all four commodities in every month covered so that there is a complete set of prices. There are no disappearing commodities, no missing prices, and no replacement commodities. These are quite strong assumptions because many of the problems encountered in practice are attributable to breaks in the continuity of the price series for the individual transactions for one reason or another. The treatment of disappearing and replacement commodities is taken up in Section B.5.

10.14 Three widely used formulas that have been, or still are, in use by statistical offices to calculate elementary price indices are illustrated in Table 10.1. It should be noted, however, that these are not the only possibilities and some alternative formulas are considered later.

- The first is the Carli index for $i = 1, \ldots, n$ commodities. It is defined as the simple, or unweighted, arithmetic mean of the price ratios, or price relatives, for the two periods, 0 and t, to be compared:

$$P_C^{0:t} = \frac{1}{n}\sum_i \left(\frac{p_i^t}{p_i^0}\right). \tag{10.1}$$

- The second is the Dutot index, which is defined as the ratio of the unweighted arithmetic mean prices:

$$P_D^{0:t} = \frac{\frac{1}{n}\sum_i p_i^t}{\frac{1}{n}\sum_i p_i^0}. \tag{10.2}$$

- The third is the Jevons index, which is defined as the unweighted geometric mean of the price ratios, which is identical with the ratio of the unweighted geometric mean prices:

$$P_J^{0:t} = \prod_i \left(\frac{p_i^t}{p_i^0}\right)^{1/n} = \frac{\prod_i (p_i^t)^{1/n}}{\prod_i (p_i^0)^{1/n}}. \tag{10.3}$$

10.15 Each *month-to-month* index shows the change in the index from one month to the next. The *chained month-to-month* index links together these month-to-month changes by successive multiplication. The *direct* index compares the prices in each successive month directly with those of the reference month, January. By simple inspection of the various indices in Table 10.1, it is clear that the choice of formula and method can make a substantial difference in the results obtained. Some results are striking—in particular, the large difference between the chained Carli index for July and each of the direct indices for July, including the direct Carli.

10.16 The properties and behavior of the different indices are summarized in the following paragraphs and explained in more detail in Chapter 21. First, the differences between the results obtained by using the different formulas tend to increase as the variance of the price relatives, or ratios, increases. The greater the change in the dispersion of the price movements, the more critical the choice of index formula and method becomes. If the elementary aggregates are defined so that the price movements within the aggregate are

Table 10.1. Calculation of Price Indices for an Elementary Aggregate[1]

	January	February	March	April	May	June	July
				Prices			
Commodity *A*	6.00	6.00	7.00	6.00	6.00	6.00	6.60
Commodity *B*	7.00	7.00	6.00	7.00	7.00	7.20	7.70
Commodity *C*	2.00	3.00	4.00	5.00	2.00	3.00	2.20
Commodity *D*	5.00	5.00	5.00	4.00	5.00	5.00	5.50
Arithmetic mean prices	5.00	5.25	5.50	5.50	5.00	5.30	5.50
Geometric mean prices	4.53	5.01	5.38	5.38	4.53	5.05	4.98
				Month-to-month price relatives			
Commodity *A*	1.00	1.00	1.17	0.86	1.00	1.00	1.10
Commodity *B*	1.00	1.00	0.86	1.17	1.00	1.03	1.07
Commodity *C*	1.00	1.50	1.33	1.25	0.40	1.50	0.73
Commodity *D*	1.00	1.00	1.00	0.80	1.25	1.00	1.10
				Current to reference month (January) price relatives			
Commodity *A*	1.00	1.00	1.17	1.00	1.00	1.00	1.10
Commodity *B*	1.00	1.00	0.86	1.00	1.00	1.03	1.10
Commodity *C*	1.00	1.50	2.00	2.50	1.00	1.50	1.10
Commodity *D*	1.00	1.00	1.00	0.80	1.00	1.00	1.10
Carli index—Arithmetic mean of price relatives							
Month-to-month index	100.00	112.50	108.93	101.85	91.25	113.21	100.07
Chained month-to-month index	100.00	112.50	122.54	124.81	113.89	128.93	129.02
Direct index on January	100.00	112.50	125.60	132.50	100.00	113.21	110.00
Dutot index—Ratio of arithmetic mean prices							
Month-to-month index	100.00	105.00	104.76	100.00	90.91	106.00	103.77
Chained month-to-month index	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Direct index on January	100.00	105.00	110.00	110.00	100.00	106.00	110.00
Jevons index—Geometric mean of price relatives or ratio of geometric mean prices							
Month-to-month index	100.00	110.67	107.46	100.00	84.09	111.45	98.70
Chained month-to-month index	100.00	110.67	118.92	118.92	100.00	111.45	110.00
Direct index on January	100.00	110.67	118.92	118.92	100.00	111.45	110.00

[1]All price indices have been calculated using unrounded figures.

minimized, the results obtained become less sensitive to the choice of formula and method.

10.17 Certain features displayed by the data in Table 10.1 are systematic and predictable and follow from the mathematical properties of the indices. For example, it is well known that an arithmetic mean is always greater than, or equal to, the corresponding geometric mean—the equality holding only in the trivial case in which the numbers being averaged are all the same. The direct Carli indices are therefore all greater than the Jevons indices, except in May and July when the four price relatives based on January are all equal. In general, the Dutot index may be greater or less than the Jevons index, but tends to be less than the Carli index.

10.18 One general property of geometric means should be noted when using the Jevons formula. If any one observation out of a set of observations is zero, its geometric mean is zero, whatever the values of the other observations. The Jevons index is sensitive to extreme changes in prices, and it may be necessary to impose upper and lower bounds on the individual price relatives of, say, 10 and 0.1, respectively, when using the Jevons. Of course, extreme observations are often the result of errors of one kind or another, and so extreme price movements should be carefully checked in any case. More details on data editing can be found in Section D.

10.19 Another important property of the indices illustrated in Table 10.1 is that the Dutot and the Jevons indices are transitive, whereas the Carli index is not. Transitivity means that the chained monthly indices are identical with the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices that link together the month-to-month-indices. The intransitivity of the Carli index is illustrated dramatically in Table 10.1, in which each of the four individual prices in May returns to the same level as it was in January, but the chained Carli index registers an increase of almost 14 percent over January. Similarly, in July, although each individual price is exactly 10 percent higher than in January, the chained Carli index registers an increase of 29 percent. These results would be regarded as perverse and unacceptable in the case of a direct index, but even in the case of the chained index, the results seems so intuitively unreasonable as to undermine the credibility of the chained Carli index. The price changes between March and April illustrate the effects of "price bouncing," in which the same four prices are observed in both periods, but they are switched between the different commodities. The monthly Carli index from March to April increases, whereas both the Dutot and the Jevons indices are unchanged.

10.20 The message emerging from this brief illustration of the behavior of just three possible formulas is that different index numbers and methods can deliver very different results. Index compilers have to familiarize themselves with the interrelationships between the various formulas at their disposal for the calculation of the elementary price indices so that they are aware of the implications of choosing one formula rather than another. However, knowledge of these interrelationships is not sufficient to determine which formula should be used, even though it makes it possible to make a more informed and reasoned choice. It is necessary to appeal to additional criteria to settle the choice of formula. Two main approaches may be used, the axiomatic and the economic approaches.

B.2.1 Sampling properties of elementary price indices

10.21 The interpretation of the elementary aggregate indices is related to the way in which the sample of commodities is drawn. Hence, if the commodities in the sample are selected with probabilities proportional to the population value shares in the price reference period,

- The sample (unweighted) Carli index provides an unbiased estimate of the population Laspeyres price index, and
- The sample (unweighted) Jevons index provides an unbiased estimate of the population geometric Laspeyres price index (see equation (10.5)).

10.22 If the commodities are sampled with probabilities proportional to population quantity shares in the price reference period, the sample (unweighted) Dutot index would provide an estimate of the population Laspeyres price index. However, if the basket for the Laspeyres index contains different kinds of products whose quantities are not additive, the quantity shares, and hence the probabilities, are undefined.

B.2.2 Axiomatic approach to elementary price indices

10.23 As explained in Chapters 17 and 21, one way to decide on an appropriate index formula is to require it to satisfy certain specified axioms or tests. The tests throw light on the properties possessed by different kinds of indices, some of which may not be intuitively obvious. Four basic tests illustrate the axiomatic approach.

Proportionality test. If all prices are λ times the prices in the price reference period (January in the example), the index should equal λ. The data for July, when every price is 10 percent higher than in January, show that all three direct indices satisfy this test. A special case of this test is the *identity test,* which requires that if the price of every commodity is the same as in the reference period, the index should be equal to unity (as in May in the example).

Changes in the units of measurement test (or commensurability test). The price index should not change if the quantity units in which the commodities are measured are changed—for example, if the prices are expressed per liter rather than per pint. The Dutot index fails this

Table 10.2. Properties of Main Elementary Aggregate Index Formulas

	Formula		
Formula Properties	Carli—Arithmetic mean of price relatives	Dutot—Relative of arithmetic mean prices	Jevons—Geometric mean of price relatives
Proportionality	yes	yes	yes
Change of units of measurement	yes	no	yes
Time reversal	no	yes	yes
Transitivity	no	yes	yes
Allows for substitution	no	no	yes

test, as explained below, but the Carli and Jevons indices satisfy the test.

Time reversal test. If all the data for the two periods are interchanged, then the resulting price index should equal the reciprocal of the original price index. The Carli index fails this test, but the Dutot and the Jevons both satisfy the test. The failure of the Carli index to satisfy the test is not immediately obvious from the example but can easily be verified by interchanging the prices in January and April, for example, in which case the backward Carli for January based on April is equal to 91.3 whereas the reciprocal of the forward Carli index is 1/132.5, or 75.5.

Transitivity test. The chained index between two periods should equal the direct index between the same two periods. The example shows that the Jevons and the Dutot indices both satisfy this test, whereas the Carli index does not. For example, although the prices in May have returned to the same levels as in January, the chained Carli index registers 113.9. This illustrates the fact that the Carli index may have a significant built-in upward bias.

10.24 Many other axioms or tests can be devised, as presented in Chapters 17 and 21, but the above (summarized in Table 10.2) are sufficient to illustrate the approach and also to throw light on some important features of the elementary indices under consideration here.

10.25 The sets of commodities covered by elementary aggregates are meant to be as homogeneous as possible. If they are not fairly homogeneous, the failure of the Dutot index to satisfy the units of measurement, or commensurability, test can be a serious disadvantage. Although defined as the ratio of the unweighted arithmetic average prices, the Dutot index may also be interpreted as a weighted arithmetic average of the price ratios in which each ratio is weighted by its price in the base period.[2] However, if the commodities are not homogeneous, the relative prices of the different commodities may depend quite arbitrarily on the quantity units in which they are measured.

10.26 Consider, for example, salt and pepper, which are found within the same Central Product Classification subclass. Suppose the unit of measurement for pepper is changed from grams to ounces while the units in which salt is measured (say, kilos) are left unchanged. Because an ounce of pepper is equal to 28.35 grams, the "price" of pepper increases by more than 28 times, which effectively increases the weight given to pepper in the Dutot index by more than 28 times. The price of pepper relative to salt is inherently arbitrary, depending entirely on the choice of units in which to measure the two goods. In general, when there are different kinds of commodities within the elementary aggregate, the Dutot index is not acceptable.

10.27 The Dutot index is acceptable only when the set of commodities covered is homogeneous, or at least nearly homogeneous. For example, the Dutot index may be acceptable for a set of apple prices, even though the apples may be of different varieties, but not for the prices of different kinds of fruits, such as apples, pineapples, and bananas, some of which may be much more expensive per item or per kilo than others. Even when the commodities are fairly homogeneous and measured in the same units, the Dutot index's implicit weights may still not be satisfactory. More weight is given to the

[2]This can be seen by rewriting equation (10.2) above as

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^0 \cdot \left(p_i^t / p_i^0\right)}{\frac{1}{n}\sum p_i^0}.$$

price changes for the more expensive commodities, but more expensive items may not account for the highest traded value share.

10.28 It may be concluded that from an axiomatic viewpoint, both the Carli and the Dutot indices, although they have been and still are widely used by statistical offices, have serious disadvantages. The Carli index fails the time reversal and transitivity tests. In principle, it should not matter whether we choose to measure price changes forward or backward in time. We would expect the same answer, but this is not the case for the Carli index. Chained Carli indices may be subject to a significant upward bias and should not be applied. The Dutot index is meaningful for a set of homogeneous commodities but becomes increasingly arbitrary as the set of commodities becomes more diverse. On the other hand, the Jevons index satisfies all the tests listed above and also emerges as the preferred index when the set of tests is enlarged, as shown in Chapter 21. From an axiomatic point of view, the Jevons index is clearly the index with the best properties, even though it may not have been used much until recently.

B.2.3 Economic approach to elementary price indices

10.29 The objective of the economic approach is to estimate an "ideal" or "true" economic index for the elementary aggregates. Consider a price index for the exports produced by resident establishments. As explained in Chapter 18, if, for example, it is assumed that the establishments behave as revenue maximizers, it follows that they would switch export production to commodities with higher relative price changes. This behavioral assumption about the firm allows something to be said about what a "true" index number formula should be and the suitability of different index number formulas as approximations to it. For example, the Laspeyres price index uses a fixed reference period for its export revenue shares to weight the price relatives and ignores the substitution of production toward products with higher relative price changes. The Laspeyres price index will thus understate aggregate price changes—that is, be biased downward against its true index. The Paasche price index uses fixed current period weights and will thus overstate aggregate price changes—that is, be biased upward against its true index.

10.30 The advantage of the economic approach is that it takes account of the interdependence between prices and quantities. The economic approach thus requires information on quantities or value shares. Index number compilation distinguishes between two stages of aggregation: the elementary level without weights and a higher level with weights. The concern here is with the elementary level. Because information on quantities or value shares is not available at the elementary level, an economic approach cannot be used at this level. However, if the items being priced are sampled with probabilities proportionate to quantity or value shares, then quantity or value information becomes attached to the sampled prices and the unweighted elementary index number formulas are implicitly weighted, as outlined in Chapter 21, Section G. The sampling should be with probabilities proportionate to *population* quantity or value shares and it is most likely in index number compilation that the available quantity or value share data will relate to the *reference,* and not current, period. Thus a prerequisite for the application of the economic approach to elementary index number formulas is that prices are sampled with probabilities proportionate to reference period population quantity or value shares. It is unusual that such formal sample designs are used for the sampling of establishments and items in survey-based XMPIs, and in their absence choice of formulas is best guided by the axiomatic approach.

10.31 Thus two things are needed to apply the economic approach: first, a specific sample design involving quantity or value shares to translate a sample unweighted elementary index into an estimator of a weighted population index; second, establishment of whether this estimated weighted index is an appropriate target, that is, one based on the behavioral assumptions of the enterprises or households responsible for exporting or importing the commodities.

10.32 Consider, for example, the Carli index, an unweighted average of a sample of price relatives. It differs from the Laspeyres index because there are no reference period weights to attach to the price relatives. Say sampling is with probabilities proportionate to population reference period value shares. Then the sample Carli index, under this sample design, is an estimator of the population Laspeyres index. Say exporters maintain fixed reference period revenue shares and ignore the substitution of exports toward items with higher relative price changes. Then Laspeyres is the appropriate target index. Information on current-period revenue shares is not relevant. Say there is some substitution behavior toward higher priced items. Then, as Chapter 18 shows, the target index will be one that takes account of substitution effects, such as the Fisher or Törnqvist index, and the sample estimator of Laspeyres will be biased. It will understate price changes. Considered below are the implications for this approach for export and import price indices in turn.

B.2.3.1 Output export price indices, XPIs

10.33 For the export price index (XPI) the commodities for which respondents provide prices are treated as a basket of goods and services sold by establishments to provide revenue. The establishments may substitute between the commodities supplied in response to changes in their relative prices. However, in the absence of information about quantities or trade volumes within an elementary aggregate, an economic index can be estimated only under assumptions about the establishments' reaction to price changes and a sample design that involves relative quantities or values.

10.34 There are two special cases of some interest. The first case is that used for illustration above, when producers continue to produce the same relative quantities irrespective of any changes in relative export prices on the market, that is, when all cross-elasticities of supply are zero. In this case, a population Laspeyres index would be an exact measure of the economic export (output) price index. The sampling approach, outlined in Chapter 21, Section G, demonstrates that a sample Carli index would provide an estimate of the economic index, provided that commodities are selected with probabilities proportional to the population value shares in the reference period. If the products were selected with probabilities proportional to the population quantity shares in the reference period, the sample Dutot would provide an estimate of the population Laspeyres.

10.35 However, even if sampling with probabilities proportionate to reference period quantity shares is used, such sampling is meaningful only for homogeneous (identical) items measured in the same units, for the units are implicitly added up. Many markets comprise differentiated branded items and it is not meaningful to add up the quantities of such heterogeneous items. The applicability of Dutot is thus relatively limited. Yet, for the case of homogeneous items, say tons of rolled steel of the same dimensions and quality sold under the same terms and conditions, the appropriate target index is not a Laspeyres index but a unit value index (Chapter 21). Thus the use of a Dutot index and sampling with probabilities proportionate to reference period quantity shares for homogeneous items would still be inappropriate.

10.36 The second case of interest is that the sample Jevons index provides an unbiased estimate of the population geometric Laspeyres index provided that the commodities are selected with probabilities proportional to the population revenue shares in the reference period (Chapter 21, Section F). If producers are assumed to vary the quantities they produce in inverse proportion to the changes in relative output prices such that the cross-elasticities of supply are all unity, so the revenue shares remain the same in both periods, then the sample Jevons index is an estimator of the population Törnqvist index. This again requires that commodities are selected with probabilities proportional to the population revenue shares in the reference period, which in this case is also equal to the current period. It may be argued that unitary cross-elasticities are unlikely as they imply that establishments should produce more of those goods whose prices are falling and less of those whose prices are increasing. However, it might be the case that producers also take into account other factors than prices, such as their market share or expected demand. In such cases, producers may focus on growing markets even if the relative prices are falling; for example, many high-technology product markets are characterized by rapid growth and falling prices, especially if quality adjusted.

10.37 In competitive, demand-led industries, producers will tend to produce more of those commodities whose relative price has increased. Under such conditions none of these indices provide a close estimate of the economic index. However, a Carli index is more likely to provide a closer approximation to the economic index than the Jevons index, which may be viewed as downward biased, provided sampling is with reference period revenue shares.

10.38 In the economic approach, the choice of index formula rests on which is likely to approximate more closely the underlying economic index—in other words, whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the cross-elasticities could take on any value. In some industries supply is relatively unresponsive to changes in demand, and producers tend to produce the same relative quantities irrespective of relative price changes. In this case, a Carli index is likely to give a closer approximation to the economic index than the Jevons index, provided prices are sampled with probability proportionate to value shares in the reference period.

10.39 While these results may at first sight add credence to the use of these formulas, they do so only if two conditions are met. First, that the appropriate sample design is used and second, that it is meaningful for the product group under consideration. As noted above, a justification for Carli requires sampling with probabilities proportionate to reference period revenue shares and zero-valued cross-elasticities.

B.2.3.2 Input import price indices, import price indices

10.40 The discussion is phrased in terms of a cost-minimizing resident purchaser for an import price index (or nonresident purchaser for an XPI). Again the choice of index formula rests on the sample design and which is likely to approximate more closely the underlying ideal economic index, that is, whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In some industries demand for inputs is relatively unresponsive to changes in prices and establishments tend to import the same relative quantities irrespective of changes in their prices. In such cases the Carli index is likely to provide a closer approximation to the ideal economic index than the Jevons index, which may be viewed as having a downward bias. If establishments tend to substitute toward cheaper inputs as a response to change in relative prices, the Jevons index may provide the better estimate of the economic index, and the Carli may be viewed as having an upward bias. However, in both instances a prerequisite for making any such calls is that the appropriate sample design has been utilized and, as noted earlier, for survey-based XMPIs, this is unlikely.

10.41 It should be noted that the Jevons index does not imply, or assume, that the trade value shares remain constant. Obviously, the Jevons can be calculated whether or not changes occur in the value shares in practice. What the economic approach shows is that *if* sampling is with probabilities proportionate to population value shares, and *if* the value shares remain constant, or roughly constant, *then* the Jevons index can be expected to provide a good estimate of the underlying ideal economic index. Similarly, *if* the same sample design holds and *if* the relative quantities remain constant, *then* the Carli index can be expected to provide a good estimate. But neither of these formulas actually implies that sampling is with probability proportionate to value or quantities and that value shares or relative quantities remain fixed over time. Quite limiting assumptions are required for the use of the economic approach at the elementary level, and if these assumptions do not hold, the axiomatic approach provides sound guidance to adopt the Jevons index.[3] Reference should be made to Chapter 18 and, in the context of elementary index numbers, Section F of Chapter 21, for a more rigorous statement of the economic approach.

B.3 Chained versus direct indices for elementary aggregates

10.42 In a direct elementary index, the prices of the current period are compared directly with those of the price reference period. In a chained elementary index, prices in each period are compared with those in the previous period, and the resulting short-term indices are then multiplied, or chained, to obtain the long-term index, as illustrated in Table 10.1.

10.43 Provided that prices are recorded for the same set of commodities in every period, as in Table 10.1, any index formula defined as the ratio of the average prices will be transitive—that is, the same result is obtained whether the index is calculated as a direct index or as a chained elementary index. In a chained elementary index, successive numerators and denominators will cancel out, leaving only the average price in the last period divided by the average price in the price reference period, which is the same as the direct index. Both the Dutot and the Jevons indices are therefore transitive. As already noted, however, a chain Carli index is not transitive and should not be used because of its upward bias. The direct Carli fails, as noted above, the time reversal test and is not generally advised. Nevertheless, the direct Carli remains an option.

10.44 Although the chained and direct versions of the Dutot and Jevons indices are identical when there are no breaks in the series for the individual commodities, they offer different ways of dealing with new and disappearing commodities, missing prices, and quality adjustments. In practice, commodities continually have to be dropped from the index and new ones included, in which case the direct and the chain indices may differ if the imputations for missing prices are made differently.

10.45 When a replacement commodity has to be included in a direct index, it often will be necessary to estimate the price of the new commodity in the price reference period, which may be some time in the past. The same happens if, as a result of an update of

[3]The Carli index failed the time reversal and transitivity tests and should not be used. The Dutot failed the commensurability test and is applicable only for homogeneous items.

the sample, new commodities have to be linked into the index. If no information exists on the price of the replacement commodity in the price reference period, it will be necessary to estimate it using price relatives calculated for the commodities that remain in the elementary aggregate, a subset of these commodities, or some other indicator. However, the direct approach should be used only for a limited period. Otherwise, most of the reference prices would end up being imputed, which would be an undesirable outcome. This effectively rules out the use of the Carli index over a long period, because the Carli index can be used only in its direct form anyway, and even then is subject to bias owing to its failure of the time reversal test, being unacceptable when chained.

10.46 In a chained elementary index, if a commodity becomes permanently missing, a replacement commodity can be linked into the index as part of the ongoing index calculation by including the commodity in the monthly index as soon as prices for two successive months are obtained. Similarly, if the sample is updated and new commodities have to be linked into the index, this will require successive old and new prices for the present and the preceding month. However, for a chain elementary index, the missing observation will affect the index for two months, because the missing observation is part of two links in the chain. This is not the case for a direct index where a single, nonestimated missing observation will affect only the index in the current period. For example, when comparing periods 0 and 3, a missing price of a commodity in period 2 means that the chained index excludes the commodity for the last link of the index in periods 2 and 3, while the direct index includes it in period 3 (because a direct index will be based on commodities with prices available in periods 0 and 3). However, in general, the use of a chained index can make the estimation of missing prices and the introduction of replacements easier from a computational point of view, whereas it may be inferred that a direct index will limit the usefulness of overlap methods for dealing with missing observations. Missing price observations are discussed further in Section B.5.

10.47 The direct and the chained elementary approaches also produce different by-products that may be used for monitoring price data. For each elementary aggregate, a chained index approach gives the latest monthly price change, which can be useful for both editing data and imputing missing prices. By the same token, however, a direct index derives average price levels for each elementary aggregate in each period, and this information may be a useful by-product. However, the availability of cheap computing power and spreadsheets allows such by-products to be calculated whether a direct or a chained approach is applied, so the choice of formula should not be dictated by considerations regarding by-products.

B.4 Consistency in aggregation

10.48 Consistency in aggregation means that if an index is calculated stepwise by aggregating lower-level indices to obtain indices at progressively higher levels of aggregation, the same overall result should be obtained as if the calculation had been made in one step. If the elementary indices are calculated using one formula, and then averaged to obtain the higher-level indices using another formula, the resulting XMPI is not consistent in aggregation. However, it may be argued that consistency in aggregation is not necessarily an important or even appropriate criterion. There may be different elasticities of substitution within elementary aggregates compared to the elasticities between elementary aggregates. This may be an argument for using a different index formula at a different level of aggregation. Also, it may be unachievable, particularly when the amount of information available on quantities and trade values is not the same at the different levels of aggregation.

10.49 The Carli index is consistent in aggregation with a higher-level Laspeyres index if the commodities are selected with probabilities proportional to trade values in the price reference period. The Dutot and the Jevons indices are not consistent in aggregation with a higher-level Laspeyres index. However, as explained below, the XMPIs actually calculated by statistical offices are usually not true Laspeyres indices anyway, even though they may be based on fixed baskets of goods and services. If the higher-level index were to be defined as a geometric Laspeyres index, consistency in aggregation could be achieved by using the Jevons index for the elementary indices at the lower level, provided that the individual commodities are sampled with probabilities proportional to trade values. Although unfamiliar, a geometric Laspeyres index has desirable properties from an economic point of view and is considered again in Section B.6.

B.5 Missing price observations

10.50 The price of a commodity may not be collected in a particular period either because the commodity is missing temporarily or because it has permanently disappeared. The two classes of missing prices require different treatments. Temporary unavailability may occur for seasonal commodities (particularly for fruit, vegetables, and clothing) because of supply shortages or possibly because of some collection difficulty (e.g., an establishment was closed or a respondent was on vacation). The treatment of seasonal commodities raises a number of particular problems. These are dealt with in Chapter 23 and are not discussed here.

B.5.1 Treatment of temporarily missing prices

10.51 In the case of temporarily missing observations for commodities, one of four actions may be taken:

- Omit the commodity for which the price is missing so that a matched sample is maintained (like is compared with like), even though the sample is depleted.
- Carry forward the last observed price.
- Impute the missing price by the average price change of the prices that are available in the elementary aggregate.
- Impute the missing price by the price change of a comparable commodity from a similar establishment.

10.52 The price development for a given commodity may be different according to the country to which it is exported or from which it is imported. This may be due to different price trends in the countries or exchange rate changes. An elementary index thus may contain commodities from several countries, and the price development of a missing commodity for a specific country may be unusual compared to the average price development of the remaining ones. When imputing a price by the price development of another commodity or a group of commodities, one should therefore give consideration to the country of origin (imports) or destination (exports).

10.53 Omitting an observation from the calculation of an elementary index is equivalent to assuming that the price would have moved in the same way as the average of the prices of the commodities that remain included in the index. Omitting an observation changes the implicit weights attached to the other prices in the elementary aggregate.

10.54 Carrying forward the last observed price should be avoided wherever possible and is acceptable for only a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and commodity turnover. Although simple to apply, carrying forward the last observed price biases the resulting index toward zero change. In addition, there is likely to be a compensating step-change in the index when the price of the missing commodity is recorded again. The adverse effect on the index will be increasingly severe if the commodity remains unpriced for some length of time. In general, carry forward is not an acceptable procedure or solution to the problem unless one is certain the price has not changed.

10.55 Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates when the prices can be expected to move in the same direction. The imputation can be made using all the remaining prices in the elementary aggregate. As already noted, this is numerically equivalent to omitting the commodity for the immediate period, but it is useful to make the imputation so that if the price becomes available again in a later period, the sample size is not reduced in that period. In some cases, depending on the homogeneity of the elementary aggregate, it may be preferable to use only a subset of commodities from the elementary aggregate to estimate the missing price. In some instances, this may even be a single comparable commodity from a similar type of establishment whose price change can be expected to be similar to the missing one.

10.56 Table 10.3 illustrates the calculation of the price index for an elementary aggregate consisting of three commodities, where one of the prices is missing in March. The upper part of Table 10.3 shows the indices where the missing price has been omitted from the calculation. The direct indices are therefore calculated on the basis of *A, B,* and *C* for all months except March, where they are calculated on basis of *B* and *C* only. The chained indices are calculated on the basis of all three prices from January to February and from April to May. From February to March and from March to April, the monthly indices are calculated on the basis of *B* and *C* only.

10.57 For both the Dutot and the Jevons, the direct and chain indices now differ from March onward. The first link in the chained index (January to February) is the same as the direct index, so that the two indices are identical numerically. The direct index for March ignores the price decrease of commodity *A* between January and February, whereas this is taken into account in the chained index. As a result, the direct index is higher than the chained index for March. On the other hand, in April and May, where all prices again are available, the direct index catches the price development, whereas the chained index fails to track the development in the prices.

Table 10.3. Imputation of Temporarily Missing Prices

	January	February	March	April	May
			Prices		
Commodity *A*	6.00	5.00		7.00	6.60
Commodity *B*	7.00	8.00	9.00	8.00	7.70
Commodity *C*	2.00	3.00	4.00	3.00	2.20
Omit missing commodity from the index calculation					
Carli index—Arithmetic mean of price relatives					
Direct index	100.00	115.87	164.29	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.67	118.18	84.62	91.67
Chained month-to-month index	100.00	106.67	126.06	106.67	97.78
Direct index	100.00	106.67	144.44	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Month-to-month index	100.00	112.62	122.47	81.65	87.31
Chained month-to-month index	100.00	112.62	137.94	112.62	98.33
Direct index	100.00	112.62	160.36	125.99	110.00
Imputation					
Carli index—Arithmetic mean of price relatives					
Impute price for A *in March as* $5\ (9/8 + 4/3)/2 = 6.15$					
Direct index	100.00	115.87	143.67	126.98	110.00
Dutot index—Ratio of arithmetic mean prices					
Impute price for A *in March as* $5\ [(9 + 4)/(8 + 3)] = 5.91$					
Month-to-month index	100.00	106.67	118.18	95.19	91.67
Chained month-to-month index	100.00	106.67	126.06	120.00	110.00
Direct index	100.00	106.67	126.06	120.00	110.00
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for A *in March as* $5\ (9/8 \times 4/3)^{0.5} = 6.12$					
Month-to-month index	100.00	112.62	122.47	91.34	87.31
Chained month-to-month index	100.00	112.62	137.94	125.99	110.00
Direct index	100.00	112.62	137.94	125.99	110.00

10.58 In the lower half of Table 10.3, the missing price for commodity *A* in March is imputed by the average price change of the remaining commodities from February to March. Although the index may be calculated as a direct index comparing the prices of the present period with the reference period prices, the imputation of missing prices should be made on the basis of the average price change from the preceding to the present period, as shown in the table. Imputation on the basis of the average price change from the price reference period to the present period should not be used because it ignores the information about the price change of the missing commodity that has already been included in the index. The treatment of imputations is discussed in more detail in Chapter 8.

10.59 A special case of "missing prices" occurs when prices are recorded with different frequency, such as if some prices are recorded monthly while others only quarterly or every half-year. If the index is compiled on a monthly basis there will then be a need to temporarily update the prices recorded with a lower frequency. The options for updating the lower frequency prices will be the same as those described above.

Table 10.4. Disappearing Commodities and Their Replacements with No Overlap

	January	February	March	April	May
			Prices		
Commodity *A*	6.00	7.00	5.00		
Commodity *B*	3.00	2.00	4.00	5.00	6.00
Commodity *C*	7.00	8.00	9.00	10.00	9.00
Commodity *D*				9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 9/[(5/3 + 10/7) 0.5] = 5.82*					
Direct index	100.00	99.21	115.08	154.76	155.38
Dutot index—Ratio of arithmetic mean prices					
Impute price for D *in March as 9/[(5 + 10)/(4 + 9)] = 7.80*					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Impute price for D *in January as 9/[(5 + 10)/(3 + 7)] = 6.00*					
Direct index	100.00	106.25	112.50	150.00	143.75
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Impute price for D *in March as* $9/((5/4*10/9)^{0.5}) = 7.64$					
Month-to-month index	100.00	96.15	117.13	117.85	98.65
Chained month-to-month index	100.00	96.15	112.62	132.73	130.94
Impute price for D *in January as* $9/((5/3*10/7)^{0.5}) = 5.83$					
Direct index	100.00	96.15	112.62	154.30	152.22
Omit the price					
Dutot index—Ratio of arithmetic mean prices					
Month-to-month index	100.00	106.25	105.88	115.38	95.83
Chained month-to-month index	100.00	106.25	112.50	129.81	124.40
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Monthly index	100.00	96.15	117.13	117.85	98.65
Chained month-to-month index	100.00	96.15	112.62	132.73	130.94

B.5.2 Treatment of commodities that have permanently disappeared and their replacements

10.60 Commodities may disappear permanently for various reasons. The commodity may disappear from the market because new commodities have been introduced or an establishment from which prices have been collected leaves the market. When commodities disappear permanently, a replacement commodity has to be sampled and included in the index. The replacement commodity should ideally be one that accounts for a significant proportion of sales, is likely to continue to be sold for some time, and is likely to be representative of the price changes of the market that the old commodity covered.

10.61 The timing of the introduction of replacement commodities is important. Many new commodities are initially sold at high prices that then gradually drop over time, especially as the value of sales increases. Alternatively, some commodities may be introduced at artificially low prices to stimulate demand. In such cases, delaying the introduction of a new or replacement commodity until a large volume of sales is achieved may miss some systematic price changes that ought to be captured by XMPIs. It may be desirable to try to avoid forced replacements caused when commodities disappear completely from the market and to try to introduce replacements when sales of the commodities they replace are decreasing and before they cease altogether.

10.62 Table 10.4 shows an example where commodity *A* disappears after March and commodity *D* is included as a replacement from April onward. Commodities *A* and *D* are not available on the market at the same time, and their price series do not overlap. To include the new commodity in the index from April onward, an imputed price needs to be calculated either for the base period (January) if a direct index is being calculated, or for the preceding period (March) if a chained index is calculated. In both cases, the imputation method ensures that the inclusion of the new commodity does not, in itself, affect the index.

10.63 In the case of an unweighted (elementary) chained formulation, imputing the missing price by the average change of the available prices gives the same result as if the commodity is simply omitted from the index calculation until it has been priced in two successive periods. This allows the chained index to be compiled by simply chaining the month-to-month index between periods $t-1$ and t, based on the matched set of prices in those two periods, on to the value of the chained index for period $t-1$. In the example, no further imputation is required after April, and the subsequent movement of the index is unaffected by the imputed price change between March and April.

10.64 In the case of a direct index, however, an imputed price is always required for the reference period to include a new commodity. In the example, the price of the new commodity in each month after April still has to be compared with the imputed price for January. As already noted, to prevent a situation in which most of the reference period prices end up being imputed, the direct approach should be used for only a limited period of time.

10.65 The situation is somewhat simpler when there is an overlap month in which prices are collected for both the disappearing and the replacement commodity. In this case, it is possible to link the price series for the new commodity to the price series for the old commodity that it replaces. Linking with overlapping prices involves making an implicit adjustment for the difference in quality between the two commodities, because it assumes that the relative prices of the new and old commodity reflect their relative qualities. For perfect or nearly perfect markets, this may be a valid assumption, but for certain markets and commodities it may not be so reasonable. The question of when to use overlapping prices is dealt with in detail in Chapter 8. The overlap method is illustrated in Table 10.5.

10.66 In the example, overlapping prices are obtained for commodities *A* and *D* in March. Their relative prices suggest that one unit of commodity *A* is worth two units of commodity *D*. If the index is calculated as a direct Carli index, the January base period price for commodity *D* can be imputed by dividing the price of commodity *A* in January by the price ratio of *A* and *D* in March.

10.67 A monthly chain index of arithmetic mean prices will be based on the prices of commodities *A*, *B*, and *C* until March, and from April onward by *B*, *C*, and *D*. The replacement commodity is not included until prices for two successive periods are obtained. Thus, the monthly chained index has the advantage that it is not necessary to carry out any explicit imputation of a reference price for the new commodity.

10.68 If a direct index is calculated as the ratio of the arithmetic mean prices, the price of the new commodity needs to be adjusted by the price ratio of *A* and *D* in March in every subsequent month, which complicates computation. Alternatively, a reference period price of commodity *D* for January may be imputed. However, this results in a different index because the price relatives are implicitly weighted by the relative reference period prices in the Dutot index, which is not the case for the Carli or the Jevons index. For the Jevons index, all three methods give the same result, which is an additional advantage of this approach.

10.69 Problems with missing prices may be particular in smaller countries or for commodity groups, even in larger countries, where the number of reporting establishments is very limited. From time to time establishments will leave the market and no replacement can be found. If there are still prices recorded, the elementary index can be continued on the basis of the remaining prices. However, in some instances all prices for an elementary aggregate may disappear. In this case, it will be necessary to assign the weight to another elementary aggregate or to impute or carry forward the elementary index until the next revision of the index.

10.70 The statistical office may try to reduce the problems associated with missing prices by defining the elementary aggregates not too narrowly. More broadly defined elementary aggregates will reduce problems with missing prices and help facilitate a smooth, regular compilation of the index. However, markets change over time and the index should reflect this. This issue is dealt with in more detail in Sections C.6.4 and C.6.5 on the introduction of new elementary and higher-level indices in the overall price index.

Table 10.5. Disappearing and Replacement Commodities with Overlapping Prices

	January	February	March	April	May
			Prices		
Commodity *A*	6.00	7.00	5.00		
Commodity *B*	3.00	2.00	4.00	5.00	6.00
Commodity *C*	7.00	8.00	9.00	10.00	9.00
Commodity *D*			10.00	9.00	8.00
Carli index—Arithmetic mean of price relatives					
Impute price for D *in January as 6/(5 /10) = 12.00*					
Direct index	100.00	99.21	115.08	128.17	131.75
Dutot index—Ratio of arithmetic mean prices					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	106.25	105.88	104.35	95.83
Chained month-to-month index	100.00	106.25	112.50	117.39	112.50
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	106.25	112.50	121.88	118.75
Impute price for D *in January as 6/(5 /10) = 12.00*					
Direct index	100.00	106.25	112.50	109.09	104.55
Jevons index—Ratio of geometric mean prices or geometric mean of price relatives					
Chain the monthly indices based on matched prices					
Month-to-month index	100.00	96.15	117.13	107.72	98.65
Chained month-to-month index	100.00	96.15	112.62	121.32	119.68
Divide D*'s price in April and May with 10/5 = 2 and use* A*'s price in January as base price*					
Direct index	100.00	96.15	112.62	121.32	119.68
Impute price for D *in January as 6/(5 /10) = 12.00*					
Direct index	100.00	96.15	112.62	121.32	119.68

B.6 Calculation of elementary price indices using weights

10.71 Whenever possible, weights that reflect the relative importance of the sampled commodities may be introduced in the calculation of the elementary indices. For certain elementary aggregates, information about the value of export or import of particular commodities may be obtained from existing trade and industry sources, or the statistical office can work with establishment respondents to obtain weighting data, as discussed in Chapter 5. In addition, the growing use of electronic recording of transactions in many countries, in which records on both prices and quantities are maintained, means that valuable new sources of information may become increasingly available to statistical offices.

10.72 For example, assume that the number of importers of a certain commodity such as gasoline is limited. The market shares of the importers may be known from business survey statistics and can be used as weights in the calculation of an elementary aggregate price index for gasoline.

10.73 A special situation occurs in the case of tariff prices. A tariff is a list of prices for the provision of a particular kind of good or service under different terms and conditions. One example is electricity for which one price is charged during the day and a lower price is charged at night. Another example may be airline passenger fares sold at one price to some passengers and at lower prices to others. In such cases, it is appropriate to assign weights to the different tariffs or prices to calculate the price index for the elementary aggregate.

10.74 Weights within elementary aggregates may be updated independently and possibly more often than the elementary aggregate weights themselves.

Table 10.6. Calculation of a Weighted Elementary Index

	Weight	December	January	February	Percent Change Dec.–Feb.
Commodity *A*	0.80	7	7	9	28.6
Commodity *B*	0.17	20	20	10	−50.0
Commodity *C*	0.03	28	28	12	−57.1
Weighted arithmetic mean of price ratios (Laspeyres)					**Index**
(0.8*(9/7) + 0.17*(10/20) + 0.03*(12/28))*100 =					112.64
Weighted geometric mean of price ratios (geometric Laspeyres)					
$((9/7)^{0.8}*(10/20)^{0.17}*(12/28)^{0.03})*100 =$					105.95
Ratio of weighted arithmetic mean prices					
Weighted arithmetic mean prices		9.84	9.84	9.26	
(9.26/9.84)*100 =					94.11

10.75 If weighting data are available for all the individual commodities within an elementary aggregate, the elementary price index can be calculated as a Laspeyres price index, or as a geometric Laspeyres index; both are discussed further in Chapter 21. The Laspeyres price index is defined as

$$P_L^{0:t} = \frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} = \sum_i w_i^0 \cdot \left(\frac{p_i^t}{p_i^0}\right),\ w_i^0 = \frac{p_i^0 q_i^0}{\sum_i p_i^0 q_i^0}. \qquad (10.4)$$

10.76 As the quantities are often unknown, the index usually will have to be calculated by weighting together the individual price ratios by their trade value shares in the price reference period, w_i^0. The available weighting data may refer to an earlier period than the price reference period, but may still provide a good estimate. A more general version of equation (10.4) would be that of a Lowe or a Young index, where the weights are not necessarily those of the price reference period. These two indices are discussed in more detail in Section C.3. Note that if all the weights were equal, equation (10.4) would reduce to the Carli index. If the weights were proportional to the prices in the reference period, it would reduce to the Dutot index.

10.77 The geometric Laspeyres index is defined as

$$P_{GL}^{0:t} = \prod_i \left(\frac{p_i^t}{p_i^0}\right)^{w_0^i} = \frac{\prod_i (p_i^t)^{w_i^0}}{\prod_i (p_i^0)^{w_i^0}},\ \sum_i w_i^0 = 1, \qquad (10.5)$$

where the weights, w_i^0, are again the trade value shares in the reference period. When the weights are all equal, equation (10.5) reduces to the Jevons index. If the trade value shares do not change much between the weight reference period and the current period, then the geometric Laspeyres index approximates a Törnqvist index. A more general version of equation (10.5) would be that of a geometric Young index, where the weights are not necessarily those of the price reference period.

10.78 The weights may be attached to the individual price observations or to groups of price observations. For example, two establishments may both report, say, five prices that enter into the calculation of an elementary aggregate price index. However, the only weighting information may refer to the overall relative market share of the two establishments rather than to the individual commodities. Thus, if the relative market shares are 40/60, the two groups of prices may be weighted according to the 40/60 shares of the establishments.

10.79 Table 10.6 provides an example of calculation of an elementary index using weights. The elementary aggregate consists of three commodities for which prices are collected monthly. The trade value shares are estimated to 0.80, 0.17, and 0.03.

10.80 One option is to calculate the index as the weighted arithmetic mean of the price ratios, which gives an index of 112.64. The individual price changes are weighted according to their explicit weights, irrespective of the price levels. This corresponds to the calculation of a Laspeyres price index, where the price ratios and the weights refer to the same reference month. The index may also be calculated as the weighted geometric mean of the price ratios, the so-called geometric Laspeyres index, which gives an index of 105.95.

10.81 A third option could be to calculate the index as the ratio of the weighted arithmetic mean prices. As already noted, an elementary index should be based on arithmetic mean prices only if it includes homogeneous products measured in the same unit; otherwise it is not meaningful to calculate an average price. In practice, this will also mean that the price level of the products should be more or less the same. Second, this approach weights the price changes according to the relative price level in the reference period. Hence, the increase of 28.6 percent on commodity *A* that accounts for 80 percent of the market is weighted down because of its relative low price, resulting in an index of 94.11. This calculation is misleading, however.

10.82 The difference between the two arithmetic methods can be illustrated by an example: Assume an elementary aggregate with two commodities, X and Y, of equal weights (50/50). The price of X is constant 90, and the price of Y increases from 10 to 12. The weighted arithmetic mean of the price ratios gives 90/90 + 12/10 = 1.10. The ratio of arithmetic weighted prices gives (90 + 12)/(90 + 10) = 1.02. In the first approach, the price increases of the two commodities are equally weighted, which gives an increase of 10 percent. The problem in the second approach is that it weights the 0 percent price increase on X by 90/100, and the 20 percent increase of Y by only 10/100, which gives an overall increase of 2 percent. This can be justified only if the weights are proportional to the relative price level in the reference period, that is, if the weight of X is 90 and that of Y is 10, which, however, contradicts the assumption of 50/50 weights. Because of the calculation method, the weights are twisted according to the relative price levels resulting in a misleading index.

10.83 Weighting information at the very detailed level demands resources to obtain and update. This has to be balanced against the possible gains in terms of a more accurate price index.

B.7 Some alternative index formulas

10.84 Another type of average is the harmonic mean. In the present context, there are two possible versions: either the harmonic mean of price ratios or the ratio of harmonic mean of prices. The harmonic mean of price ratios is defined as

$$P_{HR}^{0:t} = \frac{1}{\frac{1}{n}\sum_i \frac{p_i^0}{p_i^t}}. \tag{10.6}$$

The ratio of harmonic mean prices is defined as

$$P_{RH}^{0:t} = \frac{\sum_i n/p_i^0}{\sum_i n/p_i^t}. \tag{10.7}$$

Neither formula appears to be used much in practice, perhaps because the harmonic mean is not a familiar concept and would not be easy to explain to users. However, at an aggregate level, the widely used Paasche index is a weighted harmonic average.

10.85 The ranking of the three common types of mean is always

arithmetic mean $\geq$ *geometric mean* $\geq$ *harmonic mean.*

It is shown in Chapter 21 that, in practice, the Carli index, the arithmetic mean of the price ratios, is likely to exceed the Jevons index, the geometric mean, by roughly the same amount that the Jevons exceeds the harmonic mean, as shown in equation (10.6). The harmonic mean of the price relatives has the same kinds of axiomatic properties as the Carli but with opposite tendencies and biases. It fails the transitivity and time reversal tests discussed earlier. In addition it is very sensitive to "price bouncing," as is the Carli index. As it can be viewed conceptually as the complement, or rough mirror image, of the Carli index, it has been argued that a suitable elementary index would be provided by a geometric mean of the two, in the same way that, at an aggregate level, a geometric mean is taken of the Laspeyres and Paasche indices to obtain the Fisher index. Such an index has been proposed by Carruthers, Sellwood, and Ward (1980) and Dalén (1992)—namely,

$$I_{CSWD}^{0:t} = \sqrt{I_C^{0:t} \cdot I_{HR}^{0:t}}. \tag{10.8}$$

I_{CSWD} is shown in Chapter 20 to have very good axiomatic properties but not quite as good as Jevons index, which is transitive, whereas the I_{CSWD} is not. However, it can be shown to be approximately transitive and, empirically, it has been observed to be very close to the Jevons index.

B.8 Unit value indices

10.86 As noted in Chapter 2, unit value indices based on customs data have a history as the predominant method of compiling XMPIs. However, it was also noted in Chapter 2 that unit value indices are subject to a particular form of formula bias and that price indices based on price surveys are now considered, for the most part, to be preferable. The potential bias in unit value indices is addressed in Chapter 2. The unit value index for period *t* relative to a

$$P_{UV}^{0:t} = \left(\frac{\sum_i p_i^t q_i^t}{\sum_i q_i^t} \right) \Bigg/ \left(\frac{\sum_i p_i^0 q_i^0}{\sum_i q_i^0} \right). \qquad (10.9)$$

10.87 The unit value index is simple in form. The unit value in each period is calculated by dividing total trade value on some commodity by the related total quantity. It is clear that the quantities must be strictly additive in an economic sense, which implies that they should relate to a single homogeneous commodity. The unit value index is then defined as the ratio of unit values in the current period to those in the reference period. An example of a unit value calculation can be found in Table 10.7. The trade quantity data is assumed to be available at the level of documentation required for the international shipment. The example in Table 10.7 shows the commodity category for the Harmonized Code 6402991815: Tennis shoes, basketball shoes, gym shoes, training shoes, and the like.

10.88 It is important to recognize that the unit value index is not a price index as normally understood, because it essentially is a measure of the change in the average price of a *single* commodity when that commodity is sold at different prices to different purchasers, perhaps at different times within the same period. In Table 10.7, both the trade value and quantity increased from January to February and the resulting unit value index shows an increase. However, because the commodity category is so broad, it cannot be determined whether in fact there was a price increase or a change in the type and/or quality of footwear traded. It is concluded that unit value indices should not be calculated for sets of heterogeneous commodities. Because international trade in homogeneous commodities like raw materials makes up less and less of total world trade, the use of unit value indices is not recommended as a substitute for XMPIs.

10.89 However, in cases with strict homogeneous commodities where the commodity specifications remain constant, although unlikely with customs data, a unit value may be used to estimate a price index. Often the unit value indices will be available from foreign trade statistics, or can be calculated on the basis of data from foreign trade statistics. The unit values should be based on data on both the value of the total trade and the total quantities sold covering the whole period, for example, one month, in order to derive a unit value index. This is particularly important if the commodity is sold at a discount price for part of the period and at the "regular" price for the rest of the period. Under these conditions, neither the discount price nor the regular price is likely to be representative of the average price at which the commodity has been sold or the price change between periods. The unit value over the whole month should be used. With the possibility of collecting more and more data from electronic records, such procedures may be increasingly used. However, it should be stressed that the commodity specifications must remain constant through time. Changes in the commodity specifications could lead to unit value changes that reflect quantity, or quality, changes and should not be part of price changes.

10.90 It is possible to combine unit values or unit value indices with prices collected in the XMPI survey in the form of a hybrid index as discussed in Chapter 2. For example, one elementary aggregate may consist of three commodities where a unit value is used for the first commodity, while sampled prices are used for the other two. It may also be the case that a unit value

Table 10.7. Calculation of Unit Value Index for Sample Commodity Category[1]
(6402991815—Tennis shoes, basketball shoes, gym shoes, training shoes, and the like)

Commodity Category	January		February	
	Trade value	Quantity	Trade value	Quantity
6402991815	1000.00	20	1500.00	25
Unit value price	50.00		60.00	
Unit value index	100		120	

[1]All unit value indices have been calculated using unrounded figures.

index constitutes an elementary index on its own. It can then be aggregated into higher-level indices with other elementary indices, whether these are based on sampled prices or unit value indices.

B.9 Formulas applicable to electronic data

10.91 Respondents may well have computerized management accounting systems that include highly detailed data on sales in terms of both prices and quantities. Their primary advantages are that the number of price observations can be significantly larger and that both price and quantity information are available in real time. Much work has been undertaken on the use of scanner data as an emerging data source for consumer price index (CPI) compilation and there are parallels for XMPIs, whenever establishments have detailed electronic files on individual transactions. There are a large number of practical considerations, which are discussed and referenced in the *CPI Manual* (ILO and others, 2004a) and also in Chapter 7, Section D, of this *Manual,* but it is relevant to discuss briefly here a possible index number formula that may be applicable if electronic data are collected and used in XMPI compilation.

10.92 The existence of quantity and trade value information at the detailed transaction level increases the ability to estimate price changes accurately. It means that traditional index number approaches such as Laspeyres and Paasche can be used, and that superlative formulas such as the Fisher and Törnqvist-Theil indices can also be derived in real time. The main observation made here is that because price and quantity information are available for each period, it may be tempting to produce monthly or quarterly chained indices using one of the ideal formulas mentioned above. However, the compilation of subannual chained indices has been found in some studies to be problematic because it often results in an upward bias referred to as "chain drift."

C. Calculation of Higher-Level Indices

C.1 Target indices

10.93 A statistical office must have some target index at which to aim. Statistical offices have to consider what kind of index they would choose to calculate in the ideal hypothetical situation in which they have complete information about prices and quantities in both time periods compared. If the XMPI is meant to be an economic index, then a superlative index such as a Fisher, Walsh, or Törnqvist-Theil would have to serve as the theoretical target, because a superlative index may be expected to approximate the underlying economic index.

10.94 Many countries do not aim to calculate an economic index and prefer the concept of a *basket index*. A basket index measures the change in the total value of a given basket of goods and services between two time periods. This general category of index is described here as a *Lowe* index after the early 19th-century index number pioneer who first proposed this kind of index (see Chapter 16, Section D). The meaning of a Lowe index is clear and can be easily explained to users. It should be noted that, in general, there is no necessity for the basket to be the actual basket in one or other of the two periods compared. If the theoretical target index is to be a basket or Lowe index, the preferred basket might be one that attaches equal importance to the baskets in both periods—for example, the Walsh index.[4] Thus, the same kind of index may emerge as the theoretical target on both the basket and the economic index approaches. In practice, however, a statistical office may prefer to designate the basket index that uses the actual basket in the earlier of the two periods as its target index on grounds of simplicity and practicality. In other words, the Laspeyres index may be a target index.

10.95 The theoretical target index is a matter of choice. In practice, it is likely to be either a Laspeyres or some superlative index. However, even when the target index is the Laspeyres, there may a considerable gap between what is actually calculated and what the statistical office considers to be its target. It is now necessary to consider what statistical offices tend to do in practice.

C.1.1 Aggregation by product or activity classification

10.96 The elementary indices are aggregated into higher-level indices by use of a product or activity classification. When aggregating according to a product classification, such as the HS, each elementary aggregate is assigned a detailed product code. This enables the statistical office to aggregate the elementary indices into indices at successively higher levels of aggregation for product classes, groups, divisions, and so on. In the same way, the elementary indices can be aggregated

[4]The quantities that make up the basket in the Walsh index are the geometric means of the quantities in the two periods.

into higher-level indices by type of activity, using, for example, the ISIC or the General Industrial Classification of Economic Activities within the European Communities.

10.97 It is up to the statistical office to decide which aggregation structure to follow. However, it is recommended that a standard international classification be applied. When national classifications or national variants of the international standards are used, they should allow for international comparisons, at least down to a fairly detailed level of aggregation. The aggregation structure should be consistent so that the weights at each level of aggregation are equal to the sum of their components.

10.98 In some instances the statistical office may wish to compile higher-level indices according to both a product and activity classification. Users may have interest in both kinds of aggregates or, for example, the national accounts may need deflators according to product or activity aggregates depending on the structure in the production of national accounts statistics.

10.99 One way to deal with this would be to assign both a product and activity code to each elementary aggregate, after which aggregation would provide higher-level indices according to both classifications. This may not always be possible in practice, however. The problem is that in general it is not possible to uniquely identify the originating industry of a product because the detailed product code may identify products originating from establishments in different industries. Thus, in order to aggregate elementary aggregates defined by a product classification (HS, for example) into higher-level indices according to some activity classification (ISIC, say), it will be necessary to have a key between the elementary aggregate product codes and the higher-level activity codes.

C.2 XMPIs as weighted averages of elementary indices

10.100 Section B discussed alternative formulas for combining individual price observations to calculate the first level of indices, the elementary aggregate indices. The next step in compiling the XMPI involves taking the elementary indices and combining them, using weights, to calculate successively higher levels of indices.

10.101 A higher-level index is an index for some trade aggregate above the level of elementary aggregates, including the overall XMPIs themselves. The second stage of compiling the XMPI does not involve individual prices or quantities. Instead, the higher-level indices are calculated by averaging the elementary indices using a set of predetermined weights. The inputs into the calculation of the higher-level indices are the elementary price indices and the weights of the elementary aggregates derived primarily from trade values data.

10.102 The weights typically remain fixed for a sequence of at least 12 months. Some countries revise their weights at the beginning of each year to approximate as closely as possible to current trade patterns. However, many countries continue to use the same weights for several years. The weights may be changed only every five years or so. Owing to the volatility of international trade, it is expected that the weights for XMPIs would be updated more often than those for the CPI or producer price index. The use of fixed weights has the considerable practical advantage that the index can make repeated use of the same weights, which saves both time and money. Revising the weights can be both time consuming and costly, but is necessary to ensure the weights remain relevant. Resources permitting, updating the weights annually is preferable.

10.103 The higher-level indices are calculated as the trade share–weighted arithmetic average of the elementary price indices. The formula can be written as follows:

$$P^{0:t} = \sum_j w_j^b P_j^{0:t}\,,\ \sum_j w_j^b = 1\,, \qquad (10.10)$$

where $P^{0:t}$ denotes the overall XMPI, or any higher-level index, from period 0 to t; w_j^b is the weight attached to each of the elementary price indices; and $P_j^{0:t}$ is the corresponding elementary price index identified by the subscript j. As already noted, a higher-level index is any index, including the overall XMPIs, above the elementary aggregate level. The weights are derived from trade values in period b, which in practice precedes period 0, the price reference period.

10.104 Equation (10.10) applies at each level of aggregation above the elementary aggregate level. The index is additive. This means that any higher-level index can be calculated in one step as the weighted arithmetic average of the elementary indices of which it consists, or by weighting together the indices at the intermediate level, with the same result. For example, a higher-level index at the two-digit level may be calculated by weighting together the elementary indices or the three-digit-level indices of which it consists.

10.105 Provided the elementary aggregate indices are calculated using a transitive formula such as the Jevons or Dutot, but not the Carli, and provided that there are no new or disappearing commodities from period 0 to t, equation (10.10) is equivalent to

$$P^{0:t} = \sum w_j^b P_j^{0:t-1} P_j^{t-1:t}, \sum w_j^b = 1. \qquad (10.11)$$

The advantage of this version of the index is that it allows the sampled commodities within the elementary price index from $t - 1$ to t to differ from the sampled commodities in the periods from 0 to $t - 1$. Hence, it allows replacement commodities and new commodities to be linked into the index from period $t - 1$ without the need to estimate a price for period 0, as explained in Section B.5. For example, if one of the sampled commodities in periods 0 and $t - 1$ is no longer available in period t, and the price of a replacement commodity is available for $t - 1$ at t, the new replacement commodity can be included in the index using the overlap method.

10.106 Note also from equation (10.11) that an elementary or lower-level index from $t - 1$ to t enters into the higher-level index not by its weight, but by the weight multiplied by the price development up to period $t - 1$. In order to calculate the rate of change of the higher-level index from $t - 1$ to t it is necessary to update the weights to reflect the price changes that have taken place from period 0 to period $t - 1$.

10.107 It is useful to recall that three kinds of reference periods may be distinguished:

- *Weight Reference Period*: The period covered by the trade value statistics used to calculate the weights. Usually, the weight reference period is a year.
- *Price Reference Period*: The period whose prices are used as denominators in the index calculation.
- *Index Reference Period*: The period for which the index is set to 100.

10.108 The three periods are generally different. For example, an XMPI might have 1998 as the weight reference *year,* December 2002 as the price reference *month,* and the *year* 2000 as the index reference period. The weights typically refer to a whole year, or even two or three years, whereas the periods whose prices are compared are typically months or quarters. The weights are usually compiled from trade value statistics already collected some time before the price reference period. For these reasons, the weight and the price reference periods are invariably separate periods in practice.

10.109 The index reference period is often a year, but it could be a month or some other period. An index series may also be re-referenced to another period by simply dividing the series by the value of the index in that period, without changing the rate of change of the index. The expression "base period" can mean any of the three reference periods and can sometimes be quite ambiguous. "Base period" should be used only when it is absolutely clear in context exactly which period is referred to.

10.110 Table 10.8 illustrates the calculation of higher-level indices. The index consists of five elementary aggregate indices (A–E), which are calculated using one of the formulas presented in Section 10.B, and two intermediate higher-level indices, G and H. The overall index (Total) and the higher-level indices (G and H) are all calculated using equation (10.10). For example, the overall index for April can be calculated from the two intermediate higher-level indices of April as

$$P^{Jan:Apr} = 0.6 \times 103.92 + 0.4 \times 101.79 = 103.06,$$

or directly from the five elementary indices

$$\begin{aligned} P^{Jan:Apr} &= 0.2 \times 108.75 + 0.25 \times 100 \\ &\quad + 0.15 \times 104 + 0.1 \times 107.14 + 0.3 \times 100 \\ &= 103.06. \end{aligned}$$

C.3 Price updating of trade value weights

10.111 As already noted most, if not all, statistical offices calculate the higher-level indices by use of equation (10.10) or the equivalent equation (10.11). However, for the practical calculation of XMPIs the situation is complicated by the fact that the weight reference period usually precedes the price reference period and the duration of the weight reference period is typically much longer than the period to which the prices refer. The weights usually refer to the trade values over a year, or longer, whereas the price reference period is usually a month or a quarter in some later year. For example, a monthly index may be compiled from January 2003 onward with December 2002 as the price reference month, but the latest available weights during the year 2003 may refer to the year 2000, or even some earlier year.

10.112 This means that the statistical office has to decide if the weights should be re-referenced, or *price-updated,* from the weight reference period to the price reference period, or be applied as they stand without any price updating.

Table 10.8. The Aggregation of the Elementary Price Indices

Index	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
A	0.20	100.00	102.50	104.88	101.16	101.15	100.00
B	0.25	100.00	100.00	91.67	109.09	101.67	108.20
C	0.15	100.00	104.00	96.15	104.00	101.92	103.77
D	0.10	100.00	92.86	107.69	107.14	100.00	102.67
E	0.30	100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
B	0.25	100.00	100.00	91.67	100.00	101.67	110.00
C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00
Higher-level indices							
G = *A* + *B* + *C*	0.60	100.00	101.83	99.03	103.92	105.53	110.00
H = *D* + *E*	0.40	100.00	99.46	101.25	101.79	104.29	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00

10.113 By price updating, the weights are aligned to the same reference period as the prices. If the statistical office decides to price-update the weights, the resulting index will be a *Lowe index*. The Lowe index is a *fixed-basket index,* which from period to period measures the value of the same (annual) basket of goods and service. It is defined as follows:

$$P_{Lo}^{0:t} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^0 q_i^b}. \tag{10.12}$$

The individual quantities (q_i^b) in the weight reference period b make up the basket. The index measures the value of the period b basket in period t in relation to the value of the same basket in period 0. However, to be used in practice it is necessary to express the index as a function of value shares rather than individual quantities:

$$P_{Lo}^{0:t} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^0 q_i^b} = \sum_i \frac{p_i^0 q_i^b \dfrac{p_i^t}{p_i^0}}{\sum_i p_i^0 q_i^b} = \sum_i w_i^{b(0)} \frac{p_i^t}{p_i^0}, \tag{10.13}$$

$$w_i^{b(0)} = \frac{w_i^b \left(p_i^0 / p_i^b\right)}{\sum_i w_i^b \left(p_i^0 / p_i^b\right)}, \quad w_i^b = \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}.$$

The individual price ratios are weighted together with their *hybrid* value shares, $w_i^{b(0)}$, that is, the period b quantities valued at period 0 prices. The hybrid shares are calculated by price updating the period b value shares from period b to 0. By price updating, the quantities are implicitly kept constant and the value shares are allowed to change according to the development in the relative prices. The practical counterpart of (10.13) is

$$P_{Lo}^{0:t} = \sum_i w_i^{b(0)} P_i^{0:t}, \quad w_i^{b(0)} = \frac{w_i^b P_i^{b:0}}{\sum_i w_i^b P_i^{b:0}}. \tag{10.14}$$

Equation (10.14) shows that in practice the higher-level indices are calculated by weighting together the elementary aggregate indices by their price-updated weights. The price-updated weights are calculated by multiplying the original period b value shares by their elementary indices from period b to period 0 and rescaling to sum to unity.

10.114 The Lowe index is not a Laspeyres index, as the weight and price reference periods do not coincide. However, it reduces to the Laspeyres index when $b = 0$ and to the Paasche index when $b = t$. Further, the Lowe index can be expressed as the ratio of two Laspeyres indices, one from b to 0 and one from b to t:

$$P_{Lo}^{0:t} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^0 q_i^b} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^b q_i^b} \Bigg/ \frac{\sum_i p_i^0 q_i^b}{\sum_i p_i^b q_i^b}. \qquad (10.15)$$

From equation (10.15) it follows that the Lowe index from period 0 to t will show the same rate of change as a Laspeyres price index with period b as weight and price reference period. In other words, price updating the weights from b to 0 means that the index will show the same rate of changes as if the weights had been applied from period b.

10.115 Because it uses the fixed basket of an earlier period, the Lowe index is sometimes loosely described as a "Laspeyres-type" index, but this description is unwarranted. A true Laspeyres index would require the basket to be that purchased in the price reference month, whereas in most XMPIs the basket refers to a period different from the price reference month. When the weights are annual and the prices are monthly, it is not possible, even retrospectively, to calculate a monthly Laspeyres price index.

10.116 The statistical office may decide instead to calculate the higher-level indices without price updating the weights. This corresponds to the calculation of a share-weighted arithmetic mean of the price ratios:

$$P_{Yo}^{0:t} = \sum_i w_i^b \left(\frac{p_i^t}{p_i^0}\right), \quad w_i^b = \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}. \qquad (10.16)$$

10.117 This general type of index is described here as a *Young index*. The index is general in the sense that the shares are not restricted to refer to any particular period, but may refer to any period or an average of different periods, for example. The index is named after another 19th-century index number pioneer who advocated this type of index (see Chapter 16, Section D). In practice it is calculated simply by weighting together the elementary indices from 0 to t by their value shares as they stand without price updating:

$$P_{Yo}^{0:t} = \sum_i w_i^b P_i^{0:t} . \qquad (10.17)$$

10.118 The Young index is a *fixed-weight index* in which the focus is that the weights should be as representative as possible for the average value shares for the period in which the weights are used in the calculation of the index. A fixed-weight index is not necessarily a fixed-basket index, that is, it does not necessarily measure the change in the value of a fixed basket such as the Lowe index. The Young index measures the development in the trade value from period 0 to t with the period b trade value shares. This does not correspond to the changing value of any actual basket, unless the trade shares have remained unchanged from b to 0. In the special case where b equals *0*, the Young index reduces to the Laspeyres index.

10.119 Note that even if it is decided to price-update the weights and calculate a fixed-basket or Lowe index, the index is calculated in the form of a Young index, namely as a share-weighted arithmetic average of the elementary indices. Thus, a Lowe index is equal to a Young index in which the weights are *hybrid* value shares obtained by revaluing the period b quantities at the prices of the price reference month.

10.120 The issues involved are explained with the help of a numerical example in Table 10.9, The base period b is assumed to be the year 2000 so that the weights are the trade value shares in 2000. In the upper half of the table, 2000 is also used as the price reference period. However, in practice, weights based on 2000 cannot be introduced until after 2000 because of the time needed to collect and process the trade value data. In the lower half of the table, it is assumed that the 2000 weights are introduced in December 2002, and that this is also chosen as the new price reference base.

10.121 If the statistical office decides to use the quantities, the trade weights of 2000 have to be adjusted for the relative price changes from 2000 to December 2002 in order to preserve the quantities. This means, in practice, that the value weights of 2000 have to be multiplied by the development in their elementary aggregate indices from 2000 to December 2002, and rescaled to sum to unity. This is illustrated in the lower half of Table 10.9, where the price-updated weights are labeled $w_{00(Dec02)}$.

10.122 The resulting index with price-updated weights in the lower part of Table 10.9 is a basket, or Lowe index, in which the quantities are those of 2000. The index can be expressed as ratios of the indices in the upper part of the table. For example, the overall Lowe index for March 2003 with December 2002 as its price reference base, namely 101.97, is the ratio of the index for March 2003 based on 2000 shown in the upper part of the table, namely 106.05, divided by the index for December 2002 based on 2000, namely 104.00. Thus, the price updating preserves the movements of the indices in the upper part of the table while shifting the price reference period to December 2002.

Table 10.9. Price Updating of Weights Between Weight and Price Reference Periods

Index	Weight	2000	Nov. 02	Dec. 02	Jan. 03	Feb. 03	Mar. 03
Index with 2000 as weight and price reference period							
Elementary price indices							
	W_{00}						
A	0.20	100.00	98.00	99.00	102.00	101.00	104.00
B	0.25	100.00	106.00	108.00	107.00	109.00	110.00
C	0.15	100.00	104.00	106.00	98.00	100.00	97.00
D	0.10	100.00	101.00	104.00	108.00	112.00	114.00
E	0.30	100.00	102.00	103.00	106.00	105.00	106.00
Higher-level indices							
$G = A + B + C$	0.60	100.00	102.83	104.50	103.08	104.08	104.75
$H = D + E$	0.40	100.00	101.75	103.25	106.50	106.75	108.00
Total		100.00	102.40	104.00	104.45	105.15	106.05
Index re-referenced to December 2002 and weights price-updated to December 2002							
Elementary price indices							
	$W_{00(Dec02)}$						
A	0.190	101.01	98.99	100.00	103.03	102.02	105.05
B	0.260	92.59	98.15	100.00	99.07	100.93	101.85
C	0.153	94.34	98.11	100.00	92.45	94.34	91.51
D	0.100	96.15	97.12	100.00	103.85	107.69	109.62
E	0.297	97.09	99.03	100.00	102.91	101.94	102.91
Higher-level indices							
$G = A + B + C$	0.603	95.69	98.41	100.00	98.64	99.60	100.24
$H = D + E$	0.397	96.85	98.55	100.00	103.15	103.39	104.60
Total		96.15	98.46	100.00	100.43	101.11	101.97
Rescaled to 2000 = 100		100.00	102.40	104.00	104.45	105.15	106.05

10.123 On the other hand, it could be decided to calculate a series of Young indices using the trade value weights from 2000 as they stand without price updating. If the trade value shares were actually to remain constant, the quantities would have had to move inversely with the prices between 2000 and December 2002. As the quantities are kept constant in the price-updated Lowe index, the movements of the two indices usually will be different. In the special case where the relative prices remain unchanged from the weight to the price reference period, the price-updated weights will be unchanged and the Young and the Lowe indices will give the same result.

10.124 It is up to the statistical offices to decide for themselves whether to price-update the trade value shares or not. If the primary aim is to compile an XMPI measuring the price development of an actual past fixed basket of goods and services, the weights should be price-updated. The resulting fixed-basket, or Lowe, index will provide a good estimate of the price development if quantities tend to remain constant.

10.125 If the statistical office considers that the value shares of the weight reference period are the better estimates of the relative importance of commodities imported/exported, this may be an argument for applying the value shares as they stand and omitting price updating. The case for using value shares is stronger if they remain relatively constant over time or the weights are frequently updated.

10.126 The likely bias of the Young and Lowe indices depends on the target of the XMPI and the development in relative prices and trade shares, that is, whether the target of the statistical office is a Laspeyres index or an economic, superlative index, such as the Fisher, Törnqvist, or Walsh index. The factors determining the differences between the Lowe index and the Laspeyres index, and the Young index and the Laspeyres index, are outlined in Chapter 16, as are those determining the difference between Laspeyres and Paasche, and therefore Laspeyres and Fisher. Because both quantities and trade shares changes through time and by progressively larger amounts, the more the indices are likely to drift apart. Thus, whether the weights are price-updated or not, they should be reviewed and updated frequently to reduce potential bias.

10.127 Price updating the weights does not imply that the resulting trade value weights are necessarily more up to date. When there is a strong inverse relation between movements of price and quantities, price updating on its own could produce perverse results. For example, the price of computers has been declining rapidly in recent years. If the quantities are held fixed while the prices are updated, the resulting trade value on computers would also decline rapidly. In practice, however, the share of trade value on computers might actually be rising because of a very rapid increase in quantities of computers purchased.

10.128 When rapid changes take place in relative quantities as well as relative prices, statistical offices are effectively obliged to change their trade value weights more frequently. Price updating on its own cannot cope with this situation. The trade value weights have to be updated with respect to their quantities as well as their prices, which, in effect, implies collecting new trade value data.

C.4 Factoring the Young index

10.129 It is possible to calculate the change in a higher-level Young index between two consecutive periods, such as $t-1$ and t, as a weighted average of the individual price indices between $t-1$ and t, provided that the weights are updated to take into account the price changes between the price reference period 0 and the previous period, $t-1$. This makes it possible to factor equation (10.10) into the product of two component indices in the following way:

$$P^{0:t} = \sum_j w_j^b P_j^{0:t} \tag{10.18}$$

$$= P^{0:t-1} \sum_j \frac{w_j^b P_j^{0:t-1} P_j^{t-1:t}}{\sum_j w_j^b P_j^{0:t-1}}$$

$$= P^{0:t-1} \sum_j w_j^{b(t-1)} P_j^{t-1:t},$$

$$\text{where } w_j^{b(t-1)} = \frac{w_j^b P_j^{0:t-1}}{\sum w_j^b P_j^{0:t-1}}$$

where $P^{0:t-1}$ is the Young index for period $t-1$. The weight $w_j^{b(t-1)}$ is the original weight for elementary aggregate j, price-updated by multiplying it by the elementary price index for j between 0 and $t-1$, and rescaled to sum to unity. The price-updated weights are hybrid weights because they implicitly revalue the quantities of period b at the prices of $t-1$ instead of at the average prices of b. Such hybrid weights do not measure the actual trade value shares of any period.

10.130 The Young index for period t can thus be calculated by multiplying the already calculated Young index for $t-1$ by a separate Young index between $t-1$ and t with hybrid price-updated weights. In effect, the higher-level index is calculated as a chained index. This method gives more flexibility to introduce replacement commodities and makes it easier to monitor the movements of the recorded prices for errors, because month-to-month movements are smaller and less variable than the total changes because the price reference period.

C.5 Change of index reference period

10.131 It is possible to re-reference index series to another index reference period by dividing the series with the value of the index in the new reference period. Re-referencing changes only the period in which the index is equal to 100; it does not influence the rate of change of the index. Change of the index reference period may be useful for presentational purposes or, for example, if the index is supposed to be compared with other statistics that refer to a different period in time.

10.132 The issue of re-referencing can be illustrated by an example. Assume that an index ($P^{03:t}$) has been calculated with 2003 as reference year, but for some reason the statistical office wishes to re-reference the index to 2005. The re-referenced index with 2005 equal to 100 ($P^{05:t}$) can then be calculated by dividing the original index by its value in 2005:

$$P^{05:t} = \frac{P^{03:t}}{P^{03:05}} = \frac{\sum_j w_j^{03} P_j^{03:t}}{\sum_j w_j^{03} P_j^{03:05}} \tag{10.19}$$

$$= \frac{\sum_j w_j^{03} P_j^{03:05} P_j^{05:t}}{\sum_j w_j^{03} P_j^{03:05}}$$

$$= \sum \frac{w_j^{03} P_j^{03:05}}{\sum_j w_j^{03} P_j^{03:05}} \cdot P_j^{05:t}$$

$$= \sum w_j^{03(05)} P_j^{05:t}.$$

The only difference between the two index series is a constant, namely the value of the original index in 2005. This means that the rates of change of the two index series are identical. Equation (10.19) also shows that a re-referenced index can be calculated by price updating the weights and re-referencing the elementary, or lower-level, indices to the same period in time. Thus, simultaneous price updating of the weights and re-referencing of the index series does not change the rate of change of the index, as also illustrated in the example in Table 10.9.

C.6 Introduction of new weights and chain linking

10.133 From time to time, the weights for the elementary aggregates have to be revised to ensure that they reflect current trade value shares and business activity. When new weights are introduced, the price reference period for the new index can be the last period of the old index, the old and the new indices being linked together at this point. The old and the new indices make a chained index.

10.134 The introduction of new weights is often a complex operation because it provides the opportunity to introduce new commodities, new samples, new data sources, new compilation practices, new elementary aggregates, new higher-level indices, or new classifications. These tasks are often undertaken simultaneously at the time of reweighting to minimize overall disruption to the time series and any resulting inconvenience to users of the indices.

10.135 In many countries reweighting and chaining is carried out about every five years but some countries introduce new weights each year. However, chained indices do not have to be linked annually, and the linking may be done less frequently. The real issue is not whether to chain but how frequently to chain. Reweighting is inevitable sooner or later, as the same weights cannot continue to be used forever. Whatever the time frame, statistical offices have to address the issue of chain linking sooner or later. It is an inevitable and major task for index compilers.

C.6.1 Frequency of reweighting

10.136 It is reasonable to continue to use the same set of elementary aggregate weights as long as trade patterns at the elementary aggregate level remain fairly stable. However, over time new commodities are continually being introduced on the market while others drop out. Over the longer term, trade patterns are also influenced by several other factors. These include export new markets and country sources of imported material and semi-finished, finished, and capital inputs; rising incomes and standards of living; demographic changes in the structure of the population; changes in technology; and changes in tastes and preferences.

10.137 There is wide consensus that regular updating of weights—at least every five years, and more often if there is evidence of rapid changes in international trade patterns—is a sensible and necessary practice. In the European Union, for example, the regulation on short-term statistics requires member countries to update the weights at least every five years. Frequent (e.g., annual) updating of the weights and chain-linking can be costly to implement and maintain. On the other hand, annual chaining has the advantage that changes such as the inclusion of new goods can be introduced on a regular basis, although every index needs some ongoing maintenance, whether annually chained or not. XMPIs are advantageously placed with regard to frequently updating weights because the primary source of weighting information is from administrative customs sources. Because these data are reliable and timely, the recommendation is to update as frequently as resources allow, ideally annually, unless the pattern of trade is, and is expected to continue to be, relatively constant over time.

10.138 Both exporters and importers of certain types of commodities are strongly influenced by short-term fluctuations in the economy. For example, trade of cars, major durables, expensive luxuries, and so on may change drastically from year to year. In such cases, it may be preferable to base the weight on an average of two or more years' trade value.

C.6.2 The calculation of a chained index

10.139 Assume that a series of fixed-weight Young indices have been calculated with period 0 as the price

Table 10.10. Calculation of a Chained Index

Index	Weight 1998	1998	Nov. 02	Dec. 02	Weight 2000	Dec. 02	Jan. 03	Feb. 03	Mar. 03
		1998 = 100				Dec. 2002 = 100			
Elementary price indices									
A	0.20	100.00	120.00	121.00	0.25	100.00	100.00	100.00	102.00
B	0.25	100.00	115.00	117.00	0.20	100.00	102.00	103.00	104.00
C	0.15	100.00	132.00	133.00	0.10	100.00	98.00	98.00	97.00
D	0.10	100.00	142.00	143.00	0.18	100.00	101.00	104.00	104.00
E	0.30	100.00	110.00	124.00	0.27	100.00	103.00	105.00	106.00
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Higher-level indices									
G = *A* + *B* + *C*	0.60	100.00	120.92	122.33	0.55	100.00	100.36	100.73	101.82
H = *D* + *E*	0.40	100.00	118.00	128.75	0.45	100.00	102.20	104.60	105.20
Total		100.00	119.75	124.90		100.00	101.19	102.47	103.34
Chaining of higher-level indices to 1998 = 100									
G = *A* + *B* + *C*	0.60	100.00	120.92	122.33	0.55	122.33	122.78	123.22	124.56
H = *D* + *E*	0.40	100.00	118.00	128.75	0.45	128.75	131.58	134.67	135.45
Total		100.00	119.75	124.90		124.90	126.39	127.99	129.07

reference period, and that in a subsequent period, *k*, a new set of weights has to be introduced in the index. The new weights are likely to be based on data surveyed in a period prior to *k*, the weight reference period, and may, or may not, have been price-updated from the new weight reference period to period *k*. For ease of exposition these weights are denoted as w_i^k. A chained index is then calculated as

$$P^{0:t} = P^{0:k}\sum_j w_j^k P_j^{k:t} \quad (10.20)$$
$$= P^{0:k}P^{k:t}.$$

10.140 There are several important features of a chained index:

- The chained index formula allows weights to be updated and facilitates the introduction of new commodities and subindices and removal of obsolete ones.
- To link the old and the new series, an overlapping period (*k*) is needed in which the index has to be calculated using both the old and the new set of weights.
- A chained index may have two or more links. Between each link period, the index may be calculated as a fixed-weight index using equation (10.10) or any other index formula. The link period may be a month or a year, provided the weights and indices refer to the same period.
- Chaining is intended to ensure that the individual indices on all levels show the correct development through time.
- Chaining leads to nonadditivity. When the new series is chained onto the old as in equation (10.20), the higher-level indices cannot be obtained as the weighted arithmetic averages of individual indices using the new weights.[5] Such results need to be carefully explained and presented.

10.141 An example of the calculation of a chain index is presented in Table 10.10. From 1998 to December 2002, the index is calculated with the year 1998 as weight and price reference period. From December 2002 onward, a new set of weights is introduced. The weights may refer to the year 2000, for example, and may or may not have been price-updated to December 2002. A new fixed-weight index series is then calculated with December 2002 as the price reference month. Finally, the new index series is linked onto the old index with 1998 = 100 by multiplication to get a continuous index from 1998 to March 2003.

[5] If, on the other hand, the index reference period is changed and the index series before the link period are rescaled to the new index reference period, these series cannot be aggregated to higher-level indices by use of the new weights.

Table 10.11. Calculation of a Chained Index Using Linking Coefficients

Index		1998	Nov. 02	Dec. 02		Jan. 03	Feb. 03	Mar. 03
			Elementary price indices (1998 = 100)					
	Weight 1998				**Linking Coefficient**			
A	0.20	100.00	120.00	121.00	1.2100	121.00	121.00	123.42
B	0.25	100.00	115.00	117.00	1.1700	119.34	120.51	121.68
C	0.15	100.00	132.00	133.00	1.3300	130.34	130.34	129.01
D	0.10	100.00	142.00	143.00	1.4300	144.43	148.72	148.72
E	0.30	100.00	110.00	124.00	1.2400	127.72	130.20	131.44
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07
			Higher-level indices (1998 = 100)					
G = *A* + *B* + *C*	0.60	100.00	120.92	122.33	1.2233	122.78	123.22	124.56
H = *D* + *E*	0.40	100.00	118.00	128.75	1.2875	131.58	134.67	135.45
Total		100.00	119.75	124.90	1.2490	126.39	127.99	129.07
			Elementary price indices (December 2002 = 100)					
Index	**Linking Coefficient**	**1998**	**Nov. 02**	**Dec. 02**	**Weight 2000**	**Jan. 03**	**Feb. 03**	**Mar. 03**
A	0.82645	82.65	99.17	100.00	0.25	100.00	100.00	102.00
B	0.85470	85.47	98.29	100.00	0.20	102.00	103.00	104.00
C	0.75188	75.12	99.25	100.00	0.10	98.00	98.00	97.00
D	0.69993	69.99	99.39	100.00	0.18	101.00	104.00	104.00
E	0.80645	80.65	88.71	100.00	0.27	103.00	105.00	106.00
Total	0.80064	80.06	95.88	100.00		101.19	102.47	103.34
			Higher-level indices (December 2002 = 100)					
G = *A* + *B* + *C*	0.81746	81.75	98.85	100.00	0.55	100.36	100.73	101.82
H = *D* + *E*	0.77670	77.67	91.65	100.00	0.45	102.20	104.60	105.20
Total	0.80064	80.06	95.88	100.00		101.19	102.47	103.34

10.142 The chained higher-level indices in Table 10.10 are calculated as

$$P^{00:t} = P^{98:Dec02}\sum w_j^{00(Dec02)} P_j^{Dec02:t}. \qquad (10.21)$$

Because of the lack of additivity, the overall chained index for March 2003 (129.07), for example, cannot be calculated as the weighted arithmetic mean of the chained higher-level indices G and H using the weights from December 2002.

C.6.3 Chaining indices using linking coefficients

10.143 Table 10.11 presents an example of chaining indices with new weights to the old reference period (1998 = 100). The linking can be done several ways. As described above, one can take the current index on the new weights and multiply it by the old index level in the overlap month (December 2002). Alternatively, a linking coefficient can be calculated between the old and new series during the overlap period and this coefficient applied to the new index series to bring the index up to the level of the old series. The linking coefficient for keeping the old index reference period is the ratio of the old index in the overlap period to the new index for the same period. For example, the coefficient for the Total index is (124.90/100.00) = 1.2490. This coefficient is then applied to the Total index each month to convert it from a December 2002 reference period to the 1998 reference period. Note that a linking coefficient is needed for each index series that is being chained.

10.144 Another option is to change the index reference period at the time the new weights are introduced. In the current example, the statistical office can shift to a December 2002 reference period and link the old index to the new reference period. This is done by calculating the linking coefficient for each index as the ratio of the new index in the overlap period to the old index. For example, the coefficient for the Total index is (100.00/124.90) = 0.80064. This coefficient is applied to the old Total index series to bring it down to the level of the new index. Table 10.11 presents the linking coefficients and the resulting re-referenced price indices using the two alternative index reference periods—1998 or December 2002.

C.6.4 Introduction of new elementary aggregates

10.145 First, consider the situation in which new weights are introduced and the index is chain linked in December 2002. The overall coverage of the XMPI is assumed to remain the same, but certain commodities have increased sufficiently in importance to merit recognition as new elementary aggregates. Possible examples are the introduction of new elementary aggregates for exports of mobile telephones or a new multinational company setting up a car factory for exports.

10.146 Consider the calculation of the new index from December 2002 onward, the new price reference period. The calculation of the new index presents no special problems and can be carried out using equation (10.20). However, if the weights are price-updated from, say, 2000 to December 2002, difficulties may arise because the elementary aggregate for mobile telephones did not exist before December 2002, so there is no price index with which to price-update the weight for mobile telephones. Prices for mobile telephones may have been recorded before December 2002, possibly within another elementary aggregate (communications equipment) so that it may be possible to construct a price series that can be used for price updating. Otherwise, price information from other sources such as business surveys, trade statistics, or industry sources may have to be used. If no information is available, then movements in the price indices for similar elementary aggregates may be used as proxies for price updating.

10.147 The inclusion of a new elementary aggregate means that the next and successive higher-level indices contain a different number of elementary aggregates before and after the linking. Therefore, the rate of change of the higher-level index whose composition has changed may be difficult to interpret. However, failing to introduce new goods or services for this reason would result in an index that does not reflect the actual dynamic changes taking place in the economy. If it is customary to revise the XMPI backward, then the prices of the new commodity and their weights might be introduced retrospectively. If the XMPI is not revised backward, however, little can be done to improve the quality of the chained index. In many cases, the addition of a single elementary aggregate is unlikely to have a significant effect on the next higher-level index into which it enters. If the addition of an elementary aggregate is believed to have a significant impact on the time series of the higher-level index, it may be necessary to discontinue the old series and commence a new higher-level index. These decisions can be made only on a case-by-case basis. In all cases information should be provided on the release to explain the changes taking place and provisions for such updates explained in the XMPIs' metadata.

C.6.5 Introduction of new, higher-level indices

10.148 It may be necessary to introduce a new, higher-level index in the overall XMPI. This situation may occur if the coverage of the XMPI is enlarged or the grouping of elementary aggregates is changed. It then needs to be decided what the initial value of the new higher-level index should be when it is included in the calculation of the overall XMPI. Take as an example the situation in Table 10.10 and assume that a new higher-level index from January 2003 has to be included in the index. The question is what should be the December 2002 value to which the new higher-level index is linked. There are two options.

- Estimate the value in December 2002 that the new higher-level index would have had with 1998 as the price reference period, and link the new series from January 2003 onward to this value. This procedure will prevent any break in the index series.
- Use 100 in December 2002 as the starting point for the new higher-level index. This simplifies the problem from a calculation perspective, although there remains the problem of explaining the index break to users.

In any case, major changes such as those just described should, so far as possible, be made in connection with the regular reweighting and chaining to minimize disruptions to the index series.

10.149 A final case to consider concerns classification change. For example, a country may decide to change from a national classification to an international one, such as ISIC. The changes in the composition of the aggregates within the XMPI may then be so large that it is not meaningful to link them. In such cases, it is recommended that the XMPI with the new classification should be calculated backward for at least one year so that consistent annual rates of change can be calculated.

C.6.6 Partial reweighting and introducing new goods

10.150 The weights for the elementary aggregates may be obtained from a variety of sources over a number of periods. Consequently, it may not be possible to introduce all the new weighting information at the same time. In some cases, it may be preferable to introduce new weights for some elementary aggregates as soon as possible after the information is received. This would be the case for introducing new goods (e.g., revolutionary goods, discussed in Chapter 9) into the index when these goods fall within the existing commodity structure of the index. The introduction of new weights for a subset of the overall index is known as partial reweighting.

10.151 As an example, assume there is a four-digit industry with three major commodities (*A*, *B*, and *C*) that were selected for the sample in 2000. From the trade value data for 2000, *A* had 50 percent of trade values, *B* had 35 percent, and *C* had 15 percent. From a review of trade values conducted for 2002, the statistical office discovers that *C* now has 60 percent of the trade value and *A* and *B* each have 20 percent. When the new weights are introduced into the index, the procedures discussed in Section C.7.2 for chaining the new index onto the old index can be used. For example, the new commodity weights for 2002 are used to calculate the index in an overlap month such as April 2003 with a base price reference period of December 2002. For May 2003, the index using the new commodity weights is again calculated and the change in the new index is then applied (linked) to the old industry-level index for April 2003 (with 2000 = 100) to derive the industry index for May 2003 (2000 = 100). The formula for this calculation is the following:

$$P^{00:May03} = P^{00:Apr03} \times \left[\sum_{j=1}^{n} w_j^{02} P_j^{Dec02:May03} \Big/ \sum_{j=1}^{n} w_j^{02} P_j^{Dec02:Apr03} \right]. \quad (10.22)$$

10.152 Continuing with this example, assume the review of trade values finds the new commodity (*D*) has a significant share of trade (perhaps 15 or 20 percent), and it is expected to continue growing in relative importance. The statistical office would use the same procedure for introducing the new commodity. In this case, the calculations for the new industry index in April and May would use all four commodities instead of the original three. The price change in the new sample is linked to the old index as in equation (10.21). The only difference will be that the summations are over m (four commodities) instead of n (three) commodities.

10.153 One could also make the same calculations using the linking coefficient approach discussed in Section C.7.3. The linking coefficient is derived by taking the ratio of the old industry index (2000 = 100) to the new industry index (December 2002 = 100) in the overlap period (April 2003):

$$\textit{Linking coefficient} = \sum_{j=1}^{m} w_j^{02} P_j^{Dec02:Apr03} \Big/ \sum_{j=1}^{n} w_j^{02} P_j^{Dec02:Apr03}. \quad (10.23)$$

The linking coefficient, computed for the overlap period only, is then applied each month to the new index to adjust it to the level of the old index with an index reference period of 2000.

10.154 Another issue is the weights to use for compiling the index for the commodity groups represented by *A*, *B*, *C*, and *D*. For example, if indices for commodities *A* and *B* are combined with indices for commodities *X* and *Y* to calculate a commodity group index, the new weights for *A* and *B* present a problem because they represent trade values in a more current period than do the weights for *X* and *Y*. Also, the indices have different price reference periods. If we had weights for commodities *X* and *Y* for the same period as for *A* and *B*, then we could use the same approach as just described for compiling the industry index. Lacking new commodity weights for *X* and *Y* means the statistical office will have to take additional steps. One approach to resolve this problem is to price-update the weights for commodities *X* and *Y* from 2000 to 2002 using the change in the respective price indices. Thus, the original weight for commodity *X* is multiplied by the change in prices between 2002 and 2000 (i.e., the ratio of the average price index of *X* in 2002 to the average price index of *X* in 2000). Then one can use the same base price reference period as for *A* and *B* so that the indices for commodities *X* and *Y* are each re-referenced to December 2002. The commodity group index can then be compiled for April 2003 using the new weights for all four commodities and their indices with December 2002 = 100. Once the April 2003 index is compiled on the December 2002 price reference period, then the linking coefficient using equation (10.23) can be calculated to adjust the new index level to that of the old index. Alternatively, the price change in the new commodity group index (December 2002 = 100) can be applied to the old index level each month as shown in equation (10.22).

10.155 As this example demonstrates, partial reweighting has particular implications for the practice of price updating the weights. Weighting information may not be available for some elementary aggregates at the time of reweighting. Thus, it may be necessary to consider price updating the old weights for those elementary aggregates for which no new weights are available. The weights for the latter may have to be price-updated over a long period, which, for reasons given earlier, may give rise to some index bias if relative quantities have changed inversely with the relative price changes. Data on both quantity and price changes for the old index weights should be sought before undertaking price updating alone. The disadvantage of partial reweighting is that the implicit quantities belong to different periods than do other components of the index, so that the composition of the basket is obscure and not well defined.

10.156 One may conclude that the introduction of new weights and the linking of a new series to the old series are not difficult in principle. The difficulties arise in practice when trying to align weight and price reference periods and when deciding whether higher-level indices comprising different elementary aggregates should be chained over time. It is not possible for this *Manual* to provide specific guidance on decisions such as these, but compilers should consider carefully the economic logic and statistical reliability of the resulting chained series and also the needs of users. To facilitate the decision process, one should give careful thought to these issues in advance during the planning of a reweighting exercise, paying particular attention to which indices are to be published.

C.6.7 Long- and short-term links

10.157 Consider a long-term chained index in which the weights are changed annually. In any given year, the current monthly indices when they are first calculated have to use the latest set of available weights, which cannot be those of the current year. However, when the weights for the year in question become available subsequently, the monthly indices can then be recalculated on the basis of the weights for the same year. The resulting series can then be used in the long-term chained index rather than the original indices first published. Thus, the movements of the long-term chained index from, say, any one December to the following December, are based on weights of that same year, the weights being changed each December.[6]

10.158 Assume that each link runs from December to December. The long-term index for month *m* of year *Y* with December of year 0 as index reference period is then calculated by the formula[7]

$$P^{Dec0:mY} = \left(\prod_{y=1}^{Y-1} P^{Decy-1:Decy}\right) P^{DecY-1:mY}$$
$$= P^{Dec0:Dec1} \times P^{Dec1:Dec2} \times \dots \times P^{DecY-2:DecY-1} \times P^{DecY-1:mY}. \quad (10.24)$$

The long-term movement of the index depends on the long-term links only as the short-term links are successively replaced by their long-term counterparts. For example, let the short-term indices for January to December 2001 be calculated as

$$P^{Dec00:m01} = \sum w_j^{00(Dec00)} P_j^{Dec00:m01}, \quad (10.25)$$

where $w_j^{00(Dec00)}$ are the weights from 2000 price-updated to December 2000. At the time when weights for 2001 are available, this is replaced by the long-term link

$$P^{Dec00:Dec01} = \sum w_j^{01(Dec00)} P_j^{Dec00:Dec01}, \quad (10.26)$$

where $w_j^{01(Dec00)}$ are the weights from the 2001 price backdated to December 2000. The same set of weights from 2001 price-updated to December 2001 is used in the new short-term link for 2002,

$$P^{Dec01:m02} = \sum w_j^{01(Dec01)} P_j^{Dec01:m02}. \quad (10.27)$$

10.159 With this method, the movement of the long-term index is determined by contemporaneous weights that refer to the same period. The method is conceptually attractive because the weights that are most relevant for most users are those based on trade patterns at the time the price changes actually take place. The method takes the process of chaining to its logical conclusion, at least

[6]This method has been developed by the Central Statistical Office of Sweden, where it is applied in the calculation of the CPI. See Statistics Sweden (2001).

[7]In the actual Swedish practice, a factor scaling the index from December year 0 to the average of year 0 is multiplied onto the right-hand side of equation (10.24) to have a full year as reference period.

assuming the indices are not chained more frequently than once a year. Because the method uses weights that are continually revised to ensure that they are representative of current trade patterns, the resulting index also largely avoids the substitution bias that occurs when the weights are based on the trade patterns of some period in the past. The method may therefore appeal to statistical offices whose objective is to estimate an economic index.

10.160 Finally, it may be noted that the method involves some revision of the index first published. In some countries, there is opposition to revising a XMPI once it has been first published, but it is standard practice for other economic statistics, including the national accounts, to be revised as more up-to-date information becomes available. This point is considered below.

C.7 Decomposition of index changes

10.161 Users of the index are often interested in how much of the change in the overall index is attributable to the change in the price of some particular commodity or group of commodities, such as petroleum or agricultural goods. Alternatively, there may be interest in what the index would be if agricultural goods or petroleum were left out. Questions of this kind can be answered by decomposing the change in the overall index into its constituent parts.

10.162 Assume that the index is calculated as in equation (10.10) or (10.11). The relative change of the index from $t - m$ to t can then be written as

$$\frac{P^{0:t}}{P^{0:t-m}} - 1 = \frac{\sum_j w_j^b P_j^{0:t-m} P_j^{t-m:t}}{\sum_j w_j^b P_j^{0:t-m}} - 1. \qquad (10.28)$$

Hence, a subindex from $t - m$ to t enters the higher-level index with a weight of

$$\frac{w_j^b P_j^{0:t-m}}{\sum_j w_j^b \cdot P_j^{0:t-m}} = \frac{w_j^b \cdot P_j^{0:t-m}}{P^{0:t-m}}. \qquad (10.29)$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$\begin{aligned} \mathit{Effect} &= \frac{w_j^b P_j^{0:t-m}}{P^{0:t-m}} \cdot \left(\frac{P_j^{0:t}}{P_j^{0:t-m}} - 1 \right) \\ &= \frac{w_j^b}{P_j^{0:t-m}} \left(P_j^{0:t} - P_j^{0:t-m} \right). \end{aligned} \qquad (10.30)$$

With $m = 1$, equation (10.30) gives the effect of a monthly change; with $m = 12$, it gives the effect of the change over the past 12 months.

10.163 If the index is calculated as a chained index, as in equation (10.20), then a subindex from $t - m$ to t enters the higher-level index with a weight of

$$\frac{w_j^k P_j^{k:t-m}}{P^{k:t-m}} = \frac{w_j^k \left(P_j^{0:t-m} / P_j^{0:k} \right)}{\left(P^{0:t-m} / P^{0:k} \right)}. \qquad (10.31)$$

The effect on the higher-level index of a change in a subindex can then be calculated as

$$\begin{aligned} \text{Effect} &= \frac{w_i^k}{P^{k:t-m}} \cdot \left(P_j^{k:t} - P_j^{k:t-m} \right) \\ &= \frac{w_j^k}{\left(P^{0:t-m} / P^{0:k} \right)} \left(\frac{P_j^{0:t} - P_j^{0:t-m}}{P_j^{0:k}} \right). \end{aligned} \qquad (10.32)$$

It is assumed that $t - m$ lies in the same link (i.e., $t - m$ refers to a period at or later than k). If the effect of a subindex on a higher-level index is to be calculated across a chain, the calculation needs to be carried out in two steps, one with the old series up to the link period and one from the link period to period t.

10.164 Table 10.12 illustrates an analysis using both the percentage index point effect and contribution of each component index to the overall 12-month change. The next-to-last column in Table 10.12 is calculated using equation (10.30) to derive the effect each component index contributes to the total percentage change. For example, for agriculture the index weight (w_j^b) is 38.73, which is divided by the previous period index ($P^{0:t-m}$) or 120.2, and then multiplied by the index point change ($P_j^{t:0} - P_j^{0:t-m}$) between January 2003 and January 2002, 10.5. The result shows that agriculture's effect on the 9.1 percent overall change was 3.4 percent. The change in agriculture contributed 37.3 percent (3.4/9.1 × 100) to the total 12-month change.

C.8 Some alternatives to fixed-weight indices

10.165 Monthly XMPIs are typically arithmetic weighted averages of the price indices for the elementary aggregates in which the weights are kept fixed over a number of periods, which may range from 12 months to many years. The repeated use of the same weights relating to some past period b simplifies calculation

Table 10.12. Decomposition of Index Change from January 2002 to January 2003

	Industry Sector	2000 Weights (w_i^b)	Price Index			Percent Change from Jan. 02 to Jan. 03	Effect (Contribution)	
			2000	Jan. 02	Jan. 03		Percentage points of total price change	Percent of total price change
1	Agriculture	38.73	100	118.8	129.3	8.8	3.4	37.3
2	Mining	6.40	100	132.8	145.2	9.3	0.7	7.3
3	Manufacturing	18.64	100	109.6	120.6	10.0	1.7	18.8
4	Transport and Communication	19.89	100	126.3	131.3	4.0	0.8	9.1
5	Chemicals	16.34	100	123.4	141.3	14.5	2.4	26.8
	Total	100.00	100	120.2	131.1	9.1	9.1	100.0

procedures and reduces data collection requirements and costs. Moreover, when the weights are known in advance of the price collection, the index can be calculated immediately after the prices have been collected and processed.

10.166 However, the longer the same weights are used, the less representative of current trade patterns they become, especially in periods of rapid technical change when new kinds of goods and services are continually appearing on the market and old ones are disappearing. This may undermine the credibility of an index that purports to measure the rate of change in the value of goods and services exported and imported. Similarly, if the objective is to compile an economic index, the continuing use of the same fixed basket is likely to become increasingly unsatisfactory the longer the same basket is used. The longer the same basket is used, the greater the bias in the index is likely to become.

10.167 There are several possible ways of minimizing, or avoiding, the potential biases from the use of fixed-weight indices. These are outlined below.

10.168 *Annual chaining.* One way to minimize the potential biases from the use of fixed-weight indices is to keep the weights and the base period as up to date as possible by frequent weight updates and chaining. A number of countries have adopted this strategy and revise their weights annually. In any case, as noted earlier, it would be impossible to deal with the changing universe of commodities without some chaining of the price series within the elementary aggregates, even if the weights attached to the elementary aggregates remain fixed. Annual chaining eliminates the need to choose a base period, because the weight reference period is always the previous year, or possibly the preceding year.

10.169 *Annual chaining with current weights.* When the weights are changed annually, it is possible to replace the original weights based on the previous year, or years, by those of the current year if the index is revised retrospectively as soon as information on current-year trade value becomes available. The long-term movements in the XMPI are then based on the revised series. This is the method adopted by the Swedish Statistical Office for its CPI as explained in Section C.7.7. This method could provide unbiased results.

10.170 *Other index formulas.* When the weights are revised less frequently, say every five years, another possibility would be to use a different index formula for the higher-level indices instead of an arithmetic average of the elementary price indices. One possibility would be a weighted geometric average. This is not subject to the same potential upward bias as the arithmetic average. More generally, a weighted version of the Lloyd-Moulton formula, given in Section B.6, might be considered. This formula takes account of the substitutions that purchasers make in response to changes in relative prices and should be less subject to bias for this reason. It reduces to the geometric average when the elasticity of substitution is unity, on average. It is unlikely that such a formula could replace the arithmetic average in the foreseeable future and gain general acceptance, if only because it cannot be interpreted as measuring changes in the value of a fixed basket. However, it could be compiled on an experimental basis and might well provide a useful supplement to the main index. It could at least flag the extent to which the main index is liable to be biased and throw light on its properties.

10.171 *Retrospective superlative indices*. Finally, it is possible to calculate a superlative index retrospectively. Superlative indices such as Fisher and Törnqvist-Theil treat both periods compared symmetrically and require trade value data for both periods. Although the XMPI may have to be based on weighting data of a past period when it is first published, it may be possible to estimate a superlative index later when much more information becomes available about producers' trade values period by period. At least one office, the U.S. Bureau of Labor Statistics, is publishing such an index for its CPI. The publication of revised or supplementary indices raises matters of statistical policy, but users readily accept revisions in other fields of economic statistics.

D. Data Editing

10.172 This chapter has been concerned with the methods used by statistical offices to calculate their XMPIs. This concluding section considers the data editing carried out by statistical offices, a process closely linked to the calculation of the price indices for the elementary aggregates. Data collection, recording, and coding—the data capture processes—are dealt with in Chapter 6. The next step in the production of price indices is the data editing. Data editing is here meant to comprise two steps:

- Detecting possible errors and outliers, and
- Verifying and correcting data.

10.173 Logically, the purpose of detecting errors and outliers is to exclude errors or the effects of outliers from the index calculation. Errors may be falsely reported prices, or they may be caused by recording or coding mistakes. Also, missing prices because of nonresponse may be dealt with as errors. Possible errors and outliers are usually identified as observations that fall outside some prespecified acceptance interval or are judged to be unrealistic by the analyst on some other ground. It may also be the case, however, that even if an observation is not identified as a potential error, it may actually show up to be false. Such observations are sometimes referred to as inliers. On the other hand, the sampling may have captured an exceptional price change that falls outside the acceptance interval but has been verified as correct. In some discussions of survey data, any extreme value is described as an outlier. The term is reserved here for extreme values that have been verified as being correct.

10.174 When a possible error has been identified, whether it is in fact an error or not needs to be verified. This can usually be accomplished by asking the respondent to verify the price, or by comparing it with the price change of similar commodities. If it is an error, it needs to be corrected. This can be done easily if the respondent can provide the correct price or, where this is not possible, by imputation or omitting the price from the index calculation. If the price proves to be correct, it should be included in the index. If it proves to be an outlier, it can be accepted or corrected according to a predefined practice—for example, omitting or imputation.

10.175 Data editing involves two steps: the detection of possible errors and outliers, and the verification and correction of the data. Effective monitoring and quality control are needed to ensure the reliability of the basic price data fed into the calculation of the elementary prices indices on which the quality of the overall index depends. However, extreme values arise for traded goods and services because price changes are undertaken infrequently. There is much theory and evidence on this. For example, cost-driven or exchange rate changes may not be immediately passed through to prices but stored up and delivered as a large price increase rather than a series of smaller ones. Harsh data editing may interpret such an increase as noise, rather than the signal of an actual price change. It is advised that automatic outlier detection routines be used in conjunction with a system that allows, at least for commodities with substantial trade, an external validation, say by phone contact with the establishment responsible. This is facilitated when the source of the price data is the establishment. *However, unit value indices from customs data are by their nature volatile, and automatic outlier routines may well distort the results unless there is a follow-up procedure.*

10.176 Although the power of computers provides obvious benefits, not all of these activities have to be computerized. However, there should be a complete set of procedures and records that controls the processing of data, even though some or all of it may be undertaken without the use of computers. It is not always necessary for all of one step to be completed before the next is started. If the process uses spreadsheets, for example, with default imputations predefined for any missing data, the index can be estimated and reestimated whenever a new observation is added or modified. The ability to examine the impact of individual price observations on elementary aggregate indices and the impact of elementary indices on various higher-level aggregates is useful in all aspects of the computation and analytical processes.

10.177 It is neither necessary nor desirable to apply the same degree of scrutiny to all reported prices. The price changes recorded by some respondents carry more weight than do others, and statistical analysts should be aware of this. For example, one elementary aggregate with a weight of 2 percent, say, may contain 10 prices, whereas another elementary aggregate of equal weight may contain 100 prices. Obviously, an error in a reported price will have a much smaller effect in the latter, where it may be negligible, whereas in the former it may cause a significant error in the elementary aggregate index and even influence higher-level indices.

10.178 However, there may be an interest in the individual elementary indices as well as in the aggregates built from them. Because the sample sizes used at the elementary level may often be small, any price collected, and error in it, may have a significant impact on the results for individual commodities or industries. The verification of reported data usually has to be done on an index-by-index basis, using statistical analysts' experience. Also, for support, analysts will need the cooperation of the respondents to the survey to help explain unusual price movements.

10.179 Obviously, the design of the survey and questionnaires influences the occurrence of errors. Hence, price reports and questionnaires should be as clear and unambiguous as possible to prevent misunderstandings and errors. Whatever the design of the survey, it is important to verify that the data collected are those that were requested initially. The survey questionnaire should prompt the respondent to indicate if the requested data could not be provided. If, for example, a commodity is not produced anymore and thus is not priced in the current month, a possible replacement would be requested along with details of the extent of its comparability with the old one. If the respondent cannot supply a replacement, there are a number of procedures for dealing with missing data (see Chapter 8).

D.1 Identifying possible errors and outliers

10.180 One of the ways price surveys are different from other economic surveys is that, although prices are recorded, the measurement concern is with price *changes*. Because the index calculations consist of comparing the prices of matching observations from one period to another, editing checks should focus on the price changes calculated from pairs of observations, rather than on the reported prices themselves.

10.181 Identification of unusual price changes can be accomplished by

- Nonstatistical checking of input data,
- Statistical checking of input data, and
- Output checking.

These will be described in turn.

D.1.1 Nonstatistical checking of input data

10.182 Nonstatistical checking can be undertaken by manually checking the input data, by inspecting the data presented in comparable tables, or by setting filters.

10.183 When the price reports or questionnaires are received in the statistical office, the reported prices can be checked manually by comparing these with the previously reported prices of the same commodities or by comparing them with prices of similar commodities from other establishments. Although this procedure may detect obvious unusual price changes, it is far from ensuring that all possible errors are detected. It is also extremely time consuming, and it does not identify coding errors.

10.184 After the price data have been coded, the statistical system can be programmed to present the data in a comparable form in tables. For example, a table showing the percentage change for all reported prices from the previous to the current month may be produced and used for detection of possible errors. Such tables may also include the percentage changes of previous periods for comparison and 12-month changes. Most computer programs and spreadsheets can easily sort the observations according to, say, the size of the latest monthly rate of change so that extreme values can easily be identified. It is also possible to group the observations by elementary aggregates.

10.185 The advantage of grouping observations is that it highlights potential errors so that the analyst does not have to look through all observations. A hierarchical strategy whereby all extreme price changes are first identified and then examined in context may save time, although the price changes in elementary aggregate indices, which have relatively high weights, should also be examined in context.

10.186 Filtering is a method by which possible errors or outliers are identified according to whether the price changes fall outside some predefined limits, such as ± 20 percent or even 50 percent. This test should capture any serious data coding errors, as well as some of the cases in which a respondent has erroneously reported on a different commodity. It is usually possible to identify these errors without reference to any other observations in the survey, so this check can be carried out at the data-capture stage. The advantage of filtering is that the analyst need not look through numerous individual observations.

10.187 These upper and lower limits may be set for the latest monthly change, or change over some other period. Note that the set limits should take account of the context of the price change. They may be specified differently at various levels in the hierarchy of the indices—for example, at the commodity level, at the elementary aggregate level, or at higher levels. Larger changes for commodities whose prices are known to be volatile might be accepted without question. For example, for monthly changes, limits of ±10 percent might be set for petroleum prices, while for professional services the limits might be 0 percent to +5 percent (because any price that falls is suspect), and for computers it might be –5 percent to zero, because any price that rises is suspect. One can also change the limits over time. If it is known that petroleum prices are rising, the limits could be 10 percent to 20 percent, while if they are falling, they might be –10 percent to –20 percent. The count of failures should be monitored regularly to examine the limits. If too many observations are being identified for review, the limits will need to be adjusted or the customization refined.

10.188 The use of automatic deletion systems is not advised, however. It is a well-recorded phenomenon in pricing that price changes for many commodities, especially durables, are not undertaken smoothly over time but saved up to avoid what are termed "menu costs" associated with making a price change. These relatively substantial increases may take place at different times for different models of commodities and have the appearance of extreme, incorrect values. To delete a price change for each model of the commodity as being "extreme" at the time it occurs is to ignore all price changes for the industry.

D.1.2 Statistical checking of input data

10.189 Statistical checking of input data compares, for some time period, each price change with the change in prices in the same or a similar sample. Two examples of such filtering are given here, the first based on nonparametric summary measures and the second on the log-normal distribution of price changes.

10.190 The first method involves tests based on the median and quartiles of price changes, so they are unaffected by the impact of any single extreme observation. First, the median, first quartile, and third quartile price relatives are defined as R_M, R_{Q1}, and R_{Q3}, respectively. Then, any observation with a price ratio more than a certain multiple C of the distance between the median and the quartile is identified as a potential error. The basic approach assumes price changes are normally distributed. Under this assumption, it is possible to estimate the proportion of price changes that are likely to fall outside given bounds expressed as multiples of C. Under a normal distribution, R_{Q1} and R_{Q3} are equidistant from R_M; thus, if C is measured as $R_M - (R_{Q1} + R_{Q3})/2$, 50 percent of observations would be expected to lie within $\pm C$ from the median. From the tables of the standardized normal distribution, this is equivalent to about 0.7 times the standard deviation (σ). If, for example, C were set to 6, the distance implied is about 4σ of the sample, so about 0.17 percent of observations would be identified this way. With $C = 4$, the corresponding figures are 2.7σ, or about 0.7 percent of observations. If $C = 3$, the distance is 2.02σ, so about 4 percent of observations would be identified.

10.191 In practice, most prices may not change each month, and the share of observations identified as possible errors as a percentage of all changes would be unduly high. Some experimentation with alternative values of C for different industries or sectors may be appropriate. If this test is to be used to identify possible errors for further investigation, a relatively low value of C should be used.

10.192 To use this approach in practice, three modifications should be made. First, to make the calculation of the distance from the center the same for extreme changes on the low side as well as on the high side, a transformation of the relatives should be made. The transformed distance for the ratio of one price observation i, S_i, should be

$$S_i = 1 - R_M/R_i \quad \text{if } 0 < R_i < R_M \text{ and}$$
$$= R_i/R_M - 1 \quad \text{if } R_i \geq R_M.$$

Second, if the price changes are grouped closely together, the distances between the median and quartiles may be very small, so that many observations would be identified that had quite small price changes. To avoid this, some minimum distance, say 5 percent for monthly changes, should also be set. Third, with small samples, the impact of one observation on the distances between the median and quartiles may be too great. Because sample sizes for some elementary indices are small, samples for similar elementary indices may need to be grouped together.[8]

10.193 An alternative method can be used if it is thought that the price changes may be distributed log-normally. In this method, the standard deviation of the log of all price changes in the sample (excluding unchanged observations) is calculated and a goodness of fit test (χ^2) is undertaken to identify whether the distribution is log-normal. If the distribution satisfies the test, all price changes outside two times the exponential of the standard deviation are highlighted for further checking. If the test rejects the log-normal hypothesis, all the price changes outside three times the exponential of the standard deviation are highlighted. The same caveats mentioned before about clustered changes and small samples apply.

10.194 The second example is based on the Tukey algorithm. The set of price relatives is sorted and the highest and lowest 5 percent flagged for further attention. In addition, now that the top and bottom 5 percent are excluded, exclude the price relatives that are equal to 1 (no change). The arithmetic (trimmed) mean (AM) of the remaining price relatives is calculated. This mean is used to separate the price relatives into two sets, an upper and a lower one. The upper and lower "mid-means"—that is, the means of each of these sets (AM_L, AM_U)—are then calculated. Upper and lower Tukey limits (T_L, T_U) are then established as the mean ±2.5 times the difference between the mean and the mid-means:

$$T_U = AM + 2.5\,(AM_U - AM),$$

$$T_L = AM - 2.5\,(AM - AM_L).$$

Then, all those observations that fall above T_U and below T_L are flagged for attention.

10.195 This method is similar to but simpler than that based on the normal distribution. Because it excludes all cases of no change from the calculation of the mean, it is unlikely to produce limits that are very close to the mean, so there is no need to set a minimum difference. However, its success will also depend on there being a large number of observations on the set of changes being analyzed. Again, it will often be necessary to group observations from similar elementary indices. For any of these algorithms, the comparisons can be made for any time periods, including the latest month's changes, but also longer periods, in particular, 12-month changes.

10.196 The advantage of these two models of filtering compared with the simple method of filtering is that for each period the upper and lower limits are determined by the data itself and hence are allowed to vary over the year, given that the analyst has decided the value of the parameters entering the models. A disadvantage is that, unless one is prepared to use approximations from earlier experience, all the data have to be collected before the filtering can be undertaken. Filters should be set tightly enough so that the percentage of potential errors that turn out to be real errors is high. As with all automatic methods, the flagging of an unusual observation is for further investigation, as opposed to automatic deletion.

D.1.3 Checking by impact, or data output checking

10.197 Filtering by impact, or output editing, is based on calculating the impact an individual price change has on an index to which it contributes. This index can be an elementary aggregate index, the total index, or some other aggregate index. The impact a price change has on an index is its percentage change times its effective weight. In the absence of sample changes, the calculation is straightforward: It is the nominal (reference period) weight, multiplied by the price relative, and divided by the level of the index to which it is contributing. So the impact on the index P of the change of the price of commodity i from time t to $t + 1$ is $\pm w_i\,(p_{t+1}/p_t)\,/\,P_t$, where w_i is the nominal weight in the price reference period. A minimum value for this impact can be set, so that all price changes that cause an impact greater than this change can be flagged for review. If index P is an elementary index, then all elementary indices may be reviewed, but if P is a higher-level index, prices that change by a given percentage will be flagged or not, depending on how important the elementary index to which they contribute is in the aggregate.

10.198 However, at the lowest level, births and deaths of commodities in the sample cause the effective weight of an individual price to change quite substantially. The effective weight is also affected if a price observation is used as an imputation for other missing observations. The evaluation of effective weights in each period is possible, though complicated. However, as an aid to highlighting potential errors, the nominal weights, as a

[8]For a detailed presentation of this method the reader is referred to Hidiroglou and Berthelot (1986). The method can be expanded to also take into account the level of the prices, so that, for example, a price increase from 100 to 110 is attributed a different weight than is a price increase from 10 to 11.

percentage of their sum, will usually provide a reasonable approximation. If the impact of 12-month changes is required to highlight potential errors, approximations are the only feasible filters to use, because the effective weights will vary over the period.

10.199 One advantage of identifying potential errors this way is that it focuses on the results. Another advantage is that this form of filtering also helps the analyst to describe the contributions to change in the price indices. In fact, much of this kind of analysis is done after the indices have been calculated, because the analyst often wishes to highlight those indices that have contributed the most to overall index changes. Sometimes the analysis results in a finding that particular industries have a relatively high contribution to the overall price change, and that finding is considered unrealistic. The change is traced back to an error, but it may be late in the production cycle and jeopardize the scheduled release date. There is thus a case for identifying such unusual contributions as part of the data editing procedures. The disadvantage of this method is that an elementary index's change may be rejected at that stage. It may be necessary to override the calculated index, though this should be a stopgap measure only until the index sample is redesigned.

D.2 Verifying and correcting data

10.200 Some errors, such as data coding errors, can be identified and corrected easily. Ideally, these errors are caught at the first stage of checking, before they need to be viewed in the context of other price changes. Dealing with other potential errors is more difficult. There may be errors that are not identified in the data checking procedure and observations that have been identified as potential errors may prove to be correct, especially if the data checking limits are rather narrow. Some edit failures may be resolved only by checking the data with the respondent.

10.201 If a satisfactory explanation can be obtained from the respondent, the data can be verified or corrected. If not, procedures may differ. Rules may be established that if a satisfactory explanation is not obtained, then the reported price is omitted from the index calculation. On the other hand, it may be left to the analyst to make the best judgment as to the price change. However, if an analyst makes a correction to some reported data, without verifying it with the respondent, it may subsequently cause problems with the respondent. If the respondent is not told of the correction, the same error may persist in the future. The correct action depends on a combination of the confidence in the analysts, the revision policy in the survey, and the degree of communication with respondents. Most statistical offices do not want to unduly burden respondents.

10.202 In many organizations, a disproportionate share of activity is devoted to identifying and following up on potential errors. If the practice leads to little change in the results, because most reports end up being accepted, then the bounds on what are considered to be extreme values should be relaxed. More errors are likely introduced by respondents failing to report changes that occur than from wrongly reporting changes, and the goodwill of respondents should not be unduly undermined.

10.203 Generally, the effort spent on identifying potential errors should not be excessive. Obvious mistakes should be caught at the data capture stage. The time spent identifying observations to query, unless they are highly weighted and excessive, is often better spent treating those cases in the production cycle where things have changed—quality changes or unavailable prices—and reorganizing activities toward maintaining the relevance of the sample and checking for errors of omission.

10.204 If the price observations are collected in a way that prompts the respondent with the previously reported price, the respondent may report the same price as a matter of convenience. This can happen even though the price may have changed, or even when the particular commodity being surveyed is no longer available. Because prices for many commodities do not change frequently, this kind of error is unlikely to be spotted through normal checks. Often the situation comes to light when the contact at the responding outlet changes and the new contact has difficulty finding something that corresponds to the price previously reported. It is advisable, therefore, to keep a record of the last time a particular respondent reported a price change. When that time has become suspiciously long, the analyst should verify with the respondent that the price observation is still valid. What constitutes too long will vary from commodity to commodity and the level of overall price inflation but, in general, any price that has remained constant for more than a year is suspect.

D.2.1 Treatment of outliers

10.205 Detection and treatment of outliers (extreme values that have been verified as being correct) is an insurance policy. It is based on the fear that a particular data point collected is exceptional by chance, and that if there were a larger survey, or even a different one, the results would be less extreme. The treatment, therefore, is to reduce the impact of the exceptional observation, though not to ignore it, because, after all, it did occur. The methods to test for outliers are the same as those used to identify potential errors by statistical filtering, as described above. For example, upper and lower bounds of distances from the median price change are determined. In this case, however, when observations are found outside those bounds, they may be changed to be at the bounds or imputed by the rate of change of a comparable set of prices. This outlier adjustment is sometimes made automatically, on the grounds that the analyst by definition has no additional information on which to base a better estimate. Although such automatic adjustment methods are employed, the *Manual* proposes caution in their use. If an elementary aggregate is relatively highly weighted and has a relatively small sample, an adjustment may be made. The general prescription should be to include verified prices and the exception to dampen them.

D.2.2 Treatment of missing price observations

10.206 It is likely that not all the requested data will have been received by the time the index needs to be calculated. It is generally the case that missing data turn out to be delayed. In other cases, the respondent may report that a price cannot be reported because neither the commodity, nor any similar substitute, is being made anymore. Sometimes, of course, what started as an apparent late report becomes a permanent loss to the sample. Different actions need to be taken depending on whether the situation is temporary or permanent.

10.207 For temporarily missing prices, the most appropriate strategy is to minimize the occurrence of missing observations. Survey reports are likely to come in over a period of time before the indices need to be calculated. In many cases, they follow a steady routine—some respondents will tend to file quickly, others typically will file later in the processing cycle. An analyst should become familiar with these patterns. If there is a good computerized data capture system, it can flag reports that appear to be later than usual, well before the processing deadline. Also, some data are more important than others. Depending on the weighting system, some respondents may be particularly important, and such commodities should be flagged as requiring particular scrutiny.

10.208 For those reports for which no estimate can be made, two basic alternatives are considered here (see Chapter 8 for a full range of approaches): imputation, preferably targeted, in which the missing price change is assumed to be the same as some other set of price changes, or an assumption of no change, in which the preceding period's price is used (the carry-forward method discussed in Chapter 8). However, this latter procedure ignores the fact that some prices will prove to have changed, and if prices are generally moving in one direction, this will mean that the change in the indices will be understated. It is not advised. However, if the index is periodically revised, the carry-forward method will lead to fewer subsequent revisions than will making an imputation, because for most commodities, prices do not generally change in any given period. The standard approach to imputation is to base the estimate of the missing price observation on the change of some similar group of observations.

10.209 There will be situations where the price is permanently missing because the commodity no longer exists. Because there is no replacement for the missing price, an imputation will have to be made each period until either the sample is redesigned or a replacement can be found. Imputing prices for permanently missing sample observations is, therefore, more important than in the case of temporarily missing reports and requires closer attention.

10.210 The missing price can be imputed by the change of the remaining price observations in the elementary aggregate, which has the same effect as removing the missing observation from the sample, or by the change of a subset of other price observations for comparable commodities. The series should be flagged as being based on imputed values.

10.211 Samples are designed on the basis that the commodities chosen to observe are representative of a wider range of commodities. Imputations for permanently missing prices are indications of weakness in the sample, and their accumulation is a signal that the sample should be redesigned. For indices where there is known to be a large number of deaths in the sample, the need for replacements should be anticipated.

11. Treatment of Specific Products and Issues

A. Introduction

11.1 This chapter provides examples of how different national statistical agencies handle different commodities and explains some pricing issues important in international trade. The emphasis is on those areas in which price measurement generally is regarded as difficult; however, examples of routine commodity areas are included. *It should be kept in mind that the presentation of these methods is not intended to convey them as "best practices." In fact, it is recognized that in some cases a country's circumstances likely will necessitate deviations from these methodologies.* To underscore this point the discussion of each issue includes mention of outstanding issues—issues that point to problems in the described procedures.

11.2 A general problem in constructing export and import price indices (XMPIs) is formulating a precise characterization of the good or service to be priced. To some extent, that characterization hinges on the definition of the commodity area to which the commodity is assigned. For the purposes of this chapter, the Standard International Trade Classification (SITC) Rev. 3 will be used as a reference. The linkage between the selection of the products to be priced and their SITC assignment is independent of whether there is probability or judgmental sampling.

11.3 In addition to the SITC Rev. 3 used in this chapter, other classification systems used for the calculation and publication of XMPIs are presented in Chapter 4. These include the Harmonized Commodity Description and Coding System, Broad Economic Categories, the Central Product Classification, the Statistical Classification of Products by Activity in the European Union, and, for services trade, the Extended Balance of Payments Services Classification.

11.4 After the selection of a commodity to be priced, the difficult problem is characterizing the good in a way that not only facilitates repricing but also distinguishes between changes in quality and changes in price. The last aspect is extremely important for an accurate measure of price change. Previous chapters in this *Manual* have provided discussion of the conceptual framework underlying many aspects of constructing XMPIs. This chapter will provide some examples of different statistical agency practices.

11.5 Within the context of any economy, there will be some commodities for which a relatively straightforward application of these methods and concepts is possible and commodities for which that is not the case. In this chapter, both types will be addressed.

11.6 Generally, the commodity areas that allow for a straightforward application of the methods and concepts are ones for which the trade is countable. That is to say, a respondent's trade is physically measurable. In this case, the definition of the commodity's price is clear. Examples of commodity areas falling into this category and discussed below are agriculture (SITC 0), crude petroleum and gasoline (SITC 33), and metals (SITC 68).

11.7 Commodity areas that experience frequent technological change present some special problems. Though the trade in the computer industry (SITC 75) may be measurable, constructing price indices for computers is difficult when trying to capture quality change that arises from the technological change. Computers and motor vehicles (SITC 78) are examples provided in this chapter.

11.8 Clothing (SITC 84) presents a similar problem. The trade is measurable, but the measurement of price change is complicated by the change in the quality of the clothing and the influence of seasons. The case of the clothing industry is specifically considered.

11.9 Because service industries generally do not have easily measurable output, it is difficult to apply the concepts set out in the *Manual* to them. Accordingly, this chapter covers some of the difficulties involved with calculating XMPIs for services and considers XMPIs for services such as airfreight, air passenger fares, crude oil tanker freight, ocean liner freight, and travel and tourism.

11.10 The discussion below does not fully address issues concerning sample design or sampling methodology. These features are discussed only to the extent that they affect the establishment of a pricing strategy for the commodity or service area.

11.11 The chapter concludes with a discussion of various pricing issues that are particularly important for calculating XMPIs. These issues include the country of origin/destination, duties, currency conversion, intracompany transfers, and price bases.

B. Agriculture, SITC 0

11.12 Many countries employ secondary sources for prices of agricultural commodities that are already being collected in order to decrease the sampling and repricing burden to respondents.[1] For example, the Australian Bureau of Statistics (1995) uses the wool market indicator price from the auction market for exported greasy wool because a large percentage of greasy wool exports are sold through this market. Other examples of the use of secondary source data are the U.S. Bureau of Labor Statistics' use of export price data from the U.S. Department of Agriculture for exported grains (U.S. Bureau of Labor Statistics, various issues) and the U.K. Government Statistical Service's use of meat prices from the Meat and Livestock Commission (Richardson, 2000). The construction of a price index for agricultural products generally, and crops in particular, is more difficult because of two circumstances that sometimes combine. First, marked seasonal patterns in the trade of commodities' make prices unobservable during part of the year. Second, volatility in price and production from year to year, and sometimes within a year, are caused by external forces such as the weather or economic influences.

11.13 These two problems have to be accommodated by building into the indices a method for dealing with gaps in the supply of prices and for smoothing volatile elements while reflecting, as quickly as possible, changes in the trend of agricultural production.

[1]Note that sources of secondary price data might also be available for other homogeneous commodities, such as crude petroleum and raw metals.

B.1 Seasonal commodities in the Canadian FPPI

11.14 The following description is drawn from the recently redesigned Canadian Farm Producer Price Index (FPPI) (Baldwin, 2002) and the procedures introduced to meet these problems, which are representative of the techniques used by other countries for producer price indices (PPIs) and which may be applicable to XMPIs.

- The index follows a seasonal-basket concept in which the volume shares of the various commodities are different for each month in the year. Thus there are 12 different baskets used in calculating the months of a calendar year in the FPPI.
- The annual index number for a given year is a weighted average of the corresponding monthly indices, rather than a simple average, as is common in other indices.
- The index is an annually reweighted chain price index, so the annual weighting pattern is updated every year. Each annual weighting pattern, or basket, is based on marketing data for the five most recent years available.
- The linking of new baskets each year is done at the annual index number level, not for any one month.

B.2 Seasonal baskets

11.15 The formula for constructing the seasonal baskets in the Canadian FPPI is a variant of what usually is called the Rothwell formula, after Doris Rothwell, an economist with the U.S. Bureau of Labor Statistics, who proposed it in a 1958 paper for the U.S. consumer price index (CPI). However, the formula was originally proposed in 1924 by two economists with the U.S. Department of Agriculture, Louis H. Bean and O.C. Stine, as an index number for farm prices. Thus the formula adopted for constructing seasonal baskets was originally designed as an indicator of farm price movements.

11.16 The Rothwell formula must be used to calculate indices of fresh produce in the harmonized indices of farm product prices of the European Community, so statisticians of those countries are familiar with it. The formula also is used to calculate series for seasonal commodity groups in the CPIs of several countries, including Japan, France, and the United Kingdom.

11.17 The Rothwell formula is

$$P^{(c)}_{y,m/0} = \sum_j p^j_{y,m} q^j_{c,m} \Big/ \sum_j p^j_0 q^j_{c,m}, \qquad (11.1)$$

$$\text{where} \quad p^j_0 = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} \Big/ \sum_{m=1}^{12} q^j_{c,m} = \sum_{m=1}^{12} p^j_{0,m} q^j_{c,m} \Big/ q^j_c.$$

In the above formula, $p^j_{y,m}$ is the price of the j^{th} commodity for the m^{th} month of year y, p^j_0 is its price in base year 0, and $q^j_{c,m}$ is its quantity sold in the m^{th} month of the basket reference period c. Note that in the special case in which the basket reference period c is the same as the base year 0, the formula becomes

$$P^{(0)}_{y,m/0} = \sum_j p^j_{y,m} q^j_{0,m} \Big/ \sum_j p^j_0 q^j_{0,m}, \qquad (11.2)$$

$$\text{where} \quad p^j_0 = \sum_{m=1}^{12} p^j_{0,m} q^j_{0,m} \Big/ \sum_{m=1}^{12} q^j_{0,m} = \sum_{m=1}^{12} p^j_{0,m} q^j_{0,m} \Big/ q^j_0.$$

Also note that the average base-year price of each commodity is its base-year unit value. In the special case where $q^j_{0,m} = q_0/12$, $m = 1,2, \ldots 12$, for every commodity (i.e., quantities sold were the same in every month of the base year for every commodity), this variant reduces to the familiar Laspeyres formula.

11.18 The Rothwell formula for the annual index would be

$$P^{(c)}_{y/0} = \sum_j p^j_y q^j_c \Big/ \sum_j p^j_0 q^j_c, \qquad (11.3)$$

$$\text{where} \quad p^j_y = \sum_{m=1}^{12} p^j_{y,m} q^j_{c,m} \Big/ \sum_{m=1}^{12} q^j_{c,m} = \sum_{m=1}^{12} p^j_{y,m} q^j_{c,m} \Big/ q^j_c.$$

In the special case in which the basket reference period c is identical with the base year 0, the formula becomes

$$P^{(0)}_{y/0} = \sum_j p^j_y q^j_c \Big/ \sum_j p^J_0 q^j_c,$$

$$\text{where} \quad p^j_y = \sum_{m=1}^{12} p^j_{y,m} q^j_{0,m} \Big/ \sum_{m=1}^{12} q^j_{0,m} = \sum_{m=1}^{12} p^j_{y,m}\, q^j_{0,m} \Big/ q^j_0.$$

Note that even when the base-year prices are unit values, prices for other years are not, because they are weighted according to another period's monthly sales pattern.

11.19 In the Canadian FPPI, the monthly weighting patterns are calculated as follows: For each product, the average quantities sold for the five years from 1994 to 1998 were calculated for each month of the year. The quantities sold of most agricultural products can be measured directly: the availability of measures such as bushels or head obviates the need for deflation. The 12 monthly shares are then calculated. To obtain the monthly revenue weight for a given product, the annual revenue weight for a particular year is multiplied by the relevant monthly share. The sum of these monthly weights yields the annual weight. As described below, the annual weights change every year, but the monthly share patterns are held constant until the next major review, in about five years. This approach allows the relative importance of commodities in the 12 monthly baskets to change from year to year, reflecting the changes in the relative prices of the different commodities.

11.20 A major strength of this approach is that it accounts for highly seasonal products available for only a few months in the year. In the previous annual basket approach, such commodities had the same basket share in every month of the year. One had to impute prices in months when no quantities were sold. In a monthly basket approach, if there were no sales for a commodity in a given month from 1994 to 1998, then it simply fell out of the index basket. There was no need to impute a price for it.

11.21 Problems with changing seasonal patterns may remain. If a seasonal commodity had no sales in a given month from 1994 to 1998, but some thereafter, the prices for that month would be ignored. For example, if the season for corn lengthened to include sales in November, where before no sales have occurred after October, this shift in the overall seasonal pattern of production of an agricultural commodity would not be reflected until the next update of the seasonal patterns. Changes in the length of a season do not occur very often, and it is the *beginning or end* of the season that is being ignored. Ignoring that is much less serious than assuming all months would have about an 8 percent (one-twelfth) share of the annual sales.

11.22 Imputations cannot always be avoided. If there typically is a weight for a product in a certain month, but for some reason, such as early frost in October, no sales of that product occurred that year, an imputed price would have to be assigned to it. This kind of scenario is more likely to occur than the example of the lengthened season discussed above. In such a situation, the imputed price would be the weighted average price for the in-season months through September. Although one could argue for other solutions, such an imputation is simple, does not depend on price information external to the stratum or the commodity in question, and gives the same annual price as if one simply ignored October in calculating the annual price.

11.23 The problems of imputation, as well as the formation of the seasonal basket, are ones faced by seasonal commodities, such as clothing, discussed in Section C.

B.3 Annual price index

11.24 The annual price indices are weighted averages of the monthly index numbers. The weights are the monthly expenditure weights. In this, they differ from the simple means of the monthly index numbers. A weighted average is used because the monthly shares of trade of many farm products are highly unequal. Most occur in only two or three months of the year, and in the same two or three months of the year, year after year. One cannot have much confidence in an annual index based on equal weighting of the monthly indices if the different months have such unequal contributions to annual output.

11.25 Although they are close, the annual prices at the most detailed level are not unit values of the commodities. The annual unit value for a commodity is calculated as the total annual revenue divided by the total annual quantity sold. This amounts to a weighted average of monthly prices, weighted by *same year* quantities. The annual prices in the FPPI are weighted averages of monthly quantities for the seasonal profile reference period, currently from 1994 to 1998.

B.4 Annual chaining

11.26 The index is updated every year, from the receipts for a five-year period. The basket for 1999, for example, is based on the sales from 1993 to 1997, revalued to 1998 average prices.

11.27 Consider the updating done for the January 1999 index. The quantities sold from 1993 to 1997 are evaluated at prices for 1998 to provide a new basket. With this basket, indices are recalculated for each month from January 1998 onward; the recalculated index will automatically be on a 1998 time reference, so the ratio of this index to the previously calculated 1998 index gives the link factor. Indices for the months of 1999 are multiplied by this link factor. In January 2000, the same procedure is followed, instead of using quantities sold for the period 1994 to 1998.

B.5 Linking at the annual index

11.28 Linking series that are computed with both monthly and annual measures can be a problem because it is not possible to preserve continuity for both. Most series get linked at the monthly level so that the monthly index changes are not distorted by shifts between the baskets. This can be done by linking in December, so that December and January prices are compared in terms of the new basket.

11.29 For this index, the monthly baskets change anyway, so there is no advantage in linking by the month. Linking at the year preserves the year-to-year movement as a measure of pure price change.

B.6 Analysis of monthly price changes

11.30 Monthly baskets have the disadvantage of having no measure of pure price change between months. Even if there is no change in prices from one month to the next, a change in the index is possible because of the change in the basket. However, it is possible to decompose the monthly change in the FPPI into a pure price change component and a residual component for all months except January. The pure price change component measures what the change in the FPPI would be if there were no change in the monthly basket. This calculation may require the calculation of imputed prices for some commodities that may have gone out of season by the next month.

11.31 The decomposition is as follows:

$$\left(P^{(c)}_{y,m/y-1} - P^{(c)}_{y,m-1/y-1}\right) = \frac{\sum (p_{y,m} - p_{y,m-1}) q_{c,m-1}}{\sum p_{y-1} q_{c,m-1}} + \frac{\sum p_{y,m} (q_{c,m} - q_{c,m-1})}{\sum p_{y-1} q_{c,m}}, \tag{11.4}$$

where summation is over commodities. Therefore, the monthly percentage change in the Rothwell index can be decomposed between a pure price change component,

$$\left(\sum (p_{y,m} - p_{y,m-1})\, q_{c,m-1} \Big/ \sum p_{y,m-1}\, q_{c,m-1}\right) \times 100, \tag{11.5}$$

and a residual component,

$$\left(\sum p_{y,m}\, (q_{c,m} - q_{c,m-1}) \sum p_{y,m-1}\, q_{c,m-1}\right) \times 100. \tag{11.6}$$

(As can be seen, the residual component is not a pure quantity change component because there are different prices in the numerator and the denominator.)

11.32 Where very large basket shifts exist from one month to the next, it may not be acceptable to take the previous month's basket as appropriate for comparing prices between the previous and current month. An Edgeworth-Marshall type cross should then be calculated:

$$\left(\frac{\sum(p_{y,m} - p_{y,m-1})\overline{q}_{c,m-1\&m}}{\sum p_{y,m-1}\,\overline{q}_{c,m-1\&m}}\right) \times 100, \qquad (11.7)$$

where $\overline{q}_{c,m-1\&m} = (q_{c,m-1} + q_{c,m})/2$.

11.33 Equation (11.5) answers the question of what the monthly percentage change in the FPPI would have been if there had been no change in the monthly basket from the previous month, with the previous month's FPPI remaining as published. Equation (11.7) answers the question of what the monthly percentage change in the FPPI would have been if both previous and current month estimates had been calculated using a common monthly basket representing sales in both months. Equation (11.5) thus is more closely connected to the published FPPI than is equation (11.7). Yet the latter may be a better measure of month-to-month price change because it uses quantity weights from two time periods.

11.34 An Edgeworth-Marshall cross has the advantage of being consistent in aggregation and satisfying the property of transactions equality. (If the volume of sales in month *m* is five times larger in month *m* than in month *m* – 1, month *m* will be about five times more important in determining the basket shares of the price comparisons.)

11.35 A Fisher cross is another way to incorporate information from two time periods. However, such an index does not satisfy transactions equality. In a Fisher cross, the price comparisons are weighted using each basket, and then their geometric mean is taken; the two baskets are treated as being of about equal importance, which may be contrary to reality as in the example where sales in month *m* is 5 times the sales in month *m* – 1.

11.36 An Edgeworth-Marshall cross also has an advantage over a Walsh cross, another index that combines information from two time periods, in that it does not remove seasonally disappearing commodities from the comparison. For a Walsh cross,

$$\left(\frac{\sum(p_{y,m} - p_{y,m-1})\overline{\overline{q}}_{c,m-1\&m}}{\sum p_{y,m-1}\,\overline{\overline{q}}_{c,m-1\&m}}\right) \times 100, \qquad (11.8)$$

where the average $\overline{\overline{q}}_{c,m-1\&m} = \sqrt{q_{c,m-1} \times q_{c,m}}$.

If a commodity were missing in either month, its mean quantity sold would be zero, and it would have no impact on the measured price change; in the Edgeworth-Marshall cross, all commodities with sales in at least one of the two months would have an influence on the estimated price change.

11.37 In calculating an Edgeworth-Marshall cross using equation (11.7), one must impute prices for commodities unavailable in either month *m* – 1 or *m* (but not both) and not, as with equation (11.5), only for those unavailable in month *m*.

11.38 The December to January change is distorted not only by the switch from one monthly basket to another, but also by the switch from one annual basket to another. Because the annual basket changes every year, comparisons of 12-month changes between the same months of successive years do not provide a measure of pure price change. This problem is met by calculating each new index for 24 months, as previously described. Although the monthly index numbers are not used for the first 12 months, comparisons between them and the 12 months that follow can be used as measures of pure price change for 12 periods. In other words, the 1998 indices, on a 1998 base, are not used in the index; only those indices for 1999 are. Because they use the same basket, comparisons of the May 1998 (1998 = 100) and May 1999 indices give a pure price change measure.

B.7 Other issues

11.39 *Use of receipts in the absence of quantities sold.* For some commodities, such as maple products, quantities are not provided, though there are cash receipts. In this index, the price movements are taken from the movement of the total crops index. This ensures that each kind of product is represented in the index with an appropriate weight.

11.40 *Choice of time reference*. The FPPI is referred to 1997 = 100. As the index is a chained fixed-basket index with the basket changing every year, the choice of time reference has nothing to do with the estimated price movements over time. The base was chosen to correspond with Canada's choice of 1997 as the reference for most of its economic series, including the *System of National Accounts*.

B.8 Seasonal commodities in the U.S. XMPI

11.41 The discussion presented in the sections above relates to the methodology employed by the Canadian

FPPI; the following section describes treatment of seasonality in the U.S. XMPI.

11.42 An analysis of seasonal commodities in U.S. XMPIs was performed by Alterman, Diewart, and Feenstra (1999). Four different approaches to the treatment of seasonal commodities were described by the authors as follows.

- An *annual price index* addresses the seasonality problem by treating each commodity in each month as a different commodity to be compared. The commodity-month observations are compared as a set with the same set of observations for the previous year to obtain the annual index.
- In a *year-over-year price index,* the index is calculated by comparing the sets of commodities in one month with the same set in the same month of the previous year. For example, the price of the commodities traded in April 2005 would be compared with the price of the same commodities traded in April 2004.
- Another option is to simply exclude the seasonal commodities from the index calculation and to calculate a *month-to month index over nonseasonal commodities.*
- Finally, a *month-to-month index with maximum overlap* can be calculated by comparing the prices for all commodities available in two consecutive months. This last option is the one that the U.S. Bureau of Labor Statistics (BLS) uses in practice.

11.43 Alterman, Diewart, and Feenstra (1999) tested these four approaches using simulated and real data and recommend that XMPIs use a *month-to-month index with maximum overlap* estimated with a geometric index using monthly trade weights lagged two years. The introduction of monthly weights would ensure that seasonal commodities would drop out of the index during the months when there is no trade.

11.44 The Canadian and U.S. approaches raise issues that some countries may want to avoid by coming up with alternative methods. As described earlier, the seasonal basket combines information from annual and monthly data, creating issues about (1) how to select the market basket and (2) how to interpret the switch between annual and monthly quantity data. In addition, one must choose the appropriate base year. A fuller discussion of seasonal adjustment is provided in Chapter 23.

C. Clothing, SITC 84

11.45 The procedures used by the Australian Bureau of Statistics PPI for the clothing industry can also be applicable to the calculation of XMPIs for apparel. The broad category of apparel covers trade in a wide range of garments, from basics to high-fashion items. The commodities can be categorized in a number of ways, but commodity classifications generally adopt the traditional split of

- Women's and girls' clothing,
- Men's and boys' clothing, and
- Infants' clothing, and clothing not elsewhere classified.

11.46 A further dissection by functional type of clothing can be made within these categories. For example, women's and girls' clothing could be divided into women's dresses, girls' dresses, women's skirts, girls' skirts, women's sleepwear, girls' sleepwear, and so on.

11.47 Alternative classifications may focus on aspects such as formal or fashion wear, business wear, casual wear, or sporting wear, or on the type of material used, including cotton or polyester.

11.48 After the items to be covered by the index (e.g., women's dresses) are selected, the respondents to be included in the index and the specific items to be priced need to be selected. As is the case with all XMPIs, the selection of respondents will normally be based on data from export and import trade statistics.

11.49 Selection of the actual specifications to be priced will require contact with the respondents and may be complex. Key principles in selecting the actual specifications from any particular respondent are as follows:

- Specifications should provide adequate coverage of the types of garments traded within that commodity category. In particular, they should represent the pricing practices adopted by the respondents. That is, the factors that cause prices to move differently across specifications, such as the type of material used (e.g., cotton fabric shirts may move differently in price with polyester fabric shirts), should be taken into account.
- One should be able to price the specifications on an ongoing basis to maintain constant quality. To do so on an ongoing basis, full details of the specification need to be obtained (see below).

11.50 As was the case for agricultural goods, a general problem in pricing clothing is the distinct seasonal variations in the clothing traded because manufacturers switch from summer to winter clothing, Some garments are traded for only part of the year; therefore, some technique is required to handle the period when these seasonal items are not traded. The most common technique is to simply repeat prices for the out-of-season items.

11.51 As was mentioned regarding agricultural products, the problem of missing items is common when dealing with seasonal commodities. Imputations are therefore necessary. Section B.6 of Chapter 8 discusses imputations.

11.52 Another problem is finding the same items to price in the new season (e.g., this winter) as were priced in that season of the previous year (i.e., last winter). Items often change because of fashion, and style changes the relative costs of different fabrics (e.g., wool versus synthetics). Where the same item cannot be repriced and a different item is priced instead, it will be necessary to assess what price movement should be shown.

11.53 Quality change can be identified by any changes in the characteristics that incur costs. For a type of clothing, the quality change associated with a substitution of polyester for cotton can be handled by valuing the different cost. A wide range of factors can affect the quality of these garments. Major factors include

- Type of fabric used (e.g., pure cotton, cotton blend, polyester),
- Quality of fabric (e.g., weight, thread count, type of dyeing used), and
- Quality of make (e.g., example, type of seams, buttonholes, collar, pleats).

11.54 With clothing, a natural question is what to do about fashion changes that are generally tied to seasonal variation. Opinions differ on whether a specific quality change should be made for fashion. Some might argue that a quality adjustment should be performed because the fashion element is the key price-determining characteristic. Others might argue that fashion changes manifest themselves in changes in other characteristics, such as fabric, and therefore do not require additional adjustments. If there are no changes in any of the measurable characteristics of the article of clothing, then some imputation for the cost of design may be necessary though quite difficult. Furthermore, no such adjustments typically are made for other products traditionally redesigned every year, such as automobiles. (The quality-adjustment procedures for automobiles are discussed in Section G.) Finally, although manufacturers devote considerable efforts to establish their designs as the fashion of the season, there is no certainty of success. Accordingly, the validity of computing a quality adjustment for fashion rests to some extent on whether the fashion can be deemed successful.

11.55 The practical problems for the price statistician are, first, to detect these changes and, second, to place a value on them. To detect quality changes, it is necessary to list on the prices questionnaire the actual specifications being priced from particular respondents; for example:

> "Brand X, Men's dress shirt, style No. xxxx, 100% cotton, size 38–43, long sleeves, single cuffs, etc."

11.56 In addition to the detailed specifications, respondents specifically should be asked on the questionnaire whether there have been any changes in the quality of the specifications being priced and asked for an estimate of the cash value of the difference.

11.57 If such an estimate is not available from the respondent, it may be feasible for the commodity expert in the statistical office to estimate the value of the quality change. Hedonic regression methods may be employed if resources permit. Descriptions of hedonic techniques used for clothing in consumer price indices can be found in Liegey (1993).

11.58 Seasonal dimensions can be handled by creating checklists that are seasonally based. Thus, a list for an item could include women's summer dresses, fall dresses, and so on.

11.59 Two prominent issues arise with measuring price change for clothing. First, as mentioned above, is how to impute missing prices and quantities. Second, there is a question of whether changes in fashion should be considered as quality change. Earlier it was argued that such changes should not be considered quality change.

D. Crude Petroleum and Gasoline, SITC 33

11.60 In many countries, detailed data on crude petroleum imports are already collected by the Ministry or Department of Energy. If trade volumes and quantities are available at a detailed enough level to ensure that the crude stream is homogenous over time, then a unit value methodology can be employed to obtain prices per barrel of oil. The following methodology is used by the U.S. BLS to measure price changes for imported crude petroleum.[2]

11.61 The U.S. BLS uses two different methodologies to calculate its import crude oil index: one methodology for the initial estimate and another for the three subsequent revisions. The U.S. BLS primarily uses transaction data from the Department of Energy (DOE). The DOE collects data on the costs and quantities of virtually all foreign crude oil acquired for importation into the United States. When the crude oil index for a given month is published for the second, third, and fourth times, the U.S. BLS simply aggregates the transaction data from the DOE. However, most of these data are available only on a lagged basis. About half the data are available with a one-month lag and almost all the data are available with a two-month lag. The initial data are also subject to revision. Thus most transaction data are unavailable when the first estimate of the crude oil index is published. As a result, the U.S. BLS uses a different methodology for the first estimate of the crude oil index. The DOE transaction data are aggregated for the current month, but this short-term relative is not published; rather, it is used as an input in a regression model.

11.62 The second input in the regression model is based on prices from the U.S. Energy Information Administration (EIA). The EIA publishes weekly U.S. average prices for crude oil that are weighted by estimated import volume. The U.S. BLS computes another measure of monthly crude oil price change by calculating the monthly average of these weekly average prices. The final revision of the monthly percent change in the crude oil index (using DOE data) is regressed on the initial percent change of the DOE transaction data and the EIA monthly percent change. This regression model is used to calculate the estimate of the current month's crude oil index. In the six years that the regression model has been used, the root mean squared error of the model's estimate has been 64 percent lower than the value derived from the limited DOE transaction data and 41 percent lower than the EIA monthly percent change.

11.63 Unlike other commodities, the reference period for the crude oil index is the entire month rather than the first week of the month.

[2]The following description is found in the U.S. BLS (undated), "How the International Price Program Measures Price Change for Crude Oil and Gasoline in the U.S. Import/Export Price Indexes," Division of International Prices.

11.64 Secondary source data are also used by the U.S. BLS for the XMPIs for gasoline. The U.S. BLS uses spot prices from the first week of the month to calculate the import and export gasoline price indices. Reuters News Service is the source for the spot prices for conventional regular gasoline for New York Harbor, the U.S. Gulf Coast, and Los Angeles. The prices are published in the EIA's Weekly Petroleum Status Report.

11.65 The import and export gasoline price indices use the same price data but differ solely because of the different weights associated with the prices for New York Harbor, the U.S. Gulf Coast, and Los Angeles. Most imports come into New York and most exports leave from Los Angeles. The weights are derived from the EIA's Petroleum Supply Annual. The reference period for gasoline is the first week of the month.

E. Metals, SITC 68

11.66 Pricing of metals can rely on secondary source data from the London Metal Exchange. World prices are available for metals such as aluminum, copper, lead, nickel, tin, and zinc.

11.67 The secondary source prices may differ from transactions prices owing to differences in transportation costs. In addition, these prices are quoted in U.S. dollars so they would have to be converted into the national currency (see Section H.2). Finally, the cost of insurance included may vary (see Section H.4 for a description of different price bases used).

F. Electronic Computers, SITC 75

11.68 Some countries use data from major trading partners as a proxy for import price indices. For example, the United States is a major exporter of electronic computers, so U.S. export price indices for computers could serve as a proxy for other countries' import price index for computers.

11.69 A process for estimating price changes for electronic computers that has served as a model for many countries was developed by the U.S. PPI program, outlined in Holdway (2001). This methodology, which has also been adopted by the U.S. XMPI program, is described below.

11.70 The primary commodity traded in the computer industry is the assembly of components into general-purpose computer systems that process data according to a stored set of instructions. These instructions are contained in the computer software (operating and application) and are often included in the computer system by the manufacturer. Establishments that primarily manufacture machinery or equipment that incorporates computers for the purpose of performing functions such as measuring, displaying, or controlling process variables are classified based on the manufactured end product.

11.71 Trade in computers can be disaggregated into several categories. These categories should be broadly defined because the rapid pace of industry technological change can render narrowly defined categories obsolete. The U.S. BLS structure for computers is based on product detail collected by the U.S. Census Bureau in its Current Industrial Report survey; the categories are as follows:

- Host computers, multiusers (mainframes, super computers, medium-scale systems, UNIX servers, PC servers);
- Single-user computers, microprocessor-based, capable of supporting attached peripherals (personal computers, workstations, and portable computers); and
- Other computers (array, analog, hybrid, and special-use computers).

11.72 Rapid changes in computer technology can create the classification problem of new product classes that do not fit neatly into an existing product classification structure. For example, handheld devices would be classified under "portable computers" in the above classification structure. However, index users, including producers, have come to view the portable computer designation as including only laptops or notebooks. Therefore, the introduction of handheld devices into the index should result in revision to the commodity title such as changing "portable computers" to "portable computers, including handhelds." If revising the title of an existing product classification does not satisfy analytical requirements, then a more aggressive adaptation could include the introduction of a new more specific product category into the publication structure, such as "Handheld computers, including personal digital assistants (PDAs)."

11.73 The product classification issue described above is related to rapid postsample changes in output. Similar adjustments at the disaggregate level may be required for the trade in other high-tech industry products such as semiconductors.

11.74 In the U.S. XMPI program, computer exporters and importers were selected with a probability proportionate to size as measured by trade value, and then individual commodities representing current trade patterns were selected based on their relative importance to a respondent's value of shipments. Respondents provided detailed product specifications for each of the computers that were sampled for which the respondents provide monthly price updates. Because of rapid technological change, respondents generally are unable to maintain a matched model for more than three or four months. Therefore, new computers or updated versions of predecessor computers are continually introduced into the XMPI as sampled products become obsolete. Product substitution caused by rapid product displacement in effect provides an automatic sample update mechanism. However, new technologies or changes in characteristic quantities embodied in computer replacements challenge a statistical agency's ability to publish constant quality indices.

11.75 Hedonic methods are used to estimate quality change valuations for computers in the PPI and the XMPI. The hedonic function is based on the premise that the characteristics that make up a complex product can be unbundled and their influences on price measured.

11.76 The correct specification for a hedonic model is often a technical issue that is more dependent on product- and market-specific knowledge than econometrics. If appropriate data, including transaction costs, are available to support a model, then regressions can provide estimated coefficient values (implicit prices) for each of the independent variables described in a specification. Discussion of hedonic models is provided in Chapter 8, Section E.4.

11.77 When cost data are unavailable, then the implicit prices from a hedonic model can be used to value changes in the quantities of characteristics reported to the XMPI.

11.78 The mechanics of quality-adjusting price relatives when computer characteristics change is described below:

ICP = Implicit characteristic price from hedonic model,

P_0 = Price of predecessor computer in reference period,

P_c = Price of replacement computer in comparison period,

$PR =$ Price relative, and

$$PR = \frac{P_c - ICP}{P_0}.$$

11.79 The above example is based on an increase in the quantity of computer characteristics such as system memory or hard drive capacity. If the quantity of computer characteristics declines in period *c*, then the value of *ICP* is added to rather than subtracted from P_c.

11.80 The independent variables specified in the hedonic models include inputs such as microprocessors, memory, and disk drives, all of which exhibit extraordinarily rapid price declines. Many other inputs are also used in the regressions. Because the costs of these components change rapidly, the U.S. BLS has opted for frequently updated cross-sectional models rather than less-frequent updates of pooled data.

11.81 Ideally, the U.S. BLS would update its cross-sectional computer models on a monthly basis, but resource constraints result in quarterly updates. Nevertheless, the U.S. BLS has greater confidence in the constant quality measures provided by quarterly cross-sectional updates relative to a pooled model. Frequent updates of cross-sectional models also help the U.S. BLS estimate implicit prices for new characteristics shortly after they are introduced. The availability of a large amount of computer-related data on the Internet has aided the updating of the hedonic regressions.

11.82 Regularly updated cross-sectional models provide implicit prices that are based on market conditions at or close to the point at which a product replacement actually occurs, thereby enabling an improved approximation of constant quality indices in the U.S. BLS's real-time monthly production environment.

11.83 Because a longitudinal analysis of the relationship between prices and characteristics is the preferred way of basing quality adjustments, some agencies may want to address the manner in which a sequence of updated cross-sectional regressions approximates a longitudinal regression.

G. Motor Vehicles, SITC 78

11.84 The primary commodity of the broad motor vehicle building industry is motor vehicles and engines and parts for motor vehicles. The discussion below describes the PPI for the Australian automobile industry and the methods of the Australian Bureau of Statistics, which are also useful for XMPIs.

11.85 The commodity areas can be defined by the main activities of the industry, such as

- Motor vehicles,
- Motor vehicle engines and parts,
- Motor vehicle bodies,
- Automotive electrical and instruments, and
- Other automotive components.

11.86 The first requirement in attempting to measure price change for this sector is to establish a clear understanding of the industry. In particular, one must determine the major categories of motor vehicles.

11.87 The following discussion focuses on complete motor vehicles. The concepts discussed also will be of assistance in considering issues involved with pricing other motor vehicle–related production.

11.88 The next stage is to select potential respondents that are representative of these activities. As is the case with all XMPIs, the selection of potential respondents will normally be based on data from export and import trade volumes.

11.89 The prices should reflect market values in cases where the trading partners are related. In such cases, cross-subsidization may make it difficult to obtain the proper price. More details on intra-company transfer pricing are provided in Section H.3.

11.90 Usually match pricing on a particular day of the month (such as on the 15th) will be adequate for monthly indices, because motor vehicle prices tend not to be as volatile as those for some commodities.

11.91 A major issue for producing an index for any technologically advanced commodity, such as motor vehicles, is quality change. Although vehicle manufacturing tends to follow models that will be on the production run for at least a year (giving some opportunity to assess more fundamental technological change), motor vehicle suppliers are constantly offering packaged deals on these models. Given the array of options available for automobiles, price statisticians have the challenge of pricing to constant quality.

11.92 Examples of motor vehicle features, which may be relevant for item selection and assessment of quality change, include

- Make and model,
- General type of vehicle (e.g., sport, four-wheel drive, limousine, sedan, wagon),
- Engine size,
- Exterior dimensions,
- Interior dimensions,
- Torque,
- Anti-lock braking system,
- All-wheel drive,
- Fuel consumption (high consumption is regarded as a negative attribute; the type of fuel used has differing assessments depending on relative fuel costs and efficiencies),
- Airbags,
- Traction-control systems,
- Safety rating,
- Acceleration,
- Brake horsepower,
- Curbside weight,
- Air-conditioning,
- Cruise control,
- Compact disc player and stacker,
- Global positioning system,
- Keyless entry,
- Security system,
- Power windows,
- Electric sunroof,
- Electric mirrors, and
- Metallic paint.

11.93 One method commonly employed for change of specification is the overlap method of pricing, discussed in Chapter 8. To undertake this method, prices must be available for the old and new model at the same time, which often may not be possible. The price comparison uses the old specification price in the earlier period and the replacement specification in the next period. Implicitly, the price difference is said to represent the market's evaluation of the quality difference between the two items.

11.94 An adjustment for changes in quality also can be made by valuing the difference in production cost attributable to the change in characteristics. This method has conceptual appeal in the case of XMPIs, because assessments of quality change are best made on estimates of production cost differences in models. This method is frequently employed in the quality assessment of motor vehicles. A great deal of costing information that can be used for this purpose often is available from manufacturers. Similar sources of information may include motoring magazines or assessments made by motoring clubs or insurance companies.

11.95 Another approach is to use hedonic methods for quality adjustment purposes (see Chapters 8 and 22 for an in-depth explanation of hedonic methods). This will require an extensive data set of motor vehicles' prices with the quantities of all characteristics influencing price, preferably on the correct pricing basis (i.e., basic prices), from which to calculate the hedonic function. The implicit prices of the motor vehicle characteristics from the hedonic function are used to value the differences in new and replacement motor vehicles within the ongoing sample. Alternatively, if complete time series data sets of prices and characteristics are available, then the time dummy method could be used to directly estimate a price index from the hedonic function. The hedonic function on which these implicit characteristic prices are based should be updated at least annually. See Bodé and van Dalen (2001) on the use of hedonic methods for constructing constant quality price indices for motor vehicles.

11.96 A number of private companies collect and collate pricing data on motor vehicles. Such sources often are used for detailed hedonic analysis of quality change. Whatever the quality assessment technique used, price statisticians may find it useful to refer to websites that provide reliable and free comparisons between different models and makes. An example of such a site is www.autobytel.com.

11.97 It should be noted that the set of characteristic changes also should include those mandated by governments. Some typical examples include

- Catalytic converters to limit pollution,
- Seatbelts or airbags,

- Systems that prevent ignition without the use of seatbelts, and
- Speed-limiting or warning mechanisms.

Legally mandated features should be seen as a quality improvement because they cost extra to produce and reflect a greater volume of production. Manufacturers usually can supply estimates of the extra production costs imposed by the addition of these features.

11.98 The price statistician needs to be concerned with some issues in implementing quality adjustments for automobiles. For example, automobile purchasers often order models with options—that is, the purchased model differs from the standard model. If such options are popular in a time period, then a high percentage of the cars purchased may have those options. If, on realizing the option's popularity, the manufacturer decides to make the option standard, then care must be taken in estimating the quality adjustment. To illustrate, suppose that all of the automobiles purchased in a given time period were ordered with the option and that in the next time period the option becomes standard. In this case, no quality adjustment should be conducted in the month that the option becomes standard, because in the previous month the value of the option should have been accounted for. When dealing with options, one must take care to recognize the market penetration of the option before performing a quality adjustment should the option become standard. Another caveat applies when performing quality adjustments for changes in features that can return to the original level. For example, suppose that because of relatively stable fuel prices, engine horsepower starts increasing and quality adjustments are performed for the increase. If fuel prices rise sharply and induce reductions in horsepower to the level of the reference model, then a decision must be made on how to treat horsepower change. One the one hand, a quality erosion could be recorded (relative to the last model) but, on the other hand, there is no quality change relative to the reference model.

H. Services

11.99 Most countries that produce XMPIs publish indices only for trade in commodities; however, services make up an increasing amount of international trade. In order to obtain accurate balance of payments data in inflation-adjusted terms and to fully analyze international trade, services XMPIs are needed. The Voorburg Group, established in 1986, recognized the need for better measurement of the service sector and has been coordinating research into services measurement issues including research on measuring prices for internationally traded services.

11.100 Measurement of services is complicated by the intangible nature of the commodities traded. Often it is difficult to even define the service being traded. In many cases the transactions are unique and a stable commodity cannot be tracked over time.

11.101 Often, internationally traded services are not classified according to the classification systems used by national statistical agencies. For example, in the United States the BLS instead publishes indices according to two different definitions (U.S. BLS, 1997). First, a balance of payments definition (import and export) is used, so that the indices measure price trends for payments between domestic and foreign residents on internationally traded services. These indices are used to deflate the foreign sector of the national accounts. Second, an international services definition (inbound and outbound) is used to measure price trends regardless of the residencies. These indices are useful for the analysis of international trade and inflation.

11.102 Transportation services are perhaps one of the more straightforward types of services to measure and one for which secondary source data might be readily available. The following sections describe the methodologies used by the U.S. BLS to produce XMPIs for airfreight, air passenger fares, crude oil tanker freight, and ocean liner freight.

H.1 Airfreight

11.103 The U.S. BLS publishes both balance of payments (import/export) and international (inbound/outbound) indices for international airfreight rates. The import airfreight index (U.S. BLS, 2008a) measures changes in rates paid for the transportation of freight from foreign countries to the United States on foreign carriers. The export airfreight index measures changes in rates paid for the transportation of freight from the United States to foreign countries on U.S. carriers and freight transportation between two foreign countries on U.S. carriers.

11.104 For the international services indices, the inbound airfreight index measures changes in rates paid for the transportation of freight from foreign countries into the United States on any carrier. The outbound airfreight index measures changes in rates paid for the transportation of freight from the United States into foreign countries on any carrier.

11.105 Airfreight consists of commodities tendered to an airline for transportation, not including mail, express, or passenger baggage. The service being priced is the transport from airport to airport, so all ground transport and port services are excluded.

11.106 The sampling frame for the U.S. airfreight indices comes from data already collected by the U.S. Department of Transportation, which includes data on airports of origin and destination, air carrier name and nationality, and service class (e.g., passenger/cargo or cargo).

11.107 The price data are collected from responding air carriers. The price-determining characteristics for an airfreight item include the air carrier nationality, the airports of origin and destination, the shipment weight, the dimensions and packaging of the shipment, the type of commodity shipped, and the type of buyer. All of these must be captured in the item description. In addition, the price quotes must include data on commissions, discounts, and surcharges so that the net freight rate can be calculated.

11.108 The revenue weights are derived from a regression analysis based on data from entry documents filed with the U.S. Customs Bureau and are updated every five years.

H.2 Air passenger fares

11.109 The U.S. BLS publishes both balance of payments (import/export) and international (inbound/outbound) indices for international air passenger fares (U.S. BLS, 2008b). The import air passenger fares index measures changes in fares paid to foreign carriers by U.S. residents for international travel. The export air passenger fares index measures changes in fares paid to U.S. carriers by foreign residents for international travel.

11.110 For the international services indices, the U.S. carrier air passenger index measures changes in fares paid to U.S carriers for international travel regardless of the residency of the passengers. The foreign carrier air passenger index measures changes in fares paid to foreign carriers for trips flown between the United States and foreign countries regardless of the residency of the passengers.

11.111 All of the air passenger fares indices use data from the U.S. Department of Transportation as the sampling frame. This data set includes passenger counts, revenues, airports of origin and destination, and fare classes.

11.112 Monthly price data are obtained from an electronic reservations system that is widely used in the industry. The price-determining characteristics for an air passenger fare include the airports of origin and destination, the carrier name, the fare class (coach, business, or first class), and the fare type (one-way or round-trip), fare basis code, purchase requirements and restrictions, and routing code. Transactions for frequent flyer tickets and tickets sold by consolidators are not currently available from the repricing source so these are excluded from the index.

11.113 The revenue weights are derived from the passenger count data collected by the U.S. Department of Transportation and are updated every five years.

H.3 Crude oil tanker freight services

11.114 Crude oil tanker freight services was one of the first international service indices published by the U.S. BLS. Crude oil tanker freight consists of bulk crude oil shipments measured in barrels (U.S. BLS, 2008c). Because the index measures price changes for ocean transportation from port to port, all ground transportation and port services are excluded from the price.

11.115 The international services definition is used, so the index measures changes in the rates paid for the transportation of crude oil loaded from foreign countries and shipped to the United States on tanker vessels regardless of the nationality of the shipper or the vessel operator/owner. Currently the U.S. BLS produces the index only for inbound shipments.

11.116 As is the case for the import crude petroleum index (described in Section D), data for crude oil tanker freight are collected by the U.S. Department of Energy, so the whole universe of reported transactions is available. The index uses prices calculated as the weighted average of transactions sharing trade routes for the month. Each average price uses only those transactions from the same region of origin and delivery, such as all transactions from the Middle East to the U.S. Gulf of Mexico. Individual transactions are weighted together by quantity of barrels and price per barrel.

11.117 The trade volume weights for use in aggregating the index are derived from a regression analysis of one year of tanker transactions, and are updated every five years.

H.4 Ocean liner freight

11.118 The U.S. BLS publishes an index for inbound ocean liner freight that reflects changes in rates paid for the transportation of freight from foreign countries into the United States on ocean liner vessels regardless of the nationality of the shipper or the vessel operator/owner (U.S. BLS, 2008d). It should be noted that ocean liner vessels operate on regular schedules whereas tramp and tanker ocean shipping vessels operate on irregular schedules negotiated by the shipper and the ship owner.

11.119 The sampling frame for the inbound ocean liner freight index comes from the U.S. Maritime Administration. This data set includes date of entry, U.S. port of entry, foreign port of origin, vessel name, shipping weight, operator, service type, commodity type, and customs value of the shipment. A sample of company routes is probabilistically selected.

11.120 Price quotes are collected from responding ocean liner freight operators. The price-determining characteristics include the service route, the commodity type, the container size, the type of rate (service contract or tariff), and any applicable surcharges. All commissions, discounts, and surcharges are excluded to arrive at a net freight rate. The shipping rates include only port-to-port transport, so all ground transportation and port services are excluded from the price.

11.121 The revenue weights are derived from a regression analysis of data from the U.S. Maritime Administration and are updated every five years.

H.5 Travel and tourism

11.122 The U.S. BLS publishes an export travel and tourism index measuring price changes for lodging, food and beverage, entertainment, local transportation, gifts and souvenirs, and other goods and services purchased in the United States by foreign visitors (U.S. BLS, 2008e). Expenditures by foreign visitors traveling to the United States for business, education, or medical treatment are excluded from this index. The indices for export travel and tourism are published according to the visitor's country of residence. The index is published with a one-month lag owing to the availability of pricing data.

11.123 Items to be priced for the index are sampled from a subset of CPI data selected on the basis of expenditures by foreign visitors to selected U.S. metropolitan travel destinations across the broad categories of lodging, food and beverage, entertainment, local transportation, gifts and souvenirs, and other expenditures. The specific transactions selected adhere to CPI's sampling methodology.

11.124 Pricing data for the export travel and tourism index are obtained monthly from the U.S. BLS CPI. The pricing data are a combination of CPI basic indices and item prices. Price data from the CPI can be used because foreigners usually pay the same prices for these goods and services as U.S. residents. The CPI basic indices are not seasonally adjusted and include taxes. The item prices also include taxes.

11.125 Prices for tour packages are excluded from this index due to data limitations. Prices for international airfare also are excluded from this index because these price changes are measured by other U.S. import/export price indices.

11.126 The trade weights used in the index reflect spending by foreign travelers as collected by the in-flight survey data collected by the Office of Travel and Tourism Industries of the Department of Commerce, Statistics Canada, and the Mexican Ministry of Tourism.

H.6 Other areas for future research

11.127 Indices for export education could be calculated from secondary source data from the College Board for tuition, fees, and room and board paid by foreign students at U.S. colleges and universities. Because the College Board publishes new data every year, both prices and weights would need to be updated on an annual basis. Normally these prices are set for the year at institutions of higher education, so annual repricing is sufficient. Although secondary source data may be readily available for higher education, it is very difficult to measure output and quality change for this service.

11.128 Other important examples of internationally traded services include the broad category of financial services and royalties and license fees.

I. Pricing Issues of Importance in International Trade

11.129 Many issues arise in the calculation of XMPIs that pose difficulties to statistical agencies. This section outlines the importance of tracking country of origin and destination as a part of the commodity description, how to handle duties associated with the transaction, which exchange rates to use, how to interpret prices for intra-company trade, and on which price basis to collect the price data.

I.1 Country of origin or destination

11.130 The country of origin for imports and the country of destination for exports might be a price-determining characteristic if the market is segmented geographically. Therefore, it is important to collect this information along with the price quote each period from the respondent. If country of origin or destination is a price-determining characteristic, only price series with a consistent country of origin or destination should be used in the calculation of the XMPI.

11.131 For imports, the country of origin is defined as the geographic location of the company that held title to the commodity when it was exported to the importing country. It should be noted that the country of origin may or may not be different from the country of manufacture. For example, if a commodity was produced in Country A, sold to a company in Country B, and then exported to the importing Country C, then the country of origin should be reported as Country B. In contrast, if the commodity was produced in Country A and merely shipped through Country B on its way to the importing Country C, then Country A is the country of origin.

11.132 When the transaction is between related parties as described below in Section H.3, it may be difficult to determine which party held title to the commodity when it was exported to the importing country. If that is the case, the country of manufacture can be used as the country of origin.

11.133 Often a commodity is imported from more than one country with no price difference. In that case a group of countries, such as "Brussels and Luxembourg," should be recorded and tracked. Likewise if the commodity is imported from a larger number of countries, record the smallest geographic area that includes all the possible exporting countries, such as the European Union (EU) or Europe, as the country of origin.

11.134 As was the case for imports, if the commodity is exported to a number of countries, either list the countries of destination or record the smallest geographic area that includes all the possible importing countries, such as EU or non-EU. However, if an exporter sells at the same price regardless of the destination country, then the country of destination can be recorded as "world."

11.135 If the country of origin or destination is tracked as part of the commodity description, the review of price changes described in Chapter 10, Section D, can include a check on whether a flagged price change might be due to a change in the country of origin or destination.

11.136 In addition, XMPIs calculated by country of origin or destination can be a useful series to publish when they are important determinants of overall price trends. Vachris (1992) explained how these indices can be calculated using a subset of the data already collected for XMPIs for the United States. Before producing these indices, the statistical agency must establish whether or not trade with the countries or regions is consistent with regard to both volume and composition over time so that price data can be collected. Owing to the smaller set of data, it may be necessary to implement the trade volume weights at a higher level of aggregation than the regular XMPIs. Otherwise the methodology remains the same. Examples of series published by U.S. BLS include imports from industrialized nations versus other nations, imports from regions such as the European Union and the Pacific Rim, and imports from specific major trading partners such as Canada.

I.2 Duties

11.137 Many prices used for international transactions include duties. Whether to include duties in the measurement of XMPIs depends on the purpose of the indices. If the XMPIs are intended to measure changes in the cost of living or competitiveness, then duties should be included because they are part of the cost.

11.138 Most XMPIs, however, are primarily intended for use in the deflation of the foreign sector of the national accounts, so the procedures for handling duties should follow the Commission of the European Communities and others (2008) *System of National Accounts 2008* (*2008 SNA*), as discussed in Chapters 4 and 15 of this *Manual*.

11.139 If a respondent provides a price that includes a duty, then the statistical agency must also obtain enough information to remove the duty from the price used in the index calculation. For example, if the amount of the duty is available in a fixed amount per some specified unit of the commodity, then it is easy to subtract this amount from the price each period. However, sometimes ad valorem duties (duties that are a percentage of the transaction value) are levied. In the ad valorem case, the respondent would need to provide the statistical agency with the actual or an estimate of the duty paid each period so that the duty can be removed from the price reported.

I.3 Currency conversion

11.140 XMPIs measure price changes faced by the domestic buyers and sellers. Therefore any prices reported in foreign currencies must be converted to the national currency using an exchange rate. According to the *SNA 2008* (paragraph 3.136), the data should be converted using the exchange rate prevailing on the date of the transaction, and if that is not available then an average exchange rate for the shortest period possible may be used for the conversion. For example, the U.S. BLS price reference period is the first of the month, so it uses the average exchange rate for the previous month. Note that there are usually at least two different exchange rates for any given date, namely, the buy rate and the sell rate. To exclude the service charge represented by the difference between these rates, the midpoint between these two rates should be used. Likewise, if the rate includes a forward exchange cover, that should be removed. The exchange rates are usually not provided by the respondents, but rather are collected each period by the statistical agency from official exchange rate publications.

11.141 Some countries (currently Japan and formerly the United States) also publish XMPIs expressed in foreign currency. These indices are useful for analyzing the competitiveness of industries in a country over time because they measure price changes from the perspective of the foreign buyer (for exports) or seller (for imports). A methodology for producing indices in foreign currencies is found in Alterman, Johnson, and Goth (1987) and is summarized below.

11.142 To calculate a foreign currency price index, the statistical agency must first calculate a nominal average exchange rate index, as shown in equation (11.9) (these indices can also be published separately). Then the foreign currency price index can be calculated by multiplying the XMPI for a product category by the nominal average exchange rate index (equation (11.10)). A real average exchange rate index (equation (11.11)) may also be calculated by deflating the foreign currency price index data with CPI data from the foreign countries represented in the index.

$$\mathrm{AERI}^{n}_{y,t} = 100 \prod_{i=1}^{n}\left(ER_i^t \big/ ER_i^0\right)^{W_i^t}, \qquad (11.9)$$

$$\mathrm{NFCPI}^{n}_{y,t} = \mathrm{XMPI} * \mathrm{AERI}^{n}_{y,t}, \qquad (11.10)$$

$$\mathrm{RFCPI}^{n}_{y,t} = \mathrm{NFCPI}^{n}_{y,t} \Big/ \prod_{i=1}^{n}(CPI_i^t)^{W_i^t}, \qquad (11.11)$$

where:

AERI = nominal average exchange rate index;

NFCPI = nominal foreign currency price index;

RFCPI = real foreign currency price index;

ER_i^t/ER_i^0 = foreign currency per domestic currency exchange rate relative for country i in period t relative to the base period 0;

XMPI = import or export price index;

CPI_i^t = consumer price index for country i in period t;

W_i^t = normalized unilateral (export or import) trade weight of country i in commodity category y;

y = commodity category for which the index is calculated;

t = index reference period;

i = a particular country; and

n = total number of countries.

11.143 These indices are useful for analyzing changes in price competitiveness because they show the price trends from the perspective of the foreign buyer (for exports) and seller (for imports).

I.4. Intra-company transfer prices

11.144 Much international trade involves transactions between related parties. For example, transactions may occur between parent companies and their wholly or partially owned subsidiaries, between a licensor and a licensee, or between two companies involved in a joint venture. Because these related parties are maximizing joint profits, the intra-company transfer price might be set in order to minimize the tax burden of profit generated in the different countries or to minimize the duty paid on the transaction. Therefore, prices for these intra-company transfers may not be based upon market values. For a more detailed presentation of the theoretical and practical implications of intra-company transfers, see

Chapter 19. A practical approach to incorporating these prices into XMPIs is summarized below.

11.145 For the purpose of calculating XMPIs it is desirable to obtain "arm's-length" prices wherever possible—that is, the prices that would be used if the parties were unrelated. The Organization for Economic Cooperation and Development (OECD) Transfer Pricing Guidelines (OECD, 2001) outlines procedures for firms to use for tax and customs declarations. In order to apply the arm's-length principle, the intra-company transaction must be compared to a transaction between unrelated parties.

11.146 Several factors affect the comparability between transactions types.[3] First, the characteristics of the commodities traded must be similar in terms of quality. In addition, the functions included in the transaction must be similar. Likewise, the contractual terms concerning the responsibilities and risks incurred by the parties in both transactions must be the same. Both transactions should have been made under similar economic circumstances to ensure that the price difference is not due to different market conditions. And, finally, a comparison of business strategies of the parties must be done because sometimes prices are set to achieve a certain market penetration or expansion.

11.147 All of the factors outlined above may result in differences between the intra-company transfer price and the price used by unrelated parties. These differences must be adjusted for when estimating the arm's-length transfer price. In practice, due to the inexact nature of estimating arm's length prices, the use of an arm's-length range is acceptable.[4]

11.148 The traditional methods of estimating an arm's-length price for tax and customs purposes include the comparable uncontrolled price (CUP) method, the resale price method, and the cost plus method. Under the CUP method, the intra-company transfer (controlled) price is compared to a similar transaction between unrelated parties (uncontrolled). The uncontrolled transaction may be between the same establishment and an unrelated one (internal comparison) or between two different establishments (external comparison). For example, establishment A's intra-company transfer price might be internally compared with a similar transaction between establishment A and unrelated establishment B. If such an internal comparison transaction cannot be found, establishment A's intra-company transfer price might be externally compared with a similar transaction between the unrelated establishments B and C in the same commodity category.

11.149 The resale price method involves collecting the price of the commodity once it is resold to an unrelated establishment and then subtracting the normally earned resale price margin to arrive at the arm's-length price. The resale price margin may be derived by comparing the transaction to similar ones between unrelated parties (either internal or external as described in the CUP method). While the resale price method uses downstream transactions to estimate an arm's-length price, the cost plus method looks upstream and begins with the expenses incurred by the seller and adding a profit markup to it.

11.150 Diewert, Alterman, and Eden (2005) applied the OECD Guidelines to the specific case of obtaining prices for XMPIs and recommends the following ranking of methods for determining an arm's-length transaction. The preferred method is to obtain an internally comparable price (as described above in the CUP method). The second choice is to use an externally referenced comparable price, that is, a secondary source for the data such as a published commodity exchange market price. The third choice would be to obtain an externally comparable price (as described above in the CUP method). The fourth choice involves using downstream or upstream prices. There is an important difference between the use of downstream or upstream prices for XMPIs and the resale price and cost plus methods for estimating an arm's-length price for tax and customs purposes. Because the purpose of the XMPI is to measure price *changes* as opposed to *levels*, the downstream or upstream price can be used in the index without adjusting for profit margins if the downstream or upstream price is expected to have the same trend as the transaction price. As a last resort, the actual customs declared transfer price could be used, only if it is market-based.

11.151 Most countries exclude all non-arm's-length prices from their XMPIs. However, if the XMPIs are to be used as deflators in the national accounts, an argument can be made that because non-arm's-length transfer prices are reflected in the trade volume data, any declared transfer price should be used in the XMPI. The U.S. BLS includes both arm's-length and non-market-based intra-company transfer prices in the calculation of XMPIs owing to the large amount of intra-company trade.

[3]OECD (2001), Chapter I, paragraphs 1.19–1.35.
[4]OECD (2001), Chapter I, paragraph 1.45.

I.5 Price basis

11.152 The valuation principles used, outlined in principle to meet different user needs in Chapter 4, are often dictated by the data source. This is especially so when data are from customs sources. Export and import prices are reported using varying bases depending on which party is responsible for and pays for freight charges, loading charges, insurance, and other costs associated with shipping the goods. Establishment surveys allow for more flexibility to derive the component elements of any valuation including transportation margins. Price bases may in practice be defined according to the trade in terms developed by the International Chamber of Commerce (ICC) (Incoterm)[5] or parties may develop their own descriptions of the responsibilities of seller and buyer in international trade in non-ICC terms.

11.153 A common valuation system is to value at the market value of the goods at the point of uniform valuation (the customs frontier of the economy from which they are exported), that is, the goods are valued free on board (f.o.b.) at that frontier. In Chapters 4 and 15, the valuation principles for alternative uses of XMPIs are outlined. In practice a common f.o.b. valuation may be used. The f.o.b. value includes that of the goods and of the related distributive services up to that point, including the cost of loading onto a carrier for onward transportation, where appropriate. The ICC defines the f.o.b. price basis to mean that the goods are considered delivered when they pass the ship's rail, and the seller must bear the cost and risk of clearing the goods for export and placing them on board the ship. The risk of loss of or damage to the goods is transferred to the buyer when the goods pass the ship's rail (off the dock and placed on the ship). In some contracts this term is written as f.o.b. vessel. Another valuation is FAS (free alongside ship). Whereas a price basis of FAS requires the buyer to pay the cost of loading the goods, a price basis of f.o.b. requires this of the seller. The seller must clear the goods for export in both f.o.b. and FAS terms.

11.154 In practice, many countries (such as Australia, the United Kingdom, and Japan) designate a preferred price basis of f.o.b. domestic port for exports. For imports, side price bases of f.o.b. foreign port (e.g., in Australia) or cost, insurance, and freight (c.i.f.) (e.g., in Japan) is designated as preferred. The ICC defines a price basis of c.i.f. to mean that the seller must pay the costs, insurance, and freight necessary to bring the goods to the named port of destination and that the goods are considered delivered when they pass the ship's rail. Either the buyer or the seller may be responsible for port charges in the import country. The buyer is responsible for customs duties, import country taxes, and delivery charges from the port of importation to the final destination.

11.155 Note that the price bases described above are used only with sea and inland waterway transport. International trade also may involve ground or air transportation and there are ICC Incoterms to cover these arrangements. For example, a price basis of free carrier may be used with any mode of transport but was specifically designed to meet the requirements of multimodal transport, such as container or roll-on, roll-off traffic by trailers and ferries. It is based on the same principle as f.o.b., except the seller fulfills its obligations when the goods are delivered to the custody of the carrier (or person named by the buyer, such as a freight forwarder) at the named place. The place of delivery specified determines which party is responsible for paying to load or unload the goods. The seller is responsible for loading if the named place is the seller's premises; if the named place is not the seller's premises, then the seller is not responsible for unloading.

11.156 When countries designate a preferred price basis, prices not reported according to these preferred bases are deleted from the index calculation unless it is possible to collect enough information to convert the reported price into the preferred price basis. For example, to convert an FAS price into an f.o.b. price, the statistical agency would need to collect the cost of clearing the goods for export and the cost of loading the goods onto the ship. For imports, if the price is reported as cost and freight instead of c.i.f., then the price of the insurance would need to be added to the transactions price before it could be used.

11.157 These types of price adjustments, however, may not be necessary. Given that the purpose of the XMPI is to measure price *changes* as opposed to *levels*, it is possible to mix price bases within an elementary index calculation (as the U.S. BLS does) as long as each individual price series maintains a constant price basis over time.

11.158 One exception to the acceptance of different price bases is that any price basis that includes duties must also be accompanied by enough information to remove the duties from the price. More information about the inclusion/exclusion of duties is found in Section B.

[5]For detailed information on buyer's and seller's responsibilities and costs for any of the Incoterms 2000, see the International Chamber of Commerce website at www.iccwbo.org.

12. Errors and Bias in XMPIs

A. Introduction

12.1 A number of sources of error and bias have been discussed in the preceding chapters and will be discussed again in subsequent chapters. The purpose of this chapter is to briefly summarize such sources to provide a readily accessible overview. Both conceptual and practical issues are covered. To be aware of the limitations of export and import price indices (XMPIs), it is necessary to consider what data are required, how they are to be collected, and how they are to be used to obtain overall summary measures of price changes. The production of XMPIs is not a trivial task, and any program of improvement must match the estimated cost of a potential improvement in accuracy against the likely gain. In some instances, one may have to take into account the user requirements necessary to meet specific needs or engender more faith in the index, in spite of the relatively limited gains in accuracy matched against their cost.

12.2 The nature and extent of any errors and bias and the practicalities of reducing them are, to a large extent, dictated by the different data sources used for XMPI compilation. Chapter 2 raised concerns about the bias that can arise from the use of unit value indices based on administrative customs data as proxies for price indices. Chapter 5 outlined the use establishment price surveys, administrative data, and world commodity prices as possible source data for XMPIs. Sections C to G of this chapter outline possible types of error and bias for establishment price survey data, and then revisit these possible types of errors and bias for administrative data in Section H and world commodity price indices in Section I.

12.3 The distinction between errors and bias is first considered in Section B. In sampling, for example, the nature of the sample design (e.g., the use of cutoff sampling; see Chapter 6) may bias the sample toward larger establishments whose average item price changes are below the average of all establishments. In contrast, an unrepresentative sample with disproportionate larger establishments may be selected by chance and similarly include item prices that are, on average, below those of all establishments. This is *error* because it is equally likely that a sample might have been selected whose average price change was, on average, above those of all establishments.

12.4 Figure 12.1 outlines the potential sources of error and bias in XMPIs. The discussion of bias and errors first requires consideration of the conceptual framework on which XMPIs are to be based and the XMPIs' related use(s). This will govern a number of issues, including the decision as to the coverage or domain of the index and choice of formula. Errors and bias may arise if the coverage, valuation, and choice of the sampling unit fail to meet a conceptual need; this is discussed in Section C.

12.5 Section D examines the sources of errors and bias in the sampling of transactions. The sampling of item prices for XMPIs can be undertaken in three stages: the sampling and selection of products to be priced, the sampling of establishments producing the selected products, and the subsequent sampling of items produced (or purchased) by those establishments. Bias may arise if the products, establishments, or items selected exhibit, on average, unusual price changes. Such bias may be due to omissions in the sampling frame or a biased selection from the frame.

12.6 However, sampling error, as discussed above and in Chapter 6, can arise even if the selection is random—from an unbiased sampling frame. Sampling error will increase as the sample size decreases and as the variance of prices increases. Sampling error arises simply because estimated XMPIs are based on samples, as opposed to a complete enumeration of the populations involved. Probabilistic estimates of sampling error can be undertaken. The errors and biases discussed in Section D are for the sample on initiation.

12.7 Section E is concerned with what happens to sampling errors and bias in subsequent matched price comparisons. Once the samples of products, establishments, and their items have been selected, they will become increasingly out of date, unrepresentative, and quite possibly biased, as time progresses. The extent and nature of any such bias will vary from industry to industry. The effect of these dynamic changes in the universe of products, establishments, and the items produced on the static, fixed sample is the subject of Section E. Sample rotation will act to refresh the sample of items, while rebasing may serve to initiate a new sample of items, products, and establishments. Following rebasing, establishments will close, and items will no longer be produced, on a temporary or permanent basis. Sample augmentation and replacement attempt to forestall some of the degradation of the sample of products and establishments, although replacement occurs only when an establishment is missing. Sample augmentation tries to bring into the sample new major products and establishments. It is a more complicated process because the weighting structure of the industry or index has to be changed (Chapter 9). When item prices are missing, the sampling of items may become unrepresentative. Imputations can be used, but they do nothing to replace the sample. In fact, they lower the effective sample size, thereby increasing sampling error. Alternatively, comparable replacement items or replacements with appropriate quality adjustments may be introduced. As for new goods providing a substantively different service, the aforementioned difficulties of including new establishments extend to new goods, which are often neglected until rebasing. Even then, their inclusion is quite problematic (Chapter 9).

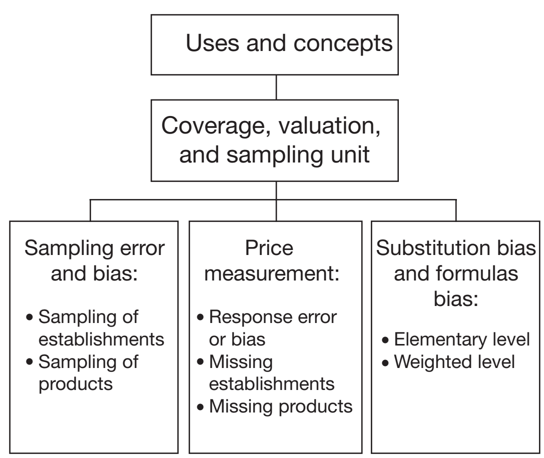
12.8 The discussion above has been concerned with how missing establishments and items may bias or increase the error in sampling. But the normal price collection procedure based on the matched-models method may have errors and bias as a result of the prices collected and recorded being different from those transacted. Such response errors and biases, along with those arising from the methods of treating temporarily and permanently missing items and goods, are outlined in Section F as errors and bias in price measurement. Section F is concerned with deficiencies in methods of replacing missing establishments and items so that the matched-models method can continue, while Section E is concerned with the effect of such missing establishments and items on the efficacy of the sampling procedure.

12.9 The final types of bias, outlined in Section G, are formula and substitution bias. Different formulas, as shown in Chapters 16 through 18, have different axiomatic properties and formulas that do not meet some of these desirable properties can be considered to be biased. For example, if a formula, when measuring price changes between periods 0 and t, has the property that it will exceed unity when the prices are unchanged between these two periods, then it can be deemed to be biased upwards. Formulas can also be shown to be "exact" if they replicate different behavioral assumptions. For example, producers may substitute production (purchases) toward (away from) items whose price changes are above average. A Laspeyres fixed-weight formula would not be exact because it requires that the basket be held fixed in period 0—Leontief preferences. At the higher level of aggregation, where weights are used, substitution effects are shown to be included in superlative formulas but excluded in the traditional Laspeyres formula (Chapter 16). Similar considerations are discussed in Chapter 21 for the lower level. Whether it is desirable to include such effects depends on the concepts of the index adopted. A pure fixed-base period concept would exclude such effects, while an economic cost-of-living approach (Chapters 18 and 20) would include them. The concepts in Figure 12.1 can be used to address definitional issues such as coverage, valuation, and sampling as well as price measurement issues such as quality adjustment and the inclusion of new goods and establishments.

12.10 It is worthwhile to list the main sources of errors and bias:

(1) Inappropriate coverage and valuation (Sections C and I);

Figure 12.1. Outline of Sources of Error and Bias



(2) Sampling error and bias, including
  (a) Sample design on initiation (Section D), and
  (b) Effect of missing items and establishments on sampling error (Section E);

(3) Matched price measurement (Sections F and H), including
  (a) Response error/bias,
  (b) Quality adjustment bias,
  (c) New goods bias, and
  (d) New establishments bias; and

(4) Substitution bias and formula bias (Sections G and H), including
  (a) Upper-level item and establishment substitution,
  (b) Lower-level item and establishment substitution, and
  (c) Formula bias.

12.11 It is not possible to judge which sources are the most serious. In some countries and industries, the increasing differentiation of items and rate of technological change make it difficult to maintain a sizable, representative matched sample, and issues of quality adjustment and the use of chained or hedonic indices might be appropriate. In other countries, a limited coverage of economic sectors in which the XMPI is used might be the major concern. Inadequacies in the sampling frame of establishments might also be a concern.

12.12 Although the above outline of sources of errors and bias has been written in the context of surveys of establishments as the source data, much is applicable to administrative sources data and world commodity price data, the subject of Sections H and I, respectively.

12.13 There is no extensive literature on the nature and extent of errors and bias in XMPI measurement. Berndt, Griliches, and Rosett (1993) is a notable exception, though their concern was with producer price indices (PPIs). However, there is substantial literature on errors and bias in consumer price index (CPI) measurement, and Diewert (1998a and 2002c) and Obst (2000) provided a review and extensive reference list. Much of this literature includes problem areas that apply to XMPIs as well as CPIs.

B. Errors and Bias

12.14 In this section, a distinction is made between *error* and *bias*. The distinction is more appropriate to the discussion of sampling, although the same framework is shown to apply to nonsampling errors and bias. Yet an error or bias can also be discussed in terms of how an existing measure corresponds to some true concept of a XMPI and will vary depending on the concept advocated, which in turn will depend on the use(s) required of the measure. These issues are discussed in turn.

B.1 Sampling error and bias

12.15 Consider the collection of a random *sample* of prices whose overall population average (arithmetic mean) is μ.[1] The *estimator* is the method used for estimating μ from sample data. An appropriate estimator for μ is the mean of a sample drawn using a random design. An *estimate* is the value obtained using a specific sample and method of estimation, let us say $\bar{x}_1$, the sample mean. The population mean μ, for example, may be 20, but the arithmetic mean from a random sample of a given size drawn in a specific way may be 19. This difference is *error,* not *bias;* it is simply by chance that a random sample was drawn with, on average, below-average prices. If an infinite number of samples were drawn using sufficiently large samples, the average of the $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_\infty$ sample means would in principle equal μ. The estimator is said to be *unbiased;* if it is not, it is called *biased*. The error caused by $\bar{x}_1$ being different from $\mu = 20$ did not arise from any systematic under- or overestimation in the way the sample was drawn and the average calculated. If an infinite number of such estimates were drawn and summarized, no error would be found, the estimator not being biased and the discrepancy being part of the usual expected sampling error.[2]

12.16 It should be stressed that any one sample may give an inaccurate result, even though the method used to draw the sample and calculate the estimate is, on average, unbiased. Improvements in the design of the sample, increases in the sample size, and less variability in the prices (more detailed price specifications for the price basis) will lead to less error, and the extent of such improvements in terms of the sample's probable accuracy is measurable. Note that the accuracy of

[1]For simplicity, the discussion is in terms of prices and not price changes.

[2]This is *sampling error,* which can be estimated as the differences between upper and lower bounds of a given probability, more usually known as *confidence intervals*. Methods and principles for calculating such bounds are explained in Cochran (1977), Singh and Mangat (1996), and most introductory statistical texts. Moser and Kalton (1971) provided a good account of the different types of errors and their distinction.

such estimates is measured in principle by confidence intervals, that is, probabilistic bounds in which μ is likely to fall. Smaller bounds at a given probability are considered to be more precise estimates. It is in the interest of statistical agencies to design their sample and use estimators in a way that leads to more precise estimates.

12.17 The calculation of such intervals is more straightforward if based on a measure of the variance of price relatives from a random sample design at all stages. The sampling of prices for XMPIs generally do not involve the sampling of establishments and items using probabilistic methods, at least at each stage. Judgmental and cutoff methods are often considered to be more feasible and less resource intensive. Yet it is feasible to develop partial (conditional) measures in which only a single source of variability (stage of sampling) is quantified (see Balk and Kerston, 1986, for a CPI example). Alternative methods for nonprobability samples were discussed in Särndal, Swensson, and Wretman (1992).

12.18 *Efficiency* gains (smaller sampling errors) may be achieved for a given sample size and population variance by using better *sample designs* (methods of selecting the sample) as outlined in Chapter 6. Yet it may be that the actual selection probabilities deviate from those specified in the sample design. Errors arising from such deviations are called *selection errors.*

12.19 Whereas an unbiased estimator may give imprecise results, especially if small samples are used, a biased estimator may give quite precise results. Consider the sampling from only large establishments. Suppose such prices were, on average, less than μ, but assume these major establishments covered a substantial share of the revenue of the industry concerned; then the mean of the estimates from all such possible samples $\underline{m}$ may be quite close to μ, even if smaller establishments had different prices. However, the difference between m and μ would be of a systematic and generally predictable nature. On average, m would exceed μ, the bias[3] being $(\mu - m)$.

B.2 Nonsampling error and bias

12.20 The above framework for distinguishing between errors and biases is also pertinent to nonsampling error. If, for example, the prices of items are incorrectly recorded, a response error results. If such errors are *unsystematic*, then prices are overrecorded in some instances but, counterbalancing this, underrecorded in others. Overall, errors in one direction should cancel out those in the other, and the net error, on average, will be expected to be small. If, however, the establishments selected and kept in the sample are older and produce at higher (quality-adjusted) prices than their newer, high-technology equivalent establishments, then there is a systematic bias. The results are biased in the sense that if an infinite number of similar random samples of older establishments were taken from the population of establishments, the average or expected value of the results would differ from the true population average, and this difference would be the bias. The distinction is important. Increasing the sample size of a biased sample, of older establishments for example, when samples are rebased, reduces the error but not the bias.

12.21 This distinction between errors and bias is for the purpose of *estimation.* When one uses the results from a sample to estimate a population parameter, both error and bias affect the accuracy of the results. Yet there is also a distinction in the statistical literature between types of errors according to their *source:* sampling versus nonsampling (response, nonresponse, processing, etc.) error. Although they are both described as errors, the distinction remains that if their magnitude cannot be estimated from the sample itself, they are biases, and some estimate of μ is required to measure them. If they can be estimated from the sample, they are errors.

B.3 Concepts of a true or good index

12.22 The discussion of errors and bias so far has been in terms of estimating μ as if it were the required measure. This has served the purpose of distinguishing between errors and bias. However, much of this *Manual* has been concerned with the choice of an appropriate index number formula. It is now necessary to consider bias in terms of the difference between the index number formula and methods used to calculate the XMPIs, and some concept of a true index. In Chapter 18, true theoretical indices will be defined from economic theory. The question is, if producers behave as optimizers and switch production toward products with relatively high price increases, which would be the appropriate formula to use? The result will be seen to be a number of *superlative* index number formulas. They do not include the Laspeyres index or the Young index (Chapter 16), which give

[3]Because μ is not known, estimates of sampling error are usually made; they are but one component of the variability of prices around μ.

unduly low weights to products with relatively high price increases, because no account is taken of substitution effects (see Chapter 18). For industries whose establishments substitute production toward items and products with above-average price increases, Laspeyres is biased downward. An understanding of bias thus requires a concept of a true index. Superlative indices make more reasonable assumptions about the optimizing (substitution) behavior of economic agents than do the Leontief preferences implicit in Laspeyres.

12.23 A good index number formula can be defined by axiomatic criteria as outlined in Chapter 17. The Young and Carli indices, for example, were argued to be biased upward because they failed the time reversal test: The product of the indices between periods 0 and 1 and periods 1 and 0 exceeded unity.

12.24 In XMPI number theory and practice there are quite different conceptual approaches. On the one hand, there is the revenue-maximizing concept of the output export price index and on the other the cost minimizing assumptions of the input import price index. Against these, there is the fixed-basket approach.[4] An index based on the latter approach would not suffer, in the strictest sense of the concept, from the biases of substitution (formula) or new goods because the concept is one of measuring the prices of a *fixed* basket of goods. However, it may be argued on the grounds of representativeness that the baskets should be updated and substitution effects incorporated.

C. Use, Coverage, and Valuation

12.25 Errors and biases can arise from the inappropriate use of XMPIs, regardless of the methodology used to compile them. Because price changes can vary considerably from product to product, the value of the price index will depend partly on which products or items are included in the index and how the item prices are determined (Chapter 16, Section B.1). In Chapter 4, different uses of XMPIs were mentioned and aligned with different domains and valuation principles. Thus, the discussion of errors and biases starts with a need to decide whether the coverage and valuation practices are appropriate for the purposes required.

12.26 The bias and errors of XMPIs depend on the uses to which they are put so, first, some major uses are noted, and the domain or coverage of the indices is considered. Second, the principles of valuation are reiterated.

C.1 Uses and coverage

12.27 The input import price index (MPI) is a short-term indicator of inflation. It tracks potential inflation as price pressure builds up for imported goods and services that enter the factory gate. Input PPIs can be disaggregated into MPI and the input price index from resident producers. Similarly, output PPIs can be disaggregated into the export price index (XPI) and the PPI for outputs to domestic markets. Together such indices provide analytical tools to help identify the sources of inflationary pressure and the effects of changes in the term of trade. There may be deficiencies in the coverage of XMPIs. If, for example, an XPI is restricted to the industrial sector, this is a source of error when examining the overall inflation, if price changes for other sectors differ from the industrial sector.

12.28 The XMPIs may be biased when used for national accounts deflation. First, their coverage may be inadequate yet still be used by national accountants. For example, if only manufacturing XMPIs are used to deflate industrial trade flows, and price changes from the missing, quarrying, and construction sectors differ in the aggregate from those of manufacturing, there is a bias. The undercoverage bias is in the use of the index, not necessarily in its construction, although statistical agencies should be sensitive to the needs of users. Second, overcoverage bias means some elements are included in the survey that do not belong to the target population. The bias surfaces if their price changes differ on aggregation from the included ones. Third, the classification of activities for the XMPI should be at an appropriately low level of disaggregation, and the system of classification should be the same as that required for the production and trade accounts under the Commission of the European Communities and others (2008) *System of National Accounts 2008* (*SNA 2008*).

12.29 Highly aggregated XMPIs are used for the macroeconomic analysis of inflation. Certain industries or products with volatile price changes may be excluded. Such indices may be excluded because they introduce substantial sampling error into the aggregate indices, and their exclusion helps with the identification of any underlying "core" trend.

[4]A discussion of the debate can be found in Triplett (2001).

12.30 The preceding discussion has considered the coverage or domain of the index in terms of the activities included. However, such issues also may extend to the geographic scope. The exclusion of establishments in rural areas, for example, may lead to bias if their price changes differ from those in urban areas. Such issues are considered in Sections D and E.

C.2 Valuation

12.31 In Chapters 4, 15, and 20, valuation issues are outlined. The appropriate valuation principle to adopt depended on the use of the XMPI. A resident producer's perspective was identified as appropriate for the use of terms of trade measurement, transmission of inflation, and productivity analysis. The valuation of an (output) XPI from a resident producer's perspective involves valuing output at basic prices with any value-added tax or similar deductible tax, invoiced to the purchaser, excluded. Such tax revenues go to the government and should be excluded because they are not part of the establishment's receipts. Transport charges and trade margins invoiced separately by the producer should also be excluded. An (input) MPI from a resident producer's perspective should value intermediate inputs with nondeductible taxes included, because they are part of the actual costs paid by the establishment. For MPIs, changes in the tax procedures—say, owing to a switch to import duties on intermediate inputs, for example—can lead to bias. In such instances, ex-tax or ex-duty indices might be produced. In any event, it is necessary to ensure that establishments treat indirect taxes in a consistently appropriate way, especially when such tax rates fluctuate. A nonresident producer's perspective requires that exports are valued at purchaser's prices and imports at basic prices. However, there are pros and cons to the consistent use of purchaser's prices and basic prices for supply and use tables in volume terms, as discussed in Chapter 4 and Chapter 15 of the *2008 SNA*. Even with these principles established and a purpose in mind, if data are from customs sources, imports are valued c.i.f. (i.e., they include the cost of carriage, insurance, and freight) at the point of entry into the importing economy and exports are valued f.o.b (free on board) at the point of exit from the exporter's economy. As outlined in Chapter 4, neither of these may meet the analytical needs for the purposes outlined above.

D. Sampling Error and Bias on Initiation

12.32 In Chapter 6, appropriate approaches to sample design were outlined. The starting point for potential bias in sample design is an inadequate sampling frame. It is one of the most pernicious sources of error because the inadequacies of a sampling frame are not immediately apparent to users. Yet a sampling frame biased to particular sizes of establishments or industrial sectors will yield a biased sample irrespective of the probity of the sample selection. Because sampling is generally in three stages—the sampling of products, establishments producing those products, and the items within such establishments—a sampling frame is required for products, establishments, and items within establishments. The sampling frame for items within establishments relies on the establishment producing data on the revenues, quantities, and prices (or revenue per unit of output) for the items produced. Bias may arise here, perhaps because some components produced are priced and recorded elsewhere, say at the head office. The sampling of establishments requires a procedure to identify exporters and importers. In some countries, licensing bodies or trade associations may provide systematic files. Alternatively, if the customs documents identify the exporter or importer and this information can then be matched to a directory of businesses, then this is likely to provide a more reliable frame, which may also take account of the value of exports and imports for stratification. In some categories, such as imports of cigarettes, there may be many, possibly illegal, importers not covered by such identification procedures. The sampling frame for products can be taken from disaggregated Harmonized System of Commodity Classification Coding (HS) categories customized to the needs of the country. Exports, unlike imports, are usually concentrated in a relatively small number of classification categories particular to the economy of the country concerned. It should be kept in mind that even when purposive sampling is used, there is an implicit frame from which the respondent selects items. It should be clear to the respondent what the frame should be.

12.33 The selection of the sample of establishments from the sampling frame should be random or, failing that, purposive. In the latter case, the aim should be to include major items whose price changes are likely to represent overall price changes. Chapter 6 provided a fairly detailed account of the principles and practice of sample selection and the biases that may ensue. The distinction has already been drawn between *bias* and sampling *error,* and the possibility has been raised that unbiased selection will be accompanied by estimates with substantial error, owing to high variability in the price (change) data and relatively low sample sizes.

E. Sampling Error and Bias: The Dynamic Universe

12.34 Chapters 8 and 9 also considered sampling issues. Under the matched-models method, prices will be missing in a period if the item is temporarily or permanently out of production. If overall imputations are used to replace the missing prices, the sample size is being effectively reduced and the sampling error increased. In a comparison between prices in period 0 and period *t,* imputation procedures (Chapter 8) ignore the prices in period 0 of items whose prices are missing in period *t*. If such old prices of items no longer produced differ from other prices in period 0, there is a bias owing to their exclusion. Similarly, new items produced after period 0, and thus not part of the matched sample, are ignored; if their prices in period *t* differ, on average, from the prices of matched items in period *t,* there is a bias. Sampling error and bias, therefore, may arise owing to the exclusion of prices introduced after initiation and dropped when they go missing. The need for the sample to be representative of the dynamic universe is over and above any errors and bias in the sample design on initiation.

12.35 As the sample of establishments and items deteriorates, rebasing the index—to update the weights and sample of establishments and items or the rotation of sample items—becomes increasingly desirable. However, rebasing involves costly and irregular procedures and, for some industries, more immediate steps are required. Rebasing and sample rotation are used to improve the sampling of establishments and items. Strategies for dealing with missing establishments and missing prices also have an effect on the sampling of establishments and items. Such strategies involve introducing *replacement* establishments and items that replenish the sample in a more limited way than do rebasing and sample rotation. Quality adjustments to prices are required if the replacement establishment or item differs from the missing ones, although this is the concern of price measurement bias in Section F. New establishments and goods may also need to be incorporated into the sample to avoid sampling bias. There is a need in such instances to *augment* the sample. Such augmentation may require a change to the weighting system and, as discussed in Chapter 9, should be undertaken only when the incorporation of major new establishments or goods is considered necessary. Thus, bias in sampling owing to differences between the dynamic universe and the static one on initiation may, to some extent, be militated by sample replacement and augmentation (Chapter 9).

12.36 Circumstances may arise in which there is a serious sample deterioration owing to missing items as differentiated items rapidly turn over. In such cases, hedonic indices or chaining based on resampling the universe each month was advised in Chapter 8, Section G.

F. Price Measurement: Response Error and Bias, Quality Change, and New Goods

F.1 Response error and bias

12.37 Errors may happen if the reporting or recording of prices is inaccurate. If the errors occur in a systematic manner, there will be bias. The item descriptions that define the price basis should be as tightly specified as is reasonably possible, so that the prices of like items are compared with like. Allowing newer models to be automatically considered comparable in quality introduces an upward bias if quality is improving. Similar considerations apply to improvements in the quality of the service that accompanies an item. The period to which the prices relate should be clearly indicated, especially where prices vary over the month in question and some average price is required (Chapter 7). Errors in valuation can be reduced by clear statements of the basis of valuation and discussions with respondents if the valuation principles of their accounting systems differ from the valuation required. This is of particular importance when there are changes in tax rates or systems. Diagnostic checks for extremely unusual price changes should be part of an automated quality assurance system, and extreme values should be checked with the respondent and not automatically deleted. For example, a change in the currency in which the price is quoted, or from transaction to list price, can lead to unusual price changes that, on detection, can be remedied. Price collectors should visit establishments on initiation and then periodically as part of a quality assurance auditing program (see Chapter 13).

12.38 For unit value indices from customs data, information on the customs declarations may be inaccurate (Chapter 2). The information on quantities shipped may be in the wrong units or simply missing. Such data should be validated by recourse to the establishment or some independent data source such as world commodity prices.

F.2 Quality change bias

12.39 Bias can arise, as discussed in Section E, because newly introduced items do not form part of the matched sample, and their (quality-adjusted) prices may differ from those in the matched sample. This *sampling* bias from items of improved quality and new goods was the subject of Section E. It was also noted that statistical agencies may deplete the sample by using imputation or use replacements to replenish the sample. The concern here is with the validity of such approaches for price measurement, not their effects on sampling bias.

12.40 In Chapter 8, a host of explicit and implicit quality adjustment methods were outlined. From a practical perspective, the quality change problem involves trying to measure price changes for a product that exhibited a quality change. The old item is no longer produced, but a replacement one or alternative is there. If the effect of quality on price is, on average, either improving or deteriorating, then a bias will result if the items are compared as if they were of the same quality, when they are not. An explicit quality adjustment may be made to the price of either of the items to make them comparable. A number of methods for such explicit adjustments were outlined in Chapter 8, including expert judgment, quantity adjustment, option and production costs, and hedonic price adjustments. If the adjustments are inappropriate, there will be errors and, if the adjustments are inappropriate in a systematic direction, there will be a bias. For example, using quantity adjustments to price very small lots of output, for which customers pay more per unit for their convenience, would yield a biased estimate of the price adjustment owing to quality change (Chapter 8, Section E.2).

12.41 There are also implicit approaches to quality adjustment. These include the overlap approach, overall and targeted mean imputation, class mean imputation, comparable substitution, spliced to show no price change, and the carry-forward approach. Imputations, whereby the price changes of missing items are assumed to be the same as those of the overall sample or some targeted group of items, are widely used. Yet such approaches increase error through the drop in effective sample size and may lead to bias if the items being dropped are at stages in their life cycle where their pricing differs from that of other items. Such bias is usually taken to overestimate price changes (Chapter 8, Section D).

12.42 The choice of appropriate quality adjustment procedure was argued in Chapter 8 to vary among industries to meet their particular features. There are some products, such as consumer durables, materials, and high-technology electronic products, in which the quality change is believed to be significant. If such products have a significant weight in the index, overall bias may arise if such changes are ignored or the effects of quality change on price are mismeasured. Whichever of the methods are used, an assumption is being made about the extent to which any price change taking place is due to quality; bias will ensue if the assumption is not valid.

12.43 The problem of quality change is more serious for unit value indices based on customs data. In this case any changes in the mix of the qualities shipped between periods leads to bias in the index. For example, two models of a make of automobile may be imported from a company. If the proportion shipped of the more expensive model increases between the two periods compared, the unit value index will overstate the price change (Chapter 2).

F.3 New-goods bias

12.44 Over time, new goods (and services) will appear. These may be quite different from what is currently produced. An index that does not adequately allow for the effect on prices of new goods may be biased. Introducing new goods into an index is problematic. First, there will be no data on weights. Second, there is no base-period price to compare the new price with. Even if the new good is linked into the index, there is no (reservation) price in the period preceding its introduction to compare with its price on introduction. Including the new good on rebasing will miss the price changes in the product's initial period of introduction, and it is in such periods that the unusual price changes are expected if the new good delivers something better for a given or lower price. Similar considerations apply to new establishments (Section G.4). New-goods and new-establishment bias is assumed to overstate price changes, on average.

F.4 Temporarily missing bias

12.45 The availability of some items, such as fruits and vegetables imported for canning, fluctuates with the seasons. A number of methods are available to impute such prices during their missing periods. Bias has been shown to arise if inappropriate imputation approaches are used. Indeed, if seasonal items constitute a large proportion of revenue, it is difficult to give meaning to month-on-month indices, although comparisons between a month and its counterpart in the next year will generally be meaningful (see Chapter 23).

G. Substitution Bias

12.46 Given the domain of an index and the valuation principles, the value of the revenue accruing to the establishment can be compared over two periods, say, 0 and 1. It is shown in Chapter 16 that the change in such values between periods 0 and 1 can be broken down into two components: the overall price and overall quantity change. An index number formula is required to provide an overall, summary measure of the price change. In practice, this may be undertaken in two stages. At the higher level, a weighted average of price changes (or change in the weighted average of prices) is compiled with information on revenues (quantities) serving as weights. At the lower level, the summary index number formulas do not use revenue or quantity weight, and use only price information to measure the elementary aggregate indices of average price changes (or changes in average prices). It is recognized that in many industries weights are available at the lowest item level and the lower-level problem does not exist. Five approaches are used in Chapters 16 through 18 to consider an appropriate formula at the higher level; a similar analysis is undertaken for lower-level elementary aggregate indices in Chapter 21.

G.1 Upper-level substitution bias

12.47 Different formulas for aggregation have different properties. At the upper-weighted level, substantial research from the axiomatic, stochastic, Divisia, fixed-base, and economic approach has led to an understanding of the bias implicit in particular formulas. Chapters 16 through 18 discuss such bias in some detail. The Laspeyres formula is generally considered to be used for XMPI construction for the practical reason that it does not require any current-period quantity information. It is also recognized that the appropriate deflator that generates estimates of output at constant prices is a Paasche one (Chapter 19). Thus, if estimates of a series of output (including that aimed at the export market) at constant prices is required, the use of the Laspeyres deflator will result in bias. In practice, for a price comparison between periods 0 and *t*, period 0 revenue weights are not available and a Young index is used, which weights period 0 to *t* price changes by an earlier period *b* revenue shares. Chapter 16 finds this index to be biased. Superlative index number formulas, in particular the Fisher and Törnqvist indices, have good axiomatic properties and can also be justified using the fixed-base, stochastic, and economic approaches. Indeed, Laspeyres can be shown to suffer from *substitution* bias if particular patterns of economic behavior are assumed. For example, producers may seek to maximize revenue from a given technology, and inputs may shift production to items with above-average relative price increases. The Laspeyres formula, in holding quantities constant in the base period, does not incorporate such effects in its weighting, giving unduly low weights to items with above-average price increases. Therefore, it suffers from a downward bias. It can be similarly argued that the fixed, current-period weighted Paasche index suffers from an upward bias, whereas the Fisher index is a symmetric mean of the two, falling within these bounds. Calculating the Fisher index retrospectively on a trailing basis will give insights into upper-level substitution bias.

12.48 The extent of the bias depends on the extent of the substitution effect. The Laspeyres index is appropriate if there is no substitution. However, the economic model assumes that the technology of production is the same for the two periods being compared. If, for example, the factory changes its technology to produce the same item at a lower cost, the assumptions that dictate the nature and extent of the bias break down.

G.2 Lower-level substitution bias

12.49 In some countries or industries, elementary aggregate indices at the lower level of aggregation that use only price information are constructed. The prices are aggregated over what should be the same item. In practice, however, item specifications may be quite loose and the price variation between items being aggregated quite substantial.

12.50 The axiomatic (test), stochastic, and economic approaches can also be applied to the choice of formula on this lower level (Chapter 21). The Carli index, as an arithmetic mean of price changes, performed badly on axiomatic grounds and is not recommended. The Dutot index, as a ratio of arithmetic means, was shown to be influenced by the units of measurements used for price changes and is not advised when items do not meet tight quality specifications. The Jevons index, as the geometric mean of price changes (and equivalently, the ratio of geometric means of prices), performed well when tested by the axiomatic approach and, assuming sampling proportionate to stable—unitary elasticity—revenue share, incorporates a substitution effect. But the substitution effect goes the opposite way of that predicted by the aforementioned economic model. For a consumer price index, the economic model is one of consumers substituting *away* from items with above-average price increases so more of the relatively cheaper items are purchased. Constant revenue shares is an appropriate assumption in these circumstances. However, producer theory requires producers to substitute *toward* items with above-average price increases, and assumptions of equal revenues are not tenable. Chapter 21 details a number of formulas with quite different properties. However, it concludes that because the axiomatic, stochastic, fixed-base, and economic approaches, as noted in Section G.1, find superlative index numbers to be superior (Chapters 16 to 18), a more appropriate course of action is to attempt to use such formulas at the lower level, rather than replicate their effects using only price data, a task to which they are unsuited. Respondents should be asked to provide revenue or quantity data as well as price data. Failing that, appropriate index number formulas are advocated depending on the expected nature of the substitution bias.

G.3 New-establishment (substitution) bias

12.51 The need to include new establishments in the sample has already been referred to in Section E. Products produced by new establishments may not only have different (usually lower) prices, arguing for their inclusion in the sample, but gain increasing acceptance as purchasers substitute goods from new establishments for goods from old establishments. Their exclusion may overstate price changes owing to substitution bias.

G.4 Formulas bias

12.52 Formulas bias can arise because of the properties of the index number formulas used. For example, as explained in Chapter 21, the Carli index number formula, the unweighted arithmetic mean of price ratios, fails the time reversal test. The product of a Carli index between periods 0 and 1, and between periods 1 and 0, exceeds unity; it is upward biased. In Chapter 17 desirable properties or *axioms* are outlined for index numbers, and the failure of some of these properties is such that the use of such formulas can be regarded as leading to bias. In some cases, a formula that is otherwise generally accepted may be biased when applied to data with particular characteristics. For example, Chapters 20 and 23 show how a chain price index can drift when applied to volatile data such as highly seasonal data.

H. Administrative Data

12.53 Chapter 5 outlined alternative data sources including the use of administrative data, that is, data taken from other administrative sources derived to serve other purposes, but useful for XMPI compilation. The primary source for such data is customs data. Such data must also be evaluated in terms of its sources of errors and bias. In particular, as discussed in Chapter 2, there is likely to be a problem with making matched price comparisons owing to the reliance on unit values. XMPIs compiled from customs data use unit values as the "prices" of goods exported and imported. The unit value is calculated for detailed headings of the HS. Yet there is scope for variation in the mix of what is imported and exported in each month, and the product descriptions on customs documents may be insufficient to even identify the change in mix. For example, a change in the quantity of higher-priced items in a load from one month to the next may be misrepresented as an increase in prices. Of critical importance is the variation in prices over the items in the customs specification. If there is no variation, the goods are homogeneous and unit values are appropriate. Many times, this will not be the case.

12.54 There may be problems of coverage. Much of the internationally traded service industry and transactions between countries (or trading zones within countries) with whom the country has a free-trade agreements may not be covered**.** Such coverage problems primarily arise if administrative customs data are used as the sole source of data. However, such coverage problems can be ameliorated by use of establishment-based survey data, though sampling and nonresponse issues may arise.

12.55 There is also a problem with regard to valuation. Customs data value imports usually with freight and insurance included as they cross the importing country's frontier, but exports, while recording values f.o.b., are valued as they leave the exporting country rather than when they arrive at the importing country. The requirement, as outlined in Chapters 4 and 15, is for the valuation to take place when there is a change of ownership with the precise principle for valuation being related to the use of the XMPIs and whether a resident's or nonresident's perspective is adopted.

12.56 Even if the contents of each load are homogeneous within an HS, there may be response bias, owing to a tendency in product groups for which tariff duties are high to undervalue specific imports and exports.

Many high-value goods are built to customers' specifications and will not be found in subsequent months for accurate price comparisons.

I. World Commodity Prices

12.57 Data on import and export price changes may be drawn from world commodity price indices on the assumption that the market for such prices is sufficiently competitive that world price changes, at least, are similar to those of the compiling country. Thus, it may be recognized that prices in the importing country are above world trade levels, say owing to higher transport costs from the countries of origin. What is assumed, however, is that price changes will be similar.

12.58 The use of world commodity price indices as a data source has some appeal. They are generally readily and freely available, and this is usually on a timely basis. They are more suitable for imports (and exports) of homogeneous commodities purchased (produced) by many small importers (or resident producers). A number of potential sources of error should be noted.

12.59 First is the coverage of the data. The compiling organization, usually a trade association, should be contacted to ensure that the data cover all exports and imports and, if not, whether the price indices relating to the excluded trade flows are likely to differ from the included ones and, more particularly, whether they are likely to differ from the price changes of the compiling country. It may be the case that separate indices are available for separate regions (or even countries); if so, commodity price indices for regions with similar expected price changes should be sought.

12.60 Second is the definition of the price series. It is necessary to establish that the commodity, or the mix of commodities, covered by the index relates to those traded by the compiling country. Different commodities may be imported or exported at different levels of quality and have different physical or financial "treatments" to, say, preserve or insure their contents during a possibly long transportation. Transport costs can be estimated to identify whether, for example, they are likely to have different changes for imports to the compiling country than the world at large and, if so, adjustments can be made.

12.61 Third is the aggregation procedure. In Chapters 16 to 21, alternative index number formulas for aggregating price changes are outlined. Some of the formulas are shown to have better properties than others, and some are shown to have demonstrable bias. It is important to ascertain whether the formulas used for world trade price indices are subject to such biases.

12.62 Fourth, there is a specific concern here that world commodity price indices act as import price indices.

12.63 A more reliable data source than world trade price indices will usually be the resident distributor, because such data require no assumption of invariance of price changes for the country purchasing (producing) the commodities. However, if there are problems obtaining data from such distributors, or if the data are in doubt, world commodities indices, at the very least, provide a benchmark against which to evaluate data from domestic businesses.

12.64 The sections above are merely an overview of the sources of error and bias and are not intended to be either exhaustive or detailed accounts. The detail is to be found in the individual chapters concerned. The multiplicity of such sources argues for statistical agencies undertaking audits of their strengths and weaknesses and formulating strategies to counter such errors and bias in a cost-effective manner.

13. Organization and Management

A. Introduction

13.1 Export and import price indices (XMPIs) are used for many purposes by government, business, labor, universities, and other kinds of organizations, as well as by members of the general public. Accuracy and reliability are paramount for a statistic as important as XMPIs. Whether XMPIs are used as a deflator of national account values, an indicator of inflation, in escalation of contracts, in assessing developments in the global economy such as exchange rate fluctuations, or in other economic analyses, the process of producing XMPIs needs to be carefully planned and executed.

13.2 Individual circumstances vary to such an extent that this *Manual* cannot be too prescriptive about timetables or critical path analysis of all the steps involved. However, the description in this chapter provides an outline of the kinds of activities that should result from a detailed examination of the logistics of the whole periodic operation of compiling the index.

13.3 The following guidance seeks to present some options in the organization of data collection and estimation. The examples given are based on experience and provide an indication of goals a country may seek. In recognizing these options, this chapter, which talks about organization and management of XMPI processes, covers the relationships between staff, who may be stationed at regional offices in large countries, and XMPI staff at the central office, covering the work carried out in the central office, the flow of information among each part of the organization, and related activities for coordinating collection and processing data. Because of the size, frequency, cost, and complexity of the collection of prices as the basis of the index, in some countries not all these operations and relationships will be appropriate. In some cases, countries may resort to using data already available such as customs data or data from other price programs such as the producer price index (PPI) or consumer price index (CPI).

13.4 The complexity of the organizational structure is necessarily correlated with the scope of the effort, in terms of resources as well as the volume of data collection. In particular, XMPI programs tend to be smaller than other price programs in any given country. Ideally a program would comprise a multifaceted staff with a variety of skills, including economists, statisticians, system analysts/programmers, field staff, and clerical support. In the case of XMPIs, however, resource limitations often may require individuals to perform more than one task.

13.5 Although the mission of XMPIs is singular—to produce a set of price indices—the process for producing these data can be broken out into several discrete activities. These activities can be separated into five major phases: sampling, initiation, repricing, estimation (compilation), and publication. Each phase, however, may require a number of different technical skills as well as a separate but coordinated management structure.

13.6 As explained in Chapter 2, XMPIs may be based on unit value indices from customs data, establishment price surveys, or a hybrid mixture of the two, along with other source data. Organizational issues relating to sampling, initiation, and repricing are primarily the concern of establishment price survey data, though issues in managing data quality and verification remain relevant.

13.7 Sampling is normally the first component of developing XMPIs. This usually entails the selection of companies and products (or product areas) that will be in the market basket. This is also the stage at which a determination is made on how much publication detail the sample can support. During the initiation phase, company cooperation is elicited and, if necessary, a final determination is made regarding the actual items that are to be repriced on a regular basis. The next phase is repricing. This is a recurring process whereby at a regular interval, usually monthly or quarterly, price data are collected from a respondent. Collection methods vary and a program may make use of more than one mode. Current modes include mail, fax, web collection, and personal visits. This component would also include a review, manual and/or automated, of the data in order to ensure its compliance with economy theory. Estimation is the process whereby the detailed price information is aggregated in order to produce a price index. Finally, the data are ready for publication, either as hard copy printouts or, as more and more often is the case, released on the organization website. These stages will be considered in turn after a general consideration of organizational and management issues.

B. Organizational Structure and Resource Management

B.1 Structuring

13.8 In practice, most countries have devoted comparatively little, if any, resources to XMPIs. Whereas some agencies such as the U.S. Bureau of Labor Statistics dedicate more than 130 employees to the production of its export and import price data, the vast majority of countries make do with only a comparative handful of employees. This smaller staff resourcing is usually due to two reasons: first, the higher priority usually given to the production of CPIs and PPIs; and second, the common practice of using readily available administrative data to construct XMPIs based on unit value indices. Consequently, XMPIs may be compiled by a unit devoted solely to XMPIs, or they might be part of a unit responsible for publishing data on the value of exports and imports. However, there are synergies with PPI compilation, especially if the primary data source for price relatives is establishment-based surveys. In such a case, the XMPI unit may well be part of a larger staff who also have responsibility for producing PPIs.

13.9 Like any data collection program, the organizational structure supporting XMPIs can be either vertical or horizontal, with various possibilities in between. A fully vertical structure would be one in which all of the staff on the program work in the same office and work only on XMPIs. A fully horizontal structure would be one in which no one in the program works solely on XMPIs. For example, a unified field data collection staff may be responsible for collecting data for every program in the agency, while a single office of statisticians may similarly be responsible for developing samples for every survey in the agency.

13.10 The key determinants for assessing the organizational structure would be availability of resources, similarity of methodologies, complexity of program, and efficiencies gained by combining or splitting functions. Usually there are advantages in combining resources between programs. In general, the more alike a processing component is between two programs, the better it is to share resources. In the case of XMPIs this usually will lead to the program sharing resources with a PPI program, where many of the concepts and data sources are similar. Consequently, much of the organization and management conventions associated with XMPIs are similar to those found in the construction of PPIs.

B.2 Managing the process

13.11 When the unit responsible for producing XMPIs is small and wholly contained, managerial issues also tend to be small because one supervisor may be responsible for every aspect of the data collection and index estimation process. Even in a small program, however, it is important to have a staff large enough to allow for the development of individual expertise in a selected area, especially in those aspects that demonstrate differences from PPI concepts and methodologies. For example, sources of data available for sampling and weighting tend to be unique in XMPIs. Also, the specialized need to handle pricing in intrafirm trade and exchange rates are unique or especially critical to these series. Finally, and perhaps most important, in addition to having general expertise in statistics, economics, and computer programming, the organization should have as much specialized expertise by industry (or product category) as resources permit. One of the key difficulties in producing accurate and up-to-date price indices is an awareness of product (and other trends) in the underlying economy. Experienced staff who specialize in sectors of the economy are best positioned to spot these trends and help make additions or adjustments in the data in order to keep the indices current and relevant.

13.12 In a larger organization, often the organization has separate functional units in which staff are split by their skills or work assignments. In order to ensure that the individual unit works together, one option is to develop a matrix structure in which different areas of the program are managed by staff from different offices. In some cases the matrix structure might be delineated by processing component; in other examples matrices have been divided into research, development, and production components. For example, in a horizontal structure staff may be categorized by their functional skills: sampling, initiation, data collection, verification, and estimation, and also by whether they are working primarily on XMPIs or PPIs. Matrices also tend to be more flexible and can be more readily rearranged as circumstances warrant.

B.3 Staffing

13.13 The appropriate mix of skills in any given XMPI office may be a function of how the statistical process is organized in a given country, as well as the approach of the nation's data collection effort (e.g., administrative vs. economic). Nonetheless, certain disciplines and tasks are vital to ensuring that each component of the index process is properly and efficiently carried out, and that the published data are accurate, timely, and relevant. A large organization may have the resources to hire separate staff for each of these disciplines. However, frequently XMPI offices are quite small. Consequently, employees are likely to have multiple responsibilities (e.g., data collection, sampling, and index review); statistical offices may need to employ more adaptable and more educated people, not only because they will have to perform more than one task, but also because they are more likely to have to serve as backups to their colleagues.

13.14 Input from economists is necessary to the process of producing XMPIs. These indices have a multitude of uses, and proper construction of these indices requires economists who are familiar with their purposes and can ensure that the construction of the indices meets the needs of the key functions of the data. In addition to competence with some of the standard subdisciplines in economics, micro- and macroeconomics as well as international economics (both finance and trade), some staff economists should also be experts in price index number theory itself, a more specialized discipline in economics. Knowledge of specific industries and technical expertise can also be critical in order to properly understand the inputs and outputs as well as the industrial organization of a given sector of the economy. Statistical offices should organize and meet frequently with users committees.

13.15 Statisticians play a key role in the sampling process as well as the index estimation process. Because no establishment-based price program can afford to collect data on the entire set of items in the universe of goods and services being measured, the program requires a method for determining a subsample of both importers and exporters and individual products that should be included in the pricing market basket. This determination process involves balancing a number of factors, such as the number of respondents that the program budget can support, the number of items necessary to sustain an accurate index, the need to trade off respondent burden with the need to include large traders in the market basket, the life of the market basket as well as the need to regularly update the mix of items in the basket, the variation in deterioration rates by industry, and the need to select only items that are traded and repriceable on an ongoing basis. Owing to the complexity of sampling, statisticians should have a good grounding in sampling theory and practice.

13.16 The past few decades have seen a growing role for computer specialists in the production of economic data. Even smaller countries rely more and more on computers for collecting and reviewing the data and calculating XMPIs. And with the increasing need for producing more indices more rapidly, this trend is expected to continue. With many series such as XMPIs collecting price data on thousands of items each month, the use of electronic data processing is central to a program's ability to produce outputs in a timely manner. The computer programmers should be skilled at evaluating user requirements and determining the proper database design and programming languages necessary to support the program. Particularly in smaller organizations, where a substantial part of the data collection and processing may be handled with comparatively simple software such as spreadsheets, it has become more common to hire technical staff who have very strong programming skills in addition to their subject expertise (usually economics or statistics).

13.17 Often the most difficult roles to fill in a price index program are those on the front lines: the data collectors who usually meet the respondents face to face. Many, if not most, price collection programs are voluntary. Thus, this staff must play many roles, including part economist, part salesperson, in the sense that they must convince establishments of the need to provide data. Literature should be prepared in advance on the usefulness of XMPIs, confidentiality of responses, and need for continuity of price information from a representative sample of establishments. Legislative backing is also useful. The major role for the data collector in XMPI programs is to initiate respondents into the program and often to collect the first set of data from the sampled respondent. This process can be quite complicated, particularly when soliciting a large company, and the data collector must be sufficiently trained to handle a multitude of different issues that may be raised by the respondent. The data collector is also frequently called upon to make split-second decisions during an initiation session, when time may be extremely limited. Note that the "salesperson" part of this job tends to be much more important in programs such as the PPI and XMPIs, where the respondents are usually responsible for supplying data at initiation as well as periodic pricing. In contrast, in consumer price programs, the periodic pricing can normally be done by data collectors from the statistical office.

B.3.1 Outsourcing

13.18 One of the decisions facing any statistical agency carrying out a price collection program is whether to use in-house staff or tender the collection to an external organization. For example, another part of the agency, another government department that specializes in surveys, or a private market research company could perform this function.

13.19 The nature of the price collection and the distribution and profile of statistical staff may help determine whether the collection is suitable for contracting out to another agency or even the private sector. Where price collection is continuous, involves complicated decision making (such as quality adjustment), or is collected from a small number of businesses, it may be advantageous to keep the collection in-house. However, if the collection takes place over just a few days per month from a large number of businesses, is relatively straightforward, and involves only routine or simple decision making (perhaps selecting from a list of codes), then contracting out to another agency can be considered. For example, if the statistical office does not have a dedicated data collection staff, it could contract with other agencies such as commerce, industry, and agriculture to collect XMPI data. Another possibility could be a private research company, if there are market research companies with suitable skills in the country. The statistical office must also take confidentiality requirements into consideration when contracting with another agency to guarantee that there are no breaches in confidentiality. This may involve national statistical laws that address the issue of data collection by contractors and enforcement of penalties for breaking confidentiality requirements.

13.20 Contracting out price collection can lead to lower costs. When price collection is carried out using electronic methods such as computer-assisted telephone interviews or computer-assisted personal interviews, the responsibility for purchasing and maintaining data-capture devices may also be transferred to the contractor. Contracting out may also allow statistical office staff to spend more time analyzing data rather than collecting it. By separating the roles of data collector and data checker, statistical staff can feel more comfortable questioning the validity of price data and the accuracy of collected data can be linked to the performance of the contractor through performance measures, which drive incentives payments (and penalties if targets are not achieved).

13.21 The same considerations may be used when deciding whether the survey division or XMPI staff should conduct the price collection. Usually, some mixed mode of operation will be in place. Staff from the survey division may handle straightforward and routine price collection; more complicated and specialized industries such as chemicals and semiconductors will require price collection by specialists, whether from XMPI staff or consultants of a statistical office.

B.3.2 Training

13.22 Effective training will motivate staff and equip them to deliver a good-quality index. At its simplest, training will give a background understanding of the nature and uses of the index and its compilation. Training and development take many different forms and may include

- Tutoring by the line manager or supervisor,
- Attending an induction course or reading a manual, and
- Accompanying an experienced price collector.

13.23 A written training plan can be useful in identifying training and development needs against the backdrop of the organization's goals and targets. It can also identify the resources required to deliver these needs and evaluate whether training has been delivered effectively and objectives have been met.

13.24 The statistical office should have a general training program for staff working on the price programs. There are six basic components of such a program:

- Fundamental (basic) training must provide general information on how to collect data, code data elements, review and edit basic price data, and compile collected data to produce indices. In addition, the training should impart to staff information on the purposes and uses of the collected prices.

- Price collectors need to be trained specifically in field procedures, including relations with businesses, a selection and definition of a valid price, special rules for certain individual price transactions (including seasonal price transactions), how to complete initiation forms, and, where appropriate, how to use computers. Compilers of the index will need to be trained specifically on the validation, consistency checking, and calculation of centrally collected indices; weighting procedures and how to aggregate prices; and treatment of seasonal price transactions and special procedures relating to some sections of the index.
- Training, especially for newer staff, may also entail a period of ongoing monitoring of performance.
- The program should highlight the need for continuous training of staff at all levels. Staff should provide feedback at all levels—from respondents to data collectors and from supervisors to staff. There should be regularly scheduled meetings between staff and supervisors at all levels to assess the program and identify current and potential problems.
- Statistical offices also need to provide professional training for staff in computer technology, economics, statistics, and even psychology (for dealing effectively with respondents).
- Regularly scheduled meetings for all or selected staff members can provide a time to discuss the strengths and weakness of the program and plans/changes for the future. This is particularly true when the program undertakes major changes in index methods, such as rebasing with new weights and new sample designs.

B.4 Process improvement

13.25 Developing, maintaining, and updating clearly delineated program goals are crucial to the ongoing success and utility of a well-run program. This process must take into account all stakeholders, both internal and external, and assume realistic estimates of resource availability.

13.26 With the rapid spread of the Internet, the development of new electronic data gathering processes and the demand for new kinds of data (e.g., those measuring intangibles) have made the job of data collectors more difficult. The need for systematic process reviews may be especially critical for XMPI programs to deal with as the rapid globalization of the world's economy and the attendant shifts in production and consumption have made measurement more difficult.

13.27 XMPI programs should have a well-understood process in place for reviewing and revising goals and should include a process for assessing changes in international trade patterns to ensure outputs, methodology, and data processing methods and procedures are continually reviewed to meet changing needs.

13.28 Staff input is an essential part of continuous quality improvement. Staff may be invited to operational reviews where all team members have the opportunity to raise concerns and, where appropriate, tackle specific issues through individual or group efforts. In order to effect this, staff schedules should include an appropriate amount of time for process improvement.

13.29 A process for improving XMPIs can best take place in an environment where a systematic quality assurance program is in place. This includes not only introducing and monitoring quality controls on the production process itself, but also effective interaction with the user community in order to ensure that their needs are being met—see Section E for further discussion on quality management.

B.5 Processing components

13.30 Although the processing methods and data review used in constructing XMPIs are discussed in other chapters, especially Chapters 6, 7, and 14, there are several organizational and management aspects of each of these processes that should also be touched upon.

13.31 One question that has become increasingly important in the development and maintenance of each of the processing components is how to take best advantage of the efficiencies that can be gained via more effective use of computers. Ideally, an XMPI organization would have its own systems staff to develop and maintain an XMPI production system. In cases in which systems staff are shared with other programs, there should be sufficient flexibility to allow for systems staff to undertake specific XMPI developmental work. If staff are to be shared, then one essential ingredient is that the programs and programming languages should be standardized.

13.32 As a general rule of thumb, the managers of XMPI programs would like to have as much programming under their direct authority as possible. If the best way to achieve this goal is to use simple high-level languages such as EXCEL or SAS (which can be programmed by economists, statisticians, or others on the immediate staff), that may be an appropriate approach. XMPI compilation methods may need to be changed in the light of experience and new developments and it is essential for the XMPI unit to have, or call upon, the expertise to do so.

C. The Sampling Process

13.33 Because many countries have detailed records on all export, and especially import, shipments, very good and detailed data on what companies have been shipping what products across borders are often available. This source of data may, or may not, be collected by the same organization that constructs XMPIs. In either case, however, it is valuable for XMPI offices to make use of these records in order to draw a sample of both respondents and products to initiate and reprice its item selection. Sampling methods were outlined in Chapter 6.

13.34 Whether the data are collected by the same organization or a different one, it is necessary to constantly monitor the quality of the data and to ensure that the data are as up-to-date as possible. While the level of data detail normally available in these records is often quite good, the quality of data being collected for administrative purposes such as duties is often uneven. Thus the management process should include systematically working with the provider office in order to ensure that the data used to construct the sampling frame are high quality. In addition, it is also important that the staff working with developing the sample have access to feedback from those staff who have had direct contact with respondents during the data collection phase.

D. The Initiation Process

13.35 This process can be very labor intensive and normally involves XMPI staff either visiting or conducting a phone interview with individual sampled businesses to establish cooperation, stress the importance of the index, and receive basic information, such as the exact goods and services produced by the business, relative importance of transactions with various clients, and individuals to contact on a recurrent basis (see Chapter 7 for more details). During this initiation visit the respondent and XMPIs staff will normally identify the actual items that will be repriced on a regular basis. In some countries these operations may be conducted by telephone. The range and number of businesses visited and the types of goods and services priced will vary between countries (see Chapter 6).

13.36 Management of the initiation process may be very important as this is usually the process that is the greatest burden to respondents. Especially when a data collection program is voluntary, it is important to minimize the burden on the reporter. While the original sample universes may differ, frequently the PPI and XMPI market basket of respondents and items will overlap. This is true in part owing to the tendency of companies engaged in foreign trade to be concentrated. Consequently, it may be appropriate to use and manage the same staff to initiate respondents in both programs, Not only might this minimize the duplication of technical knowledge, but it can also serve to minimize burden and enable more efficient scheduling of contacts with an individual reporter.

13.37 Note that in some cases, the data collectors responsible for initiating new respondents into the process may also be responsible for repricing the data on a regular basis.

13.38 Although the precise method of current price collection will vary (see Chapter 7), each price collector may be responsible for the initiation and collection from a certain business or from certain types of businesses. This would enable the collector to specialize in certain subject areas of the index. In other cases, particularly in larger countries, price collectors may specialize by regions. Although price collection is usually done monthly or quarterly, the frequency can change if the prices for certain transactions change at known intervals. Items that are traded only in certain seasons may be collected only in specified months. In other cases respondents may indicate that owing to burden concerns, the data will need to be collected less frequently. Sometimes these prices may be collected directly by XMPI staff in the main office based on external information such as contact with other government offices or through the media. In any case, checks must be in place to ensure all price data are reported. (See Chapter 7, Section B, for information on timing and frequency of price collection.)

13.39 *Quality* is an important part of price collection; a high-quality price collection enables a statistical agency to have confidence in the index it produces and ensure that observed price changes are genuine and not the result of collector error. Procedures must be developed to ensure that a high standard of collection is maintained for every collection period. These procedures will form the basis of collector training and should be included in any training material developed for price collectors. Guidance should cover price index principles, organizational issues, and validation procedures for both technology and price collection methods.

13.40 *Transaction descriptions.* During the initiation phase of data collection, accurate price transaction descriptions are critical to ensuring price transaction continuity. Descriptions should be comprehensive to ensure that collectors or reporters can price the same transaction in each collection period. Collectors must record all information that uniquely defines the price transaction selected. So, for example, in price collection for production of clothes, color, size, and fabric composition must be specified to ensure that the same price transaction is priced each month (see Chapter 7 for details).

13.41 Accurate price transaction descriptions will assist the price collector, respondent, and XMPI staff in choosing a replacement for a price transaction that has been terminated and will also help to identify changes in quality. XMPI staff should be encouraged to spend some time, each collection period, going through reported descriptions to ensure that the correct price transactions are being priced. Collectors or respondents should also be encouraged to review their descriptions to ensure that they contain all the relevant information, and it may be useful to ask collectors occasionally to switch collections with another collector as part of a peer review.

13.42 *Continuity* is one of the most important principles of price collection. Because a price index measures price changes, the same price transaction must be priced every month so that a true picture of price changes is established. It is not possible to be prescriptive because the concept of equivalence will vary among countries, but for practical purposes a detailed description of the price transactions must be kept. Some guidelines may be drawn up by the statistical office's XMPI Head Office staff to cover different price transactions. All transaction specifications, and especially price-determining characteristics, must be obtained and monitored.

13.43 *Feedback.* When price collectors are used, they should be encouraged to give feedback to XMPI Head Office staff on their experiences. Collectors are a valuable source of information and often give good early feedback on changes in the different industries. Collectors can often warn of size or product changes before XMPI Head Office staff can derive this information from other sources, such as trade magazines or the business press. Collector feedback can form the basis of a collector newsletter and can support observed price movements and provide supplementary briefing material. Significant changes in price transactions within a business may require an additional visit by XMPI analysts to the business to update the price transaction descriptions.

13.44 *Auditing.* Because the routine of initiating respondents in the field may be the most critical and labor-intensive component of the entire process, it needs to be carefully planned and managed. Circumstances vary, and it is not appropriate to be too prescriptive. Some of the measures mentioned below may be irrelevant if XMPI analysts in the Head Office collect the prices centrally. However, it is vitally important to check that the information sent in is accurate and complete.

13.45 One way to monitor the work of price collectors is to employ auditors to occasionally accompany collectors during field collection—whether data are collected by phone or personal visits—or to carry out a retrospective check on data that have been collected.

E. The Repricing Process

13.46 The repricing process for XMPI is the single most complex process in constructing price indices as it consists of numerous steps including (1) determining the set of items to be collected in the current period; (2) notifying respondents (or data collectors) of the data they need to supply; (3) receiving the data from respondents; (4) verifying the data; (5) if data are incomplete, missing, or do not appear to be correct, taking remedial action; and (6) preparing the data for estimation. Because of the increasing desire from policymakers for more timely economic data, this process may have to occur in a very short time frame.

13.47 Data can be collected using a number of different methods, such as (1) an online web collection application, (2) e-mail, (3) fax, (4) phoned in, or (5) mailed in. Given advances in computer technology over years, this may be an area where management may want to focus its data processing expertise.

13.48 Although data collection via field staff is often necessary during the initiation phase, especially when a program is voluntary and a certain amount of convincing respondents to participate is needed, field collection via a personal visit is expensive and XMPI programs should try to avoid it during the periodic repricing.

13.49 In any voluntary data collection program, the single biggest issue facing management is nonresponse. In cases in which the organization must rely upon companies to supply the data, it is incumbent upon the program to ensure that there is in place a systematic process for collecting the data and for following up when the information has not been received.

13.50 Offering respondents a variety of modes for collecting data is one aid to garnering respondent cooperation. Reminders using e-mail and other points of contact may also prove to be effective. Because phone contacts are the most labor intensive, it may be more efficient to use clerical staff to make phone calls in cases where a simple checklist or computer-assisted telephone interview process can be developed.

13.51 The key for management is to develop a systematic method for monitoring response rates on a regular basis. Often a sudden change in response rates may indicate a problem somewhere in the data collection process.

13.52 In addition, for any number of reasons (such as volatility of trade, growing complexity of goods, and shorter product life cycle), the data that are collected during repricing may also require some type of follow-up. Consequently, a key to successfully managing this process is to obtain the data as fast and as clean (accurate) as possible, and to ensure there is an effective method for recontacting the respondent and amending the data as appropriate.

13.53 Collectors or respondents should report prices at similar times within each collection period. This is particularly important when pricing volatile price transactions with sharp fluctuations.

E.1 Data entry queries

13.54 Once the price data have been entered in the system, a series of *validation checks* may be run. In deciding which checks should be carried out, take into account the validation checks carried out in the field, whether by price collectors in the regional office, survey division officials in the main office, or XMPI analysts. When data entry is automated (e.g., collection via the web) then these checks can and should be built directly into the data collection application.

13.55 The range of tests carried out for all collection methods may include the following:

13.56 *Missing data*. The most common check is for data completeness. Often, a number of price factors besides the price are collected. These may include country of origin or destination, currency of invoicing, and discount structure. When a data element is missing, it should be flagged and reviewed.

13.57 In XMPIs, however, one of the major reasons for missing price data is attributable to the volatility of trade. Particularly in XMPIs, which collect data monthly, respondents will often indicate that there was no trade in the reference month. However, in a PPI, though the respondent may be able to estimate the price of his or her establishment's product, it is more difficult to do so for an importer. Consequently, it is very important for management to develop a consistent approach to handling data that are temporarily unavailable, because this can represent up to 20 percent or more of a given month's market basket (see Chapter 8 for the treatment of temporarily missing items).

13.58 *Specification change*. For any number of reasons, an item may have had a change in its specification. It may be as simple as a change in color or, as in the case of PCs, it may be significant, such as a more powerful microprocessor or larger hard drive. Any time there is a change in the specification, it should be reviewed by a product specialist in order to ensure that the price change from the first period to the second period is appropriately adjusted for any change in the quality of the good.

13.59 *Price change*. The price entered may be compared with the price for the same defined transaction in the same business in the previous month and queries raised where this is outside preset percentage limits. The latter may vary depending on the price transaction or group of price transactions and may be determined by looking at historical evidence of price variation. If there is no valid price for the previous month, for example, because the produced good was not traded that month, the check can be made against the price two or three months ago. The price may also be compared with other transactions conducted by the same business in the current month. Checks may also be put in place to reveal instances where prices have not been changed for an inordinate amount of time, because this may sometimes indicate instances where the data are not being properly updated.

13.60 *Maximum/minimum prices.* A query may be raised if the price entered exceeds a maximum or is below a minimum price for a group of goods or services of which the particular product is representative. The range may be derived from the validated maximum and minimum values observed for that price transaction in the previous month expanded by a standard scaling factor. This factor may vary between price transactions.

13.61 If computer-assisted techniques are used, these tests can be easily implemented to take place at the time of collection; otherwise, they will need to be conducted by XMPI Head Office staff as soon as possible after collection and before prices are processed on the main system. If a price given by a respondent fails the computer-assisted test, this should not result in collectors being unable to price the price transaction, but it should prompt them to check and confirm their entries and prompt for an explanatory comment.

13.62 Queries raised may be dealt with either by staff analysts at the XMPI Head Office or by the price collector or respondent contacted for resolution. For example, scrutiny of a form might show that a big price difference has arisen because the transaction priced was a new product replacing one that had been discontinued. In this case, there may be no need to raise a query with the price collector unless there is evidence to suggest that labeling the transaction as a "new product" is incorrect.

13.63 One noticeable difference between collecting data for a PPI versus collecting data for an XMPI is the role of transfer prices (transactions between related parties). Often, transfer prices are valued at cost instead of an arm's-length market price. Sometimes transfer prices may remain unchanged for what appears to be an inordinate length of time, while in other instances they may fluctuate wildly and for no apparent reason. Consequently, XMPI programs should track these data and take special measures in handling them (see Chapter 19).

E.2 Quality control

13.64 As in the initiation process, regular quality checking of the ongoing repricing data is critical to the ability to publish accurate price indices. Although the concepts associated with the quality control of repricing data are similar to those pertaining to the earlier initiation process, the specific type of audits and controls may differ. For example, whereas the initiation process usually involves a staff member from the collection organization in the documenting and verifying of the initial data, the reporting of the ongoing repricing data would normally be done by an employee of the importer or exporter. Regardless of how these data are reported, the information must be subject to a rigorous review. This would include such steps as

- Ensuring that the price reports are sent in when they are due. If not, it is necessary to find out the reason and take appropriate action;
- Confirming that the reports contain what they are supposed to contain—that is, that required fields have not been left blank, that numeric fields contain numbers, and that nonnumeric fields do not;
- Reviewing and editing each return. Substitutions may have to be made centrally or those made by the collectors may have to be approved. Unusual (or simply large) price changes may need to be queried. Transactions priced in multiple units or varying weights may have to be converted to price per standard unit. Missing prices must be dealt with according to standard rules relating to the cause; and
- Identifying and correcting errors introduced when keying the numbers into the computer or transcribing them onto worksheets.

13.65 As stated above, logical checks conducted in the field by an automated process can reduce the amount of checks and errors handled by XMPI Head Office staff.

13.66 Some of these tasks can be done by computer, others, manually. Therefore, no general suggestion can be made about the sequence of the work or about its division into different parts. Procedures should be in place to check that all documents, messages, or files are returned from the field so that price collectors can be contacted about missing returns. Initial checks should then be carried out to ensure that data are complete and correct. If any prices fail these checks, a query should be raised with the price collector for clarification. Because some of the checking may require reference back to the price collectors (or to their supervisors or respondents when direct-mail questionnaires are used), the timetable for producing the index must allow for this communication to take place.

13.67 Following the price data checks, a series of validation checks may be run. In deciding which checks should be carried out, account should be taken of the validation checks carried out in the field. For example, computers will increase the potential for validation at the time of price collection and reduce the need for detailed scrutiny at the XMPI Head Office. In addition, it would not be productive or cost-effective to repeat all the tests already carried out, except as a secondary audit or random check.

13.68 *Reports* (on paper or computer) should be generated routinely for most representative price transactions. Reports help the analyst pick out particular prices that are different from those reported for similar firms elsewhere or that lie outside certain specified limits. A computer printout can list all cases that either fall well outside the range of prices obtained earlier for that representative price transaction or show a marked percentage change from last time for the same price transaction in the same business. The limits used will vary from price transaction to price transaction and can be amended. The analyst can study the printout, first ascertaining whether there has been a keying error, then examining whether the explanation furnished by the collector adequately explains the divergent price behavior, and finally determining whether a query should be sent back to the supervisor or collector. The timetable should allow for this step, and anomalous observations should be discarded where an acceptable explanation or correction cannot be obtained in time. (Also see Chapter 10, Section D, on editing data.)

13.69 *The quote report* consists of a range of information on a price transaction that the index dispersion report has highlighted as warranting further investigation. Information listed may include current price, recent previous prices, and base price, together with similar quotes from other reporting businesses. The report can be used to identify the quotes that require further investigation and also to investigate rejected prices.

13.70 *Algorithms* can be created that identify and invalidate price movements that differ significantly from the norm for a price transaction. For some seasonal price transactions for which price movements are erratic, it may be more appropriate to construct an algorithm to look at price level rather than price change.

13.71 Although algorithms can be an efficient way to highlight problematical data, a word of caution should be expressed about using them. Analysts will want to assure themselves that their use does not result in systematic bias in the index. This issue may also need to be addressed in any editing routines (as presented in Chapter 10, Section D), although it is less likely to be problematical in the context of manual editing.

F. The Estimation Process

13.72 The estimation process is the component where all of the data come together to produce program outputs. Although the index calculation process is usually highly automated, the index review process can be quite labor intensive, albeit with a number of automated summary outputs that facilitate a quick review. One approach to take during the index review process is to have it looked at both by someone who is knowledgeable about the specific product area covered by the index and also by someone who is looking more broadly to ensure consistency across indices. The latter review, for example, may be appropriate if there is a widespread shock to the economy such as a dramatic devaluation or revaluation of the home country's currency.

13.73 Usually the calculation of price indices is handled by a computer program, either one customized by the agency or one that is off-the-shelf. In the latter case, such software can range from a simple spreadsheet program, such as Excel, to a powerful statistical program such as SAS (which originally stood for Statistical Analysis Software).

13.74 Because the algorithm used in the construction of the index can be fairly complex, it is vital that statisticians and economists play a major role in developing the formula and also in verifying that the formula has been properly converted into the appropriate calculation module.

13.75 Although data collection agencies may have different rules for when to publish and when not to publish data (owing to confidentiality or index quality constraints), it is important that indices be reviewed every time they are calculated in order to ensure they meet some standards for publication. These rules can be as complex as use of variance estimates or as simple as verifying the index is constructed using a minimum number of respondents and items. Part of management's responsibility is to ensure that that these rules are consistently applied and that they dovetail with publication rules in similar programs. It is important, however, to keep in mind that these rules need to be flexible enough to allow for the fact that commodity areas may manifest very different pricing traits. For example, price trends in fuels tend to be very uniform, whereas price trends for individual miscellaneous electronic components can vary dramatically. Another approach is to look for similar data, such as PPIs or CPIs, that can be used to verify price trends.

G. The Publication and Documentation Process

G.1 Publication

13.76 In recent years the publication process appears to have drawn closer attention for two reasons: first, with the rise of the Internet, there is greater concern regarding the security of data; and two, published economic data appear to be playing a larger role in the movement of prices in the stock market. Consequently, management should now give greater scrutiny to the publication process, the subject of Chapter 14. This has proven to be an area where good documentation has become especially important. There is usually a set amount of time between the time the indices are finalized and the time they are published. Frequently, there are a number of steps during the interim, including writing any appropriate text and possibly transmitting the data to a publication database.

13.77 In addition to tightening up documentation, agencies have also had to expend significant resources to ensure the security of their external and internal websites. Rules and procedures covering the electronic handling of sensitive data are usually established by the agency producing the data. However, the individual office that produces the data may have responsibility for making these procedures operational. Although XMPIs have traditionally not received the attention given to other price indices, an organization cannot afford to make the mistake of prematurely releasing any of its data.

13.78 Depending on the structure of the organization, successively higher levels of management and any other affected office as appropriate may need to be made aware of any impending news release. Often an agency may have a separate publication office responsible for coordinating agency outputs. If so, it is also good practice to work closely with this office in order to ascertain the types of questions that they may receive from users of the data.

13.79 Because of the similarity of XMPIs and PPIs, an organization may choose to release these data simultaneously. In such instances, it may be ideal to have the same staff produce the release and be prepared to answer questions from the media and other users.

13.80 Compilers of the index also may wish to visit the field occasionally and participate in or observe the price collection. This will provide them with a better appreciation of the practical problems associated with price collection, a better feel for data and index quality, and the skills required to help with price collection in the event of an emergency. In XMPIs, this may be of great importance, especially when collecting price transaction data in more complicated economic branches.

G.2 Documentation

13.81 The larger an organization, the more critical is good documentation. Staff, especially junior staff, should not have to rely on their memory when faced with a question of how to handle a specific issue.

13.82 Smaller organizations, too, need good documentation. Because a smaller organization may not have a large enough staff to have sufficient overlap of staff as new employees start and old ones leave, there may be a lack of what is called *institutional memory,* used to orally pass on data collection practices.

13.83 Offices with large systematized computer programming staff usually produce adequate system documentation. However, in small programs where any automated data processing may be handled by a statistician or economist, special care must be taken to ensure good documentation, because the training in those disciplines is less likely to include the appropriate type of documentation skills.

13.84 Good documentation should also include production schedules. Each aspect of the processing environment should have its own production schedule that needs to be integrated with each of the other schedules. This is true even though the production schedules may be on very different cycles. Much of the ongoing data collection and index calculation schedules are usually monthly (or quarterly), but the production or initiation of a sample of respondents may take a year or more. A process should be documented and in place that facilitates the overall development of an integrated schedule. Often unforeseen circumstances, such as weather or computer downtime, may force a change in the schedule. By having one process for scheduling changes, it ensures that the overall impact of any change can be effectively synchronized.

13.85 Manuals and other documents such as desk instructions may serve for initial training and later on should enable the collectors and compilers to remind themselves of all the relevant XMPI rules and procedures. Documents should be well organized and well indexed so that answers to problems can quickly be found.

13.86 All concerned should check the documentation and update it regularly; the pile of paper containing amendments should never grow large and should be replaced by a new consolidated version. One way of achieving this, for hard copies, is to have a loose-leaf manual so that individual pages can be replaced whenever necessary.

13.87 However, the use of a centralized electronic storage of documentation is increasing. This facilitates a systematic and controlled method for maintaining documentation. A variety of available software can help the statistical office. The benefits of using standard electronic software for documentation are threefold:

- More efficient production of documentation, because software helps with initial compilation and reduces the need to print and circulate paper copies;
- Better informed staff, because they have immediate electronic access to the latest documentation, including desk instructions with search facility by subject and author; and
- Better quality control, because authors can readily amend and date-stamp updates, and access is restricted to "read only" nonauthors.

13.88 In a more sophisticated technological environment, documentation can be centrally maintained, with updates automatically promulgated to PCs or other electronic devices, thus obviating the need for manual updating.

13.89 Documentation includes not only price collectors' and compilers' manuals but also details of the many routines used in collection and compilation including features of product markets that have given rise to a particular method for use when there are missing values or a need for quality adjustments—such metadata are outlined in Chapter 9. It further includes metadata for users that should be posted on the statistical authority's website. There should be a short user-friendly account for lay users along with frequently asked questions (and answers) and a more detailed technical manual along with technical articles on issues such as rebasing and, of course, data.

H. Quality Assurance

13.90 Though this chapter touched on some specific quality control techniques in the earlier data collection sections, there are a number of more general quality management areas and approaches one may also want to investigate. Statistical offices are faced with the continuous challenge of providing a wide range of outputs and services to meet user, that is, customer, needs. Thus, a key element of quality is customer focus and the effective dissemination of relevant, accurate, and timely statistics. In addition, a quality program should include effective customer education on the use of such statistics. In these terms, success can be measured by the achievement of a high level of satisfaction among well-informed users. The IMF has developed the Dissemination Standards Bulletin Board[1] that provides dissemination standards and a data quality reference site.

13.91 For the quality management of XMPIs, it can be argued that the priority area is quality control of the production process itself. For most statistical offices, this will be an area that represents a high risk, given the complexity of the process and the financial implications of an error in the index. If the principles of organizing and managing the collection of data, and subsequent processing of information to produce XMPIs, are to be adopted, then it is vital that a quality management system is in place. This will ensure the data obtained, the processes involved in achieving the specified outputs, and the formulation of policies and strategies that drive them are managed in an effective, consistent manner. The data systems should, wherever possible, be open to verification and mechanisms put in place to ensure outputs meet requirements—in other words, customer satisfaction.

13.92 A number of examples and case studies of quality systems in practice illustrate how different models may be applied. Some models may be more suitable than others, depending on the exact mode of XMPI operations in different countries:

13.93 *Total Quality Management*, or TQM, is most closely identified with a management philosophy rather than a highly specified and structured system. The characteristics associated with TQM and an effective quality culture in an organization includes *benchmarking,* a process of comparing with, and learning from, others about what you do and how well you do it, with the aim of creating improvements. The Australian Bureau of Statistics has been particularly active in this area and undertook an exercise in 1998–2000 in partnership with the United Kingdom. Benchmarking projects have also been undertaken in New Zealand, the United States, and Scandinavian countries.

[1]Available at http://dsbb.imf.org/Applications/web/dsbbhome/.

13.94 *The European Foundation for Quality Management Excellence Model* is a self-assessment diagnostic tool that is becoming widely used by government organizations across Europe to improve quality and performance. It may be described as a tool that drives the philosophy of TQM. It focuses on general business areas and assesses performance against five criteria covering what the business area does (the enablers: leadership, people, policy/strategy, partnership/resources, process) and four criteria covering what the business area achieves (the results: people results, customer results, society results, key performance results). Evidence based on feedback from focus groups, questionnaires, and personal interviews is used to score performance and a resulting action plan for improvement is introduced, which is then included in the business plan.

13.95 *ISO 9000* is the International Standards Organization (ISO) international quality standard for management systems. The quality system is a commonsense, well-documented business management system that is applicable to all business sectors and that helps to ensure consistency and improvement of working practices, including the products and services produced. The ISO standards have been fully revised to match current philosophies of quality management and to provide the structures needed to ensure continuous improvement is maintained.

13.96 *An independent advisory board* is a common approach to addressing quality issues. Depending on membership, a board (or boards) can serve a multitude of purposes. Some boards may consist of technical experts who can provide advice on specific techniques used in the construction of price indices, whereas others may serve as more of a forum for users who want to ensure that the indices meet their needs. Boards can also serve as a method for helping to publicize new outputs to the community at large.

14. Publication, Dissemination, and User Relations

A. Introduction

14.1 As discussed in Chapter 3, export and import price indices (XMPIs) are an important statistical series for monitoring inflation and assisting in the measurement of GDP volume series. It follows, therefore, that the XMPIs must be published, and otherwise disseminated, according to the policies, codes of practice, and standards set for such data.

14.2 The XMPIs, therefore, should be

- Released as soon as possible (noting the trade-off between timelines and quality);
- Made available to all users at the same time;
- Released according to preannounced, credible, and independent timetables;
- Released separately from ministerial comment;
- Made available in convenient form for users and include analysis of the main contributors to overall change;
- Released by a single agency. If, for example, some institution such as the central bank publishes unit value indices it can undermine the efforts of statistical agencies doing surveys;
- In accordance with any legal requirements including the level of classification;
- Open to criticism and willing to adjust or expand to cover any new activities and stay relevant;
- Accompanied by any issues concerning the quality of the indices;
- Accompanied by methodological explanation to maintain transparency and advice as to where more detailed metadata can be found; and
- Backed up by professional statisticians or economists who can answer questions and provide further information.

14.3 Above all, the XMPIs should meet the United Nation's (UN's) *Fundamental Principles of Official Statistics*, which is published in several languages on the website of the United Nations.[1] The *Principles* refer to dissemination and to all aspects of statistical work. In addition, the data dissemination standards developed by the IMF should be reviewed and followed by statistical offices. These and other standards are discussed in this chapter.

B. Types of Presentation

B.1 Time-series presentation of level and change

14.4 It is common to give prominence to indices that show changes in aggregate prices between the month or quarter for which the most up-to-date data are available, the change from the same period one year earlier, and the one-period change. It is also usual to compare the annual change with the annual change shown one month or quarter previously. The model presentation in Section B.6 provides examples of these.

14.5 The arguments for the first presentation shown in the example are as follows. The 12-month comparison provides an indication of price changes in a reasonably long time frame by referring to periods that are unlikely to be influenced by seasonal factors. The one-month change might include one-off changes in either of the two months that can have an influence on the index.

14.6 Data on the one-month change, especially for some components of the XMPIs, need to be treated with caution to avoid, for example, suggesting that a 2 percent change in one month is similar to a 24 percent change over a year. (See the second presentation in the example.)

[1]http://unstats.un.org/unsd/methods/statorg/Principles_stat_activities/principles_stat_activities.asp.

14.7 It is normal practice to set a reference period (usually a year, though a shorter period, such as a month, may be used) for which the price index is set or based at 100.0. Index numbers for all subsequent periods are percentages of the value for the reference period. Indeed, this index is used as the basic figure from which the other changes are calculated.

14.8 The base period is generally considered to be the period with which other periods are compared and whose values provide the weights for the price index. However, the term "base period" is not used in a precise and consistent manner. Three types of base periods may be distinguished: First, the *price reference period,* that is, the period whose prices appear in the denominators of the price relatives used to calculate the index. Second, the *weight reference period,* that is, the period, usually a year, whose values serve as weights for the index. However, when hybrid expenditure weights are used—that is, when the quantities of one period are valued at the prices of some other period—there is no unique weight reference period. Finally, the *index reference period,* that is, the period for which the index is set equal to 100.0. The three reference periods may coincide but frequently do not. The status of the reference period should be made clear in the methodological explanation. For technical reasons, a reference period that is abnormal (e.g., in terms of absolute or relative price levels, industry structure) should be avoided. In order to maintain the usability of the indices, the reference period should be changed infrequently. Under some circumstances, namely if index values have moved too far away from 100.0, consideration should be made to rebase the index series to 100.0 in a more recent period.

14.9 Generally, if an index series is discontinued and then resumed, the series should be disseminated as a new series, based equal to 100.0 in the period it is resumed, and any historical data previously published should be removed. Otherwise, the index movement over the period the index was not published would be derivable. However, exceptions might be made in cases where the index is discontinued for only a short period of time (less than a year), such as with a commodity that traded during only part of the year.

14.10 These indices are usually shown to only one decimal place, as are the other changes mentioned here, so figures have to be rounded.[2] Rounding in these circumstances can, however, give a false impression of comparative change and must therefore be explained, especially where price changes are small.

14.11 Care also has to be taken to differentiate between changes in index points and percentage changes between one period and the next. If in one month the index is, for example, 200 and the following month it is 201, then the change can be described as one index point (above the previous level of 200) or as an increase of half a percent. Both measures are valid, but they require careful specification.

14.12 XMPIs are, by definition, indices; they are, therefore, not an absolute value of a level or a series of absolute changes in prices. Nevertheless, in the process of presenting the indices, average prices are sometimes calculated for categories of goods and services. This would be particularly useful for exports or imports of major commodities such as coffee or oil. It is thus possible to publish some average prices for groups of goods or services and also to show the upper and lower bands of the prices from which the averages have been calculated. Some users of the index find average price levels useful, and they should be made available to researchers who may want them.[3] It must be noted, however, that price-level data may be less reliable than the price change indices for any given group of goods or services because of the sampling strategies used. This is especially true in the case of XMPIs where, in general, sample size is smaller than the samples used to derive the consumer price index (CPI) and producer price index (PPI). Further, quality changes can distort comparisons over time.

14.13 So far this chapter has referred to only the broadest aggregates without reference to subgroups of prices or to variants of the XMPIs that may include or exclude certain items. Nor does it refer to price indices with underlying concepts that may differ from those underlying the XMPIs. Some of these considerations are discussed later in this chapter.

14.14 All of the above can refer to the most common form of the XMPIs, which is usually intended to refer to the average price change in a specific country and to include high coverage of import and export prices in that country. But it can equally refer to countries or regions of origin (for imports) or destination (for exports), or to subcomponents (such as raw materials versus manufactured

[2]Recent work by Williams (2006) has suggested the desirability of rounding to three decimal places rather than one.

[3]When releasing data on average prices, confidentiality requirements must be maintained. See Section C.4.

goods), different commodity groups, or related or alternative measures of price change. Subaggregate indices are discussed in Section B.5.

B.2 Seasonal adjustment and smoothing of indices

14.15 The treatment of seasonal commodities and the estimation of seasonal effects are discussed in Chapter 23. In the present chapter, the dissemination of such adjusted or smoothed series is discussed.

14.16 Many economic statistics are shown seasonally adjusted as well as unadjusted. Normally, however, XMPIs are not seasonally adjusted. In cases where there are seasonal factors, statistical series are frequently recalculated using the latest data. As a result, seasonally adjusted series can be retrospectively revised. Unadjusted XMPIs are not necessarily revised, although in some countries, there is an explicit revision policy to publish preliminary XMPIs and then revise that index over a fixed period (usually one to three months). This occurs because the entire sample is not received by the index cutoff date, so the index is released on a preliminary basis; but, after a few months, practically the entire sample is received and a revised index is published. XMPIs are more commonly revised than is the CPI or PPI because of generally smaller sample sizes, which lead to a greater sensitivity to late data.

14.17 In comparing one month with the same month a year earlier, it is implicitly assumed that seasonal patterns are much the same from one year to the next. However, there may be exceptional months when the usual seasonal change is advanced or delayed, in which case the advance or delay should be identified as one of the likely contributors to a change in the XMPIs or one of their components.

14.18 Changes over periods of less than a year are subject to seasonal influence. To differentiate them from other factors, it is necessary to try to quantify seasonal effects and identify them as contributing to changes in the index.

14.19 Although the XMPIs themselves are not seasonally adjusted normally, some variants of the XMPIs may be seasonally adjusted (such as the XMPIs for raw materials or agricultural commodities) because they are more subject to seasonality and can be revised retrospectively if necessary. If such variants are seasonally adjusted, it is important to explain why.

14.20 Seasonal adjustment usually leads to a smoother series than the original unadjusted one. But there are other ways of smoothing a monthly series, such as using three-month moving averages. However, statistical offices do not usually smooth the XMPI series in their published presentations. Import and export price changes are not usually so erratic from month to month that they disguise price trends. To the extent that there might be an erratic change, the compilers of the index can usually explain the reasons for any sharp fluctuation.

14.21 In cases where any seasonally adjusted or smoothed XMPI series are published, it is important to publish the unadjusted series as well, so that the effects of the adjustment process are clear to users who may wish to know what has happened to the actual transaction prices and whether the changes can be attributed to seasonal factors. Similarly, a full explanation should be given for the reasons why a particular seasonal adjustment procedure is followed.

B.3 Analysis of contributions to change

14.22 XMPIs are aggregates of many different commodities, whose prices are changing at different rates and possibly in different directions. Many users of the index want to know which commodities have contributed most to changes in the aggregate index and which prices may be out of step with general price trends. The index compilers are well placed to provide analyses of the contributions to the price change in the current press release and current issue of the XMPI publication.

14.23 Sufficient detail should be made available to the users of the index, so they can see for themselves what has happened to various groups of prices. However, because of the time constraints facing many users, the statistician should indicate which prices are the main contributors to the aggregate XMPI and which ones may be most different from the aggregate. They can be presented in the form of tables and charts, so that trends may be compared.

14.24 Similarly, statisticians should indicate any reasons for price changes that may not be immediately obvious but are nevertheless discernable from the published figures. For example, if there had been a sharp price rise or fall one year earlier, then it will affect the current year-on-year change, regardless of what happens to the current-period prices.

14.25 Analysis of contributions to change should also refer to any preannounced price changes, or major changes since the last price-reporting date, that will affect the outlook for the index over the following months.

B.4 Economic commentary and interpretation of index

14.26 In undertaking analysis such as that described above, statisticians must be objective, so that users of the data may differentiate clearly between the figures themselves and the interpretation of them. It is, therefore, essential to avoid expressing any judgment of the policy causes or possible implications for future policies. Whether the figures should be seen as good news or bad news is for the users to decide. The statistician's role is to make it as easy as possible for users to form their own judgments from their own particular economic or political perspective. Care should also be taken when deciding to mention the reasons for index movement when those reasons involve unpublished micro data. In general, if the price movement in any unpublished series is general knowledge, such as a price change for something traded in the commodities market, then it is all right to include that in the overall analysis of price movement. If the price movement in unpublished micro data is not common knowledge, it should not be mentioned in any analysis.

14.27 There are several ways of avoiding any apparent or real lapses in objectivity in the analysis. The first and perhaps most important way is to publish the figures independently of any ministerial or other kind of political comment. Another is to be consistent in the way the analysis is presented. That is to say, the data should be presented in much the same format every month (see Section B.6). For example, tables and charts should cover the same periods every month and use the same baselines.

B.5 Presentation of related or alternative measures

14.28 One type of alternative index is to produce XMPIs in foreign currency terms rather than the home currency of the country. Movements in foreign currency import indices can be used to observe fluctuations in revenues received by exporters, whereas foreign currency export indices show the price movement of exports from the perspective of foreign buyers. XMPIs are derived by using average exchange rate indices to convert the home currency indices into foreign currency terms, much the same way an exchange rate is used to convert an item price from one currency to another. Average exchange rate indices measure the change in the price of trade-weighted baskets of currencies against the home currency.

14.29 A second example of an alternative index is the XMPI by locality or origin (for imports) or destination (for exports), rather than commodity type. Generally, indices of this type are aggregated at the commodity level of detail but broken out by various countries or regions of interest. These index types can be published at an aggregate level of detail for each locality or be subdivided by commodity type or industry. These indices could also be published in foreign or local currencies. Generally, however, unless separate provisions are made in the sample for the import or export locality, there are limitations as to the robustness of these indices below aggregate levels.

14.30 Another alternative index is terms of trade indices, which measure the ratio of an export price index over an import price index. These could help build interest in XMPIs by providing a measure of changes in a country's economic welfare. Terms of trade indices could be presented by commodity or industry as well as by major trading partners. The indices could be published in foreign or local currencies. Similar to producing indices by locality of origin or destination, one limitation to producing terms of trade indices by trading partners would be the size of the sample and whether separate provisions are made to sample for import or export localities.

14.31 In addition to producing XMPIs using a standard index formula, data can also be published using an alternative index formula. If the primary XMPIs are derived using a Laspeyres formula, retrospective indices can be produced using either a Fisher or Törnqvist formula as an alternative. Although such indices would not be as timely, that should not be an issue if these indices served as a supplement to the Laspeyres indices. These could be useful to analyze any upper-level substitution bias existing in the primary indices.

14.32 Further examples are XMPIs for services. No country, at this time, has complete coverage of all goods and services in their XMPIs. Many countries started by developing XMPIs for goods and then progressively added service activities over time. This results in the availability of a range of XMPIs for different sectors of the economy. However, not all sectors of the national economy have their own XMPIs.

B.5.1 Subaggregate indices

14.33 Countries commonly calculate price indices for hundreds of commodities (e.g., bread or footwear) based on thousands of individual price records. Therefore, the number of possible subaggregates is quite large.

14.34 One kind of subaggregation is by groups of commodities (products) that, when aggregated, comprise the whole of the XMPIs. An important consideration here is the relationship between different commodities within the subgroups. For example, an index may be presented for food; then, under the "food" heading, indices may be presented for subgroups such as breads, cereals, vegetables, and so on.

14.35 Another type of aggregation is by industry. Indices for each four-digit industry aggregate to three-digit, two-digit, and one-digit groupings. For each aggregate grouping, there are subgroups that represent the industries within the grouping. Another important consideration is that the XMPIs by industry and the XMPIs by commodity produce the same aggregate price change in the overall XMPI, so that the weighting structure used in the commodity and industry aggregations is consistent.

14.36 One of the first considerations in presenting such subaggregate data for related commodities or by industry is consistency over time. That is, there should be a set of subaggregates for which indices are calculated and presented each month. Users commonly attach great importance to being able to continue their analysis from month to month.

14.37 Another consideration is international standardization of the division of the index into groups of goods and services, which enables comparison among countries. Some countries also have their own subaggregate groupings that may predate the current international standard. The generally accepted international standards for the presentation of subaggregates are the International Standard Industrial Classification (ISIC revision 3.4), the Harmonized Commodity Description and Coding System, the Standard International Trade Classification, and the Central Product Classification. These classifications are important because they define groups of industries or products (commodities) by the technology used for production or the purpose for which they are produced (e.g., manufactured commodities or transport services). Many national classifications are derived from these international standards by adapting them to local circumstances. Locally, it is important to identify and include certain modifications that make the classification more useful and better understood within the country.

14.38 A further type of subaggregate index is an index that is essentially the same as the XMPI except that it excludes certain items. The underlying inflation index discussed earlier is an example. In the presentation of all related or alternative measures, the concepts and definitions should be made clear, and it is advisable to give the reasons for the alternative presentation. Most important, it should not be suggested that the subaggregate index is more meaningful than, or superior to, the XMPIs themselves.

B.6 Model press release, bulletin, and methodological statement

14.39 An example of a press release for a fictitious country appears at the end of this chapter. The example provides only text and charts. It does not include data tables that would normally be attached to support the analysis in the text. Other formats are possible; for example, it might include a seasonally adjusted index.

14.40 Note that the example press release contains the following information:

1. Details of issuing office,
2. Date and time of release,
3. Percentage change in new month versus a year ago,
4. Comparison of percentage in a new month with that of the previous month,
5. Information on commodity (product) groups that contributed to change and on any significant component price,
6. Reference to where more information can be found, and
7. Any announcements pertaining to changes in the presentation of the data such as the addition of new series or changes to the reference base period.

14.41 Note also that

- No judgments are offered on policy or economic reasons for the price change, and
- No judgment is given on whether the change is good or bad.

14.42 What is not obvious from just one example is that the format should be the same in all releases from period to period. Using a consistent format is important to avoid appearing to indicate a preference. A format with a selected starting date, for example, might indicate a preferred trend.

14.43 Other pages of the press release should give the monthly or quarterly indices (reference period that equals 100) from which the percentage changes are calculated. Similar indices should also be given for major groups of goods and services. Charts may also be used to illustrate, for example, which prices have contributed most or least to the overall XMPIs.

14.44 If any other import and export price variant is also being published, then the differences between the indices, including any methodological differences, should be briefly explained. Variants that require explanation include unit value indices, indices by locality of origin or destination, or indices in foreign currency terms. More detailed explanation can be given in handbooks.

14.45 In addition, the press release should include a short note on methodology similar to the following:

What is the import and export price index (XMPI) measuring and how is it done?

For example:

> The all-items import price index is an overall measure of the change in prices received by residents of other countries from residents of the home country. The all-items export price index is an overall measure of the change in prices received by residents of the home country from residents of other countries. Both are valued at border transaction prices. The XMPIs are a key indicator of price movements that contribute to inflation. They measure the average change in prices, from period to period, of the goods and services traded with residents of other countries.
>
> Prices are collected from establishments that import or export goods and services. The trade value for these goods and services is derived from a regular census of establishments. The prices and trade values are then combined to calculate the price indices for divisions and groups of industries, and for the all-items index.
>
> The overall index, with all of its component indices, is published each month in our *XMPI Bulletins*. The *Bulletins* also contain more information on the methodology used in calculating XMPIs. A small booklet can also be made available. For a detailed account of the methodology used in calculating the XMPIs, the user is referred to the *XMPI Technical Manual* published by this statistical office. For more information on these publications and how they may be obtained, please refer to our website at . . . . or call the numbers listed on the front of this press notice.

An example of a press release is given in Section E.

B.7 UN Fundamental Principles of Official Statistics, IMF Data Standards, and International Labour Organization Standards

14.46 Many international standards apply, in general terms or specifically, to the XMPI. One very general but fundamental standard is the UN's "Fundamental Principles of Official Statistics" (1994). As indicated earlier, it is available in several languages on the UN's website.[4] It refers not just to dissemination but to all aspects of statistical work. The introduction to this chapter lists some of the broad principles that are reflected in many of the international standards in some form.

14.47 IMF standards are particularly pertinent in this context because they are specifically aimed at dissemination issues. There are two that refer to statistics, including import and export price indices. One is the General Data Dissemination System (GDDS), and the other is the Special Data Dissemination Standard (SDDS). The GDDS provides a general framework with some specific indicators defined as *core* and others defined as *encouraged*. The SDDS is based on the GDDS framework but is more demanding and applies only to those countries that choose to subscribe to it in writing to the IMF Board. Both are available on the IMF Data Dissemination Bulletin Board (www.ddsb.org).

14.48 The GDDS has several dimensions for dissemination standards. Under the heading of *quality,* the GDDS refers to the necessity to provide information on sources, methods, component detail, and checking procedures. Under *integrity* it refers to declared standards of confidentiality, internal government access before data release, identification of ministerial commentary information on revision, and advance notice of changes in methodology. Under *access by the public,* it refers to the need for preannounced release dates

[4]http://unstats.un.org/unsd/methods/statorg/Principles_stat_activities/principles_stat_activities.asp.

and simultaneous access for all users. In the tables of data categories, it refers to the XMPI as *core indicators* that should be issued monthly or quarterly, within one to two months of the data collection date. All of these standards are reflected in the present manual. The International Labour Organization also has guidelines on the dissemination of labor statistics on its website.[5]

C. Dissemination Issues

C.1 Timing of the release

14.49 The XMPIs should be released as soon as possible, but it is equally important to release the index according to a strict timetable with an unambiguous embargo time to ensure simultaneous access. It is also important to publish the timetable of release dates as far in advance as possible. Having a fixed release date, published well in advance, is important for several reasons. First, it reduces the scope for the manipulation of the release date for political expediency. Second, it gives confidence to users that the release date is as early as possible and has not been delayed (or brought forward) for purely political reasons. A third advantage is that users know when to expect the figures and can be prepared to use them.

14.50 Ideally, the XMPIs—complete with any essential metadata—are released simultaneously to the press, government users, and all others. Also, some statistical offices bring the journalists together, perhaps half an hour before the official release time, to provide them with the printed press release, explain the data, and answer any questions. Then, at release time, the journalists are permitted to transmit the figures to their offices for wider distribution.

C.2 Timeliness of the release versus data accuracy

14.51 There is a basic tradeoff between accuracy (ensuring the credibility of the data by maintaining the high quality of the indices) and timeliness (releasing the data in a time frame that is useful to data consumers). Maintaining accuracy means waiting to publish indices until enough data have been reported and reviewed. Timeliness means releasing the indices as close as possible to the reference period.

14.52 The accuracy of the index is particularly important because so much depends on the XMPIs. In addition to the economic policy implications of the index, its components are used in many countries as deflators in the foreign sector of national accounts to derive constant price GDP; they are also used in a variety of commercial contracts.

14.53 It follows that although timeliness is important, the timetable must allow time for the data to be properly prepared and thoroughly checked. If a revision policy is in effect or the XMPI series are revised on an ad hoc basis, then the policy or the changes must be fully described and explained when the new data are released. If there is any methodological change, then users should be advised several months before the change occurs.

C.3 Confidentiality vs. accessibility

14.54 For administrative data from customs authorities, the data provided to the statistical authority are a secondary source. Data from establishment surveys are, however, obtained under assurances of confidentiality given by the statistical authority. There is a tradeoff between confidentiality (the assurance given to data providers that the micro data they provide won't be released to anyone else without their permission) and accessibility (making as much micro data as possible available to data customers). Confidentiality is important because data providers need to feel confident the micro data they produce won't be revealed to anyone outside the agency. Yet if the data are to be useful to researchers, the data need to be made available.

14.55 Although as much data as possible should generally be made available to users as explained above, there are reasons why confidentiality is important in most instances.

14.56 First, most data supplied by establishments are provided on the understanding that the data will be used only for the purpose of aggregation with other data and will not be released in any other form. This can be especially important when the data are supplied voluntarily, as they often are. Most statistical offices make a pledge that the price data are strictly confidential, or confidentiality requirements may be included in statistical legislation. In such cases, the statistical office will not provide the information to any other organizations or publish the data in a manner that

[5]www.ilo.org/global/What_we_do/Statistics/standards/guidelines/lang--en/WCMS_087614/index.htm.

would reveal the individual respondent's information. Many agencies have rules about the minimum number of establishments (e.g., three or more) that must report before data can be published or released. In addition, many statistical offices have rules about dominant enterprises within an industry (e.g., 75 percent of production), so that data for large firms are not divulged without the firm's consent.

14.57 Second, only a sample of particular commodity transactions is priced as representative of a much larger group of commodities. If it were known which varieties are included in the index and which are not, then it might be possible to bias components of the index by manipulating a small number of prices.

14.58 Even the knowledge that price data are or might be collected on one particular day in the month could enable some component price indices to be biased by respondents choosing to change prices on a particular day. However, this provides only a short-run advantage and cannot be sustained.

14.59 With the XMPIs, as with other statistics, users should be allowed access to as much data as possible for two main reasons. First, some users find the detailed data very useful in their analysis. Second, access to the detail increases the understanding of and the confidence in the data.

14.60 There are, however, limits on the amount of data that can be made available to users. One constraint is confidentiality. Another is the limited volume of data that most users can absorb. Still another reason is the cost of publishing large amounts of data that few users need.

14.61 In general, the XMPIs and their major components are deemed to be of such wide importance that they are freely available through press releases and statistical office websites. More detailed data are often published only in statistical office bulletins and other media, and users are charged fees so the statistical office can recover some of the dissemination costs. Similarly, special analyses made at the request of particular users are usually charged using a rate commensurate with the work involved.

14.62 In some cases, additional micro data might be disseminated to researchers beyond what is released to the general public. However, researchers receiving unpublished micro data should be required to sign confidentiality agreements that specify they will not give out or publish any micro data beyond what is available to the public.

C.4 Electronic release format—utilizing technology versus fairness

14.63 Deciding how to utilize electronic release formats involves a tradeoff between fully utilizing the available technology to disseminate data and being fair to all users, some of whom might not be able to receive the data in the same time frame as others. Statistical agencies should seek to disseminate data to customers as quickly as possible and in formats that provide the greatest flexibility for users. However, though it is preferable that data be made available to all users at the same time, not everyone has the same access to electronic means.

14.64 The World Wide Web has several advantages as a dissemination medium. For the data producer, distribution costs are relatively small, involving no printing or mailing. As soon as the data are on the Web, they are available to all Web users at the same time. Putting a large amount of data on the Web costs little more than doing the same with a much smaller amount. Web users can download the data without rekeying, which thus increases speed and reduces transmission or transposition errors.

14.65 One disadvantage is that all data users may not in some countries have equal access to the Web. Another is that users may go straight to the data without reading the metadata that may be crucial to understanding the data. Also, it may be as easy for a user to disseminate the XMPIs widely by electronic means as it is for the statistical office to do so, thus enabling users to preempt the statistical agency by providing statistics without the metadata that may prevent a misunderstanding of the figures. Another disadvantage is that the Web makes it more difficult to track who the users of the data are.

14.66 Care must be taken to ensure that the XMPIs are available at the same time to all users regardless of the dissemination medium used.

C.5 Revision policy—using all data versus the stability of initial measures

14.67 The XMPIs may be subject to revision, depending on the data collection method used and the timeliness of source price data. XMPI data differ from CPI data, which are collected through personal visits and for which the source prices are practically all available by the end of the month. For that reason, it is rare for the CPI to be revised after first publication. Most XMPI source data are collected by a mail survey, for which the returns arrive more slowly and may not all be available at the time of first publication. In such instances, the statistical office may institute a revision policy in which the monthly XMPIs are first published on a preliminary basis, then revised for one to three months when practically all sample returns have been received. If data are revised, it is important to have a transparent and set policy as well as a published revision calendar.

14.68 Revising indices is also more important for XMPIs because the sample size is generally small relative to either the CPIs or PPIs, obviating the need to use the maximum available data to calculate indices (see the discussion in the following section).

C.6 Limitations on data dissemination owing to sample size

14.69 Generally, owing to budgetary constraints, the sample size for XMPIs is much smaller than the samples used to produce CPIs and PPIs. The size of the sample puts limitations on both the number of published indices and the level of aggregation.

14.70 The smaller sample size also makes XMPIs more sensitive to data outliers. If possible, measures of standard errors should be included with the indices. Standard errors are useful measures to include with any indices, but are especially important in cases where the sample size is small.

14.71 XMPIs are generally sampled by commodity or industry type rather than by locality of importation or exportation. Unless separate provisions are made, there are limitations as to the robustness of indices disaggregated by locality below aggregate levels of commodity types.

D. User Consultation

D.1 Explanation of different uses of XMPIs

14.72 The different uses of XMPIs are discussed in some detail in Chapter 3. It is important to explain to potential users of the XMPIs what are suitable uses and what are not. To this end, it is important to explain how the XMPIs are constructed, in terms of its sources and methods (see Section D.2).

14.73 It is also important to make readily available explanations of alternative indices or subindices such as foreign currency indices, indicating how their uses are different from or supplement the uses of the headline XMPIs.

D.2 Presentation of methodology

14.74 When the XMPIs are published each period, users are anxious to see the main figures and to use them. They do not generally want to be burdened with explanations of the methodology underlying the data. Nevertheless, methodological explanations must be accessible to those who may want them and in forms that are comprehensible to users with different levels of expertise and interest.

14.75 Any significant changes in methodology must be fully explained, and users must be notified as far in advance as possible of the change being made.

14.76 In addition to a brief statement in press releases (see Section B.6), methodological explanations should be available on at least two levels. Nonexperts should be able to refer to a booklet that explains the history, principles, and practice underlying the XMPIs and any alternative measures that may be available. A more thorough explanation of sources and methods should be readily available to those users who are sufficiently interested to use it. An example would be statisticians in training who may be new to working in the production of the XMPIs. The information must also be kept up to date despite the pressures to devote time to the output at the expense of documentation.

14.77 As noted earlier, the ready availability of a full explanation of sources and methods is essential to confidence and trust in the XMPIs.

D.3 Role of advisory committees

14.78 For a statistical series as important as the XMPIs, it is essential that there is an advisory committee or set of committees that is representative of users and producers. There are many contentious issues in the construction of the XMPIs. The role of an advisory committee is to consider and advise on issues contentious and otherwise. But perhaps its equally important role is *presentational,* that it provides evidence that the XMPIs can be trusted and is not a tool of government propaganda.

14.79 In those countries where advisory committees have not been the norm, there may be a fear on the part of statisticians that by including nongovernment participants, their expectations may be raised beyond what the statisticians can deliver, thus increasing their dissatisfaction. On the other hand, the inclusion of nongovernment users can lead to a greater understanding of the realities and practical constraints of meeting theoretical needs. This is the usual experience of statistical offices that already have advisory bodies that include representatives of all the major constituencies inside and outside government.

14.80 It is, therefore, important that the advisory committee should be composed of academics, employers, trade unions, and others who have an interest in the index from differing points of view. It is also important that its reports are made available to the public fully and without undue delay.

D.4 Presentation of issues concerning index quality

14.81 The XMPIs may be regarded with suspicion at many different levels. As countries continue to move to more open economies, international trade is becoming an increasingly more important component of those respective economies. Therefore, unless the XMPIs are expanded to cover more economic activities, it will be criticized for being less relevant than appropriate. Also, there may be criticism of the index because of suspicion that it does not keep track of newer types of goods and services or changes in the quality of commodities. In transition economies, there is also the concern about the ability of the XMPIs to measure changes in trade patterns including newly emerging industries in the home country or the movement of existing industries abroad.

14.82 In the light of such suspicion, it is important for the producers of the index to be willing to discuss these issues and explain how they are being dealt with. As with other issues discussed here, the producers of the index must be open about their methods and the extent to which they can overcome the theoretical and actual problems that have been identified.

14.83 It follows that the statisticians who produce the index should publish explanations of quality issues, whether or not the quality of the index is being questioned currently.

E. Press Release Example

National Statistical Office of [name of country]

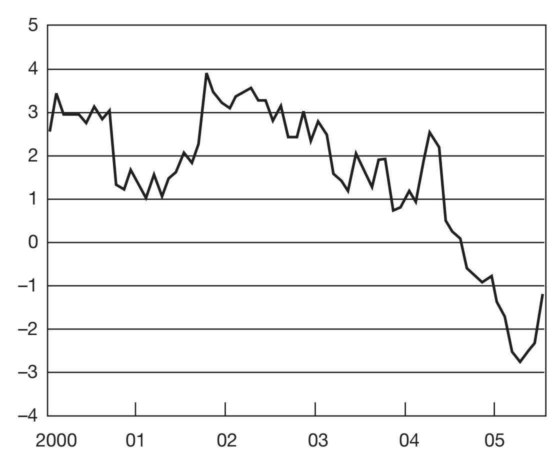
Friday August 19, 2005
for release at 11:00 a.m.

IMPORT/EXPORT (XMPI)

JULY 2005: PRESS RELEASE

In July 2005, prices were 1.2 percent lower than in July 2004 for overall imports and were 2.3 percent higher for overall exports. These 12-month changes followed 12-month changes recorded in June (−2.3 percent for imports and +1.9 percent for exports) and May (−2.5 percent for imports and +2.4 percent for exports).

Figure 1. Percentage Change in the Import Price Index: 2000–05
(Relative to the same month of the previous year)



Main contributors to the overall 1.2 percent decrease

Over the past 12 months, import raw materials prices declined 3.3 percent, led by a 5.4 percent decrease in energy prices. In contrast, the import price index for import finished goods rose 0.9 percent for the year ended in July. Import food prices declined 1.0 percent over the past year.

Figure 2. Percentage Change in the Export Price Index: 2000–05
(Relative to the same month of the previous year)

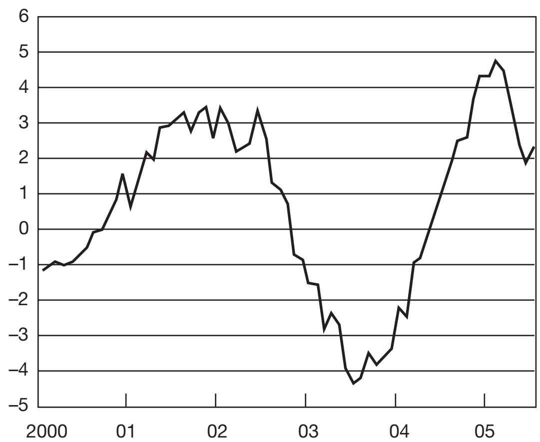
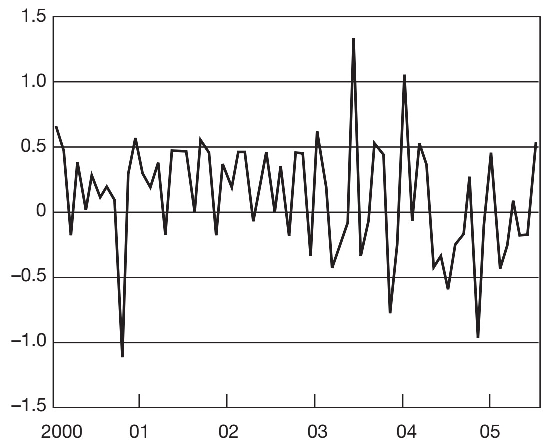


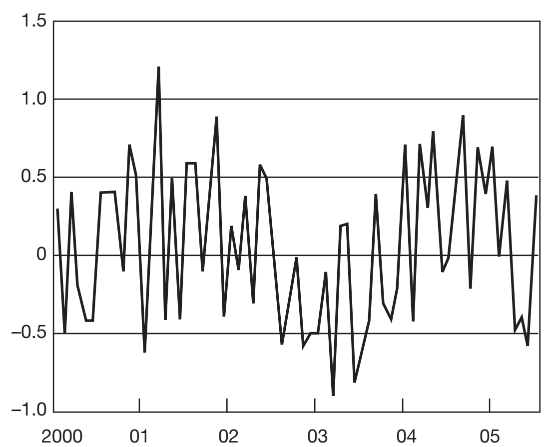
Figure 3. Percentage Change in the Import Price Index: 2000–05
(Relative to the previous month)



Main contributors to the overall 2.3 percent increase

The price index for export raw materials declined 1.2 percent over the past 12 months as energy prices fell 2.3 percent over the period. Export finished goods prices rose 3.1 percent for the year ended in July. Over the past 12 months, export food prices increased 2.1 percent.

Figure 4. Percentage Change in the Export Price Index: 2000–05
(Relative to the previous month)



Current Period Changes

The price index for overall imports increased 0.5 percent in July from the June level, compared to a 0.7 percent decline the previous month. The increase was led by a 2.3 percent advance in import raw materials prices, which previously declined 0.9 percent in June. Food prices also rose in July, increasing 1.2 percent for the month. In contrast, the price index for import finished goods declined for the second consecutive month, decreasing 1.0 percent in July after a 0.2 percent drop in June.

Overall export prices rose 0.4 percent in July, a downturn from the 0.8 percent increase the previous month. Raw material prices also contributed to the July increase in export prices, rising 1.2 percent for the month following a 0.5 percent increase in June. The price index for export finished goods declined 0.2 percent in July after recording no change for the month of June. Export food prices increased 0.7 percent in June.

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fax number, and e-mail address.
Background notes on the XMPI are given in the annex to this note.
More notes and details are given on our website.

15. The System of Price Statistics

A. Introduction

15.1 This chapter is about value aggregates and their associated price indices in an integrated system of economic statistics. To understand why value aggregates are important, we foreshadow the next chapter, which addresses concepts for decomposing value aggregates into price and volume components. Chapter 16 begins with defining a value aggregate in equation (16.1) as the sum of the products of the prices and quantities of goods and services. Equations (16.2) and (16.3) characterize a price index as the factor, given the relative change in the value aggregate, arising from changes in prices. Not surprisingly, then, to define a price index, it is first necessary to define precisely the associated value aggregate.

15.2 Four of the principal price indices in the system of economic statistics—the consumer price index (CPI), the producer price index (PPI), and the export and import price indices (XMPIs)—are well known and closely watched indicators of macroeconomic performance. They are direct indicators of the purchasing power of money in various types of transactions and other flows involving goods and services. As such, they also are used as deflators to provide summary measures of the volume of goods and services produced, consumed, and traded. Consequently, these indices are important tools in the design and conduct of the monetary and fiscal policy of governments, and they are also useful in informing economic decisions throughout the private sector. They do not, or should not, comprise merely a collection of unrelated price indicators but provide instead an integrated and consistent view of price developments pertaining to production, consumption, and international transactions in goods and services. By implication, the meaning of all of these indices derives in no small measure from the significance of the value aggregates to which each refers.

15.3 Section B of this chapter establishes the relationships among the four major price series, as well as their relationships with a number of supporting or derivative price indices. It does this by associating them with certain aspects of the interlocking aggregates defined in the Commission of the European Communities and others (2008), *System of National Accounts 2008* (*2008 SNA*).[1] Section C briefly considers purchasing power parities in the system of economic statistics.

15.4 The reader interested in a survey of the goods and services accounts of the *2008 SNA* and how it interrelates to the full range of price indices in the economy will find the entire chapter of interest. Readers engaged principally in compiling XMPIs should focus on Sections B.1.1, B.1.2, B.1.3.1, B.1.3.5, B.1.3.6, B.2.2, and B.3, because these deal directly with the XMPIs. This sequence of sections skips over explanations of how the *2008 SNA* builds up the consumption, capital formation, and external trade flows in the supply and use table (SUT) from the accounts of individual economic agents. Also skipped in this sequence are the total economy price indices for total supply, final uses, and GDP, and the price index for labor services.

15.5 Section B.5 also may be of interest to compilers, because it focuses on how the XMPIs relate to other major price indices. Chapter 5 of this *Manual* on weights and their sources cross-references the current chapter, which defines the institutional unit and transactions scope of the XMPIs. This chapter also lays out the conceptual framework for the weights of the XMPIs. Chapters 4 and 7 on price collection discuss the practical dimensions of defining the price to be collected, cross-referencing the current chapter regarding the price valuation basis for the XMPIs.

[1]At the time of writing, the *2008 SNA*, in draft stage, was available on the United Nations Statistics Division website at http://unstats.un.org/unsd/sna1993/draftingphase/ChapterList.asp. The *2008 SNA* is an updated version of the *1993 SNA* (Commission of the European Communities and others, 1993).

B. Major Goods and Services Price Statistics and National Accounts

B.1 National accounts as a framework for the system of price statistics

15.6 The significance of a price index derives from its referent value aggregate.[2] This chapter considers the core system of value aggregates for transactions in goods and services that is clearly of broad economic interest: the system of national accounts. The major price and quantity indices should, in principle, cover those value aggregates in the national accounts system representing major flows of goods and services and levels of tangible and intangible stocks. If the coverage of the major indices is not complete relative to the national accounts aggregates, then it should be compatible with and clearly related to the components of these aggregates. This chapter shows how the national accounts positions headline price indices such as the CPI, PPI, and XMPIs and these indices can be logically linked.

15.7 The *2008 SNA*, Paragraph 1.1, describes itself as follows:

> 1.1 The System of National Accounts is the internationally agreed standard set of recommendations on how to compile measures of economic activity in accordance with strict accounting conventions based on economic principles. The recommendations are expressed in terms of a set of concepts, definitions, classifications and accounting rules that comprise the internationally agreed standard for measuring such items as gross domestic product (GDP), the most frequently quoted indicator of economic performance. The accounting framework of the System allows economic data to be compiled and presented in a format that is designed for purposes of economic analysis, decision-taking and policy-making. The accounts themselves present in a condensed way a great mass of detailed information, organized according to economic principles and perceptions, about the working of an economy. They provide a comprehensive and detailed record of the complex economic activities taking place within an economy and of the interaction between the different economic agents, and groups of agents, that takes place on markets or elsewhere. The framework of the System provides accounts that are:
>
> a. comprehensive, in that all designated activities and the consequences for all agents in an economy are covered;
>
> b. consistent because identical values are used to establish the consequences of a single action on all parties concerned using the same accounting rules;
>
> c. integrated in that all the consequences of a single action by one agent are necessarily reflected in the resulting accounts, including the impact on measurement of wealth captured in balance sheets.

15.8 The accounts cover the major activities taking place within an economy, such as production, consumption, trade, financing, and the accumulation of capital goods. Some of the flows involved, such as income, saving, lending, and borrowing, do not relate to goods and services, so not all of them can be factored into price and quantity components.[3] However, the *2008 SNA* also contains a comprehensive framework, the SUT, discussed in more detail below, within which the relationships between the main flows of goods and services in the economy are established and displayed. The coverage and contents of these flows are defined, classified, and measured in a conceptually consistent manner. The SUT clearly shows the linkages between major flows of goods and services associated with activities such as production, consumption, distribution, importing, and exporting. It provides an ideal framework for designing and organizing a system of internally consistent price statistics that relate to a set of economically interdependent flows of goods and services. Not only are the relationships between consumer, producer, import, and export prices established within such a table, but so are their linkages with price indices for major macroeconomic aggregates such as GDP.

15.9 This overview of price indices first takes a top-level view of the major national accounts aggregates. It then reviews the underlying construction of these aggregates. It first considers the types of economic agents that the national accounting system recognizes. It then considers the economic accounts kept on transactions that build up to the main aggregates. As

[2]As noted in Chapter 3, price indices may be used for various purposes as deflators and general economic indicators. They also may be used in the calculation of escalators for adjusting contract, government pension, and transfer payments. A distinction may be drawn between a price index, which is defined in this chapter as the price component of relative change in a value aggregate, and an escalator, which is one of the uses of a price index. Although an escalator may be chosen as equal to a selected price index, the optimal determination of escalators can lead to more complex functions of price indices than a simple identity relationship.

[3]Some income flows, such as real national income and terms of trade effects, are defined in terms of the purchasing power of the bundle of goods the deflator represents. For example, if the import price index is used as the numeraire deflator to determine terms of trade effects, the magnitude of, say, the gain is determined in terms of changes in the volume of imports that could be purchased from the terms of trade gain.

these accounts are built from their foundations, precise relationships emerge between the well-known headline price indicators—the CPI, PPI, and XMPIs—and the closely watched national accounts aggregates.

B.1.1 Supply and use of goods and services in the aggregate

15.10 At the most aggregate level, the supply and use of goods and services in the national accounts is the simple textbook macroeconomic identity equating total supply with total uses. Total supply is the sum of output Y, imports M, and taxes less subsidies on products T. Total uses is the sum of intermediate consumption Z, the final consumption of households C and government G, capital formation I, and exports X:[4]

$$Y + M + T = Z + C + G + I + X. \tag{15.1}$$

15.11 Rearranging this identity by subtracting intermediate consumption and imports from both sides reveals the familiar alternative expressions for GDP from the production (*value-added*) and expenditure approaches:

$$\begin{aligned}(Y - Z) + T &= \text{value added} + T \\ &\equiv C + G + I + X - M \\ &= \text{GDP}. \end{aligned} \tag{15.2}$$

GDP is internationally recognized as the central national accounts aggregate for measuring national economic performance. It is essentially a measure of production, distinct from final demand. More precisely, it measures the value added of the productive activity carried out by all the units resident in an economy. Because imports are not included in GDP, a price index for GDP tracks internally generated inflation. Compiling indices for tracking the parts of relative change in GDP and its components that can be attributed to price and volume change is one of the primary objectives for developing price statistics in modern statistical systems.

15.12 As explained in more detail later, the SUT in the *2008 SNA* is a comprehensive matrix covering the economy that exploits the identities, equations (15.1) and (15.2), at a disaggregated level. Each row of the matrix shows the total uses of a commodity, or group of commodities; each column shows the total supplies from domestic industries and imports. The SUT provides an accounting framework that imposes the discipline of both conceptual and numerical consistency on data on flows of goods and services drawn from different sources. The flows have to be defined, classified, and valued in the same way, and any errors have to be reconciled. The SUT provides a good basis for compiling a set of interdependent price and quantity indices. In the following sections, the various elements or building blocks that make up the SUT are outlined. Then reference will be made to SUTs in volume terms to which XMPIs naturally relate.

B.1.2 Institutional units and establishments: Economic agents and units of analysis in the national accounts

15.13 In building the accounting system and the major aggregates $Y, M, T, Z, C, G, I,$ and X of equations (15.1) and (15.2), the *2008 SNA* first organizes the economy of a country into the kinds of entities or agents that undertake economic activity. These agents are called *institutional units* and comprise five types of units that are resident in the economy, as well as a single nonresident category—the rest of the world. An institutional unit is *resident* in an economy if its primary center of economic interest is located there.[5] The five types of resident institutional sectors are nonfinancial corporations, financial corporations, general government, households, and nonprofit institutions serving households (NPISHs). Generally speaking, the *2008 SNA* associates with institutional units the ability to hold title to productive assets, and thus they represent the smallest units on which complete balance sheets can be compiled.[6]

15.14 For analyzing production, the *2008 SNA* identifies a unit or agent smaller than an institutional unit called an *establishment* or *local kind of activity unit* (LKAU). Within an institutional unit, the establishment is the smallest unit organized for production whose costs and output can be identified separately. Generally, establishments specialize in the production of only a few types of output at a single location.

15.15 In addition to production activity, institutional units may engage in final consumption of goods and

[4]This chapter uses the standard terminology of the *2008 SNA*, in which net accumulation of current output to enable future production is called *capital formation* rather than *investment*.

[5]For example, this is determined by physical domicile for households, according to whether the household has been living within the geographic boundaries of a country for a year or more.

[6]The *2008 SNA* classification or sectoring of institutional units does not strictly follow the legal status of institutional units, but rather follows their function. Hence, a government-owned nonfinancial enterprise producing output sold at prices substantially covering its costs and for which a balance sheet can be compiled would be classified as a nonfinancial corporation, along with nonfinancial legal corporations. For further details, see *2008 SNA*, Chapter IV.

Box 15.1. Institutional Sectors in the *System of National Accounts 2008*

S1 Total economy

S11	Nonfinancial corporations
S12	Financial corporations
S121	Central bank
S122	Deposit-taking corporations, except the Central Bank
S123	Money market funds
S124	Non–money market investment funds
S125	Other financial intermediaries, except insurance corporations and pension funds
S126	Financial auxiliaries
S127	Captive financial institutions and money lenders
S128	Insurance corporations
S129	Pension funds
S13	General government
S131[1]	Central government excluding social security on all levels of government
S1311	Central government excluding social security
S1312	State government excluding social security
S1313	Local government excluding social security
S1314	Social security
S132[1]	Central government including social security on all levels of government
S1321	Central government including social security
S1322	State government including social security
S1323	Local government including social security
S14	Households
S141	Employers
S142	Own account workers
S143	Employees
S144	Recipients of property and transfer income
S1441	Recipients of property income
S1442	Recipients of pensions
S1443	Recipients of other transfers
S15	Nonprofit institutions serving households

S2 Rest of the world

[1]S131 and S132 are alternative coding systems.

services and in capital formation, represented by the accumulation of goods and services as productive assets. The *2008 SNA* classification of institutional units into sectors is shown in Box 15.1. Notice that the *2008 SNA* institutional sectors represent the units typically covered in economic and household censuses and surveys. The *2008 SNA*, as indicated by its name, focuses on the activities of institutional units that are resident in a nation. A provision for the rest of the world (S.2 in Box 15.1) is made to capture the transactions of resident institutional units with nonresidents. This enables balanced accounts to be derived for transactions that include that part of output directed to the rest of the world as exports and parts of intermediate consumption, final consumption, and capital formation arising from the rest of the world as imports. Transactions of nonresidents with other nonresidents are out of scope for the national or regional accounts of a given country or region.

B.1.3 Constructing the system of supply and use flows from accounting data on institutional units

15.16 Equations (15.1) and (15.2) identified the basic aggregates comprising the total supply and uses of goods and services in the economy, and derived GDP in terms of these aggregates. To separate the price and volume components of supply and use, it is necessary to build these basic aggregates from the institutional sector accounts of the economy's economic agents. One must detail the production and consumption activities of these agents, as well as the types of goods and services they produce and consume. The framework within which this information is organized is the SUT in the national accounts. As it is built, the SUT also effectively begins to accumulate data on the price (or volume) index weights considered in Chapter 5. The basic accounts of the *2008 SNA* in which all of these aggregates are recorded at the level of institutional units are the *production*, *use of income*, *capital,* and *external goods and services* accounts. These accounts organize the information for the following top-level aggregates:

- Production account: output Y, intermediate consumption Z, and value added $Y - Z$;
- Use of income account: household consumption C and government consumption G;
- Capital account: capital formation K; and
- External goods and services account: exports X and imports M.

B.1.3.1 Recording transactions in goods and services

15.17 Before this chapter further elaborates on these four goods and services accounts, it is important to specify how each entry in the value aggregates comprising them is to be recorded. The items in the value aggregate equation (15.1) represent detailed goods and services flows classified into categories of transactions. There are two defining aspects of recording transactions: timing and valuation.

B.1.3.1.1 Timing of transactions covered

15.18 To associate each transaction with a date, the national accounts consider a transaction to have been consummated when the event takes place that creates the liability to pay. In the case of flows of goods and services, this occurs when the ownership of the good is exchanged or when the service is delivered. When change of ownership occurs or the service is delivered, a transaction is said to have *accrued*. In general, this time need not be the same as the moment at which the payment actually takes place.

B.1.3.1.2 Valuation

15.19 There are two valuation principles in the national accounts, one for suppliers and one for users. For suppliers, transactions in goods and services are valued at *basic prices*. The basic price is the price per unit of good or service *receivable* by the producer.[7] Because the producer does not receive taxes (if any) on products but does receive subsidies (if any) on products, taxes on products are excluded from the basic price, whereas subsidies on products are included.[8] The producer also does not receive invoiced transportation and insurance charges provided by other suppliers, or any distribution margins added by other retail or wholesale service producers, and these also are excluded from the basic price. On the other hand, the user, as purchaser, pays all of these charges. Users' purchases are therefore valued at *purchasers' prices,* which add taxes net of subsidies on products and margins for included transportation, insurance, and distribution services to the basic price.

15.20 Accordingly, output Y and imports M in equations (15.1) and (15.2) are valued at *basic prices*, to which are added taxes less subsidies on products T to arrive at total supply.[9] The components of total uses are

[7]The term *receivable* is used to indicate that the price refers to an *accrued* transaction for the seller, and the term *payable* is used to indicate a transaction that has *accrued* to the purchaser.

[8]The *2008 SNA* distinguishes between *taxes on products* and *other taxes on production*. Taxes net of subsidies on products T includes all taxes payable per unit or as a fraction of the value of goods or services transacted. Included in T are excise, sales, and the nonrefundable portion of value-added taxes, duties on imports, and taxes on exports. Subsidies on products include all subsidies receivable per unit or as a fraction of the value of goods or services produced, including in particular subsidies paid on imports and exports. *Other taxes on production* comprise, for example, taxes on real property and taxes on profits. *Other subsidies on production* include, for example, regular payments by the government to cover the difference between the costs and revenues of loss-making enterprises. Of total taxes and subsidies on production, only taxes and subsidies on *products* are considered in defining basic and purchasers' prices. By implication, there are no taxes payable on products included in either of the aggregates Y or M, whereas subsidies receivable on products are included in these aggregates.

[9]The reader may have noted that transportation, insurance, and distribution margins have somehow disappeared after having been introduced. Whether these services are included with the good or invoiced separately does not affect the total expenditure on goods and services by the purchaser. For the economy as a whole, these transactions cancel out, but when industry activity and product detail are considered, they will have redistributive effects among goods and services products. This point is revisited in the discussion of the SUT below.

valued at purchasers' prices. This is clearly interpreted for the final consumption of households and government. For capital formation expenditures, the notion of purchasers' prices also includes the costs of setting up fixed capital equipment. For exports, purchasers' prices include export taxes net of subsidies. Now each of the four major goods and services accounts is discussed in turn.

B.1.3.2 Production

15.21 An institutional unit engaged in production is said to be an *enterprise*. By implication, any of the five types of resident institutional units can be an enterprise. The *production account* for enterprises in the *2008 SNA* appears, with minor reordering of elements, essentially as shown in Table 15.1. An identical presentation also applies to the establishments or LKAUs owned by enterprises and, in fact, an establishment can be defined operationally as the smallest unit for which a production account can be constructed. There are cases in which an establishment or LKAU is synonymous with, or at least inseparable from, the institutional unit that owns it. This is true of single establishment corporations and of household unincorporated enterprises, for example. In other cases, an enterprise may own multiple establishments. The production account also can be produced for various establishment and enterprise groupings, including institutional sectors, and for establishment industry activity groups. In the production account and throughout the *2008 SNA*, the transaction codes beginning with "P" refer to entries for transactions in goods and services. The codes beginning with "B" refer to so-called "balancing and net worth items," which are defined residually as the difference between a resources total and the sum of itemized uses of those resources. B1g, for example is the balancing value added, the "g" referring to it being gross value added.

15.22 For classifying an establishment or LKAU, output is broken down into market output and two types of nonmarket output. Market output (P11) is sold at economically significant prices substantially covering the cost of production. Nonmarket output is provided without charge or at prices so low they bear no relationship to production cost. The two types of nonmarket output are output for own final use (P12) and other nonmarket output (P13). Output for own final use includes the production of, for example, machine tools and structures (fixed capital formation items) by an establishment for the sole use of the establishment itself or other establishments in the same enterprise; the imputed rental value of certain productive assets owned by households, such as (and currently limited to) owner-occupied dwellings; and the production of certain other unincorporated household enterprises, such as agricultural products produced by a farmer for consumption by his or her own family or employees. Other nonmarket output comprises the output of general government and NPISHs distributed free or sold at prices that are not economically significant. For price index construction, only those transactions of establishment units that involve economically significant prices, and thus market output (P11), are relevant. However, the prices collected for market output items also may

Table 15.1. Production Account for an Establishment, Institutional Unit, or Institutional Sector

Uses		Resources	
P2	**Intermediate consumption (purchasers' prices)**	**P1**	**Output (basic prices)**
B.1g	*Gross value added* (balances the account; i.e., it is the difference between output [P1] and intermediate consumption [P2])		
			Of which, memorandum items breaking down total output for classifying the market/nonmarket status of the producer unit:
			P11 Market output
			P12 Output for own final use
			P13 Other nonmarket output

Note: *2008 System of National Accounts* (*2008 SNA*) items in bold refer to flows in goods and services.

be used to value the own final use portion of nonmarket output (P12). Our scope of coverage for price indices thus extends to cover this component of nonmarket output as well.

15.23 A production unit's resources derive from the value of its output, and its uses of resources are the costs it incurs in carrying out production. The production account therefore uses both the basic price and purchasers' price methods of valuation, as appropriate to a production unit in its roles as a supplier and a user of products. For the *supply (resources)* of goods and services, products are valued at *basic prices*, the national currency value *receivable* by the producer for each unit of a product. They include subsidies and exclude the taxes on products and additional charges or margins on products to pay for included retail and wholesale trade services, and for included transportation and insurance. For *uses* of goods and services, products are valued at *purchasers' prices,* the national currency value *payable* by the user for each unit of a product, including taxes on products, and bundled but separately invoiced services, such as trade and transport margins, and excluding subsidies on products.

15.24 *Product detail in the production account.* In addition to breaking down output into its market and nonmarket components, output and intermediate consumption also can be broken down by type of product. With product types classified using, for example, the international standard Central Product Classification (CPC, version 2.0), the production account for each establishment appears as in Table 15.2.

15.25 *Industry detail in the production account.* The entries in Table 15.3 of total output by product and total market and nonmarket outputs for each establishment allow us to classify establishments by their principal activity or industry *and* market/nonmarket status. To reflect the information required for this

Table 15.2. Production Account with Product Detail for an Establishment or Local Kind of Activity Unit

Note: Establishment ID: eeeeeeee. Activity/industry code (ISIC): aaaa.	Institutional unit ID: uuuuuuuu Institutional sector code: S.nnnnn Market status: P1n.
Uses	Resources
P2 Intermediate consumption (purchasers' prices), *of which*	**P1 Output (basic prices), *of which***
CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products
CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment
CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment
CPC 5 Constructions and construction services	CPC 5 Constructions and construction services
CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services
CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services
B.1g *Gross value added*	
	Memorandum items breaking down total output for classifying the market/nonmarket status of the producer:
	P11 Market output **P12 Output for own final use** **P13 Other nonmarket output**

Note: *2008 SNA* items in bold refer to flows in goods and services.

Table 15.3. Industry/Activity Production Account with Detail for Products and Market/Nonmarket

Note: Activity/industry code (ISIC): aaaa.

Uses			Resources		
P2 Intermediate consumption (purchasers' prices), market, *of which*	**P2 Intermediate consumption (purchasers' prices), own final use, *of which***	**P2 Intermediate consumption (purchasers' prices), other nonmarket, *of which***	**P11 Output (basic prices), market, *of which***	**P12 Output (basic prices), own final use, *of which***	**P13 Output (basic prices), other nonmarket, *of which***
			PPI output aggregate		
CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry and fishery products
CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and mineral; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages and tobacco; textiles, apparel and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery and equipment
CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery and equipment
CPC 5 Construction and construction services	CPC 5 Construction and construction services	CPC 5 Construction and construction services	CPC 5 Construction and construction services	CPC 5 Construction and construction services	CPC 5 Constructions and construction services
CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food and beverage serving services; transport services; and utilities distribution services
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services
CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social and personal services
B.1 *Gross value added, market output establishments*	B.1 *Gross value added, output for own final consumption establishments*	B.1 *Gross value added, other nonmarket output establishments*			

Note: *2008 SNA* items in bold refer to flows in goods and services.

classification, positions for the activity and market or nonmarket classification codes of the establishment are shown in the first line of Table 15.2.[10] The activity classification involves principally, if not exclusively, sorting establishments according to the types of product (CPC code *ccccc* or other product code, such as the Classification of Products by Activity) for which the total output is greatest. The major categories of the International Standard Industrial Classification of All Economic Activity (ISIC), Revision 4, are shown in Box 15.2.

15.26 The associated products are grouped in the production accounts by activity and output transaction status, and each entry of the accounts is summed across all establishments within each industry *and* output transaction status category. Table 15.3 shows a model production account for an industry (identified by activity code *aaaa*). This account is an aggregate of the production accounts of establishments classified into that industry and according to whether they are principally market, own final use, or other nonmarket producers. In most cases, both the establishment and industry production accounts would show higher product detail than has been shown here, preferably at the four- or five-digit CPC level, or higher with country-specific extensions.

15.27 *Output aggregate for the PPI in the production account.* The PPI is an index of the prices of the outputs of establishments. The position of the PPI in the *2008 SNA* is defined by the relationship of its output value aggregate to those defined in the national accounts. Box 15.2 considers the formation of the PPI value aggregate according to its industry coverage, arguing that the PPI's industry coverage should be complete. The coverage of the PPI across the type of output by market status is shown under the column of Table 15.3 labeled P11 Output (basic prices), market. For most establishments, output for own final use, P12, comprises only capital formation, such as acquisition of machine tools or construction. Household establishments also may produce goods for households' own consumption, such as food, and this activity is included within the *2008 SNA* production boundary. Large portions of P12, output for own final use, may be valued at market prices if close market substitutes are available but otherwise at the cost of production (*2008 SNA*, Paragraph 6.91). In principle the weighting of items in the PPI could be extended to cover the market-valued portion of P12. The scope of the PPI would not extend to P13, other nonmarket output, because this is almost without exception valued at production cost because rarely are market equivalents available, and thus there is no basis for constructing an explicit price index.

B.1.3.3 Consumption

15.28 Final consumption of goods and services in the *2008 SNA* is shown in the *use of income account*, which appears essentially as in Table 15.4 for each institutional unit. Recall that the *2008 SNA* designates goods and services items with the codes "P*n*." These goods and services flows can be decomposed into price and volume components and thus would draw our interest as price index compilers. Items of final consumption are designated by P3 with extensions. P3 comprises individual consumption expenditure (P31) and collective consumption expenditure (P32).[11]

15.29 *Individual consumption, actual consumption, and household consumption expenditures.* The *2008 SNA* distinguishes individual from collective goods and services, a distinction equivalent to that between private and public goods in economic theory. It is mainly relevant to services. Individual services are provided to individual households and benefit those particular households, whereas collective services are provided to the community—for example, public order, administration, security, and defense. Many individual services such as education, health, housing, and transportation may be financed

[10]As indicated in Table 15.3, the *2008 SNA* recommends use of the International Standard Industrial Classification of All Economic Activities (ISIC) for activities or industries, the CPC for domestic products, and the closely related Harmonized Commodity Description and Coding System for exported and imported products. Each country may adapt the international standard to its specific circumstances. If the adaptation amounts to adding further detail, the classification is said to be derived from the international standard. The Nomenclature génerale des Activités économiques dans les Communautés Européennes, or the General Industrial Classification of Economic Activities within the European Communities, is an industrial classification derived from the ISIC. If the adaptation reorganizes the way in which detailed categories are grouped compared with the international standard but provides for a cross-classification at some level of detail, it is said to be related. The North American Industrial Classification System being implemented by Canada, Mexico, and the United States is an industrial classification related to the ISIC. The European Union's PRODCOM classification of industrial products is derived from its Classification of Products by Activity, which, in turn, is related to the international standard CPC through a cross-classification defined at a high level of product detail.

[11]Final consumption expenditure (P3) is made by institutional units classified in the general government (S13), household (S14), and NPISH (S15) institutional sectors only. Corporations (S11) and (S12) do not have final consumption expenditure, and thus for these units operating surplus (B2g) is equal to saving (B8g) in the use of income account (Table 15.4).

Box 15.2. Industry/Activity Coverage of the Producer Price Index Output Value Aggregate

The principal economic activities of the International Standard Industrial Classification of All Economic Activities (ISIC), Revision 4, are

A	Agriculture, forestry, and fishing
B	Mining and quarrying
C	Manufacturing
D	Electricity, gas, steam, and air conditioning supply
E	Water supply; sewerage, waste management, and remediation activities
F	Construction
G	Wholesale and retail trade; repair of motor vehicles and motorcycles
H	Transportation and storage
I	Accommodation and food service activities
J	Information and communication
K	Financial and insurance activities
L	Real estate activities
M	Professional, scientific, and technical activities
N	Administrative and support service activities
O	Public administration and defense; compulsory social security
P	Education
Q	Human health and social work activities
R	Arts, entertainment, and recreation
S	Other service activities
T	Activities of households as employers; undifferentiated goods and services
U	Activities of extraterritorial organizations and bodies

These are characteristic of the activities identified in most national industrial classifications. In assembling data on the supply and use flows in the economy, a detailed industry production account such as that in Table 15.3 is effectively constructed for each type of activity in the economy. The major activity categories are shown in the ISIC list above. (More is said about the comprehensive presentation of supply and use for the total economy later in Section B.1.3.) With the product output and expenditure detail, Table 15.3 shows more explicitly the typical goods and services coverage of the producer price index (PPI) within the output aggregate P1 of the production account for each industry. In most countries, PPIs cover goods-producing industries, such as the "mining and manufacturing" activities B and C and sometimes also agriculture, forestry, and fishing (A) and construction (F). Most PPIs also cover the "industrial" service activities electricity, gas, steam, and air conditioning supply (D); water supply and sewerage, waste management, and remediation activities (E); transportation and storage (H); and information and communication (J). In principle, the PPI should cover the market output of all activities, and a number of countries are working on rounding out PPI coverage of other service-producing activities beyond transportation and utilities.

and paid for by government or nonprofit institutions and provided free or at a nominal price to individual households. A large part of government consumption expenditure is not on public goods but on goods or services supplied to individual households. These individual consumption expenditures by governments and NPISHs are described as *social transfers in kind* in the *2008 SNA*.

15.30 Household consumption can have three distinct meanings. First, it can mean the total set of individual consumption goods and services *acquired* by households, including those received as social transfers in kind. Second, it can mean the subset that households actually *pay for* themselves. To distinguish between these two sets, the *2008 SNA* describes the first as the *actual final consumption* of households and the second as *household final consumption expenditures*. A third possible interpretation of household consumption is the actual physical process of consuming the goods and services. It is this process from which utility is derived and that determines

Table 15.4. Use of Income Account for Institutional Units and Sectors

Note : Institutional unit ID: uuuuuuuu, Institutional sector code: S.nnnnn.

Uses		Resources	
P3	**Final consumption expenditure (purchasers' prices)**[1]	B.6	*Disposable income*[2]
P31	**Individual consumption expenditure**		
	P311 Monetary consumption expenditure		
	P312 Imputed expenditure on owner-occupied housing services		
	P313 Financial Intermediation Services Implicitly Measured (FISIM)		
	P314 Other individual consumption expenditure		
P32	**Collective consumption expenditure (general government sector S13 only)**		
D8	Adjustment for the change in the net equity of households in pension funds[3]		
B.8g	*Saving* (balances the account; i.e., it is the difference between disposable income [B.6g] and the sum of expenditures [P3] and adjustment [D8])		

Note: *2008 SNA* items in bold refer to flows in goods and services.

[1]By definition, corporations have no final consumption in the *2008 SNA*. Thus, item P3 and its subdivisions appear with nonzero entries only for household, government, and NPISH units.

[2]The *2008 SNA* derives disposable income in a sequence of accounts producing the balancing items *Value added* B.1g (production account), *Operating surplus* B.2g and *Mixed income* B.3g (generation of income account), *Balance of primary incomes* B.5g (allocation of primary income account), and *Disposable income* B.6g (secondary distribution of income account). Collapsing all of these steps, *Disposable income* B.6g is *Value added* B.1g *less* (net) taxes on production and imports (payable) D2 *plus* (net) subsidies D3 (receivable), *plus* compensation of employees receivable, D1, *plus* (net) property income (receivable) D4, *less* (net) taxes on income and wealth (payable) D5, *less* (net) social contributions (payable) D61, *plus* (net) social benefits (receivable) D62, *less* (net) other transfers (payable) D7.

[3]This adjustment reflects the treatment by the *2008 SNA* of privately funded pensions as owned by the household beneficiaries of such plans. It maintains consistency between the income and accumulation accounts in the system. It is not relevant to price and volume measurement, and the reader is referred to the *2008 SNA*, Chapter IX, Section A.4, for further details.

households' standard of living. The process of consuming or using the goods or services can take place some time after the goods or services are acquired, because most consumer goods can be stored. The distinction between acquisition and use is most pronounced in the case of consumer durables that may be used over a long time. The treatment of durables is discussed further in Box 15.3.

15.31 The existence of social transfers in kind is not recognized in CPIs, although one should take account of them, especially when considering changes in the cost of living. Moreover, governments may start to charge for services that previously were provided free, a practice that has become increasingly common in many countries. The goods and services provided free as social transfers could, in principle, be regarded also as being part of household consumption expenditures but having a zero price. The shift from a zero to positive price is then a price increase that could, at least in principle, be captured by a CPI.

15.32 *Monetary and imputed expenditures.* Not all household expenditures are monetary. A monetary expenditure is one in which the counterpart to the good or service acquired is the creation of some kind of financial liability. This liability may be immediately extinguished by a cash payment, but many monetary expenditures are made on credit. Household consumption expenditures also include certain imputed expenditures on goods or services that households produce for themselves. These are treated as expenditures because households incur the costs of producing them (in contrast to social transfers in kind, which are paid for by government or nonprofit institutions).

15.33 The imputed household expenditures recognized in the *2008 SNA* include all of those on goods that households produce for themselves (mainly agricultural goods in practice) but exclude all household services produced for own consumption *except* for housing services produced by owner occupiers. The imputed prices

Box 15.3. The Treatment of Housing and Consumer Durables in the *2008 SNA* and CPIs

Dwellings are fixed assets. Purchases of dwellings by households therefore constitute household gross fixed capital formation and are not part of household consumption. They cannot enter into a price index for household consumption. Fixed assets are used for purposes of production, not consumption. Dwellings therefore have to be treated as fixed assets that are used by their owners to produce housing services. The *2008 SNA* actually sets up a production account in which this production is recorded. The services are consumed by the owners. The expenditures on the services are imputed, the services being valued by the estimated rentals payable on the market for equivalent accommodation. The rentals have to cover both the depreciation on the dwellings and the associated interest charges or capital costs.

The existence of these imputed expenditures on owner-occupied housing services has always been recognized in national accounts, and most countries also have included them in their consumer price indices (CPIs), even though other imputed expenditures are not included.

Consumer durables, such as automobiles, cookers, and freezers, also are assets used by their owners over long periods of time. In principle, they could be treated in the same way as dwellings and be reclassified as fixed assets that produce flows of services consumed by their owners. For certain analytic purposes, it may be desirable to treat them this way. However, to do so in the *2008 SNA* would not simply be a matter of estimating the market rentals that would be payable for hiring the assets. It also would be necessary to set up production accounts in which the durables are used as fixed assets. This has traditionally been regarded as too difficult and artificial. There also are objections to extending further the range of imputed flows included in the *2008 SNA* and GDP. In practice, therefore, expenditures on durables are classified in the *2008 SNA* as consumption expenditures and not as gross fixed capital formation, a practice carried over into CPIs.

at which the included goods and services are valued are their estimated prices on the market. In the case of housing services, these are imputed market rentals. In practice, most countries follow the *2008 SNA* by including owner-occupied housing in the CPI. Some countries include other imputed expenditures, such as for vegetables, fruit, dairy, or meat products produced for own consumption.

15.34 *A hierarchy of household consumption aggregates.* The following hierarchy of household consumption aggregates that are relevant to CPIs may be distinguished in the *2008 SNA*. It is worth noting that all household consumption expenditures are individual expenditures, by definition.

P41 Actual individual consumption, *of which*

- D6 Social transfers in kind, comprising individual consumption expenditure P31 of general government S13 and NPISHs S15.
- P31 Individual consumption expenditure of households S14, *of which*
  - P311 Monetary consumption expenditure;
  - P312 Imputed expenditure on owner-occupied housing services;
  - P313 Financial Intermediation Services Implicitly Measured (FISIM);
  - P314 Other individual consumption expenditure:
    - Expenditures on nonhousing production for own consumption;
    - Expenditures on goods and services received by employees as income in-kind.

15.35 The codes P311, P312, P313, and P314 do not exist in the *2008 SNA* but are introduced for convenience here. These four subcategories of household consumption expenditures are separately specified in Tables 15.4, 15.5, and 15.6. As already noted, D6 and P314 are usually excluded from the calculation of CPIs.

15.36 It is worth noting that the treatment of financial services in the *2008 SNA* would imply an augmented treatment of financial services consumption expenditures to include expenditures on bank services not

Table 15.5. Use of Income Account with Product Detail for Institutional Units and Sectors

Note: Institutional unit ID: uuuuuuuu. Institutional sector code: S.nnnnn.

Uses					Resources
P31 Individual consumption expenditure			**P32 Collective consumption expenditure**	**P3 Final consumption expenditure (total, purchasers' prices)**	B.6g *Disposable income*
Monetary individual consumption expenditure (P311)	*Owner-occupied housing services (P312) and FISIM (P313)*	*Other individual consumption expenditure (P314)*	*Entries for general government sector (S13) only*		
CPC 0 Agriculture, forestry, and fishery products		CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	
CPC 1 Ores and minerals; electricity, gas, and water		CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products		CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	
CPC 3 Other transportable goods, except metal products, machinery, and equipment		CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	
CPC 4 Metal products, machinery, and equipment		CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	
CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services		CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services[1]	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	
CPC 8 Business and production services		CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	
CPC 9 Community, social, and personal services		CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	
				D7 Change in pension entitlements (households)	
				B8g *Saving*	

Note: Left columns show detail of far right column. *2008 SNA* items in bold refer to flows in goods and services. Sector titles in italics indicate whether the column appears in the use of income account for that sector.

[1]In addition to the real estate, rental, and leasing services of homeowners, the *2008 SNA* treats financial services consumption expenditure as the sum of measured and imputed components. Measured expenditures comprise explicit service charges levied by financial institutions for deposit, loan, advisory services, and the like, whereas imputed expenditures reflect the income forgone because the household does not lend (keep deposits with a financial institution) or borrow at a reference rate—see Chapter 11. In principle, these imputed expenditures, as well as those for other imputed consumption, are of the same market-equivalent valued type as for owner-occupied housing services and could be covered in the consumer price index.

Table 15.6. Use of Income Account with Product Detail for the Total Economy

Note: Institutional unit ID: uuuuuuuu.	Institutional sector code: S.nnnnn.				B6g *Disposable income*, total economy S1*, with uses comprising*
P31 Individual consumption expenditure, Total economy S1 (purchasers' prices), *comprising*				**P32 Collective consumption expenditure, total economy S1 (purchasers' prices),** *comprising*	**P3 Final consumption expenditure, Total economy S1,** ***of which***
P31 Individual consumption expenditure, Household sector S14			**P31 Individual consumption expenditure, General government S13 and NPISH S15 sectors**	**P32 Collective consumption expenditure, General government sector S13**	
Consumer price index reference aggregate #1					
CPI reference aggregate #2[1]					
Monetary individual consumption expenditure (P311)	*Owner-occupied housing services (P312) and FISIM (P313)*	*Other individual consumption expenditure (P314)*			
CPC 0 Agriculture, forestry, and fishery products		CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry and fishery products
CPC 1 Ores and minerals; electricity, gas, and water		CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products		CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages and tobacco; textiles, apparel and leather products
CPC 3 Other transportable goods, except metal products, machinery and equipment		CPC 3 Other transportable goods, except metal products, machinery and equipment	CPC 3 Other transportable goods, except metal products, machinery and equipment	CPC 3 Other transportable goods, except metal products, machinery and equipment	CPC 3 Other transportable goods, except metal products, machinery and equipment
CPC 4 Metal products, machinery, and equipment		CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment
CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services		CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services
CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services	CPC 7 Financial and related services; real estate services; and rental and leasing services
CPC 8 Business and production services		CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social, and personal services		CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social, and personal services	CPC 9 Community, social and personal services
					D7 Change in pension entitlements (households *S14*)
					B8 *Saving, total economy S1*

Note: Left columns show detail of far right column. *2008 SNA* items in bold refer to flows in goods and services.
[1]See also Table 15.7, Capital Account.

separately distinguished from interest charge, as well as the explicit expenditures on service charges charged directly. This is indicated in the footnote to the CPC 7 item in Table 15.5.

15.37 *Product detail in the use of income account.* As with the production accounts of establishments owned by institutional units, the product detail of goods and services consumption can be expanded in the use of income account according to the type of product consumed. To maintain the integration of the system of price and volume statistics on consumption with those that have just been covered on production, products would be classified according to the same system as in the production account. Table 15.5 shows the major categories of the CPC, version 2.0, within the components of final consumption expenditure.[12]

15.38 *The expenditure aggregate of CPI in the use of income account.* The detailed use of income accounts for institutional sectors can be assembled into a consolidated framework by choosing columns from Table 15.5 for each sector and displaying them together as in Table 15.6. Table 15.6 shows an economy-wide presentation of final consumption and saving. It also shows that total economy individual consumption comprises the individual consumption entries (P31) of the household, NPISH, and general government sector use of income accounts. Table 15.6 separately shows the final collective consumption of government (P32) and consolidates the disposable income (B6g) of all three. The account in Table 15.6 has been arranged specifically to show the consumption coverage of the typical CPI, which comprises the first and second columns.

B.1.3.4 Capital formation

15.39 Capital formation comprises the accumulation of fixed tangible and intangible assets, such as equipment, structures, and software; changes in inventories and works in progress; and acquisitions less disposals of valuables, such as works of art. These items are accounted for in the *2008 SNA capital account*, which appears essentially as in Table 15.7 for each institutional unit.

15.40 B9g *Net lending* (+)/*net borrowing* (−) is the balancing item of the capital account. It makes the uses on the left, comprising net acquisitions of stocks of various tangible and intangible items, add up to the resources on the right, comprising the sources of income financing them. From the section on institutional units and establishments, it would be easy to conclude that the smallest economic unit to which the capital account can apply is the institutional unit. It was asserted earlier that only institutional units maintain balance sheets and can monitor stock variables that are the focus of this account. However, the physical capital assets whose changes are tracked in the capital account can and should be compiled, if possible, at the establishment or LKAU level to allow production of data on capital formation by industry. Such data are particularly useful for productivity analysis, even though complete capital accounts cannot be compiled at the establishment level. As with the other goods-and-services-related accounts in the *2008 SNA*, the capital account's goods and services items, designated by the codes P5 with extensions, can be exploded by product type.[13] The account, therefore, can be rearranged to show this goods and services detail as in Table 15.8, which, as for Table 15.7, may pertain to an institutional unit, an institutional sector aggregate, or the total economy.

B.1.3.5 External trade

B.1.3.5.1 The external account of goods and services: The nonresident's view of exports and imports

15.41 The external account of goods and services is shown in Table 15.9. It contains the transactions of nonresident institutional units sector—S2 rest of the world—with the five types of resident units taken together. The external goods and services account is generally taken from the balance of payments, which uses adjusted merchandise trade information from the customs records for goods on P61 and P71, and assembles services data on P62 and P72 from various sources. Note, however that the *2008 SNA*

[12]Although the discussion in this chapter maintains a consistent product classification of expenditure across all goods and services accounts, alternative, functional classifications of expenditure have been developed for each institutional sector for specific purposes. The international standard versions of these classifications included in the *2008 SNA* comprise the Classification of Individual Consumption by Purpose (COICOP), the Classification of the Purposes of Nonprofit Institutions Serving Households, the Classification of the Functions of Government, and the Classification of the Purposes of Producers. The first column of Tables 15.5 and 15.6 is often compiled from household expenditure survey data, which are collected using functional classifications such as COICOP rather than product classifications. To facilitate constructing the cross-economy framework of the *2008 SNA* considered in this chapter, there is a concordance between the CPC and the COICOP.

[13]In addition to the CPC, version 2.0, shown here, the *2008 SNA*, Annex V, contains a nonfinancial assets classification identifying the specific tangible, intangible, produced, and nonproduced fixed assets, as well as inventory and valuables items, recognized by the *2008 SNA*.

Table 15.7. Capital Account

Note: Institutional unit ID: uuuuuuu. Institutional sector: S.nnnnn.

Uses	Resources
P5 Gross capital formation	B101 *Changes in net worth due to saving and capital transfers, of which*
P51g Gross fixed capital formation	
P511 Acquisitions less disposals of fixed assets	B8n *Saving, net*
P5111 Acquisitions of new fixed assets	B8g *Saving (from use of income account)*
Of which, residential dwellings	P51c Consumption of fixed capital (–)
CPI reference aggregate #2	
P5112 Acquisitions of existing fixed assets	
Of which, residential dwellings	
CPI reference aggregate #2	
P5113 Disposals of existing fixed assets	
Of which, residential dwellings	
CPI reference aggregate #2	
P512 Costs of ownership transfer on non-produced assets	
P52 Changes in inventories	
P53 Acquisitions less disposals of valuables	D9r Capital transfers receivable (+)
	D92r Investment grants receivable
P51c Consumption of fixed capital (–)	D99r Other capital transfers receivable
	D9p Capital transfers payable (–)
NP Acquisitions less disposals of non-produced assets	D91p Capital taxes payable
NP1 Acquisitions less disposals of natural resources	D92p Investment grants payable
NP2 Acquisitions less disposals of contracts, leases, and licenses	D99p Other capital transfers payable
NP3 Purchases less sales of goodwill and marketing assets	
B9 *Net lending (+)/net borrowing (−)*	

Note: *2008 SNA* items in bold refer to flows of goods and services.

differs from the IMF's *Balance of Payments and International Investment Position Manual, Sixth Edition* (*BPM6*) in compiling the external accounts from the nonresident's point of view rather than the resident's point of view: For a given cross-border transaction, a *BPM6* resident's goods and services credit (export) is a *2008 SNA* nonresident's use (export) and a *BPM6* resident's debit (import) is a *2008 SNA* nonresident's resource (import). As with the other accounts, the external goods and services account can be enlarged to show product detail, as in Table 15.10.

B.1.3.5.2 External trade as a component of output, intermediate and final consumption, and capital formation: The resident's view of exports and imports

15.42 Besides the direct contribution of imports to total supply and exports to total uses, users of national accounts statistics are interested in knowing the contribution of exports and imports to developments in, respectively, the output component of total supply and the part of total uses comprising intermediate consumption, final consumption, and capital formation (excluding

Table 15.8. Capital Account with Product Detail

Note : Institutional unit ID: uuuuuuuu.	*Institutional sector code:* **S***nnnnn.*				B101 *Change in net worth due to saving and capital transfers, with uses comprising*
P 51g Gross fixed capital formation			**P52 Change in inventories**	**P53 Acquisitions less disposals of valuables**	**P5 Gross capital formation**
AN111 Dwellings **AN112 Other buildings and structures** **AN113 Machinery and equipment** **AN114 Weapons systems** **AN115 Cultivated biological resources**	**AN116 Costs of ownership transfer on non-produced assets**	**AN117 Intellectual property products**	**AN12 Inventories by type of inventory**	**AN13 Valuables**	
CPC 0 Agriculture, forestry, and fishery products			CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products
CPC 1 Ores and minerals; electricity, gas, and water			CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products			CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment			CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment
CPC 4 Metal products, machinery, and equipment			CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery, and equipment
CPC 5 Constructions and construction services	CPC 5 Constructions and construction services		CPC 5 Constructions and construction services		CPC 5 Constructions and construction services *Of which Dwellings*
	CPC 7 Financial and related services; real estate services; and rental and leasing services				*CPI ref aggregate #2*
	CPC 8 Business and production services				P6 Consumption of fixed capital
					NP Acquisitions less disposals of nonproduced assets
					B9 *Net net lending/borrowing*

Note: *2008 SNA* items in bold refer to flows of goods and services.

Table 15.9. External Account of Goods and Services

Uses	Resources
P6 Exports of goods and services	**P7 Imports of goods and services**
P61 Exports of goods	**P71 Imports of goods**
P62 Exports of services	**P72 Imports of services**
B11 *External balance of goods and services*	

Note: All resident institutional units S1.nnnn with nonresident institutional units S.2. *2008 SNA* goods and services items shown in bold.

Table 15.10. External Account of Goods and Services with Product Detail

Uses	Resources
P6 Exports of goods and services	**P7 Imports of goods and services**
Export Price Index uses aggregate	***Import Price Index supply aggregate***
P61 Exports of goods	**P71 Imports of goods**
At purchasers' prices of which	*At basic prices of which*
CPC 0 Agriculture, forestry, and fishery products	CPC 0 Agriculture, forestry, and fishery products
CPC 1 Ores and minerals; electricity, gas, and water	CPC 1 Ores and minerals; electricity, gas, and water
CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products	CPC 2 Food products, beverages, and tobacco; textiles, apparel, and leather products
CPC 3 Other transportable goods, except metal products, machinery, and equipment	CPC 3 Other transportable goods, except metal products, machinery, and equipment
CPC 4 Metal products, machinery, and equipment	CPC 4 Metal products, machinery and equipment
	Less: *Adjustment to total imports of goods CIF for insurance and freight provided by both residents and nonresidents for delivery to the first resident recipient.*
P62 Exports of services	**P72 Imports of services**
CPC 5 Constructions and construction services[1]	CPC 5 Constructions and construction services[1]
CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services, *of which*	CPC 6 Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services, *of which*
• Distributive trade services; accommodation, food and beverage-serving services; transport services; and utilities distribution services; *except* Transport services on imports and exports rendered by residents	• Distributive trade services; accommodation, food, and beverage-serving services; transport services; and utilities distribution services; *except* transport services on imports rendered by nonresidents
• Transport services on imports and exports rendered by residents	• Transport services on imports and exports rendered by nonresidents
CPC 7 Financial and related services; real estate services; and rental and leasing services, *of which*	CPC 7 Financial and related services; real estate services; and rental and leasing services, *of which*
• Financial and related services; real estate services; and rental and leasing services; *except* Insurance services on imports rendered by residents	• Financial and related services; real estate services; and rental and leasing services; *except* insurance services on imports rendered by nonresidents
• Insurance services on imports rendered by residents	• Insurance services on imports rendered by nonresidents
CPC 8 Business and production services	CPC 8 Business and production services
CPC 9 Community, social ,and personal services	CPC 9 Community, social, and personal services
B.11 *External balance of goods and services*	

Note: All resident institutional units S1 with nonresident institutional units S.2; *2008 SNA* goods and services items shown in bold.
[1]Construction services only.

from final uses, by implication, exports). The valuation of these main supply and use aggregates follows basic prices for domestic suppliers' and purchasers' prices for domestic users in the accounts where these aggregates appear. The affected accounts are the production account for output and intermediate consumption, the use of income account for consumption, and the capital account for capital formation. Thus, as noted in Section B.1.5.3.1, the export and import subcomponents of these aggregates follow a *resident supplier-user orientation* coincident with that of the *BPM6*. Exports are then valued at basic prices as a subcomponent of output in the production account and of disposals of fixed capital, inventories, and valuables in the capital account.

15.43 Consider now the uses of goods at purchasers' prices in the international trade context of this *Manual*. When taking the resident's view of imports as an intermediate or final use of goods and services, it is necessary to distinctly consider the import status of separately invoiced items as well as the distribution margins in the same purchase transaction. These separately invoiced "subtransactions" for goods, transport services, insurance services, distribution services, and the like must be classified as imported or domestically supplied to fully account for the contribution imports make to a given goods and services flow for the purchasers' values of intermediate uses by product and industrial activity and of final uses by product.

15.44 Although codes for the resident's view of export and import aggregates do not exist in the *2008 SNA*, this *Manual* creates them to clarify the role of external trade in the goods and services accounts. These created codes are shown below in bold to distinguish them from the *2008 SNA* codes.

P1 Output [production account]

P11 Market output

P11D Market output for domestic users

P11X Exported market output

P12 Output for own final use

P13 Nonmarket output

P13D Nonmarket output for domestic users

P13X Exported nonmarket output

P2 Intermediate consumption [production account]

P2D Intermediate consumption from domestic suppliers

P2M Imported intermediate consumption

P3 Final consumption [use of income account]

P3D Final consumption from domestic suppliers

P3M Imported final consumption

P5 Capital formation [capital account]

P51n Net fixed capital formation

P51g Gross fixed capital formation

P511 Acquisitions less disposals of fixed assets

P5111 Acquisitions of new fixed assets

P5111D Acquisitions of new fixed assets from domestic suppliers

P5111M Imports of new (transportable) fixed assets

P5112 Acquisitions of existing fixed assets

P5112D Acquisitions of existing fixed assets from domestic suppliers

P5112M Imports of existing (transportable) fixed assets

P5113 Disposals of existing fixed assets

P5113D Domestic disposals of existing fixed assets

P5113X Exports of existing (transportable) fixed assets

P512 Costs of ownership transfer on nonproduced assets

P52 Changes in inventories

P52AD Additions to inventories from domestic suppliers

P52AM Imports of additions to inventories

P52WD Withdrawals from inventories to domestic users

P52WX Exports of withdrawals from inventories

P53 Acquisitions less disposals of valuables

P53AD Acquisitions of valuables from domestic suppliers

P53AM Imports of valuables

P53WD Disposals of valuables to domestic users

P53WX Exports of valuables

15.45 This breakdown of exports and imports from the resident's point of view provides a road map to developing economic index numbers from the resident's view for these aggregates in Chapter 18. To illustrate the main principles of economic index numbers in the external trade context, that chapter deals only with exports as a subaggregate of output P1 (P11X + P13X) and imports as a subaggregate of intermediate consumption P2 (P2M). The economic index for imports in final consumption (P3M) flows from household theory (see the *Consumer Price Index Manual* (ILO and others, 2004a), Chapter 17), augmented by a utility theory of government consumption. Economic index numbers for the fixed capital and inventory items are not yet well developed. Consequently, neither are their export (P5113X + P52WX + P53WX) and import (P5111M + P5112M + P52AM + P53AM) subindices. Although it is not dealt with in this *Manual*, economic index numbers of exports P6 and imports P7 from the nonresident view are the mirror image of economic index numbers for exports and import from the resident view. The economic price index number for exports P6 is the rest of the world import price index (at rest of world purchasers' values) and for imports P7 is the rest of the world export price index (at rest of world basic values).

B.1.3.6 The Supply and Use Table

15.46 The SUT arrays the industries side by side first for market producers, then for own account producers, and then for other nonmarket producers under *Resources* and *Uses*. A SUT is shown in Table 15.11. It arrays various accounts relevant to monitoring developments in production and consumption within a country according to the *supply* of goods and services (with reference to the *2008 SNA* codes labeling the regions of Table 15.11)

- From resident establishments (arranged in industries) in the form of domestic output (P1)**,** given by Y in equations (15.1) and (15.2);
- From the rest of the world as imports (P7), given by M in equations (15.1) and (15.2);
- Adjusted for trade and transport margins[14] and taxes less subsidies on products (D21 through D31), given by T in equations (15.1 and 15.2);

and the ***uses*** of goods and services

- For current inputs into production by resident producers (arranged in industries) in the form of intermediate consumption (P2), given by Z in equations (15.1) and (15.2);
- For final domestic consumption, including individual consumption by resident households, resident NPISHs, and the government (P31), and collective consumption by the government (P32), given by, respectively, C and G in equations (15.1) and (15.2);
- Capital formation by resident enterprises (P5) (comprising fixed capital formation (P51g), inventory change (P52), and acquisitions less disposals of valuables (P53)), given by I in equations (15.1) and (15.2); and
- For export (P6) and use by the rest of the world, given by X in equations (15.1) and (15.2).

15.47 The aggregates P6 (exports) and P7 (imports) describe flows and incorporate valuation principles consistent with the view *nonresidents* take of these flows, namely, that exports are a *use* by the rest of the world of domestically supplied goods and services, and imports are a *supply* by the rest of the world to domestic users. Exports thus are valued at the rest of the world's purchasers' prices and imports at the rest of the world's basic prices, as noted above in Section B.1.3.5.1.

15.48 To reflect the *resident* view of exports discussed in Section B.1.3.5.2, the supply (output) matrix P1 of the SUT would be "delaminated" element by element into flows to domestic destinations and flows to rest of world destinations (exports), all at domestic basic prices. For the *resident* view of imports, the intermediate uses matrix P2 and the final uses matrices P3 and P5 also would be "delaminated" element by element into flows from domestic sources and flows from rest of the world sources (imports), all at domestic purchasers' prices. Further, for the "goods" product rows of the uses matrix, the imports layer of this delamination would have to account in each product-industry cell for (1) the portion of "goods" imported, (2) the imported portion of separately invoiced transportation services used in

[14]Trade and transport margins do not appear in the standard sequence of accounts in the *2008 SNA* because these accounts are not shown with product detail. Although these margins are nonzero for individual products, they sum to zero in total, because the amount added to the domestic supply of *goods* comes from the domestic supply of distribution, insurance, and transport *services*. Margins are thus shown in Table 15.11, separately for margins on domestic production and imports (carriage, insurance, and freight (c.i.f.)/free on board (f.o.b.) adjustment), because the SUT displays product detail down the columns. In the aggregate, of course, these adjustments for trade and transport margins on domestic production and the c.i.f./f.o.b. adjustment for imports cancel each other out.

Table 15.11. The Supply and Use Table (SUT)

Supply							Use						
P1 Output at basic prices *of establishments producing for:*			Transport and distribution margin adjustment[1]	D21–D31 Taxes less subsidies products[2]	Imports c.i.f.[3]		P2 Intermediate consumption at purchasers' prices *of establishments producing for:*			P31 Individual consumption	P32 Collective consumption	P5 Gross capital formation	P6 Exports f.o.b.
P11 Market	P12 Own use	P13 Other nonmarket			+ P7 Imports f.o.b.	+ c.i.f./f.o.b. adjustment.[4]	P11 Market	P12 Own use	P13 Other nonmarket				
Product × *Industry*	*Product* × *Industry*	*Product* × *Industry*	+ **Goods (+)** **Services (−)**	+ ***Product* × 1**	***Goods f.o.b.*** ***Services***[6]	**Goods (+)**[5] **Services (−)**	= *Product* × *Industry*	*Product* × *Industry*	*Product* × *Industry*	+ *Product* × 1	*Product* × 1	*Product* × 1	+ *Product* × 1
							+	+	+				
							B.1 Value added 1 × *Industry*	B.1 Value added 1 × *Industry*	B.1 Value added 1 × *Industry*				
=	=	=					=	=	=				
P11 Output at basic prices 1 × *Industry*	P12 Output at basic prices 1 × *Industry*	P13 Output at basic prices 1 × *Industry*					= P11 Output at basic prices 1 × *Industry*	P12 Output at basic Prices 1 × *Industry*	P13 Output at basic prices 1 × *Industry*				

Note: Production account: double outlines and no fill; use of income account: single outlines and no fill; capital account: diagonal fill; external account of goods and services: vertical fill.

[1]The sum of items in this column is zero. It appears in the SUT but does not appear in any *2008 SNA* account. Included in this adjustment are transportation, insurance, and distribution charges on goods within the geographical boundaries of the national economy only. This column disappears if uses are recorded unbundled in producers' prices.

[2]Taxes and subsidies on products are shown in the *2008 SNA.* Allocation of primary income account for the general government institutional sector S.13, which derives B.5g *Balance of primary incomes* as the balancing item (*2008 SNA,* Annex V, Table A.V.5). B.5g is the sum of B.2g operating surplus, D2 taxes on production and imports, less D3 subsidies, plus D4 property income (net). This account is the source of data for construction of this column in the SUT when exploded to show product detail for the items D21 and D31, taxes and subsidies on products.

[3]As noted elsewhere, the *2008 SNA* values goods imports c.i.f. at the product level of detail but f.o.b. in total. Thus, the *2008 SNA* presentation of goods imports in the supply matrix is the sum of PP8 imports f.o.b. and the c.i.f./f.o.b. adjustment on goods imports. To simplify the presentation of this diagram of the SUT and clarify the nature of the negative adjustment to services, assume that insurance and freight provided on imports can be compiled by product, and thus imports f.o.b. can be compiled by product. Insurance and freight provided on imports by residents already are included in the insurance and transportation rows of the P1 matrix.

[4]The sum of items in this column is zero. It appears in the SUT but does not appear in any *2008 SNA* account.

[5]Insurance and freight on imports of goods by product provided by both residents and nonresidents.

[6]Including insurance and freight on imports provided by nonresidents. Insurance and freight provided on imports by residents are included in the insurance and transportation rows of the P1 matrix.

moving those goods,[15] (3) the portion of distribution services applied to those goods that was supplied by nonresident producers,[16] and (4) the portion of taxes less subsidies on products comprising import taxes levied by the government of the economic territory. The domestic/import source delamination of the "services" rows of the above uses matrices at purchasers' prices would need to account only for components (3) and (4), where (3) would comprise the service product transacted rather than a distributive service associated with a transaction in a good.

15.49 There are transportation, distribution, and other services entering into trade in their own right, without being attached to a goods transaction (change of ownership). The *2008 SNA* and the *BPM6* consider the specific case of nonresident distributors or "merchants" who accept goods on consignment—that is, goods intended for sale but not actually sold at the time they cross the frontier—from residents of an economic territory for subsequent resale. The services of "merchants" are imports to the economic territory where the owners of the "merchanted" goods reside. Exports of the goods the nonresident merchant is handling for the resident owner are not recognized until the merchant actually sells the goods and change of ownership occurs. At that time, exports of goods are equal to the portion of the sale proceeds transferred to the former owner, and imports of distributive services are equal to the merchant's gross margin on the sale. These exports and imports are recorded for the economic territory where the first mentioned owner of the goods resides. The *2008 SNA* and *BPM6* also recognize more general "goods for processing" arrangements that involve movement of goods without change of ownership between a resident institutional unit and a unit in the rest of the world, processing of those goods, and subsequent return of the processed product to the owner. The result of an international arrangement of this type is an import of processing services to the economic territory but no export or import of goods, because no change of ownership has occurred. On the other hand, transportation related to merchanted or processed goods is recorded when the transport services are provided to the merchant or processor, which generally differs from the date of the change of ownership (sale) of the goods. These transportation services would be recognized as international transactions for transport services in the trade statistics without associated goods transactions.

15.50 The relationship between a resident's view and nonresident's view of exports thus is:

P6 Exports [nonresident's view, *uses* at purchasers' prices]

− **Trade and invoiced transport margin adjustment for exports**

− **Taxes less subsidies on exported products**

= **P11X Exports of market output**

+ **P13X Exports of nonmarket output**

+ **P5113X Exports of existing (transportable) fixed assets**

+ **P52WX Exports of withdrawals from inventories**

+ **P53WX Exports of valuables**

= **Exports** [resident's view, *supply* at basic prices].

The "Trade and invoiced transport margin adjustment for exports" merely reallocates domestic trade and transport margins on goods destined for export between goods and services. It is, in the aggregate, equal to zero, adding margins for domestically produced distribution and invoiced transport services to exports at basic prices and subtracting the same from the specific lines for domestically produced distribution and invoiced transport services. Thus, in the aggregate, the difference between the nonresident's and resident's view of exports is the valuation wedge introduced by taxes less subsidies on exported products.

15.51 The relationship between resident's view and nonresident's view of imports thus is:

P7 Imports [nonresident's view, *supply* at basic prices]

+ Trade and invoiced transport margin adjustment for imports

+ Taxes less subsidies on imported products

[15]An imported good may be transported from the source country by a domestic carrier or by a foreign carrier. If it is transported by a foreign carrier, this part of the purchasers' value of the goods used constitutes an import of transportation services.

[16] In the flows for uses of goods, data on the value of imports from, for example, customs sources will include distribution margins for retail and wholesale services supplied by producers in the rest of the world. The values of imports reported by enterprises and their establishments will normally include these imported distribution margins as well. However, there are cases, discussed in the next paragraph, in which these margins are recorded separately in time from the goods flows and should appear in the distribution services product lines of the SUT.

= **P2M Imported intermediate consumption**

+ **P3M Imported final consumption**

+ **P5111M Imported new (transportable) fixed assets**

+ **P5112M Imported existing (transportable) fixed assets**

+ **P52AM Imported additions to inventories**

+ **P53AM Imported valuables**

= **Imports** [resident's view, *uses* at purchaser's prices].

The "Trade and invoiced transport margin adjustment for imports" merely reallocates international transport margins on imported goods between goods and services. It is, in the aggregate, equal to zero, subtracting margins for invoiced transport services from imports at purchasers' prices and adding the same to the specific lines for either imported or domestically produced transport services. Thus, in the aggregate, the difference between the nonresident's and resident's view of imports is the valuation wedge introduced by the taxes less subsidies on imported products.

15.52 A full international accounting of these tax wedges would take account of taxes less subsidies on products imposed by governments in the rest of the world as well. On a product-by-product basis, this might be determined by looking at the exports and imports accounts of a country as well as the imports and exports accounts of its trading partners, assuming all are compiled with separate accounting for taxes less subsidies on internationally traded products. Such data would support, for example, trade restrictiveness analyses assessing the degree to which import tariffs and export subsidies affect the mix and volume of international trade.

15.53 The SUT is a matrix of flows of goods and services designed to highlight the relationship between the production and consumption of institutional units and institutional sectors. For example, households may undertake production in unincorporated enterprises whose activity appears in the production for own final use part of the SUT, but also may consume goods and services, as represented in "individual consumption." The current production transactions of the establishments of all institutional units are grouped together and summarized in one part of the SUT, and the remaining transactions are summarized and organized in another part. The SUT deals principally with flows of transactions in goods and services. Associated with these monetary flows are price and volume components. It is of central interest in monitoring the economy with national accounts statistics to be able to assess the price and volume components of flows of goods and services exchanged for money or credit in market transactions in the SUT. Movements in the price components are of use for estimating volume changes, terms of trade changes and effects, and the transmission of inflation, and assessing changes in the purchasing power of incomes, as well as in influencing the rate of general price change through monetary policy. Finally, price movements in the various national accounts aggregates are used in private sector decision making and in the escalation of contracts. Movements in the price components of national accounts aggregates are, as discussed at the beginning of this section, measured by price indices.

B.1.3.7 Supply and use tables in volume terms

15.54 As outlined in Chapter 4, Section 3.2.1, price deflators are applied to value aggregates at a detailed product level. However, this should be undertaken within the framework of SUTs in volume terms to enable the detailed price indices of export and imports to benefit from being meaningfully reconciled with those of output, consumption, and investment to form commodity balances in volume terms. The *2008 SNA* advises that SUTs be developed in volume terms at the same time as, and be consistent with, the SUT at current prices. The valuation requirements for nominal value aggregates for exports and imports in SUTs thus carry over to the price indices used as their deflators. Chapter 14 of the *2008 SNA* stresses the need for a common valuation for supply and use so that the product rows can balance and thus be useful for reconciliation. For SUTs valued at purchasers' prices, and the needs of deflators at the product level, Chapter 4, Section B.3.1, outlines how the valuation of the supply of imports is at basic prices with taxes and margins added subsequently to raise it to purchasers' prices and the value of the use of exports is at purchasers' prices (*2008 SNA*, Paragraph 14.75).

15.55 It also follows that to derive SUTs at basic prices it is necessary to lower the use table from purchasers' to basic prices. *2008 SNA* notes that there are arguments for (and against) the use of basic prices, as against purchasers' prices, for valuing SUTs. The principal argument for a valuation at basic prices is that it facilitates the compilation of SUTs in volume terms, a matter that is of interest to this *Manual*. The case for deflating the components of a SUT at basic prices[17] is that PPIs, used at a detailed level for deflating both output and intermediate consumption, are compiled at basic prices and thus are well suited for deflating the rows of a basic price SUT. Further, a use, such as intermediate consumption of a product valued at purchasers' prices can be decomposed into that part of its value at basic prices derived from domestic production, that is, derived from imports,[18] trade margins, transport margins, and taxes and subsidies on products. This decomposition enables separate deflators to be applied to the constituent elements.

15.56 The case against valuing the SUT at basic prices is first, as outlined in *2008 SNA* Section D.2, the arduous task of the decomposition of the use table (at purchasers' prices) into its six constituent components: basic price domestic production, imports, trade margins, transport margins, taxes, and subsidies. Six matrices must be set up to enable the decomposition of each element of the use table. This is a much more resource-intensive task that setting up the six columns on the supply side to raise supply at basic prices to purchasers' prices. Second, the deflation of margins and taxes is problematic on conceptual as well as practical grounds. Third, while it has been argued that PPIs are usefully available for deflation at basic prices, it is also the case that the CPI is suitable for deflating detailed components of household consumption expenditure at purchasers' prices.

B.2 XMPIs and other major price series

B.2.1 PPI variants

B.2.1.1 Price indices for intermediate consumption

15.57 In considering total economy and industry intermediate consumption price indices (IPIs), the weights correspond to a column-wise reading of the intermediate consumption part of the SUT's use matrix. The intermediate consumption matrix derives from the production account in Table 15.3. It is shown in Tables 15.11 and 15.12 as the region labeled P2. Because the various margins on basic prices inherent in prevailing purchasers' prices may vary from industry to industry, the ideal sources for purchasers' prices for IPIs would be enterprise surveys. Such surveys are generally burdensome and expensive. Instead, as noted in the discussion on price indices for total supply, the price index of intermediate consumption by industry can be derived from detailed product components of the supply price index (SPI). This index will be acceptably accurate if the variation in the total tax, subsidy, transport, and distribution margin is not too great from industry to industry within product class. For the total economy, the price index of intermediate consumption is obtained as a weighted average of industries' intermediate input price indices. The weights are the share of each industry's intermediate consumption in the total intermediate consumption in the economy. As noted in Sections B.1.3.5.2 and B.1.3.6, the imports subindex of the PPI for intermediate consumption is an important component of the price index for imports from the resident's view.

B.2.1.2 Net output PPIs and value-added deflators

15.58 The PPI has been defined in terms of the total market or market-valued output aggregates of the *2008 SNA*, but PPIs sometimes are produced for net output as well as total output. The argument for net output PPIs is that, for a given aggregate of establishments, total output PPIs overweight or "double count" the output of goods used in intermediate consumption within the aggregate. Net output PPIs may be produced for various narrow or broad aggregations of establishments, from detailed industries to the entire population of establishments resident in the economy. The value aggregate of net output PPIs subtracts from total output the value of goods and services used within the aggregate and of the same types as produced for output by establishments in the aggregate. With one exception, net output is not value added, because it does *not* exclude the intermediate consumption of goods and services used by establishments of the aggregate that are *not* of the same types as produced for output. The exception is when the aggregate is all resident establishments.

15.59 By implication, the net output PPI for all establishments resident in the economy must be closely related to the value-added price index or deflator discussed in Chapter 18. The value aggregate for the all

[17]See *2008 SNA,* Section D.3, and United Nations (1999b).

[18]*2008 SNA*, Paragraph 14.132, advises that for the individual product rows of each use at basic prices, imports are separated from domestic production. In some product groups a use may be only domestically produced or only imported and in other cases, in the absence of hard data, informed judgment may have to be used to allocate the respective shares.

items net output PPI would be value added (Section B.1) as defined in the *2008 SNA* and shown in the production account (Tables 15.1, 15.2, and 15.3). In fact, if the PPI has complete product coverage, including all service products, then net output and value added are the same thing for the total economy.[19] They may be the same even at the industry level under an alternative definition of "net output." See this issue in a stage of processing context in Section B.2.1.3.

15.60 The principal issue in interpreting net output PPIs is the definition of the intermediate consumption prices netted from output to arrive at the net output aggregate. These prices should be defined with a view toward the valuation principle inherent in the value aggregates to which they refer. Recall that output (P1) is valued at *basic prices* whereas intermediate consumption is valued at *purchasers' prices*. Ideally, the net output PPI would be a type of double-deflation price index, similar in principle to the value-added deflator described in Chapter 17 of the *PPI Manual* (ILO and others, 2004b). In such an index, the prices of the goods and services in intermediate consumption would be defined inclusive of taxes on products and charges for included transportation and distribution services, and exclusive of subsidies on products. The prices of goods and services in output would be defined as exclusive of taxes on products and separately invoiced charges for transportation and distribution, and inclusive of subsidies on products. Net output PPIs generally do not attempt the *purchasers' price* valuation of the intermediate consumption of output-type goods and services within the industry aggregate in question. They compromise the concept of the net output index, should there be a change in any component of the purchasers' prices of intermediate consumption goods and services other than the underlying basic prices of products. See Section B.2.1.3 regarding the scope of intermediate consumption in the net output PPI and its alignment with intermediate consumption in value added.

B.2.1.3 Stage of processing PPIs

15.61 *Product-based stage of processing indices.* The simplest method of forming a set of stage of processing PPIs is first to determine an ordering of products a priori, on the basis of judgment, from primary to finished goods. The second step is to produce PPIs for goods grouped by this intrinsic stage of processing classification. Such indices are referred to as product- or commodity-based stage of processing indices. They may employ the so-called "end-use" product classifications associated with the commodity flow methods often used in compiling the national accounts.

15.62 *Industry-based stage of processing indices.* Industry net output PPIs are associated with industry stage of processing PPIs. They are produced in an effort to measure the contribution of the basic prices of goods and services to the change in value added for the economy. They also provide an analytical tool to measure the transmission of inflation through stages of processing, from primary goods and services to those sold for final uses. Industry stage of processing PPIs involve a sorting of the product rows and industry columns of the use matrix so the matrix is roughly triangular. In other words, any given *product row* in the stage of processing-sorted use matrix comprises all zero uses to the left of a particular industry sorted by industry stage of processing. It would have mostly positive uses for that industry and other industries to the right of (and thus at higher stages of processing relative to) that industry. Further, within any given industry column, products earlier in the stage of processing sort (above the product in question in the industry column) would tend to have positive uses. There would be zero use of products later in the stage of processing sort (below the product in question) in the industry column. Stages of processing are meaningful in this context for goods, but triangularizing the use matrix tends to classify business services in the primary production category because all industries use them in varying degrees. In this definition of stage of processing, they are primary output because they are produced mainly with labor and capital primary inputs, rather than the outputs of other industries.

15.63 The PPIs constructed for such stage of processing-sorted use matrices are compiled as net output indices, exclusive of uses of output-type goods and services within the industry aggregate in question. Hence, net output PPIs generally are associated with industry stage of processing PPIs. For most aggregates, total economy industry net output is equivalent to value added. Unfortunately, when the coverage of services is incomplete, output and intermediate consumption prices cannot be fully characterized except for goods, the largest industry for which the net output aggregation is feasible. Here only the price index for the net

[19]Note, however, the equivalence between net output and value-added price indices for the total economy presumes variations in taxes on products, and charges for included (not separately invoiced) transportation and distribution charges on outputs used as inputs are part of the prices of those inputs. The practice of compiling net output PPIs should, but sometimes does not, take this into account.

output of goods can be characterized, which differs from the value added of the goods industry because intermediate consumption of services is still not netted from the net output of goods.

15.64 There is a second interpretation of stage of processing PPIs. In this view, they are conceptually the same as value-added price indices or deflators for industries that have been sorted by stage of processing according to the above diagonalization process. By implication, a PPI for an industry at a late stage of processing would expressly exclude the price change of primary products from the price change of the tertiary or finished products of the late-stage industry. Again, universal product coverage, including services, is needed for output and intermediate consumption, and many countries are lacking particularly in the coverage of the prices of service products. When there are no service price indices, value-added deflators cannot be computed even for goods-producing industries, because the services component of intermediate consumption is missing.

B.2.2 Relationship of the PPI to other major price indices

15.65 It is instructive at this point to associate the four major, headline price indices compiled by most countries with the component aggregates and matrices of the SUT. The four main price indices and their associated national accounts aggregates and matrices in the SUT are

(1) Output of resident producers (P1): Producer Price Index (PPI),[20]

(2) Individual consumption expenditure on goods and services (P31), except consumption from own production but including the imputed rent of owner-occupied dwellings, of the household sector (S13) only: Consumer Price Index,

(3) Exports (P6): Export Price Index, and

(4) Imports (P7): Import Price Index.

15.66 The location and coverage of these major price indicators as they directly apply to goods and services value aggregates in the national accounts are shown in Table 15.12. Recall that Section A of this chapter characterized a price index as a function of price relatives and weights, noting that, other than the formula for the index itself, the requisite features of the relatives and weights would be determined by the value aggregate. These factors were

- What items to include in the index,
- How to determine the item prices,
- Which transactions that involve these items to include in the index, and
- From what source to draw the weights used in the selected index formula.

Table 15.13 summarizes these particulars for each of the four major indices based on our survey of the goods and services accounts of the *2008 SNA* culminating in the SUT.

B.2.3 CPI versus PPI as a measure of inflation in market transactions

15.67 Central banks take an interest in the major price indices, particularly if they implement an “inflation targeting” monetary policy. The CPI is the most widely available macroeconomic price statistic, and in many countries it may be the only available option for inflation measurement. When available, the PPI ordinarily is produced monthly on a timetable similar to that of the CPI. It is useful, therefore, to compare the two indices as candidates for inflation measurement.

15.68 Both reference aggregates for the CPI (consumption plus capital formation) are important components of total final expenditure and GDP in virtually all countries. Indeed, reference aggregate 2 (consumption plus capital formation) has been promoted by some analysts as a better measure of change in the prices of actual transactions in goods and services than CPIs based on reference aggregate 1 (consumption), which gives substantial weight to the imputed rent of owner-occupied housing. On the other hand, the total value of transactions in goods and services also includes intermediate consumption and acquisitions and disposals of tangible and intangible capital assets, so as an inflation index for total goods and services transactions, the CPI’s coverage is rather limited under either definition 1 or 2. The CPI’s purchasers’ price valuation principle also includes taxes less subsidies on products, which may not be desired in an inflation indicator for underlying price change.

[20]This chapter has also described net output PPIs, whose associated value aggregate is value added (B1g) for the economy as a whole, as well as for individual industries, under the assumption that all products including services are covered in the PPI. As noted earlier, if product coverage (e.g., of services) is incomplete, then the net output concept deviates from value added because the intermediate consumption of noncovered goods is not subtracted from output.

Table 15.12. Location and Coverage of the Major Price Indices in the Supply and Use Table

Supply							Use						
P1 Output at basic prices *of establishments producing for:*			*Transport and distribution margin adjustment*[1]	D21–D31 Taxes less subsidies on products	Imports c.i.f.		P2 Intermediate consumption at purchasers' prices *of establishments producing for:*			P31 Individual consumption	P32 Collective consumption	P5 Gross capital formation	P6 Exports f.o.b.
P11 Market	P12 Own use	P13 Other nonmarket			P7 Imports f.o.b.	c.i.f./f.o.b. adj.	P11 Market	P12 Own use	P13 Other nonmarket				
PPI	***PPI***	*Product × Industry*	+ *Goods (+)* / *Services (−)*	+ *Product × 1*	+ ***MPI***	+ *Goods (+)* / *Services (−)*	= *Product × Industry*	*Product × Industry*	*Product × Industry*	+ ***CPI***[2]	*Product × 1*	***PPI***[3]	+ ***XPI***
							+	+	+				
							PPI net output/ Value-added deflator	*PPI net output/ Value-added deflator*	B.1g Value added 1 × *Industry*				
=	=	=					=	=	=				
PPI	***PPI***	P13 Output at basic prices 1 × *Industry*					= ***PPI***	***PPI***	P13 Output at basic prices 1 × *Industry*				

Note: The effective coverage of the major indices is shown by areas with gray fill. CPI = consumer price index. PPI = producer price index. XPI = export price index.

[1]This column disappears if uses are unbundled and recorded in producers' prices.

[2]Covering the Individual consumption expenditure P31 of the Household sector S.13 only and excluding consumption of goods produced by households for own final use, with the exception of imputed rental of dwellings of owner occupants.

[3]Covering the finished goods component of Change in inventories (P52).

Table 15.13. Definition of Scope, Price Relatives, Coverage, and Weights for Major Price Indices

Index	Items to Include	Price Determination for Relatives	Transactions Coverage	Sources of Weights
PPI	All types of domestically produced or processed goods and services that are valued at market prices.	Basic prices, determined for goods as the date when available for sale (available for change of ownership) or service price when service rendered.	Output of resident enterprises, comprising sales plus change in finished goods inventories for goods, and sales for services.	The product by industry matrices of market output (P11) and Output for own final use (P12) in the expanded industry production account and in the SUT.
CPI	All types of goods and services purchased by households for individual consumption.	Purchasers' prices, determined for goods and services on the date when used, including taxes on products, excluding subsidies on products, and including transportation and distribution margins.	Consumption expenditures of the households sector (S14) of institutional units, excluding consumption from own production, except for imputed expenditures for rental of owner-occupied dwellings.	The product column of the CPI consumption subaggregate of individual consumption (P31) of the household sector (S14) in the expanded Use of Income account and in the SUT.
XPI	All types of transportable goods and services purchased by nonresidents from residents. Goods exported without change of ownership for significant processing by nonresidents and subsequent reimport are included.	*Nonresident view,* purchasers' prices at the border of the exporting country, including taxes less subsidies on exports, and including transport and insurance from the production location to the border of the economic territory. *Resident view,* basic prices at the border of the exporting country, excluding taxes less subsidies on exports and non-invoiced additional services.	All transportable goods and services produced or processed by residents and purchased by nonresidents, except goods in transit or goods exported and minimally processed by nonresidents for reimport.	The product column of exports (P6) in the expanded External account of goods and services and in the SUT.
MPI (import price index)	All types of transportable goods and services purchased by residents from nonresidents. Goods imported without change of ownership for significant processing by residents and subsequent reexport are included.	*Nonresident view,* basic prices at the border of the exporting country. *Resident view,* purchasers' prices at the border of the using country, including international transport and insurance as well as taxes less subsidies on imports.	All transportable goods and services produced or processed by nonresidents and purchased by residents, except goods in transit or goods imported and minimally processed by residents for reexport.	The product column of imports (P7) in the expanded External account of goods and services and in the SUT.

15.69 In contrast, the PPI covers, in principle, total output, which by definition implicitly includes intermediate consumption as well as value added.[21] A second desirable feature of the PPI is that it provides some information on the transmission of inflation through the economy by stage of processing. As noted earlier, product-based stage of processing PPIs may be used to provide information on transmission of inflation through the economy from primary products to finished products. If industry value-added indices are compiled, then industry-based stage of processing net output price indices can be used to inform on the transmission of inflation from primary activity to tertiary activity. As noted earlier, the latter indices require price indices for intermediate consumption, which most often are estimated using available information on basic prices, trade and

[21]However, progress in extending the industry coverage of the PPI to cover all output-producing activities, services in particular, has proceeded slowly owing to the technical difficulty of specifying service products and measuring the associated prices.

transport margins, and taxes and subsidies on products, rather than from direct surveys, although the latter may be used and are preferable if the survey resources are available.[22]

B.3 The two variants of export and import price indices in the national accounts and their relationship to other price indices

15.70 The nonresident's view export and import price indices allow decomposition of the SNA aggregates for exports (P6) and imports (P7) to produce volume estimates for GDP compiled from the expenditure approach as the sum of consumption, capital formation, and net exports. The resident's view export and import price indices allow decomposition of the SNA aggregates for output (P1) and intermediate consumption (P2) into domestic and internationally traded components, allowing compilation of the contribution international trade makes to developments in GDP compiled from the production approach as value added equals output less intermediate consumption. It also allows decomposition of gross capital formation (P5) into domestic and internationally supplied components. In combination with the aforementioned contribution of international trade to net output (value added) volume, this decomposition of capital formation additionally allows compilation of statistics on the contributions international trade makes to developments in multifactor productivity.

B.4 Other goods and services price indicators in national accounts

B.4.1 Price indices for total supply

15.71 Consistent with our earlier discussion of the PPI coverage, total market-valued output is the sum of market output (P11) and output for own final use (P12). Total output (P1) is the sum of market-valued output and other nonmarket output (P13). Total supply at basic prices is the sum of output and imports (P7). Markup adjustments at the product level for trade and transport margins on domestic production, insurance and freight on imports, and taxes (D21) less subsidies (D31) on products would be added to total supply at basic prices to produce total supply at purchasers' prices.

15.72 In decomposing total supply into price and volume components, the total SPI at basic prices can be seen to be a weighted mean of the total output price index (YPI) and the import price index (MPI). The YPI comprises in turn the PPI and an implicit deflator index for other nonmarket output. To obtain the deflator for total supply at purchasers' prices, the SPI would be multiplied by an index of the total markup for trade, insurance, and transport margins,[23] and taxes net of subsidies on products.

15.73 Total SPIs at product levels of detail are useful in compiling and reconciling discrepancies in supply and use tables expressed in volume terms. In addition, SPIs are employed in producing industry price indices for intermediate consumption (P20), which are useful for compiling GDP volume measures from the production approach. Although principally used as a compilation aid and in deflation of value added at basic prices via the double deflation approach (see Section B.4.2), SPIs could also serve as analytical indicators in their own right because of their coverage of all goods and services transactions in the economy relating to production and external trade. As such, they may be useful as indicators for economic policy analysis and evaluation requiring broad transaction coverage, in monetary policy formulation, for example.

B.4.2 Price indices for final uses

15.74 The price indices for final uses comprise deflators for individual consumption (P31), collective consumption (P32), gross fixed capital formation (P51g), change in inventories (P52), acquisitions less disposals of valuables (P53), and exports (P6). Of the major price indices discussed above, the CPI is the principal source of detailed (product-level) information for P31, and the PPI is a significant source of detailed information for P51g and the principal source for the finished goods component of P52. The SPI may be the principal source for the input inventories component of P52 in the absence of a detailed intermediate inputs purchase price survey, and the XPI is

[22]Although it is possible to produce something similar to industry-based stage of processing indices with information only on basic prices deriving from the output-based PPI in conjunction with a product by industry intermediate consumption matrix, such indices do not capture changes in trade and transportation margins or taxes less subsidies on production. To the extent that such changes are occurring, such indices measure the value-added deflators with an error. However, for inflation measurement, particularly with a view toward an inflation targeting monetary policy, it may be desirable to remove the contribution to change in such industry-based stage of processing indices that arises from changes in taxes net of subsidies on products.

[23]These margins matter only when developing SPIs at purchasers' prices for individual products and product subaggregates. For all products they cancel out, leaving only taxes less subsidies on products contributing to the total markup on total supply at basic prices.

the deflator for P6. The SPI can serve, as well, as a source of detailed product information for P32, P51g, and P53. The deflator for total final uses is designated as the final uses price index, or the FPI. It would be computed as a weighted mean (formula to be determined) of the component indices just discussed.

B.4.3 GDP deflator

15.75 As noted above in the discussion of the SPI and the IPI, the GDP price deflator[24] can be compiled in two ways, corresponding to the two goods and services methods of compiling GDP: the production approach and the expenditure approach. Recall that the production approach derives from the definition of value added, which is the difference between output (P1) (at basic prices) and intermediate consumption (P2) (at purchasers' prices). The *2008 SNA* recommends the use of double deflation for value added, by which output at basic prices *Y* is deflated by the all items YPI to obtain output volume, and intermediate purchases are deflated by an intermediate purchases price index to obtain intermediate input volume. Real value added is then computed as the difference between output volume and intermediate input volume.[25] This operation is equivalent to deflating value added in current prices with a double-deflation-type price index having a positive weight on the YPI and a negative weight on the IPI (see Chapter 18 of the *PPI Manual*).[26] The total value added at current basic prices divided by real value added obtained via double deflation yields the implicit deflator for value added at basic prices. Finally, the GDP deflator at purchasers' prices is the value-added price index (at basic prices for output and purchasers' prices for intermediate input) multiplied by the index of the markup on value added of output taxes less output subsidies on products.

[24]The terminology "GDP price index" could be used here with no confusion of meaning, but we follow conventional usage as set out in Chapter 18 of the *PPI Manual* and Section B of Chapter 20 of this *Manual*. This does not imply that a price index that declines with increases in some prices is in fact not a price index—this *Manual* considers a price index to be that part of the relative change in a value aggregate that can be attributed to the associated change in prices, whether such a change increases or decreases the aggregate.

[25]See *2008 SNA*, Chapter XVI.

[26]In the usual case just described, the value-added deflator is a Paasche index (Chapter 16, equation (16.6)) of the output price index $YPI^{s,t}$ and the intermediate input price index $IPI^{s,t}$, where the weight on the $IPI^{s,t}$ is $w_I^t = \dfrac{-P2^t}{P1^t - P2^t}$.

As noted in Chapter 16, equation (16.11), the corresponding volume index has the Laspeyres or "constant price" form, which is equivalent to the double deflated real value-added volume measure described in the text divided by base-period value added.

15.76 Alternatively, the final expenditure deflator FPI may be combined with the MPI using a double-deflation-type approach. GDP volume is calculated from expenditure data by deflating imports (P7) by the MPI and subtracting it from the volume of final uses, calculated by deflating final uses by the FPI. The implicit GDP deflator would be the ratio of GDP at current prices with GDP volume so calculated.

B.4.4 Labor services price indices

15.77 The *2008 SNA* provides for the income components comprising value added in the *generation of income account*, shown in Table 15.14. The largest of the income components itemized in this account is compensation of employees (D1), comprising wages and salaries (D11) and employers' social contributions (D12). D1 represents a value aggregate for a flow of labor services and thus is susceptible to decomposition into price and volume components. Table 15.15 shows the same account exploded by type of labor service (occupation) for an establishment or industry. The price index of labor services or employment cost index (ECI) measures developments in total compensation by occupation within industry. The price of labor services in total compensation terms is of particular interest when compared with the GDP deflator, which indicates the relative purchasing power of labor compensation in terms of production for final consumption. This comparison is useful in assessing cost-push pressures on output prices and as an input into compiling measures of the productivity of labor. A second useful comparison is between the wages and salaries subindex of the ECI[27] with the CPI. The ratio of the ECI with the CPI indicates the purchasing power of wages in terms of consumption goods and services, and tracks the material welfare particularly of the employees subsector (S143) of the household institutional subsector (S14) (see Box 15.1).

B.5 A framework for a system of price statistics

15.78 To summarize this section's overview of the main price indicators and the national accounts, Table 15.16 shows the price indices needed for the value aggregates in the national accounts and their relation to

[27]In the ECI, the price of labor services comprises all of the components of compensation of employees, including employers' social contributions (benefits) as well as wages and salaries. The wages and salaries subindex of the ECI would be another example of a price index adjusted by a markup index. Analogously with the price index for total supply at purchasers' prices or for GDP by production in Table 15.12, the ECI would be adjusted in this case by a "markdown index" taking off employers' social contributions.

Table 15.14. Generation of Income Account for Establishment, Institutional Unit, or Institutional Sector

Uses		Resources
D1	**Compensation of employees**	B.1g *Value added*[1]
	D11 Wages and salaries	
	D12 Employers' social contributions	
	D121 Employers' actual social contributions	
	D122 Employers' imputed social contributions	
D2	Taxes on production and imports	
	D29 Other taxes on production[2]	
D3	Subsidies	
	D39 Other subsidies on production (−)[3]	
B.2g	*Operating surplus*[4]	

Note: 2008 SNA goods and services items shown in bold.
[1]From the production account.
[2]Taxes on production unrelated to products.
[3]Subsidies on production unrelated to products.
[4]Balancing item of the generation of income account.

Table 15.15. Generation of Income Account for Establishment and Industry with Labor Services (Occupational)[1] Detail

Note: Establishment ID: eeeeeeee.	Activity/Industry code (ISIC): aaaa. Market status: P1n.	Institutional unit ID: uuuuuuuu. Institutional sector code: Snnnnn.	
Uses			Resources
D11 Wages and salaries	**D12 Employers' social contributions**	**D1 Compensation of employees**	B1g *Value added*[2]
1: Legislators, senior officials, and managers	1: Legislators, senior officials, and managers	1: Legislators, senior officials, and managers	
2: Professionals	2: Professionals	2: Professionals	
3: Technicians and associate professionals	3: Technicians and associate professionals	3: Technicians and associate professionals	
4: Clerks	4: Clerks	4: Clerks	
5: Service workers and shop and market sales workers	5: Service workers and shop and market sales workers	5: Service workers and shop and market sales workers	
6: Skilled agricultural and fishery workers	6: Skilled agricultural and fishery workers	6: Skilled agricultural and fishery workers	
7: Craft and related trades workers	7: Craft and related trades workers	7: Craft and related trades workers	
8: Plant and machine operators and assemblers	8: Plant and machine operators and assemblers	8: Plant and machine operators and assemblers	
9: Elementary occupations	9: Elementary occupations	9: Elementary occupations	
0: Armed forces	0: Armed forces	0: Armed forces	
		D2 Taxes on production and imports	
		D29 Other taxes on production	
		D3 Subsidies (−)	
		D39 Other subsidies on production	
		B2g *Operating surplus*[3]	

Note: 2008 *SNA goods* and services items shown in bold.
[1]Shown are major groups of the International Standard Classification of Occupations 1988 (ISCO-88), International Labour Organization.
[2]From the production account.
[3]Balancing item of the generation of income account.

Table 15.16. A Framework for Price Statistics

2008 SNA Aggregate	*2008 SNA* Transaction Codes[1]	Valuation and Needed Detail	*2008 SNA* Source Account	Price Index[2]	Derivation from Other Price Indices
			Supply		
Market-valued output	**P11 + P12**	**Basic prices, product by industry**	**Production account with industry and product detail, total economy (S1)**	***Producer Price Index (PPI)***	
Other nonmarket output[3]	P13	Basic prices (cost of production), product by industry	Production account with industry and product detail, total economy (S1)	*Implicit Deflator Index for other nonmarket output*	Derived from volume indicator
Total output	P1 = P11 + P12 + P13	Basic prices, by product	Production account with industry and product detail, total economy (S1)	*Output Price Index*	$YPI = f(PPI, IDI; w_m),\ w_m = \frac{P13}{P1}$
Imports	**P7 [nonresident view]**	**Basic prices of the rest of the world, by product**	**External transactions in goods and services account with product detail, total economy (S1)**	***Import Price Index (MPI)***	Laspeyres lies below economic Laspeyres index, Paasche lies above economic Paasche index
	P2M + P3M + P5111M + P5112M + P52AM + P53AM [resident view]	Purchasers' prices, by product and industry	Production account (S1), use of income account (S1), and capital account (S1)	*Import subindices of PPI, Intermediate Consumption Price Index (below), HPI (below), GPI (below), KPI (below), NPI (below), VPI (below)*	May be aggregated into imports at purchasers' prices, but supplied essentially to provide imports contribution to change in various indices for SNA uses aggregates for, e.g., productivity analysis, inflation analysis, etc. In the aggregate, equal to P7 plus taxes less subsidies on imports. Laspeyres lies above economic Laspeyres index, Paasche lies below economic Paasche.
Total supply, basic prices	P1 + P7	Basic prices, by product	Supply and Use Table, total economy (S1)	*Supply Price Index*	$SPI = f(MPI, YPI; w_y),\ w_y = \frac{P1}{P1 + P7}$
Domestic trade, insurance, and transport margin adjustment		Basic prices, for services provided for transportation and distributio n within national frontiers, by product	Supply and Use Table, total economy (S1)	*Supply Markup Index (SMI)*	
Freight and insurance on imports adjustment		Basic prices (for services provided from exporter frontier to domestic frontier, regardless of residency of provider), by product	Supply and Use Table, total economy (S1)		$SMI = \frac{\frac{P1^t + P7^t + D21^t - D31^t}{P1^t + P7^t}}{\frac{P1^s + P7^s + D21^s - D31^s}{P1^s + P7^s}}$ *(in the aggregate)*. Product level total output markup indices also would include trade and transport margins in the numerator of the above expression.
Taxes less subsidies on products	D21 – D31	Payable, by product	Allocation of primary income account, general government sector (S13)		
Total supply, purchasers' prices	P11 + P12 + P7 + D21 – D31	Purchasers' prices			$SPI \times SMI$

(Continued)

Table 15.16. A Framework for Price Statistics *(continued)*

2008 SNA Aggregate	*2008 SNA* Transaction Codes[1]	Valuation and Needed Detail	*2008 SNA* Source Account	Price Index[2]	Derivation from Other Price Indices
			Uses		
Intermediate consumption	P2	Purchasers' prices, products by industries	Production account with product and industry detail, total economy (S1)	*Intermediate Consumption Price Index*	Usually incorporates product-level information from the Total supply price index at purchasers' prices.
Individual consumption	P31	Purchasers' prices, by product	Use of income account with product detail, total economy (S1)	*Household Consumption Price Index (HPI)*	Incorporates the CPI, and may incorporate product level information from the CPI and PPI regarding goods and services produced from own consumption and provided to individuals by NPISHs and General Government.
Household sector S14	**CPI reference aggregate #1: P31, except employers' social contributions and consumption from production for own final use, but including imputed rent of homeowners**	**Purchasers' prices, by product**	**Reference aggregate #1: Use of income account with product detail, household sector (S14), with special subclassification of P3**	***Consumer Price Index, consumption basis***	
	CPI reference aggregate #2: P31, except employers' social contributions and consumption from production for own final use (and excluding by implication imputed rent of homeowners)	**Purchasers' prices, by product**	**CPI reference aggregate #2: Use of income account with product detail, household sector (S14), with special subclassification of P31**	*Consumer Price Index, transactions or inflation basis*	
Collective consumption	P32	Purchasers' prices, by product	Use of income account with product detail, general government sector (S13)	*Government Price Index (GPI)*	*May incorporate product indices from the CPI and PPI.*
Gross fixed capital formation	P51g	Purchasers' prices, by product	Capital account with product detail, total economy (S1)	*Fixed Capital Formation Price Index (KPI)*	*May incorporate product indices from the PPI.*
Household sector S14	**CPI reference aggregate #2: Gross capital formation in residential dwellings (P51g)**	**Purchasers' prices, by product**	**CPI Reference aggregate #2: Capital account regarding acquisitions (P511) less disposals (P5113) of residential dwellings**	*Consumer Price Index (CPI), transactions or inflation basis*	
Change in inventories	P52	Purchasers' prices, by product	Capital account with product detail, total economy (S1)	*Inventory Price Index (NPI)*	*Price index of inventory stocks*
Acquisitions less disposals of valuables	P53	Purchasers' prices, by product	Capital account with product detail, total economy (S1)	*Valuables Price Index (VPI)*	*Price index of valuables stocks*

Exports	**P6**	**Purchasers' prices of rest of the world, by product**	**External transactions in goods and services account with product detail, Total economy (S1)**	***Export Price Index (XPI)***	Laspeyres lies above economic Laspeyres index, Paasched lies below economi Paasche index.
	P11X + P13X + P5113X + P52WX + P53WX	Basic prices of domestic supplier, by product and industry	Production account with product detail, Capital accont with product detail (S1)	*PPI, KPI, NPI, VPI*	May be aggregated into exports at basic prices, but supplied essentially to provide exports contribution to change in various indices for SNA supply aggregates for, e.g., productivity analysis, inflation analysis, etc. In the aggregate, equal to P6 minus taxes less subsidies on exports. Laspeyres lies below economic Laspeyres index, Paasche lies above economic Paasche index.
Total final uses	P3 + P5 + P6	Purchasers' prices, by product	Supply and Use Table, total economy (S1)	*Final Uses Price Index (FPI)*	$FPI = f(HPI, GPI, KPI, NPI, VPI, XPI, \vec{w})$ *where* $\vec{w} = [w_G, w_K, w_N, w_V, w_X]^4$ *and* $w_G = \frac{P32}{P3 + P4 + P5 + P7}$ $w_K = \frac{P51}{P3 + P4 + P5 + P7}$ $w_G = \frac{P32}{P3 + P4 + P5 + P7}$ $w_V = \frac{P53}{P3 + P4 + P5 + P7}$ $w_X = \frac{P6}{P3 + P4 + P5 + P7}$
			Gross Domestic Product		
Value added (Net output PPI)	V = P1 – P2 + D21 – D31	By industry, product and institutional sector, with industry and total value-added price indices adjusted by a markup factor for taxes net of subsidies on products. Net output PPI may exclude from intermediate consumption products, particularly services, on which there may be no price information.	Supply and Use Table, total economy (S1)	*Value-added deflator*	*Value-added deflator* $= f(SPI, IPI; w_I)$ *where*[5] $w_M = \frac{-P8}{GDP}$ $w_I = \frac{-P2}{GDP}$

(Continued)

Table 15.16. A Framework for Price Statistics *(continued)*

2008 SNA Aggregate	*2008 SNA* Transaction Codes[1]	Valuation and Needed Detail	2008 SNA Source Account	Price Index[2]	Derivation from Other Price Indices
Gross Domestic Product	GDP = P3 + P5 + P6 − P7	By product and institutional sector.	Supply and Use Table, total economy (S1)	*GDP deflator*	$GDP\ deflator = f(FPI, MPI; w_M) = SMI^* \times f(SPI, IPI; w_I)'$ *where* $SMI^* = \dfrac{\dfrac{P1^t - P2^t + D21^t - D31^t}{P1^t - P2^t}}{\dfrac{P1^s - P2^s + D21^s - D31^s}{P1^s - P2^s}}$ *(in the aggregate)* Industry level value-added markup indices *SMI** would include the total trade and transport margins on output in the numerator.
		Other Price Indices Related to Net Output			
Compensation of employees	D1	By occupation, industry and institutional sector	Generation of income account, total economy (S1)	*Employment Cost Index*	

[1]P11 = Market output, P12 = Output for own final use, D21 = Taxes on products, and D31 = Subsidies on products.

[2]The four major price indices are shown in bold.

[3]This category comprises public services output provided free of charge or at economically insignificant prices by general government and nonprofit institutions serving households (NPISHs). This output is valued at cost because it has no market comparator. A price index cannot be directly constructed for this aggregate because there are no economically significant prices for other nonmarket output. The implicit deflator for Other nonmarket output (P13) is derived by dividing a directly compiled volume indicator into the value of other nonmarket output.

[4]Unlike the other aggregations of indices that involve the combination of two component indices, it is shown that the FPI is a simultaneous aggregation of six price indices for the components of final uses. Again, *f* can be any of the indices introduced in this chapter, and with the weight of the first item (here of Individual consumption (P31) determined as one minus the rest of the weights, and the price relatives given by the list of index arguments).

[5]The negative weights of the second index arguments of both of these formulae for GDP is an indication that they represent a double deflation-type price index. See *2008 SNA,* Chapter XVI, Section E.

the four main price indicators. Indices that are functions of two other indices are shown with the general notation $f(I_1,I_2;w)$, where f is an index formula, I_1 and I_2 are price indices (e.g., MPI and YPI), and w is the weight of the second index, with the weight of the first argument in f understood to be $1 - w$. For example, if f is the Laspeyres formula then the output price index *YPI* would be calculated by making the following substitutions: $P_L^{s,t} = YPI^{s,t}$ $r_1^{s,t} = PPI^{s,t}$, $w_1^s = 1 - w_X^s$, $r_2^{s,t} = XPI^{s,t} \times \Delta^{s,t}$, $w_2^s = w_X^s$. f also could be chosen as a Paasche formula (with the same substitutions except for change in the time superscript on the weights $w_1^t = 1 - w_X^t$ and $w_2^t = w_D^t$), Fisher ideal formula, or other index formula.

C. International Comparisons of Expenditure on Goods and Services

15.79 The main price statistics discussed thus far trace price developments of goods and services through time. Purchasing power parities (PPPs) compare price levels expressed in a numeraire currency, such as the U.S. dollar or the euro, of detailed goods and services between different countries or geographical areas for a given accounting period. They eliminate the effect of prices when comparing the levels of GDP between two countries or areas. The price relatives in bilateral PPPs comprise the ratios of the local prices, converted to a numeraire currency, of identical goods and services between the two countries or areas. The weights are proportional to the shares of these items in expenditure on GDP, expressed in a numeraire currency, between the two countries or areas. PPPs thus follow the same scope and valuation concepts as GDP in Table 15.16, with the superscript t referring to an area or country rather than month, quarter, or year.

15.80 The sources of price relatives are the same as those for the final uses GDP deflator, and the weights are simply the total final uses, net of imports free on board, by product. To ensure the PPP between area A and area B is the reciprocal of the PPP between B and A, bilateral PPPs need to be computed using symmetric index numbers such as the Fisher or Törnqvist indices.[28]

15.81 A matrix of bilateral PPPs provides a means of making not only direct bilateral comparisons but also bilateral comparisons between any two areas as the product of a sequence of bilateral PPPs through any set of intervening areas, beginning with the first area and ending with the second. To ensure the consistency of such comparisons (e.g., that a chain beginning with a given area and ending with the same area produces a PPP of unity), bilateral PPPs are adjusted to produce a transitive set of comparisons. The methods for imposing transitivity on a system of bilateral parities compare each area or country's goods and services prices and shares in GDP to a regional set of reference prices and reference shares.

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[28]Note that in the international comparisons case the superscripts s and t of the price and volume decompositions in Section A of this chapter refer to two countries rather than two time periods.

16. Basic Index Number Theory

A. Introduction

> The answer to the question what is the Mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as there are purposes; and we may almost say in the matter of prices as many purposes as writers. Hence much vain controversy between persons who are literally at cross purposes. (Edgeworth, 1888, p. 347)

16.1 The number of physically distinct goods and unique types of services that consumers can purchase is in the millions. On the business or production side of the economy, there are even more products that are actively traded. The reason is that firms produce not only products for final consumption but also exports and intermediate products that are demanded by other producers. Firms collectively also use millions of imported goods and services, thousands of different types of labor services, and hundreds of thousands of specific types of capital. If we further distinguish physical products by their geographic location or by the season or time of day that they are produced or consumed, then there are *billions* of products that are traded within each year in any advanced economy. For many purposes, it is necessary to summarize this vast amount of price and quantity information into a much smaller set of numbers. The question that this chapter addresses is the following: *How exactly should the microeconomic information involving possibly millions of prices and quantities be aggregated into a smaller number of price and quantity variables?* This is the basic *index number problem.*

16.2 It is possible to pose the index number problem in the context of microeconomic theory; that is, given that we wish to implement some economic model based on producer or consumer theory, what is the best method for constructing a set of aggregates for the model? However, when constructing aggregate prices or quantities, other points of view (that do not rely on economics) are possible. Some of these alternative points of view are considered in this chapter and the next chapter. Economic approaches are pursued in Chapter 18.

16.3 The index number problem can be framed as the problem of decomposing the value of a well-defined set of transactions in a period of time into an aggregate price multiplied by an aggregate quantity term. It turns out that this approach to the index number problem does not lead to any useful solutions. Therefore, Section B addresses the problem of decomposing a *value ratio* pertaining to two periods of time into a component that measures the overall *change in prices* between the two periods (this is the *price index*) multiplied by a term that measures the overall *change in quantities* between the two periods (this is the *quantity index*). The simplest price index is *a fixed-basket index.* In this index, fixed amounts of the *n* quantities in the value aggregate are chosen, and then this fixed basket of quantities at the prices of period 0 and period 1 are calculated. The fixed-basket price index is simply the ratio of these two values, where the prices vary but the quantities are held fixed. Two natural choices for the fixed basket are the quantities transacted in the base period, period 0, or the quantities transacted in the current period, period 1. These two choices lead to the Laspeyres (1871) and Paasche (1874) price indices, respectively.

16.4 Unfortunately, the Paasche and Laspeyres measures of aggregate price change can differ, sometimes substantially. Thus Section C considers taking an average of these two indices to come up with a single measure of price change. Section C.1 argues that the best average to take is the geometric mean, which is Irving Fisher's (1922) ideal price index. In Section C.2, instead of averaging the Paasche and Laspeyres measures of price change, taking an average of the two baskets is considered. This fixed-basket approach to index number theory leads to a price index advocated by Walsh (1901, 1921a). However, other fixed-basket approaches are also possible. Instead of choosing the basket of period 0 or 1 (or an average of these two baskets), it is possible to choose a basket that pertains to an entirely different period, say period *b*. In fact, it is typical statistical agency practice to pick a basket that pertains to an entire year (or even two years) of transactions in a year before period 0, which is usually a month. Indices of this type, where the weight reference period differs from the price reference period, were originally proposed by Joseph Lowe (1823), and in Section D indices of this type are studied. They are also evaluated from the axiomatic perspective in Chapter 17 and from the economic perspective in Chapter 18.[1]

16.5 In Section E, another approach to the determination of the *functional form* or the *formula* for the price index is considered. This approach, introduced by the French economist, François Divisia (1926), is based on the assumption that price and quantity data are available as continuous functions of time. The theory of differentiation is used in order to decompose the rate of change of a continuous time value aggregate into two components that reflect aggregate price and quantity change. Although Divisia's approach offers some insights,[2] it does not offer much guidance to statistical agencies in terms of leading to a *definite* choice of index number formula.

16.6 In Section F, the advantages and disadvantages of using a *fixed-base* period in the bilateral index number comparison are considered versus always comparing the current period with the previous period, which is called the *chain system*. In the chain system, a *link* is an index number comparison of one period with the previous period. These links are multiplied to make comparisons over many periods.

B. Decomposition of Value Aggregates into Price and Quantity Components

B.1 Decomposition of value aggregates and the product test

16.7 A *price index* is a measure or function that summarizes the *change* in the prices of many products from one situation 0 (a time period or place) to another situation 1. More specifically, for most practical purposes, a price index can be regarded as a weighted mean of the change in the relative prices of the products under consideration in the two situations. To determine a price index, it is necessary to know

(1) Which products or items to include in the index,

(2) How to determine the item prices,

(3) Which transactions that involve these items to include in the index,

(4) How to determine the weights and from which sources should these weights be drawn, and

(5) What formula or type of mean should be used to average the selected item relative prices.

All of the above price index definition questions except the last can be answered by appealing to the definition of the *value aggregate* to which the price index refers. A *value aggregate V* for a given collection of items and transactions is computed as

$$V = \sum_{i=1}^{n} p_i q_i, \qquad (16.1)$$

where p_i represents the price of the ith item in national currency units, q_i represents the corresponding quantity transacted in the time period under consideration, and the subscript i identifies the ith elementary item in the group of n items that make up the chosen value aggregate V. Included in this definition of a value aggregate is the specification of the group of included products (which items to include) and of the economic agents engaging in transactions involving those products (which transactions to include), as well as the valuation and time of recording principles motivating the behavior of the economic agents undertaking the transactions (determination of prices). The included elementary items, their valuation (the p_i), the eligibility of the transactions, and the item weights (the q_i) are all within the domain of definition of the value aggregate. The precise determination of the p_i and q_i is discussed in more detail in Chapter 3 and other chapters.[3]

16.8 The value aggregate V defined by equation (16.1) referred to a certain set of transactions pertaining to a single (unspecified) time period. Now, consider the

[1] Indices of this type do not appear in Chapter 19, where most of the index number formulas exhibited in Chapters 15–18 are illustrated using an artificial data set. However, indices where the weight reference period differs from the price reference period are illustrated numerically in Chapter 22, where the problem of seasonal products is discussed.

[2] In particular, it can be used to justify the chain system of index numbers, which is discussed in Section E.

[3] Turvey and others (1989) has noted that some values may be difficult to decompose into unambiguous price and quantity components. Some examples of values difficult to decompose are bank charges, gambling expenditures, and life insurance payments.

same value aggregate for two places or time periods, periods 0 and 1. For the sake of definiteness, period 0 is called the *base period* and period 1 is called the *current period*. Assume that observations on the base-period price and quantity vectors, $p^0 \equiv [p_1{}^0, \ldots, p_n{}^0]$ and $q^0 \equiv [q_1{}^0, \ldots, q_n{}^0]$, respectively, have been collected.[4] The value aggregates in the two periods are defined in the obvious way as

$$V^0 \equiv \sum_{i=1}^{n} p_i^0 q_i^0; \quad V^1 \equiv \sum_{i=1}^{n} p_i^1 q_i^1. \tag{16.2}$$

16.9 In the previous paragraph, a price index was defined as a function or measure that summarizes the *change* in the prices of the n products in the value aggregate from situation 0 to situation 1. In this paragraph, a *price index* $P(p^0, p^1, q^0, q^1)$ along with the corresponding *quantity index* (or *volume index*) $Q(p^0, p^1, q^0, q^1)$ is defined as two functions of the $4n$ variables p^0, p^1, q^0, q^1 (these variables describe the prices and quantities pertaining to the value aggregate for periods 0 and 1), where these two functions satisfy the following equation:[5]

$$V^1/V^0 = P(p^0, p^1, q^0, q^1)\, Q(p^0, p^1, q^0, q^1). \tag{16.3}$$

If there is only one item in the value aggregate, then the price index P should collapse to the single-price ratio $p_1{}^1/p_1{}^0$ and the quantity index Q should collapse to the single-quantity ratio $q_1{}^1/q_1{}^0$. In the case of many items, the price index P is to be interpreted as some sort of weighted average of the individual price ratios, $p_1{}^1/p_1{}^0, \ldots, p_n{}^1/p_n{}^0$.

16.10 Thus, the first approach to index number theory can be regarded as the problem of *decomposing* the change in a value aggregate, V^1/V^0, into the product of a part that is due to *price change*, $P(p^0, p^1, q^0, q^1)$, and a part that is due to *quantity change*, $Q(p^0, p^1, q^0, q^1)$. This approach to the determination of the price index is the approach taken in the national accounts, where a price index is used to *deflate* a value ratio to obtain an estimate of quantity change. Thus, in this approach to index number theory, the primary use for the price index is as a *deflator*. Note that once the functional form for the price index $P(p^0, p^1, q^0, q^1)$ is known, then the corresponding quantity or volume index $Q(p^0, p^1, q^0, q^1)$ is completely determined by P; that is, rearranging equation (16.3),

$$Q(p^0, p^1, q^0, q^1) = (V^1/V^0)/P(p^0, p^1, q^0, q^1). \tag{16.4}$$

Conversely, if the functional form for the quantity index $Q(p^0, p^1, q^0, q^1)$ is known, then the corresponding price index $P(p^0, p^1, q^0, q^1)$ is completely determined by Q. Thus, using this deflation approach to index number theory, separate theories for the determination of the price and quantity indices are not required: If either P or Q is determined, then the other function is implicitly determined by the product test equation (16.4).

16.11 In the next subsection, two concrete choices for the price index $P(p^0, p^1, q^0, q^1)$ are considered, and the corresponding quantity indices $Q(p^0, p^1, q^0, q^1)$ that result from using equation (16.4) are also calculated. These are the two choices used most frequently by national accountants.

B.2 Laspeyres and Paasche indices

16.12 One of the simplest approaches to determining the price index formula was described in great detail by Joseph Lowe (1823). His approach to measuring the price change between periods 0 and 1 was to specify an approximate *representative product basket*,[6] which is a quantity vector $q \equiv [q_1, \ldots, q_n]$ that is representative of purchases made during the two periods under consideration, and then to calculate the level of prices in period 1 relative to period 0 as the ratio of the period 1 cost of the basket, $\sum_{i=1}^{n} p_i^1 q_i$, to the period 0 cost of the basket, $\sum_{i=1}^{n} p_i^0 q_i$. This *fixed-basket approach* to the determination of the price index leaves open the following question: How exactly is the fixed-basket vector q to be chosen?

16.13 As time passed, economists and price statisticians demanded a bit more precision with respect to the specification of the basket vector q. There are two natural choices for the reference basket: the base period 0 product vector q^0 or the current period 1 product vector q^1. These two choices led to the

[4]Note that it is assumed that there are no new or disappearing products in the value aggregates. Approaches to the "new goods problem" and the problem of accounting for quality change are discussed in Chapters 7, 8, and 21.

[5]The first person to suggest that the price and quantity indices should be jointly determined to satisfy equation (16.3) was Irving Fisher (1911, p. 418). Frisch (1930, p. 399) called equation (16.3) the *product test*.

[6]Joseph Lowe (1823, Appendix 95) suggested that the product basket vector q should be updated every five years. Lowe indices are studied in more detail in Section D.

Laspeyres (1871) price index[7] P_L defined by equation (16.5) and the Paasche (1874) price index[8] P_P defined by equation (16.6)[9]:

$$P_L(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}; \tag{16.5}$$

$$P_P(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^1}. \tag{16.6}$$

16.14 The above formulas can be rewritten in a manner that is more useful for statistical agencies. Define the period t value share on product i as follows:

$$s_i^t \equiv p_i^t q_i^t \Big/ \sum_{j=1}^{n} p_j^t q_j^t \quad \text{for } i = 1,\ldots,n \text{ and } t = 0, 1. \tag{16.7}$$

Then, the Laspeyres index equation (16.5) can be rewritten as[10]

$$\begin{aligned} P_L(p^0, p^1, q^0, q^1) &= \sum_{i=1}^{n} p_i^1 q_i^0 \Big/ \sum_{j=1}^{n} p_j^0 q_j^0 \\ &= \sum_{i=1}^{n} \left(p_i^1/p_i^0\right) p_i^0 q_i^0 \Big/ \sum_{j=1}^{n} p_j^0 q_j^0 \\ &= \sum_{i=1}^{n} \left(p_i^1/p_i^0\right) s_i^0 \end{aligned} \tag{16.8}$$

using definitions in equation (16.7). Thus, the Laspeyres price index P_L can be written as a base-period value share-weighted arithmetic average of the n price ratios, p_i^1/p_i^0. The Laspeyres formula (until the very recent past) has been widely used as the intellectual base for export and import price indices (XMPIs) around the world. To implement it, a statistical agency needs only to collect information on trade shares $s_n{}^0$ for the index domain of definition for the base period 0 and then collect information on item *prices* alone on an ongoing basis. *Thus, the Laspeyres XMPI can be produced on a timely basis without current-period quantity information.*

16.15 The Paasche index can also be written in value share and price ratio form as[11]

$$\begin{aligned} P_P(p^0, p^1, q^0, q^1) &= 1 \Big/ \left\{ \sum_{i=1}^{n} p_i^0 q_i^1 \Big/ \sum_{j=1}^{n} p_j^1 q_j^1 \right\} \\ &= 1 \Big/ \left\{ \sum_{i=1}^{n} \left(p_i^0/p_i^1\right) p_i^1 q_i^1 \Big/ \sum_{j=1}^{n} p_j^1 q_j^1 \right\} \\ &= 1 \Big/ \left\{ \sum_{i=1}^{n} \left(p_i^1/p_i^0\right)^{-1} s_i^1 \right\} \\ &= \left\{ \sum_{i=1}^{n} \left(p_i^1/p_i^0\right)^{-1} s_i^1 \right\}^{-1}, \end{aligned} \tag{16.9}$$

using definitions in equation (16.7). Thus, the Paasche price index P_P can be written as a period 1 (or current-period) trade share–weighted *harmonic* average of the n item price ratios $p_i{}^1/p_i{}^0$.[12] The lack of information on current-period quantities prevents statistical agencies from producing Paasche indices on a timely basis.

16.16 The quantity index that corresponds to the Laspeyres price index using the product test, equation (16.3), is the Paasche quantity index; that is, if P in equation (16.4) is replaced by P_L defined by equation (16.5), then the following quantity index is obtained:

$$Q_P(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i{}^1 q_i{}^1}{\sum_{i=1}^{n} p_i{}^1 q_i{}^0}. \tag{16.10}$$

[7]This index was actually introduced and justified by Drobisch (1871a, p. 147) slightly earlier than by Laspeyres. Laspeyres (1871, p. 305) in fact explicitly acknowledged that Drobisch showed him the way forward. However, the contributions of Drobisch were forgotten for the most part by later writers because Drobisch aggressively pushed for the ratio of two unit values as being the best index number formula. Although this formula has some excellent properties, if all the n products being compared have the same unit of measurement, the formula is useless when, say, both goods and services are in the index basket.

[8]Again, Drobisch (1871b, p. 424) appears to have been the first to explicitly define and justify this formula. However, he rejected this formula in favor of his preferred formula, the ratio of unit values, and so again he did not get any credit for his early suggestion of the Paasche formula.

[9]Note that $P_L(p^0, p^1, q^0, q^1)$ does not actually depend on q^1, and $P_P(p^0, p^1, q^0, q^1)$ does not actually depend on q^0. However, it does no harm to include these vectors, and the notation indicates that the reader is in the realm of bilateral index number theory; that is, the prices and quantities for a value aggregate pertaining to two periods are being compared.

[10]This method of rewriting the Laspeyres index (or any fixed-basket index) as a share-weighted arithmetic average of price ratios is due to Irving Fisher (1897, p. 517; 1911, p. 397; 1922, p. 51) and Walsh (1901, p. 506; 1921a, p. 92).

[11]This method of rewriting the Paasche index (or any fixed-basket index) as a share-weighted harmonic average of the price ratios is due to Walsh (1901, p. 511; 1921a, p. 93) and Irving Fisher (1911, pp. 397–98).

[12]Note that the derivation in equation (16.9) shows how harmonic averages arise in index number theory in a very natural way.

Note that Q_P is the value of the period 1 quantity vector valued at the period 1 prices, $\sum_{i=1}^{n} p_i^1 q_i^1$, divided by the (hypothetical) value of the period 0 quantity vector valued at the period 1 prices, $\sum_{i=1}^{n} p_i^1 q_i^0$. Thus, the period 0 and 1 quantity vectors are valued at the same set of prices, the current-period prices, p^1.

16.17 The quantity index that corresponds to the Paasche price index using the product test, equation (16.3), is the Laspeyres quantity index; that is, if P in equation (16.4) is replaced by P_P defined by equation (16.6), then the following quantity index is obtained:

$$Q_L(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}. \qquad (16.11)$$

Note that Q_L is the (hypothetical) value of the period 1 quantity vector valued at the period 0 prices, $\sum_{i=1}^{n} p_i^0 q_i^1$, divided by the value of the period 0 quantity vector valued at the period 0 prices, $\sum_{i=1}^{n} p_i^0 q_i^0$. Thus, the period 0 and 1 quantity vectors are valued at the same set of prices, the base-period prices, p^0.

16.18 The problem with the Laspeyres and Paasche index number formulas is that they are equally plausible, but, in general, they will give *different* answers. For most purposes, it is not satisfactory for the statistical agency to provide *two* answers to this question:[13] What is the best overall summary measure of price change for the value aggregate over the two periods in question? Thus, in the following section, it is considered how "best" averages of these two estimates of price change can be constructed. Before doing this, we ask, What is the normal relationship between the Paasche and Laspeyres indices? Under normal economic conditions, when the price ratios pertaining to the two situations under consideration are negatively correlated with the corresponding quantity ratios, it can be shown that the Laspeyres price index will be larger than the corresponding Paasche index.[14] In Appendix 16.1 a precise statement of this result is presented.[15] This divergence between P_L and P_P suggests that if a *single estimate* for the price change between the two periods is required, then some sort of evenly weighted average of the two indices should be taken as the final estimate of price change between periods 0 and 1. This strategy is pursued in the following section. For indices such as the consumer price index, producer price index, and services XMPIs, statistical agencies will not have information on current value weights and, hence, averages of Paasche and Laspeyres indices can be produced only on a delayed basis (perhaps using national accounts information) or not at all. Reasonably timely Paasche indices for the goods XMPIs, however, may be feasible because monthly trade weights are available on a timely basis in many countries.[16]

C. Symmetric Averages of Fixed-Basket Price Indices

C.1 Fisher index as an average of the Paasche and Laspeyres indices

16.19 As was mentioned in the previous paragraph, because the Paasche and Laspeyres price indices are equally plausible but often give different estimates of the amount of aggregate price change between periods 0 and 1, it is useful to consider taking an evenly weighted average of these fixed-basket price indices as a single estimator of price change between the two periods. Examples of such *symmetric averages*[17] are the arithmetic mean, which leads to the Drobisch (1871b,

[13]In principle, instead of averaging the Paasche and Laspeyres indices, the statistical agency could think of providing both (the Paasche index on a delayed basis). This suggestion would lead to a matrix of price comparisons between every pair of periods instead of a time series of comparisons. Walsh (1901, p. 425) noted this possibility: "In fact, if we use such direct comparisons at all, we ought to use all possible ones."

[14]Peter Hill (1993, p. 383) summarized this inequality as follows: "It can be shown that relationship (13) [that is, that P_L is greater than P_P] holds whenever the price and quantity relatives (weighted by values) are negatively correlated. Such negative correlation is to be expected for price takers who react to changes in relative prices by substituting goods and services that have become relatively less expensive for those that have become relatively more expensive. In the vast majority of situations covered by index numbers, the price and quantity relatives turn out to be negatively correlated so that Laspeyres indices tend systematically to record greater increases than Paasche with the gap between them tending to widen with time."

[15]There is another way to see why P_P will often be less than P_L. If the period 0 value shares s_i^0 are exactly equal to the corresponding period 1 value shares s_i^1, then by Schlömilch's (1858) Inequality (see Hardy, Littlewood, and Pólya, 1934, p. 26), it can be shown that a weighted harmonic mean of n numbers is equal to or less than the corresponding arithmetic mean of the n numbers and the inequality is strict if the n numbers are not all equal. If value shares are approximately constant across periods, then it follows that P_P will usually be less than P_L under these conditions; see Section D.3.

[16] However, the monthly goods trade figures from customs services likely will be subject to revision, so, by implication, the Paasche goods trade price index also would be subject to revision, at least in principle. If revisions to trade by commodity and destination/source country usually are small, these revisions will be very small, however.

[17]For a discussion of the properties of symmetric averages, see Diewert (1993c). Formally, an average $m(a,b)$ of two numbers a and b is symmetric if $m(a,b) = m(b,a)$. In other words, the numbers a and b are treated in the same manner in the average. An example of a nonsymmetric average of a and b is $(1/4)a + (3/4)b$. In general, Walsh (1901, p. 105) argued for a symmetric treatment if the two periods (or countries) under consideration were to be given equal importance.

p. 425), Sidgwick (1883, p. 68), Bowley (1901, p. 227)[18] index, $P_D \equiv (1/2)P_L + (1/2)P_P$, and the geometric mean, which leads to the Irving Fisher[19] (1922) ideal index, P_F, defined as

$$P_F(p^0, p^1, q^0, q^1) \equiv \{P_L(p^0, p^1, q^0, q^1) \times P_P(p^0, p^1, q^0, q^1)\}^{1/2}. \quad (16.12)$$

At this point, the fixed-basket approach to index number theory is transformed into the *test approach* to index number theory; that is, in order to determine which of these fixed-basket indices or which averages of them might be best, desirable *criteria* or *tests* or *properties* are needed for the price index. This topic is pursued in more detail in the next chapter, but an introduction to the test approach is provided in the present section because a test is used to determine which average of the Paasche and Laspeyres indices might be best.

16.20 What is the best symmetric average of P_L and P_P to use as a point estimate for the theoretical cost-of-living index? It is very desirable for a price index formula that depends on the price and quantity vectors pertaining to the two periods under consideration to satisfy the *time reversal test.*[20] An index number formula $P(p^0, p^1, q^0, q^1)$ satisfies this test if

$$P(p^1, p^0, q^1, q^0) = 1/P(p^0, p^1, q^0, q^1); \quad (16.13)$$

that is, if the period 0 and period 1 price and quantity data are interchanged and the index number formula is evaluated, then this new index $P(p^1, p^0, q^1, q^0)$ is equal to the reciprocal of the original index $P(p^0, p^1, q^0, q^1)$. It seems desirable that the measure of aggregate price change should satisfy this property so that it does not matter which period is chosen as the base period. Put another way, the index number comparison between any two points of time should not depend on the choice of which period we regard as the base period: If the other period is chosen as the base period, then the new index number should simply equal the reciprocal of the original index. It should be noted that the Laspeyres and Paasche price indices *do not* satisfy this time reversal property.

16.21 Having defined what it means for a price index P to satisfy the time reversal test, then it is possible to establish the following result:[21] The Fisher ideal price index defined by equation (16.12) above is the *only* index that is a homogeneous[22] symmetric average of the Laspeyres and Paasche price indices, P_L and P_P, and satisfies the time reversal test in equation (16.13). Thus the Fisher ideal price index emerges as perhaps the best evenly weighted average of the Paasche and Laspeyres price indices.

16.22 It is interesting to note that this *symmetric basket approach* to index number theory dates back to one of the early pioneers of index number theory, Arthur L. Bowley, as the following quotations indicate:

> If [the Paasche index] and [the Laspeyres index] lie close together there is no further difficulty; if they differ by much they may be regarded as inferior and superior limits of the index number, which may be estimated as their arithmetic mean . . . as a first approximation. (Bowley, 1901, p. 227)

> When estimating the factor necessary for the correction of a change found in money wages to obtain the change in real wages, statisticians have not been content to follow Method II only [to calculate a Laspeyres price index], but have worked the problem backwards [to calculate a Paasche price index] as well as forwards. . . . They have then taken the arithmetic, geometric or harmonic mean of the two numbers so found. (Bowley, 1919, p. 348)[23]

16.23 The quantity index that corresponds to the Fisher price index using the product test, equation (16.3), is the Fisher quantity index; that is, if P in equation (16.4) is replaced by P_F defined by (16.12), the following quantity index is obtained:

$$Q_F(p^0, p^1, q^0, q^1) \equiv \{Q_L(p^0, p^1, q^0, q^1) \times Q_P(p^0, p^1, q^0, q^1)\}^{1/2}. \quad (16.14)$$

[18]Walsh (1901, p. 99) also suggested this index. See Diewert (1993a, p. 36) for additional references to the early history of index number theory.

[19]Bowley (1899, p. 641) appears to have been the first to suggest the use of this index. Walsh (1901, pp. 428–29) also suggested this index while commenting on the big differences between the Laspeyres and Paasche indices in one of his numerical examples: "The figures in columns (2) [Laspeyres] and (3) [Paasche] are, singly, extravagant and absurd. But there is order in their extravagance; for the nearness of their means to the more truthful results shows that they straddle the true course, the one varying on the one side about as the other does on the other."

[20]See Diewert (1992a, p. 218) for early references to this test. If we want the price index to have the same property as a single price ratio, then it is important to satisfy the time reversal test. However, other points of view are possible. For example, we may want to use our price index for compensation purposes, in which case satisfaction of the time reversal test may not be so important.

[21]This is formally derived in Diewert (1997) p. 138).

[22]An average or mean of two numbers a and b, $m(a,b)$, is *homogeneous* if, when both numbers a and b are multiplied by a positive number λ, then the mean is also multiplied by λ; that is, m satisfies the following property: $m(\lambda a,\lambda b) = \lambda m(a,b)$.

[23]Irving Fisher (1911, pp. 417–18; 1922) also considered the arithmetic, geometric, and harmonic averages of the Paasche and Laspeyres indices.

Thus, the Fisher quantity index is equal to the square root of the product of the Laspeyres and Paasche quantity indices. It should also be noted that $Q_F(p^0, p^1, q^0, q^1) = P_F(q^0, q^1, p^0, p^1)$; that is, if the role of prices and quantities is interchanged in the Fisher price index formula, then the Fisher quantity index is obtained.[24]

16.24 Rather than take a symmetric average of the two basic fixed-basket price indices pertaining to two situations, P_L and P_P, it is also possible to return to Lowe's basic formulation and choose the basket vector q to be a symmetric average of the base- and current-period basket vectors, q^0 and q^1. The following subsection pursues this approach to index number theory.

C.2 Walsh index and theory of "pure" price index

16.25 Price statisticians tend to be very comfortable with a concept of the price index based on pricing out a constant representative basket of products, $q \equiv (q_1, q_2, \ldots, q_n)$, at the prices of period 0 and 1, $p^0 \equiv (p_1{}^0, p_2{}^0, \ldots, p_n{}^0)$ and $p^1 \equiv (p_1{}^1, p_2{}^1, \ldots, p_n{}^1)$, respectively. Price statisticians refer to this type of index as a *fixed-basket index* or a *pure price index,*[25] and it corresponds to Knibbs' (1924, p. 43) *unequivocal price index*.[26] Because Joseph Lowe (1823) was the first person to describe systematically this type of index, it is referred to as a *Lowe index*. Thus, the general functional form for the *Lowe price index* is

$$P_{Lo}(p^0, p^1, q) \equiv \sum_{i=1}^{n} p_i^1 q_i \Big/ \sum_{i=1}^{n} p_i^0 q_i = \sum_{i=1}^{n} s_i \left(p_i^1 / p_i^0\right), \tag{16.15}$$

where the (hypothetical) *hybrid value shares* s_i[27] corresponding to the quantity weights vector q are defined by

$$s_i \equiv p_i^0 q_i \Big/ \sum_{j=1}^{n} p_j^0 q_j \quad \text{for } i = 1, 2, \ldots, n. \tag{16.16}$$

16.26 The main reason price statisticians might prefer a member of the family of Lowe or fixed-basket price indices defined by equation (16.15) is that *the fixed-basket concept is easy to explain to the public*. Note that the Laspeyres and Paasche indices are special cases of the pure price concept if we choose $q = q^0$ (which leads to the Laspeyres index) or if we choose $q = q^1$ (which leads to the Paasche index).[28] The practical problem of picking q remains to be resolved, and that is the problem addressed in this section.

16.27 It should be noted that Walsh (1901, p. 105; 1921a) also saw the price index number problem in the above framework:

> Commodities are to be weighted according to their importance, or their full values. But the problem of axiometry always involves at least two periods. There is a first period, and there is a second period which is compared with it. Price variations have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period? Or those of the second? Or should there be a combination of the two sets? There is no reason for preferring either the first or the second. Then the combination of both would seem to be the proper answer. And this combination itself involves an averaging of the weights of the two periods. (Walsh, 1921a, p. 90)

Walsh's suggestion will be followed, and thus the *i*th quantity weight, q_i, is restricted to be an average or *mean* of the base-period quantity $q_i{}^0$ and the current-period quantity for product i $q_i{}^1$, say $m(q_i{}^0, q_i{}^1)$, for $i = 1, 2, \ldots, n$.[29] Under this assumption, the Lowe price index (16.15) becomes

[24]Irving Fisher (1922, p. 72) said that P and Q satisfied the *factor reversal test* if $Q(p^0, p^1, q^0, q^1) = P(q^0, q^1, p^0, p^1)$ and P and Q satisfied the product test in equation (16.3) as well.

[25]See Section 7 in Diewert (2001a).

[26]"Suppose, however, that for each commodity, $Q' = Q$, the fraction, $\Sigma(P'Q)/\Sigma(PQ)$, viz., the ratio of aggregate value for the second unit-period to the aggregate value for the first unit-period is no longer merely a ratio of totals; it also shows unequivocally the effect of the change in price. Thus, it is an unequivocal price index for the quantitatively unchanged complex of commodities, *A, B, C,* et cetera.

"It is obvious that if the quantities were different on the two occasions, and if at the same time the prices had been unchanged, the preceding formula would become $\Sigma(PQ')/\Sigma(PQ)$. It would still be the ratio of the aggregate value for the second unit-period to the aggregate value for the first unit-period. But it would be also more than this. It would show in a generalized way the ratio of the quantities on the two occasions. Thus it is an unequivocal quantity index for the complex of commodities, unchanged as to price and differing only as to quantity.

"Let it be noted that the mere algebraic form of these expressions shows at once the logic of the problem of finding these two indices is identical." (Knibbs, 1924, pp. 43–44)

[27]Irving Fisher (1922, p. 53) used the terminology "weighted by a hybrid value," whereas Walsh (1932, p. 657) used the term "hybrid weights."

[28]Note that the *i*th share defined by equation (16.16) in this case is the hybrid share $s_i = p_i^0 q_i^1 / \sum_{i=1}^{n} p_i^0 q_i^1$ which uses the prices of period 0 and the quantities of period 1.

[29]Note that the mean function $m(q_i{}^0, q_i{}^1)$ has been chosen to be the same for each item i. It is assumed that $m(a,b)$ has the following two properties: $m(a,b)$ is a positive and continuous function, defined for all positive numbers a and b, and $m(a,a) = a$ for all $a > 0$.

$$P_{Lo}(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 m(q_i^0, q_i^1)}{\sum_{j=1}^{n} p_j^0 m(q_j^0, q_j^1)}. \quad (16.17)$$

16.28 In order to determine the functional form for the mean function m, it is necessary to impose some *tests* or *axioms* on the pure price index defined by equation (16.17). As in Section C.1, we ask that P_{Lo} satisfy the *time reversal test,* equation (16.13) above. Under this hypothesis, it is immediately obvious that the mean function m must be a *symmetric mean*;[30] that is, m must satisfy the following property: $m(a,b) = m(b,a)$ for all $a > 0$ and $b > 0$. This assumption still does not pin down the functional form for the pure price index defined by equation (16.17) above. For example, the function $m(a,b)$ could be the *arithmetic mean,* $(1/2)a + (1/2)b$, in which case equation (16.17) reduces to the *Marshall Edgeworth* (Marshall, 1887; Edgeworth, 1925) *price index* P_{ME}, which was the pure price index preferred by Knibbs (1924, p. 56):

$$P_{ME}(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 \left\{(q_i^0 + q_i^1)/2\right\}}{\sum_{j=1}^{n} p_j^0 \left\{(q_j^0 + q_j^1)/2\right\}}. \quad (16.18)$$

16.29 On the other hand, the function $m(a,b)$ could be the *geometric mean,* $(ab)^{1/2}$, in which case equation (16.17) reduces to the *Walsh* (1901, p. 398; 1921a, p. 97) *price index,* P_W[31]:

$$P_W(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 \sqrt{q_i^0 q_i^1}}{\sum_{j=1}^{n} p_j^0 \sqrt{q_j^0 q_j^1}}. \quad (16.19)$$

16.30 There are many other possibilities for the mean function m, including the mean of order r, $[(1/2)a^r + (1/2)b^r]^{1/r}$ for $r \neq 0$. To completely determine the functional form for the pure price index P_{Lo}, it is necessary to impose at least one additional test or axiom on $P_{Lo}(p^0, p^1, q^0, q^1)$.

16.31 There is a potential problem with the use of the Edgeworth-Marshall price index, equation (16.18), that has been noticed in the context of using the formula to make international comparisons of prices. If the price levels of a very large country are compared with the price levels of a small country using equation (16.18), then the quantity vector of the large country may totally overwhelm the influence of the quantity vector corresponding to the small country.[32] In technical terms, the Edgeworth-Marshall formula is not homogeneous of degree 0 in the components of both q^0 and q^1. To prevent this problem from occurring in the use of the pure price index $P_K(p^0, p^1, q^0, q^1)$ defined by equation (16.17), it is asked that P_{Lo} satisfy the following *invariance to proportional changes in current quantities test:*[33]

$$P_{Lo}(p^0, p^1, q^0, \lambda q^1) = P_{Lo}(p^0, p^1, q^0, q^1)$$
$$\text{for all } p^0, p^1, q^0, q^1 \text{ and all } \lambda > 0. \quad (16.20)$$

The two tests, the time reversal test in equation (16.13) and the invariance test in equation (16.20), enable one to determine the precise functional form for the pure price index P_{Lo} defined by equation (16.17) above: The pure price index P_K must be the Walsh index P_W defined by equation (16.19).[34]

16.32 To be of practical use by statistical agencies, an index number formula must be able to be expressed as a function of the base-period value shares, s_i^0; the current-period value shares, s_i^1; and the n price ratios, p_i^1/p_i^0. The Walsh price index defined by equation (16.19) above can be rewritten in this format:

$$P_W(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} p_i^1 \sqrt{q_i^0 q_i^1}}{\sum_{j=1}^{n} p_j^0 \sqrt{q_j^0 q_j^1}} = \frac{\sum_{i=1}^{n} \left(p_i^1/\sqrt{p_i^0 p_i^1}\right)\sqrt{s_i^0 s_i^1}}{\sum_{j=1}^{n} \left(p_j^0/\sqrt{p_j^0 p_j^1}\right)\sqrt{s_j^0 s_j^1}}$$

[30] For more on symmetric means, see Diewert (1993c, p. 361).

[31] Walsh endorsed P_W as being the best index number formula: "We have seen reason to believe formula 6 better than formula 7. Perhaps formula 9 is the best of the rest, but between it and Nos. 6 and 8 it would be difficult to decide with assurance" (Walsh, 1921a, p. 103). His formula 6 is P_W defined by equation (16.19) and his 9 is the Fisher ideal defined by equation (16.12) above. The *Walsh quantity index*, $Q_W(p^0, p^1, q^0, q^1)$, is defined as $P_W(q^0, q^1, p^0, p^1)$; that is, prices and quantities in equation (16.19) are interchanged. If the Walsh quantity index is used to deflate the value ratio, an implicit price index is obtained, which is Walsh's formula 8.

[32] This is not likely to be a severe problem in the time-series context where the change in quantity vectors going from one period to the next is small.

[33] This is the terminology used by Diewert (1992a, p. 216). Vogt (1980) was the first to propose this test.

[34] See Section 7 in Diewert (2001a).

$$= \frac{\sum_{i=1}^{n} \sqrt{s_i^0 s_i^1} \sqrt{p_i^1 / p_i^0}}{\sum_{j=1}^{n} \sqrt{s_j^0 s_j^1} \sqrt{p_j^0 / p_j^1}}. \qquad (16.21)$$

C.3 Conclusion

16.33 The approach taken to index number theory in this section was to consider averages of various fixed-basket price indices. The first approach was to take an evenhanded average of the two primary fixed-basket indices: the Laspeyres and Paasche price indices. These two primary indices are based on pricing out the baskets that pertain to the two periods (or locations) under consideration. Taking an average of them led to the Fisher ideal price index P_F defined by equation (16.12) above. The second approach was to average the basket quantity weights and then price out this average basket at the prices pertaining to the two situations under consideration. This approach led to the Walsh price index P_W defined by equation (16.19) above. Both these indices can be written as a function of the base-period value shares, s_i^0; the current-period value shares, s_i^1; and the n price ratios, p_i^1/p_i^0. Assuming that the statistical agency has information on these three sets of variables, which index should be used? Experience with normal time-series data has shown that these two indices will not differ substantially, and thus it is a matter of choice which of these indices is used in practice.[35] Both these indices are examples of "superlative indices."[36] However, note that both treat the data pertaining to the two situations in a *symmetric* manner. Peter Hill (1993; see also Hill, 1988) commented on superlative price indices and the importance of a symmetric treatment of the data as follows:

> Thus economic theory suggests that, in general, a symmetric index that assigns equal weight to the two situations being compared is to be preferred to either the Laspeyres or Paasche indices on their own. The precise choice of superlative index—whether Fisher, Törnqvist, or other superlative index—may be of only secondary importance as all the symmetric indices are likely to approximate each other, and the underlying theoretic index fairly closely, at least when the index number spread between the Laspeyres and Paasche is not very great. (P. Hill, 1993, p. 384)

[35]Diewert (1978, pp. 887–89) showed that these two indices will approximate each other to the second order around an equal price and quantity point. Thus, for normal time-series data where prices and quantities do not change much going from the base period to the current period, the indices will approximate each other quite closely.

[36]Superlative indices are defined in Chapter 18.

D. Annual Weights and Monthly Price Indices

D.1 Lowe index with monthly prices and annual base-year quantities

16.34 It is now necessary to discuss a major practical problem with the theory of basket-type indices. Up to now, it has been assumed that the quantity vector $q \equiv (q_1, q_2, \ldots, q_n)$ that appeared in the definition of the Lowe index, $P_{Lo}(p^0, p^1, q)$ defined by equation (16.15), is either the base-period quantity vector q^0 or the current-period quantity vector q^1 or an average of the two. In fact, in terms of actual statistical agency practice, the quantity vector q is usually taken to be an annual quantity vector that refers to a *base year b*, say, the year before the base period for the prices, period 0. Typically, a statistical agency will produce XMPIs at a monthly or quarterly frequency but, for the sake of definiteness, a monthly frequency will be assumed in what follows. Thus, a typical price index will have the form $P_{Lo}(p^0, p^t, q^b)$, where p^0 is the price vector pertaining to the base-period month for prices, month 0, p^t is the price vector pertaining to the current-period month for prices, month t, say, and q^b is a reference basket quantity vector that refers to the base year b, which is equal to or before month 0.[37] Note that this Lowe index $P_{Lo}(p^0, p^t, q^b)$ is *not* a true Laspeyres index (because the annual quantity vector q^b is not equal to the monthly quantity vector q^0 in general).[38]

16.35 The question is this: Why do statistical agencies *not* pick the reference quantity vector q in the Lowe formula to be the monthly quantity vector q^0 that pertains to transactions in month 0 (so that the index would reduce to an ordinary Laspeyres price index)? There are two main reasons:

- Most economies are subject to seasonal fluctuations, and so picking the quantity vector of month 0 as the reference quantity vector for all months of the year would not be representative of transactions made throughout the year.

[37]Month 0 is called the price reference period, and year b is called the weight reference period.

[38]Triplett (1981, p. 12) defined the Lowe index, calling it a Laspeyres index, and calling the index that has the weight reference period equal to the price reference period a pure Laspeyres index. Triplett also noted the hybrid share representation for the Lowe index defined by equation (16.15) and equation (16.16). Triplett noted that the ratio of two Lowe indices using the same quantity weights was also a Lowe index.

- Monthly household quantity or value weights are usually collected by the statistical agency using an establishment survey with a relatively small sample. Hence, the resulting weights are usually subject to very large sampling errors, and so standard practice is to average these monthly value or quantity weights over an entire year (or in some cases, over several years), in an attempt to reduce these sampling errors. In other instances, where an establishment census is used, the reported value weights are for an annual period.

The index number problems that are caused by seasonal monthly weights are studied in more detail in Chapter 23. For now, it can be argued that the use of annual weights in a monthly index number formula is simply a method for dealing with the seasonality problem.[39]

16.36 One problem with using annual weights corresponding to a perhaps distant year in the context of monthly XMPIs must be noted at this point. If there are systematic (but divergent) trends in product prices, and consumers or businesses increase their purchases of products that decline (relatively) in price and decrease their purchases of products that increase (relatively) in price, then the use of distant quantity weights will tend to lead to an upward bias in this Lowe index compared with one that uses more current weights, as is shown below. This observation suggests that statistical agencies should get up-to-date weights on an ongoing basis.

16.37 It is useful to explain how the annual quantity vector q^b could be obtained from monthly values on each product during the chosen base year b. Let the month m value of the reference population in the base year b for product i be $v_i^{b,m}$, and let the corresponding price and quantity be $p_i^{b,m}$, and $q_i^{b,m}$, respectively. Value, price, and quantity for each product are related by the following equations:

$$v_i^{b,m} = p_i^{b,m}\, q_i^{b,m};\; i = 1, \ldots, n;\; m = 1, \ldots, 12. \quad (16.22)$$

For each product i, the annual total q_i^b can be obtained by price-deflating monthly values and summing over months in the base year b as follows:

$$q_i^b = \sum_{m=1}^{12} \frac{v_i^{b,m}}{p_i^{b,m}} = \sum_{m=1}^{12} q_i^{b,m};\; i = 1, \ldots, n, \quad (16.23)$$

where equation (16.22) was used to derive equation (16.23). In practice, the above equations will be evaluated using aggregate values over closely related products, and the price $p_i^{b,m}$ will be the month m price index for this elementary product group i in year b relative to the first month of year b.

16.38 For some purposes, it is also useful to have annual prices by product to match up with the annual quantities defined by equation (16.23). Following national accounting conventions, a reasonable[40] price p_i^b to match the annual quantity q_i^b is the value of total value for product i in year b divided by q_i^b. Thus, we have

$$p_i^b \equiv \sum_{m=1}^{12} v_i^{b,m}/q_i^b;\; i = 1, \ldots, n$$

$$= \frac{\sum_{m=1}^{12} v_i^{b,m}}{\sum_{m=1}^{12} v_i^{b,m}/p_i^{b,m}};\ \text{using equation (16.23)}$$

$$= \left[\sum_{m=1}^{12} s_i^{b,m}(p_i^{b,m})^{-1}\right]^{-1}, \quad (16.24)$$

where the share of annual value on product i in month m of the base year is

$$s_i^{b,m} \equiv \frac{v_i^{b,m}}{\sum_{k=1}^{12} v_i^{b,k}};\; i = 1, \ldots, n. \quad (16.25)$$

Thus, the annual base-year price for product i, p_i^b, turns out to be a monthly value weighted *harmonic mean* of the monthly prices for product i in the base year, $p_i^{b,1}$, $p_i^{b,2}, \ldots, p_i^{b,12}$.

16.39 Using the annual product prices for the base year defined by equation (16.24), a vector of these prices can be defined as $p^b \equiv [p_1^b, \ldots, p_n^b]$. Using this definition, the Lowe index can be expressed as a ratio of two Laspeyres indices where the price vector p^b

[39]In fact, using the Lowe index $P_{Lo}(p^0, p^t, q^b)$ in the context of seasonal products corresponds to Bean and Stine's (1924, p. 31) Type A index number formula. Bean and Stine made three additional suggestions for price indices in the context of seasonal products. Their contributions are evaluated in Chapter 22.

[40]Hence, these annual product prices are essentially unit value prices. Under conditions of high inflation, the annual prices defined by equation (16.24) may no longer be reasonable or representative of prices during the entire base year because the values in the final months of the high-inflation year will be somewhat artificially blown up by general inflation. Under these conditions, the annual prices and annual product value shares should be interpreted with caution. For more on dealing with situations where there is high inflation within a year, see Hill (1996).

plays the role of base-period prices in each of the two Laspeyres indices:

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \frac{\sum_{i=1}^{n} p_i^t q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b \Big/ \sum_{i=1}^{n} p_i^b q_i^b} = \frac{\sum_{i=1}^{n} s_i^b \left(p_i^t / p_i^b\right)}{\sum_{i=1}^{n} s_i^b \left(p_i^0 / p_i^b\right)}$$

$$= P_L(p^b, p^t, q^b)/P_L(p^b, p^0, q^b), \qquad (16.26)$$

where the Laspeyres formula P_L was defined by equation (16.5) above. Thus, the above equation shows that the Lowe monthly price index comparing the prices of month 0 with those of month t using the quantities of base year b as weights, $P_{Lo}(p^0, p^t, q^b)$, is equal to the Laspeyres index that compares the prices of month t with those of year b, $P_L(p^b, p^t, q^b)$, divided by the Laspeyres index that compares the prices of month 0 with those of year b, $P_L(p^b, p^0, q^b)$. Note that the Laspeyres index in the numerator can be calculated if the base-year product value shares, s_i^b, are known along with the price ratios that compare the prices of product i in month t, p_i^t, with the corresponding annual average prices in the base year b, p_i^b. The Laspeyres index in the denominator can be calculated if the base-year product value shares, s_i^b, are known along with the price ratios that compare the prices of product i in month 0, p_i^0, with the corresponding annual average prices in the base year b, p_i^b.

16.40 Another convenient formula for evaluating the Lowe index, $P_{Lo}(p^0, p^t, q^b)$, uses the hybrid weights formula, equation (16.15). In the present context, the formula becomes

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b},$$

$$= \frac{\sum_{i=1}^{n} \left(p_i^t / p_i^0\right) p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^{0b}, \qquad (16.27)$$

where the hybrid weights s_i^{0b} using the prices of month 0 and the quantities of year b are defined by

$$s_i^{0b} \equiv \frac{p_i^0 q_i^b}{\sum_{j=1}^{n} p_j^0 q_j^b}; \; i = 1, \ldots, n$$

$$= \frac{p_i^b q_i^b \left(p_i^0 / p_i^b\right)}{\sum_{j=1}^{n} \left[p_j^b q_j^b \left(p_j^0 / p_j^b\right)\right]}. \qquad (16.28)$$

Equation (16.28) shows how the base-year values, $p_i^b q_i^b$, can be multiplied by the product price indices, p_i^0/p_i^b, to calculate the hybrid shares.

16.41 One additional formula for the Lowe index, $P_{Lo}(p^0, p^t, q^b)$, is exhibited. Note that the Laspeyres decomposition of the Lowe index defined by the third line in equation (16.26) involves the very long-term price relatives, p_i^t/p_i^b, that compare the prices in month t, p_i^t, with the possibly distant base-year prices, p_i^b. Further, the hybrid share decomposition of the Lowe index defined by the third line in equation (16.27) involves the long-term monthly price relatives, p_i^t/p_i^0, which compare the prices in month t, p_i^t, with the base-month prices, p_i^0. Both these formulas are not satisfactory in practice because of the problem of sample attrition: Each month, a substantial fraction of products disappears from the marketplace and thus it is useful to have a formula for updating the previous month's price index using just month-over-month price relatives. In other words, long-term price relatives disappear at a rate that is too large in practice to base an index number formula on their use. The Lowe index for month $t + 1$, $P_{Lo}(p^0, p^{t+1}, q^b)$, can be written in terms of the Lowe index for month t, $P_{Lo}(p^0, p^t, q^b)$, and an updating factor as follows:

$$P_{Lo}(p^0, p^{t+1}, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}\right] \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}\right]$$

$$= P_{Lo}(p^0, p^t, q^b) \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}\right]$$

$$= P_{Lo}(p^0, p^t, q^b)\left[\frac{\sum_{i=1}^{n}\left(\frac{p_i^{t+1}}{p_i^t}\right)p_i^t q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}\right]$$

$$= P_{Lo}(p^0, p^t, q^b)\left[\sum_{i=1}^{n}\left(\frac{p_i^{t+1}}{p_i^t}\right)s_i^{tb}\right], \qquad (16.29)$$

where the hybrid weights s_i^{tb} are defined by

$$s_i^{tb} \equiv \frac{p_i^t q_i^b}{\sum_{j=1}^{n} p_j^t q_j^b}; i = 1, \ldots, n. \qquad (16.30)$$

Thus, the required updating factor, going from month t to month $t + 1$, is the chain link index $\sum_{i=1}^{n} s_i^{tb}(p_i^{t+1}/p_i^t)$, which uses the hybrid share weights s_i^{tb} corresponding to month t and base year b.

16.42 The Lowe index $P_{Lo}(p^0, p^t, q^b)$ can be regarded as an approximation to the ordinary Laspeyres index, $P_L(p^0, p^t, q^0)$, that compares the prices of the base month 0, p^0, to those of month t, p^t, using the quantity vector of month 0, q^0, as weights. There is a relatively simple formula that relates these two indices. To explain this formula, it is first necessary to make a few definitions. Define the ith price relative between month 0 and month t as

$$r_i \equiv p_i^t/p_i^0; i = 1, \ldots, n. \qquad (16.31)$$

The ordinary Laspeyres price index, going from month 0 to t, can be defined in terms of these price relatives as follows:

$$P_L(p^0, p^t, q^0) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \frac{\sum_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)s_i^0$$

$$= \sum_{i=1}^{n} s_i^0 r_i \equiv r^*, \qquad (16.32)$$

where the month 0 value shares s_i^0 are defined as follows:

$$s_i^0 \equiv \frac{p_i^0 q_i^0}{\sum_{j=1}^{n} p_j^0 q_j^0}; i = 1, \ldots, n. \qquad (16.33)$$

16.43 Define the ith quantity relative t_i as the ratio of the quantity of product i used in the base year b, q_i^b, to the quantity used in month 0, q_i^0, as follows:

$$t_i \equiv q_i^b/q_i^0; i = 1, \ldots, n. \qquad (16.34)$$

The Laspeyres quantity index, $Q_L(q^0, q^b, p^0)$, that compares quantities in year b, q^b, with the corresponding quantities in month 0, q^0, using the prices of month 0, p^0, as weights can be defined as a weighted average of the quantity ratios t_i as follows:

$$Q_L(q^0, q^b, p^0) \equiv \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \frac{\sum_{i=1}^{n}\left(\frac{q_i^b}{q_i^0}\right)p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

$$= \sum_{i=1}^{n}\left(\frac{q_i^b}{q_i^0}\right)s_i^0; \text{ using equation (16.34)}$$

$$= \sum_{i=1}^{n} s_i^0 t_i \equiv t^*. \qquad (16.35)$$

16.44 Using equation (A16.2.4) in Appendix 16.2, the relationship between the Lowe index $P_{Lo}(p^0, p^t, q^b)$ that uses the quantities of year b as weights to compare the prices of month t to month 0 and the corresponding ordinary Laspeyres index $P_L(p^0, p^t, q^0)$ that uses the quantities of month 0 as weights is defined as

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

$$= P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0}{Q_L(q^0, q^b, p^0)}. \qquad (16.36)$$

Thus, the Lowe price index using the quantities of year b as weights, $P_{Lo}(p^0, p^t, q^b)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0, p^t, q^0)$, plus a covariance term $\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0$ between the price relatives $r_i \equiv p_i^t/p_i^0$ and the quantity relatives $t_i \equiv q_i^b/q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0, q^b, p^0)$ between month 0 and base year b.

16.45 Equation (16.36) shows that the Lowe price index will coincide with the Laspeyres price index if the covariance or correlation between the month 0 to t price relatives $r_i \equiv p_i^t/p_i^0$ and the month 0 to year b quantity relatives $t_i \equiv q_i^b/q_i^0$ is zero. Note that this covariance will be zero under three different sets of conditions:

- If the month t prices are proportional to the month 0 prices so that all $r_i = r^*$,
- If the base year b quantities are proportional to the month 0 quantities so that all $t_i = t^*$, and
- If the distribution of the relative prices r_i is independent of the distribution of the relative quantities t_i.

The first two conditions are unlikely to hold empirically, but the third is possible, at least approximately, if purchasers do not systematically change their purchasing habits in response to changes in relative prices.

16.46 If this covariance in equation (16.36) is negative, then the Lowe index will be less than the Laspeyres and, finally, if the covariance is positive, then the Lowe index will be greater than the Laspeyres index. Although the sign and magnitude of the covariance term is ultimately an empirical matter, it is possible to make some reasonable conjectures about its likely sign. If the base year b precedes the price reference month 0 *and there are long-term trends in prices*, then it is likely that this covariance is *positive,* and hence that the Lowe index will exceed the corresponding Laspeyres price index;[41] that is,

$$P_{Lo}(p^0, p^t, q^b) > P_L(p^0, p^t, q^0). \tag{16.37}$$

To see why this covariance is likely to be positive, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t/p_i^0) - r^*$ is positive. With normal substitution responses,[42] q_i^t/q_i^0 less an average quantity change of this type (t^*) is likely to be negative or, upon taking reciprocals, q_i^0/q_i^t less an average quantity change of this (reciprocal) type is likely to be positive. But if the long-term upward trend in prices has persisted back to the base year b, then $t_i - t^* \equiv (q_i^b/q_i^0) - t^*$ is also likely to be positive. Hence, the covariance will be positive under these circumstances. Moreover, the more distant the weight reference year b is from the price reference month 0, the bigger the residuals $t_i - t^*$ will likely be and the bigger will be the positive covariance. Similarly, the more distant is the current-period month t from the base-period month 0, the bigger the residuals $r_i - r^*$ will likely be and the bigger will be the positive covariance. *Thus, under the assumptions that there are long-term trends in prices and normal substitution responses, the Lowe index will normally be greater than the corresponding Laspeyres index.*

16.47 Define the Paasche index between months 0 and t as follows:

$$P_P(p^0, p^t, q^t) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t}. \tag{16.38}$$

As was discussed in Section C.1 above, a reasonable target index to measure the price change going from month 0 to t is some sort of symmetric average of the Paasche index $P_P(p^0, p^t, q^t)$ defined by equation (16.38) and the corresponding Laspeyres index $P_L(p^0, p^t, q^0)$ defined by equation (16.32). Adapting equation (A16.1.5) in Appendix 16.1, the relationship between the Paasche and Laspeyres indices can be written as follows:

$$P_P(p^0, p^t, q^t) = P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(u_i - u^*)s_i^0}{Q_L(q^0, q^t, p^0)}, \tag{16.39}$$

where the price relatives $r_i \equiv p_i^t/p_i^0$ are defined by equation (16.31) and their share-weighted average r^* by equation (16.32) and the u_i, u^*, and Q_L are defined as follows:

$$u_i \equiv q_i^t/q_i^0;\ i = 1, \ldots, n, \tag{16.40}$$

$$u^* \equiv \sum_{i=1}^{n} s_i^0 u_i = Q_L(q^0, q^t, p^0), \tag{16.41}$$

and the month 0 value shares s_i^0 are defined by equation (16.33). Thus, u^* is equal to the Laspeyres quantity index between months 0 and t. This means that the Paasche price index that uses the quantities of month t as weights,

[41]It is also necessary to assume that purchasers have normal substitution effects in response to these long-term trends in prices; that is, if a product increases (relatively) in price, its quantity purchased will decline (relatively), and if a product decreases relatively in price, its quantity purchased will increase relatively. This reflects the normal "market equilibrium" response to changes in supply.

[42]Walsh (1901, pp. 281–82) was well aware of substitution effects, as can be seen in the following comment which noted the basic problem with a fixed-basket index that uses the quantity weights of a single period: "The argument made by the arithmetic averagist supposes that we buy the same quantities of every class at both periods in spite of the variation in their prices, which we rarely, if ever, do. As a rough proposition, we—a community—generally spend more on articles that have risen in price and get less of them, and spend less on articles that have fallen in price and get more of them."

$P_P(p^0, p^t, q^t)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0, p^t, q^0)$, plus a covariance term $\sum_{i=1}^{n}(r_i - r^*)(u_i - u^*)s_i^0$ between the price relatives $r_i \equiv p_i^t/p_i^0$ and the quantity relatives $u_i \equiv q_i^t/q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0, q^t, p^0)$ between month 0 and month t.

16.48 Although the sign and magnitude of the covariance term is again an empirical matter, it is possible to make a reasonable conjecture about its likely sign. *If there are long-term trends in prices, and purchasers respond normally to price changes in their purchases*, then it is likely that this covariance is *negative,* and hence the Paasche index will be less than the corresponding Laspeyres price index; that is,

$$P_P(p^0, p^t, q^b) < P_L(p^0, p^t, q^0). \qquad (16.42)$$

To see why this covariance is likely to be negative, suppose that there is a long-term upward trend in the price of product i[43] so that $r_i - r^* \equiv (p_i^t/p_i^0) - r^*$ is positive. With normal substitution responses, q_i^t/q_i^0 less an average quantity change of this type (u^*) is likely to be negative. Hence, $u_i - u^* \equiv (q_i^t/q_i^0) - u^*$ is likely to be negative. Thus, the covariance will be negative under these circumstances. Moreover, the more distant the base month 0 is from the current month t, the bigger in magnitude the residuals $u_i - u^*$ will likely be and the bigger in magnitude will be the negative covariance.[44] Similarly, the more distant the current-period month t is from the base-period month 0, the bigger the residuals $r_i - r^*$ will likely be and the bigger in magnitude will be the covariance. *Thus, under the assumptions that there are long-term trends in prices and normal substitution responses, the Laspeyres index will be greater than the corresponding Paasche index*, with the divergence likely growing as month t becomes more distant from month 0.

16.49 Putting the arguments in the three previous paragraphs together, it can be seen that under the assumptions that there are long-term trends in prices and normal substitution responses, the Lowe price index between months 0 and t will exceed the corresponding Laspeyres price index, which in turn will exceed the corresponding Paasche price index; that is, under these hypotheses,

$$P_{Lo}(p^0, p^t, q^b) > P_L(p^0, p^t, q^0) > P_P(p^0, p^t, q^t). \qquad (16.43)$$

Thus, if the long-run target price index is an average of the Laspeyres and Paasche indices, it can be seen that the Laspeyres index will have an *upward bias* relative to this target index, and the Paasche index will have a *downward bias.* In addition, *if the base year* b *is prior to the price reference month, month 0, then the Lowe index will also have an upward bias relative to the Laspeyres index and hence also to the target index.*

D.2 Lowe index and midyear indices

16.50 The discussion in the previous paragraph assumed that the base year b for quantities preceded the base month for prices, month 0. However, if the current-period month t is quite distant from the base month 0, then it is possible to think of the base year b as referring to a year *that lies between months 0 and* t. If the year b does fall between months 0 and t, then the Lowe index becomes a *midyear index*.[45] The Lowe midyear index no longer has the upward biases indicated by the inequalities in equation (16.43) under the assumption of long-term trends in prices and normal substitution responses by quantities.

16.51 It is now assumed that the base-year quantity vector q^b corresponds to a year that lies between months 0 and t. Under the assumption of long-term trends in prices and normal substitution effects so that there are also long-term trends in quantities (in the opposite direction to the trends in prices so that if the ith product price is trending up, then the corresponding ith quantity is trending down), it is likely that the intermediate-year quantity vector will lie between the

[43]The reader can carry through the argument if there is a long-term relative decline in the price of the ith product. The argument required to obtain a negative covariance requires that there be some differences in the long-term trends in prices; that is, if all prices grow (or fall) at the same rate, we have price proportionality, and the covariance will be zero.

[44]However, $Q_L = u^*$ may also be growing in magnitude, so the net effect on the divergence between P_L and P_P is ambiguous.

[45]This concept can be traced to Peter Hill (1998, p. 46): "When inflation has to be measured over a specified sequence of years, such as a decade, a pragmatic solution to the problems raised above would be to take the middle year as the base year. This can be justified on the grounds that the basket of goods and services purchased in the middle year is likely to be much more representative of the pattern of consumption over the decade as a whole than baskets purchased in either the first or the last years. Moreover, choosing a more representative basket will also tend to reduce, or even eliminate, any bias in the rate of inflation over the decade as a whole as compared with the increase in the CoL [cost-of-living] index." Thus, in addition to introducing the concept of a midyear index, Hill also introduced the idea of *representativity bias*. For additional material on midyear indices, see Schultz (1999) and Okamoto (2001). Note that the midyear index concept could be viewed as a close competitor to Walsh's (1901, p. 431) multiyear fixed-basket index, where the quantity vector was chosen to be an arithmetic or geometric average of the quantity vectors in the period.

monthly quantity vectors q^0 and q^t. The midyear Lowe index, $P_{Lo}(p^0, p^t, q^b)$, and the Laspeyres index going from month 0 to t, $P_L(p^0, p^t, q^0)$, will still satisfy the exact relationship given by equation (16.36). Thus, $P_{Lo}(p^0, p^t, q^b)$ will equal $P_L(p^0, p^t, q^0)$ plus the covariance term $\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0/Q_L(q^0, q^b, p^0)$, where $Q_L(q^0, q^b, p^0)$ is the Laspeyres quantity index going from month 0 to t. This covariance term is likely to be negative, so that

$$P_L(p^0, p^t, q^0) > P_{Lo}(p^0, p^t, q^b). \tag{16.44}$$

To see why this covariance is likely to be negative, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv \left(p_i^t/p_i^0\right) - r^*$ is positive. With normal substitution responses, q_i will tend to decrease relatively over time, and because q_i^b is assumed to be between q_i^0 and q_i^t, q_i^b/q_i^0 less an average quantity change of this type, r^* is likely to be negative. Hence $u_i - u^* \equiv \left(q_i^b/q_i^0\right) - t^*$ is likely to be negative. Thus, the covariance is likely to be negative under these circumstances. *Under the assumptions that the quantity base year falls between months 0 and* t *and that there are long-term trends in prices and normal substitution responses, the Laspeyres index will normally be larger than the corresponding Lowe midyear index*, with the divergence likely growing as month t becomes more distant from month 0.

16.52 It can also be seen that under the above assumptions, the midyear Lowe index is likely to be greater than the Paasche index between months 0 and t; that is,

$$P_{Lo}(p^0, p^t, q^b) > P_P(p^0, p^t, q^t). \tag{16.45}$$

To see why the above inequality is likely to hold, think of q^b starting at the month 0 quantity vector q^0 and then trending smoothly to the month t quantity vector q^t. When $q^b = q^0$, the Lowe index $P_{Lo}(p^0, p^t, q^b)$ becomes the Laspeyres index $P_L(p^0, p^t, q^0)$. When $q^b = q^t$, the Lowe index $P_{Lo}(p^0, p^t, q^b)$ becomes the Paasche index $P_P(p^0, p^t, q^t)$. Under the assumption of trending prices and normal substitution responses to these trending prices, it was shown earlier that the Paasche index will be less than the corresponding Laspeyres price index; that is, that $P_P(p^0, p^t, q^t)$ was less than $P_L(p^0, p^t, q^0)$; recall equation (16.42). Thus, under the assumption of smoothly trending prices and quantities between months 0 and t, and *assuming that* q^b *is between* q^0 *and* q^t, we will have

$$P_P(p^0, p^t, q^t) < P_{Lo}(p^0, p^t, q^b) < P_L(p^0, p^t, q^0). \tag{16.46}$$

Thus, if the base year for the Lowe index is chosen to be in between the base month for the prices, month 0, and the current month for prices, month t, and there are trends in prices with corresponding trends in quantities that correspond to normal substitution effects, then the resulting Lowe index is likely to lie between the Paasche and Laspeyres indices going from months 0 to t. If the trends in prices and quantities are smooth, then choosing the base year halfway between periods 0 and t should give a Lowe index that is approximately halfway between the Paasche and Laspeyres indices and hence will be very close to an ideal target index between months 0 and t. This basic idea has been implemented by Okamoto (2001) using Japanese consumer data and he found that the resulting midyear indices approximated the corresponding Fisher ideal indices very closely.

16.53 It should be noted that these midyear indices can be computed only on a retrospective basis; that is, they cannot be calculated in a timely fashion as can Lowe indices that use a base year before month 0. Thus, midyear indices cannot be used to replace the more timely Lowe indices. However, these timely Lowe indices are likely to have an upward bias even bigger than the usual Laspeyres upward bias compared with an ideal target index, which was taken to be an average of the Paasche and Laspeyres indices.

16.54 All of the inequalities derived in this section rest on the assumption of long-term trends in prices (and corresponding economic responses in quantities). If there are no systematic long-run trends in prices and only random fluctuations around a common trend in all prices, then the above inequalities are not valid and the Lowe index using a prior base year will probably provide a perfectly adequate approximation to both the Paasche and Laspeyres indices. However, there are some reasons for believing that some long-run trends in prices exist:

(1) The computer chip revolution of the past 40 years has led to strong downward trends in the prices of products that use these chips intensively. As new uses for chips are developed, the share of products that are chip-intensive grows, which implies that what used to be a relatively minor problem has become a more major problem.

(2) Other major scientific advances have had similar effects. For example, the invention of fiber-optic cable (and lasers) has led to a downward

trend in telecommunications prices as obsolete technologies based on copper wire are gradually replaced.

(3) Since the end of World War II, a series of international trade agreements have dramatically reduced tariffs around the world. These reductions, combined with improvements in transportation technologies, have led to a rapid growth of international trade and remarkable improvements in international specialization. Manufacturing activities in the more developed economies have gradually been outsourced to lower-wage countries, leading to deflation in goods prices in most countries. However, many services cannot be readily outsourced,[46] and so on average the price of services trends upward while the price of goods trends downward.

(4) At the microeconomic level, there are tremendous differences in growth rates of firms. Successful firms expand their scale, lower their costs, and cause less successful competitors to wither away with their higher prices and lower volumes. This leads to a systematic negative correlation between changes in item prices and the corresponding changes in item volumes, which can be very large.

Thus, there is some a priori basis for assuming long-run divergent trends in prices and hence some basis for concern that a Lowe index that uses a base year for quantity weights that is prior to the base month for prices may be upward biased, compared to a more ideal target index.

D.3 Young index

16.55 Recall the definitions for the base-year quantities, q_i^b, and the base-year prices, p_i^b, given by equation (16.23) and equation (16.24). The base-year value shares can be defined in the usual way as follows:

$$s_i^b \equiv \frac{p_i^b q_i^b}{\sum_{k=1}^{n} p_k^b q_k^b}; i = 1, \ldots, n. \tag{16.47}$$

Define the vector of base-year value shares in the usual way as $s^b \equiv [s_1^b, \ldots, s_n^b]$. These base-year value shares were used to provide an alternative formula for the base year b Lowe price index going from month 0 to t defined in equation (16.26) as $P_{Lo}(p^0, p^t, q^b) = \sum_{i=1}^{n} s_i^b (p_i^t/p_i^b)/\sum_{i=1}^{n} s_i^b(p_i^0/p_i^b)$. Rather than using this index as their short-run target index, many statistical agencies use the following closely related index:

$$P_Y(p^0, p^t, s^b) \equiv \sum_{i=1}^{n} s_i^b(p_i^t/p_i^0). \tag{16.48}$$

This type of index was first defined by the English economist Arthur Young (1812).[47] Note that there is a change in focus when the Young index is used compared with when the indices proposed earlier in this chapter are used. Up to this point, the indices proposed have been of the fixed-basket type (or averages of such indices), where a *product basket* that is somehow representative for the two periods being compared is chosen and then "purchased" at the prices of the two periods and the index is taken to be the ratio of these two costs. On the other hand, for the Young index, one instead chooses *representative value shares* that pertain to the two periods under consideration and then uses these shares to calculate the overall index as a share-weighted average of the individual price ratios, p_i^t/p_i^0. Note that this share-weighted average of price ratios view of index number theory is a bit different from the view taken at the beginning of this chapter, which viewed the index number problem as the problem of decomposing a value ratio into the product of two terms, one of which expresses the amount of price change between the two periods and the other expresses the amount of quantity change.[48]

[46] However, some services, such as call centers, computer programming, and airline maintenance, can be internationally outsourced.

[47] Walsh (1901, p. 536; 1932, p. 657) attributed this formula to Young.

[48] Irving Fisher's 1922 book is famous for developing the value ratio decomposition approach to index number theory, but his introductory chapters took the share-weighted average point of view: "An index number of prices, then shows the *average percentage change* of prices from one point of time to another" (1922, p. 3). Fisher went on to note the importance of economic weighting: "The preceding calculation treats all the commodities as equally important; consequently, the average was called 'simple'. If one commodity is more important than another, we may treat the more important as though it were two or three commodities, thus giving it two or three times as much 'weight' as the other commodity" (1922, p. 6). Walsh considered both approaches: "We can either (1) draw some average of the total money values of the classes during an epoch of years, and with weighting so determined employ the geometric average of the price variations [ratios]; or (2) draw some average of the mass quantities of the classes during the epoch, and apply to them Scrope's method" (1901, pp. 430–31). Scrope's method is the same as using the Lowe index. Walsh (1901, pp. 88–90) consistently stressed the importance of weighting price ratios by their economic importance (rather than using equally weighted averages of price relatives). Both the value ratio decomposition approach and the share-weighted average approach to index number theory are studied from the axiomatic perspective in the following chapter; see also Sections C and E in Chapter 16.

16.56 Statistical agencies sometimes regard the Young index defined above as an approximation to the Laspeyres price index $P_L(p^0, p^t, q^0)$. Hence, it is of interest to see how the two indices compare. Defining the long-term monthly price relatives going from month 0 to t as $r_i \equiv p_i^t/p_i^0$ and using equations (16.32) and (16.48),

$$\begin{aligned} &P_Y(p^0, p^t, s^b) - P_L(p^0, p^t, q^0) \\ &\equiv \sum_{i=1}^n s_i^b \left(\frac{p_i^t}{p_i^0}\right) - \sum_{i=1}^n s_i^0 \left(\frac{p_i^t}{p_i^0}\right) \\ &= \sum_{i=1}^n \left[s_i^b - s_i^0\right]\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i=1}^n \left[s_i^b - s_i^0\right] r_i \\ &= \sum_{i=1}^n \left[s_i^b - s_i^0\right][r_i - r^*] + r^* \sum_{i=1}^n \left[s_i^b - s_i^0\right] \\ &= \sum_{i=1}^n \left[s_i^b - s_i^0\right][r_i - r^*], \end{aligned} \tag{16.49}$$

because $\sum_{i=1}^n s_i^b = \sum_{i=1}^n s_i^0 = 1$ and defining $r^* \equiv \sum_{i=1}^n s_i^0 r_i = P_L(p^0, p^t, q^0)$. Thus, the Young index $P_Y(p^0, p^t, s^b)$ is equal to the Laspeyres index $P_L(p^0, p^t, q^0)$ plus the covariance between the difference in the annual shares pertaining to year b and the month 0 shares, $s_i^b - s_i^0$, and the deviations of the relative prices from their mean, $r_i - r^*$.

16.57 It is no longer possible to guess the likely sign of the covariance term. The question is no longer whether the *quantity* demanded goes down as the price of product i goes up (the answer to this question is usually yes) but whether the *share* of traded value goes down as the price of product i goes up. The answer depends on the elasticity of demand/supply for the product. However, let us provisionally assume that, for, say, an import price index, there are long-run trends in product prices and if the trend in prices for product i is above the mean, then the value share for the product trends *down* (and vice versa). Thus, we are assuming high elasticities or very strong substitution effects. Assuming also that the base year b is before month 0, then under these conditions, suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t/p_i^0) - r^*$ is positive. With the assumed very elastic purchaser substitution responses, s_i will tend to decrease relatively over time. Because s_i^b is assumed to be before s_i^0, s_i^0 is expected to be less than s_i^b or, in other words, $s_i^b - s_i^0$ will likely be positive. Thus, the covariance is likely to be *positive* under these circumstances. *Hence with long-run trends in prices and very elastic responses of purchasers to price changes, the Young index is likely to be greater than the corresponding Laspeyres index.*

16.58 Assume that there are long-run trends in product prices. If the trend in prices for product i is above the mean, then suppose that the value share for the product trends *up* (and vice versa). Thus, we are assuming low elasticities or very weak substitution effects. Assume also that the base year b is before month 0, and suppose that there is a long-term upward trend in the price of product i so that $r_i - r^* \equiv (p_i^t/p_i^0) - r^*$ is positive. With the assumed very inelastic substitution responses, s_i will tend to increase relatively over time and because s_i^b is assumed to be before s_i^0, we will have s_i^0 greater than s_i^b or, in other words, $s_i^b - s_i^0$ will likely be negative. Thus, the covariance is likely to be *negative* under these circumstances. *Hence with long-run trends in prices and very inelastic responses of purchasers to price changes, the Young index is likely to be less than the corresponding Laspeyres index.*

16.59 The previous two paragraphs indicate that a priori, it is not known what the likely difference between the Young index and the corresponding Laspeyres index will be. If elasticities of substitution are close to 1, then the two sets of value shares, s_i^b and s_i^0, will be close to each other and the difference between the two indices will be close to zero. However, if monthly value shares have strong seasonal components, then the annual shares s_i^b could differ substantially from the monthly shares s_i^0.

16.60 It is useful to have a formula for updating the previous month's Young price index using only month-over-month price relatives. The Young index for month $t + 1$, $P_Y(p^0, p^{t+1}, s^b)$, can be written in terms of the Lowe index for month t, $P_Y(p^0, p^t, s^b)$, and an updating factor as follows:

$$\begin{aligned} P_Y(p^0, p^{t+1}, s^b) &\equiv \sum_{i=1}^n s_i^b \left(\frac{p_i^{t+1}}{p_i^0}\right) \\ &= P_Y(p^0, p^t, s^b) \frac{\sum_{i=1}^n s_i^b \,(p_i^{t+1}/p_i^0)}{\sum_{i=1}^n s_i^b (p_i^t/p_i^0)} \\ &= P_Y(p^0, p^t, s^b) \frac{\sum_{i=1}^n p_i^b\, q_i^b (p_i^{t+1}/p_i^0)}{\sum_{i=1}^n p_i^b\, q_i^b (p_i^t/p_i^0)}; \end{aligned}$$

using equation (16.47)

$$= P_Y(p^0, p^t, s^b) \frac{\sum_{i=1}^n p_i^b q_i^b \left(\frac{p_i^t}{p_i^0}\right)\left(\frac{p_i^{t+1}}{p_i^t}\right)}{\sum_{i=1}^n p_i^b\, q_i^b (p_i^t/p_i^0)}$$

$$= P_Y(p^0, p^t, s^b)\left[\sum_{i=1}^{n} s_i^{b0t}\left(p_i^{t+1}/p_i^t\right)\right], \quad (16.50)$$

where the hybrid weights s_i^{b0t} are defined by

$$s_i^{b0t} \equiv \frac{p_i^b q_i^b\left(p_i^t/p_i^0\right)}{\sum_{k=1}^{n} p_k^b q_k^b\left(p_k^t/p_k^0\right)} = \frac{s_i^b\left(p_i^t/p_i^0\right)}{\sum_{k=1}^{n} s_k^b\left(p_k^t/p_k^0\right)};\ i = 1, \ldots, n. \quad (16.51)$$

Thus, the hybrid weights s_i^{b0t} can be obtained from the base-year weights s_i^b by updating them, that is, by multiplying them by the price relatives (or indices at higher levels of aggregation), p_i^t/p_i^0. Thus, the required updating factor, going from month t to month $t + 1$, is the chain link index, $\sum_{i=1}^{n} s_i^{b0t}(p_i^{t+1}/p_i^t)$, which uses the hybrid share weights s_i^{b0t} defined by equation (16.51).

16.61 Even if the Young index provides a close approximation to the corresponding Laspeyres index, it is difficult to recommend the use of the Young index as a final estimate of the change in prices going from period 0 to t, just as it was difficult to recommend the use of the Laspeyres index as the *final* estimate of inflation going from period 0 to t. Recall that the problem with the Laspeyres index was its lack of symmetry in the treatment of the two periods under consideration; that is, using the justification for the Laspeyres index as a good fixed-basket index, there was an identical justification for the use of the Paasche index as an equally good fixed-basket index to compare periods 0 and t. The Young index suffers from a similar lack of symmetry with respect to the treatment of the base period. The problem can be explained as follows. The Young index, $P_Y(p^0, p^t, s^b)$, defined by equation (16.48), calculates the price change between months 0 and t, treating month 0 as the base. But there is no particular reason other than convention to treat month 0 as the base month. Hence, if we treat month t as the base and use the same formula to measure the price change from month t back to month 0, the index $P_Y(p^0, p^t, s^b) = \sum_{i=1}^{n} s_i^b\left(p_i^0/p_i^t\right)$ would be appropriate. This estimate of price change can then be made comparable to the original Young index by taking its reciprocal, leading to the following *rebased Young index,*[49] $P_Y{}^*(p^0, p^t, s^b)$, defined as

$$P_Y{}^*(p^0, p^t, s^b) \equiv 1\Big/\sum_{i=1}^{n} s_i^b\left(p_i^0/p_i^t\right) = \left[\sum_{i=1}^{n} s_i^b\left(p_i^t/p_i^0\right)^{-1}\right]^{-1}. \quad (16.52)$$

Thus, the rebased Young index, $P_Y{}^*(p^0, p^t, s^b)$, that uses the current month as the initial base period is a *share-weighted harmonic mean* of the price relatives going from month 0 to month t, whereas the original Young index, $P_Y(p^0, p^t, s^b)$, is a *share-weighted arithmetic mean* of the same price relatives.

16.62 Fisher argued that an index number formula should give the same answer no matter which period was chosen as the base:

> Either one of the two times may be taken as the "base". Will it make a difference which is chosen? Certainly, it *ought* not and our Test 1 demands that it shall not. More fully expressed, the test is that the formula for calculating an index number should be such that it will give the same ratio between one point of comparison and the other point, *no matter which of the two is taken as the base.* (I. Fisher, 1922, p. 64)

16.63 The problem with the Young index is that not only does it not coincide with its rebased counterpart, but there is a definite inequality between the two indices, namely

$$P_Y^*(p^0, p^t, s^b) \le P_Y(p^0, p^t, s^b), \quad (16.53)$$

with a strict inequality provided that the period t price vector p^t is not proportional to the period 0 price vector p^0.[50] Thus, a statistical agency that uses the direct Young index $P_Y(p^0, p^t, s^b)$ will generally show a higher inflation rate than will a statistical agency that uses the same raw data but uses the rebased Young index, $P_Y{}^*(p^0, p^t, s^b)$.

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[49]Using Irving Fisher's (1922, p. 118) terminology, $P_Y{}^*(p^0, p^t, s^b) \equiv 1/[P_Y(p^t, p^0, s^b)]$ is the *time antithesis* of the original Young index, $P_Y(p^0, p^t, s^b)$.

[50]These inequalities follow from the fact that a harmonic mean of M positive numbers is always equal to or less than the corresponding arithmetic mean; see Walsh (1901, p. 517) or Irving Fisher (1922, pp. 383–84). This inequality is a special case of Schlömilch's (1858) Inequality; see Hardy, Littlewood, and Pólya (1934, p. 26). Walsh (1901, pp. 330–32) explicitly noted the inequality in equation (16.53) and also noted that the corresponding geometric average would fall between the harmonic and arithmetic averages. Walsh (1901, p. 432) computed some numerical examples of the Young index and found big differences between it and his best indices, even using weights that were representative for the periods being compared. Recall that the Lowe index becomes the Walsh index when geometric mean quantity weights are chosen, and so the Lowe index can perform well when representative weights are used. This is not necessarily the case for the Young index, even using representative weights. Walsh summed up his numerical experiments with the Young index as follows: "In fact, Young's method, in every form, has been found to be bad" (1901, p. 433).

16.64 The inequality in equation (16.53) does not tell us by how much the Young index will exceed its rebased time antithesis. However, in Appendix 16.3, it is shown that to the accuracy of a certain second-order Taylor series approximation, the following relationship holds between the direct Young index and its time antithesis:

$$P_Y(p^0, p^t, s^b) \approx P_Y^*(p^0, p^t, s^b) + P_Y(p^0, p^t, s^b) \times \text{Var } e, \quad (16.54)$$

where Var e is defined as

$$\text{Var } e \equiv \sum_{i=1}^{n} s_i^b [e_i - e^*]^2. \quad (16.55)$$

The deviations e_i are defined by $1 + e_i = r_i / r^*$ for $i = 1, \ldots, n$ where the r_i and their weighted mean r^* are defined by

$$r_i \equiv p_i^t / p_i^0;\ i = 1, \ldots, n, \quad (16.56)$$

$$r^* \equiv \sum_{i=1}^{n} s_i^b r_i, \quad (16.57)$$

which turns out to equal the direct Young index, $P_Y(p^0, p^t, s^b)$. The weighted mean of the e_i is defined as

$$e^* \equiv \sum_{i=1}^{n} s_i^b e_i, \quad (16.58)$$

which turns out to equal 0. Hence, the more dispersion there is in the price relatives p_i^t/p_i^0, to the accuracy of a second-order approximation, the more the direct Young index will exceed its counterpart that uses month t as the initial price reference period rather than month 0.

16.65 Given two a priori equally plausible index number formulas that give different answers, such as the Young index and its time antithesis, Irving Fisher (1922, p. 136) generally suggested taking the geometric average of the two indices.[51] A benefit of this averaging is that the resulting formula will satisfy the time reversal test. Thus, rather than using *either* the base period 0 Young index, $P_Y(p^0, p^t, s^b)$, *or* the current period t Young index, $P_Y^*(p^0, p^t, s^b)$, which is always below the base period 0 Young index if there is any dispersion in relative prices, it seems preferable to use the following index, which is the *geometric average* of the two alternatively based Young indices:[52]

$$P_Y^{**}(p^0, p^t, s^b) \equiv [P_Y(p^0, p^t, s^b) P_Y^*(p^0, p^t, s^b)]^{1/2}. \quad (16.59)$$

If the base-year shares s_i^b happen to coincide with both the month 0 and month t shares, s_i^0 and s_i^t respectively, the time-rectified Young index $P_Y^{**}(p^0, p^t, s^b)$ defined by (16.59) will coincide with the Fisher ideal price index between months 0 and t, $P_F(p^0, p^t, q^0, q^t)$, which will also equal the Laspeyres and Paasche indices under these conditions. Note also that the index P_Y^{**} defined by equation (16.59) can be produced on a timely basis by a statistical agency.

E. Divisia Index and Discrete Approximations

E.1 Divisia price and quantity indices

16.66 The second broad approach to index number theory relies on the assumption that price and quantity data change in a more or less continuous way.

16.67 Suppose that the price and quantity data on the n products in the chosen domain of definition can be regarded as continuous functions of (continuous) time, say $p_i(t)$ and $q_i(t)$ for $i = 1, \ldots, n$. Suppose further that the traded value at time t is $V(t)$ defined in the obvious way as

$$V(t) \equiv \sum_{i=1}^{n} p_i(t) q_i(t). \quad (16.60)$$

16.68 Now suppose that the functions $p_i(t)$ and $q_i(t)$ are differentiable. Then both sides of equation (16.60) can be differentiated with respect to time to obtain

$$V'(t) = \sum_{i=1}^{n} p_i'(t) q_i(t) + \sum_{i=1}^{n} p_i(t) q_i'(t). \quad (16.61)$$

Divide both sides of equation (16.61) by $V(t)$ and, using equation (16.60), obtain the following equation:

$$V'(t)/V(t) = \frac{\sum_{i=1}^{n} p_i'(t) q_i(t) + \sum_{i=1}^{n} p_i(t) q_i'(t)}{\sum_{j=1}^{n} p_j(t) q_j(t)}$$

[51]"We now come to a third use of these tests, namely, to 'rectify' formulae, i.e., to derive from any given formula which does not satisfy a test another formula which does satisfy it; . . . This is easily done by 'crossing', that is, by averaging antitheses. If a given formula fails to satisfy Test 1 [the time reversal test], its time antithesis will also fail to satisfy it; but the two will fail, as it were, in opposite ways, so that a cross between them (obtained by *geometrical* averaging) will give the golden mean which does satisfy" (Irving Fisher, 1922, p. 136). Actually, the basic idea behind Fisher's rectification procedure was suggested by Walsh, who was a discussant for Fisher (1921), where Fisher gave a preview of his 1922 book: "We merely have to take any index number, find its antithesis in the way prescribed by Professor Fisher, and then draw the geometric mean between the two" (Walsh, 1921b, p. 542).

[52]This index is a base-year weighted counterpart to an equally weighted index proposed by Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992, p. 140) in the context of elementary index number formulas. See Chapter 20 for further discussion of this unweighted index.

$$= \sum_{i=1}^{n} \frac{p_i{}'(t)}{p_i(t)} s_i(t) + \sum_{i=1}^{n} \frac{q_i{}'(t)}{q_i(t)} s_i(t), \quad (16.62)$$

where the time t value share on product i, $s_i(t)$, is defined as

$$s_i(t) \equiv \frac{p_i(t)q_i(t)}{\sum_{m=1}^{n} p_m(t)q_m(t)} \text{ for } i = 1, \ldots, n. \quad (16.63)$$

16.69 François Divisia (1926, p. 39) argued as follows: suppose the aggregate value at time t, $V(t)$, can be written as the product of a time t price-level function, $P(t)$, say, multiplied by a time I quantity-level function, $Q(t)$, say; that is, we have

$$V(t) = P(t)Q(t). \quad (16.64)$$

Suppose, further, that the functions $P(t)$ and $Q(t)$ are differentiable. Then, differentiating (16.64) yields

$$V'(t) = P'(t)Q(t) + P(t)Q'(t). \quad (16.65)$$

Dividing both sides of equation (16.65) by $V(t)$ and using equation (16.64) leads to the following equation:

$$\frac{V'(t)}{V(t)} = \frac{P'(t)}{P(t)} + \frac{Q'(t)}{Q(t)}. \quad (16.66)$$

16.70 Divisia compared the two expressions for the logarithmic value derivative, $V'(t)/V(t)$, given by equation (16.62) and equation (16.66). He simply defined the logarithmic rate of change of the *aggregate price level*, $P'(t)/P(t)$, as the first set of terms on the right-hand side of equation (16.62), and he simply defined the logarithmic rate of change of the *aggregate quantity level*, $Q'(t)/Q(t)$, as the second set of terms on the right-hand side of equation (16.62); that is, he made the following definitions:

$$\frac{P'(t)}{P(t)} \equiv \sum_{i=1}^{n} s_i(t) \frac{p_i{}'(t)}{p_i(t)}; \quad (16.67)$$

$$\frac{Q'(t)}{Q(t)} \equiv \sum_{i=1}^{n} s_i(t) \frac{q_i{}'(t)}{q_i(t)}. \quad (16.68)$$

16.71 Equations (16.67) and (16.68) are reasonable definitions for the proportional changes in the aggregate price and quantity (or quantity) levels, $P(t)$ and $Q(t)$.[53] The problem with these definitions is that economic data are not collected in *continuous* time; they are collected in *discrete* time. In other words, even though transactions can be thought of as occurring in continuous time, no producer records his or her purchases as they occur in continuous time; rather, purchases over a finite time period are cumulated and then recorded. A similar situation occurs for producers or sellers of products; firms cumulate their sales over discrete periods of time for accounting or analytical purposes. If one attempts to approximate continuous time by shorter and shorter discrete time intervals, empirical price and quantity data can be expected to become increasingly erratic, because consumers make purchases only at discrete points of time (and producers or sellers of products make sales only at discrete points of time). However, it is still of some interest to approximate the continuous time price and quantity levels, $P(t)$ and $Q(t)$ defined implicitly by equations (16.67) and (16.68), by discrete time approximations. This can be done in two ways. Either methods of numerical approximation can be used or assumptions about the path taken by the functions $p_i(t)$ and $q_i(t)$ ($i = 1, \ldots, n$) through time can be made. The first strategy is used in the following section. For discussions of the second strategy, see Van Ijzeren (1987, pp. 8–12), Vogt (1977 and 1978), Vogt and Barta (1997), and Balk (2000a).

16.72 There is a connection between the Divisia price and quantity levels, $P(t)$ and $Q(t)$, and the economic approach to index number theory. However, this connection is best made after one has studied the economic approach to index number theory in Chapter 18.

E.2 Discrete approximations to continuous time Divisia index

16.73 To make operational the continuous time Divisia price and quantity levels, $P(t)$ and $Q(t)$ defined by the differential equations (16.67) and (16.68), it is necessary to convert to discrete time. Divisia (1926, p. 40) suggested a straightforward method for doing this conversion, which we now outline.

16.74 Define the following price and quantity (forward) differences:

$$\Delta P \equiv P(1) - P(0); \quad (16.69)$$

$$\Delta p_i \equiv p_i(1) - p_i(0); \; i = 1, \ldots, n. \quad (16.70)$$

[53] If these definitions are applied (approximately) to the Young index studied in the previous section, then it can be seen that for the Young price index to be consistent with the Divisia price index, the base-year shares should be chosen to be average shares that apply to the entire time period between months 0 and t.

Using the above definitions,

$$\frac{P(1)}{P(0)} = \frac{P(0) + \Delta P}{P(0)}$$

$$= 1 + \frac{\Delta P}{P(0)} \approx 1 + \frac{\sum_{i=1}^{n} \Delta p_i q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)}, \qquad (16.71)$$

using equation (16.67) when $t = 0$ and approximating $p_i(0)$ by the difference Δp_i

$$= \frac{\sum_{i=1}^{n} \{p_i(0) + \Delta p_i\} q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)}$$

$$= \frac{\sum_{i=1}^{n} p_i(1) q_i(0)}{\sum_{m=1}^{n} p_m(0) q_m(0)}$$

$$= P_L(p^0, p^1, q^0, q^1),$$

where $p^t \equiv [p_1(t), \ldots, p_n(t)]$ and $q^t \equiv [q_1(t), \ldots, q_n(t)]$ for $t = 0, 1$. Thus, it can be seen that Divisia's discrete approximation to his continuous time price index is just the Laspeyres price index, P_L, defined by equation (16.5).

16.75 But now a problem noted by Frisch (1936, p. 8) occurs: Instead of approximating the derivatives by the discrete (forward) differences defined by equation (16.69) and (16.70), other approximations could be used and a wide variety of discrete time approximations can be obtained. For example, instead of using forward differences and evaluating the index at time $t = 0$, one could use backward differences and evaluate the index at time $t = 1$. These backward differences are defined as

$$\Delta_b p_i \equiv p_i(0) - p_i(1); \; i = 1, \ldots, n. \qquad (16.72)$$

This use of backward differences leads to the following approximation for $P(0)/P(1)$:

$$\frac{P(0)}{P(1)} = \frac{P(1) + \Delta_b P}{P(1)}$$

$$= 1 + \frac{\Delta_b P}{P(1)} \approx 1 + \frac{\sum_{i=1}^{n} \Delta_b p_i q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}, \qquad (16.73)$$

using equation (16.67) when $t = 1$ and approximating $p_i(1)$ by the difference $\Delta_b p_i$:

$$= \frac{\sum_{i=1}^{n} \{p_i(1) + \Delta_b p_i\} q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}$$

$$= \frac{\sum_{i=1}^{n} p_i(0) q_i(1)}{\sum_{m=1}^{n} p_m(1) q_m(1)}$$

$$= \frac{1}{P_P(p^0, p^1, q^0, q^1)},$$

where P_P is the Paasche index defined by equation (16.6). Taking reciprocals of both sides of equation (16.73) leads to the following discrete approximation to $P(1)/P(0)$:

$$\frac{P(1)}{P(0)} \approx P_P. \qquad (16.74)$$

16.76 Thus, as Frisch[54] noted, both the Paasche and Laspeyres indices can be regarded as (equally valid) approximations to the continuous time Divisia price index.[55] Because the Paasche and Laspeyres indices can differ considerably in some empirical applications, it can be seen that Divisia's idea is not all that helpful in determining a *unique* discrete time index number formula.[56] What is useful about the Divisia indices is the idea that as the discrete unit of time gets smaller, discrete approximations to the Divisia indices can approach meaningful economic indices under certain conditions. Moreover, if the Divisia concept is accepted as the correct one for index number theory, then the corresponding correct discrete time counterpart might be taken as a weighted average of

[54]"As the elementary formula of the chaining, we may get Laspeyres' or Paasche's or Edgeworth's or nearly any other formula, according as we choose the approximation principle for the steps of the numerical integration" (Frisch, 1936, p. 8).

[55]Diewert (1980, p. 444) also obtained the Paasche and Laspeyres approximations to the Divisia index using a somewhat different approximation argument. He also showed how several other popular discrete time index number formulas could be regarded as approximations to the continuous time Divisia index.

[56]Trivedi (1981) systematically examined the problems involved in finding a best discrete time approximation to the Divisia indices using the techniques of numerical analysis. However, these numerical analysis techniques depend on the assumption that the true continuous time micro price functions, $p_i(t)$, can be adequately represented by a polynomial approximation. Thus, we are led to the conclusion that the best discrete time approximation to the Divisia index depends on assumptions that are difficult to verify.

the chain price relatives pertaining to the adjacent periods under consideration, where the weights are somehow representative for the two periods under consideration.

F. Fixed-Base Versus Chain Indices

16.77 This section[57] discusses the merits of using the chain system for constructing price indices in the time-series context versus using the fixed-base system.[58]

16.78 The chain system[59] measures the change in prices going from one period to another using a bilateral index number formula involving the prices and quantities pertaining to the two adjacent periods. These one-period rates of change (the links in the chain) are then cumulated to yield the relative levels of prices over the entire period under consideration. Thus, if the bilateral price index is P, the chain system generates the following pattern of price levels for the first three periods:

$$1, P(p^0, p^1, q^0, q^1), \quad P(p^0, p^1, q^0, q^1)P(p^1, p^2, q^1, q^2). \tag{16.75}$$

16.79 On the other hand, the fixed-base system of price levels using the same bilateral index number formula P simply computes the level of prices in period t relative to the base period 0 as $P(p^0, p^t, q^0, q^t)$. Thus, the fixed-base pattern of price levels for periods 0, 1, and 2 is

$$1, P(p^0, p^1, q^0, q^1), \quad P(p^0, p^2, q^0, q^2). \tag{16.76}$$

16.80 Note that in both the chain system and the fixed-base system of price levels defined by equations (16.75) and (16.76), the base-period price level is equal to 1. The usual practice in statistical agencies is to set the base-period price level equal to 100. If this is done, then it is necessary to multiply each of the numbers in equations (16.75) and (16.76) by 100.

16.81 Because of the difficulties involved in obtaining current-period information on traded quantities (or, equivalently, on values), many statistical agencies loosely base their XMPIs on the Laspeyres formula in equation (16.5) and the fixed-base system. Therefore, it is of some interest to look at the possible problems associated with the use of fixed-base Laspeyres indices.

16.82 The main problem with the use of fixed-base Laspeyres indices is that the period 0 fixed basket of products that is being priced out in period t often can be quite different from the period t basket. Thus, if there are systematic *trends* in at least some of the prices and quantities[60] in the index basket, the fixed-base Laspeyres price index, $P_L(p^0, p^t, q^0, q^t)$, can be quite different from the corresponding fixed-base Paasche price index, $P_P(p^0, p^t, q^0, q^t)$.[61] This means that both indices are likely to be an inadequate representation of the movement in average prices over the time period under consideration.

16.83 The fixed-base Laspeyres quantity index cannot be used forever; eventually, the base-period quantities q^0 are so far removed from the current-period quantities q^t that the base must be changed. Chaining is merely the limiting case where the base is changed each period.[62]

16.84 The main advantage of the chain system is that under normal conditions, chaining will reduce the spread between the Paasche and Laspeyres indices.[63] These indices provide an asymmetric perspective on the amount of price change that has occurred between the two periods under consideration, and it could be expected that a single point estimate of the aggregate price change should lie between these two estimates.

[57]This section is based largely on the work of Peter Hill (1988; 1993, pp. 385–90).

[58]The results in Appendix 17.1 provide some theoretical support for the use of chain indices in that it is shown that under certain conditions, the Divisia index will equal an economic index. Hence, any discrete approximation to the Divisia index will approach the economic index as the time period gets shorter. Thus, under certain conditions, chain indices will approach an underlying economic index.

[59]The chain principle was introduced independently into the economics literature by Lehr (1885, pp. 45–46) and Marshall (1887, p. 373). Both authors observed that the chain system would mitigate the difficulties because of the introduction of new products into the economy, a point also mentioned by Peter Hill (1993, p. 388). Irving Fisher (1911, p. 203) introduced the term "chain system."

[60]Examples of rapidly downward trending prices and upward trending quantities can be found with computers, electronic equipment of all types, Internet access, and telecommunication charges.

[61]Note that $P_L(p^0, p^t, q^0, q^t)$ will equal $P_P(p^0, p^t, q^0, q^t)$ if *either* the two quantity vectors q^0 and q^t are proportional *or* the two price vectors p^0 and p^t are proportional. Thus, to obtain a difference between the Paasche and Laspeyres indices, nonproportionality in *both* prices and quantities is required.

[62]Regular seasonal fluctuations can cause monthly or quarterly data to "bounce," using Szulc's (1983) term and chaining bouncing data can lead to a considerable amount of index drift; that is, if after 12 months, prices and quantities return to their levels of a year earlier, then a chained monthly index will usually not return to unity. Hence, the use of chained indices for "noisy" monthly or quarterly data is not recommended without careful consideration.

[63]See Diewert (1978, p. 895) and Peter Hill (1988; 1993, pp. 387–88).

Thus, the use of either a chained Paasche or Laspeyres index will usually lead to a smaller difference between the two and hence to estimates that are closer to the "truth."[64]

16.85 Peter Hill (1993, p. 388), drawing on his earlier research (1988, pp. 136–37) and that of Szulc (1983), noted that it is not appropriate to use the chain system when prices oscillate, or "bounce," to use Szulc's (1983, p. 548) term. This phenomenon can occur in the context of regular seasonal fluctuations or in the context of price wars. However, in the context of roughly monotonically changing prices and quantities, Peter Hill (1993, p. 389) recommended the use of chained symmetrically weighted indices (see Section C). The Fisher and Walsh indices are examples of symmetrically weighted indices.

16.86 It is possible to be a bit more precise regarding under which conditions one should or should not chain. Basically, one should chain if the prices and quantities pertaining to adjacent periods are *more similar* than the prices and quantities of more distant periods, because this strategy will lead to a narrowing of the spread between the Paasche and Laspeyres indices at each link.[65] One needs a measure of how similar are the prices and quantities pertaining to two periods. The similarity measures could be *relative* or *absolute*. In the case of absolute comparisons, two vectors of the same dimension are similar if they are identical and dissimilar otherwise. In the case of relative comparisons, two vectors are similar if they are proportional and dissimilar if they are nonproportional.[66] Once a similarity measure has been defined, the prices and quantities of each period can be compared using this measure, and a "tree" or path that links all of the observations can be constructed where the most similar observations are compared using a bilateral index number formula.[67] R.J. Hill (1995) defined the price structures between the two countries to be more dissimilar the bigger the spread is between P_L and P_P, that is, the bigger max $\{P_L/P_P, P_P/P_L\}$. The problem with this measure of dissimilarity in the price structures of the two countries is that it could be the case that $P_L = P_P$ (so that the R.J. Hill measure would register a maximal degree of similarity), but p^0 could be very different from p^t. Thus, there is a need for a more systematic study of similarity (or dissimilarity) measures to pick the best one that could be used as an input into R.J. Hill's (1999a, 1999b, 2001) spanning tree algorithm for linking observations.

16.87 The method of linking observations explained in the previous paragraph based on the similarity of the price and quantity structures of any two observations may not be practical in a statistical agency context, because the addition of a new period may lead to a reordering of the previous links. However, the above scientific method for linking observations may be useful in deciding whether chaining is preferable or whether fixed-base indices should be used while making month-to-month comparisons within a year.

16.88 Some index number theorists have objected to the chain principle on the grounds that it has no counterpart in the spatial context:

> They [chain indices] only apply to intertemporal comparisons, and in contrast to direct indices they are not applicable to cases in which no natural order or sequence exists. Thus, the idea of a chain index, for example, has no counterpart in interregional or international price comparisons, because countries

[64]This observation will be illustrated with an artificial data set in Chapter 19.

[65]Walsh, in discussing whether fixed-base or chained index numbers should be constructed, took for granted that the precision of all reasonable bilateral index number formulas would improve, provided that the two periods or situations being compared were more similar and hence, for this reason, favored the use of chained indices: "The question is really, in which of the two courses [fixed-base or chained index numbers] are we likely to gain greater exactness in the comparisons actually made? Here the probability seems to incline in favor of the second course; for the conditions are likely to be less diverse between two contiguous periods than between two periods say fifty years apart" (Walsh, 1901, p. 206). Walsh (1921a, pp. 84–85) later reiterated his preference for chained index numbers. Fisher also made use of the idea that the chain system would usually make bilateral comparisons between price and quantity data that were more similar and hence the resulting comparisons would be more accurate: "The index numbers for 1909 and 1910 (each calculated in terms of 1867–1877) are compared with each other. But direct comparison between 1909 and 1910 would give a different and more valuable result. To use a common base is like comparing the relative heights of two men by measuring the height of each above the floor, instead of putting them back to back and directly measuring the difference of level between the tops of their heads" (1911, p. 204). "It seems, therefore, advisable to compare each year with the next, or, in other words, to make each year the base year for the next. Such a procedure has been recommended by Marshall, Edgeworth and Flux. It largely meets the difficulty of non-uniform changes in the Q's [quantities], for any inequalities for successive years are relatively small" (I. Fisher, 1911, pp. 423–24).

[66]Diewert (2002b) took an axiomatic approach to defining various indices of absolute and relative dissimilarity.

[67]Irving Fisher (1922, pp. 271–76) hinted at the possibility of using spatial linking; that is, of linking countries that are similar in structure. However, the modern literature has grown owing to the pioneering efforts of R.J. Hill (1995; 1999a; 1999b; and 2001). Hill (1995) used the spread between the Paasche and Laspeyres price indices as an indicator of similarity and showed that this criterion gives the same results as a criterion that looks at the spread between the Paasche and Laspeyres quantity indices.

> cannot be sequenced in a "logical" or "natural" way (there is no $k + 1$ nor $k - 1$ country to be compared with country k). (von der Lippe, 2001, p. 12)[68]

This is correct, but R.J. Hill's approach does lead to a natural set of spatial links. Applying the same approach to the time-series context will lead to a set of links between periods that may not be month to month, but it will in many cases justify year-over-year linking of the data pertaining to the same month. This problem will be reconsidered in Chapter 23.

16.89 It is of some interest to determine if there are index number formulas that give the same answer when either the fixed-base or chain system is used. Comparing the sequence of chain indices defined by equation (16.75) above to the corresponding fixed-base indices, it can be seen that we will obtain the same answer in all three periods if the index number formula P satisfies the following functional equation for all price and quantity vectors:

$$P(p^0, p^2, q^0, q^2) = P(p^0, p^1, q^0, q^1)P(p^1, p^2, q^1, q^2). \quad (16.77)$$

If an index number formula P satisfies equation (16.77), then P satisfies the *circularity test*.[69]

16.90 If it is assumed that the index number formula P satisfies certain properties or tests in addition to the circularity test above,[70] then Funke, Hacker, and Voeller (1979) showed that P must have the following functional form credited originally to Konüs and Byushgens[71] (1926, pp. 163–66)[72]:

$$P_{KB}(p^0, p^1, q^0, q^1) \equiv \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0}\right)^{\alpha_i}, \quad (16.78)$$

where the n constants α_i satisfy the following restrictions:

$$\sum_{i=1}^{n} \alpha_i = 1 \text{ and } \alpha_i > 0 \text{ for } i = 1, \ldots, n. \quad (16.79)$$

Thus, under very weak regularity conditions, the only price index satisfying the circularity test is a weighted geometric average of all the individual price ratios, the weights being constant through time.

16.91 An interesting special case of the family of indices defined by equation (16.78) occurs when the weights α_i are all equal. In this case, P_{KB} reduces to the Jevons (1865) index:

$$P_J(p^0, p^1, q^0, q^1) \equiv \prod_{i=1}^{n} \left(\frac{p_i^1}{p_i^0}\right)^{\frac{1}{n}}. \quad (16.80)$$

16.92 The problem with the indices defined by Konüs and Byushgens and Jevons is that the individual price ratios, p_i^1/p_i^0, have weights (either α_i or $1/n$) that are *independent* of the economic importance of product i in the two periods under consideration. Put another way, these price weights are independent of the quantities of product i or the values on product i traded during the two periods. Hence, these indices are not really suitable for use by statistical agencies at higher levels of aggregation when value share information is available.

16.93 The above results indicate that it is not useful to ask that the price index P satisfy the circularity test *exactly*. However, it is of some interest to find index number formulas that satisfy the circularity test to some degree of *approximation*, because the use of such an index number formula will lead to measures of aggregate price change that are more or less the same whether we use the chain or fixed-base systems. Irving Fisher (1922) found that deviations from circularity using his data set and the Fisher ideal price index P_F defined by equation (16.12) above were quite small.

[68]It should be noted that von der Lippe (2001, pp. 56–58) is a vigorous critic of all index number tests based on symmetry in the time-series context, although he is willing to accept symmetry in the context of making international comparisons. "But there are good reasons *not* to insist on such criteria in the *intertemporal* case. When no symmetry exists between 0 and *t*, there is no point in interchanging 0 and *t*" (von der Lippe, 2001, p. 58).

[69]The test name is credited to Irving Fisher (1922, p. 413), and the concept was originally credited to Westergaard (1890, pp. 218–19).

[70]The additional tests are (1) positivity and continuity of $P(p^0, p^1, q^0, q^1)$ for all strictly positive price and quantity vectors p^0, p^1, q^0, q^1; (2) the identity test; (3) the commensurability test; (4) $P(p^0, p^1, q^0, q^1)$ is positively homogeneous of degree 1 in the components of p^1; and (5) $P(p^0, p^1, q^0, q^1)$ is positively homogeneous of degree zero in the components of q^1.

[71]Konüs and Byushgens (1926) show that the index defined by equation (16.78) is exact for Cobb and Douglas (1928) preferences; see also Pollak (1983, pp. 119–20). The concept of an exact index number formula is explained in Chapter 17.

[72]This result can be derived using results in Eichhorn (1978, pp. 167–68) and Vogt and Barta (1997, p. 47). A simple proof can be found in Balk (1995). This result vindicates Irving Fisher's intuition. He asserted that "the only formulae which conform perfectly to the circular test are index numbers which have *constant weights*" (1922, p. 274). Irving Fisher went on to assert; "But, clearly, constant weighting is not theoretically correct. If we compare 1913 with 1914, we need one set of weights; if we compare 1913 with 1915, we need, theoretically at least, another set of weights. . . . Similarly, turning from time to space, an index number for comparing the United States and England requires one set of weights, and an index number for comparing the United States and France requires, theoretically at least, another" (1922, p. 275).

This relatively high degree of correspondence between fixed-base and chain indices has been found to hold for other symmetrically weighted formulas like the Walsh index P_W defined by equation (16.19).[73] Thus, in most time-series applications of index number theory where the base year in fixed-base indices is changed every five years or so, it will not matter very much whether the statistical agency uses a fixed-base price index or a chain index, provided that a symmetrically weighted formula is used.[74] This, of course, depends on the length of the time series considered and the degree of variation in the prices and quantities as we go from period to period. The more prices and quantities are subject to large fluctuations (rather than smooth trends), the less the correspondence.[75]

16.94 It is possible to give a theoretical explanation for the approximate satisfaction of the circularity test for symmetrically weighted index number formulas. Another symmetrically weighted formula is the Törnqvist index P_T.[76] The natural logarithm of this index is defined as follows:

$$\ln P_T(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}(s_i^0 + s_i^1) \ln \left(\frac{p_i^1}{p_i^0}\right), \tag{16.81}$$

where the period t value shares s_i^t are defined by equation (16.7) above. Alterman, Diewert, and Feenstra (1999, p. 61) showed that if the logarithmic price ratios $\ln (p_i^t/p_i^{t-1})$ trend linearly with time t, and the value shares s_i^t also trend linearly with time, then the Törnqvist index P_T will satisfy the circularity test exactly.[77] Because many economic time series on prices and quantities satisfy these assumptions approximately, then the Törnqvist index will satisfy the circularity test approximately. As is shown in Chapter 20, generally the Törnqvist index closely approximates the symmetrically weighted Fisher and Walsh indices, so that for many economic time series (with smooth trends), all three of these symmetrically weighted indices will satisfy the circularity test to a high enough degree of approximation so that it will not matter whether we use the fixed-base or chain principle.

16.95 Walsh (1901, p. 401; 1921a, p. 98; and 1921b, p. 540) introduced the following useful variant of the circularity test:

$$1 = P(p^0, p^1, q^0, q^1)P(p^1, p^2, q^1, q^2) \ldots P(p^T, p^0, q^T, q^0). \tag{16.82}$$

The motivation for this test is the following. Use the bilateral index formula $P(p^0, p^1, q^0, q^1)$ to calculate the change in prices going from period 0 to 1. Use the same formula evaluated at the data corresponding to periods 1 and 2, $P(p^1, p^2, q^1, q^2)$, to calculate the change in prices going from period 1 to 2, . . . , use $P(p^{T-1}, p^T, q^{T-1}, q^T)$ to calculate the change in prices going from period $T - 1$ to T. Introduce an artificial period $T + 1$ that has exactly the price and quantity of the initial period 0 and use $P(p^T, p^0, q^T, q^0)$ to calculate the change in prices going from period T to 0. Finally, multiply all these indices and, because we end up where we started, then the product of all of these indices should ideally be 1. Diewert (1993a, p. 40) called this test a *multiperiod identity test*.[78] Note that if $T = 2$ (so that the number of periods is 3 in total), then Walsh's test reduces to Irving Fisher's (1921, p. 534; 1922, p. 64) *time reversal test*.[79]

16.96 Walsh (1901, pp. 423–33) showed how his circularity test could be used in order to evaluate the worth of a bilateral index number formula. He invented artificial price and quantity data for five periods and added a sixth period that had the data of the first period. He then evaluated the right-hand side of equation (16.82) for various formulas, $P(p^0, p^1, q^0, q^1)$, and determined how far from unity the results were. His best formulas had products that were close to 1.[80]

16.97 This same framework is often used to evaluate the efficacy of chained indices versus their direct counterparts. Thus, if the right-hand side of equation (16.82)

[73]See, for example, Diewert (1978, p. 894). Walsh (1901, pp. 424 and 429) found that his three preferred formulas all approximated each other very well, as did the Fisher ideal for his artificial data set.

[74]More specifically, most superlative indices (which are symmetrically weighted) will satisfy the circularity test to a high degree of approximation in the time-series context. See Chapter 17 for the definition of a superlative index. It is worth stressing that fixed-base Paasche and Laspeyres indices are very likely to diverge considerably over a five-year period if computers (or any other product that has price and quantity trends different from the trends in the other products) are included in the value aggregate under consideration. See Chapter 19 for some empirical evidence on this topic.

[75]Again, see Szulc (1983) and Peter Hill (1988).

[76]This formula was implicitly introduced in Törnqvist (1936) and explicitly defined in Törnqvist and Törnqvist (1937).

[77]This exactness result can be extended to cover the case when there are monthly proportional variations in prices and the value shares have constant seasonal effects in addition to linear trends; see Alterman, Diewert, and Feenstra (1999, p. 65).

[78]Walsh (1921a, p. 98) called his test the *circular test,* but because Irving Fisher also used this term to describe his transitivity test defined earlier by equation (16.77), it seems best to stick to Fisher's terminology because it is well established in the literature.

[79]Walsh (1921b, pp. 540–541) noted that the time reversal test was a special case of his circularity test.

[80]This is essentially a variant of the methodology that Irving Fisher (1922, p. 284) used to check how well various formulas corresponded to his version of the circularity test.

turns out to be different from unity, the chained indices are said to suffer from "chain drift." If a formula does suffer from chain drift, it is sometimes recommended that fixed-base indices be used in place of chained ones. However, this advice, if accepted, would *always* lead to the adoption of fixed-base indices, provided that the bilateral index formula satisfies the identity test, $P(p^0, p^0, q^0, q^0) = 1$. Thus, it is not recommended that Walsh's circularity test be used to decide whether fixed-base or chained indices should be calculated. However, it is fair to use Walsh's circularity test as he originally used it; that is, as an approximate method for deciding the utility of a particular index number formula. To decide whether to chain or use fixed-base indices, one should decide on the basis of how similar are the observations being compared and choose the method that will best link up the most similar observations.

16.98 Various properties, axioms, or tests that an index number formula could satisfy were introduced in this chapter. In the following chapter, the test approach to index number theory is studied in a more systematic manner.

Appendix 16.1 Relationship Between Paasche and Laspeyres Indices

16.99 Recall the notation used in Section B.2. Define the ith relative price or price relative r_i and the ith quantity relative t_i as follows:

$$r_i \equiv \frac{p_i^1}{p_i^0}; \quad t_i \equiv \frac{q_i^1}{q_i^0}; \quad i = 1, \ldots, n. \qquad \text{(A16.1.1)}$$

Using equation (16.8) above for the Laspeyres price index P_L and equation (A16.1.1), we have

$$P_L = \sum_{i=1}^{n} r_i s_i^0 \equiv r^* ; \qquad \text{(A16.1.2)}$$

that is, we define the "average" price relative r^* as the base-period value share-weighted average of the individual price relatives, r_i.

16.100 Using equation (16.6) for the Paasche price index P_P, we have

$$P_P \equiv \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{m=1}^{n} p_m^0 q_m^1} \qquad \text{(A16.1.3)}$$

$$= \frac{\sum_{i=1}^{n} r_i t_i p_i^0 q_i^0}{\sum_{m=1}^{n} t_m p_m^0 q_m^0} \quad \text{using equation (A16.1.1)}$$

$$= \frac{\sum_{i=1}^{n} r_i\, t_i\, s_i^0}{\sum_{m=1}^{n} t_m s_m^0}$$

$$= \left\{ \frac{1}{\sum_{m=1}^{n} t_m s_m^0} \sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0 \right\} + r^*,$$

using equation (A16.1.2) and $\sum_{i=1}^{n} s_i^0 = 1$ and where the average quantity relative t^* is defined as

$$t^* \equiv \sum_{i=1}^{n} t_i\, s_i^0 = Q_L, \qquad \text{(A16.1.4)}$$

where the last equality follows using equation (16.11), the definition of the Laspeyres quantity index Q_L.

16.101 Taking the difference between P_P and P_L and using equation (A16.1.2), equation (A16.1.4) yields

$$P_P - P_L = \frac{1}{Q_L} \sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0. \qquad \text{(A16.1.5)}$$

16.102 Now let r and t be discrete random variables that take on the n values r_i and t_i, respectively. Let $s_i{}^0$ be the joint probability that $r = r_i$ and $t = t_i$ for $i = 1, \ldots, n$, and let the joint probability be 0 if $r = r_i$ and $t = t_j$ where $i \neq j$. It can be verified that the summation $\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0$ on the right-hand side of equation (A16.1.5) is the covariance between the price relatives r_i and the corresponding quantity relatives t_i. This covariance can be converted into a correlation coefficient.[81] If this covariance is negative, which is the usual case in the consumer (import) context, then P_P will be less than P_L. If it is positive, which will occur in the situations where supply conditions are fixed (as in the fixed-input *output* price index), but demand is changing, then P_P will be greater than P_L.

Appendix 16.2 Relationship Between Lowe and Laspeyres Indices

16.103 Recall the notation used in Section D.1. Define the ith relative price relating the price of product i of month t to month 0, r_i, and the ith quantity relative, t_i, relating quantity of product i in base year b to month 0, t_i, as follows:

$$r_i \equiv \frac{p_i^t}{p_i^0}; \; t_i \equiv \frac{q_i^b}{q_i^0}; \; i = 1, \ldots, n. \qquad \text{(A16.2.1)}$$

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[81]See Bortkiewicz (1923, pp. 374–75) for the first application of this correlation coefficient decomposition technique.

16.104 As in Appendix 16.1, the Laspeyres price index $P_L(p^0, p^t, q^0)$ can be defined as r^*, the month 0 expenditure share-weighted average of the individual price relatives r_i defined in equation (A16.2.1), except that the month t price, p_i^t, now replaces period 1 price, p_i^1, in the definition of the ith price relative r_i:

$$r^* \equiv \sum_{i=1}^{n} r_i s_i^0 = P_L. \tag{A16.2.2}$$

16.105 The average quantity relative t^* relating the quantities of base year b to those of month 0 is defined as the month 0 expenditure share-weighted average of the individual quantity relatives t_i defined in equation (A16.2.1):

$$t^* \equiv \sum_{i=1}^{n} t_i s_i^0 = Q_L, \tag{A16.2.3}$$

where $Q_L = Q_L(q^0, q^b, p^0)$ is the Laspeyres quantity index relating the quantities of month 0, q^0, to those of the year b, q^b, using the prices of month 0, p^0, as weights.

16.106 Using equation (16.26), the Lowe index comparing the prices in month t with those of month 0, using the quantity weights of the base year b, is equal to

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \tag{A16.2.4}$$

$$= \frac{\sum_{i=1}^{n} p_i^t t_i q_i^0}{\sum_{i=1}^{n} p_i^0 t_i q_i^0} \quad \text{using equation (A16.2.1)}$$

$$= \left\{\frac{\sum_{i=1}^{n} p_i^t t_i q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}\left\{\frac{\sum_{i=1}^{n} p_i^0 t_i q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}^{-1}$$

$$= \left\{\frac{\sum_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right) t_i p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}\Big/ t^* \quad \text{using equation (A16.2.3)}$$

$$= \left\{\frac{\sum_{i=1}^{n} r_i t_i p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}\right\}\Big/ t^* \quad \text{using equation (A16.2.1)}$$

$$= \frac{\sum_{i=1}^{n} r_i t_i s_i^0}{t^*} = \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{\sum_{i=1}^{n} r^* t_i s_i^0}{t^*}$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{r^*\left[\sum_{i=1}^{n} t_i s_i^0\right]}{t^*}$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*) t_i s_i^0}{t^*} + \frac{r^*[t^*]}{t^*} \quad \text{using equation (A16.2.3)}$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{t^*} + \frac{t^*\left[\sum_{i=1}^{n} r_i s_i^0 - r^*\right]}{t^*} + r^*$$

$$= \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{t^*} + r^* \quad \text{since } \textstyle\sum_{i=1}^{n} r_i s_i^0 = r^*$$

$$= P_L(p^0, p^t, q^0) + \frac{\sum_{i=1}^{n} (r_i - r^*)(t_i - t^*) s_i^0}{Q_L(q^0, q^b, p^0)}$$

because using equation (A16.2.2), r^* equals the Laspeyres price index, $P_L(p^0, p^t, q^0)$, and using equation (A16.2.3), t^* equals the Laspeyres quantity index, $Q_L(q^0, q^b, p^0)$. Thus, equation (A16.2.4) tells us that the Lowe price index using the quantities of year b as weights, $P_{Lo}(p^0, p^t, q^b)$, is equal to the usual Laspeyres index using the quantities of month 0 as weights, $P_L(p^0, p^t, q^0)$, plus a covariance term $\sum_{i=1}^{n}(r_i - r^*)(t_i - t^*)s_i^0$ between the price relatives $r_i \equiv p_i^t/p_i^0$ and the quantity relatives $t_i \equiv q_i^b/q_i^0$, divided by the Laspeyres quantity index $Q_L(q^0, q^b, p^0)$ between month 0 and base year b.

Appendix 16.3 Relationship Between Young Index and Its Time Antithesis

16.107 Recall that the direct Young index, $P_Y(p^0, p^t, s^b)$, was defined by equation (16.48) and its time antithesis, $P_Y{}^*(p^0, p^t, s^b)$, was defined by equation (16.52). Define the ith relative price between months 0 and t as

$$r_i \equiv p_i^t / p_i^0,\ i = 1, \ldots, n, \tag{A16.3.1}$$

and define the weighted average (using the base-year weights s_i^b) of the r_i as

$$r^* \equiv \sum_{i=1}^{n} s_i^b r_i, \tag{A16.3.2}$$

which turns out to equal the direct Young index, $P_Y(p^0, p^t, s^b)$. Define the deviation e_i of r_i from their weighted average r^* using the following equation:

$$r_i = r^*(1 + e_i);\ i = 1, \ldots, n. \qquad \text{(A16.3.3)}$$

16.108 If equation (A16.3.3) is substituted into equation (A16.3.2), the following equations are obtained:

$$r^* \equiv \sum_{i=1}^{n} s_i^b\, r^*(1 + e_i) \qquad \text{(A16.3.4)}$$

$$= r^* + r^* \sum_{i=1}^{n} s_i^b e_i \quad \text{since} \sum_{i=1}^{n} s_i^b = 1$$

$$e^* \equiv \sum_{i=1}^{n} s_i^b\, e_i = 0. \qquad \text{(A16.3.5)}$$

Thus, the weighted mean e^* of the deviations e_i equals 0.

16.109 The direct Young index, $P_Y(p^0, p^t, s^b)$, and its time antithesis, $P_Y{}^*(p^0, p^t, s^b)$, can be written as functions of r^*, the weights $s_i{}^b$, and the deviations of the price relatives e_i as follows:

$$P_Y(p^0, p^t, s^b) = r^*; \qquad \text{(A16.3.6)}$$

$$P_Y^*(p^0, p^t, s^b) = \left[\sum_{i=1}^{n} s_i^b \{r^*(1 + e_i)\}^{-1}\right]^{-1}$$

$$= r^*\left[\sum_{i=1}^{n} s_i^b\,(1 + e_i)^{-1}\right]^{-1}. \qquad \text{(A16.3.7)}$$

16.110 Now, regard $P_Y{}^*(p^0, p^t, s^b)$ as a function of the vector of deviations, $e \equiv [e_1, \ldots, e_n]$, say $P_Y{}^*(e)$. The second-order Taylor series approximation to $P_Y{}^*(e)$ around the point $e = 0_n$ is given by the following expression:[82]

$$P_Y^*(\mathrm{e}) \approx r^* + r^* \sum_{i=1}^{n} s_i^b e_i + r^* \sum_{i=1}^{n} \sum_{j=1}^{n} s_i^b\, s_j^b\, e_i\, e_j - r^* \sum_{i=1}^{n} s_i^b\,[e_i]^2 \qquad \text{(A16.3.8)}$$

$$= r^* + r^* 0 + r^* \sum_{i=1}^{n} s_i^b \left[\sum_{j=1}^{n} s_j^b e_j\right] e_i - r^* \sum_{i=1}^{n} s_i^b\,[e_i - e^*]^2 \quad \text{using equation (A16.3.5)}$$

$$= r^* + r^* \sum_{i=1}^{n} s_i^b\,[0] e_i - r^* \sum_{i=1}^{n} s_i^b [e_i - e^*]^2 \quad \text{using equation (A16.3.5)}$$

$$= P_Y(p^0, p^t, s^b) - P_Y(p^0, p^t, s^b) \times \sum_{i=1}^{n} s_i^b [e_i - e^*]^2 \quad \text{using equation (A16.3.6)}$$

$$= P_Y(p^0, p^t, s^b) - P_Y(p^0, p^t, s^b) \mathrm{Var}\, e$$

where the weighted sample variance of the vector e of price deviations is defined as

$$\mathrm{Var}\, e \equiv \sum_{i=1}^{n} s_i^b [e_i - e^*]^2. \qquad \text{(A16.3.9)}$$

16.111 Rearranging equation (A16.3.8) gives the following approximate relationship between the direct Young index $P_Y(p^0, p^t, s^b)$ and its time antithesis $P_Y{}^*(p^0, p^t, s^b)$, to the accuracy of a second-order Taylor series approximation about a price point where the month t price vector is proportional to the month 0 price vector:

$$P_Y(p^0, p^t, s^b) \approx P_Y^*(p^0, p^t, s^b) + P_Y(p^0, p^t, s^b)\, \mathrm{Var}\, e. \qquad \text{(A16.3.10)}$$

Thus, to the accuracy of a second-order approximation, the direct Young index will exceed its time antithesis by a term equal to the direct Young index times the weighted variance of the deviations of the price relatives from their weighted mean. Thus, the bigger the dispersion in relative prices, the more the direct Young index will exceed its time antithesis.

[82]This type of second-order approximation is credited to Dalén (1992, p. 143) for the case $r^* = 1$ and to Diewert (1995a, p. 29) for the case of a general r^*.

17. Axiomatic and Stochastic Approaches to Index Number Theory

A. Introduction

17.1 As Chapter 16 demonstrated, it is useful to be able to evaluate various index number formulas that have been proposed in terms of their properties. If a formula turns out to have rather undesirable properties, then doubt is cast on its suitability as a target index that could be used by a statistical agency. Looking at the mathematical properties of index number formulas leads to the *test* or *axiomatic approach to index number theory*. In this approach, desirable properties for an index number formula are proposed; then it is determined whether any formula is consistent with these properties or tests. An ideal outcome is that the proposed tests are desirable and completely determine the functional form for the formula.

17.2 The axiomatic approach to index number theory is not completely straightforward, because choices have to be made in two dimensions:

- The index number framework must be determined, and
- Once the framework has been decided upon, the tests or properties that should be imposed on the index number need to be determined.

The second point is straightforward: Different price statisticians may have different ideas about what tests are important, and alternative sets of axioms can lead to alternative best index number functional forms. This point must be kept in mind while this chapter is read, because there is no universal agreement on what the best set of reasonable axioms is. Hence, the axiomatic approach can lead to more than one best index number formula.

17.3 The first point about the choices listed above requires further discussion. In the previous chapter, for the most part, the focus was on *bilateral index number theory*; that is, it was assumed that prices and quantities for the same n commodities were given for two periods, and the object of the index number formula was to compare the overall level of prices in one period with that of the other period. In this framework, both sets of price and quantity vectors were regarded as variables that could be *independently varied*, so that, for example, variations in the prices of one period did not affect the prices of the other period or the quantities in either period. The emphasis was on comparing the overall cost of a fixed basket of quantities in the two periods or taking averages of such fixed-basket indices. This is an example of an index number framework.

17.4 But other index number frameworks are possible. For example, instead of decomposing a value ratio into a term that represents price change between the two periods times another term that represents quantity change, one could attempt to decompose a value aggregate for one period into a single number that represents the price level in the period times another number that represents the quantity level in the period. In the first variant of this approach, the price index number is supposed to be a function of the n product prices pertaining to that aggregate in the period under consideration, and the quantity index number is supposed to be a function of the n product quantities pertaining to the aggregate in the period. The resulting price index function was called an *absolute index number* by Frisch (1930, p. 397), a *price level* by Eichhorn (1978, p. 141), and a *unilateral price index* by Anderson, Jones, and Nesmith (1997, p. 75). In a second variant of this approach, the price and quantity functions are allowed to depend on both the price and quantity vectors pertaining to the period under consideration.[1] These two variants of unilateral index number theory are considered in Section B.[2]

[1]Eichhorn (1978, p. 144) and Diewert (1993d, p. 9) considered this approach.

[2]In these unilateral index number approaches, the price and quantity vectors are allowed to vary independently. In yet another index number framework, prices are allowed to vary freely, but quantities are regarded as functions of the prices. This leads to the *economic approach to index number theory,* which is considered in more depth in Chapters 17 and 18.

17.5 The remaining approaches in this chapter are largely bilateral approaches; that is, the prices and quantities in an aggregate are compared for two periods. In Sections C and E below, the value ratio decomposition approach is taken.[3] In Section C, the bilateral price and quantity indices, $P(p^0, p^1, q^0, q^1)$ and $Q(p^0, p^1, q^0, q^1)$, are regarded as functions of the price vectors pertaining to the two periods, p^0 and p^1, and the two quantity vectors, q^0 and q^1. Not only do the axioms or tests that are placed on the price index $P(p^0, p^1, q^0, q^1)$ reflect reasonable price index properties, but some of them have their origin as reasonable tests on the quantity index $Q(p^0, p^1, q^0, q^1)$. The approach in Section C simultaneously determines the best price and quantity indices.

17.6 In Section D, attention is shifted to the *price ratios* for the n commodities between periods 0 and 1, $r_i \equiv p_i^1/p_i^0$ for $i = 1, \ldots, n$. In the *unweighted stochastic approach to index number theory,* the price index is regarded as an evenly weighted average of the n price relatives or ratios, r_i. Carli (1804) and Jevons (1863, 1865) were the early pioneers in this approach to index number theory, with Carli using the arithmetic average of the price relatives and Jevons endorsing the geometric average (but also considering the harmonic average). This approach to index number theory is covered in Section D.1. This approach is consistent with a statistical approach that regards each price ratio r_i as a random variable with a mean equal to the underlying price index.

17.7 A major problem with the unweighted average of price relatives approach to index number theory is that it does not take into account the economic importance of the individual commodities in the aggregate. Arthur Young (1812) did advocate some form of rough weighting of the price relatives according to their relative value over the period being considered, but the precise form of the required value weighting was not indicated.[4] However, it was Walsh (1901, pp. 83–121; 1921a, pp. 81–90) who stressed the importance of weighting the individual price ratios, where the weights are functions of the associated values for the commodities in each period and each period is to be treated symmetrically in the resulting formula:

> What we are seeking is to average the variations in the exchange value of one given total sum of money in relation to the several classes of goods, to which several variations [price ratios] must be assigned weights proportional to the relative sizes of the classes. Hence the relative sizes of the classes at both the periods must be considered. (Walsh, 1901, p. 104)

> Commodities are to be weighted according to their importance, or their full values. But the problem of axiometry always involves at least two periods. There is a first period and there is a second period which is compared with it. Price variations[5] have taken place between the two, and these are to be averaged to get the amount of their variation as a whole. But the weights of the commodities at the second period are apt to be different from their weights at the first period. Which weights, then, are the right ones—those of the first period or those of the second? Or should there be a combination of the two sets? There is no reason for preferring either the first or the second. Then the combination of both would seem to be the proper answer. And this combination itself involves an averaging of the weights of the two periods. (Walsh, 1921a, p. 90)

17.8 Thus, Walsh was the first to examine in some detail the rather intricate problems[6] in deciding how to weight the price relatives pertaining to an aggregate, taking into account the economic importance of the commodities in the two periods being considered. Note that the type of index number formulas that he was considering was of the form $P(r, v^0, v^1)$, where r is the vector of price relatives that has ith component $r_i = p_i^1/p_i^0$

[3]See Section B in Chapter 16 for an explanation of this approach.

[4]Walsh referred to Young's contributions as follows: "Still, although few of the practical investigators have actually employed anything but even weighting, they have almost always recognized the theoretical need of allowing for the relative importance of the different classes ever since this need was first pointed out, near the commencement of the century just ended, by Arthur Young. . . . Arthur Young advised simply that the classes should be weighted according to their importance" (1901, p. 84).

[5]A price variation is a price ratio or price relative in Walsh's terminology.

[6]Walsh (1901, pp. 104–105) realized that it would not do to simply take the arithmetic average of the values in the two periods, $[v_i^0 + v_i^1]/2$, as the correct weight for the ith price relative r_i because, in a period of rapid inflation, this would give too much importance to the period that had the highest prices, and he wanted to treat each period symmetrically: "But such an operation is manifestly wrong. In the first place, the sizes of the classes at each period are reckoned in the money of the period, and if it happens that the exchange value of money has fallen, or prices in general have risen, greater influence upon the result would be given to the weighting of the second period; or if prices in general have fallen, greater influence would be given to the weighting of the second period. Or in a comparison between two countries greater influence would be given to the weighting of the country with the higher level of prices. But it is plain that *the one period, or the one country, is as important, in our comparison between them, as the other, and the weighting in the averaging of their weights should really be even*." However, Walsh was unable to come up with Theil's (1967) solution to the weighting problem, which was to use the average value share $[s_i^0 + s_i^1]/2$ as the correct weight for the ith price relative in the context of using a weighted geometric mean of the price relatives.

and v^t is the period t value vector that has ith component $v_i^t = p_i^t q_i^t$ for $t = 0, 1$. His suggested solution to this weighting problem was not completely satisfactory, but he did at least suggest a useful framework for a price index as a value-weighted average of the n price relatives. The first satisfactory solution to the weighting problem was obtained by Theil (1967, pp. 136–37), and his solution is explained in Section D.2.

17.9 It can be seen that one of Walsh's approaches to index number theory[7] was an attempt to determine the best weighted average of the price relatives, r_i. This is equivalent to using an axiomatic approach to try and determine the best index of the form $P(r, v^0, v^1)$. This approach is considered in Section E.[8]

17.10 Recall that in Chapter 16, the Young and Lowe indices were introduced. These indices do not fit precisely into the bilateral framework because the value or quantity weights used in these indices do not necessarily correspond to the values or quantities that pertain to either of the periods that correspond to the price vectors p^0 and p^1. In Section F, the axiomatic properties of these two indices with respect to their price variables are studied.

B. The Levels Approach to Index Number Theory

B.1 Axiomatic approach to unilateral price indices

17.11 Denote the price and quantity of product n in period t by p_i^t and q_i^t, respectively, for $i = 1, 2, \ldots, n$ and $t = 0, 1, \ldots, T$. The variable q_i^t is interpreted as the total amount of product i transacted within period t. In order to conserve the value of transactions, it is necessary that p_i^t be defined as a unit value; that is, p_i^t must be equal to the value of transactions in product i for period t divided by the total quantity transacted, q_i^t. In principle, the period of time should be chosen so that variations in product prices within a period are quite small compared to their variations between periods.[9] For $t = 0, 1, \ldots, T$, and $i = 1, \ldots, n$, define the value of transactions in product i as $v_i^t = p_i^t q_i^t$ and define the *total value of transactions in period t* as

$$V^t \equiv \sum_{i=1}^{n} v_i^t = \sum_{i=1}^{n} p_i^t q_i^t, \quad t = 0, 1, \ldots, T. \qquad (17.1)$$

17.12 Using the notation above, the following *levels version of the index number problem* is defined as follows: For $t = 0, 1, \ldots, T$, find scalar numbers P^t and Q^t such that

$$V^t = P^t Q^t, \; t = 0, 1, \ldots, T. \qquad (17.2)$$

The number P^t is interpreted as an aggregate period t price level, while the number Q^t is interpreted as an aggregate period t quantity level. The aggregate price level P^t is allowed to be a function of the period t price vector, p^t, while the aggregate period t quantity level, Q^t, is allowed to be a function of the period t quantity vector, q^t. As a result we have the following:

$$P^t = c(p^t) \text{ and } Q^t = f(q^t), \; t = 0, 1, \ldots, T. \qquad (17.3)$$

17.13 The functions c and f are to be determined somehow. Note that equation (17.3) requires that the functional forms for the price aggregation function c and for the quantity aggregation function f be independent of time. This is a reasonable requirement, because there is no reason to change the method of aggregation as time changes.

[7]Walsh also considered basket-type approaches to index number theory, as was seen in Chapter 16.

[8]In Section E, rather than starting with indices of the form $P(r, v^0, v^1)$, indices of the form $P(p^0, p^1, v^0, v^1)$ are considered. However, if the invariance to changes in the units of measurement test is imposed on this index, it is equivalent to studying indices of the form $P(r, v^0, v^1)$. Vartia (1976a) also used a variation of this approach to index number theory.

[9]This treatment of prices as unit values over time follows Walsh (1901, p. 96; and 1921a, p. 88) and Irving Fisher (1922, p. 318). Fisher and Hicks both had the idea that the length of the period should be short enough so that variations in price within the period could be ignored as the following quotations indicate: "Throughout this book 'the price' of any commodity or 'the quantity' of it for any one year was assumed given. But what is such a price or quantity? Sometimes it is a single quotation for January 1 or July 1, but usually it is an average of several quotations scattered throughout the year. The question arises: On what principle should this average be constructed? The *practical* answer is *any* kind of average since, ordinarily, the variations during a year, so far, at least, as prices are concerned, are too little to make any perceptible difference in the result, whatever kind of average is used. Otherwise, there would be ground for subdividing the year into quarters or months until we reach a small enough period to be considered practically a point. The quantities sold will, of course, vary widely. What is needed is their sum for the year (which, of course, is the same thing as the simple arithmetic average of the per annum rates for the separate months or other subdivisions). In short, the simple arithmetic average, both of prices and of quantities, may be used. Or, if it is worth while to put any finer point on it, we may take the weighted arithmetic average for the prices, the weights being the quantities sold" (I. Fisher, 1922, p. 318). "I shall define a week as that period of time during which variations in prices can be neglected. For theoretical purposes this means that prices will be supposed to change, not continuously, but at short intervals. The calendar length of the week is of course quite arbitrary; by taking it to be very short, our theoretical scheme can be fitted as closely as we like to that ceaseless oscillation which is a characteristic of prices in certain markets" (Hicks, 1946, p. 122).

17.14 Substituting equations (17.3) and (17.2) into equation (17.1) and dropping the superscripts t means that c and f must satisfy the following functional equation for all strictly positive price and quantity vectors:

$$c(p)f(q) = \sum_{i=1}^{n} p_i q_i, \tag{17.4}$$

for all $p_i > 0$ and for all $q_i > 0$.

17.15 It is natural to assume that the functions $c(p)$ and $f(q)$ are positive if all prices and quantities are positive:

$$c(p_1, \ldots, p_n) > 0; f(q_1, \ldots, q_n) > 0 \tag{17.5}$$

if all $p_i > 0$ and for all $q_i > 0$.

17.16 Let 1_n denote an n dimensional vector of ones. Then equation (17.5) implies that when $p = 1_n$, $c(1_n)$ is a positive number, a for example, and when $q = 1_n$, then $f(1_n)$ is also a positive number, b for example; that is, equation (17.5) implies that c and f satisfy

$$c(1_n) = a > 0\,; f(1_n) = b > 0. \tag{17.6}$$

17.17 Let $p = 1_n$ and substitute the first expression in equation (17.6) into (17.4) in order to obtain the following equation:

$$f(q) = \sum_{i=1}^{n} \frac{q_i}{a} \quad \text{for all } q_i > 0. \tag{17.7}$$

17.18 Now let $q = 1_n$ and substitute the second part of equation (17.6) into (17.4) in order to obtain the following equation:

$$c(p) = \sum_{i=1}^{n} \frac{p_i}{b} \quad \text{for all } p_i > 0. \tag{17.8}$$

17.19 Finally, substitute equations (17.7) and (17.8) into the left-hand side of equation (17.4) and the following equation is obtained:

$$\left(\sum_{i=1}^{n} \frac{p_i}{b}\right)\left(\sum_{i=1}^{n} \frac{q_i}{a}\right) = \sum_{i=1}^{n} p_i q_i, \tag{17.9}$$

for all $p_i > 0$ and for all $q_i > 0$. If n is greater than one, it is obvious that equation (17.9) cannot be satisfied for all strictly positive p and q vectors. Thus, if the number of commodities n exceeds one, then there are no functions c and f that satisfy equations (17.4) and (17.5).[10]

17.20 Thus, this levels test approach to index number theory comes to an abrupt halt; it is fruitless to look for price- and quantity-level functions, $P^t = c(p^t)$ and $Q^t = f(q^t)$, that satisfy (17.2) or (17.4) and also satisfy the very reasonable positivity requirements in equation (17.5).

17.21 Note that the levels price index function, $c(p^t)$, did not depend on the corresponding quantity vector q^t, and the levels quantity index function, $f(q^t)$, did not depend on the price vector p^t. Perhaps this is the reason for the rather negative result obtained above. As a result, in the next section, the price and quantity functions are allowed to be functions of both p^t and q^t.

B.2 A second axiomatic approach to unilateral price indices

17.22 In this section, the goal is to find functions of $2n$ variables, $c(p, q)$ and $f(p, q)$, such that the following counterpart to equation (17.4) holds:

$$c(p, q)f(p, q) = \sum_{i=1}^{n} p_i q_i, \tag{17.10}$$

for all $p_i > 0$ and for all $q_i > 0$.

17.23 Again, it is natural to assume that the functions $c(p, q)$ and $f(p, q)$ are positive if all prices and quantities are positive:

$$c(p_1, \ldots, p_n, q_1, \ldots, q_n) > 0;$$
$$f(p_1, \ldots, p_n; q_1, \ldots, q_n) > 0 \tag{17.11}$$

if all $p_i > 0$ and for all $q_i > 0$.

17.24 The present framework does not distinguish between the functions c and f, so it is necessary to require that these functions satisfy some reasonable properties. The first property imposed on c is that this function be homogeneous of degree one in its price components:

$$c(\lambda p, q) = \lambda c(p, q) \quad \text{for all } \lambda > 0. \tag{17.12}$$

Thus if all prices are multiplied by the positive number λ, then the resulting price index is λ times the initial price index. A similar linear homogeneity property is imposed on the quantity index f; that is, f is to be homogeneous of degree one in its quantity components:

$$f(p, \lambda q) = \lambda f(p, q) \quad \text{for all } \lambda > 0. \tag{17.13}$$

17.25 Note that the properties in equations (17.10), (17.11), and (17.13) imply that the price index $c(p, q)$

[10]Eichhorn (1978, p. 144) established this result.

has the following homogeneity property with respect to the components of q:

$$c(p, \lambda q) = \sum_{i=1}^{n} \frac{p_i \lambda q_i}{f(p, \lambda q)} \text{ where } \lambda > 0. \quad (17.14)$$
$$= \sum_{i=1}^{n} \frac{p_i \lambda q_i}{\lambda f(p, q)} \text{ using (17.13)}$$
$$= \sum_{i=1}^{n} \frac{p_i q_i}{f(p, q)}$$
$$= c(p, q) \text{ using equations (17.10) and (17.11).}$$

Thus $c(p, q)$ is homogeneous of degree 0 in its q components.

17.26 A final property that is imposed on the levels price index $c(p, q)$ is the following: Let the positive numbers d_i be given. Then it is asked that the price index be invariant to changes in the units of measurement for the n commodities, so that the function $c(p, q)$ has the following property:

$$c(d_1p_1, \ldots, d_np_n; q_1/d_1, \ldots, q_n/d_n) = c(p_1, \ldots, p_n; q_1, \ldots, q_n). \quad (17.15)$$

17.27 It is now possible to show that the properties in equations (17.10), (17.11), (17.12), (17.14), and (17.15) on the price levels function $c(p, q)$ are inconsistent; that is, there is no function of $2n$ variables $c(p, q)$ that satisfies these quite reasonable properties.[11]

17.28 To see why this is so, apply (17.15), setting $d_i = q_i$ for each i, to obtain the following equation:

$$c(p_1, \ldots, p_n; q_1, \ldots, q_n) = c(p_1q_1, \ldots, p_nq_n; 1, \ldots, 1). \quad (17.16)$$

If $c(p, q)$ satisfies the linear homogeneity property in equation (17.12) so that $c(\lambda p, q) = \lambda c(p, q)$, then (17.16) implies that $c(p, q)$ is also linearly homogeneous in q, so that $c(p, \lambda q) = \lambda c(p, q)$. But this last equation contradicts equation (17.14), which establishes the impossibility result.

17.29 The rather negative results obtained in Section B.1 and this section indicate that it is fruitless to pursue the axiomatic approach to the determination of price and quantity levels, where both the price and quantity vector are regarded as independent variables.[12] Therefore, in the following sections of this chapter, the axiomatic approach to the determination of a *bilateral price index* of the form $P(p^0, p^1, q^0, q^1)$ is pursued.

C. First Axiomatic Approach to Bilateral Price Indices

C.1 Bilateral indices and some early tests

17.30 In this section, the strategy will be to assume that the bilateral price index formula, $P(p^0, p^1, q^0, q^1)$, satisfies a sufficient number of reasonable tests or properties so that the functional form for p is determined.[13] The word "bilateral"[14] refers to the assumption that the function p depends only on the data pertaining to the two situations or periods being compared; that is, p is regarded as a function of the two sets of price and quantity vectors, (p^0, p^1, q^0, q^1), that are to be aggregated into a single number that summarizes the overall change in the n price ratios, $p_1^1/p_1^0, \ldots, p_n^1/p_n^0$.

17.31 In this section, the value ratio decomposition approach to index number theory is taken; that is, along with the price index $P(p^0, p^1, q^0, q^1)$, there is a companion quantity index $Q(p^0, p^1, q^0, q^1)$ such that the product of these two indices equals the value ratio between the two periods.[15] Thus, throughout this section, it is assumed that p and q satisfy the following *product test:*

$$V^1/V^0 = P(p^0, p^1, q^0, q^1)Q(p^0, p^1, q^0, q^1). \quad (17.17)$$

The period t values, V^t, for $t = 0, 1$ are defined by equation (17.1). Equation (17.17) means that as soon as the functional form for the price index p is determined, then equation (17.17) can be used to determine the functional form for the quantity index Q. However, a further advantage of assuming that the product test holds is that if a reasonable test is imposed on the quantity index Q, then equation (17.17) can be used to translate this test

[11]This proposition is from Diewert (1993d, p. 9), but his proof is an adaptation of a closely related result from Eichhorn (1978, pp. 144–45).

[12]Recall that in the economic approach, the price vector p is allowed to vary independently, but the corresponding quantity vector q is regarded as being determined by p.

[13]Much of the material in this section is drawn from Sections 2 and 3 of Diewert (1992a). For more recent surveys of the axiomatic approach, see Balk (1995) and von Auer (2001).

[14]Multilateral index number theory refers to the case in which there are more than two situations whose prices and quantities need to be aggregated.

[15]See Section B of Chapter 16 for more on this approach, which was initially from Irving Fisher (1911, p. 403; 1922).

on the quantity index into a corresponding test on the price index P.[16]

17.32 If $n = 1$, so that there is only one price and quantity to be aggregated, then a natural candidate for p is $p_1{}^1/p_1{}^0$, the single price ratio, and a natural candidate for q is $q_1{}^1/q_1{}^0$, the single quantity ratio. When the number of products or items to be aggregated is greater than 1, index number theorists have proposed over the years properties or tests that the price index p should satisfy. These properties are generally multidimensional analogues to the one good price index formula, $p_1{}^1/p_1{}^0$. In Sections C.2 through C.6, 20 tests are listed that turn out to characterize the Fisher ideal price index.

17.33 It will be assumed that every component of each price and quantity vector is positive; that is, $p^t >> 0_n$ and $q^t >> 0_n$[17] for $t = 0, 1$. If it is desired to set $q^0 = q^1$, the common quantity vector is denoted by q; if it is desired to set $p^0 = p^1$, the common price vector is denoted by p.

17.34 The first two tests are not very controversial, so they are not discussed in detail.

T1—*Positivity:*[18] $P(p^0, p^1, q^0, q^1) > 0$.

T2—*Continuity:*[19] $P(p^0, p^1, q^0, q^1)$ is a continuous function of its arguments.

17.35 The next two tests are somewhat more controversial.

T3—*Identity or Constant Prices Test:*[20]
$P(p, p, q^0, q^1) = 1$.

That is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the quantity vectors are. The controversial part of this test is that the two quantity vectors are allowed to be different.[21]

T4—*Fixed-Basket or Constant Quantities Test:*[22]

$$P(p^0, p^1, q, q) = \frac{\sum_{i=1}^{n} p_i^1 q_i}{\sum_{i=1}^{n} p_i^0 q_i}.$$

That is, if quantities are constant during the two periods so that $q^0 = q^1 \equiv q$, then the price index should equal the value generated by trading the constant basket in period 1, $\sum_{i=1}^{n} p_i^1 q_i$, divided by the value generated by trading the basket in period 0, $\sum_{i=1}^{n} p_i^0 q_i$.

17.36 If the price index p satisfies test T4 and p and q jointly satisfy the product test, equation (17.17), then q must satisfy the identity test $Q(p^0, p^1, q, q) = 1$ for all strictly positive vectors p^0, p^1, q.[23] This *constant quantities test* for q is also somewhat controversial, because p^0 and p^1 are allowed to be different.

C.2 Homogeneity tests

17.37 The following four tests restrict the behavior of the price index p as the scale of any one of the four vectors p^0, p^1, q^0, q^1 changes.

T5—*Proportionality in Current Prices:*[24]
$P(p^0, \lambda p^1, q^0, q^1) = \lambda P(p^0, p^1, q^0, q^1)$ for $\lambda > 0$.

That is, if all period 1 prices are multiplied by the positive number λ, then the new price index is λ times the old price index. Put another way, the price index function $P(p^0, p^1, q^0, q^1)$ is (positively) homogeneous of degree one in the components of the period 1 price

[16] This observation was first made by Irving Fisher (1911, pp. 400–06). Vogt (1980) and Diewert (1992a) also pursued this idea.

[17] Notation: $q >> 0_n$ means that each component of the vector q is positive; $q \geq 0_n$ means each component of q is nonnegative; and $q > 0_n$ means $q \geq 0_n$ and $q \neq 0_n$.

[18] Eichhorn and Voeller (1976, p. 23) suggested this test.

[19] Irving Fisher (1922, pp. 207–15) informally suggested the essence of this test.

[20] Laspeyres (1871, p. 308), Walsh (1901, p. 308), and Eichhorn and Voeller (1976, p. 24) have all suggested this test. Laspeyres came up with this test or property to discredit the ratio of unit values index of Drobisch (1871a), which does not satisfy this test. This test is also a special case of Irving Fisher's (1911, pp. 409–10) price proportionality test.

[21] Usually, economists assume that given a price vector p, the corresponding quantity vector q is uniquely determined. Here, the same price vector is used, but the corresponding quantity vectors are allowed to be different.

[22] The origins of this test go back at least 200 years to the Massachusetts legislature, which used a constant basket of goods to index the pay of Massachusetts soldiers fighting in the American Revolution; see Willard Fisher (1913). Other researchers who have suggested the test over the years include Lowe (1823, Appendix, p. 95), Scrope (1833, p. 406), Jevons (1865), Sidgwick (1883, pp. 67–68), Edgeworth (1925, p. 215), Marshall (1887, p. 363), Pierson (1895, p. 332), Walsh (1901, p. 540; and 1921b, pp. 543–44), and Bowley (1901, p. 227). Vogt and Barta (1997, p. 49) correctly observed that this test is a special case of Irving Fisher's (1911, p. 411) proportionality test for quantity indexes that Fisher (1911, p. 405) translated into a test for the price index using the product test in equation (16.3).

[23] This is shown in Vogt (1980, p. 70).

[24] This test was proposed by Walsh (1901, p. 385), Eichhorn and Voeller (1976, p. 24), and Vogt (1980, p. 68).

vector p^1. Most index number theorists regard this property as a fundamental one that the index number formula should satisfy.

17.38 Walsh (1901) and Irving Fisher (1911, p. 418; 1922, p. 420) proposed the related proportionality test $P(p, \lambda p, q^0, q^1) = \lambda$. This last test is a combination of T3 and T5; in fact, Walsh (1901, p. 385) noted that this last test implies the identity test T3.

17.39 In the next test, instead of multiplying all period 1 prices by the same number, all period 0 prices are multiplied by the number λ.

T6—*Inverse Proportionality in Base-Period Prices:*[25]
$P(\lambda p^0, p^1, q^0, q^1) = \lambda^{-1}P(p^0, p^1, q^0, q^1)$ for $\lambda > 0$.

That is, if all period 0 prices are multiplied by the positive number λ, then the new price index is $1/\lambda$ times the old price index. Put another way, the price index function $P(p^0, p^1, q^0, q^1)$ is (positively) homogeneous of degree minus one in the components of the period 0 price vector p^0.

17.40 The following two homogeneity tests can also be regarded as invariance tests.

T7—*Invariance to Proportional Changes in Current Quantities:*
$P(p^0, p^1, q^0, \lambda q^1) = P(p^0, p^1, q^0, q^1)$ for all $\lambda > 0$.

That is, if current-period quantities are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0, p^1, q^0, q^1)$ is (positively) homogeneous of degree zero in the components of the period 1 quantity vector q^1. Vogt (1980, p. 70) was the first to propose this test,[26] and his derivation of the test is of some interest. Suppose the quantity index q satisfies the quantity analogue to the price test T5; that is, suppose q satisfies $Q(p^0, p^1, q^0, \lambda q^1) = \lambda Q(p^0, p^1, q^0, q^1)$ for $\lambda > 0$. Then using the product test in equation (17.17), it can be seen that p must satisfy T7.

T8—*Invariance to Proportional Changes in Base Quantities:*[27]
$P(p^0, p^1, \lambda q^0, q^1) = P(p^0, p^1, q^0, q^1)$ for all $\lambda > 0$.

That is, if base-period quantities are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0, p^1, q^0, q^1)$ is (positively) homogeneous of degree zero in the components of the period 0 quantity vector q^0. If the quantity index q satisfies the following counterpart to T8: $Q(p^0, p^1, \lambda q^0, q^1) = \lambda^{-1}Q(p^0, p^1, q^0, q^1)$ for all $\lambda > 0$, then using equation (17.17), the corresponding price index p must satisfy T8. This argument provides some additional justification for assuming the validity of T8 for the price index function P.

17.41 T7 and T8 together impose the property that the price index p does not depend on the *absolute* magnitudes of the quantity vectors q^0 and q^1.

C.3 Invariance and symmetry tests

17.42 The next five tests are invariance or symmetry tests. Irving Fisher (1922, pp. 62–63, 458–60) and Walsh (1901, p. 105; 1921b, p. 542) seem to have been the first researchers to appreciate the significance of these kinds of tests. Fisher (1922, pp. 62–63) spoke of fairness, but it is clear that he had symmetry properties in mind. It is perhaps unfortunate that he did not realize that there were more symmetry and invariance properties than the ones he proposed; if he had realized this, it is likely that he would have been able to provide an axiomatic characterization for his ideal price index, as is done in Section C.6. The first invariance test is that the price index should remain unchanged if the *ordering* of the commodities is changed:

T9—*Commodity Reversal Test* (or invariance to changes in the ordering of commodities):
$P(p^{0*}, p^{1*}, q^{0*}, q^{1*}) = P(p^0, p^1, q^0, q^1)$,

where p^{t*} denotes a permutation of the components of the vector p^t, and q^{t*} denotes the same permutation of the components of q^t for $t = 0, 1$. This test is from Irving Fisher (1922, p. 63);[28] it is one of his three famous reversal tests. The other two are the time reversal test and the factor reversal test, which are considered below.

17.43 The next test asks that the index be invariant to changes in the units of measurement.

[25]Eichhorn and Voeller (1976, p. 28) suggested this test.

[26]Fisher (1911, p. 405) proposed the related test $P(p^0, p^1, q^0, \lambda q^0) = P(p^0, p^1, q^0, q^0) = \sum_{i=1}^{n} p_i^1 q_i^0 / \sum_{i=1}^{n} p_i^0 q_i^0$.

[27]This test was proposed by Diewert (1992a, p. 216).

[28]"This [test] is so simple as never to have been formulated. It is merely taken for granted and observed instinctively. Any rule for averaging the commodities must be so general as to apply interchangeably to all of the terms averaged" (I. Fisher, 1922, p. 63).

T10—*Invariance to Changes in the Units of Measurement* (commensurability test):

$$P\left(\alpha_1 p_1^0, \ldots, \alpha_n p_n^0; \alpha_1 p_1^1, \ldots, \alpha_n p_n^1; \alpha_1^{-1} q_1^0, \ldots, \alpha_n^{-1} q_n^0; \alpha_1^{-1} q_1^1, \ldots, \alpha_n^{-1} q_n^1\right) = P\left(p_1^0, \ldots, p_n^0; p_1^1, \ldots, p_n^1; q_1^0, \ldots, q_n^0; q_1^1, \ldots, q_n^1\right)$$

for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

That is, the price index does not change if the units of measurement for each product are changed. The concept of this test comes from Jevons (1863, p. 23) and the Dutch economist Pierson (1896, p. 131), who criticized several index number formulas for not satisfying this fundamental test. Fisher (1911, p. 411) first called this test *the change of units test*, and later he called it the *commensurability test* (Fisher, 1922, p. 420).

17.44 The next test asks that the formula be invariant to the period chosen as the base period.

T11—*Time Reversal Test:*
$P(p^0, p^1, q^0, q^1) = 1/P(p^1, p^0, q^1, q^0)$.

That is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index. In the one good case when the price index is simply the single price ratio, this test will be satisfied (as are all of the other tests listed in this section). When the number of goods is greater than one, many commonly used price indices fail this test; for example, the Laspeyres (1871) price index, P_L, defined by equation (16.5) in Chapter 16, and the Paasche (1874) price index, P_P, defined by equation (16.6) in Chapter 16, both fail this fundamental test. The concept of the test comes from Pierson (1896, p. 128), who was so upset with the fact that many of the commonly used index number formulas did not satisfy this test that he proposed that the entire concept of an index number should be abandoned. More formal statements of the test were made by Walsh (1901, p. 368; 1921b, p. 541) and Irving Fisher (1911, p. 534; 1922, p. 64).

17.45 The next two tests are more controversial, because they are not necessarily consistent with the economic approach to index number theory. However, these tests are quite consistent with the weighted stochastic approach to index number theory discussed later in this chapter.

T12—*Quantity Reversal Test* (quantity weights symmetry test): $P(p^0, p^1, q^0, q^1) = P(p^0, p^1, q^1, q^0)$.

That is, if the quantity vectors for the two periods are interchanged, then the price index remains invariant. This property means that if quantities are used to weight the prices in the index number formula, then the period 0 quantities q^0 and the period 1 quantities q^1 must enter the formula in a symmetric or evenhanded manner. Funke and Voeller (1988, p. 3) introduced this test; they called it the *weight property*.

17.46 The next test is the analogue to T12 applied to quantity indices:

T13—*Price Reversal Test* (price weights symmetry test):[29]

$$\left(\frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}\right) \Big/ P(p^0, p^1, q^0, q^1) = \left(\frac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^0}\right) \Big/ P(p^1, p^0, q^0, q^1). \quad (17.18)$$

Thus, if we use equation (17.17) to define the quantity index Q in terms of the price index P, then it can be seen that T13 is equivalent to the following property for the associated quantity index Q:

$$Q(p^0, p^1, q^0, q^1) = Q(p^1, p^0, q^0, q^1). \quad (17.19)$$

That is, if the price vectors for the two periods are interchanged, then the quantity index remains invariant. Thus, if prices for the same good in the two periods are used to weight quantities in the construction of the quantity index, then property T13 implies that these prices enter the quantity index in a symmetric manner.

C.4 Mean value tests

17.47 The next three tests are mean value tests.

T14—*Mean Value Test for Prices:*[30]

$$\min_i(p_i^1/p_i^0 : i = 1, \ldots, n) \le P(p^0, p^1, q^0, q^1) \le \max_i(p_i^1/p_i^0 : i = 1, \ldots, n). \quad (17.20)$$

That is, the price index lies between the minimum price ratio and the maximum price ratio. Because the price index is supposed to be interpreted as a kind of

[29]This test was proposed by Diewert (1992a, p. 218).

[30]This test seems to have been first proposed by Eichhorn and Voeller (1976, p. 10).

average of the n price ratios, p_i^1/p_i^0, it seems essential that the price index p satisfy this test.

17.48 The next test is the analogue to T14 applied to quantity indices:

T15—*Mean Value Test for Quantities:*[31]

$$\min_i(q_i^1/q_i^0 : i = 1, \ldots, n) \le \frac{(V^1/V^0)}{P(p^0, p^1, q^0, q^1)} \le \max_i(q_i^1/q_i^0 : i = 1, \ldots, n), \quad (17.21)$$

where V^t is the period t value for the aggregate defined by equation (17.1) above. Using the product test equation (17.17) to define the quantity index Q in terms of the price index P, one can see that T15 is equivalent to the following property for the associated quantity index Q:

$$\min_i(q_i^1/q_i^0 : i = 1, \ldots, n) \le Q(p^0, p^1, q^0, q^1) \le \max_i(q_i^1/q_i^0 : i = 1, \ldots, n). \quad (17.22)$$

That is, the implicit quantity index Q defined by P lies between the minimum and maximum rates of growth q_i^1/q_i^0 of the individual quantities.

17.49 In Section C of Chapter 16, it was argued that it was reasonable to take an average of the Laspeyres and Paasche price indices as a single best measure of overall price change. This point of view can be turned into a test:

T16—*Paasche and Laspeyres Bounding Test:*[32] The price index P lies between the Laspeyres and Paasche indices, P_L and P_P, defined by equations (16.5) and (16.6) in Chapter 16.

A test could be proposed where the implicit quantity index Q that corresponds to P via equation (17.17) is to lie between the Laspeyres and Paasche quantity indices, Q_P and Q_L, defined by equations (16.10) and (16.11) in Chapter 16. However, the resulting test turns out to be equivalent to test T16.

C.5 Monotonicity tests

17.50 The final four tests are monotonicity tests; that is, how should the price index $P(p^0, p^1, q^0, q^1)$ change as any component of the two price vectors p^0 and p^1 increases or as any component of the two quantity vectors q^0 and q^1 increases?

T17—*Monotonicity in Current Prices:*
$P(p^0, p^1, q^0, q^1) < P(p^0, p^2, q^0, q^1)$ if $p^1 < p^2$.

That is, if some period 1 price increases, then the price index must increase, so that $P(p^0, p^1, q^0, q^1)$ is increasing in the components of p^1. This property was proposed by Eichhorn and Voeller (1976, p. 23), and it is a reasonable property for a price index to satisfy.

T18—*Monotonicity in Base Prices:*
$P(p^0, p^1, q^0, q^1) > P(p^2, p^1, q^0, q^1)$ if $p^0 < p^2$.

That is, if any period 0 price increases, then the price index must decrease, so that $P(p^0, p^1, q^0, q^1)$ is decreasing in the components of p^0. This quite reasonable property was also proposed by Eichhorn and Voeller (1976, p. 23).

T19—*Monotonicity in Current Quantities:* If $q^1 < q^2$, then

$$\left(\frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}\right) \Big/ P(p^0, p^1, q^0, q^1) < \left(\frac{\sum_{i=1}^{n} p_i^1 q_i^2}{\sum_{i=1}^{n} p_i^0 q_i^0}\right) \Big/ P(p^0, p^1, q^0, q^2). \quad (17.23)$$

T20—*Monotonicity in Base Quantities:* If $q^0 < q^2$, then

$$\left(\frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}\right) \Big/ P(p^0, p^1, q^0, q^1) > \left(\frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^2}\right) \Big/ P(p^0, p^1, q^2, q^1). \quad (17.24)$$

17.51 Let Q be the implicit quantity index that corresponds to P using equation (17.17). Then it is found that T19 translates into the following inequality involving Q:

$$Q(p^0, p^1, q^0, q^1) < Q(p^0, p^1, q^0, q^2) \text{ if } q^1 < q^2. \quad (17.25)$$

[31]This test was proposed by Diewert (1992a, p. 219).
[32]Bowley (1901, p. 227) and Irving Fisher (1922, p. 403) both endorsed this property for a price index.

That is, if any period 1 quantity increases, then the implicit quantity index Q that corresponds to the price index P must increase. Similarly, we find that T20 translates into

$$Q(p^0, p^1, q^0, q^1) > Q(p^0, p^1, q^2, q^1) \text{ if } q^0 < q^2. \quad (17.26)$$

That is, if any period 0 quantity increases, then the implicit quantity index Q must decrease. Tests T19 and T20 are from Vogt (1980, p. 70).

17.52 This concludes the listing of tests. In the next section, it is asked whether any index number formula $P(p^0, p^1, q^0, q^1)$ exists that can satisfy all 20 tests.

C.6 Fisher ideal index and test approach

17.53 It can be shown that the only index number formula $P(p^0, p^1, q^0, q^1)$ that satisfies tests T1–T20 is the Fisher ideal price index P_F, defined as the geometric mean of the Laspeyres and Paasche indices:[33]

$$P_F(p^0, p^1, q^0, q^1) \equiv \{P_L(p^0, p^1, q^0, q^1)\, P_P(p^0, p^1, q^0, q^1)\}^{1/2}. \quad (17.27)$$

This assertion can be proven by showing that the Fisher index satisfies all 20 tests.

17.54 The more difficult part of the proof is showing that it is the *only* index number formula that satisfies these tests. This part of the proof follows from the fact that if P satisfies the positivity test T1 and the three reversal tests, T11–T13, then P must equal P_F. To see this, rearrange the terms in the statement of test T13 into the following equation:

$$\frac{\sum_{i=1}^{n} p_i^1 q_i^1 \Big/ \sum_{i=1}^{n} p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^1 \Big/ \sum_{i=1}^{n} p_i^1 q_i^0} = \frac{P(p^0, p^1, q^0, q^1)}{P(p^1, p^0, q^0, q^1)}$$

$$= \frac{P(p^0, p^1, q^0, q^1)}{P(p^1, p^0, q^0, q^1)} \quad \text{using T12, the quantity reversal test}$$

$$= P(p^0, p^1, q^0, q^1)^2 \quad \text{using T11, the time reversal test.} \quad (17.28)$$

Now take positive square roots on both sides of equation (17.28); it can be seen that the left-hand side of the equation is the Fisher index $P_F(p^0, p^1, q^0, q^1)$ defined by equation (17.27) and the right-hand side is $P(p^0, p^1, q^0, q^1)$. Thus, if P satisfies T1, T11, T12, and T13, it must equal the Fisher ideal index P_F.

17.55 The quantity index that corresponds to the Fisher price index using the product test equation (17.17) is Q_F, the Fisher quantity index, defined by equation (16.14) in Chapter 16.

17.56 It turns out that P_F satisfies yet another test, T21, which was Irving Fisher's (1921, p. 534; 1922, pp. 72–81) third reversal test (the other two being T9 and T11):

T21—*Factor Reversal Test* (functional form symmetry test):

$$P(p^0, p^1, q^0, q^1)\, P(q^0, q^1, p^0, p^1) = \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}. \quad (17.29)$$

A justification for this test is the following: Assume $P(p^0, p^1, q^0, q^1)$ is a good functional form for the price index; then if the roles of prices and quantities are reversed, $P(q^0, q^1, p^0, p^1)$ ought to be a good functional form for a quantity index (which seems to be a correct argument). The product, therefore, of the price index $P(q^0, q^1, p^0, p^1)$ and the quantity index $Q(q^0, q^1, p^0, p^1) = P(q^0, q^1, p^0, p^1)$ ought to equal the value ratio, V^1/V^0. The second part of this argument does not seem to be valid; consequently, many researchers over the years have objected to the factor reversal test. However, for those willing to embrace T21 as a basic test, Funke and Voeller (1988, p. 180) showed that the only index number function $P(q^0, q^1, p^0, p^1)$ that satisfies T1 (positivity), T11 (time reversal test), T12 (quantity reversal test), and T21 (factor reversal test) is the Fisher ideal index P_F defined by equation (17.27). Thus, the price reversal test T13 can be replaced by the factor reversal test in order to obtain a minimal set of four tests that lead to the Fisher price index.[34]

C.7 Test performance of other indices

17.57 The Fisher price index P_F satisfies all 20 of the tests listed in Sections C.1–C.5. Which tests do other commonly used price indices satisfy? Recall the Laspeyres index P_L defined by equation (16.5), the Paasche index P_P defined by equation (16.6), the Walsh index P_W defined by equation (16.19), and the Törnqvist index P_T defined by equation (16.81) in Chapter 16.

[33]This is demonstrated in Diewert (1992a, p. 221).

[34]Other characterizations of the Fisher price index can be found in Funke and Voeller (1988) and Balk (1985, 1995).

17.58 Straightforward computations show that the Paasche and Laspeyres price indices, P_L and P_P, fail only the three reversal tests, T11, T12, and T13. Because the quantity and price reversal tests, T12 and T13, are somewhat controversial and can be discounted, the test performance of P_L and P_P seems at first glance to be quite good. However, the failure of the time reversal test, T11, is a severe limitation associated with the use of these indices.

17.59 The Walsh price index, P_W, fails four tests: T13, the price reversal test; T16, the Paasche and Laspeyres bounding test; T19, the monotonicity in current quantities test; and T20, the monotonicity in base quantities test.

17.60 Finally, the Törnqvist price index P_T fails nine tests: T4, the fixed-basket test; T12 and T13, the quantity and price reversal tests; T15, the mean value test for quantities; T16, the Paasche and Laspeyres bounding test; and T17–T20, the four monotonicity tests. Thus, the Törnqvist index is subject to a rather high failure rate from the viewpoint of this axiomatic approach to index number theory.[35]

17.61 The tentative conclusion that can be drawn from these results is that from the viewpoint of this particular bilateral test approach to index numbers, the Fisher ideal price index P_F appears to be best because it satisfies all 20 tests.[36] The Paasche and Laspeyres indices are next best if we treat each test as being equally important. However, both of these indices fail the very important time reversal test. The remaining two indices, the Walsh and Törnqvist price indices, both satisfy the time reversal test, but the Walsh index emerges as the better one because it passes 16 of the 20 tests, whereas the Törnqvist satisfies only 11 tests.

C.8 Additivity test

17.62 There is an additional test that many national income accountants regard as very important: the *additivity test*. This is a test or property that is placed on the implicit quantity index $Q(q^0, q^1, p^0, p^1)$ that corresponds to the price index $P(q^0, q^1, p^0, p^1)$ using the product test in equation (17.17). This test states that the implicit quantity index has the following form:

$$Q(p^0, p^1, q^0, q^1) = \frac{\sum_{i=1}^{n} p_i^* q_i^1}{\sum_{m=1}^{n} p_m^* q_m^0}, \tag{17.30}$$

where the common across-periods *price* for product *i*, p_i* for $i = 1, \ldots, n$, can be a function of all 4n prices and quantities pertaining to the two periods or situations under consideration, p^0, p^1, q^0, q^1. In the literature on making multilateral comparisons (i.e., comparisons between more than two situations), it is quite common to assume that the quantity comparison between any two regions can be made using the two regional quantity vectors, q^0 and q^1, and a common reference price vector, $p^* \equiv (p_1^*, \ldots, p_n^*)$.[37]

17.63 Different versions of the additivity test can be obtained if further restrictions are placed on precisely which variables each reference price p_i* depends on. The simplest such restriction is to assume that each p_i* depends only on the product *i* prices pertaining to each of the two situations under consideration, p_i^0 and p_i^1. If it is further assumed that the functional form for the weighting function is the same for each product, so that $p_i^* = m(p_i^0, p_i^1)$ for $i = 1, \ldots, n$, then we are led to the *unequivocal quantity index* postulated by Knibbs (1924, p. 44).

17.64 The theory of the *unequivocal quantity index* (or the *pure quantity index*[38]) parallels the theory of the pure price index outlined in Section C.2 of Chapter 16. An outline of this theory is now given. Let the pure quantity index Q_K have the following functional form:

$$Q_K(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} q_i^1 m\left(p_i^0, p_i^1\right)}{\sum_{k=1}^{n} q_k^0 m\left(p_k^0, p_k^1\right)}. \tag{17.31}$$

It is assumed that the price vectors p^0 and p^1 are strictly positive, and the quantity vectors q^0 and q^1 are nonnegative but have at least one positive component.[39]

[35]However, it is shown in Chapter 19 that the Törnqvist index approximates the Fisher index quite closely using normal time-series data that are subject to relatively smooth trends. Under these circumstances, the Törnqvist index can be regarded as passing the 20 tests to a reasonably high degree of approximation.

[36]This assertion needs to be qualified: There are many other tests that we have not discussed, and price statisticians could differ on the importance of satisfying various sets of tests. Some references that discuss other tests are Eichhorn and Voeller (1976), Balk (1995), Vogt and Barta (1997), and von Auer (2001 and 2002). In Section E, it is shown that the Törnqvist index is ideal for a different set of axioms.

[37]Hill (1993, pp. 395–97) termed such multilateral methods *the block approach*, while Diewert (1996a, pp. 250–51) used the term *average price approaches*. Diewert (1999b, p. 19) used the term *additive multilateral system*. For axiomatic approaches to multilateral index number theory, see Balk (1996a, 2001) and Diewert (1999b).

[38]Diewert (2001a) used this term.

[39]It is assumed that $m(a, b)$ has the following two properties: $m(a, b)$ is a positive and continuous function, defined for all positive numbers a and b; and $m(a, a) = a$ for all $a > 0$.

The problem is to determine the functional form for the averaging function m if possible. To do this, it is necessary to impose some tests or properties on the pure quantity index Q_K. As was the case with the pure price index, it is reasonable to ask that the quantity index satisfy the *time reversal test:*

$$Q_K(p^1, p^0, q^1, q^0) = \frac{1}{Q_K(p^0, p^1, q^0, q^1)}. \tag{17.32}$$

17.65 As was the case with the theory of the unequivocal price index, it can be seen that if the unequivocal quantity index Q_K is to satisfy the time reversal test of equation (17.32), the mean function in equation (17.31) must be *symmetric*. It is also asked that Q_K satisfy the following *invariance to proportional changes in current prices test.*

$$Q_K(p^0, \lambda p^1, q^0, q^1) = Q_K(p^0, p^1, q^0, q^1)$$
$$\text{for all } p^0, p^1, q^0, q^1 \text{ and all } \lambda > 0. \tag{17.33}$$

17.66 The idea behind this invariance test is this: The quantity index $Q_K(p^0, p^1, q^0, q^1)$ should depend only on the *relative* prices in each period. It should not depend on the amount of general inflation between the two periods. Another way to interpret equation (17.33) is to look at what the test implies for the corresponding implicit price index, P_{IK}, defined using the product test of equation (17.17). It can be shown that if Q_K satisfies equation (17.33), then the corresponding implicit price index P_{IK} will satisfy test T5, the *proportionality in current prices test.* The two tests in equations (17.32) and (17.33) determine the precise functional form for the pure quantity index Q_K defined by equation (17.31): The *pure quantity index* or Knibbs' *unequivocal quantity index* Q_K must be the Walsh quantity index Q_W[40] defined by

$$Q_W(p^0, p^1, q^0, q^1) \equiv \frac{\sum_{i=1}^{n} q_i^1 \sqrt{p_i^0 p_i^1}}{\sum_{k=1}^{n} q_k^0 \sqrt{p_k^0 p_k^1}}. \tag{17.34}$$

17.67 Thus, with the addition of two tests, the pure price index P_K must be the Walsh price index P_W defined by equation (16.19) in Chapter 16. With the addition of the same two tests (but applied to quantity indices instead of price indices), the pure quantity index Q_K must be the Walsh quantity index Q_W defined by equation (17.34). However, note that the product of the Walsh price and quantity indices is *not* equal to the value ratio, V^1/V^0. Thus, believers in the pure or unequivocal price and quantity index concepts *have to choose one of these two concepts;* they cannot apply both simultaneously.[41]

17.68 If the quantity index $Q(q^0, q^1, p^0, p^1)$ satisfies the additivity test in equation (17.30) for some price weights $p_i{}^*$, then the percentage change in the quantity aggregate, $Q(q^0, q^1, p^0, p^1) - 1$, can be rewritten as follows:

$$Q(p^0, p^1, q^0, q^1) - 1 = \frac{\sum_{i=1}^{n} p_i^* q_i^1}{\sum_{m=1}^{n} p_m^* q_m^0} - 1 = \frac{\sum_{i=1}^{n} p_i^* q_i^1 - \sum_{m=1}^{n} p_m^* q_m^0}{\sum_{m=1}^{n} p_m^* q_m^0} = \sum_{i=1}^{n} w_i (q_i^1 - q_i^0), \tag{17.35}$$

where the *weight* for product *i*, w_i, is defined as

$$w_i \equiv \frac{p_i^*}{\sum_{m=1}^{n} p_m^* q_m^0}; \quad i = 1, \ldots, n. \tag{17.36}$$

Note that the change in product i going from situation 0 to situation 1 is $q_i^1 - q_i^0$. Thus, the ith term on the right-hand side of equation (17.35) is the *contribution of the change in product i to the overall percentage change in the aggregate going from period 0 to 1.* Business analysts often want statistical agencies to provide decompositions like equation (17.35) so they can decompose the overall change in an aggregate into sector-specific components of change.[42] Thus, there is a demand on the part of users for additive quantity indices.

17.69 For the Walsh quantity index defined by equation (17.34), the ith weight is

$$w_{W_i} \equiv \frac{\sqrt{p_i^0 p_i^1}}{\sum_{m=1}^{n} q_m^0 \sqrt{p_m^0 p_m^1}}; \quad i = 1, \ldots, n. \tag{17.37}$$

Thus, the Walsh quantity index Q_W has a percentage decomposition into component changes of the form in equation (17.35) where the weights are defined by equation (17.37).

[40]This is the quantity index that corresponds to the price index 8 defined by Walsh (1921a, p. 101).

[41]Knibbs (1924) did not notice this point!

[42]Business and government analysts also often demand an analogous decomposition of the change in price aggregate into sector-specific components that add up.

17.70 It turns out that the Fisher quantity index Q_F defined by equation (16.14) in Chapter 16 also has an additive percentage change decomposition of the form given by equation (17.35).[43] The ith weight w_{Fi} for this Fisher decomposition is rather complicated and depends on the Fisher quantity index $Q_F(p^0, p^1, q^0, q^1)$ as follows:[44]

$$w_{F_i} \equiv \frac{w_i^0 + (Q_F)^2 w_t^1}{1 + Q_F}; \quad i = 1, \ldots, n, \qquad (17.38)$$

where Q_F is the value of the Fisher quantity index, $Q_F(p^0, p^1, q^0, q^1)$, and the period t normalized price for product i, w_i^t, is defined as the period i price p_i^t divided by the period t value on the aggregate:

$$w_i^t \equiv \frac{p_i^t}{\sum_{m=1}^{n} p_m^t q_m^t}; \; t = 0, 1; \; i = 1, \ldots, n. \qquad (17.39)$$

17.71 Using the weights w_{Fi} defined by equations (17.38) and (17.39), one obtains the following exact decomposition for the Fisher ideal quantity index:[45]

$$Q_F(p^0, p^1, q^0, q^1) - 1 = \sum_{i=1}^{n} w_{F_i}(q_i^1 - q_i^0). \qquad (17.40)$$

Thus, the Fisher quantity index has an additive percentage change decomposition.

17.72 Owing to the symmetric nature of the Fisher price and quantity indices, it can be seen that the Fisher price index P_F defined by equation (17.27) also has the following additive percentage change decomposition:

$$P_F(p^0, p^1, q^0, q^1) - 1 = \sum_{i=1}^{n} v_{F_i}(p_i^1 - p_i^0), \qquad (17.41)$$

where the product i weight v_{Fi} is defined as

$$v_{F_i} \equiv \frac{v_i^0 + (P_F)^2 v_i^1}{1 + P_F}; \quad i = 1, \ldots, n, \qquad (17.42)$$

where P_F is the value of the Fisher price index, $P_F(p^0, p^1, q^0, q^1)$, and the period t normalized quantity for product i, v_i^t, is defined as the period i quantity q_i^t divided by the period t value on the aggregate:

$$v_i^t \equiv \frac{q_i^t}{\sum_{m=1}^{n} p_m^t q_m^t}; \quad t = 0, 1; \quad i = 1, \ldots, n. \qquad (17.43)$$

17.73 The above results show that the Fisher price and quantity indices have exact additive decompositions into components that give the contribution to the overall change in the price (or quantity) index of the change in each price (or quantity).[46]

D. Stochastic Approach to Price Indices

D.1 Early unweighted stochastic approach

17.74 The stochastic approach to the determination of the price index can be traced back to the work of Jevons (1863, 1865) and Edgeworth (1888) more than a hundred years ago.[47] The basic idea behind the (unweighted) stochastic approach is that each price relative, p_i^1/p_i^0, for $i = 1, 2, \ldots, n$ can be regarded as an estimate of a common inflation rate α between periods 0 and 1;[48] that is, it is assumed that

$$\frac{p_i^1}{p_i^0} = \alpha + \varepsilon_i; \quad i = 1, 2, \ldots, n, \qquad (17.44)$$

where α is the common inflation rate and the ε_i are random variables with mean 0 and variance σ^2. The least squares or maximum likelihood estimator for α is the *Carli* (1804) *price index* P_C defined as

$$P_C(p^0, \mathrm{p}^1) \equiv \sum_{i=1}^{n} \frac{1}{n} \frac{p_i^1}{p_i^0}. \qquad (17.45)$$

A drawback of the Carli price index is that it does not satisfy the time reversal test, that is, $P_C(p^1, p^0) \neq 1/P_C(p^0, p^1)$.[49]

[43]The Fisher quantity index also has an additive decomposition of the type defined by equation (17.30) thanks to Van Ijzeren (1987, p. 6). The ith reference price p_i^* is defined as $p_i^* \equiv (1/2)p_i^0 + (1/2)p_i^1/P_F(p^0, p^1, q^0, q^1)$ for $i = 1, \ldots, n$ and where P_F is the Fisher price index. This decomposition was also independently derived by Dikhanov (1997). The Van Ijzeren decomposition for the Fisher quantity index is currently being used by the Bureau of Economic Analysis; see Moulton and Sullivan (1999, p. 16) and Ehemann, Katz, and Moulton (2002).

[44]This decomposition was obtained by Diewert (2002a) and Reinsdorf, Diewert, and Ehemann (2002). For an economic interpretation of this decomposition, see Diewert (2002a).

[45]To verify the exactness of the decomposition, substitute equation (17.38) into equation (17.40) and solve the resulting equation for Q_F. It is found that the solution is equal to Q_F defined by equation (16.14) in Chapter 16.

[46] An overview of additive and multiplicative decompositions of the Fisher indices was given by Balk (2004).

[47]For references to the literature, see Diewert (1993a, pp. 37–38; and 1995a; 1995b).

[48]"In drawing our averages the independent fluctuations will more or less destroy each other; the one required variation of gold will remain undiminished" (Jevons, 1863, p. 26).

[49]In fact, Irving Fisher (1922, p. 66) noted that $P_C(p^0, p^1)P_C(p^1, p^0) \geq 1$ unless the period 1 price vector p^1 is proportional to the period 0 price vector p^0. He urged statistical agencies not to use this formula. Walsh (1901, pp. 331 and 530) also discovered this result for the case $n = 2$.

17.75 Now change the stochastic specification and assume that the logarithm of each price relative, $\ln(p_i^1/p_i^0)$, is an unbiased estimate of the logarithm of the inflation rate between periods 0 and 1, β, say. The counterpart to equation (17.44) is

$$\ln\left(\frac{p_i^1}{p_i^0}\right) = \beta + \varepsilon_i; \quad i = 1, 2, \ldots, n, \qquad (17.46)$$

where $\beta \equiv \ln \alpha$ and the ε_i are independently distributed random variables with mean 0 and variance σ^2. The least squares or maximum likelihood estimator for β is the logarithm of the geometric mean of the price relatives. Hence, the corresponding estimate for the common inflation rate α[50] is the *Jevons price index* (1865) P_J defined as follows:

$$P_J(p^0, p^1) \equiv \prod_{i=1}^{n} \sqrt[n]{\frac{p_i^1}{p_i^0}}. \qquad (17.47)$$

17.76 The Jevons price index P_J does satisfy the time reversal test and thus is much more satisfactory than the Carli index P_C. However, both the Jevons and Carli price indices suffer from a fatal flaw: Each price relative p_i^1/p_i^0 is regarded as *being equally important* and is given an equal weight in the index number equations (17.45) and (17.47). Keynes was particularly critical of this *unweighted stochastic approach* to index number theory.[51] He directed the following criticism toward this approach, which was vigorously advocated by Edgeworth (1923):

> Nevertheless I venture to maintain that such ideas, which I have endeavoured to expound above as fairly and as plausibly as I can, are root-and-branch erroneous. The "errors of observation", the "faulty shots aimed at a single bull's eye" conception of the index number of prices, Edgeworth's "objective mean variation of general prices", is the result of confusion of thought. There is no bull's eye. There is no moving but unique centre, to be called the general price level or the objective mean variation of general prices, round which are scattered the moving price levels of individual things. There are all the various, quite definite, conceptions of price levels of composite commodities appropriate for various purposes and inquiries which have been scheduled above, and many others too. There is nothing else. Jevons was pursuing a mirage.
>
> What is the flaw in the argument? In the first place it assumed that the fluctuations of individual prices round the "mean" are "random" in the sense required by the theory of the combination of independent observations. In this theory the divergence of one "observation" from the true position is assumed to have no influence on the divergences of other "observations". But in the case of prices, a movement in the price of one product necessarily influences the movement in the prices of other commodities, whilst the magnitudes of these compensatory movements depend on the magnitude of the change in revenue on the first product as compared with the importance of the revenue on the commodities secondarily affected. Thus, instead of "independence", there is between the "errors" in the successive 'observations' what some writers on probability have called "connexity", or, as Lexis expressed it, there is "sub-normal dispersion".
>
> We cannot, therefore, proceed further until we have enunciated the appropriate law of connexity. But the law of connexity cannot be enunciated without reference to the relative importance of the commodities affected—which brings us back to the problem that we have been trying to avoid, of weighting the items of a composite commodity. (Keynes, 1930, pp. 76–77)

The main point Keynes seemed to be making in the quotation above is that prices in the economy are not independently distributed from each other and from quantities. In current macroeconomic terminology, Keynes can be interpreted as saying that a macroeconomic shock will be distributed across all prices and quantities in the economy through the normal interaction between supply and demand, that is, through the workings of the general equilibrium system. Thus, Keynes seemed to be leaning towards the economic approach to index number theory (even before it was developed to any great extent), where quantity movements are functionally related to price movements. A second point that Keynes made in the above quotation is that there is no such thing as *the* inflation rate; there are only price changes that pertain to well-specified sets of commodities or transactions; that is, the domain of definition of the price index must be carefully specified.[52] A final point that Keynes made is that price

[50]Greenlees (1999) pointed out that although $\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{p_i^1}{p_i^0}\right)$ is an unbiased estimator for β, the corresponding exponential of this estimator, P_J, defined by equation (17.47), will generally *not* be an unbiased estimator for α under our stochastic assumptions. To see this, let $x_i = \ln(p_i^1/p_i^0)$. Taking expectations, we have $\mathrm{E}x_i = \beta = \ln\alpha$. Define the positive, convex function f of one variable x by $f(x) \equiv e^x$. By Jensen's (1906) inequality, $\mathrm{E}f(x) \geq f(\mathrm{E}x)$. Letting x equal the random variable x_i, this inequality becomes $\mathrm{E}(p_i^1/p_i^0) = \mathrm{E}f(x_i) \geq f(\mathrm{E}x_i) = f(\beta) = e^{\beta} = e^{\ln\alpha} = \alpha$. Thus, for each n, $\mathrm{E}(p_i^1/p_i^0) \geq \alpha$, and it can be seen that the Jevons price index will generally have an upward bias under the usual stochastic assumptions.

[51]Walsh (1901, p. 83) also stressed the importance of proper weighting according to the economic importance of the commodities in the periods being compared: "But to assign uneven weighting with approximation to the relative sizes, either over a long series of years or for every period separately, would not require much additional trouble; and even a rough procedure of this sort would yield results far superior to those yielded by even weighting. It is especially absurd to refrain from using roughly reckoned uneven weighting on the ground that it is not accurate, and instead to use even weighting, which is much more inaccurate."

[52]See Section B in Chapter 16 for additional discussion on this point.

movements must be weighted by their economic importance, that is, by quantities or values.

17.77 In addition to the above theoretical criticisms, Keynes also made the following strong empirical attack on Edgeworth's unweighted stochastic approach:

> The Jevons–Edgeworth "objective mean variation of general prices", or 'indefinite" standard, has generally been identified, by those who were not as alive as Edgeworth himself was to the subtleties of the case, with the purchasing power of money—if only for the excellent reason that it was difficult to visualise it as anything else. And since any respectable index number, however weighted, which covered a fairly large number of commodities could, in accordance with the argument, be regarded as a fair approximation to the indefinite standard, it seemed natural to regard any such index as a fair approximation to the purchasing power of money also.
>
> Finally, the conclusion that all the standards "come to much the same thing in the end" has been reinforced "inductively" by the fact that rival index numbers (all of them, however, of the wholesale type) have shown a considerable measure of agreement with one another in spite of their different compositions. . . . On the contrary, the tables given above (pp. 53, 55) supply strong presumptive evidence that over long period as well as over short period the movements of the wholesale and of the consumption standards respectively are capable of being widely divergent. (Keynes, 1930, pp. 80–81)

In the quotation above, Keynes noted that the proponents of the unweighted stochastic approach to price change measurement were comforted by the fact that all of the then-existing (unweighted) indices of wholesale prices showed broadly similar movements. However, Keynes showed empirically that his wholesale price indices moved quite differently than did his consumer price indices.

17.78 In order to overcome these criticisms of the unweighted stochastic approach to index numbers, it is necessary to

- Have a definite domain of definition for the index number, and
- Weight the price relatives by their economic importance.[53]

17.79 In the following section, alternative methods of weighting will be discussed.

D.2 Weighted stochastic approach

17.80 Walsh (1901, pp. 88–89) seems to have been the first index number theorist to point out that a sensible stochastic approach to measuring price change means that individual price relatives should be weighted according to their economic importance or *their transactions' value* in the two periods under consideration:

> It might seem at first sight as if simply every price quotation were a single item, and since every commodity (any kind of commodity) has one price-quotation attached to it, it would seem as if price-variations of every kind of commodity were the single item in question. This is the way the question struck the first inquirers into price-variations, wherefore they used simple averaging with even weighting. But a price-quotation is the quotation of the price of a generic name for many articles; and one such generic name covers a few articles, and another covers many. . . . A single price-quotation, therefore, may be the quotation of the price of a hundred, a thousand, or a million dollar's worths, of the articles that make up the commodity named. Its weight in the averaging, therefore, ought to be according to these money-unit's worth. (Walsh, 1921a, pp. 82–83)

However, Walsh did not give a convincing argument on exactly how these economic weights should be determined.

17.81 Theil (1967, pp. 136–37) proposed a solution to the lack of weighting in the Jevons index, P_J, defined by equation (17.47). He argued as follows: Suppose we draw price relatives at random in such a way that each dollar of value in the base period has an equal chance of being selected. Then the probability that we will draw the ith price relative is equal to $s_i^0 \equiv p_i^0 q_i^0 / \sum_{k=1}^{n} p_k^0 q_k^0$, the period 0 value share for product i. Then the overall mean (period 0 weighted) logarithmic price change is $\sum_{i=1}^{n} s_i^0 \ln(p_i^1/p_i^0)$.[54] Now repeat the above mental experiment and draw price relatives at random in such a way that each dollar of value in period 1 has an equal probability of being selected. This leads to the overall mean (period 1 weighted) logarithmic price change of $\sum_{i=1}^{n} s_i^1 \ln(p_i^1/p_i^0)$.[55] Each of these measures of overall logarithmic price change seems equally valid, so we could argue for taking a symmetric average of the two

[53]Walsh (1901, pp. 82–90; 1921a, pp. 82–83) also objected to the lack of weighting in the unweighted stochastic approach to index number theory.

[54]In Chapter 19, this index is called the *geometric Laspeyres index*, P_{GL}. Vartia (1978, p. 272) referred to this index as the *logarithmic Laspeyres index*. Yet another name for the index is the *base weighted geometric index*.

[55]In Chapter 19, this index is called the *geometric Paasche index*, P_{GP}. Vartia (1978, p. 272) referred to this index as the *logarithmic Paasche index*. Yet another name for the index is the *current-period weighted geometric index*.

measures in order to obtain a final single measure of overall logarithmic price change. Theil[56] argued that a nice, symmetric index number formula can be obtained if the probability of selection for the nth price relative is made equal to the arithmetic average of the period 0 and 1 value shares for product n. Using these probabilities of selection, Theil's final measure of overall logarithmic price change was

$$\ln P_T(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{p_i^1}{P_i^0}\right). \quad (17.48)$$

Note that the index P_T defined by equation (17.48) is equal to the Törnqvist index defined by equation (16.81) in Chapter 16.

17.82 A statistical interpretation of the right-hand side of equation (17.48) can be given. Define the ith logarithmic price ratio r_i by

$$r_i \equiv \ln\left(\frac{p_i^1}{p_i^0}\right) \quad \text{for } i = 1, \ldots, n. \quad (17.49)$$

Now define the discrete random variable—we will call it R—as the random variable that can take on the values r_i with probabilities $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1, \ldots, n$. Note that because each set of value shares, s_i^0 and s_i^1, sums to one over i, the probabilities ρ_i will also sum to one. It can be seen that the expected value of the discrete random variable R is

$$\begin{aligned} \mathrm{E}[R] &\equiv \sum_{i=1}^{n} \rho_i r_i = \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{p_i^1}{p_i^0}\right) \\ &= \ln P_T(p^0, p^1, q^0, p^1). \quad (17.50) \end{aligned}$$

Thus, the logarithm of the index P_T can be interpreted as *the expected value of the distribution of the logarithmic price ratios* in the domain of definition under consideration, where the n discrete price ratios in this domain of definition are weighted according to Theil's probability weights, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1, \ldots, n$.

17.83 Taking antilogs of both sides of equation (17.48), one can obtain the Törnqvist (1936; Törnqvist and Törnqvist, 1937) Theil price index, P_T.[57] This index number formula has a number of good properties. In particular, P_T satisfies the proportionality in current prices test (T5) and the time reversal test (T11) discussed in Section C. These two tests can be used to justify Theil's (arithmetic) method of forming an average of the two sets of value shares in order to obtain his probability weights, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1, \ldots, n$. Consider the following *symmetric mean class of logarithmic index number formulas:*

$$\ln P_s(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} m\left(s_i^0, s_i^1\right) \ln\left(\frac{p_i^1}{p_i^0}\right), \quad (17.51)$$

where $m(s_i^0, s_i^1)$ is a positive function of the period 0 and 1 value shares on product i, s_i^0 and s_i^1 respectively. In order for P_S to satisfy the time reversal test, it is necessary for the function m to be symmetric. Then for P_S to satisfy test T5, m must be the arithmetic mean.[58] This provides a reasonably strong justification for Theil's choice of the mean function.

17.84 The stochastic approach of Theil has another advantageous symmetry property. Instead of considering the distribution of the price ratios $r_i = \ln(p_i^1/p_i^0)$, we could also consider the distribution of the *reciprocals* of these price ratios, say

$$t_i \equiv \ln\frac{p_i^0}{p_i^1} = \ln\left(\frac{p_i^1}{p_i^0}\right)^{-1} = -\ln\frac{p_i^1}{p_i^0} = -r_i$$
$$\text{for } i = 1, \ldots, n. \quad (17.52)$$

The symmetric probability, $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$, can still be associated with the ith reciprocal logarithmic price ratio t_i for $i = 1, \ldots, n$. Now define the discrete random variable, T, say, as the random variable that can take on the values t_i with probabilities $\rho_i \equiv (1/2)[s_i^0 + s_i^1]$ for $i = 1, \ldots, n$. It can be seen that the expected value of the discrete random variable T is

$$\begin{aligned} \mathrm{E}[T] &\equiv \sum_{i=1}^{n} \rho_i t_i \\ &= -\sum_{i=1}^{n} r_i t_i \quad \text{using equation (16.52)} \\ &= -\mathrm{E}[R] \quad \text{using equation (16.50)} \\ &= -\ln P_T(p^0, p^1, q^0, q^1). \quad (17.53) \end{aligned}$$

Thus, it can be seen that the distribution of the random variable T is equal to minus the distribution of the random variable R. Hence, it does not matter whether

[56] "The price index number defined in (1.8) and (1.9) uses the n individual logarithmic price differences as the basic ingredients. They are combined linearly by means of a two stage random selection procedure: First, we give each region the same chance ½ of being selected, and second, we give each dollar spent in the selected region the same chance $(1/m_a$ or $1/m_b)$ of being drawn" (Theil, 1967, p. 138).

[57] The sampling bias problem studied by Greenlees (1999) does not occur in the present context because there is no sampling involved in equation (17.50): the sum of the $p_i^t q_i^t$ over i for each period t is assumed to equal the value aggregate V^t for period t.

[58] This is shown in Diewert (2000) and Balk and Diewert (2001).

the distribution of the original logarithmic price ratios, $r_i \equiv \ln(p_i^1/p_i^0)$, is considered or the distribution of their reciprocals, $t_i \equiv \ln(p_i^1/p_i^0)$, is considered; essentially the same stochastic theory is obtained.

17.85 It is possible to consider weighted stochastic approaches to index number theory where the distribution of the price ratios, p_i^1/p_i^0, is considered rather than the distribution of the logarithmic price ratios, $\ln(p_i^1/p_i^0)$. Thus, again following in the footsteps of Theil, suppose that price relatives are drawn at random in such a way that each dollar of value in the *base period* has an equal chance of being selected. Then the probability that the *i*th price relative will be drawn is equal to s_i^0, the period 0 value share for product *i*. Thus, the overall mean (period 0 weighted) price change is

$$P_L(p^0, p^1, q^0, q^1) = \sum_{i=1}^{n} s_i^0 \frac{p_i^1}{p_i^0}, \tag{17.54}$$

which turns out to be the Laspeyres price index, P_L. This stochastic approach is the natural one for studying *sampling problems* associated with implementing a Laspeyres price index.

17.86 Take the same hypothetical situation and draw price relatives at random in such a way that each dollar of value in period 1 has an equal probability of being selected. This leads to the overall mean (period 1 weighted) price change equal to

$$P_{Pal}(p^0, p^1, q^0, q^1) = \sum_{i=1}^{n} s_i^1 \frac{p_i^1}{p_i^0}. \tag{17.55}$$

This is known as the Palgrave (1886) index number formula.[59]

17.87 It can be verified that neither the Laspeyres nor the Palgrave price indices satisfy the time reversal test, T11. Thus, again following in the footsteps of Theil, one might attempt to obtain a formula that satisfied the time reversal test by taking a symmetric average of the two sets of shares. Thus, consider the following class of *symmetric mean index number formulas:*

$$P_m(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} m\left(s_i^0, s_i^1\right) \frac{p_i^1}{p_i^0}, \tag{17.56}$$

where $m(s_i^0, s_i^1)$ is a symmetric function of the period 0 and 1 value shares for product *i*, s_i^0 and s_i^1, respectively. In order to interpret the right-hand side of equation (17.56) as an expected value of the price ratios p_i^1/p_i^0, it is necessary that

$$\sum_{i=1}^{n} m\left(s_i^0, s_i^1\right) = 1. \tag{17.57}$$

However, in order to satisfy equation (17.57), *m* must be the arithmetic mean.[60] With this choice of *m*, equation (17.56) becomes the following (unnamed) index number formula, P_u:

$$P_u(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \frac{p_i^1}{p_i^0}. \tag{17.58}$$

Unfortunately, the unnamed index P_u does not satisfy the time reversal test either.[61]

17.88 Instead of considering the distribution of the price ratios, p_i^1/p_i^0, one can consider the distribution of the *reciprocals* of these price ratios. The counterparts to the asymmetric indices defined earlier by equations (17.54) and (17.55) are now $\sum_{i=1}^{n} s_i^0\left(p_i^0/p_i^1\right)$ and $\sum_{i=1}^{n} s_i^1\left(p_i^0/p_i^1\right)$, respectively. These are (stochastic) price indices going *backward* from period 1 to 0. In order to make these indices comparable with other previous forward-looking indices, take the reciprocals of these indices (which lead to harmonic averages) and the following two indices are obtained:

$$P_{HL}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} s_i^0 \frac{p_i^0}{p_i^1}}, \tag{17.59}$$

$$P_{HP}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} s_i^1 \frac{p_i^0}{p_i^1}}, = \frac{1}{\sum_{i=1}^{n} s_i^1 \left(\frac{p_i^1}{p_i^0}\right)^{-1}}$$
$$= P_P(p^0, p^1, q^0, q^1), \tag{17.60}$$

using equation (16.9) in Chapter 16. Thus, the reciprocal stochastic price index defined by equation (17.60) turns out to equal the fixed-basket Paasche price index, P_P. This stochastic approach is the natural one for studying *sampling problems* associated with implementing a Paasche price index. The other asymmetrically weighted reciprocal stochastic price index defined by equation (17.59) has no author's name associated with it, but it was noted by Irving Fisher (1922, p. 467) as his

[59]It is formula number 9 in Irving Fisher's (1922, p. 466) listing of index number formulas.

[60]For a proof of this assertion, see Balk and Diewert (2001).

[61]In fact, this index exhibits the same property as the Carli index in that $P_u(p^0, p^1, q^0, q^1)P_u(p^1, p^0, q^1, q^0) \geq 1$. To prove this, note that the previous inequality is equivalent to $[P_u(p^1, p^0, q^1, q^0)]^{-1} \leq P_u(p^0, p^1, q^0, q^1)$ and this inequality follows from the fact that a weighted harmonic mean of *n* positive numbers is equal to or less than the corresponding weighted arithmetic mean; see Hardy, Littlewood, and Pólya (1934, p. 26).

index number formula 13. Vartia (1978, p. 272) called this index the *harmonic Laspeyres index* and his terminology will be used.

17.89 Now consider the class of symmetrically weighted reciprocal price indices defined as

$$P_{mr}(p^0, p^1, q^0, q^1) \equiv \frac{1}{\sum_{i=1}^{n} m\left(s_i^0, s_i^1\right)\left(\frac{p_i^1}{p_i^0}\right)^{-1}}, \qquad (17.61)$$

where, as usual, $m(s_i^0, s_i^1)$ is a homogeneous symmetric mean of the period 0 and 1 value shares on product i. However, none of the indices defined by equations (17.59) through (17.61) satisfy the time reversal test.

17.90 The fact that Theil's index number formula P_T satisfies the time reversal test leads to a preference for Theil's index as the best weighted stochastic approach.

17.91 The main features of the weighted stochastic approach to index number theory can be summarized as follows. It is first necessary to pick two periods and a transaction's domain of definition. As usual, each value transaction for each of the n commodities in the domain of definition is split up into price and quantity components. Then, assuming there are no new commodities or no disappearing commodities, there are n price relatives p_i^1/p_i^0 pertaining to the two situations under consideration along with the corresponding $2n$ value shares. The weighted stochastic approach just assumes that these n relative prices, or some transformation of these price relatives, $f(p_i^1/p_i^0)$, have a discrete statistical distribution, where the ith probability, $\rho_i = m(s_i^0, s_i^1)$, is a function of the value shares pertaining to product i in the two situations under consideration, s_i^0 and s_i^1. Different price indices result, depending on how one chooses the functions f and m. In Theil's approach, the transformation function f was the natural logarithm, and the mean function m was the simple unweighted arithmetic mean.

17.92 There is a third aspect to the weighted stochastic approach to index number theory: One must decide what *single number* best summarizes the distribution of the n (possibly transformed) price relatives. In the analysis above, the *mean* of the discrete distribution was chosen as the best summary measure for the distribution of the (possibly transformed) price relatives, but other measures are possible. In particular, the *weighted median* or various *trimmed means* are often suggested as the best measure of central tendency because these measures minimize the influence of outliers. However, a detailed discussion of these alternative measures of central tendency is beyond the scope of this chapter. Additional material on stochastic approaches to index number theory and references to the literature can be found in Clements and Izan (1981 and 1987), Selvanathan and Rao (1994), Diewert (1995b), Cecchetti (1997), and Wynne (1997, and 1999).

17.93 Instead of taking the above stochastic approach to index number theory, it is possible to take the same raw data that are used in this approach but use them with an axiomatic approach. Thus, in the following section, the price index is regarded as a value-weighted function of the n price relatives, and the test approach to index number theory is used in order to determine the functional form for the price index. Put another way, the axiomatic approach in the next section looks at the *properties* of alternative descriptive statistics that aggregate the individual price relatives (weighted by their economic importance) into summary measures of price change in an attempt to find the best summary measure of price change. Thus, the axiomatic approach pursued in Section E below can be viewed as a branch of the theory of descriptive statistics.

E. Second Axiomatic Approach to Bilateral Price Indices

E.1 Basic framework and some preliminary tests

17.94 As was mentioned in Section D.2, one of Walsh's approaches to index number theory was an attempt to determine the best weighted average of the price relatives, r_i.[62] This is equivalent to using an axiomatic approach to try and determine the best index

[62]Irving Fisher also took this point of view when describing his approach to index number theory: "An index number of the prices of a number of commodities is an average of their price relatives. This definition has, for concreteness, been expressed in terms of prices. But in like manner, an index number can be calculated for wages, for quantities of goods imported or exported, and, in fact, for any subject matter involving divergent changes of a group of magnitudes. Again, this definition has been expressed in terms of time. But an index number can be applied with equal propriety to comparisons between two places or, in fact, to comparisons between the magnitudes of a group of elements under any one set of circumstances and their magnitudes under another set of circumstances" (1922, p. 3). However, in setting up his axiomatic approach, Fisher imposed axioms on the price and quantity indices written as functions of the two price vectors, p^0 and p^1, and the two quantity vectors, q^0 and q^1; that is, he did not write his price index in the form $P(r, v^0, v^1)$ and impose axioms on indices of this type. Of course, in the end, his ideal price index turned out to be the geometric mean of the Laspeyres and Paasche price indices and as was seen in Chapter 16, each of these indices can be written as value share-weighted averages of the n price relatives, $r_i \equiv p_i^1/p_i^0$.

of the form $P(r, v^0, v^1)$, where v^0 and v^1 are the vectors of values on the n commodities during periods 0 and 1.[63] However, rather than starting off with indices of the form $P(r, v^0, v^1)$, indices of the form $P(p^0, p^1, v^0, v^1)$ will be considered, because this framework will be more comparable to the first bilateral axiomatic framework taken in Section C. If the invariance to changes in the units of measurement test is imposed on an index of the form $P(p^0, p^1, v^0, v^1)$, then $P(p^0, p^1, v^0, v^1)$ can be written in the form $P(r, v^0, v^1)$.

17.95 Recall that the product test, equation (17.17), was used in order to define the quantity index, $Q(p^0, p^1, q^0, q^1) \equiv V^1/[V^0 P(p^0, p^1, q^0, q^1)]$, that corresponded to the bilateral price index $P(p^0, p^1, q^0, q^1)$. A similar product test holds in the present framework; that is, given that the functional form for the price index $P(p^0, p^1, v^0, v^1)$ has been determined, then the corresponding *implicit quantity index* can be defined in terms of p as follows:

$$Q(p^0, p^1, v^0, v^1) = \frac{\sum_{i=1}^{n} v_i^1}{\left(\sum_{i=1}^{n} v_i^0\right) P(p^0, p^1, v^0, v^1)}. \quad (17.62)$$

17.96 In Section C, the price and quantity indices $P(p^0, p^1, q^0, q^1)$ and $Q(p^0, p^1, q^0, q^1)$ were determined *jointly*; that is, not only were axioms imposed on $P(p^0, p^1, q^0, q^1)$, but they were also imposed on $Q(p^0, p^1, q^0, q^1)$ and the product test in equation (17.17) was used to translate these tests on q into tests on P. In Section E, this approach will not be followed: Only tests on $P(p^0, p^1, v^0, v^1)$ will be used in order to determine the best price index of this form. Thus, there is a parallel theory for quantity indices of the form $Q(q^0, q^1, v^0, v^1)$ where it is attempted to find the best value-weighted average of the quantity relatives, q_i^1/q_i^0.[64]

17.97 For the most part, the tests that will be imposed on the price index $P(p^0, p^1, v^0, v^1)$ in this section are counterparts to the tests that were imposed on the price index $P(p^0, p^1, q^0, q^1)$ in Section C. It will be assumed that every component of each price and value vector is positive; that is, $p^t >> 0_n$ and $v^t >> 0_n$ for $t = 0, 1$. If it is desired to set $v^0 = v^1$, the common value vector is denoted by v; if it is desired to set $p^0 = p^1$, the common price vector is denoted by p.

17.98 The first two tests are straightforward counterparts to the corresponding tests in Section C.

T1—*Positivity:* $P(p^0, p^1, v^0, v^1) > 0$.

T2—*Continuity:* $P(p^0, p^1, v^0, v^1)$ is a continuous function of its arguments.

T3—*Identity or Constant Prices Test:*
$P(p, p, v^0, v^1) = 1$.

That is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the value vectors are. Note that the two value vectors are allowed to be different in the above test.

E.2 Homogeneity tests

17.99 The following four tests restrict the behavior of the price index p as the scale of any one of the four vectors p^0, p^1, v^0, v^1 changes.

T4—*Proportionality in Current Prices:*
$P(p^0, \lambda p^1, v^0, v^1) = \lambda P(p^0, p^1, v^0, v^1)$ for $\lambda > 0$.

That is, if all period 1 prices are multiplied by the positive number λ, then the new price index is λ times the old price index. Put another way, the price index function $P(p^0, p^1, v^0, v^1)$ is (positively) homogeneous of degree one in the components of the period 1 price vector p^1. This test is the counterpart to test T5 in Section C.

17.100 In the next test, instead of multiplying all period 1 prices by the same number, all period 0 prices are multiplied by the number λ.

T5—*Inverse Proportionality in Base Period Prices:*
$P(\lambda p^0, p^1, v^0, v^1) = \lambda^{-1} P(p^0, p^1, v^0, v^1)$ for $\lambda > 0$.

That is, if all period 0 prices are multiplied by the positive number λ, then the new price index is $1/\lambda$ times the old price index. Put another way, the price index function $P(p^0, p^1, v^0, v^1)$ is (positively) homogeneous of degree minus one in the components of the period 0 price vector p^0. This test is the counterpart to test T6 in Section C.

17.101 The following two homogeneity tests can also be regarded as invariance tests.

[63]Chapter 3 in Vartia (1976a) considered a variant of this axiomatic approach.

[64]It turns out that the price index that corresponds to this best quantity index, defined as $P^*(p^0, p^1, v^0, v^1) \equiv \sum_{i=1}^{n} \ln v_i^1 / \left[\sum_{i=1}^{n} \ln v_i^0 Q(q^0, q^1, v^0, v^1)\right]$, will not equal the best price index, $P(p^0, p^1, v^0, v^1)$. Thus, the axiomatic approach in Section E generates separate best price and quantity indices whose product does not equal the value ratio in general. This is a disadvantage of the second axiomatic approach to bilateral indices compared to the first approach studied in Section C.

T6—*Invariance to Proportional Changes in Current Period Values:*
$P(p^0, p^1, v^0, \lambda v^1) = P(p^0, p^1, v^0, v^1)$ for all $\lambda > 0$.

That is, if current-period values are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0, p^1, v^0, v^1)$ is (positively) homogeneous of degree zero in the components of the period 1 value vector v^1.

T7—*Invariance to Proportional Changes in Base-Period Values:*
$P(p^0, p^1, \lambda v^0, v^1) = P(p^0, p^1, v^0, v^1)$ for all $\lambda > 0$.

That is, if base-period values are all multiplied by the number λ, then the price index remains unchanged. Put another way, the price index function $P(p^0, p^1, v^0, v^1)$ is (positively) homogeneous of degree zero in the components of the period 0 value vector v^0.

17.102 T6 and T7 together impose the property that the price index p does not depend on the *absolute* magnitudes of the value vectors v^0 and v^1. Using test T6 with $\lambda = 1/\sum_{i=1}^n v_i^1$ and using test T7 with $\lambda = 1/\sum_{i=1}^n v_i^0$, it can be seen that p has the following property:

$$P(p^0, p^1, v^0, v^1) = P(p^0, p^1, s^0, s^1), \qquad (17.63)$$

where s^0 and s^1 are the vectors of value shares for periods 0 and 1; that is, the ith component of s^t is $s_i^t \equiv v_i^t/\sum_{i=1}^n v_k^t$ for $t = 0, 1$. Thus, tests T6 and T7 imply that the price index function p is a function of the two price vectors p^0 and p^1 and the two vectors of value shares, s^0 and s^1.

17.103 Walsh suggested the spirit of tests T6 and T7 in the following quotation:

> What we are seeking is to average the variations in the exchange value of one given total sum of money in relation to the several classes of goods, to which several variations [i.e., the price relatives] must be assigned weights proportional to the relative sizes of the classes. Hence the relative sizes of the classes at both the periods must be considered. (Walsh, 1901, p. 104)

17.104 Walsh also realized that weighting the ith price relative r_i by the arithmetic mean of the value weights in the two periods under consideration, $(1/2)[v_i^0 + v_i^1]$, would give too much weight to the values of the period that had the highest level of prices:

> At first sight it might be thought sufficient to add up the weights of every class at the two periods and to divide by two. This would give the (arithmetic) mean size of every class over the two periods together. But such an operation is manifestly wrong. In the first place, the sizes of the classes at each period are reckoned in the money of the period, and if it happens that the exchange value of money has fallen, or prices in general have risen, greater influence upon the result would be given to the weighting of the second period; or if prices in general have fallen, greater influence would be given to the weighting of the first period. Or in a comparison between two countries, greater influence would be given to the weighting of the country with the higher level of prices. But it is plain that *the one period, or the one country, is as important, in our comparison between them, as the other, and the weighting in the averaging of their weights should really be even.* (Walsh, 1901, pp. 104–05)

17.105 As a solution to the above weighting problem, Walsh (1901, p. 202; 1921a, p. 97) proposed the following *geometric price index:*

$$P_{GW}(p^0, p^1, v^0, v^1) \equiv \prod_{i=1}^n \left(\frac{p_i^1}{p_i^0}\right)^{w(i)}, \qquad (17.64)$$

where the ith weight in the above formula was defined as

$$w(i) \equiv \frac{\left(v_i^0 v_i^1\right)^{1/2}}{\sum_{k=1}^n \left(v_k^0 v_k^1\right)^{1/2}} = \frac{\left(s_i^0 s_i^1\right)^{1/2}}{\sum_{k=1}^n \left(s_k^0 s_k^1\right)^{1/2}}, \quad i = 1, \ldots, n. \qquad (17.65)$$

The second part of equation (17.65) shows that Walsh's geometric price index $P_{GW}(p^0, p^1, v^0, v^1)$ can also be written as a function of the value share vectors, s^0 and s^1; that is, $P_{GW}(p^0, p^1, v^0, v^1)$ is homogeneous of degree 0 in the components of the value vectors v^0 and v^1 and so $P_{GW}(p^0, p^1, v^0, v^1) = P_{GW}(p^0, p^1, s^0, s^1)$. Thus, Walsh came very close to deriving the Törnqvist Theil index defined earlier by equation (17.48).[65]

E.3 Invariance and symmetry tests

17.106 The next five tests are *invariance* or *symmetry tests*, and four of them are direct counterparts to similar tests in Section C. The first invariance test is that the price index should remain unchanged if the *ordering* of the commodities is changed.

[65]One could derive Walsh's index using the same arguments as Theil except that the geometric average of the value shares $(s_i^0 s_i^1)^{1/2}$ could be taken as a preliminary probability weight for the ith logarithmic price relative, $\ln r_i$. These preliminary weights are then normalized to add up to unity by dividing by their sum. It is evident that Walsh's geometric price index will closely approximate Theil's index using normal time series data. More formally, regarding both indices as functions of p^0, p^1, v^0, v^1, it can be shown that $P_W(p^0, p^1, v^0, v^1)$ approximates $P_T(p^0, p^1, v^0, v^1)$ to the second order around an equal price (that is, $p^0 = p^1$) and quantity (that is, $q^0 = q^1$) point.

T8—*Commodity Reversal Test* (or invariance to changes in the ordering of commodities):
$P(p^{0*}, p^{1*}, v^{0*}, v^{1*}) = P(p^0, p^1, v^0, v^1)$

where p^{t*} denotes a permutation of the components of the vector p^t and v^{t*} denotes the same permutation of the components of v^t for $t = 0, 1$.

17.107 The next test asks that the index be invariant to changes in the units of measurement.

T9—*Invariance to Changes in the Units of Measurement* (commensurability test):

$$P\left(\alpha_1 p_1^0, \ldots, \alpha_n p_n^0; \alpha_1 p_1^1, \ldots, \alpha_n p_n^1; v_1^0, \ldots, v_n^0; v_1^1, \ldots, v_n^1\right) = P\left(p_1^0, \ldots, p_n^0; p_1^1, \ldots, p_n^1; v_1^0, \ldots, v_n^0; v_1^1, \ldots, v_n^1\right)$$

for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

That is, the price index does not change if the units of measurement for each product are changed. Note that the value on product i during period t, v_i^t, does not change if the unit by which product i is measured changes.

17.108 Test T9 has a very important implication. Let $\alpha_1 = 1/p_1{}^0, \ldots, \alpha_n = 1/p_n{}^0$ and substitute these values for the α_i into the definition of the test. The following equation is obtained:

$$P(p^0, p^1, v^0, v^1) = P(1_n, r, v^0, v^1) \equiv P^*(r, v^0, v^1), \tag{17.66}$$

where 1_n is a vector of ones of dimension n and r is a vector of the price relatives, that is, the ith component of r is $r_i \equiv p_i{}^1/p_i{}^0$. Thus, if the commensurability test T9 is satisfied, then the price index $P(p^0, p^1, v^0, v^1)$, which is a function of $4n$ variables, can be written as a function of $3n$ variables, $P^*(r, v^0, v^1)$, where r is the vector of price relatives and $P^*(r, v^0, v^1)$ is defined as $P(1_n, r, v^0, v^1)$.

17.109 The next test asks that the formula be invariant to the period chosen as the base period.

T10—*Time Reversal Test:*
$P(p^0, p^1, v^0, v^1) = 1/P(p^1, p^0, v^1, v^0)$.

That is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index. Obviously, in the one good case when the price index is simply the single price ratio, this test will be satisfied (as are all of the other tests listed in this section).

17.110 The next test is a variant of the circularity test that was introduced in Section F of Chapter 16.[66]

T11—*Transitivity in Prices for Fixed Value Weights:*
$P(p^0, p^1, v^r, v^s)P(p^1, p^2, v^r, v^s) = P(p^0, p^2, v^r, v^s)$.

In this test, the value-weighting vectors, v^r and v^s, are held constant while making all price comparisons. However, given that these weights are held constant, then the test asks that the product of the index going from period 0 to 1, $P(p^0, p^1, v^r, v^s)$, times the index going from period 1 to 2, $P(p^1, p^2, v^r, v^s)$, should equal the direct index that compares the prices of period 2 with those of period 0, $P(p^0, p^2, v^r, v^s)$. Clearly, this test is a many-product counterpart to a property that holds for a single price relative.

17.111 The final test in this section captures the idea that the value weights should enter the index number formula in a symmetric manner.

T12—*Quantity Weights Symmetry Test:* $P(p^0, p^1, v^0, v^1) = P(p^0, p^1, v^1, v^0)$.

That is, if the value vectors for the two periods are interchanged, then the price index remains invariant. This property means that if values are used to weight the prices in the index number formula, then the period 0 values v^0 and the period 1 values v^1 must enter the formula in a symmetric or evenhanded manner.

E.4 Mean value test

17.112 The next test is a *mean value test.*

T13—*Mean Value Test for Prices:*

$$\min_i\left(p_i^1/p_i^0 : i = 1, \ldots, n\right) \le P(p^0, p^1, v^0, v^1) \le \max_i\left(p_i^1/p_i^0 : i = 1, \ldots, n\right). \tag{17.67}$$

That is, the price index lies between the minimum price ratio and the maximum price ratio. Because the price index is to be interpreted as an average of the n price ratios, $p_i{}^1/p_i{}^0$, it seems essential that the price index P satisfy this test.

E.5 Monotonicity tests

17.113 The next two tests in this section are *monotonicity tests*; that is, how should the price index $P(p^0, p^1, v^0, v^1)$ change as any component of the two price vectors p^0 and p^1 increases?

[66]See equation (16.77) in Chapter 16.

T14—*Monotonicity in Current Prices:*
$P(p^0, p^1, v^0, v^1) < P(p^0, p^2, v^0, v^1)$ if $p^1 < p^2$.

That is, if some period 1 price increases, then the price index must increase (holding the value vectors fixed), so that $P(p^0, p^1, v^0, v^1)$ is increasing in the components of p^1 for fixed p^0, v^0, and v^1.

T15—*Monotonicity in Base Prices:*
$P(p^0, p^1, v^0, v^1) > P(p^2, p^1, v^0, v^1)$ if $p^0 < p^2$.

That is, if any period 0 price increases, then the price index must decrease, so that $P(p^0, p^1, v^0, v^1)$ is decreasing in the components of p^0 for fixed p^1, v^0, and v^1.

E.6 Weighting tests

17.114 The preceding tests are not sufficient to determine the functional form of the price index; for example, it can be shown that both Walsh's geometric price index $P_{GW}(p^0, p^1, v^0, v^1)$ defined by equation (17.65) and the Törnqvist Theil index $P_T(p^0, p^1, v^0, v^1)$ defined by equation (17.48) satisfy all of the above axioms. At least one more test, therefore, will be required in order to determine the functional form for the price index $P(p^0, p^1, v^0, v^1)$.

17.115 The tests proposed thus far do not specify exactly how the value share vectors s^0 and s^1 are to be used in order to weight, for example, the first price relative, $p_1{}^1/p_1{}^0$. The next test says that only the value shares $s_1{}^0$ and $s_1{}^1$ pertaining to the first product are to be used in order to weight the prices that correspond to product 1, $p_1{}^1$ and $p_1{}^0$.

T16—*Own Share Price Weighting:*

$$P\left(p_1^0, 1, \ldots, 1; p_1^1, 1, \ldots, 1; v^0, v^1\right) = f\left(p_1^0, p_1^1, \left[v_1^0 \Big/ \sum_{k=1}^n v_k^0\right], \left[v_1^1 \Big/ \sum_{k=1}^n v_k^1\right]\right). \quad (17.68)$$

Note that, $v_1^t / \sum_{k=1}^n v_k^t$ equals $s_1{}^t$, the value share for product 1 in period t. This test says that if all of the prices are set equal to 1 except the prices for product 1 in the two periods, but the values in the two periods are arbitrarily given, then the index depends only on the two prices for product 1 and the two value shares for product 1. The axiom says that a function of $2 + 2n$ variables is actually only a function of four variables.[67]

[67]In the economics literature, axioms of this type are known as separability axioms.

17.116 If test T16 is combined with test T8, the commodity reversal test, then it can be seen that P has the following property:

$$P\left(1, \ldots, 1, p_i^0, 1, \ldots, 1; 1, \ldots, p_i^1, 1, \ldots, 1; v^0; v^1\right) = f\left(p_1^0, p_1^1, \left[v_1^0 \Big/ \sum_{k=1}^n v_k^0\right], \left[v_1^1 \Big/ \sum_{k=1}^n v_k^1\right]\right), \quad i = 1, \ldots, n. \quad (17.69)$$

Equation (17.69) says that if all of the prices are set equal to 1 except the prices for product i in the two periods, but the values in the two periods are arbitrarily given, then the index depends only on the two prices for product i and the two value shares for product i.

17.117 The final test that also involves the weighting of prices is the following:

T17—*Irrelevance of Price Change with Tiny Value Weights:*

$$P\left(p_1^0, 1, \ldots, 1; p_1^1, 1, \ldots, 1; 0, v_2^0, \ldots, v_n^0; 0, v_2^1, \ldots, v_n^1\right) = 1. \quad (17.70)$$

The test T17 says that if all of the prices are set equal to 1 except the prices for product 1 in the two periods, and the values on product 1 are zero in the two periods but the values on the other commodities are arbitrarily given, then the index is equal to 1.[68] Roughly speaking, if the value weights for product 1 are tiny, then it does not matter what the price of product 1 is during the two periods.

17.118 Of course, if test T17 is combined with test T8, the product reversal test, then it can be seen that P has the following property: for $i = 1, \ldots, n$:

$$P\left(1, \ldots, 1, p_1^0, 1, \ldots, 1; 1, \ldots, 1, p_1^1, 1, \ldots, 1; v_1^0, \ldots, 0, \ldots, v_n^0; v_1^1, \ldots, 0, \ldots, v_n^1\right) = 1. \quad (17.71)$$

Equation (17.71) says that if all of the prices are set equal to 1 except the prices for product i in the two periods, and the values on product i are 0 during the two periods but the other values in the two periods are arbitrarily given, then the index is equal to 1.

17.119 This completes the listing of tests for the weighted average of price relatives approach to bilateral index number theory. It turns out that these tests are sufficient to imply a specific functional form for the price index as will be seen in the next section.

[68]Strictly speaking, because all prices and values are required to be positive, the left-hand side of equation (17.70) should be replaced by the limit as the product 1 values, $v_1{}^0$ and $v_1{}^1$, approach 0.

E.7 Törnqvist Theil price index and second test approach to bilateral indices

17.120 In Appendix 17.1, it is shown that if the number of commodities n exceeds two and the bilateral price index function $P(p^0, p^1, v^0, v^1)$ satisfies the 17 axioms listed above, then P must be the Törnqvist Theil price index $P_T(p^0, p^1, v^0, v^1)$ defined by equation (17.48).[69] Thus, the 17 properties or tests listed in Section E provide an axiomatic characterization of the Törnqvist Theil price index, just as the 20 tests listed in Section C provided an axiomatic characterization of the Fisher ideal price index.

17.121 There is a parallel axiomatic theory for quantity indices of the form $Q(q^0, q^1, v^0, v^1)$ that depend on the two quantity vectors for periods 0 and 1, q^0 and q^1, as well as on the corresponding two value vectors, v^0 and v^1. Thus, if $Q(q^0, q^1, v^0, v^1)$ satisfies the quantity counterparts to tests T1–T17, then q must be equal to the Törnqvist Theil quantity index $Q_T(q^0, q^1, v^0, v^1)$, defined as follows:

$$\ln Q_T(q^0, q^1, v^0, v^1) \equiv \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{q_i^1}{q_i^0}\right), \quad (17.72)$$

where, as usual, the period t value share on product i, s_i^t, is defined as $v_1^t / \sum_{k=1}^{n} v_k^t$ for $i = 1, \ldots, n$ and $t = 0, 1$.

17.122 Unfortunately, the implicit Törnqvist Theil price index $P_{IT}(q^0, q^1, v^0, v^1)$, which corresponds to the Törnqvist Theil quantity index Q_T defined by equation (17.72) using the product test, is *not* equal to the direct Törnqvist Theil price index $P_T(p^0, p^1, v^0, v^1)$ defined by equation (17.48). The product test equation that defines P_{IT} in the present context is given by the following equation:

$$P_{IT}(q^0, q^1, v^0, v^1) \equiv \frac{\sum_{i=1}^{n} v_i^1}{\left(\sum_{i=1}^{n} v_i^0\right) Q_T(q^0, q^1, v^0, v^1)}. \quad (17.73)$$

The fact that the direct Törnqvist Theil price index P_T is not in general equal to the implicit Törnqvist Theil price index P_{IT} defined by equation (17.73) is a bit of a disadvantage compared to the axiomatic approach outlined in Section C, which led to the Fisher ideal price and quantity indices as being "best." Using the Fisher approach meant that it was not necessary to decide whether one wanted a best price index or a best quantity index: The theory outlined in Section C determined both indices simultaneously. However, in the Törnqvist Theil approach outlined in this section, it is necessary to *choose* whether one wants a best price index or a best quantity index.[70]

17.123 Other tests are, of course, possible. A counterpart to test T16 in Section C, the Paasche and Laspeyres bounding test, is the following *geometric Paasche and Laspeyres bounding test:*

$$\begin{aligned} &P_{GL}(p^0, p^1, v^0, v^1) \le P(p^0, p^1, v^0, v^1) \\ &\quad \le P_{GP}(p^0, p^1, v^0, v^1) \quad \text{or} \\ &P_{GP}(p^0, p^1, v^0, v^1) \le P(p^0, p^1, v^0, v^1) \\ &\quad \le P_{GL}(p^0, p^1, v^0, v^1), \end{aligned} \quad (17.74)$$

where the logarithms of the geometric Laspeyres and geometric Paasche price indices, P_{GL} and P_{GP}, are defined as follows:

$$\ln P_{GL}(p^0, p^1, v^0, v^1) \equiv \sum_{i=1}^{n} s_i^0 \ln\left(\frac{p_i^1}{p_i^0}\right), \quad (17.75)$$

$$\ln P_{GP}(p^0, p^1, v^0, v^1) \equiv \sum_{i=1}^{n} s_i^1 \ln\left(\frac{p_i^1}{p_i^0}\right). \quad (17.76)$$

As usual, the period t value share on product i, s_i^t, is defined as $v_1^t / \sum_{k=1}^{n} v_k^t$ for $i = 1, \ldots, n$ and $t = 0, 1$. It can be shown that the Törnqvist Theil price index $P_T(p^0, p^1, v^0, v^1)$ defined by equation (17.48) satisfies this test, but the geometric Walsh price index $P_{GW}(p^0, p^1, v^0, v^1)$ defined by equation (17.65) does not satisfy it. The geometric Paasche and Laspeyres bounding test was not included as a primary test in Section E because, a priori, it was not known what form of averaging of the price relatives (e.g., geometric or arithmetic or harmonic) would turn out to be appropriate in this test framework. The test equation (17.74) is an appropriate one if it has been decided that geometric averaging of the price relatives is the appropriate framework. The geometric Paasche and Laspeyres indices correspond to extreme forms of value weighting in the context of geometric averaging, and it is natural to require that the best price index lie between these extreme indices.

[69]The Törnqvist Theil price index satisfies all 17 tests, but the proof in Appendix 17.1 did not use all of these tests to establish the result in the opposite direction: tests T5, T13, T15, and either T10 or T12 were not required in order to show that an index satisfying the remaining tests must be the Törnqvist Theil price index. For alternative characterizations of the Törnqvist Theil price index, see Balk and Diewert (2001) and Hillinger (2002).

[70]Hillinger (2002) suggested taking the geometric mean of the direct and implicit Törnqvist Theil price indices in order to resolve this conflict. Unfortunately, the resulting index is not best for either set of axioms that were suggested in this section.

17.124 Walsh (1901, p. 408) pointed out a problem with his geometric price index P_{GW} defined by equation (17.65), which also applies to the Törnqvist Theil price index $P_T(p^0, p^1, v^0, v^1)$ defined by equation (17.48): These geometric type indices do not give the right answer when the quantity vectors are constant (or proportional) over the two periods. In this case, Walsh thought that the right answer must be the Lowe index, which is the ratio of the costs of purchasing the constant basket during the two periods. Put another way, the geometric indices P_{GW} and P_T do not satisfy T4, the fixed-basket test, in Section C above. What then was the argument that led Walsh to define his geometric average type index P_{GW}? It turns out that he was led to this type of index by considering another test, which will now be explained.

17.125 Walsh (1901, pp. 228–31) derived his test by considering the following simple framework. Let there be only two commodities in the index and suppose that the value share on each product is equal in each of the two periods under consideration. The price index under these conditions is equal to $P(p_1^0, p_2^0; p_1^1, p_2^1; v_1^0, v_2^0; v_1^1, v_2^1 = P^*(r_1, r_2; 1/2, 1/2; 1/2, 1/2) \equiv m(r_1, r_2)$ where $m(r_1, r_2)$ is a symmetric mean of the two price relatives, $r_1 \equiv p_1^1/p_1^0$ and $r_2 \equiv p_1^1/p_1^0$.[71] In this framework, Walsh then proposed the following *price relative reciprocal test:*

$$m\left(r_1, r_1^{-1}\right) = 1. \tag{17.77}$$

Thus, if the value weighting for the two commodities is equal over the two periods and the second price relative is the reciprocal of the first price relative I_1, then, as Walsh (1901, p. 230) argued, the overall price index under these circumstances ought to equal one, because the relative fall in one price is exactly counterbalanced by a rise in the other, and both commodities have the same values in each period. He found that the geometric mean satisfied this test perfectly, but the arithmetic mean led to index values greater than one (provided that r_1 was not equal to one) and the harmonic mean led to index values that were less than one, a situation that was not at all satisfactory.[72] Thus, he was led to some form of geometric averaging of the price relatives in one of his approaches to index number theory.

17.126 A generalization of Walsh's result is easy to obtain. Suppose that the mean function, $m(r_1, r_2)$, satisfies Walsh's reciprocal test, equation (17.77), and in addition, m is a homogeneous mean, so that it satisfies the following property for all $r_1 > 0$, $r_2 > 0$ and $\lambda > 0$:

$$m(\lambda r_1, \lambda r_2) = \lambda m(r_1, r_2). \tag{17.78}$$

Let $r_1 > 0$, $r_2 > 0$. Then

$$\begin{aligned} m(r_1, r_2) &= \left(\frac{r_1}{r_1}\right) m(r_1, r_2) \\ &= r_1 m\left(\frac{r_1}{r_1}, \frac{r_2}{r_1}\right), \text{ using equation (16.78) with } \lambda = \frac{1}{r_1} \\ &= r_1 m\left(1, \frac{r_2}{r_1}\right) = r_1 f\left(\frac{r_2}{r_1}\right), \end{aligned} \tag{17.79}$$

where the function of one (positive) variable $f(z)$ is defined as

$$f(z) \equiv m(1, z). \tag{17.80}$$

Using equation (17.77):

$$\begin{aligned} 1 &= m\left(r_1, r_1^{-1}\right) \\ &= \left(\frac{r_1}{r_1}\right) m\left(r_1, r_1^{-1}\right) \\ &= r_1 m\left(1, r_1^{-2}\right), \text{ using equation (16.78)} \\ &\quad \text{with } \lambda = \frac{1}{r_1}. \end{aligned} \tag{17.81}$$

Using equation (17.80), equation (17.81) can be rearranged in the following form:

$$f\left(r_1^{-2}\right) = r_1^{-1}. \tag{17.82}$$

Letting $z \equiv r_1^{-2}$ so that $z^{1/2} = r_1^{-1}$, equation (17.82) becomes

$$f(z) = z^{1/2}. \tag{17.83}$$

Now substitute equation (17.83) into equation (17.79) and the functional form for the mean function $m(r_1, r_2)$ is determined:

$$m(r_1, r_2) = r_1 f\left(\frac{r_2}{r_1}\right) = r_1 \left(\frac{r_2}{r_1}\right)^{1/2} = r_1^{1/2} r_2^{1/2}. \tag{17.84}$$

Thus, the geometric mean of the two price relatives is the only homogeneous mean that will satisfy Walsh's price relative reciprocal test.

17.127 One additional test should be mentioned. Irving Fisher (1911) introduced this test in his first book that dealt with the test approach to index number

[71] Walsh considered only the cases where m was the arithmetic, geometric, and harmonic means of r_1 and r_2.

[72] "This tendency of the arithmetic and harmonic solutions to run into the ground or to fly into the air by their excessive demands is clear indication of their falsity" (Walsh, 1901, p. 231).

theory. He called it the *test of determinateness as to prices* and described it as follows:

> A price index should not be rendered zero, infinity, or indeterminate by an individual price becoming zero. Thus, if any product should in 1910 be a glut on the market, becoming a "free good", that fact ought not to render the index number for 1910 zero. (I. Fisher, 1911, p. 401)

In the present context, this test could be interpreted to mean the following one: If any single price p_i^0 or p_i^1 tends to zero, then the price index $P(p^0, p, v^0, v^1)$ should not tend to zero or plus infinity. However, with this interpretation of the test, which regards the values v_i^t as remaining constant as the p_i^0 or p_i^1 tends to zero, none of the commonly used index number formulas would satisfy this test. As a result, this test should be interpreted as a test that applies to price indices $P(p^0, p^1, q^0, q^1)$ of the type that were studied in Section C above, which is how Fisher intended the test to apply. Thus, Fisher's price determinateness test should be interpreted as follows: If any single price p_i^0 or p_i^1 tends to zero, then the price index $P(p^0, p, q^0, q^1)$ should not tend to zero or plus infinity. With this interpretation of the test, it can be verified that Laspeyres, Paasche, and Fisher indices satisfy this test, but the Törnqvist Theil price index will not satisfy this test. Thus when using the Törnqvist Theil price index, *one must take care to bound the prices away from zero in order to avoid a meaningless index number value.*

17.128 Walsh was aware that geometric average type indices like the Törnqvist Theil price index P_T or Walsh's geometric price index P_{GW} defined by equation (17.64) become somewhat unstable[73] as individual price relatives become very large or small:

> Hence in practice the geometric average is not likely to depart much from the truth. Still, we have seen that when the classes [i.e., values] are very unequal and the price variations are very great, this average may deflect considerably. (Walsh, 1901, p. 373)

> In the cases of moderate inequality in the sizes of the classes and of excessive variation in one of the prices, there seems to be a tendency on the part of the geometric method to deviate by itself, becoming untrustworthy, while the other two methods keep fairly close together. (Walsh, 1901, p. 404)

17.129 Weighing all of the arguments and tests presented in Sections C and E of this chapter, one might detect a slight preference for the use of the Fisher ideal price index as a suitable target index for a statistical agency, but opinions can differ on which set of axioms is the most appropriate to use in practice.

[73]That is, the index may approach zero or plus infinity.

F. Test Properties of Young and Lowe Indices

17.130 In Chapter 16, the Young and Lowe indices were defined. In the present section, the axiomatic properties of these indices with respect to their price arguments will be developed.[74]

17.131 Let $q^b \equiv \left[q_1^b, \ldots, q_n^b\right]$ and $p^b \equiv \left[p_1^b, \ldots, p_n^b\right]$ denote the quantity and price vectors pertaining to some base year. The corresponding *base-year value shares* can be defined in the usual way as

$$s_i^b \equiv \frac{p_i^b q_i^b}{\sum_{k=1}^{n} p_k^b q_k^b}, \quad i = 1, \ldots, n. \tag{17.85}$$

Let $s^b \equiv \left[s_1^b, \ldots, s_n^b\right]$ denote the vector of base-year value shares. The Young (1812) price index between periods 0 and t is defined as follows:

$$P_Y(p^0, p^t, s^b) \equiv \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0}\right). \tag{17.86}$$

The Lowe (1823, p. 316) price index[75] between periods 0 and t is defined as follows:

$$P_{Lo}(p^0, p^t, q^b) \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{k=1}^{n} p_k^0 q_k^b} = \frac{\sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^b}\right)}{\sum_{k=1}^{n} s_k^b \left(\frac{p_k^0}{p_k^b}\right)}. \tag{17.87}$$

17.132 Drawing on those that have been listed in Sections C and E, we highlight 12 desirable axioms for price indices of the form $P(p^0, p^1)$. The period 0 and t price vectors, p^0 and p^t, are presumed to have strictly positive components.

[74]Baldwin (1990, p. 255) worked out a few of the axiomatic properties of the Lowe index.

[75]This index number formula is also precisely Bean and Stine's (1924, p. 31) Type A index number formula. Walsh (1901, p. 539) initially mistakenly attributed Lowe's formula to G. Poulett Scrope, who wrote *Principles of Political Economy* in 1833 and suggested Lowe's formula without acknowledging Lowe's priority. But in his discussion of Irving Fisher's (1921) paper, Walsh corrected his mistake on assigning Lowe's formula: "What index number should you then use? It should be this: $\sum q\, p_1 / \sum q\, p_0$. This is the method used by Lowe within a year or two of one hundred years ago. In my [1901] book, I called it Scrope's index number; but it should be called Lowe's. Note that in it are used quantities neither of a base year nor of a subsequent year. The quantities used should be rough estimates of what the quantities were throughout the period or epoch" (1921b, pp. 543–44).

T1—*Positivity Test:* $P(p^0, p^t) > 0$ if all prices are positive.

T2—*Continuity Test:* $P(p^0, p^t)$ is a continuous function of prices.

T3—*Identity Test:* $P(p^0, p^0) = 1$.

T4—*Homogeneity Test for Period t Prices:* $P(p^0, \lambda p^t) = \lambda P(p^0, p^t)$ for all $\lambda > 0$.

T5—*Homogeneity Test for Period 0 Prices:* $P(\lambda p^0, p^t) = \lambda^{-1} P(p^0, p^t)$ for all $\lambda > 0$.

T6—*Commodity Reversal Test:* $P(p^t, p^0) = P(p^{0*}, p^{t*})$ where p^{0*} and p^{t*} denote the same permutation of the components of the price vectors p^0 and p^t.[76]

T7—*Invariance to Changes in the Units of Measurement or the Commensurability Test.*

$$P\left(\alpha_1 p_1^0, \ldots, \alpha_n p_n^0; \alpha_1 p_1^t, \ldots, \alpha_n p_n^t\right) = P\left(p_1^0, \ldots, p_n^0; p_1^t, \ldots, p_n^t\right)$$

for all $\alpha_1 > 0, \ldots, \alpha_n > 0$.

T8—*Time Reversal Test:* $P(p^t, p^0) = 1/P(p^0, p^t)$.

T9—*Circularity or Transitivity Test:* $P(p^0, p^2) = P(p^0, p^1)P(p^1, p^2)$.

T10—*Mean Value Test:* $\min\left\{p_i^t/p_i^0 : i = 1, \ldots, n\right\} \le P(p^t, p^0) \le \max\left\{p_i^t/p_i^0 : i = 1, \ldots, n\right\}$.

T11—*Monotonicity Test with Respect to Period t Prices:* $P(p^0, p^t) < P(p^0, p^{t*})$ if $p^t < p^{t*}$.

T12—*Monotonicity Test with Respect to Period 0 Prices:* $P(p^0, p^t) > P(p^{0*}, p^t)$ if $p^0 < p^{0*}$.

17.133 It is straightforward to show that the Lowe index defined by equation (17.87) satisfies all 12 of the axioms or tests listed above. Hence, the Lowe index has very good axiomatic properties with respect to its price variables.[77]

17.134 It is straightforward to show that the Young index defined by equation (17.86) satisfies 10 of the 12 axioms, failing T8, the time reversal test, and T9, the circularity test. Thus, the axiomatic properties of the Young index are definitely inferior to those of the Lowe index.

[76]In applying this test to the Lowe and Young indices, it is assumed that the base-year quantity vector q^b and the base-year share vector s^b are subject to the same permutation.

[77]From the discussion in Chapter 16, it will be recalled that the main problem with the Lowe index occurs if the quantity weight vector q^b is not representative of the quantities that were purchased during the time interval between periods 0 and 1.

Appendix 17.1 Proof of Optimality of Törnqvist Theil Price Index in Second Bilateral Test Approach

17.135 Define $r_i \equiv p_i^1/p_i^0$ for $i = 1, \ldots, n$. Using T1, T9, and equation (17.66), $P(p^0, p^1, v^0, v^1) = P^*(r, v^0, v^1)$. Using T6, T7, and equation (17.63):

$$P(p^0, p^1, v^0, v^1) = P^*(r, s^0, s^1), \qquad \text{(A17.1)}$$

where s^t is the period t value share vector for $t = 0, 1$.

17.136 Let $x \equiv (x_1, \ldots, x_n)$ and $y \equiv (y_1, \ldots, y_n)$ be strictly positive vectors. The transitivity test T11 and equation (A17.1) imply that the function P^* has the following property:

$$P^*(x; s^0, s^1)P^*(y; s^0, s^1) = P^*(x_1 y_1, \ldots, x_n y_n; s^0, s^1). \qquad \text{(A17.2)}$$

17.137 Using T1, $P^*(r, s^0, s^1) > 0$ and using T14, $P^*(r, s^0, s^1)$ is strictly increasing in the components of r. The identity test T3 implies that

$$P^*(1_n, s^0, s^1) = 1, \qquad \text{(A17.3)}$$

where 1_n is a vector of ones of dimension n. Using a result due to Eichhorn (1978, p. 66), it can be seen that these properties of P^* are sufficient to imply that there exist positive functions $\alpha_i(s^0, s^1)$ for $i = 1, \ldots, n$ such that P^* has the following representation:

$$\ln P^*(r, s^0, s^1) = \sum_{i=1}^{n} \alpha_i(s^0, s^1) \ln r_i. \qquad \text{(A17.4)}$$

17.138 The continuity test T2 implies that the positive functions $\alpha_i(s^0, s^1)$ are continuous. For $\lambda > 0$, the linear homogeneity test T4 implies that

$$\begin{aligned} &\ln P^*(\lambda r, s^0, s^1) \\ &= \sum_{i=1}^{n} \alpha_i(s^0, s^1) \ln \lambda r_i, \qquad \text{using equation (A17.4)} \\ &= \sum_{i=1}^{n} \alpha_i(s^0, s^1) \ln \lambda + \sum_{i=1}^{n} \alpha_i(s^0, s^1) \ln r_i \\ &= \sum_{i=1}^{n} \alpha_i(s^0, s^1) \ln \lambda + \ln P^*(r, s^0, s^1), \\ &\qquad \text{using equation (A17.4)} \end{aligned} \qquad \text{(A17.5)}$$

17.139 Equating the right-hand sides of the first and last lines in (A17.5) shows that the functions $\alpha_i(s^0, s^1)$ must satisfy the following restriction:

$$\sum_{i=1}^{n} \alpha_i(s^0, s^1) = 1, \qquad \text{(A17.6)}$$

for all strictly positive vectors s^0 and s^1.

17.140 Using the weighting test T16 and the commodity reversal test T8, equation (17.69) holds. Equation (17.69) combined with the commensurability test T9 implies that P^* satisfies the following equation:

$$P^*(1, \ldots, 1, r_i, 1, \ldots, 1; s^0, s^1) = f(1, r_i, s^0, s^1); \quad i = 1, \ldots, n, \qquad \text{(A17.7)}$$

for all $r_i > 0$ where f is the function defined in test T16.

17.141 Substitute equation (A17.7) into equation (A17.4) in order to obtain the following system of equations:

$$P^*(1, \ldots, 1, r_i, 1, \ldots, 1; s^0, s^1) = f(1, r_i, s^0, s^1) = \alpha_i(s^0, s^1) \ln r_i ; \quad i = 1, \ldots, n. \qquad \text{(A17.8)}$$

But the first part of equation (A17.8) implies that the positive continuous function of $2n$ variables $\alpha_i(s^0, s^1)$ is constant with respect to all of its arguments except s_i^0 and s_i^1, and this property holds for each i. Thus, each $\alpha_i(s^0, s^1)$ can be replaced by the positive continuous function of two variables $\beta_i(s_i^0, s_i^1)$ for $i = 1, \ldots, n$.[78] Now replace the $\alpha_i(s^0, s^1)$ in equation (A17.4) by the $\beta_i(s_i^0, s_i^1)$ for $i = 1, \ldots, n$ and the following representation for P^* is obtained:

$$\ln P^*(r, s^0, s^1) = \sum_{i=1}^{n} \beta_i(s_i^0, s_i^1) \ln r_i. \qquad \text{(A17.9)}$$

[78]More explicitly, $\beta_1(s_1^0, s_1^1) \equiv \alpha_1(s_1^0, 1, \ldots, 1; s_1^1, 1, \ldots, 1$ and so on. That is, in defining $\beta_1(s_1^0, s_1^1)$, the function $\alpha_1(s_1^0, 1, \ldots, 1; s_1^1, 1, \ldots, 1)$ is used where all components of the vectors s^0 and s^1 except the first are set equal to an arbitrary positive number like 1.

17.142 Equation (A17.6) implies that the functions $\beta_i(s_i^0, s_i^1)$ also satisfy the following restrictions:

$$\sum_{i=1}^{n} s_i^0 = 1; \sum_{i=1}^{n} s_i^1 = 1 \text{ implies } \sum_{i=1}^{n} \beta_i\left(s_i^0, s_i^1\right) = 1. \qquad \text{(A17.10)}$$

Assume that the weighting test T17 holds and substitute equation (17.71) into (A17.9) in order to obtain the following equation:

$$\beta_i(0, 0) \ln\left(\frac{p_i^1}{p_i^0}\right) = 0; \quad i = 1, \ldots, n. \qquad \text{(A17.11)}$$

Because the p_i^1 and p_i^0 can be arbitrary positive numbers, it can be seen that equation (A17.11) implies

$$\beta_i(0, 0) = 0; \quad i = 1, \ldots, n. \qquad \text{(A17.12)}$$

17.143 Assume that the number of commodities n is equal to or greater than 3. Using equations (A17.10) and (A17.12), Theorem 2 in Aczél (1987, p. 8) can be applied and the following functional form for each of the $\beta_i(s_i^0, s_i^1)$ is obtained:

$$\beta_i\left(s_i^0, s_i^1\right) = \gamma s_i^0 + (1 - \gamma) s_i^1; \quad i = 1, \ldots, n, \qquad \text{(A17.13)}$$

where γ is a positive number satisfying $0 < \gamma < 1$.

17.144 Finally, the time reversal test T10 *or* the quantity weights symmetry test T12 can be used to show that γ must equal ½. Substituting this value for γ back into equation (A17.13) and then substituting those equations back into equation (A17.9), the functional form for P^* and hence p is determined as

$$\ln P(p^0, p^1, v^0, v^1) = \ln P^*(r, s^0, s^1) = \sum_{i=1}^{n} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{p_i^1}{p_i^0}\right). \qquad \text{(A17.14)}$$

18. Economic Approach

A. Introduction

18.1 The economic approach differs from the fixed-basket, axiomatic, and stochastic approaches outlined in Chapters 16 and 17 in an important respect: Quantities are no longer assumed to be independent of prices. Consider a price index for the output produced by establishments. If, for example, it is assumed that the establishments behave as (export) revenue maximizers, it follows that they would switch production to commodities with higher relative price changes. This behavioral assumption about the firm allows something to be said about what a "true" index number formula should be and the suitability of different index number formulas as approximations to it. For example, the Laspeyres price index uses fixed-period (export) revenue shares to weight its price relatives and ignores the substitution of production toward products with higher relative price changes. The Laspeyres price index will thus understate aggregate price changes—be biased downward against its true index. The Paasche price index uses fixed current-period weights and will thus overstate aggregate price changes—be biased upward against its true index.

18.2 Alternatively, exporters may try to anticipate demand changes by producing less of products with above-average price changes. A Laspeyres price index will thus overstate aggregate price changes—be biased upward against its true index and a Paasche price index will understate aggregate price changes—be biased downward against its true index.

18.3 Consider a price index for the (imported) intermediate inputs to establishments. If, for example, it is assumed that firms behave as cost minimizers, it follows that they would purchase more of products with below-average price changes and again the behavioral assumption would have implications for the nature of substitution bias in Laspeyres and Paasche price index number formulas. The economic approach can be very powerful, because it has identified a type of bias in Laspeyres and Paasche indices not apparent from other approaches: *substitution bias.*

18.4 The approach from the economic theory of production is thus first to develop theoretical index number formulas based on what are considered to be reasonable models of economic behavior on the part of the producer. A mathematical representation of the production activity—whereby capital and labor conjoin to turn intermediate inputs into outputs—is required. An assumption of optimizing behavior (cost minimization or revenue maximization) is also required. A theoretical index is then derived that is "true" for both the form of the representation of the production activity and behavioral assumption. The economic approach then examines practical index number formulas such as Laspeyres, Paasche, Fisher, and Törnqvist, and considers how they compare with the "true" formulas defined under different assumptions. Important findings are that (1) Laspeyres and Paasche price indices act as bounds on their true indices and, under certain conditions, also are bounds for more generally applicable theoretical true indices that adequately incorporate *substitution effects;* (2) such generally applicable theoretical indices fall between the Laspeyres and Paasche price indices arguing for an index that is a symmetric mean of the two—the Fisher price index formula is the only symmetric average of Laspeyres and Paasche that satisfies the *time reversal test;* (3) index number formulas correspond exactly to specific functional forms of the mathematical representation of the revenue/cost functions—for example, the Törnqvist index is exact for a revenue function represented by a translogarithmic functional form; and (4) a class of *superlative* index number formulas exist that are exact for flexible functional forms giving strong support to their use, because flexible functional forms incorporate substitution behavior. The Fisher, Törnqvist, and Walsh index formulas are all superlative.

18.5 Section B sets the stage for the analysis. First, two approaches are distinguished, each serving different analytical purposes, the resident's and nonresident's approaches. The *resident's approach* identifies exports as outputs from domestic economic producers—the behavioral assumption would be one of revenue maximization and the Laspeyres price index would be expected to be biased downward against its true index and the Paasche price index biased upward against its true index. The *nonresident's approach* identifies exports from the domestic economy as imports to the rest of the world and the perspective taken is the importer's whose behavioral assumption is cost minimization: The biases in the Laspeyres and Paasche price index number formulas would be reversed. The resident's approach to *imports* as inputs to the domestic economic may take cost minimization as its behavioral assumption—Laspeyres would be expected to be biased upward against its true index and the Paasche index biased downward, with the position reversed from the nonresident's perspective.

18.6 The use of a symmetric means of Laspeyres and Paasche price index formulas would accord with both the resident's and nonresident's perspectives because their mean is unaffected by the direction of the bounds. Thus for deliberations of the nature of the bias in Laspeyres and Paasche price indices it is necessary to consider the behavioral assumption of the economic agents which in turn require consideration of the perspective from which exports and imports are regarded.

B. Economic Theory and the Resident's and Nonresident's Approach

18.7 This chapter considers two perspectives on export and import price indices (XMPIs):

- *Nonresident perspective:* Exports of an economic territory are viewed from the nonresident establishment or household user's perspective as an input, and imports of an economic territory are viewed from the nonresident producer's or supplier's perspective as an output—see Dridi and Zieschang (2004) for details; and
- *Resident perspective:* Exports of an economic territory are viewed from the resident producer's or supplier's perspective as an output, and imports of an economic territory are viewed from the resident establishment or household user's perspective as an input.

18.8 The *2008 System of National Accounts* (*2008 SNA*) adopts a "non-residents' perspective" to the treatment of exports and imports in external account of goods and services. Exports and imports are treated as "uses" and "resources/supply" respectively. Exports are the nonresidents' use of goods and services produced by residents, and imports are the nonresidents' supply of goods and services to the residents of an economic territory. Thus the appropriate economic theory underlying imports and exports in the *2008 SNA*'s external account of goods and services should be based on this nonresident perspective and, as such, would carry over to be the appropriate economic theory for the price indices used to deflate these aggregates.[1] The behavioral assumptions for the economic theory of such price indices would be of cost-minimizing nonresident economic agents, including establishments, households, and government; purchasing exports; and revenue-maximizing nonresident economic agents supplying imports.

18.9 However, a "resident's perspective" would have the exports of an economic territory viewed from the resident producer's or supplier's perspective as an output, and imports of an economic territory viewed from the resident establishment or household user's perspective as an input. The resident's perspective would be appropriate for import and export price and volume series used for the analysis of (the resident country's) productivity change, changes in the terms of trade, and transmission of inflation. The counterpart aggregates to such price and volume measures would be for imports as uses to the residents, and exports as supply. The *2008 SNA*'s production account includes intermediate consumption from the resident producer's perspective and a component of this is served by imports. The *2008 SNA*'s production account also includes output as a supply, some of which is a supply to domestic markets and some to nondomestic markets—that is, exports—again from the resident producer's perspective. Thus the appropriate economic theory underlying imports and exports for the analysis of (the resident country's) productivity change, changes in the terms of trade, and transmission of inflation would be based on this resident's perspective and, as such, would carry over to be the appropriate economic theory for the price indices used to deflate these aggregates. The behavioral assumptions for the economic theory of such price indices would be of cost-minimizing resident economic agents purchasing imports and revenue-maximizing resident economic agents supplying exports.

[1]However, see Chapters 4 and 15 on supply and use tables in volume terms and the corresponding valuation requirements for price indices.

Table 18.1. Behavioral Assumptions for Resident's and Nonresident's Approaches

	Exports	Imports
Resident's approach	Revenue maximizer	Cost minimizer
Nonresident's approach	Cost minimizer	Revenue maximizer

18.10 The perspective taken dictates the behavioral assumptions applied. Section D1 of this chapter demonstrates how the behavior assumptions in turn dictate the direction of the substitution bias in terms of the relationship between Laspeyres and its theoretical "true" counterpart and the Paasche price index in relation to its theoretical "true" counterpart.

18.11 The analysis equates cost-minimizing behavior with purchasers substituting away from commodities with above-average price increases and revenue-maximizing behavior with producers substituting output toward commodities with above-average price increases. However, these general patterns need not hold for all commodities. It may well be that, for example, some resident exporters produce more of commodities with relatively low or falling price changes owing to changes in preferences and technological change that allow producers to both cut prices and increase demand. The strength of the economic analysis is that it identifies a type of bias and demonstrates how answers to questions as to its nature and extent depend on assumptions as to the behavior of exporting and importing economic agents.

18.12 Section C sets the stage for the economic analysis by defining the economic agents involved and some assumptions implicit in the analysis. *The analysis proceeds from the resident's perspective.* This is for two reasons: First, the treatment from the nonresident's perspective is well documented in Dridi and Zieschang (2004). Second, the distinguishing feature of the two approaches for the purpose of economic theory is the behavioral assumptions. There are essentially two sets of theory—those from cost-minimizing behavioral assumptions and those from revenue-maximizing assumptions. As is apparent from Table 18.1, the findings for exports from the resident's approach apply to those of imports from the nonresident's approach, and findings for imports from the resident's approach apply to those of exports from the nonresident's approach. There is simply no need to replicate the outline of the theory from one perspective given that it has been undertaken from the other. As is demonstrated in Sections D1 and F2 for exports and imports respectively, the nature of these assumptions affects the results for the direction of the bounds on the theoretical "true" indices. However, the behavioral assumptions, and thus distinction between resident's and nonresident's perspectives, do not affect the validity of superlative indices; as averages of these bounds it does not matter which direction they take.

18.13 Sections D2 and E2 respectively demonstrate how Fisher and Törnqvist price indices can be justified as appropriate export price index number formulas using economic theory. Section E outlines the justification for superlative export price index number formulas and Section F considers the economic theory of import price index number formulas.

C. Setting the Stage

C.1 The production accounts of the *2008 SNA*

18.14 In the remainder of this chapter, the resident approach is pursued, though as noted above in Table 18.1, there is an immediately apparent correspondence to the results from this perspective and those from the nonresident's perspective; details are available in Dridi and Zieschang (2004). Establishments undertake the basics of international goods and services trade but, as Chapter 15 notes, households may undertake trade for final consumption in the form of cross-border shopping and of rentals of housing units, and general government units also undertake international procurements and asset sales.[2] The economic approach to the XMPIs thus begins not at the industry or institutional sector level, but at the *establishment* and *household* level. Readers of the *Producer Price Index Manual* (ILO and others, 2004b) will note in its Chapter 17 a parallel approach to the economic index

[2]The establishments, of course, may be owned or controlled by units in any institutional sector: nonfinancial and financial corporations, general government, households, and nonprofit institutions serving households (NPISHs). By definition, establishments, including those owned by general government, households, and NPISHs, combine nonfinancial assets and intermediate consumption to produce output, and they can engage in capital formation, but do not make final consumption expenditures. The "use of income accounts" of these noncorporation institutional units thus includes not only the capital formation expenditure these noncorporations have made, but also final consumption expenditure. This chapter distinguishes between the international trade noncorporation institutional units undertake for their own intermediate consumption and capital formation, and the international trade they undertake for final consumption.

number theory of input and output price indices for the part of trade flows establishments undertake. Similarly, readers of the *Consumer Price Index Manual* (ILO and others, 2004a) will note in its Chapter 17 a parallelism with the economic theory of price indices for consumption for the part of trade flows households undertake. As outlined in Chapter 15, in order to provide a coherent theoretical framework for implementing the resident approach, it turns out that the main production accounts in the *2008 SNA* require some elaboration.

18.15 There are a number of reasons why the main production accounts in the *2008 SNA* require some modifications so that the modified accounts can provide a theoretical framework for XMPIs. The main reason is that exports and imports enter the main supply and use tables (Table 15.1) as additions (or subtractions) to total net supply or to total domestic final demand in the familiar $C + I + G + X - M$ setup, where C and G are household and government final consumption expenditures, I is gross capital formation, X is exports, and M is imports. This means that Table 15.1 in the main production accounts of the *2008 SNA* does not elaborate on which industries are actually using the imports or on which industries are actually doing the exporting by commodity.[3] Hence, for *XMPI Manual* purposes, the main additions to the *2008 SNA*, Chapter 15, are tables for the main production accounts that provide industry by commodity detail on exports and imports. With these additional tables on the industry by commodity allocation of exports and imports, the resident's approach to collecting XMPIs can be embedded in the Commission of the European Communities and others (2008) *2008 SNA* framework.

18.16 A second main reason for expanding the existing SNA production accounts is that the present set of accounts does not allow the XMPIs to be related to the producer price index (PPI) for gross outputs and the PPI for intermediate imports by industry. Thus for the purposes of this chapter a modification of the SNA production accounts is considered so that export price indices by industry become subindices of the gross output PPI for that industry and import price indices by industry become subindices of the intermediate input PPI for that industry and thus the augmented accounts provide an integrated approach to all of the price indices that affect producers.

[3]It should be noted that *2008 SNA* does have a recommended optional Table 14.19, which is suited to our present needs; that is, this table provides the detail for imports by industry by way of a use table in basic prices that separates imports from domestic production. However, *2008 SNA* does not provide a recommendation for a corresponding commodity by industry table for exports.

18.17 The *2008 SNA* distinguishes between market and nonmarket goods and services. As noted in Chapter 15, market goods and services are transacted at "economically significant prices," largely covering their cost of production, whereas nonmarket goods and services are transacted at lower prices, including zero. Nonmarket goods and services are defined to include both "output produced for own final use" (*SNA* transaction code P.12) and "other non-market output" (*SNA* transaction code P.13). The former is output retained by households for own consumption or by establishments for capital formation, and is not traded internationally and thus of no concern to XMPIs. Other nonmarket goods and services include those produced by government or nonprofit institutions serving households that are supplied free or at a price of no economic significance. These, for the large part, will be aimed at residents, though government, for example, may have as its output some goods and services that benefit nonresidents and these will be exports. Because the share of nonmarket output in exports and imports is generally small, our theory focuses on market output. For any relevant nonmarket output the *SNA* values production by imputing the prices of similarly dated market transactions in comparable goods and services.

18.18 As foreshadowed at the beginning of this section, the XMPIs cover both household and establishment units in both their production and consumption activities. XMPIs thus comprise subindices of the output, intermediate consumption, final consumption, and capital formation price indices of units resident in the economic territory. Our theory of international trade price indices thus is a theory of international trade subindices of the consumer price index and PPI, as well as the other price indices for the supply and use of goods and services mentioned in Chapter 15.

C.2 The price data

18.19 There are two major problems with making the definition of an establishment operational. The first is that many production units at specific geographic locations do not have the capability of providing basic accounting information on inputs used and outputs produced. These production units may be only a division or single plant of a large firm, and detailed accounting information on prices may be available only at the head office (or not at all). If this is the case, the definition of an establishment is modified to include production units at a number of specific geographic locations in the country instead of just one location. The important aspect of the definition of an establishment is that it be able to provide accounting information on prices and quantities.[4] A second problem is that while the establishment may be able to report accurate quantity information, its price information may be based on *transfer prices* set by a head office. These transfer prices are *imputed prices* and may not be very closely related to market prices.[5] Potentially large shares of international trade occur between related enterprises resident in different countries at such transfer prices. This problem is deferred until Chapter 19, which addresses the issue squarely.

18.20 Thus the problems involved in obtaining the correct commodity prices for establishments are generally more difficult than the corresponding problems associated with obtaining market prices for households. However, in this chapter, these problems are ignored for the most part, and it is assumed that representative market prices are available for each output produced by an establishment and for each intermediate input used by the same establishment for at least two accounting periods.[6] Price indices for the supply aggregates of goods and services (output price indices) follow valuation at basic prices, which is what the producer would receive for output excluding taxes on products and including subsidies on products.

18.21 For price indices of the aggregates for the establishment and household users of goods and services (input price indices), the economic approach to price indices requires that input prices follow valuation at purchasers' prices, adding taxes to, and subtracting subsidies on, products from the basic prices producers receive. The indirect taxes are included because users pay them, even though the producing establishments may collect them for government. The subsidies on products are excluded because the cost of goods and services purchased by establishments and households is lowered by these payments. Chapter 15 and Section B in this chapter consider in more detail these national accounting and microeconomic conventions on the treatment of indirect taxes and subsidies on production.

18.22 In this chapter, an *export price index* and an *import price index* are defined for a *single establishment* or *household* from the economic perspective of a producer in Sections D and F. Household import and export price indices are defined in Section F.2.

18.23 Note it is assumed that the list of commodities produced by the establishment and the list of inputs used by the establishment *remain the same* over the two periods of a price comparison. In real life, the list of commodities used and produced by an establishment does not remain constant over time. New commodities appear and old commodities disappear. The reasons for this churning of commodities include the following:

(1) Producers substitute new technologies for older ones that may reduce the prices of exiting varieties, but may also enable some new varieties to be (technologically and/or economically) feasible and some old ones to be no longer so. Such "technologies" may involve new capital formation, a change in the way production is organized, and/or a change in the primary and intermediate inputs used to generate the outputs. The introduction of new technologies may be in response to changes in relative prices, households' tastes, or strategic marketing.

(2) Existing processes are sufficiently flexible to produce newly differentiated varieties in addition to, or as a replacement for, existing varieties. The introduction of new varieties may be in response to changes in relative prices, households' tastes, or strategic marketing.

[4]In this modified definition of an establishment, it is generally a smaller collection of production units than a *firm* because a firm may be multinational. Thus, another way of defining an establishment for our practical purposes is as follows: An establishment is the smallest aggregate of national production units able to provide accounting information on its inputs and outputs for the time period under consideration.

[5]For many highly specialized intermediate inputs in a multistage production process using proprietary technologies, market prices may simply not exist. Furthermore, several alternative concepts could be used to define transfer prices; see Diewert (1985) and Eden (1998) and Chapter 18 of this *Manual*. The *2008 SNA* (Paragraph 3.128) noted that: "In some cases actual exchange values may not represent market prices. Examples are transactions involving transfer prices between affiliated enterprises, manipulative agreements with third parties, and certain non-commercial transactions, including concessional interest (that is, interest payable at a reduced rate as a matter of policy). Prices may be under- or over-invoiced, in which case an assessment of a market-equivalent price needs to be made. Although adjustment should be made when actual exchange values do not represent market prices, this may not be practical in many cases." It continued (Paragraph 3.129): "Such transactions should be made explicit if their value is considerable and would hinder a proper interpretation of the accounts. In some cases, transfer pricing may be motivated by income distribution or equity build-ups or withdrawals. Replacing book values (transfer prices) with market-value equivalents is desirable in principle, when the distortions are large and when availability of data (such as adjustments by customs or tax officials or from partner economies) makes it feasible to do so. Selection of the best market-value equivalents to replace book values is an exercise calling for cautious and informed judgment."

[6]These pricing problems are pursued in Chapter 6, where the concept of a market price for each product produced by an establishment during the accounting period under consideration is the value of production for that product divided by the quantity produced during that period; that is, the price is the average price for that product. There are also practical difficulties in separating domestic transport costs out of the prices of imported goods and services.

(3) Seasonal fluctuations in the demand (or supply) of commodities cause some commodities to be unavailable in certain periods of the year.

18.24 The introduction of new commodities or different varieties of existing ones is dealt with in Chapters 8, 9, and 22 and the problems associated with seasonal commodities are dealt with in Chapter 23. In the present chapter, these complications are ignored, and it is assumed that the list of commodities remains the *same* over the two periods under consideration. It also is assumed that all establishments are present in both periods under consideration; that is, there are no new or disappearing establishments.[7] When convenient, the notation is simplified to match the notation used in Chapters 16 and 17.

18.25 To most practitioners in the field, our basic framework, which assumes that detailed price and quantity data are available for each of the possibly millions of establishments in the economy, will seem to be utterly unrealistic. However, two responses can be directed at this very valid criticism:

- The spread of the computer and the ease of storing transaction data have made the assumption that the statistical agency has access to detailed price and quantity data less unrealistic. With the cooperation of businesses, it is now possible to calculate price and quantity indices of the type studied in Chapters 16 and 17 using very detailed data on prices and quantities.[8]
- Even if it is not realistic to expect to obtain detailed price and quantity data for every transaction made by every establishment in the economy on a monthly or quarterly basis, it is still necessary to accurately specify the *universe* of transactions in the economy. Once the target universe is known, sampling techniques can be applied in order to reduce data requirements. The principles and practice of sampling establishments for XMPIs are outlined in Chapter 6.

C.3 An overview of the chapter

18.26 This subsection gives a brief overview of the contents of this chapter. In Chapters 15 and 20 the present system of production accounts in the *2008 SNA* is extended to accommodate exports and imports in the resident framework. With this expanded system of production accounts in hand, in Section D, economic approaches to the *export price index* for a single establishment are developed. These approaches are basically an adaptation of the theory of the *output price index* from Fisher and Shell (1972) and Archibald (1977) and it follows closely the exposition of the export price index made by Alterman, Diewert, and Feenstra (1999). Section E follows up on this material with Diewert's (1976) theory of *superlative indices*. A superlative index can be evaluated using observable price and quantity data, but under certain conditions it can give exactly the same answer as does the theoretical output price index. Section F.1 presents an economic approach to an *import price index* for a single establishment. This theory is again from Alterman, Diewert, and Feenstra (1999). It can also be regarded as an adaptation of the theory of the *intermediate input price index* for an establishment that was developed in Chapter 17 of the *Producer Price Index Manual* (ILO and others, 2004b) and, in fact, the establishment import price index can simply be regarded as a subindex of the entire intermediate input price index for an establishment, using the expanded system of production accounts that is explained in Section B below. Section F.2 concludes by developing an economic approach to the *household import price index* for imported goods and services that do not pass through the domestic production sector. This theory is an adaptation of the standard *cost-of-living index* theory that originated with Konüs (1924).[9] Thus the theories of the XMPIs that are developed in Sections C through E are substantially the same as corresponding theories developed in the *Producer Price Index Manual* and *Consumer Price Index Manual*.

[7]Rowe was one of the first economists to appreciate the difficulties statisticians faced when attempting to construct price or quantity indices of production: "In the construction of an index of production there are three inherent difficulties which, inasmuch as they are almost insurmountable, impose on the accuracy of the index, limitations, which under certain circumstances may be somewhat serious. The first is that many of the products of industry are not capable of quantitative measurement. This difficulty appears in its most serious form in the case of the engineering industry. . . . The second inherent difficulty is that the output of an industry, even when quantitatively measurable, may over a series of years change qualitatively as well as quantitatively. Thus during the last twenty years there has almost certainly been a tendency towards an improvement in the average quality of the yarn and cloth produced by the cotton industry. . . . The third inherent difficulty lies in the inclusion of new industries which develop importance as the years go on" (1927, pp. 174–75). These three difficulties still exist today: Think of the difficulties involved in measuring the outputs of the insurance and gambling industries; an increasing number of industries produce outputs that are one of a kind, and, hence, price and quantity comparisons are necessarily difficult if not impossible; and, finally, the huge increases in research and development expenditures by firms and governments have led to ever increasing numbers of new products and industries. Chapter 8 considers the issues for index compilation arising from new and disappearing goods and services, as well as establishments.

[8]An early study that computed Fisher ideal indices for a distribution firm in western Canada for seven quarters aggregating over 76,000 inventory items is found in Diewert and Smith (1994).

[9]The theory may be found in Chapter 17 of the *Consumer Price Index Manual* (ILO and others, 2004a).

18.27 In the previous two chapters, the Fisher (1922) ideal price index and the Törnqvist (1936) price index emerged as very good choices because they are supported by both the test and stochastic approaches to index number theory. These two indices also emerge as very good choices from the economic perspective, as shown later in this chapter. However, a practical drawback to their use is that current-period information on quantities is required, and the statistical agency usually does not have this information on a current-period basis. An important recommendation of this *Manual* is that if responding establishments or administrative sources can provide current-period quantity/value share data in a timely manner, they should be used to enable the compilation of such indices.

D. The Export Price Index for a Single Establishment

D.1 The export price index and observable bounds

18.28 In this subsection, an outline of the theory of the export price index is presented for a single establishment. This theory, which was developed by Alterman, Diewert, and Feenstra (1999, pp. 10–16), was in turn based on the theory of the output price index developed by Fisher and Shell (1972) and Archibald (1977). This theory is the producer theory counterpart to the theory of the cost-of-living index for a single consumer (or household) that was first developed by the Russian economist, A.A. Konüs (1924). These economic approaches to price indices rely on the assumption of (competitive) *optimizing behavior* on the part of economic agents (consumers or producers). Thus in the case of the export price index, given a vector of output or export prices that the agent faces in a given time period t, it is assumed that the corresponding period t quantity vector is the solution to a revenue maximization problem that involves the producer's production function f or production possibilities set. The export price index considered in this section is defined using the theory of the producer and is referred to as an export (output) price index to reinforce the fact that the approach takes a resident producer's perspective.

18.29 In contrast to the axiomatic approach to index number theory, the economic approach does *not* assume that the two export quantity vectors pertaining to periods 0 and 1 are independent of the corresponding two export price vectors. In the economic approach, the period 0 export quantity vector is determined by the producer's period 0 production function and the period 0 vector of export prices that the producer faces, and the period 1 export quantity vector is determined by the producer's period 1 production function and the period 1 vector of export prices.

18.30 Before the export price index for an establishment can be defined, it is first necessary to describe the establishment's technology in period t. In the economics literature, it is traditional to describe the technology of a firm or industry in terms of a production function, which tells the maximum amount of output that can be produced using a given vector of inputs. However, because most establishments produce more than one output, it is more convenient to describe the establishment's technology in period t by means of a *production possibilities set,* S^t. The set S^t describes what output vectors $[y, x]$ can be produced in period t if the establishment has at its disposal the vector of inputs $[z, m, v]$ where y is a vector of domestic outputs produced by the establishment, x is a vector of exports produced by the establishment, z is a vector of domestic intermediate inputs utilized by the establishment, m is a vector of imported intermediate inputs utilized by the establishment, and v is a vector of primary inputs utilized by the establishment. Thus if $[y, x, z, m, v]$ belongs to S^t, then the nonnegative output vectors y and z can be produced by the establishment in period t if it can utilize the nonnegative vectors z, m, and v of inputs. Note the relationship of this establishment production structure with the industrial structure that was explained in Section B above; the only differences are that primary inputs are now introduced into the establishment production possibilities sets and establishments have replaced industries.

18.31 Let $p_x \equiv (p_{x1}, \ldots, p_{xN})$ denote a vector of positive export prices that the establishment might face in period t[10] and let y be a vector of domestic outputs that the establishment is asked to produce, z be a vector of domestic intermediate inputs that the establishment has available during the period, m be a vector of imports that the establishment can utilize during the period, and v be a vector of primary inputs that are available to the establishment. Then the establishment's *conditional export revenue function* using period t technology is defined as the solution to the following revenue maximization problem:

$$R^t(p_x, y, z, m, v) \equiv \max_x \left\{ \sum_{n=1}^{N} p_{xn} x_n : x \equiv (x_1, \ldots, x_N) \text{ and } (y, x, z, m, v) \text{ belongs to } S^t \right\}. \quad (18.1)$$

[10]Depending on the context, these export prices may be either the per unit amounts that foreign demanders pay to the establishment or these prices may be adjusted for commodity tax or subsidy payments as in Section B.

Thus $R^t(p_x, y, z, m, v)$ is the maximum value of exports, $p_x{\cdot}x \equiv \Sigma_{n=1}{}^N p_{xn}x_n$, that the establishment can produce, given that it faces the vector of export prices p and is given the vector y of domestic output targets to produce and given that the input vectors z, m, and v are available for use, using the period t technology.[11] Note that the export revenue function is conditioned on domestic export targets being given. This has the merit of allowing the behavioral assumption of exports revenue maximization to be invoked, and economic export output indices to be defined, without confounding the theory with substitution effects between the domestic and foreign markets. The reader must, however, bear in mind that this is also a limitation of the theory.

18.32 The period t revenue function R^t can be used to define the establishment's *period t technology export output price index* P^t between any two periods, say period 0 and period 1, as follows:

$$P^t\left(p_x^0, p_x^1, y, z, m, v\right) = R^t\left(p_x^1, y, z, m, v\right)/R^t\left(p_x^0, y, z, m, v\right), \qquad (18.2)$$

where p_x^0 and p_x^1 are the vectors of export prices that the establishment faces in periods 0 and 1 respectively and y, z, m, and v are reference vectors of domestic outputs, domestic intermediate inputs, imports, and primary inputs respectively.[12] If $N = 1$ so that there is only one output that the establishment produces, then it can be shown that the output price index collapses down to the single output price relative between periods 0 and 1, p_{x1}^1/p_{x1}^0. In the general case, note that the output price index defined by (18.2) is a ratio of hypothetical revenues that the establishment could realize, given that it has the period t technology, the set of domestic output targets y, and the vectors of inputs z, m, and v to work with. The numerator in (18.2) is the maximum export revenue that the establishment could attain if it faced the output prices of period 1, p_x^1, while the denominator in (18.2) is the maximum export revenue that the establishment could attain if it faced the export prices of period 0, p_x^0. Note that all of the variables in the numerator and denominator functions are exactly the same, except that the export price vectors differ. This is a defining characteristic of an economic price index: All environmental variables are held constant with the exception of the prices in the domain of definition of the price index.

18.33 Note that there are a wide variety of price indices of the form (18.2) depending on which reference technology t and reference input vector v are chosen. Thus there is not a single economic price index of the type defined by (18.2): There is an entire *family* of indices.

18.34 In order to simplify the notation in what follows, define the composite vector of *reference quantities* u as follows:

$$u \equiv (y, z, m, v). \qquad (18.3)$$

As an additional notational simplification, let p^t denote the vector of export prices, $p_x{}^t$, for periods $t = 0, 1$.

18.35 Usually, interest lies in two special cases of the general definition of the export price index (18.2): (1) $P^0(p^0, p^1, u^0)$, which uses the period 0 technology set and the reference quantity vector $u^0 \equiv (y^0, z^0, m^0, v^0)$ that was actually produced and used by the establishment in period 0 and (2) $P^1(p^0, p^1, u^1)$, which uses the period 1 technology set and the reference quantity vector $u^1 \equiv (y^1, z^1, m^1, v^1)$ that was actually produced and used by the establishment in period 1. Let x^0 and x^1 be the observed export vectors for the establishment in periods 0 and 1 respectively. If there is revenue-maximizing behavior on the part of the establishment in periods 0 and 1, then observed revenue in periods 0 and 1 should be equal to $R^0(p^0, u^0)$ and $R^1(p^1, u^1)$ respectively; that is, the following equalities should hold:

$$R^0(p^0, u^0) = \sum_{n=1}^{N} p_n^0 x_n^0 \quad \text{and} \quad R^1(p^1, u^1) = \sum_{n=1}^{N} p_n^1 x_n^1. \qquad (18.4)$$

[11]The function R^t is closely related to the *GDP function* or the *national product function* in the international trade literature; see Kohli (1978, 1991) or Woodland (1982). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include (1) the *gross profit function*; see Gorman (1968); (2) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (3) the *variable profit function*; see Diewert (1973, 1974a). The mathematical properties of the conditional revenue function are laid out in these references.

[12]This concept of the export price index was defined in Alterman, Diewert, and Feenstra (1999, pp.10–13) and it is closely related to output price indices defined by Fisher and Shell (1972, pp. 56–58), Samuelson and Swamy (1974, pp. 588–92), Archibald (1977, pp. 60–61), Diewert (1980, pp. 460–61; 1983a, p. 1055), and Balk (1998a, pp. 83–89). Readers who are familiar with the theory of the true cost-of-living index will note that the output price index defined by (18.2) is analogous to the *true cost-of-living index* which is a ratio of cost functions, say $C(p^1, u)/C(p^0, u)$, where u is a reference utility level: R replaces C and the reference utility level u is replaced by the vector of reference variables (t, y, z, m, v). The optimizing behavior for the cost-of-living index is one of minimization while that for the export output price index is revenue maximization. For references to the theory of the true cost-of-living index, see Konüs (1924), Pollak (1983), or the consumer price index counterpart to this manual (ILO and others, 2004a).

18.36 Under these revenue-maximizing assumptions, Alterman, Diewert, and Feenstra (1999, p. 11), adapting the arguments of Fisher and Shell (1972, pp. 57–58) and Archibald (1977, p. 66), have shown that the two theoretical indices, $P^0(p^0, p^1, u^0)$ and $P^1(p^0, p^1, u^1)$, described above, satisfy the following inequalities (18.5) and (18.6):

$$P^0(p^0, p^1, u^0) \equiv R^0(p^1, u^0)/R^0(p^0, u^0) \quad \text{using definition (18.2)}$$

$$= R^0(p^1, u^0) \Big/ \sum_{n=1}^{N} p_n^0 x_n^0 \quad \text{using (18.4)}$$

$$\geq \sum_{n=1}^{N} p_n^1 x_n^0 \Big/ \sum_{n=1}^{N} p_n^0 x_n^0$$

because x^0 is feasible for the maximization problem which defines $R^0(p^1, u^0)$ and so $R^0(p^1, u^0) \geq \sum_{n=1}^{N} p_n^1 x_n^0$

$$\equiv P_L(p^0, p^1, x^0, x^1), \qquad (18.5)$$

where P_L is the Laspeyres (1871) price index. Similarly,

$$P^1(p^0, p^1, u^1) \equiv R^1(p^1, u^1)/R^1(p^0, u^1) \quad \text{using definition (18.2)}$$

$$= \sum_{n=1}^{N} p_n^1 x_n^1 / R^1(p^0, u^1) \quad \text{using (18.4)}$$

$$\leq \sum_{n=1}^{N} p_n^1 x_n^1 \Big/ \sum_{n=1}^{N} p_n^0 x_n^1$$

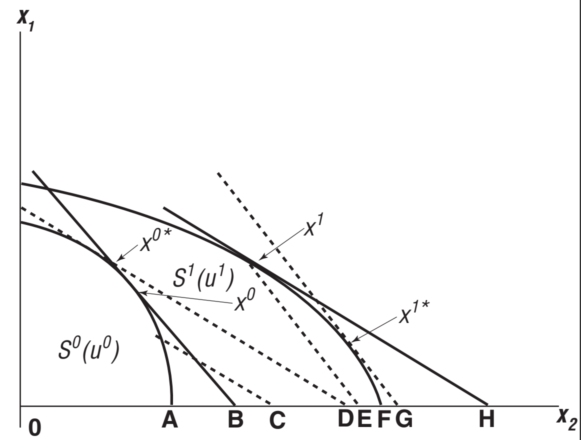
because x^1 is feasible for the maximization problem which defines $R^1(p^0, u^1)$ and so $R^1(p^0, u^1) \geq \sum_{n=1}^{N} p_n^0 x_n^1$

$$\equiv P_P(p^0, p^1, x^0, x^1), \qquad (18.6)$$

where P_P is the Paasche (1874) price index. Thus the inequality (18.5) says that the observable Laspeyres index of output prices P_L is a *lower bound* to the theoretical export output price index $P^0(p^0, p^1, u^0)$ and the inequality (18.6) says that the observable Paasche index of export output prices P_P is an *upper bound* to the theoretical export output price index $P^1(p^0, p^1, u^1)$. Note that these inequalities are in the *opposite direction* compared to their counterparts in the theory of the true cost-of-living index.[13]

[13]This is due to the fact that the optimization problem in the cost-of-living theory is a cost *minimization* problem as opposed to our present revenue *maximization* problem. The method of proof used to derive equations (18.5) and (18.6) dates back to Konüs (1924), Hicks (1940), and Samuelson (1950).

Figure 18.1. Laspeyres and Paasche Bounds to the Output Price Index



18.37 It is possible to illustrate the two inequalities (18.4) and (18.5) if there are only two commodities; see Figure 18.1, which is based on diagrams by Hicks (1940, p. 120) and Fisher and Shell (1972, p. 57).

18.38 First, the inequality (18.5) is illustrated for the case of two exports that are both produced in both periods. The solution to the period 0 export revenue maximization problem is the period 0 export vector x^0 and the straight line through B represents the revenue line that is just tangent to the period 0 export production possibilities set, $S^0(u^0) \equiv \{(x_1, x_2, u^0)$ belongs to $S^0\}$. The curved line through x^0 and A is the frontier to the producer's period 0 export production possibilities set, $S^0(u^0)$. The solution to the period 1 revenue maximization problem is the vector x^1 and the straight line through H represents the export revenue line that is just tangent to the period 1 export production possibilities set, $S^1(u^1) \equiv \{(x_1, x_2, u^1)$ belongs to $S^1\}$. The curved line through x^1 and F is the frontier to the producer's period 1 export production possibilities set, $S^1(u^1)$. The point x^{0*} solves the hypothetical maximization problem of maximizing export revenues when facing the period 1 price vector $p^1 = (p_1{}^1, p_2{}^1)$ but using the period 0 technology and reference quantity vector u^0. This hypothetical export revenue is given by $R^0(p^1, u^0) = p_1{}^1 x_1{}^{0*} + p_2{}^1 x_2{}^{0*}$ and the dashed line through D is the corresponding isorevenue line $p_1{}^1 x_1 + p_2{}^1 x_2 = R^0(p^1, u^0)$. Note that the hypothetical export revenue line through D is parallel to the actual period 1 revenue line through H.

From equation (18.5), the hypothetical export price index, $P^0(p^0, p^1, u^0)$, is $R^0(p^1, u^0)/[p_1^0x_1^0 + p_2^0x_2^0]$ while the ordinary Laspeyres export price index is $[p_1^1x_1^0 + p_2^1x_2^0]/[p_1^0x_1^0 + p_2^0x_2^0]$. Because the denominators for these two indices are the same, the difference between the indices is due to the differences in their numerators. In Figure 18.1, this difference in the numerators is expressed by the fact that the revenue line through C lies *below* the parallel revenue line through D. Now if the producer's period 0 export production possibilities set were block shaped with the vertex at x^0, then the producer would not change his or her production pattern in response to a change in the relative export prices of the two commodities while using the period 0 technology and inputs. In this case, the hypothetical vector x^{0*} would coincide with x^0, the dashed line through D would coincide with the dashed line through C, and the true export price index $P^0(p^0, p^1, u^0)$ would *coincide* with the ordinary Laspeyres export price index. However, block-shaped production possibilities sets are not generally consistent with producer behavior; that is, when the price of a commodity increases, producers generally supply more of it. Thus in the general case, there will be a gap between points C and D. The magnitude of this gap represents the amount of *substitution bias* between the true index and the corresponding Laspeyres index; that is, the Laspeyres index will generally be *less* than the corresponding true export price index, $P^0(p^0, p^1, u^0)$.

18.39 Figure 18.1 can also be used to illustrate the inequality (18.6) for the two export cases. Note that technical progress or increases in input availability have caused the period 1 export production possibilities set $S^1(u^1) \equiv \{(x_1, x_2) : (x_1, x_2, u^1)$ belongs to $S^1\}$ to be much bigger than the corresponding period 0 export production possibilities set $S^0(u^0) \equiv \{(x_1, x_2): (x_1, x_2, u^0)$ belongs to $S^0\}$.[14] Second, note that the dashed lines through E and G are parallel to the period 0 isorevenue line through B. The point x^{1*} solves the hypothetical revenue maximization problem of maximizing export revenue using the period 1 technology and inputs when facing the period 0 export price vector $p^0 = (p_1^0, p_2^0)$. This is given by $R^1(p^0, u^1) = p_1^0x_1^{1*} + p_2^0x_2^{1*}$ and the dashed line through G is the corresponding isorevenue line $p_1^1x_1 + p_2^1x_2 = R^1(p^0, u^1)$. From equation (18.6), the theoretical export price index using the period 1 technology and inputs is $[p_1^1x_1^1 + p_2^1x_2^1]/R^1(p^0, u^1)$ while the ordinary Paasche export price index is $[p_1^1x_1^1 + p_2^1x_2^1]/[p_1^0x_1^1 + p_2^0x_2^1]$. Because the numerators for these two indices are the same, the difference between the indices is due to the differences in their denominators. In Figure 18.1, this difference in the denominators is expressed by the fact that the revenue line through E lies *below* the parallel cost line through G. The magnitude of this difference represents the amount of *substitution bias* between the true index and the corresponding Paasche index; that is, the Paasche index will generally be *greater* than the corresponding true export price index using current-period technology and inputs, $P^1(p^0, p^1, u^1)$. Note that this inequality goes in the opposite direction to the previous inequality (18.5). This change in direction occurs because one set of differences between the two indices takes place in the numerators of the two indices (the Laspeyres inequalities) while the other set takes place in the denominators of the two indices (the Paasche inequalities).

18.40 There are two problems with the inequalities (18.5) and (18.6):

- There are *two* equally valid economic price indices, $P^0(p^0, p^1, u^0)$ and $P^1(p^0, p^1, u^1)$, that could be used to describe the amount of price change that took place between periods 0 and 1 whereas the public will demand that the statistical agency produce a *single* estimate of price change between the two periods.
- Only *one-sided* observable bounds to these two theoretical price indices[15] result from this analysis but *two-sided* bounds are required for most practical purposes.

The following subsection shows how a possible solution to these two problems can be found.

D.2 The Fisher ideal index as an approximation to an economic export output price index

18.41 It is possible to define a theoretical export price index that falls *between* the observable Paasche and Laspeyres export price indices. To do this, first define

[14] However, validity of the inequality (18.6) does not depend on the relative position of the two output production possibilities sets. To obtain the strict inequality version of equation (18.6), it is necessary that two conditions be satisfied: (1) the frontier of the period 1 output production possibilities set needs to be "curved" and (2) relative output prices must change going from period 0 to 1 so that the two price lines through G and H in Figure 18.1 are tangent to *different* points on the frontier of the period 1 output production possibilities set.

[15] The Laspeyres export price index is a lower bound to the theoretical index $P^0(p^0, p^1, u^0)$ while the Paasche output price index is an upper bound to the theoretical index $P^1(p^0, p^1, u^1)$.

a hypothetical export revenue function, $R(p, \alpha)$, that corresponds to the use of an α weighted average of the technology sets S^0 and S^1 for periods 0 and 1 as the reference technology and that uses an α weighted average of the period 0 and period 1 reference input and export output vectors u^0 and u^1 as the reference quantity vector:

$$R(p, \alpha) \equiv \max_x \left\{ \sum_{n=1}^{N} p_n x_n : (x, (1-\alpha)u^0 + \alpha u^1) \text{ belongs to } (1-\alpha)S^0 + \alpha S^1 \right\}. \quad (18.7)$$

Thus the revenue maximization problem in equation (18.7) corresponds to the use of a weighted average of the period 0 and 1 reference quantity vectors u^0 and u^1 where the period 0 vector gets the weight $1 - \alpha$ and the period 1 vector gets the weight α and an "average" is used of the period 0 and period 1 technology sets where the period 0 set gets the weight $1 - \alpha$ and the period 1 set gets the weight α, where α is a number between 0 and 1.[16] The meaning of the weighted average technology set in definition (18.7) can be explained in terms of Figure 18.1 as follows. As α changes continuously from 0 to 1, the export output production possibilities set changes in a continuous manner from the set $S^0(u^0)$ (whose frontier is the curve that ends in the point A) to the set $S^1(u^1)$ (whose frontier is the curve that ends in the point F). Thus for any α between 0 and 1, a hypothetical establishment export output production possibilities set that lies between the base-period set $S^0(u^0)$ and the current-period set $S^1(u^1)$ is obtained. For each α, this hypothetical output production possibilities set can be used as the constraint set for a theoretical export output price index.

18.42 The new revenue function defined by equation (18.7) is now used in order to define the following family (indexed by α) of theoretical net export output price indices:

$$P(p^0, p^1, \alpha) \equiv R(p^1, \alpha)/R(p^0, \alpha). \quad (18.8)$$

The important advantage that theoretical export output price indices of the form defined by equation (18.2) or (18.8) have over the traditional Laspeyres and Paasche export output price indices P_L and P_P is that these theoretical indices deal adequately with *substitution effects*; that is, when an export output price increases, the producer supply should increase, with inputs and the technology held constant.[17]

18.43 Diewert (1983a, pp. 1060–61) showed that, under certain conditions,[18] there exists an α between 0 and 1 such that the theoretical export output price index defined by equation (18.8) lies between the observable (in principle) Paasche and Laspeyres export output indices, P_P and P_L; that is, there exists an α such that

$$P_L \le P(p^0, p^1, \alpha) \le P_P \quad \text{or}$$
$$P_P \le P(p^0, p^1, \alpha) \le P_L. \quad (18.9)$$

18.44 The fact that the Paasche and Laspeyres export output price indices provide upper and lower bounds to a "true" export output price $P(p^0, p^1, \alpha)$ in equation (18.8) is a more useful and important result than the one-sided bounds on the "true" indices that were derived in equations (18.5) and (18.6). If the observable (in principle) Paasche and Laspeyres indices are not too far apart, then taking a symmetric average of these indices should provide a good approximation to an economic export output price index where the reference technology is somewhere between the base- and current-period technologies. The precise symmetric average of the Paasche and Laspeyres indices was determined in Section C.1 of Chapter 16 above on axiomatic grounds and led to the geometric mean, the Fisher price index, P_F:

$$P_F(p^0, p^1, x^0, x^1) \equiv \left[P_L(p^0, p^1, x^0, x^1) P_P(p^0, p^1, x^0, x^1)\right]^{1/2}. \quad (18.10)$$

[16]When $\alpha = 0$, $R(p, 0) = R^0(p, u^0)$ and when $\alpha = 1$, $R(p, 1) = R^1(p, u^1)$.

[17]This is a normal output substitution effect. However, empirically, it will often happen that observed period-to-period decreases in price are not accompanied by corresponding decreases in supply. However, these abnormal "substitution" effects can be rationalized as the effects of technological progress. For example, suppose the price of computer chips decreases substantially from period 0 to 1. If the technology were constant over these two periods, we would expect domestic producers to decrease their supply of chips going from period 0 to 1. In actual fact, the opposite happens. The fall in price is driven by technological progress arising from a reduction in the cost of producing chips which is passed on to demanders of chips. Thus the effects of technological progress should not be ignored in the theory of the output price index. The counterpart to technological change in the theory of the cost-of-living index is taste change, which is often ignored.

[18]Diewert adapted a method of proof originally developed by Konüs (1924) in the consumer context. Sufficient conditions on the period 0 and 1 technology sets for the result to hold are given in Diewert (1983a, p. 1105). Our exposition of the material in Sections D. E, and F.1 also draws on Alterman, Diewert, and Feenstra (1999).

Thus the Fisher ideal price index receives a fairly strong justification as a good approximation to an unobservable theoretical export output price index.[19]

18.45 The bounds given by equations (18.5), (18.6), and (18.9) are the best bounds that can be obtained on economic export output price indices without making further assumptions. In the next subsection, further assumptions are made on the two technology sets S^0 and S^1 or equivalently, on the two revenue functions, $R^0(p, u)$ and $R^1(p, u)$. With these extra assumptions, it is possible to determine the geometric average of the two theoretical export output price indices that are of primary interest, $P^0(p^0, p^1, u^0)$ and $P^1(p^0, p^1, u^1)$.

D.3 The Törnqvist index as an approximation to an economic export output price index

18.46 An alternative to the Laspeyres and Paasche or the Fisher index defined by equation (18.10) is to use the Törnqvist Theil (Törnqvist 1936, 1937; and Theil, 1967) price index P_T, whose natural logarithm is defined as follows:

$$\ln P_T(p^0, p^1, q^0, q^1) = \sum_{n=1}^{N} (1/2)(s_n^0 + s_n^1) \ln \left(p_n^1/p_n^0\right), \qquad (18.11)$$

where $s_n^t \equiv p_n^t x_n^t / \sum_{j=1}^{N} p_j^t x_j^t$ is the revenue share of commodity n in the total value of export sales in period t.

18.47 Recall the definition of the period t revenue function, $R^t(p, u)$, defined earlier by equation (18.1). Now assume that the period t revenue function has the following *translog functional form*:[20] for $t = 0, 1$,[21]

$$\begin{aligned}\ln R^t(p, u) = {} & \alpha_0^t + \sum_{n=1}^{N} \alpha_n^t \ln p_n + \sum_{m=1}^{M} \beta_m^t \ln u_m \\ & + (1/2) \sum_{n=1}^{N} \sum_{j=1}^{N} \alpha_{nj}^t \ln p_n \ln p_j \\ & + \sum_{n=1}^{N} \sum_{m=1}^{M} \beta_{nm}^t \ln p_n \ln u_m \\ & + (1/2) \sum_{m=1}^{M} \sum_{k=1}^{M} \gamma_{mk}^t \ln u_m \ln u_k, \qquad (18.12)\end{aligned}$$

where the α_n^t coefficients satisfy the restrictions:

$$\sum_{n=1}^{N} \alpha_n^t = 1 \quad \text{for } t = 0, 1, \qquad (18.13)$$

and the α_{nj}^t and the β_{nm}^t coefficients satisfy the following restrictions:[22]

$$\sum_{j=1}^{N} \alpha_{nj}^t = 0 \quad \text{for } t = 0, 1 \text{ and } n = 1, 2, \ldots, N; \qquad (18.14)$$

$$\sum_{n=1}^{N} \beta_{nm}^t = 0 \quad \text{for } t = 0, 1 \text{ and } m = 1, 2, \ldots, M. \qquad (18.15)$$

The restrictions (18.14) through (18.15) are necessary to ensure that $R^t(p, u)$ is linearly homogeneous in the components of the export price vector p, which is a property that a revenue function must satisfy.[23] Note that at this stage of the argument, the coefficients that characterize the technology in each period (the α's, β's, and γ's) are allowed to be completely different in each period. It should also be noted that the translog functional form is an example of a *flexible* functional form;[24] that is, it can approximate an arbitrary technology to the second order.

18.48 A result in Caves, Christensen, and Diewert (1982b, p. 1410) can now be adapted to the present context: If the quadratic price coefficients in equation (18.12) are equal across the two periods of the index number comparison (i.e., $\alpha_{nj}^0 = \alpha_{nj}^1$ for all n, j), then the geometric mean of the economic export price index that uses period 0 technology and the period 0 reference vector u^0, $P^0(p^0, p^1, u^0)$, and the economic export price index that uses period 1 technology and the period 1 reference quantity vector u^1, $P^1(p^0, p^1, u^1)$,

[19]It should be noted that Fisher (1922) constructed Laspeyres, Paasche, and Fisher output price indices for his U.S. data set. Fisher also adopted the view that the product of the price and quantity index should equal the value ratio between the two periods under consideration, an idea that he already formulated in Fisher (1911, p. 403). He did not consider explicitly the problem of deflating value added but by 1930, his ideas on deflation and the measurement of quantity growth being essentially the same problem had spread to the problem of deflating nominal value added; see Burns (1930).

[20]This functional form was introduced and named by Christensen, Jorgenson, and Lau (1971). It was adapted to the revenue function or profit function context by Diewert (1974a).

[21]Recall that the vector of reference quantities u was defined by (18.3) and is equal to (y, z, m, v). If the same commodity classification is used for domestically produced goods y, for domestic intermediate inputs z, and for imports and if the number of primary inputs v is K, then the u vector will have dimension $3N + K$, which we denote by M.

[22]It is also assumed that the symmetry conditions $\alpha_{nj}^t = \alpha_{jn}^t$ for all n, j and for $t = 0, 1$ and $\gamma_{mk}^t = \gamma_{km}^t$ for all m, k and for $t = 0, 1$ are satisfied.

[23]See Diewert (1973 and 1974a) for the regularity conditions that a revenue or profit function must satisfy.

[24]The concept of flexible functional form was introduced by Diewert (1974a, p. 113).

is *exactly equal* to the Törnqvist export output price index P_T defined by equation (18.11); that is,

$$P_T(p^0, p^1, x^0, x^1) = \left[P^0(p^0, p^1, u^0)P^1(p^0, p^1, u^1)\right]^{1/2}, \quad (18.16)$$

where $P^0(p^0, p^1, u^0)$ takes the form here as the period 0 export sales share-weighted geometric mean of price relatives and $P^1(p^0, p^1, u^1)$ the period 1 export sales share-weighted geometric mean of price relatives. The assumptions required for this result seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period and our assumptions are consistent with technological progress occurring between the two periods being compared. Because the index number formula P_T is *exactly* equal to the geometric mean of two theoretical economic export output price indices and it corresponds to a flexible functional form, the Törnqvist export output price index number formula is said to be *superlative,* following the terminology used by Diewert (1976).

18.49 For the reader who has read Chapter 17 in the *Producer Price Index Manual* (ILO and others, 2004b), the above economic theories of the export price index for an establishment will seem very similar to the economic approaches to the *gross output price index* that appeared in that manual. In fact, the theories are exactly the same; only some of the terminology has changed. Also, another way of viewing the establishment export price index is as a *subindex* of a gross output price index that encompasses both domestically produced outputs as well as outputs that are exported. Thus once the commodity by industry production accounts for the SNA are expanded along the lines suggested in Chapter 15 and Section B above, the establishment export output price index can be viewed as a subindex of a more complete system of industry by commodity output price indices.

18.50 In the following section, additional superlative export output price formulas are derived. However, this section concludes with a few words of caution on the applicability of the economic approach to export price indices. The above economic approaches to the theory of export price indices have been based on the assumption that producers take the prices of their exports as given fixed parameters that they cannot affect by their actions. However, a *monopolistic exporter* of a commodity will be well aware that the average price that can be obtained in the market for their commodity will depend on the number of units supplied during the period. Thus under noncompetitive conditions when outputs are monopolistically supplied (or when intermediate inputs are monopsonistically demanded), the economic approach to producer price indices breaks down. The problem of modeling noncompetitive behavior does not arise in the economic approach to consumer price indices because, usually, a single household does not have much control over the prices it faces in the marketplace. The economic approach to producer output price indices can be modified to deal with certain monopolistic situations. The basic idea, developed by Frisch (1936, pp. 14–15), involves linearizing the demand functions a producer faces in each period around the observed equilibrium points in each period and then calculating shadow prices that replace market prices. Alternatively, one can assume that the producer is a markup monopolist and simply adds a markup or premium to the relevant marginal cost of production.[25] However, in order to implement these techniques, econometric methods will usually have to be employed and hence, these methods are not really suitable for use by statistical agencies, except in very special circumstances when the problem of noncompetitive behavior is thought to be very significant and the agency has access to econometric resources.

18.51 The approach is a conditional one; it is assumed that the output of similar commodities to domestic and foreign markets is independent of changes in the relative prices of these similar commodities between the two markets. A revenue-maximizing producer would, for example, shift output to the export market if the price in that market relative to the domestic market increased. However, the expectation is that such a response may be "sticky" because changes in relative prices may be due to exchange rate changes, which may be relatively volatile. Further, costs will be attached to shifting output between markets, including the loss of customer loyalty.

E. Superlative Export Output Price Indices

18.52 Section D.2 demonstrated that the Paasche and Laspeyres export output price indices provide upper and lower bounds to a "true" export output price, $P(p^0, p^1, \alpha)$, in equation (18.8). Given no preference for Laspeyres and Paasche, or their theoretical counterparts $P^0(p^0, p^1, u^0)$ and $P^1(p^0, p^1, u^1)$, a symmetric average of Laspeyres and Paasche was advocated as an approximation to a true index. More particularly, the Fisher price index, as a geometric mean of Laspeyres

[25]See Diewert (1993b, pp. 169–74) for a more detailed description of these techniques for modeling monopolistic behavior and for additional references to the literature.

and Paasche price indices, was justified on the basis of its axiomatic properties, which are superior to other symmetric averages. In this section economic theory is used to justify the Fisher index formula as one of a class of superlative index number formulas. An index number is said to be *exact* when it equals its theoretical true counterpart defined for a particular functional form of its reference quantity vector, $u \equiv (y, z, m, v)$. A *superlative* index is defined as an index that is exact for a flexible functional form that can provide a second order approximation to other twice-differentiable functions around the same point. Flexible functional forms allow different outputs to be realized in response to relative price changes and thus a more realistic representation of revenue-maximizing behavior is realized: Producers substitute away from commodities with below-average price increases. To develop an economic theory of superlative indices it is first necessary to outline in Section E.1 separability conditions that allow an aggregate export output price index to be defined. Two results are then required that enable specific functional forms for the aggregator function to be related to specific index number formulas; Wold's Identity and Hotelling's Lemma are outlined in Section E.2. Fisher as a superlative index number formula is derived in Section E.3 and other superlative formulas are derived in Section E.4, and their properties for two-stage aggregation are considered in Section E.5.

E.1 Homogeneous separability and the export output price index

18.53 Instead of representing the period t technology by a set S^t, the period t technology is now represented by a *factor requirements function* F^t; that is, $v_1 = F^t(x, y, z, m, v_2, v_3, \ldots, v_K)$ is set equal to the minimum amount of primary input 1 that is required in period t in order to produce the vector of exports x and domestic outputs y, given that the vector of imports m and the amounts $v_2, v_3, \ldots, v_K$ of the remaining primary inputs are available for use. It is assumed that a linearly homogeneous aggregator function f exists for exports; that is, assume that functions f and G^t exist such that[26]

$$F^t(x, y, z, m, v_2, v_3, \ldots, v_K) = G^t(f(x), y, z, m, v_2, v_3, \ldots, v_K); \quad t = 0, 1. \quad (18.17)$$

In technical terms, period t exports are said to be *homogeneously weakly separable* from the other commodities in the technology.[27] The intuitive meaning of the separability assumption that is defined by equation (18.17) is that an export aggregate $Q \equiv f(x_1, \ldots, x_N)$ *exists*; that is, a measure of the aggregate contribution to production of the amounts x_1 of the first export, x_2 of the second export, . . . , and x_N of the Nth export is the number $Q = f(x_1, x_2, \ldots, x_N)$. Note that it is assumed that the linearly homogeneous output aggregator function f does not depend on t. These assumptions are quite restrictive from the viewpoint of empirical economics[28] but strong assumptions are required in order to obtain the existence of export aggregates from the viewpoint of this variant of economic approach.[29]

18.54 It turns out that the *export aggregator function* f has a corresponding *unit revenue function*, r, defined as follows:

$$r(p) \equiv \max_x \left\{ \sum_{n=1}^{N} p_n x_n : f(x) = 1 \right\}, \quad (18.18)$$

where $p \equiv [p_1, \ldots, p_N]$ and $x \equiv [x_1, \ldots, x_N]$. Thus $r(p)$ is the maximum export revenue that the establishment can make, given that it faces the vector of export prices p and is asked to produce a combination of exports $[x_1, \ldots, x_N] = x$ that will produce a unit level of aggregate exports.[30]

18.55 Let $Q > 0$ be an aggregate level of exports. Then it is straightforward to show that[31]

[26]This method for justifying aggregation over commodities was developed by Shephard (1953, pp. 61–71). It is assumed that $f(q)$ is an increasing, positive, and convex function of q for positive q. Samuelson and Swamy (1974) and Diewert (1980, pp. 438–42) also developed this approach to index number theory.

[27]This terminology follows that used by Geary and Morishima (1973). The concept of weak separability dates back to Sono (1945). A survey of separability concepts can be found in Blackorby, Primont, and Russell (1978).

[28]Suppose that in period 0, the vector of inputs v^0 produces the vector of outputs q^0. Our separability assumptions imply that the same vector of inputs v^0 could produce *any* vector of outputs q such that $f(q) = f(q^0)$. In real life, as q varied, we would expect that the corresponding input requirements would also vary instead of remaining fixed.

[29]The assumptions about the technology of the establishment that are made in Section D of this chapter are considerably stronger than the assumptions that were made in Section C above, where we made no separability assumptions at all. However, in the previous section, the export aggregates were conditional on a reference vector of quantities u, whereas in the present section, unconditional export aggregates are obtained.

[30]It can be shown that $r(p)$ has the following mathematical properties: $r(p)$ is a nonnegative, nondecreasing, convex, and positively linearly homogeneous function for strictly positive p vectors; see Diewert (1974b) or Samuelson and Swamy (1974). A function $r(p)$ is *convex* if for every strictly positive p^1 and p^2 and number λ such that $0 \le \lambda \le 1$, $r(\lambda p^1 + (1 - \lambda)p^2) \le \lambda r(p^1) + (1 - \lambda)r(p^2)$. A function $r(p)$ is *positively linearly homogeneous* if for every positive vector p and positive number λ, we have $r(\lambda p) = \lambda r(p)$.

[31]For additional material on revenue and factor requirements functions, see Diewert (1974b).

$$\max_x \left\{ \sum_{n=1}^{N} p_n x_n : f(x) = Q \right\}$$
$$= \max_x \left\{ \sum_{n=1}^{N} p_n x_n : (1/\text{Q}) f(x) = 1 \right\}$$
$$= \max_x \left\{ \sum_{n=1}^{N} p_n x_n : f(x/Q) = 1 \right\}$$

using the linear homogeneity of f

$$= Q \max_x \left\{ \sum_{n=1}^{N} p_n x_n / Q : f(x/Q) = 1 \right\}$$
$$= Q \max_q \left\{ \sum_{n=1}^{N} p_n q_n / Q : f(q) = 1 \right\}$$

letting $q \equiv x/Q$

$$= Q\, r(p) \quad \text{using definition (18.18).} \qquad (18.19)$$

Thus $r(p)Q$ is the maximum export revenue that the establishment can make, given that it faces the vector of output prices p and is asked to produce a combination of exports $[x_1, \ldots, x_N] = x$ that will produce the level Q of aggregate exports.

18.56 Now recall the export revenue maximization problem defined by equation (18.1). Using the factor requirements function defined by equation (18.17) in place of the period t production possibilities set S^t, this revenue maximization problem can be rewritten as follows:

$$R^t(p, u) = \max_q \left\{ \sum_{n=1}^{N} p_n x_n : v_1 = G^t(f(x), y, z, m, v_2, v_3, \ldots, v_K) \right\}$$
$$= \max_{q,Q} \left\{ \sum_{n=1}^{N} p_n x_n : v_1 = G^t(f(x), y, z, m, v_2, v_3, \ldots, v_K); Q = f(x) \right\}$$
$$= \max_Q \{ r(p)Q : v_1 = G^t(Q, y, z, m, v_2, v_3, \ldots, v_K) \}, \qquad (18.20)$$

where the last equality follows using equation (18.19). Now make assumptions (18.4); that is, that the observed period t export vector q^t solves the period t export revenue maximization problems, which are given by equation (18.20) under our separability assumption (18.17), with $(p, u) = (p^t, u^t)$ for $t = 0,1$. Using equation (18.20), the following equalities result:

$$Q^t = f(q^t);\ t = 0, 1; \qquad (18.21)$$

$$R^t(p^t, v^t) = r(p^t)Q^t;\ t = 0, 1. \qquad (18.22)$$

18.57 Consider the following export revenue maximization problem that uses the period 0 technology, the period 1 export price vector p^1, and conditions on the period 0 reference quantity vector u^0:

$$R^0(p^1, u^0)$$
$$= \max_{x,Q} \left\{ \sum_{n=1}^{N} p_n^1 x_n : v_1^0 = G^0(Q, y^0, z^0, m^0, v_2^0, v_3^0, \ldots, v_{M+K}^0); Q = f(q) \right\}$$
$$= \max_{x,Q} \left\{ \sum_{n=1}^{N} p_n^1 x_n : v_1^0 = G^0(Q^0, y^0, z^0, m^0, v_2^0, v_3^0, \ldots, v_{M+K}^0); Q^0 = f(q) \right\}$$

because Q^0 will be the only Q that satisfies the constraint $v_1{}^0 = G^0(Q, y^0, z^0, m^0, v_2{}^0, v_3{}^0, \ldots, v_{M+K}{}^0)$

$$= \max_x \left\{ \sum_{n=1}^{N} p_n^1 x_n : Q^0 = f(x) \right\} = r(p^1)Q^0$$

using equation (18.20) with $p = p^1$ and $Q = Q^0$. (18.23)

18.58 Now using the first equality in equation (18.22) and the last equality in equation (18.23) in order to evaluate *the base-period version of the theoretical export price index*, $P^0(p^0, p^1, u^0)$, defined in equation (18.5):

$$P^0(p^0, p^1, u^0) \equiv R^0(p^1, u^0)/R^0(p^0, u^0) = r(p^1)Q^0/r(p^0)Q^0 = r(p^1)/r(p^0). \qquad (18.24)$$

Note that the base-period export price index $P^0(p^0, p^1, v^0)$ no longer depends on the base-period reference quantity vector u^0; it is now simply a ratio of export unit revenue functions evaluated at the period 1 prices p^1 in the numerator and at the period 0 prices p^0 in the denominator. This is the simplification that the separability assumptions on the technologies for the two periods imply.

18.59 Using the same technique of proof that was used to establish equation (18.23), it can be shown that under the separability assumptions (18.17),

$$R^1(p^0, u^0) = r(p^0)Q^1. \qquad (18.25)$$

18.60 Now the second equality in equations (18.22) and (18.25) can be used in order to evaluate *the* *current-period version of the theoretical export price index* $P^1(p^0, p^1, u^1)$ defined above in equation (18.6):

$$\begin{aligned} P^1(p^0, p^1, u^1) &\equiv R^1(p^1, u^1)/R^1(p^0, u^1) \\ &= r(p^1)Q^1/r(p^0)Q^1 \\ &= r(p^1)/r(p^0). \end{aligned} \quad (18.26)$$

Again, the current-period export price index $P^1(p^0, p^1, u^1)$ no longer depends on the current-period reference quantity vector u^1; it is again the ratio of unit export revenue functions evaluated at the period 1 prices p^1 in the numerator and at the period 0 prices p^0 in the denominator.

18.61 Note that under the present homogeneous weak separability assumptions, both theoretical export price indices defined in equations (18.5) and (18.6) collapse down to the same thing, the ratio of unit export revenues pertaining to the two periods under consideration, $r(p^1)/r(p^0)$.[32]

18.62 Under the separability assumptions (18.17) on the establishment technologies for periods 0 and 1, the following decompositions for establishment export revenues in periods 0 and 1 can be obtained:

$$R^0(p^0, u^0) = \sum_{n=1}^{N} p_n^0 q_n^0 = r(p^0)f(q^0); \quad (18.27)$$

$$R^1(p^1, u^1) = \sum_{n=1}^{N} p_n^1 q_n^1 = r(p^1)f(q^1). \quad (18.28)$$

The ratio of unit revenues, $r(p^1)/r(p^0)$, has already been identified as the economic output price index under our separability assumptions, (18.17), so if the ratio of establishment export revenues in period 1 to revenues in period 0, $\sum_{n=1}^{N} p_n{}^1 x_n{}^1 / \sum_{n=1}^{N} p_n{}^0 x_n{}^0$, is divided by the export price index, the corresponding *implicit export quantity index*, $Q(p^0, p^1, x^0, x^1)$, is obtained:

$$\begin{aligned} Q(p^0, p^1, q^0, q^1) &\equiv \left[\sum_{n=1}^{N} p_n^1 x_n^1 \Big/ \sum_{n=1}^{N} p_n^0 x_n^0\right] / [r(p^1)/r(p^0)] \\ &= f(x^1)/f(x^0). \end{aligned} \quad (18.29)$$

Thus under the separability assumptions, the economic export quantity index is found to be equal to $f(x^1)/f(x^0)$.[33]

18.63 Now a position has been reached to apply the theory of exact index numbers. In the following subsections, some specific assumptions are made about the functional form for the export unit revenue function $r(p)$ or the export aggregator function $f(x)$[34] and these specific assumptions enable price index number formulas that are exactly equal to the theoretical output price index, $r(p^1)/r(p^0)$, to be determined. However, it is first necessary to develop the mathematics of the revenue maximization problems for periods 0 and 1 in a bit more detail. This is done in the next subsection.

E.2 The mathematics of the revenue maximization problem

18.64 In subsequent material, two additional results from economic theory will be needed: Wold's Identity and Hotelling's Lemma. These two results follow from the assumption that the establishment is maximizing export revenue during the two periods under consideration subject to the constraints of technology. Wold's Identity tells us that the partial derivative of an export aggregator function with respect to an export quantity is proportional to its export price while Hotelling's Lemma tells us that the partial derivative of an export unit revenue function with respect to an export price is proportional to the equilibrium export quantity. These two results enable specific functional forms for the aggregator function $f(q)$ or for the unit revenue function r(p) to be related to bilateral price and quantity indices, $P(p^0, p^1, q^0, q^1)$ and $Q(p^0, p^1, q^0, q^1)$, that depend on the observable price and quantity vectors pertaining to the two periods under consideration. In particular, Wold's Identity, equation (18.31), is used to establish equations (18.41) in Section E.3 and (18.54) in Section E.4 while Hotelling's Lemma, (18.33), is used to establish equations (18.37) in Section E.2 and (18.58) in Section E.4. The less mathematically inclined reader can simply note these results and skip to Section E.3.

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[32]The separability assumptions (18.44) play the same role in the economic theory of output price indices as the assumption of homothetic preferences does in the economic theory of cost-of-living indices.

[33]Note that under the separability assumptions (18.17), the family of export price indices defined by (18.2) simplifies to the unit export revenue function ratio $r(p^1)/r(p^0)$, which depends *only* on export prices (and not the reference quantity vector u) and the corresponding export quantity index is $f(x^1)/f(x^0)$, which depends *only* on quantities of exports produced during the two periods under consideration.

[34]In the following section, in order to make the notation more comparable with the notation used in previous chapters, the export quantity vector x will be replaced by the quantity vector q.

18.65 *Wold's Identity* (1944, pp. 69–71; and 1953, p. 145) is the following:[35] Assume that the establishment technologies satisfy the separability assumptions (18.17) for periods 0 and 1. Assume in addition that the observed period t export vector q^t solves the period t export revenue maximization problems, which are defined by equation (18.20) under our separability assumptions, with $(p, u) = (p^t, u^t)$ for $t = 0, 1$. Finally, assume that the export aggregator function $f(q)$ is differentiable with respect to the components of q at the points q^0 and q^1. Then it can be shown[36] that the following equations hold:

$$p_n^t \Big/ \sum_{k=1}^{N} p_k^t q_k^t = [\partial f(q^t)/\partial q_n] \Big/ \sum_{k=1}^{N} p_k q_k^t \, \partial f(q^t)/\partial q_k; \quad t = 0, 1; \quad n = 1, \ldots, N \quad (18.30)$$

where $\partial f(q^t)/\partial q_n$ denotes the partial derivative of the export revenue function f with respect to the nth export quantity q_n evaluated at the period t quantity vector q^t.

18.66 Because the export aggregator function $f(q)$ has been assumed to be linearly homogeneous, Wold's Identity (equation 18.30) simplifies into the following equations, which will prove to be very useful:[37]

$$p_n{}^t \Big/ \sum_{k=1}^{N} p_k^t q_k^t = [\partial f(q^t)/\partial q_n]/f(q^t); \quad n = 1, \ldots, N; \ t = 0, 1. \quad (18.31)$$

In words, equation (18.31) says that the vector of period t establishment export prices p^t divided by period t establishment export revenues $\sum_{k=1}^{N} p_k^t q_k^t$ is equal to the vector of first order partial derivatives of the establishment export aggregator function $\nabla f(p^t) \equiv [\partial f(q^t)/\partial q_1, \ldots, \partial f(q^t)/\partial q_N]$ divided by the period t export aggregator function $f(q^t)$.

[35] Actually, Wold derived his result in the context of a consumer utility maximization problem but his result carries over to the present production context.

[36] To prove this, consider the first order necessary conditions for the strictly positive vector q^t to solve the period t export revenue maximization problem, $\max_q \{\sum_{n=1}^{N} p_n^t q_n : f(q_1, \ldots, q_N) = f(q_1^t, \ldots, q_N^t) \equiv Q^t\}$. The necessary conditions of Lagrange for q^t to solve this problem are $p^t = \lambda^t \nabla f(q^t)$ wherel λ^t is the optimal Lagrange multiplier and $\nabla f(q^t)$ is the vector of first order partial derivatives of f evaluated at q^t. Now take the inner product of both sides of this equation with respect to the period t quantity vector q^t and solve the resulting equation for λ^t. Substitute this solution back into the vector equation $p^t = \lambda^t \nabla f(q^t)$ to obtain equation (18.30).

[37] Differentiate both sides of the equation $f(\lambda q) = \lambda f(q)$ with respect to λ and then evaluate the resulting equation at $\lambda = 1$. The equation $\sum_{n=1}^{N} f_n(q) q_n = f(q)$ results where $f_n(q) \equiv \partial f(q)/\partial q_n$.

18.67 Under assumptions (18.4) and our separability assumptions (18.17), q^t solves the following export revenue maximization problem:

$$\max_q \left\{ \sum_{n=1}^{N} p_n^t q_n : f(q_1, \ldots, q_N) = f(q_1^t, \ldots, q_N^t) \right\} = r(p^t)Q^t; \quad t = 0, 1 \quad (18.32)$$

where $Q^t \equiv f(q^t)$ and the last equality follows using equation (18.22). Consider the period t export revenue maximization problem defined by equation (18.20). *Hotelling's Lemma* (1932, p. 594) is the following: If the unit export revenue function $r(p^t)$ is differentiable with respect to the components of the price vector p, then the period t export quantity vector q^t is equal to the period t export aggregate Q^t times the vector of first order partial derivatives of the unit export revenue function with respect to the components of p evaluated at the period t price vector p^t; that is,

$$q_n^t = Q^t \partial r(p^t)/\partial p_n; \qquad n = 1, \ldots, N; \ t = 0, 1. \quad (18.33)$$

To explain why equation (18.33) holds, consider the following argument. Because it is being assumed that the observed period t export quantity vector q^t solves the export revenue maximization problem that corresponds to $r(p^t)Q^t$, then q^t must be feasible for this maximization problem so it is necessary that $f(q^t) = Q^t$. Thus q^t is a feasible solution for the following export revenue maximization problem where the general export price vector p has replaced the specific period t export price vector p^t:

$$r(p^t)Q^t \equiv \max_q \left\{ \sum_{n=1}^{N} p_n q_n : f(q_1, \ldots, q_N) = Q^t \right\} \geq \sum_{n=1}^{N} p_n q_n^t, \quad (18.34)$$

where the inequality follows from the fact that $q^t \equiv (q_1{}^t, \ldots, q_N{}^t)$ is a feasible (but usually not optimal) solution for the export revenue maximization problem in equation (18.34). Now for each strictly positive export price vector p, define the function $g(p)$ as follows:

$$g(p) \equiv \sum_{n=1}^{N} p_n q_n^t - r(p)Q^t, \quad (18.35)$$

where as usual, $p \equiv (p_1, \ldots, p_N)$. Using equations (18.32) and (18.34), it can be seen that $g(p)$ is maximized (over all strictly positive price vectors p) at $p = p^t$. Thus the first order necessary conditions for maximizing a differentiable function of N variables hold, and simplify to equation (18.33).

18.68 Combining equations (18.21), (18.27), and (18.28) yields the following:

$$\sum_{n=1}^{N} p_n^t q_n^t = r(p^t)f(q^t) = r(p^t)Q^t \quad \text{for } t = 0, 1. \quad (18.36)$$

Combining equations (18.33) and (18.36) yields the following system of equations:

$$q_n^t \Big/ \sum_{k=1}^{N} p_k^t q_k^t = [\partial r(p^t)/\partial p_n]/r(p^t);$$

$$n = 1, \ldots, N;\ t = 0, 1. \quad (18.37)$$

In words, equation (18.37) says that the vector of period t establishment exports q^t divided by period t establishment export revenues $\sum_{k=1}^{N} p_k^t q_k^t$ is equal to the vector of first order partial derivatives of the establishment unit export revenue function $\nabla r(p^t) \equiv [\partial r(p^t)/\partial p_1, \ldots, \partial r(p^t)/\partial p_N]$ divided by the period t unit export revenue function $r(p^t)$.

18.69 Note the symmetry of equations (18.37) with equations (18.31). It is these two sets of equations that shall be used in subsequent material.

E.3 Superlative indices: The Fisher ideal index

18.70 Suppose the producer's export aggregator function has the following functional form:

$$f(q_1, \ldots, q_N) \equiv \left[\sum_{i=1}^{N} \sum_{k=1}^{N} a_{ik}\, q_i q_k\right]^{1/2};$$

$$a_{ik} = a_{ki} \text{ for all } i \text{ and } k. \quad (18.38)$$

Differentiating the $f(q)$ defined by equation (18.38) with respect to q_i yields the following equations:

$$f_i(q) = (1/2)\left[\sum_{j=1}^{N} \sum_{k=1}^{N} a_{jk}\, q_j q_k\right]^{-1/2} 2\sum_{k=1}^{n} a_{ik}\, q_k;$$

$$i = 1, \ldots, N$$

$$= \sum_{k=1}^{N} a_{ik}\, q_k / f(q) \quad \text{using equation (18.38)} \quad (18.39)$$

where $f_i(q) \equiv \partial f(q^t)/\partial q_i$. In order to obtain the first equation in (18.39), the symmetry conditions, $a_{ik} = a_{ki}$, are needed. Now evaluate the second equation in (18.39) at the observed period t quantity vector $q^t \equiv (q_1{}^t, \ldots, q_N{}^t)$ and divide both sides of the resulting equation by $f(q^t)$. We obtain the following equation:

$$f_i(q^t)/f(q^t) = \sum_{k=1}^{N} a_{ik}\, q_k{}^t \,/\, [f(q^t)]^2$$

$$t = 0, 1;\ i = 1, \ldots, N. \quad (18.40)$$

Assume export revenue-maximizing behavior for the producer in periods 0 and 1. Because the aggregator function f defined by equation (18.38) is linearly homogeneous and differentiable, equation (18.31) will hold (Wold's Identity). Now recall the definition of the Fisher ideal price index, P_F, defined by equation (18.10) above. If the period 1 export revenues are divided by the period 0 export revenues and then this value ratio is divided by P_F, then the Fisher ideal quantity index, Q_F, results:

$$Q_F(p^0, p^1, q^0, q^1)$$

$$\equiv \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right] / P_F(p^0, p^1, q^0, q^1)$$

$$= \left[\sum_{i=1}^{N} p_i^0 q_i^1 \Big/ \sum_{k=1}^{N} p_k^0 q_k^0\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^1 q_k^0\right]^{1/2}$$

$$= \left[\sum_{i=1}^{N} f_i(q^0) q_i^1 / f(q^0)\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^1 q_k^0\right]^{1/2}$$

using equation (18.31)

for $t = 0$

$$= \left[\sum_{i=1}^{N} f_i(q^0) q_i^1 / f(q^0)\right]^{1/2} \Big/ \left[\sum_{k=1}^{N} p_k^1 q_k^0 \Big/ \sum_{i=1}^{N} p_i^1 q_i^1\right]^{1/2}$$

$$= \left[\sum_{i=1}^{N} f_i(q^0) q_i^1 / f(q^0)\right]^{1/2} \Big/ \left[\sum_{i=1}^{N} f_i(q^1) q_i^0 / f(q^1)\right]^{1/2}$$

using equation (18.31)

for $t = 1$

$$= \left[\sum_{i=1}^{N} \sum_{k=1}^{N} a_{ik}\, q_k^0 q_i^1 / [f(q^0)]^2\right]^{1/2} \Big/$$

$$\left[\sum_{i=1}^{N} \sum_{k=1}^{N} a_{ik}\, q_k^1 q_i^0 / [f(q^1)]^2\right]^{1/2} \quad \text{using equation (18.40)}$$

$= [1/[f(q^0)]^2]^{1/2} / [1/[f(q^1)]^2]^{1/2}$ using equation (18.38) and canceling terms

$$= f(q^1)/f(q^0). \quad (18.41)$$

Thus under the assumption that the producer engages in export revenue-maximizing behavior during periods 0 and 1 and has technologies in periods 0 and 1 that satisfy the separability assumptions (18.17), then the

Fisher ideal quantity index Q_F is *exactly* equal to the true quantity index, $f(q^1)/f(q^0)$.[38]

18.71 As was noted in earlier chapters, the price index that corresponds to the Fisher quantity index Q_F using the product test is the Fisher price index P_F defined by equation (18.10). Let $r(p)$ be the export unit revenue function that corresponds to the homogeneous quadratic export aggregator function f defined by equation (18.38). Then using equations (18.27), (18.28), and (18.41), one can see that

$$P_F(p^0, p^1, q^0, q^1) = r(p^1)/r(p^0). \tag{18.42}$$

Thus under the assumption that the producer engages in export revenue–maximizing behavior during periods 0 and 1 and has production technologies that satisfy the separability assumptions (18.17) during periods 0 and 1, then the Fisher ideal export price index P_F is exactly equal to the true price index, $r(p^1)/r(p^0)$.

18.72 A twice continuously differentiable function $f(q)$ of N variables $q \equiv (q_1, \ldots, q_N)$ can provide a *second order approximation* to another such function $f^*(q)$ around the point q^* if the level and all of the first and second order partial derivatives of the two functions coincide at q^*. It can be shown[39] that the homogeneous quadratic function f defined by equation (18.38) can provide a second order approximation to an arbitrary f^* around any (strictly positive) point q^* in the class of linearly homogeneous functions. Thus the homogeneous quadratic functional form defined by equation (18.38) is a *flexible functional form*.[40] Diewert (1976, p. 117) termed an index number formula $Q_F(p^0, p^1, q^0, q^1)$ that was *exactly* equal to the true quantity index $f(q^1)/f(q^0)$ (where f is a flexible functional form) *a superlative index number formula*.[41] Equation (18.41) and the fact that the homogeneous quadratic function f defined by equation (18.38) is a flexible functional form shows that the Fisher ideal quantity index Q_F is a superlative index number formula. Because the Fisher ideal price index P_F also satisfies equation (18.42) where $r(p)$ is the unit export revenue function that is generated by the homogeneous export quadratic aggregator function, P_F is also a superlative index number formula.

18.73 It is possible to show that the Fisher ideal price index is a superlative index number formula by a different route. Instead of starting with the assumption that the producer's export aggregator function is the homogeneous quadratic function defined by equation (18.38), start with the assumption that the producer's unit export revenue function is a homogeneous quadratic.[42] Thus suppose that the producer has the following unit export revenue function:

$$r(p_1, \ldots, p_N) \equiv \left[\sum_{i=1}^{N} \sum_{k=1}^{N} b_{ik}\, p_i p_k\right]^{1/2}, \tag{18.43}$$

where the parameters b_{ik} satisfy the following symmetry conditions:

$$b_{ik} = b_{ki} \text{ for all } i \text{ and } k. \tag{18.44}$$

Differentiating $r(p)$ defined by equation (18.43) with respect to p_i yields the following equations:

$$r_i(p) = (1/2)\left[\sum_{j=1}^{N} \sum_{k=1}^{N} b_{jk}\, p_j p_k\right]^{-1/2} 2\sum_{k=1}^{N} b_{ik}\, p_k; \quad i = 1, \ldots, N. \tag{18.45}$$

$$= \sum_{k=1}^{N} b_{ik}\, p_k / r(p) \text{ using equation (18.43),}$$

where $r_i(p) \equiv \partial r(p^t)/\partial p_i$. In order to obtain the first equation in (18.45), it is necessary to use the symmetry conditions in equation (18.44). Now evaluate the second equation in (18.45) at the observed period t price vector $p^t \equiv (p_1{}^t, \ldots, p_N{}^t)$ and divide both sides of the resulting equation by $r(p^t)$. The following equations result:

$$r_i(p^t)/r(p^t) = \sum_{k=1}^{N} b_{ik}\, p_k{}^t/[r(p^t)]^2 \quad t = 0, 1;\ i = 1, \ldots, N. \tag{18.46}$$

As export revenue–maximizing behavior is assumed for the producer in periods 0 and 1 and because the unit export revenue function r defined by equation (18.43)

[38]For the early history of this result in the consumer context, see Diewert (1976, p. 184).

[39]See Diewert (1976, p. 130) and let the parameter r equal 2.

[40]Diewert (1974a, p. 133) introduced this term to the economics literature.

[41]Fisher (1922, p. 247) used the term *superlative* to describe the Fisher ideal price index. Thus Diewert adopted Fisher's terminology but attempted to give some precision to Fisher's definition of superlativeness. Fisher defined an index number formula to be superlative if it approximated the corresponding Fisher ideal results using his data set.

[42]Given the producer's unit export revenue function $r(p)$, it is possible to modify a technique in Diewert (1974a, p. 112) and show that the corresponding export aggregator function $f(q)$ can be defined as follows: for a strictly positive quantity vector q, $f(q) \equiv \max_p \left\{\sum_{i=1}^{N} p_i q_i : r(p) = 1\right\}$.

is differentiable, equation (18.37) will hold (Hotelling's Lemma). Now recall the definition of the Fisher ideal price index, P_F, defined by equation (18.10):

$$P_F(p^0, p^1, q^0, q^1)$$
$$= \left[\sum_{i=1}^{N} p_i^1 q_i^0 \Big/ \sum_{k=1}^{N} p_k^0 q_k^0\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^0 q_k^1\right]^{1/2}$$
$$= \left[\sum_{i=1}^{N} p_i^1 r_i(p^0)/r(p^0)\right]^{1/2} \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{k=1}^{N} p_k^0 q_k^1\right]^{1/2}$$

using equation (18.37)

for $t = 0$

$$= \left[\sum_{i=1}^{N} p_i^1 r_i(p^0)/r(p^0)\right]^{1/2} \Big/ \left[\sum_{k=1}^{N} p_k^0 q_k^1 \Big/ \sum_{i=1}^{N} p_i^1 q_i^1\right]^{1/2}$$
$$= \left[\sum_{i=1}^{N} p_i^1 r_i(p^0)/r(p^0)\right]^{1/2} \Big/ \left[\sum_{i=1}^{N} p_i^0 r_i(p^1)/r(p^1)\right]^{1/2}$$

using equation (18.38) for $t = 1$

$$= \left[\sum_{i=1}^{N} \sum_{k=1}^{N} b_{ik}\, p_k^0 p_i^1/[r(p^0)]^2\right]^{1/2} \Big/ \left[\sum_{i=1}^{N} \sum_{k=1}^{N} b_{ik}\, p_k^1 p_i^0/[r(p^1)]^2\right]^{1/2}$$

using equation (18.46)

$$= [1/[r(p^0)]^2]^{1/2}/[1/[r(p^1)]^2]^{1/2}$$

using equation (18.44) and canceling terms

$$= r(p^1)/r(p^0). \qquad (18.47)$$

Thus under the assumption that the producer engages in revenue-maximizing behavior during periods 0 and 1 and has technologies that satisfy the separability assumptions (18.17) and the functional form for the unit revenue function that corresponds to the output aggregator function $f(q)$ given by equation (18.43), then the Fisher ideal price index P_F is *exactly* equal to the true price index, $r(p^1)/r(p^0)$.[43]

18.74 Because the homogeneous quadratic unit revenue function $r(p)$ defined by equation (18.43) is also a flexible functional form, the fact that the Fisher ideal price index P_F exactly equals the true export price index $r(p^1)/r(p^0)$ means that P_F is a *superlative index number formula*.[44]

18.75 Suppose that the b_{ik} coefficients in equation (18.43) satisfy the following restrictions:

$$b_{ik} = b_i b_k \qquad \text{for } i, k = i, \ldots, N, \qquad (18.48)$$

where the N numbers b_i are nonnegative. In this special case of equation (18.43), it can be seen that the unit export revenue function simplifies as follows:

$$r(p_1, \ldots, p_N) \equiv \left[\sum_{i=1}^{N} \sum_{k=1}^{N} b_i\, b_k\, p_i\, p_k\right]^{1/2}$$
$$= \left[\sum_{i=1}^{N} b_i\, p_i \sum_{k=1}^{N} b_k\, p_k\right]^{1/2}$$
$$= \sum_{i=1}^{N} b_i\, p_i. \qquad (18.49)$$

Substituting equation (18.49) into Hotelling's Lemma (equation 18.33) yields the following expressions for the period t quantity vectors, q^t:

$$q_n{}^t = Q^t\, \partial r(p^t)/\partial p_n = b_n\, Q^t \qquad i = 1, \ldots, N;\ t = 0, 1. \qquad (18.50)$$

Thus if the producer has the export aggregator function that corresponds to the unit export revenue function defined by equation (18.43) where the b_{ik} satisfy the restrictions (18.48), then the period 0 and 1 quantity vectors are equal to a multiple of the vector $b \equiv (b_1, \ldots, b_N)$; that is, $q^0 = bQ^0$ and $q^1 = bQ^1$. Under these assumptions, the Fisher, Paasche, and Laspeyres indices, P_F, P_P, and P_L, *all coincide*. However, the export aggregator function $f(q)$ which corresponds to this unit export revenue function is not consistent with normal producer behavior because the output production possibilities set in this case are block shaped and hence the producer will not substitute toward producing more expensive commodities from cheaper commodities if relative prices change going from period 0 to 1.

[43]This result was obtained by Diewert (1976, pp. 133–34) in the consumer context.

[44]Note that we have shown that the Fisher index P_F is exact for the output aggregator function defined by equation (18.38) as well as the output aggregator function that corresponds to the unit revenue function defined by equation (18.43). These two output aggregator functions do not coincide in general. However, if the N by N symmetric matrix A of the a_{ik} has an inverse, then it can readily be shown that the N by N matrix B of the b_{ik} will equal A^{-1}.

E.4 Quadratic mean of order *r* superlative indices

18.76 It turns out that there are many other superlative index number formulas; that is, there exist many export quantity indices $Q(p^0, p^1, q^0, q^1)$ that are exactly equal to $f(q^1)/f(q^0)$ and many export price indices $P(p^0, p^1, q^0, q^1)$ that are exactly equal to $r(p^1)/r(p^0)$ where the export aggregator function f or the export unit revenue function r is a flexible functional form. Two families of superlative indices are defined below.

18.77 Suppose that the producer's output aggregator function is the *following quadratic mean of order* r *aggregator function*:[45]

$$f^r(q_1, \ldots, q_N) \equiv \left[\sum_{i=1}^{N}\sum_{k=1}^{N} a_{ik}\, q_i^{r/2}\, q_k^{r/2}\right]^{1/r}, \qquad (18.51)$$

where the parameters a_{ik} satisfy the symmetry conditions $a_{ik} = a_{ki}$ for all i and k and the parameter r satisfies the restriction $r \neq 0$. Diewert (1976, p. 130) showed that the aggregator function f^r defined by equation (18.51) is a flexible functional form; that is, it can approximate an arbitrary twice continuously differentiable linearly homogeneous functional form to the second order. Note that when $r = 2$, f^r equals the homogeneous quadratic function defined by equation (18.38) above.

18.78 Define the quadratic mean of order r export quantity index Q^r by

$$Q^r(p^0, p^1, q^0, q^1) \equiv \left\{\sum_{i=1}^{N} s_i^0\,(q_i^1/q_i^0)^{r/2}\right\}^{1/r}\left\{\sum_{i=1}^{N} s_i^1\,(q_i^1/q_i^0)^{-r/2}\right\}^{-1/r}, \qquad (18.52)$$

where $s_i^t \equiv p_i^t q_i^t / \sum_{k=1}^{N} p_k^t q_k$ is the period t export revenue share for export output i as usual. It can be verified that when $r = 2$, Q^r simplifies into Q_F, the Fisher ideal quantity index.

18.79 Using exactly the same techniques as were used in Section E.3 above, it can be shown that Q^r is exact for the aggregator function f^r defined by equation (18.51); that is,

$$Q^r(p^0, p^1, q^0, q^1) = f^r(q^1)/f^r(q^0). \qquad (18.53)$$

Thus under the assumption that the producer engages in export revenue–maximizing behavior during periods 0 and 1 and has technologies that satisfy assumptions (18.17) where the output aggregator function $f(q)$ is defined by equation (18.51), then the quadratic mean of order r quantity index Q_F is *exactly* equal to the true quantity index, $f^r(q^1)/f^r(q^0)$.[46] Because Q^r is exact for f^r and f^r is a flexible functional form, the quadratic mean of order r quantity index Q^r is a *superlative index* for each $r \neq 0$. Thus there are an infinite number of superlative quantity indices.

18.80 For each quantity index Q^r, the product test can be used in order to define the corresponding *implicit quadratic mean of order* r *price index* P^{r*}:

$$P^{r*}(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \left\{\sum_{i=1}^{N} p_i^0 q_i^0\, Q^r(p^0, p^1, q^0, q^1)\right\} = r^{r*}(p^1)/r^{r*}(p^0) \qquad (18.54)$$

where r^{r*} is the unit revenue function that corresponds to the aggregator function f^r defined by equation (18.51) above. For each $r \neq 0$, the implicit quadratic mean of order r price index P^{r*} is also a superlative index.

18.81 When $r = 2$, Q^r defined by equation (18.52) simplifies to Q_F, the Fisher ideal quantity index, and P^{r*} defined by equation (18.54) simplifies to P_F, the Fisher ideal price index. When $r = 1$, Q^r defined by equation (18.52) simplifies to

$$\begin{aligned} Q^1(p^0, p^1, q^0, q^1) &\equiv \left\{\sum_{i=1}^{N} s_i^0\left(q_i^1/q_i^0\right)^{1/2}\right\}\Big/\left\{\sum_{i=1}^{N} s_i^1\left(q_i^1/q_i^0\right)^{-1/2}\right\} \\ &= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right]\left\{\sum_{i=1}^{N} p_i^0 q_i^0\left(q_i^1/q_i^0\right)^{1/2}\right\}\Big/ \left\{\sum_{i=1}^{N} p_i^1 q_i^1\left(q_i^1/q_i^0\right)^{-1/2}\right\} \\ &= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right]\left\{\sum_{i=1}^{N} p_i^0\left(q_i^0 q_i^1\right)^{1/2}\right\}\Big/ \left\{\sum_{i=1}^{N} p_i^1\left(q_i^0 q_i^1\right)^{1/2}\right\} \end{aligned}$$

[45] This terminology was used by Diewert (1976, p. 129).

[46] This is formally outlined in Diewert (1976, p. 130).

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right] \Big/ \left\{\sum_{i=1}^{N} p_i^1 (q_i^0 q_i^1)^{1/2} \Big/ \sum_{i=1}^{N} p_i^0 (q_i^0 q_i^1)^{1/2}\right\}$$

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right] \Big/ P_W(p^0, p^1, q^0, q^1), \quad (18.55)$$

where P_W is the *Walsh price index* defined by equation (16.19) in Chapter 16. Thus P^{1*} is equal to P_W, the *Walsh price index*, and hence it is also a superlative price index.

18.82 A quadratic mean of order r unit revenue function is given by[47]

$$r^r(p_1, \ldots, p_n) = \left[\sum_{i=1}^{N} \sum_{k=1}^{N} b_{ik}\, p_i^{r/2} p_k^{r/2}\right]^{1/r}, \quad (18.56)$$

where the parameters b_{ik} satisfy the symmetry conditions $b_{ik} = b_{ki}$ for all i and k and the parameter r satisfies the restriction $r \neq 0$. Diewert (1976, p. 130) showed that the unit revenue function r^r defined by equation (18.56) is a flexible functional form; that is, it can approximate an arbitrary twice continuously differentiable linearly homogeneous functional form to the second order. Note that when $r = 2$, r^r equals the homogeneous quadratic function defined by equation (18.43).

18.83 Define the quadratic mean of order r price index P^r by

$$P^r(p^0, p^1, q^0, q^1) \equiv \left\{\sum_{i=1}^{N} s_i^0 \left(p_i^1/p_i^0\right)^{r/2}\right\}^{1/r} \left\{\sum_{i=1}^{N} s_i^1 \left(p_i^1/p_i^0\right)^{-r/2}\right\}^{-1/r} \quad (18.57)$$

where $s_i^t \equiv p_i^t q_i^t / \sum_{k=1}^{N} p_k^t q_k^t$ is the period t revenue share for output i as usual. It can be verified that when $r = 2$, P^r simplifies into P_F, the Fisher ideal price index.

18.84 Using exactly the same techniques as were used in Section D.3 above, we can show that P^r is exact for the unit revenue function r^r defined by equation (18.56); that is,

$$P^r(p^0, p^1, q^0, q^1) = r^r(p^1)/r^r(p^0). \quad (18.58)$$

Thus under the assumption that the producer engages in export revenue–maximizing behavior during periods 0 and 1 and has technologies that satisfy assumptions (18.17) where the output aggregator function $f(q)$ corresponds to the unit revenue function $r^r(p)$ defined by equation (18.56), then the quadratic mean of order r price index P^r is *exactly* equal to the true export price index, $r^r(p^1)/r^r(p^0)$.[48] Because P^r is exact for r^r and r^r is a flexible functional form, that the quadratic mean of order r price index P^r is a *superlative index* for each $r \neq 0$. Thus there are an infinite number of superlative price indices.

18.85 For each price index P^r, the product test (equation 16.3 in Chapter 16) can be used in order to define the corresponding *implicit quadratic mean of order* r *quantity index* Qr^*:

$$Q^{r*}(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \left\{\sum_{i=1}^{N} p_i^0 q_i^0\, P^r(p^0, p^1, q^0, q^1)\right\} = f^{r*}(p^1)/f^{r*}(p^0), \quad (18.59)$$

where f^{r*} is the aggregator function that corresponds to the unit revenue function r^r defined by equation (18.56).[49] For each $r \neq 0$, the implicit quadratic mean of order r quantity index Q^{r*} is also a superlative index.

18.86 When $r = 2$, P^r defined by equation (18.57) simplifies to P_F, the Fisher ideal price index, and Q^{r*} defined by equation (18.59) simplifies to Q_F, the Fisher ideal quantity index. When $r = 1$, P^r defined by equation (18.57) simplifies to

$$P^1(p^0, p^1, q^0, q^1) \equiv \left\{\sum_{i=1}^{N} s_i^0 \left(p_i^1/p_i^0\right)^{1/2}\right\} \Big/ \left\{\sum_{i=1}^{N} s_i^1 \left(p_i^1/p_i^0\right)^{-1/2}\right\}$$

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right] \left\{\sum_{i=1}^{N} p_i^0 q_i^0 \left(p_i^1/p_i^0\right)^{1/2}\right\} \Big/ \left\{\sum_{i=1}^{N} p_i^1 q_i^1 \left(p_i^1/p_i^0\right)^{-1/2}\right\}$$

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0\right] \left\{\sum_{i=1}^{N} q_i^0 \left(p_i^0 p_i^1\right)^{1/2}\right\} \Big/ \left\{\sum_{i=1}^{N} q_i^1 \left(p_i^0 p_i^1\right)^{1/2}\right\}$$

[47]This terminology was used by Diewert (1976, p. 130). This functional form was first defined by Denny (1974) as a unit cost function.

[48]Further details are in Diewert (1976, pp. 133–34).

[49]The function f^{r*} can be defined by using r^r as follows: $f^{r*}(q) \equiv \max_p \left\{\sum_{i=1}^{N} p_i q_i : r^r(p) = 1\right\}$.

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 / \sum_{i=1}^{N} p_i^0 q_i^0\right] \Bigg/ \left\{\sum_{i=1}^{N} q_i^1 \left(p_i^0 p_i^1\right)^{1/2} \Bigg/ \sum_{i=1}^{N} q_i^0 \left(p_i^0 p_i^1\right)^{1/2}\right\}$$

$$= \left[\sum_{i=1}^{N} p_i^1 q_i^1 \Bigg/ \sum_{i=1}^{N} p_i^0 q_i^0\right] \Bigg/ Q_W(p^0, p^1, q^0, q^1), \quad (18.60)$$

where Q_W is the *Walsh quantity index* defined previously by equation (16.34) in Chapter 16. Thus Q^{1*} is equal to Q_W, the Walsh (1901, 1921a) quantity index, and hence it is also a superlative quantity index.

18.87 Essentially, the economic approach to index number theory provides reasonably strong justifications for the use of the Fisher price index P_F, the Törnqvist Theil price index P_T defined by equation (16.48) or (18.11), the implicit quadratic mean of order r price indices P^{r*} defined by equation (18.54) (when $r = 1$, this index is the Walsh price index defined by equation (16.19) in Chapter 16), and the quadratic mean of order r price indices P^r defined by equation (18.57). It is now necessary to ask if it matters which one of these formula is chosen as "best."

E.5 The approximation properties of superlative indices

18.88 The results in Sections E.2, E.3, E.3, and E.4 provide a large number of superlative index number formulas that appear to have good properties from the viewpoint of the economic approach to index number theory.[50] Two questions arise as a consequence of these results:

- Does it matter which of these formulas is chosen?
- If it does matter, which formula should be chosen?

18.89 With respect to the first question, Diewert (1978, p. 888) showed that all of the superlative index number formulas listed in Sections E.3 and E.4 approximate each other to the second order around any point where the two price vectors, p^0 and p^1, are equal and where the two quantity vectors, q^0 and q^1, are equal. In particular, this means that the following equalities exist for all r and s not equal to 0 provided that $p^0 = p^1$ and $q^0 = q^1$.[51]

$$P_T(p^0, p^1, q^0, q^1) = P^r(p^0, p^1, q^0, q^1) = P^{s*}(p^0, p^1, q^0, q^1); \quad (18.61)$$

$$\partial P_T(p^0, p^1, q^0, q^1)/\partial p_i^t = \partial P^r(p^0, p^1, q^0, q^1)/\partial p_i^t = \partial P^{s*}(p^0, p^1, q^0, q^1)/\partial p_i^t; \quad i = 1, \ldots, N;\ t = 0, 1; \quad (18.62)$$

$$\partial P_T(p^0, p^1, q^0, q^1)/\partial q_i^t = \partial P^r(p^0, p^1, q^0, q^1)/\partial q_i^t = \partial P^{s*}(p^0, p^1, q^0, q^1)/\partial q_i^t; \quad i = 1, \ldots, N;\ t = 0, 1; \quad (18.63)$$

$$\partial^2 P_T(p^0, p^1, q^0, q^1)/\partial p_i^t \partial p_k^t = \partial^2 P^r(p^0, p^1, q^0, q^1)/\partial p_i^t \partial p_k^t = \partial^2 P^{s*}(p^0, p^1, q^0, q^1)/\partial p_i^t \partial p_k^t; \quad i, k = 1, \ldots, N;\ t = 0,1; \quad (18.64)$$

$$\partial^2 P_T(p^0, p^1, q^0, q^1)/\partial p_i^t \partial q_k^t = \partial^2 P^r(p^0, p^1, q^0, q^1)/\partial p_i^t \partial q_k^t = \partial^2 P^{s*}(p^0, p^1, q^0, q^1)/\partial p_i^t \partial q_k^t; \quad i, k = 1, \ldots, N;\ t = 0, 1; \quad (18.65)$$

$$\partial^2 P_T(p^0, p^1, q^0, q^1)/\partial q_i^t \partial q_k^t = \partial^2 P^r(p^0, p^1, q^0, q^1)/\partial q_i^t \partial q_k^t = \partial^2 P^{s*}(p^0, p^1, q^0, q^1)/\partial q_i^t \partial q_k^t; \quad i, k = 1, \ldots, N;\ t = 0, 1; \quad (18.66)$$

where the Törnqvist Theil price index P_T is defined by equation (18.11), the implicit quadratic mean of order r price indices P^{s*} is defined by equation (18.34) and the quadratic mean of order r price indices P^r is defined by equation (18.57). Using the above results, Diewert (1978, p. 884) concluded that "all superlative indices closely approximate each other."

18.90 However, the above conclusion is not true even though the equations (18.61) through (18.66) are true. The problem is that the quadratic mean of order r price indices P^r and the implicit quadratic mean of order s price indices P^{s*} are (continuous) functions of the parameters r and s respectively. Hence, as r and s become very large in magnitude, the indices P^r and P^{s*} can differ substantially from, say, $P^2 = P_F$, the Fisher ideal index. In fact, using definition (18.57) and the limiting properties of means of order r,[52] Robert Hill (2006) showed that P^r has the following limit as r approaches plus or minus infinity:

$$\lim_{r\to+\infty} P^r(p^0, p^1, q^0, q^1) = \lim_{r\to-\infty} P^r(p^0, p^1, q^0, q^1) = \left[\min_i\{p_i^1/p_i^0\} \max_i\{p_i^1/p_i^0\}\right]^{1/2}. \quad (18.67)$$

[50]The justifications for the Fisher and Törnqvist indices presented in Sections D.2 and D.3 are stronger than the justifications for the other superlative indices presented in Sections E.3 and E.4 because the arguments in Sections D.2 and D.3 did not rely on restrictive separability assumptions.

[51]To prove the equalities in equations (18.62) through (18.66), simply differentiate the various index number formulae and evaluate the derivatives at $p^0 = p^1$ and $q^0 = q^1$. Actually, equations (18.61) through (18.66) are still true provided that $p^1 = \lambda p^0$ and $q^1 = \mu q^0$ for any numbers $\lambda > 0$ and $\mu > 0$, that is, provided that the period 1 price vector is proportional to the period 0 price vector and that the period 1 quantity vector is proportional to the period 0 quantity vector.

[52]See Hardy, Littlewood, and Pólya (1934). Actually, Allen and Diewert (1981, p. 434) obtained the result in equation (18.67), but they did not appreciate its significance.

Using Hill's method of analysis, we can show that the implicit quadratic mean of order r price index has the following limit as r approaches plus or minus infinity:

$$\lim_{r\to+\infty} P^{r*}(p^0, p^1, q^0, q^1) = \lim_{r\to-\infty} P^{r*}(p^0, p^1, q^0, q^1) = \sum_{i=1}^{N} p_i^1 q_i^1 \Big/ \sum_{i=1}^{N} p_i^0 q_i^0 \left[\min_i\{q_i^1/q_i^0\} \max_i\{q_i^1/q_i^0\}\right]^{1/2}. \quad (18.68)$$

Thus for r large in magnitude, P^r and P^{r*} can differ substantially from P_T, P^1, $P^{1*} = P_W$ (the Walsh price index) and $P^2 = P^{2*} = P_F$ (the Fisher ideal index).[53]

18.91 Although Robert Hill's theoretical and empirical results demonstrate conclusively that all superlative indices will not necessarily closely approximate each other, there is still the question of how well the more commonly used superlative indices will approximate each other. All of the commonly used superlative indices, P^r and P^{r*}, fall into the interval $0 \le r \le 2$.[54] Robert Hill (2006) summarized how far apart the Törnqvist and Fisher indices were, making all possible bilateral comparisons between any two data points for his time-series data set as follows:

> The superlative spread S(0,2) is also of interest since, in practice, Törnqvist ($r = 0$) and Fisher ($r = 2$) are by far the two most widely used superlative indices. In all 153 bilateral comparisons, $S(0,2)$ is less than the Paasche-Laspeyres spread and on average, the superlative spread is only 0.1 percent. It is because attention, until now, has focused almost exclusively on superlative indices in the range $0 \le r \le 2$ that a general misperception has persisted in the index number literature that all superlative indices approximate each other closely.

Thus for Hill's time-series data set covering 64 components of U.S. GDP from 1977 to 1994 and making all possible bilateral comparisons between any two years, the Fisher and Törnqvist price indices differed by only 0.1 percent on average. This close correspondence is consistent with the results of other empirical studies using annual time-series data.[55] Additional evidence on this topic may be found in Chapter 20.

[53]Robert Hill (2006) documented this for two data sets. His time-series data consist of annual expenditure and quantity data for 64 components of U.S. gross domestic product from 1977 to 1994. For this data set, Hill (2006, p. 16) found that "superlative indices can differ by more than a factor of two (i.e., by more than 100 percent), even though Fisher and Törnqvist never differ by more than 0.6 percent."

[54]Diewert (1980, p. 451) showed that the Törnqvist index P_T is a limiting case of P^r as r tends to 0.

[55]See, for example, Diewert (1978, p. 894) or Fisher (1922), which is reproduced in Diewert (1976, p. 135).

18.92 A reasonably strong justification has been provided by the economic approach for a small group of index numbers: the *Fisher ideal index* $P_F = P^2 = P^{2*}$ defined by equation (18.10), the *Törnqvist Theil index* P_T defined by equation (18.11), and the *Walsh index* P_W defined by equation (16.19) (which is equal to the implicit quadratic mean of order r price indices P^{r*} defined by equation (18.54) when $r = 1$). They share the property of being *superlative* and approximate each other to the second order around any point. This indicates that for "normal" time-series data, these three indices will give virtually the same answer. The economic approach gave particular support to the Fisher and Törnqvist Theil indices, albeit on different grounds. The Fisher index was advocated as the only symmetrically weighted average of Laspeyres and Paasche bounds that satisfied the time reversal test. Economic theory argued for the existence of Laspeyres and Paasche bounds on a suitable "true" theoretical index. The support for the Törnqvist Theil index arose from it requiring less restrictive assumptions to show it was superlative than did the Fisher and Walsh indices. The Törnqvist Theil index seemed to be best from the stochastic viewpoint, and the Fisher ideal index was supported from the axiomatic viewpoint, in that it best satisfied the quite reasonable tests presented. The Walsh index seemed to be best from the viewpoint of the "pure" price index. To determine precisely which one of these three alternative indices to use as a theoretical target or actual index, the statistical agency will have to decide which approach to bilateral index number theory is most consistent with its goals. It is reassuring that, as illustrated in Chapter 20, for "normal" time series data, these three indices gave virtually the same answer.

E.6 Superlative indices and two-stage aggregation

18.93 Most statistical agencies use the Laspeyres formula to aggregate prices in two stages. At the first stage of aggregation, the Laspeyres formula is used to aggregate components of the overall index (e.g., agricultural output prices, other primary industry output prices, manufacturing prices, service output prices) and then at the second stage of aggregation, these component subindices are further combined into the overall index. The following question then naturally arises: Does the index computed in two stages coincide with the index computed in a single stage? This question is initially addressed in the context of the Laspeyres formula.[56]

[56]Much of the initial material in this section is adapted from Diewert (1978) and Alterman, Diewert, and Feenstra (1999).

18.94 Now suppose that the price and quantity data for period t, p^t, and q^t, can be written in terms of J subvectors as follows:

$$p^t = (p^{t1}, p^{t2}, \ldots, p^{tJ}); \quad q^t = (q^{t1}, q^{t2}, \ldots, q^{tJ}); \quad t = 0, 1, \qquad (18.69)$$

where the dimensionality of the subvectors p^{tj} and q^{tj} is N_j for $j = 1,2, \ldots ,J$ with the sum of the dimensions N_j equal to N. These subvectors correspond to the price and quantity data for subcomponents of the export output price index for period t. Construct subindices for each of these components going from period 0 to 1. For the base period, the price for each of these subcomponents, say, P_j^0 for $j = 1,2, \ldots ,J$, is set equal to 1 and each corresponding base-period subcomponent quantities, say, Q_j^0 for $j = 1,2, \ldots ,J$, is set equal to the base-period value of production for that subcomponent. For $j = 1,2, \ldots ,J$, that is,

$$P_j^0 = 1; \quad Q_j^0 \equiv \sum_{i=1}^{Nj} p_i^{0j} q_i^{0j} \quad \text{for } j = 1, 2, \ldots, J. \qquad (18.70)$$

Now use the Laspeyres formula in order to construct a period 1 price for each subcomponent, say, P_j^1 for $j = 1, 2, \ldots, J$, of the export price index. Because the dimensionality of the subcomponent vectors, p^{tj} and q^{tj}, differs from the dimensionality of the complete period t vectors of prices and quantities, p^t and q^t, different symbols for these subcomponent Laspeyres indices will be used, say, P_L^j for $j = 1, 2, \ldots, J$. Thus the period 1 subcomponent Laspeyres price indices are defined as follows:

$$P_j^1 \equiv P_L^j(p^{0j}, p^{1j}, q^{0j}, q^{1j}) \equiv \sum_{i=1}^{Nj} p_i^{1j} q_i^{0j} \Big/ \sum_{i=1}^{Nj} p_i^{0j} q_i^{0j} \quad \text{for } j = 1, 2, \ldots, J. \qquad (18.71)$$

Once the period 1 J price subindices have been defined by equation (18.71), then corresponding J subcomponent period 1 quantity indices Q_j^1 for $j = 1, 2, \ldots, J$ can be defined by deflating the period 1 subcomponent values $\sum_{i=1}^{Nj} p_i^{1j} q_i^{1j}$ by the price indices P_j^1 defined by equation (18.71); that is,

$$Q_j^1 \equiv \sum_{i=1}^{Nj} p_i^{1j} q_i^{1j} / P_j^1 \quad \text{for } j = 1, 2, \ldots, J. \qquad (18.72)$$

Subcomponent price and quantity vectors for each J in each period $t = 0,1$ can now be defined using equations (18.70) to (18.72). Define the period 0 and 1 subcomponent price vectors P^0 and P^1 as follows:

$$P^0 = \left(P_1^0, P_2^0, \ldots, P_J^0\right) \equiv 1_J;$$
$$P^1 = \left(P_1^1, P_2^1, \ldots, P_J^1\right), \qquad (18.73)$$

where 1_J denotes a vector of ones of dimension J and the components of P^1 are defined by equation (18.71). The period 0 and 1 subcomponent quantity vectors Q^0 and Q^1 are defined as follows:

$$Q^0 = \left(Q_1^0, Q_2^0, \ldots, Q_J^0\right);$$
$$Q^1 = \left(Q_1^1, Q_2^1, \ldots, Q_J^1\right) \qquad (18.74)$$

where the components of Q^0 are defined in equation (18.70) and the components of Q^1 are defined by equation (18.72). The price and quantity vectors in equations (18.73) and (18.74) represent the results of the first stage aggregation. These vectors can now be used as inputs into the second stage aggregation problem; that is, the Laspeyres price index formula can be applied using the information in equations (18.73) and (18.74) as inputs into the index number formula. Because the price and quantity vectors that are inputs into this second stage aggregation problem have dimension J instead of the first stage formula, each of which utilized vectors of dimension N_j, a different symbol is needed for our new Laspeyres price index which is chosen to be P_L^*. Thus the Laspeyres price index computed in two stages can be denoted as $P_L^*(P^0, P^1, Q^0, Q^1)$. It is now appropriate to ask whether this two-stage Laspeyres price index equals the corresponding single-stage price index P_L that was studied in the previous sections of this chapter; that is, whether

$$P_L^*(P^0, P^1, Q^0, Q^1) = P_L(p^0, p^1, q^0, q^1). \qquad (18.75)$$

If the Laspeyres formula is used at each stage of each aggregation, the answer to the above question is yes: Straightforward calculations show that the Laspeyres index calculated in two stages equals the Laspeyres index calculated in one stage. However, the answer is yes if the Paasche formula is used at each stage of aggregation; that is, the Paasche formula is consistent in aggregation just like the Laspeyres formula.

18.95 Now suppose the Fisher or Törnqvist formula is used at each stage of the aggregation; that is, in equation (18.71), suppose the Laspeyres formula

See also Vartia (1976a and 1976b) and Balk (1996b) for a discussion of alternative definitions for the two-stage aggregation concept and references to the literature on this topic.

$P_L{}^j(p^{0j}, p^{1j}, q^{0j}, q^{1j})$ is replaced by the Fisher formula $P_F{}^j(p^{0j}, p^{1j}, q^{0j}, q^{1j})$ or by the Törnqvist formula $P_T{}^j(p^{0j}, p^{1j}, q^{0j}, q^{1j})$, and in equation (18.75), $P_L{}^*(P^0, P^1, Q^0, Q^1)$ is replaced by $P_F{}^*$ (or by $P_T{}^*$) and $P_L(p^0, p^1, q^0, q^1)$ replaced by P_F (or by P_T). Then do counterparts to the two-stage aggregation result for the Laspeyres formula, (18.75), hold? The answer is no; it can be shown that, in general,

$$P_F{}^*(P^0, P^1, Q^0, Q^1) \neq P_F(p^0, p^1, q^0, q^1) \text{ and}$$
$$P_T{}^*(P^0, P^1, Q^0, Q^1) \neq P_T(p^0, p^1, q^0, q^1). \qquad (18.76)$$

Similarly, it can be shown that the quadratic mean of order r index number formula P^r defined by equation (18.57) and the implicit quadratic mean of order r index number formula P^{r*} defined by equation (18.54) are also not consistent in aggregation.

18.96 However, even though the Fisher and Törnqvist formulas are not *exactly* consistent in aggregation, it can be shown that these formulas are *approximately* consistent in aggregation. More specifically, it can be shown that the two-stage Fisher formula $P_F{}^*$ and the single-stage Fisher formula P_F in equation (18.76), both regarded as functions of the $4N$ variables in the vectors p^0, p^1, q^0, q^1, approximate each other to the second order around a point where the two price vectors are equal (so that $p^0 = p^1$) and where the two quantity vectors are equal (so that $q^0 = q^1$) and a similar result holds for the two-stage and single-stage Törnqvist indices in equation (18.76).[57] As was shown in the previous section, the single-stage Fisher and Törnqvist indices have a similar approximation property so all four indices in equation (18.76) approximate each other to the second order around an equal (or proportional) price and quantity point. Thus for normal time series data, single-stage and two-stage Fisher and Törnqvist indices will usually be numerically very close.[58] This result is illustrated in Chapter 20 for an artificial data set.

18.97 A similar approximate consistency in aggregation results (to the results for the Fisher and Törnqvist formulas explained in the previous paragraph) can be derived for the quadratic mean of order r indices, P^r, and for the implicit quadratic mean of order r indices, P^{r*}; see Diewert (1978, p. 889). However, the results of Robert Hill (2006) again imply that *the second order approximation property of the single-stage quadratic mean of order* r *index* P[r] *to its two-stage counterpart will break down as* r *approaches either plus or minus infinity.* To see this, consider a simple example where there are only four commodities in total. Let the first price relative $p_1{}^1/p_1{}^0$ be equal to the positive number a, let the second two price relatives $p_i{}^1/p_i{}^0$ equal b, and let the last price relative $p_4{}^1/p_4{}^1$ equal c where it is assumed that $a < c$ and $a \leq b \leq c$. Using a result from Robert Hill (2006), we determine that the limiting value of the single-stage index is

$$\begin{aligned}\lim_{r\to+\infty} P^r(p^0, p^1, q^0, q^1) &= \lim_{r\to-\infty} P^r(p^0, p^1, q^0, q^1)\\ &= [\min_i\{p_i^1/p_i^0\}\max_i\{p_i^1/p_i^0\}]^{1/2}\\ &= [ac]^{1/2}. \qquad (18.77)\end{aligned}$$

Now if commodities 1 and 2 are aggregated into a subaggregate and commodities 3 and 4 into another subaggregate, using Hill's result again, we find that the limiting price index for the first subaggregate is $[ab]^{1/2}$ and the limiting price index for the second subaggregate is $[bc]^{1/2}$. Now apply the second stage of aggregation and use Hill's result once again to conclude that the limiting value of the two-stage aggregation using P^r as the index number formula is $[ab^2c]^{1/4}$. Thus the limiting value as r tends to plus or minus infinity of the single-stage aggregate over the two-stage aggregate is $[ac]^{1/2}/[ab^2c]^{1/4} = [ac/b^2]^{1/4}$. Now b can take on any value between a and c and so the ratio of the single stage limiting P^r to its two-stage counterpart can take on any value between $[c/a]^{1/4}$ and $[a/c]^{1/4}$. Because c/a is less than 1 and a/c is greater than 1, it can be seen that the ratio of the single-stage to the two-stage index can be arbitrarily far from 1 as r becomes large in magnitude with an appropriate choice of the numbers a, b, and c.

18.98 The results in the previous paragraph show that caution is required in assuming that *all* superlative indices will be approximately consistent in aggregation. However, for the three most commonly used superlative indices (the Fisher ideal P_F, the Törnqvist Theil P_T, and the Walsh P_W), the available empirical evidence indicates that these indices satisfy the consistency in aggregation property to a sufficiently high enough degree of approximation that users will not be unduly troubled by any inconsistencies.[59]

[57]See Diewert (1978, p. 889), who utilized some of Vartia's (1976a, 1976b) results. In other words, a string of equalities similar to (18.61) through (18.66) holds between the two-stage indices and their single-stage counterparts. In fact, these equalities are still true provided that $p^1 = \lambda p^0$ and $q^1 = \mu q^0$ for any numbers $\lambda > 0$ and $\mu > 0$.

[58]For an empirical comparison of the four indices, see Diewert (1978, pp. 894–95). For the Canadian consumer data considered there, the chained two-stage Fisher in 1971 was 2.3228 and the corresponding chained two-stage Törnqvist was 2.3230, the same values as for the corresponding single-stage indices.

[59]See Chapter 19 for some additional evidence on this topic.

F. Import Price Indices

F.1 The economic import price index for an establishment

18.99 Attention is now turned to the economic theory of the *import input price index for an establishment*. Note the nomenclature: It is an import price index that treats imports as inputs to a resident producing unit. This theory is analogous to the economic theory of the export output price index explained in Sections D and E above but now uses the *joint cost function* or the *conditional cost function* C in place of the revenue function R that was used in Section D and the behavioral assumption of minimizing costs as opposed to maximizing revenue. Our approach in this section turns out to be analogous to the Konüs (1924) theory for the true cost-of-living index in consumer theory.

18.100 Recall that in Section D above, the set S^t described the technology of the establishment. Thus if (y, x, z, m, v) belongs to S^t, then the nonnegative output vectors y of domestic outputs and x of exports can be produced by the establishment in period t if it can utilize the nonnegative vectors of z of domestic intermediate inputs, m of imported intermediate inputs, and v of primary inputs.

18.101 Let $p_m \equiv (p_{m1}, \ldots, p_{mN})$ denote a positive vector of import prices that the establishment might face in period t,[60] and let y be a nonnegative vector of domestic output targets, x be a vector of export targets, and z and v be nonnegative vectors of domestic intermediate inputs and primary inputs respectively that the establishment might have available for use during period t. Then the establishment's *conditional import cost function* using period t technology is defined as the solution to the following import cost minimization problem:

$$C^t(p_x, y, x, z, v) \equiv \min_x \left\{ \sum_{n=1}^{N} p_{xn} m_n : (y, x, z, m, v) \text{ belongs to } S^t \right\}. \quad (18.78)$$

Thus $C^t(p_x, y, x, z, v)$ is the minimum import cost, $\Sigma_n p_{xn} m_m$, that the establishment must pay in order to produce the vectors of outputs y and x, given that it faces the vector of intermediate input prices p_x and given that it has the input vectors z and v available for use, using the period t technology.[61]

18.102 In order to make the notation for the import price index comparable to the notation used in previous chapters for price and quantity indices, in the remainder of this subsection, the import price vector p_m is replaced by the vector p and the vector of import quantities m is replaced by the vector q. Thus $C^t(p_m, y, x, z, v)$ is rewritten as $C^t(p, y, x, z, v)$. In order to further simplify the notation, the entire vector of reference quantities, $[y, x, z, v]$, will be written as the composite quantity reference vector u. Thus $C^t(p, y, x, z, v)$ is rewritten as $C^t(p, u)$.

18.103 The period t conditional import input cost function C^t can be used to define the economy's *period* t *technology import price index* P^t between any two periods, say, period 0 and period 1, as follows:

$$P^t(p^0, p^1, u) = C^t(p^1, u)/C^t(p^0, u), \quad (18.79)$$

where p^0 and p^1 are the vectors of import prices that the establishment faces in periods 0 and 1 respectively and u is the reference vector of establishment quantities defined in the previous paragraph.[62] If $N = 1$ so that there is only one imported commodity that the establishment uses, then it can be shown that the import price index collapses down to the single import price relative between periods 0 and 1, $p_1{}^1/p_1{}^0$. In the general case, note that the import price index defined by equation (18.79) is a ratio of hypothetical import costs that the establishment must pay in order to produce the vector of domestic outputs y and the vector of exports x, given that it has the period t technology, the vector of domestic intermediate inputs z, and the vector of primary inputs v to work with. The numerator in equation (18.79) is the minimum import cost that the establishment could attain if it faced the import prices of period 1, p^1, whereas the denominator in equation (18.79) is the minimum import cost that the establishment could attain if it faced the import prices of period 0, p^0. Note

[60]From the viewpoint of economic theory, these prices should include all taxes and transportation margins, because when the establishment chooses its cost minimizing import quantities, what is relevant is the total cost of delivering these inputs to the establishment door. However, as was seen in Section B above, it often does no harm if these total import cost prices are decomposed into two or more separate terms, with the foreign price shown as one term and the tax and transportation terms shown as additional terms. However, these tax and transportation margin terms will affect establishment behavior according to the economic approach to price indices and so these terms cannot be ignored.

[61]See McFadden (1978) for the mathematical properties of a conditional cost function. Alternatively, we note that $-C^t(p_m,y,x,z,v)$ has the same mathematical properties as the revenue function R^t defined earlier in this chapter.

[62]This concept of the import price index is the same as the concept defined in Alterman, Diewert, and Feenstra (1999). This concept is related to the physical production cost index defined by Court and Lewis (1942–43, p. 30).

that all variables in the numerator and denominator of equation (18.79) except the vectors of intermediate import input prices are held constant.

18.104 As was the case with the theory of the export price index, there are a wide variety of price indices of the form of equation (18.79) depending on which (t, y, x, z, v) reference quantity vector is chosen (the reference technology is indexed by t, the reference domestic output vector is indexed by y, the reference export vector is indexed by x, the reference domestic intermediate input vector is indexed by z, and the reference primary input vector is indexed by v). As in the theory of the export price index, two special cases of the general definition of the import price index (equation 18.79) are of interest: (1) $P^0(p^0, p^1, u^0)$, which uses the period 0 technology set, the output vector y^0 that was actually produced in period 0, the export vector x^0 that was produced in period 0 by the establishment, the domestic intermediate vector z^0 that was used in period 0, and the primary input vector v^0 that was used in period 0 and (2) $P^1(p^0, p^1, u^1)$, which uses the period 1 technology set and reference quantities u^1. Let q^0 and q^1 be the observed import quantity vectors for the establishment in periods 0 and 1 respectively. If there is import cost-minimizing behavior on the part of the producer in periods 0 and 1, then the observed import cost in periods 0 and 1 should be equal to $C^0(p^0, u^0)$ and $C^1(p^1, u^1)$ respectively; that is, the following equalities should hold:

$$C^0(p^0, u^0) = \sum_{m=1}^{M} p_m^0 q_m^0 \quad \text{and}$$

$$C^1(p^1, u^1) = \sum_{m=1}^{M} p_m^1 q_m^1. \qquad (18.80)$$

18.105 Under these cost-minimizing assumptions, the arguments of Fisher and Shell (1972, pp. 57–58) and Archibald (1977, p. 66) can again be adapted to show that the two theoretical indices, $P^0(p^0, p^1, u^0)$ and $P^1(p^0, p^1, u^1)$, described in (1) and (2) above, satisfy the following inequalities (18.81) and (18.82):

$$P^0(p^0, p^1, y^0, z^0) \equiv C^0(p^1, y^0, z^0)/C^0(p^0, y^0, z^0) \quad \text{using definition (18.79)}$$

$$= C^0(p^1, y^0, z^0)/\sum_{m=1}^{M} p_m^0 q_m^0 \quad \text{using equation (18.80)}$$

$$\leq \sum_{m=1}^{M} p_m{}^1 q_m{}^0 \Big/ \sum_{m=1}^{M} p_m^0 q_m^0 \quad \text{because } q^0 \text{ is feasible for the minimization problem that defines } C^0(p^1, u^0) \text{ and so } C^0(p^1, u^0) \leq \textstyle\sum_{m=1}^{M} p_m^1 q_m^0$$

$$\equiv P_L(p^0, p^1, q^0, q^1), \qquad (18.81)$$

where P_L is the Laspeyres import input price index. Similarly,

$$P^1(p^0, p^1, y^1, z^1) \equiv C^1(p^1, u^1)/C^1(p^0, u^1) \quad \text{using definition (18.79)}$$

$$= \sum_{m=1}^{M} p_m^1 q_m^1 / C^1(p^0, u^1) \quad \text{using equation (18.80)}$$

$$\geq \sum_{m=1}^{M} p_m^1 q_m^1 / \sum_{m=1}^{M} p_m^0 q_m^1 \quad \text{because } q^1 \text{ is feasible for the minimization problem that defines } C^1(p^0, u^1) \text{ and so } C^1(p^0, u^1) \leq \sum_{m=1}^{M} p_m^0 q_m^1$$

$$\equiv P_P(p^0, p^1, q^0, q^1), \qquad (18.82)$$

where P_P is the Paasche import price index. Thus the inequality (18.81) says that the observable Laspeyres index of import prices P_L is an *upper bound* to the theoretical import index $P^0(p^0, p^1, u^0)$ and the inequality (18.82) says that the observable Paasche index of import prices P_P is a *lower bound* to the theoretical import price index $P^1(p^0, p^1, u^1)$. Note that these inequalities are the reverse of the earlier inequalities (18.5) and (18.6) that were found for the export price index but the new inequalities are analogous to their counterparts in the theory of the true cost-of-living index.

18.106 As was the case in Section D.2, it is possible to define a theoretical import price index that falls *between* the observable Paasche and Laspeyres intermediate input price indices. To do this, first define a *hypothetical import cost function*, $C(p,\alpha)$, that corresponds to the use of an α weighted average of the technology sets S^0 and S^1 for periods 0 and 1 as the reference technology and that uses an α weighted average of the period 0 and period 1 reference quantity vectors, u^0 and u^1:

$$C(p, \alpha) \equiv \min_q \left\{ \sum_{m=1}^{M} p_m q_m : [q, (1-\alpha)u^0 + \alpha u^1] \text{ belongs to } (1-\alpha)S^0 + \alpha S^1 \right\}. \qquad (18.83)$$

Thus the intermediate import input cost minimization problem in equation (18.83) corresponds to the α and $(1-\alpha)$ weighted average of the reference quantity target vectors, $(1-\alpha)u^0 + \alpha u^1$, where the period 0 reference quantity vector u^0 gets the weight $1-\alpha$ and the period 1 reference quantity vector u^1 gets the weight α, where α is a number between 0 and 1. The new import cost function defined by equation (18.83) can now be used to define the following *family of theoretical intermediate import input price indices*:

$$P(p^0, p^1, \alpha) \equiv C(p^1, \alpha)/C(p^0, \alpha). \quad (18.84)$$

18.107 Adapting the proof of Diewert (1983a, pp. 1060–61) shows that there exists an α between 0 and 1 such that the theoretical import price index defined by equation (18.84) lies between the observable (in principle) Paasche and Laspeyres import price indices, P_P and P_L; that is, there exists an α such that

$$P_L \le P(p^0, p^1, \alpha) \le P_P \quad \text{or}$$
$$P_P \le P(p^0, p^1, \alpha) \le P_L. \quad (18.85)$$

18.108 If the Paasche and Laspeyres indices are numerically close to each other, then equation (18.85) tells us that a "true" economic import price index is fairly well determined and a reasonably close approximation to the "true" index can be found by taking a symmetric average of P_L and P_P such as the geometric average which again leads to Irving Fisher's (1922) ideal price index, P_F, defined earlier by equation (18.10).

18.109 It is worth noting that the above theory of an economic import price index was very general; in particular, no restrictive functional form or separability assumptions were made about the technology.

18.110 The arguments used in Section D.3 to justify the use of the Törnqvist Theil export price index as an approximation to a theoretical export price index can be adapted to yield a justification for the use of the Törnqvist Theil import price index as an approximation to a theoretical import price index. Recall the definition of the period t conditional import cost function, $C^t(p_x, y, x, z, v) \equiv C^t(p, u)$, defined by equation (18.78) above. Now assume that the period t conditional import cost function has the following *translog functional form* for $t = 0, 1$:

$$\ln C^t(p, u) = \alpha_0^t + \sum_{n=1}^{N} \alpha_n^t \ln p_n + \sum_{k=1}^{3N+K} \beta_k^t \ln u_k$$
$$+ (1/2) \sum_{n=1}^{N} \sum_{j=1}^{N} \alpha_{nj}^t \ln p_n \ln p_j$$
$$+ \sum_{n=1}^{N} \sum_{k=1}^{3N+K} \beta_{nk}^t \ln p_n \ln u_k$$
$$+ (1/2) \sum_{k=1}^{3N+K} \sum_{j=1}^{3N+K} \gamma_{kj}^t \ln u_k \ln u_j \quad (18.86)$$

where the coefficients satisfy the following restrictions:

$$\alpha_{nj}^t = \alpha_{jn}^t \quad \text{for all } n, j \text{ and for } t = 0, 1; \quad (18.87)$$
$$\gamma_{kj}^t = \gamma_{jk}^t \quad \text{for all } k, j \text{ and for } t = 0, 1; \quad (18.88)$$
$$\sum_{n=1}^{N} \alpha_n^t = 1 \quad \text{for } t = 0, 1; \quad (18.89)$$
$$\sum_{j=1}^{N} \alpha_{nj}^t = 0 \quad \text{for } t = 0, 1 \text{ and } n = 1, 2, \ldots, N; \quad (18.90)$$
$$\sum_{n=1}^{N} \beta_{nk}^t = 0 \quad \text{for } t = 0, 1 \text{ and } k = 1, 2, \ldots, 3N + K. \quad (18.91)$$

The restrictions (18.89), (18.90), and (18.91) are necessary to ensure that $C^t(p,u)$ is linearly homogeneous in the components of the import price vector p (which is a property that a conditional cost function must satisfy). Note that at this stage of the argument the coefficients that characterize the technology in each period (the α's, β's, and γ's) are allowed to be completely different in each period.

18.111 If we adapt again the result in Caves, Christensen, and Diewert (1982b, p. 1410) to the present context:[63] If the quadratic price coefficients in equation (18.86) are equal across the two periods where an index number comparison (i.e., $\alpha_{nj}{}^0 = \alpha_{nj}{}^1$ for all n, j) is being made, then the geometric mean of the economic import price index that uses period 0 technology and period 0 reference quantities, $P^0(p^0, p^1, u^0)$, and the economic import price index that uses period 1 technology and period 1 reference quantities, $P^1(p^0, p^1, u^1)$, is *exactly* equal to the Törnqvist import price index P_T defined by equation (18.11);[64] that is,

$$P_T(p^0, p^1, q^0, q^1) = [P^0(p^0, p^1, u^0)\, P^1(p^0, p^1, u^1)]^{1/2}. \quad (18.92)$$

18.112 As was the case with the previous result (equation 18.85), the assumptions required for the result (equation 18.92) seem rather weak; in particular, there is no requirement that the technologies exhibit constant returns to scale in either period and the assumptions are

[63]The Caves, Christensen, and Diewert translog exactness result is slightly more general than a similar translog exactness result that was obtained earlier by Diewert and Morrison (1986, p. 668); Diewert and Morrison assumed that all of the quadratic terms in equation (18.86) were equal to each other during the two periods under consideration whereas Caves, Christensen, and Diewert assumed only that $\alpha_{nj}{}^0 = \alpha_{nj}{}^1$ for all n, j. See Kohli (1990) for closely related results.

[64]Of course, in the present context, export prices are replaced by import prices.

consistent with technological progress occurring between the two periods being compared. Because the index number formula P_T is *exactly* equal to the geometric mean of two theoretical economic import price indices and this corresponds to a flexible functional form, the Törnqvist import index number formula is said to be *superlative*.

18.113 It is possible to adapt the analysis of the output price index that was developed in Sections E.3 and E.4 above to the import price index and show that the two families of superlative output price indices, P^{r*} defined by equation (18.54) and P^r defined by equation (18.57), are also superlative import price indices. However, the details are omitted here because in order to derive these results, rather restrictive separability restrictions are required on the technology of the establishment.[65]

18.114 For the reader who has read Chapter 17 in the *Producer Price Index Manual* (ILO and others, 2004b), the above economic theories for the import price index for an establishment will seem very similar to the economic approaches to the *intermediate input price index* that appeared in that manual. In fact, the theories are exactly the same; only some of the terminology has changed. Also, another way of viewing the establishment import price index is as a *subindex* of a comprehensive intermediate input price index that encompasses both domestically and foreign-sourced intermediate inputs that are used by the establishment.

18.115 In the following section, the analysis presented in this section is modified to provide an economic approach to determining a household import price index.

F.2 The economic import price index for a household

18.116 The theory of the cost-of-living index for a single consumer (or household) was first developed by the Russian economist A.A. Konüs (1924). This theory relies on the assumption of *optimizing behavior* on the part of households. Thus given a vector of commodity prices p^t that the household faces in a given time period t, this approach assumes that the corresponding observed quantity vector q^t is the solution to a cost minimization problem that involves the consumer's preference or utility function F.[66] Thus in contrast to the axiomatic approach to index number theory, the economic approach does *not* assume that the two quantity vectors q^0 and q^1 are independent of the two price vectors p^0 and p^1. In the economic approach, the period 0 quantity vector q^0 is determined by the consumer's preference function F and the period 0 vector of prices p^0 that the consumer faces and the period 1 quantity vector q^1 is determined by the consumer's preference function f and the period 1 vector of prices p^1.

18.117 This household cost-of-living approach to an import price index is necessary in the present context because a small proportion of household consumption does not pass through the domestic production sector of the economy. The main expenditures of this type are tourist expenditures made abroad by domestic residents. In some countries expenditure on cross-border shopping may be a significant proportion of aggregate household consumption expenditure.

18.118 It is assumed that a household has preferences over combinations of imported goods and services, $m \equiv (m_1, \ldots, m_N)$, and domestically supplied goods and services, $y \equiv (y_1, \ldots, y_N)$, and these preferences can be represented by the utility function, $u = F(m, y)$, where u is the utility the household receives if it consumes the services of the import vector m and the domestically supplied commodities y.

18.119 Given a target utility level u and a vector of domestic commodity availabilities, y, and given that the household faces the import price vector p_m, the *consumer's conditional import cost function* is defined as follows:

$$C(p_m, y, u) \equiv \min_m \left\{ \sum_{n=1}^{N} p_{mn} m_n : F(m, y) = u \right\}. \qquad (18.93)$$

18.120 As usual, in order to make the notation in this chapter more comparable to the notation used in previous chapters, the import vector m will be replaced by the quantity vector q and the import price vector p_m will be replaced by the vector p.

18.121 Suppose the household faces the import price vector p^0 in period 0 and p^1 in period 1. Suppose also that the household has available the domestic quantity vector y for use in both periods. Finally, suppose that the household wants to achieve the same standard of living in each period; that is, the household wants to achieve the utility level u in each period at minimum

[65]The counterpart to our earlier separability assumption (18.17) is now $v_1 = F^t(y, x, z, m, v_2, \ldots, v_K) = G^t(y, x, z, f(m), v_2, \ldots, v_K)$ for $t = 0, 1$ where the import aggregator function f is linearly homogeneous and independent of t.

[66]For a description of the economic theory of the input and output price indexes, see Balk (1998a). In the economic theory of the output price index, q^t is assumed to be the solution to a revenue maximization problem involving the output price vector p^t.

import cost. Under these conditions, the household's conditional import cost function defined above can be used in order to define the following family of *household import price indices*:

$$P(p^0, p^1, y, u) \equiv C(p^1, y, u)/C(p^0, y, u). \quad (18.94)$$

18.122 There is a family of household import price indices; that is, as the standard of living indexed by the utility level u changes and as the reference vector of domestic quantity availabilities y changes, the import price index defined by equation (18.94) will change.

18.123 It is natural to choose two specific reference quantity vectors y and reference utility levels in definition (18.94): the observed base period domestic quantity vector y^0 that the household had available in period 0 along with the period 0 level of utility that was achieved by the household, u^0, and the period 1 counterparts, y^1 and u^1. It is also reasonable to assume that the household period 0 observed import vector $m^0 = q^0$ solves the following period 0 conditional cost minimization problem:

$$C(p^0, y^0, u^0) \equiv \min_q \left\{ \sum_{n=1}^{N} p_n^0 q_n : F(q, y^0) = u^0 \right\} = \sum_{n=1}^{N} p_n^0 q_n^0. \quad (18.95)$$

18.124 Similarly, it is reasonable to assume that the household period 1 observed import vector $m^1 = q^1$ solves the following period 1 conditional cost minimization problem:

$$C(p^1, y^1, u^1) \equiv \min_q \left\{ \sum_{n=1}^{N} p_n^1 q_n : F(q, y^1) = u^1 \right\} = \sum_{n=1}^{N} p_n^1 q_n^1. \quad (18.96)$$

Using assumptions (18.95) and (18.96), it is easy to establish the following bounds on two special cases of the family of import price indices defined by equation (18.94).

18.125 Consider the import price index that results when u is set equal to u^0 and y is set equal to y^0:

$$P(p^0, p^1, y^0, u^0) \equiv C(p^1, y^0, u^0)/C(p^0, y^0, u^0)$$

$$= C(p^1, y^0, u^0) \Big/ \sum_{n=1}^{N} p_n^0 q_n^0 \quad \text{using equation (18.95)}$$

$$= \min_q \left\{ \sum_{n=1}^{N} p_n^1 q_n : F(q, y^0) = u^0 \right\} \Big/ \sum_{n=1}^{N} p_n^0 q_n^0 \quad \text{using definition (18.93)}$$

$$\leq \sum_{n=1}^{N} p_n^1 q_n^0 \Big/ \sum_{n=1}^{N} p_n^0 q_n^0$$

because $q^0 \equiv (q_1{}^0, \ldots, q_N{}^0)$ is feasible for the minimization problem

$$= P_L(p^0, p^1, q^0, q^1), \quad (18.97)$$

where P_L is the Laspeyres price index defined in earlier chapters.[67]

18.126 The second of the two natural choices for a reference domestic quantity vector y and utility level u in definition (18.94) is y^1 and u^1. In this case the household import price index becomes

$$P(p^0, p^1, y^1, u^1) \equiv C(p^1, y^1, u^1)/C(p^0, y^1, u^1)$$

$$= \sum_{n=1}^{N} p_n^1 q_n^1 / C(p^0, y^1, u^1) \quad \text{using equation (18.96)}$$

$$= \sum_{n=1}^{N} p_n^1 q_n^1 / \min_q \left\{ \sum_{n=1}^{N} p_n^0 q_n : F(q, y^1) = u^1 \right\} \quad \text{using definition (18.93)}$$

$$\geq \sum_{n=1}^{N} p_n^1 q_n^1 \Big/ \sum_{n=1}^{N} p_n^0 q_n^1$$

because $q^1 \equiv (q_1{}^1, \ldots, q_N{}^1)$ is feasible for the minimization problem

$$= P_P(p^0, p^1, q^0, q^1),$$

where P_P is the Paasche price index defined earlier.[68]

18.127 At this stage, the reader will realize that the household theory of the import price index is more or less isomorphic to the establishment theory of the import price index that was developed in the previous section: The household conditional cost function replaces the establishment conditional cost function and the household price index concept defined by equation (18.94) replaces the establishment price index concept defined by equation (18.79). The same type of results that were established in the previous section can be established in the household context. Again, the Fisher and Törnqvist import price indices can be given strong justifications. The quadratic mean of order r price indices can also be justified in the present context with an appropriate separability assumption.[69]

[67]This type of inequality was first obtained by Konüs (1924, 1939, p. 17). See also Pollak (1983).

[68]This type of inequality is also from Konüs (1924, 1939, p. 19). See also Pollak (1983).

[69]The counterpart to the earlier separability assumption (18.44) is now $u = F(y, m) = G(y, f(m))$, where the import aggregator function f is linearly homogeneous.

19. Transfer Prices

A. The Transfer Price Problem

A.1 What is a transfer price?

19.1 When there is an international transaction between, say, two divisions of a multinational enterprise that has establishments in two or more countries, then the value of the transaction to the exporting division will be equal to the value of the transaction for the importing division. Thus when the multinational enterprise works out its profits worldwide for the quarter when the transaction took place, the export value will equal the import value and hence will cancel out, leaving the company's overall profits unchanged, no matter what price it chooses to value the transaction.[1] The price chosen to value the transaction is called a *transfer price*. Hence, at first glance, it appears that the multinational firm could choose the transfer price for the transaction to be practically anything. However, in a world where there are taxes on international transactions and where the rates of business income taxation differ across countries, then, as is seen below, the situation is actually worse: In this situation, the multinational will have definite financial incentives to *strategically choose* the transfer price to minimize the amount of taxation paid to both jurisdictions. It is this element of strategic choice that casts doubt on the usefulness of simply collecting transfer prices as if they were ordinary prices between unrelated parties.

19.2 This chapter is devoted to an exposition of alternative transfer pricing concepts to suit different purposes and advice to statistical agencies on possible methods for collecting the type of transfer price to suit their primary purpose.[2] In the remainder of Section A, some background information detailing the importance and complexity of pricing intra-firm trade is presented and then a brief outline of the remainder of the chapter is presented.

A.2 The prevalence of transfer pricing in international trade

19.3 The widespread use of transfer pricing is illustrated by the U.S. experience. In calendar year 2000, the latest year for which data are available, the U.S. Bureau of Economic Analysis (BEA) reported that $241 billion (or 31 percent) of export goods and $452 billion (or 37 percent) of imports were between related parties. During the past 20 years, these percentages have tended to fluctuate somewhat. The value for exports has ranged between 31 and 40 percent, while the comparable range for imports is between 37 and 44 percent. Regardless of the actual percentage, intra-firm shipments continue to represent a substantial portion of U.S. trade.

19.4 It should be noted that the characteristics of intra-firm trade could be different from trade between unrelated parties. For example, in the United States in 2001, only 13 percent of exports to both China and Korea were intra-firm, while 41 percent of sales to Mexico were between related parties. On the import side, fully 74 percent of U.S. imports from Japan were related party trade, while the comparable figure for China was just 18 percent. Similar differences crop up when looking at the data by industry, with especially high proportions of intra-firm trade in transportation equipment, computers, and chemicals.[3] Even within intra-firm trade, there can be significant differences. For U.S. multinationals, 65 percent of their exports in 1999 consisted of intermediate products exported to overseas affiliates for further processing or assembly. In contrast, for foreign multinationals, 76 percent of their shipments to the United States in 1998 were

[1]This assertion requires the proviso that there are no trade taxes on the transaction and that business income tax rates are equal in both countries.

[2]The material in this chapter is largely based on Diewert, Alterman, and Eden (2005) and Eden (1998).

[3]These data are from the Bureau of the Census (2002). Although the census data are not considered as accurate as data from BEA, the aggregate numbers are fairly consistent.

finished goods ready for final sale.[4] Given these types of variations, simply excluding intra-firm trade when constructing export and import price indices (XMPIs) would not be appropriate.

19.5 Who decides if the transfer prices chosen by multinational firms are appropriate? In the United States, there are two relevant authorities: the Internal Revenue Service (IRS) and the U.S. Customs Service. Although U.S. (and most foreign) regulations call for the use of an *arm's-length standard* in valuing intra-firm trade, these two agencies historically have worked independently in deriving values for intra-firm trade, in part because they have differing objectives.[5] Tariff officials, who are attempting to maximize duty assessments, tend to want to raise the value of imported goods, while the IRS has a tendency to want to lower the value of imports in order to maximize the amount of domestic profits.[6] The reason for these differing points of view is discussed further below. Furthermore, whereas customs values tend to be finalized comparatively quickly, final valuations associate with IRS audits and subsequent court procedures can drag on for years.[7] The laws, procedures, and documentation covering transfer prices are complicated and substantial. Both agencies devote significant resources to pricing intra-firm trade. This brief summary of the U.S. situation, of course, does not address the transfer price regulations associated with other countries but typically, they are just as complex.

19.6 Because of the complexity of transfer price regulations, corporations can end up devoting substantial resources to valuing intra-firm trade. In a recent survey by an accounting firm, 59 percent of multinationals indicated that they had undergone an audit of their transfer pricing practices in 2001.[8] Increasingly multinational traders avail themselves of any number of accounting firms that specialize in assisting corporations in navigating the multitude of both domestic and foreign regulations associated with pricing intra-firm trade. Thus the transfer price problem is a problem not only for price statisticians but also for multinationals engaged in international trade.

[4]See Zeile (1997). In addition, more recent data are available directly from BEA.

[5]Under an arm's-length standard, the transfer price should equal the price that would be settled on if the two trading firms were unrelated. This is discussed in more detail below.

[6]See Eden (1998, pp. 395–96) for further details.

[7]See Eden (1998, p. 684) for mention of four cases that each dragged on for over a decade.

[8]The survey indicated that the rate was even higher when looking at just U.S. trade; see Ernst and Young (2001).

A.3 An overview of the chapter

19.7 In the early sections of the chapter, some of the theoretical concepts for various types of transfer prices are presented. Then in the later sections of the chapter, "practical" methods for estimating transfer prices are discussed. The economic approach is the same economic approach that was used in Alterman, Diewert, and Feenstra (1999), who developed a producer price index (PPI)-oriented approach to the import price index (part of the intermediate input PPI conceptually) and the export price (part of the gross output PPI conceptually). This approach is consistent with the "resident's perspective" adopted in Chapter 18, though the theory of transfer pricing remains the same for the "nonresident's perspective."

19.8 In Section B, the four main types of transfer prices that have been considered in the theoretical literature on transfer prices are defined in general terms. In the following two sections, the transfer price problem is studied in more detail in the context of two affiliated establishments in two countries trading a single commodity. While this framework is very simple, it suffices to illustrate the various theoretical constructs for transfer prices that have been suggested in the academic literature.

19.9 Section C.1 studies the simplest case in which there are no trade taxes, business income taxes do not exist (or are the same in the two jurisdictions), and there is an external market price for the traded commodity. Section C.2 considers the case in which there are no trade or business income tax distortions but an external market for the internationally traded commodity does not exist.

19.10 In Section D, the analysis is extended to the case in which the rates of business income taxation differ in the two countries (Section D.1) and to the case in which the rates of business taxation are the same but there are trade taxes (Section D.2). Although the modeling framework is rather simple, most of the complexities of transfer pricing can be illustrated using it.

19.11 Section E looks at the main questions that are of interest to price statisticians: Namely, what are the *practical alternatives* for collecting transfer prices and can they be ordered in terms of their desirability?

19.12 Section F concludes.

B. Alternative Transfer Pricing Concepts

19.13 There are four main theoretical concepts for a transfer price. These concepts are

- the *external market* or *arm's-length* transfer price,[9]
- the *efficient* transfer price,
- the *profit maximizing* transfer price, and
- the *economic* transfer price that is suitable for collection by a statistical agency.

19.14 The first concept for a transfer price is feasible if there is a well-defined *external market price* for the traded commodity and units can be bought or sold at a common price (let us call it w). Then the transfer price for the commodity is just this price w. This is the *arm's-length transfer price*.

19.15 The second concept for a transfer price arises if there is no external market for the commodity that is traded between two production units (or establishments) of a multinational that are located in different countries. The *efficient transfer price* is generated by solving a joint profit maximization problem involving the two establishments, and it is a Lagrange multiplier, or shadow price, that corresponds to the constraint that says the output of the producing establishment must equal the input of the purchasing establishment. If there are no tax distortions,[10] then this transfer price can also be generated by setting up two profit maximization problems for the two establishments involving the traded commodity being sold by one unit at the price w, say, and being purchased by the other production unit at the price w. This artificial price is then varied so that the supply of the one establishment equals the demand of the other establishment and the resulting transfer price is called the *optimal decentralized transfer price*.[11] If there are no tax distortions and the establishments take all other input and output prices as fixed, this transfer price will also be a socially efficient one.

19.16 The *profit maximizing transfer price* is the third main concept for a transfer price. With no taxes on trade and no taxation of business income in the two jurisdictions, the profit maximizing transfer price[12] is the same as the efficient transfer price. But with tax distortions in either of the two jurisdictions, then the profit maximizing transfer price will generally be different from the efficient transfer price. In fact, with tax distortions and no constraints on the behavior of the multinational, the profit maximizing transfer price will usually be zero or an arbitrarily large number. However, usually the tax authorities will not allow such extreme transfer prices and either they will impose a transfer price or the multinational will choose a strategic transfer price that the tax authorities will accept.

19.17 The *economic transfer price* that is suitable, in theory, for collection by a statistical agency will in all cases be a marginal cost (for the exporting establishment) or a marginal revenue (for the importing establishment). In the case of no tax distortions, the economic transfer price will coincide with the external market transfer price or the efficient transfer price.

19.18 In Sections C and D below, it is shown how these concepts for a transfer price can be defined for the case where there are only *two* establishments of a multinational trading in a *single* commodity. This very simple framework will suffice to illuminate the problems involved in constructing transfer prices.

[9]This price corresponds to Eden's (1998, p. 37; and 2001, p. 32) *comparable uncontrolled price (CUP)* concept for a transfer price. Eden (2001, p. 32) followed U.S. Internal Revenue Service conventions and further distinguished an *external CUP* (also referred to as an *external comparable*) as the price set between two unrelated parties for the same or similar product sold under the same or similar circumstances and an *internal CUP* (also referred to as an *internal or in-house comparable*) where the multinational enterprise simultaneously buys or sells the same or similar product with an unrelated party. The IRS recommends internal comparables as preferable to external comparables for income tax purposes; see Feinschreiber (2004, p. 4). This concept for a transfer price also roughly corresponds to the U.S. Customs Service *transactions value* concept for a transfer price; see Eden (2001, pp. 35–36).

[10]We also need to rule out increasing returns to scale in both establishments in order to get the existence of the decentralized transfer price.

[11]This concept of a transfer price was also called an *arm's-length transfer price* by Hirshleifer (1956); see also Diewert (1985, p. 61). Under the no-tax-distortions assumption and a no-increasing-returns-to-scale assumption for each establishment, this second concept for a transfer price is equal to Diewert's (1985, pp. 49–66) efficient, arm's-length, and decentralized transfer price concepts. Note that the external market transfer price is also efficient and, in fact, multinational profits will always be greater (or at least not less than) in the situation where an external market for the product exists than in the situation where no such market exists.

[12]The profit maximizing transfer price is indeterminate under these conditions; it could be any positive price because it cancels out of the objective function of the multinational's global profit maximization problem.

C. Transfer Price Concepts When There Are No Trade or Income Taxes

C.1 Transfer pricing when an external market exists

19.19 Transfer pricing when there is a well-defined external market for the traded commodity and no tax distortions in the two countries where the two establishments of the multinational are located is very simple. Assume that establishment 1 in country 1 imports the commodity from establishment 2 in country 2. Let $x^1 \geq 0$ denote the total quantity of the commodity used by establishment 1 and let $x^2 \geq 0$ denote the production of the commodity by establishment 2. In this section, it is assumed that there are no tax distortions in order to simplify the analysis.

19.20 Suppose that establishment 1 has a technology set S^1 that is defined to be a set of feasible net output vectors[13], y^1, that can be produced if the amount x^1 of the imported commodity is available. Suppose further that the establishment faces the positive vector of prices p^1 for these net outputs. Then the *net revenue function* of establishment 1, r^1, can be defined as follows:[14]

$$r^1(p^1, x^1) \equiv \max_y \{p^1 \cdot y : (y, x^1) \text{ belongs to } S^1\} \quad (19.1)$$

where $p^1 \cdot y \equiv \sum_{i=1}^{I} p_i^1 y_i$ is the inner product between the vectors p^1 and y. Thus $r^1(p^1, x^1)$ is the net revenue establishment 1 can achieve if it faces the price vector p^1 for its outputs and non-x inputs and it has available for use x^1 units of the imported commodity.

19.21 Suppose now that establishment 2 has a technology set S^2 that is defined to be a set of feasible net input vectors,[15] z^2, that can be used to produce the amount x^2 of the commodity that is exported to establishment 1 or that is sold on the general market. Suppose further that this establishment faces the positive vector of prices p^2 for these net inputs. Then the *net cost function* for establishment 2, c^2, can be defined as follows:[16]

$$c^2(p^2, x^2) \equiv \min_z \{p^2 \cdot z : (z, x^2) \text{ belongs to } S^2\}. \quad (19.2)$$

Thus $c^2(p^2, x^2)$ is the minimum net cost establishment 2 can achieve if it faces the price vector p^2 for its net inputs and it is asked to produce x^2 units of the commodity which can be exported to establishment 1 or sold on the general market.

19.22 Given that the multinational faces the price $w > 0$ for the x commodity, the multinational's *joint profit maximization problem* is[17]

$$\max_{x,s} \{r^1(p^1, x^1) - c^2(p^2, x^2) - w[x^1 - x^2]\}. \quad (19.3)$$

19.23 If r^1 and c^2 are differentiable with respect to their x arguments, then the first order necessary conditions for x^{1*} and x^{2*} to solve (19.3) are

$$\partial r^1(p^1, x^{1*})/\partial x = w; \quad (19.4)$$

$$\partial c^2(p^2, x^{2*})/\partial x = w. \quad (19.5)$$

Equation (19.4) says that at an optimal allocation of resources between the two establishments, the *marginal revenue* generated by the last unit of x that is used by establishment 1 should be equal to the external market price of the x commodity, which is w. Equation (19.5) says that at an optimal allocation of resources between the two establishments, the *marginal cost* of establishment 2 for producing the last unit of x should be equal to the external market price of the x commodity, which is again w.

19.24 The second order sufficient conditions for x^{1*} and x^{2*} to solve (19.3)[18] are conditions (19.4) and (19.5) and the following conditions:

$$\partial^2 r^1(p^1, x^{1*})/\partial x^2 < 0; \quad (19.6)$$

$$\partial^2 c^2(p^2, x^{2*})/\partial x^2 > 0. \quad (19.7)$$

[13]If the ith component of y^1 is positive, then the ith commodity is an output produced by the establishment while if the ith component of y^1 is negative, then the ith commodity is an input used by the establishment.

[14]See Diewert (1974a, pp. 133–46; and 1993b, pp. 165–69) for the properties of net revenue or profit functions. It should be noted that definition 2 assumes competitive behavior on the part of the firm in the y markets. However, this assumption is not essential for our analysis. The firm could be behaving in a monopolistic or monopsonistic manner in these other markets but the revenue function as a function of the amount of imported commodity x can still be defined; see Diewert (1993b, pp. 169–74) for alternative methods for defining the revenue function in this case. Diewert, Alterman, and Eden (2005) considered transfer pricing problems in the monopolistic context.

[15]If the ith component of z^2 is positive, then the ith commodity is an input used by the establishment while if the ith component of z^2 is negative, then the ith commodity is an output produced by the establishment.

[16]See Diewert (1993b, p. 167) for the properties of net cost or joint cost functions.

[17]The prices in country 2 must be converted into the prices of country 1 using prevailing exchange rates (or vice versa). Henceforth, it is assumed that all prices are expressed in a common currency.

[18]Actually, conditions (19.6) and (19.7) guarantee only that x^{1*} and x^{2*} locally maximize (19.3) but if conditions (19.6) and (19.7) hold for all $x^1 > 0$ and $x^2 > 0$, then x^{1*} and x^{2*} will be a global maximum for (19.3).

Condition (19.6) says that marginal revenue is falling and condition (19.7) says that marginal cost is increasing. Basically, these two conditions rule out increasing returns to scale in both establishments in a neighborhood of the optimal allocation.

19.25 If $x^{2*} > x^{1*}$, then the multinational sells $x^{2*} - x^{1*}$ units of the internationally traded commodity to the external market while if $x^{2*} < x^{1*}$, then the multinational purchases $x^{1*} - x^{2*}$ units of the internationally traded commodity from the external market.

19.26 The external market case is relatively easy to deal with empirically: The *external market price* w is the appropriate transfer price for statistical agencies to use to value the transactions between the two production units of the multinational.[19]

19.27 In Section C.2, the more difficult case is considered where no external market for the traded commodity exists. This case is fairly common.

C.2 Transfer pricing with no external market and no trade or profit taxes

19.28 If no external market for the internationally traded commodity exists, then the amount demanded by establishment 1, x^1, must equal the amount supplied by establishment 2, x^2. Thus replacing x^1 and x^2 in (19.3) by a common x leads to the following (efficient) *global multinational profit maximization problem*:

$$\max_x \{r^1(p^1, x) - c^2(p^2, x)\}. \qquad (19.8)$$

19.29 The first order necessary condition for x^{**} to solve (19.8) is

$$\partial r^1(p^1, x^{**})/\partial x = \partial c^2(p^2, x^{**})/\partial x \equiv w^{**}. \qquad (19.9)$$

The first equation in (19.9) says that at an optimal allocation of resources between the two establishments, the *marginal revenue* generated by the last unit of x that is used by establishment 1 should be equal to the *marginal cost* of establishment 2 for producing the last unit of x. The second equation in (19.9) defines this common marginal cost and marginal revenue as w^{**}. Note that the allocation of resources generated by the solution to problem (19.8) will not in general be equal to the solution to problem (19.3) unless the solution to (19.3) had the property that $x^{1*} = x^{2*}$, so that there were no external sales or purchases of x at this solution to (19.3).

19.30 The second order sufficient conditions for x^{**} to solve equation (19.8) are conditions (19.9) and the following condition:

$$\partial^2 r^1(p^1, x^{**})/\partial x^2 - \partial^2 c^2(p^2, x^{**})/\partial x^2 > 0. \qquad (19.10)$$

Condition (19.10) is actually weaker than the earlier second order conditions (19.6) and (19.7): The new condition is consistent with increasing returns to scale in one of the two establishments.

19.31 In order to obtain an interpretation for the transfer price w^{**} defined by (19.9), consider the following constrained maximization problem, which is equivalent to (19.8):

$$\max_{x,s} \{r^1(p^1, x^1) - c^2(p^2, x^2) : x^1 - x^2 = 0\}. \qquad (19.11)$$

It turns out that w^{**} is the optimal Lagrange multiplier for the constraint in (19.11). Hence, following Diewert (1985, p. 51), Samuelson's (1947, p. 132) standard interpretation for a Lagrange multiplier may be used and the efficient transfer price w^{**} may be interpreted as the marginal increase in the worldwide net output of the multinational firm (valued at the reference prices p^1 and p^2) owing to an exogenous gift to the multinational firm of a marginal unit of the intermediate input. Note that Copithorne (1976, p. 346) used the term *opportunity cost transfer price* in place of the present term, *efficient transfer price*.

19.32 If the earlier second order conditions (19.6) and (19.7) are satisfied globally, then it can be shown that the efficient allocation of resources, that is, the x^{**} solution to equation (19.8), can be *decentralized* if the w^{**} defined by equation (19.9) is used as a transfer price. To see this, consider the following profit maximization problems for establishments 1 and 2 using the transfer price w^{**}:

$$\max_x \{r^1(p^1, x) - w^{**}x\}; \qquad (19.12)$$

$$\max_x \{w^{**}x - c^2(p^2, x)\}. \qquad (19.13)$$

[19]This is what is called a CUP in the business literature on transfer pricing. "The CUP method looks for a comparable product to the transaction in question, either in terms of the same product being bought or sold by the MNE [multinational enterprise] in a comparable transaction with an unrelated party, or the same or similar product being traded between two unrelated parties under the same or similar circumstances" (Eden, 1998, p. 37). Obviously, the same concept is applicable in a tax-distorted context as well.

It can be seen that the first order necessary conditions for equations (19.12) and (19.13) are

$$\partial r^1(p^1, x^{**})/\partial x - w^{**} = 0; \quad (19.14)$$

$$w^{**} - \partial c^2(p^2, x^{**})/\partial x = 0. \quad (19.15)$$

It can be seen that equations (19.14) and (19.15) are equivalent to the conditions (19.9), which are the first order conditions for x^{**} to solve equation (19.8). The second order sufficient conditions for equations (19.12) and (19.13) are

$$\partial^2 r^1(p^1, x^{**})/\partial x^2 < 0; \quad (19.16)$$

$$-\partial^2 c^2(p^2, x^{**})/\partial x^2 < 0, \quad (19.17)$$

and these conditions will hold if the earlier second order conditions (19.6) and (19.7) hold globally. Thus under stronger conditions on the technology of establishments 1 and 2 (i.e., no increasing returns to scale in either establishment), the above argument shows that the efficient transfer price is also the *decentralized arm's-length transfer price* introduced by Hirshleifer (1956) that equates the supply of establishment 2 to the demand of establishment 1.

19.33 To sum up, the efficient transfer price w^{**} was defined as the solution to equation (19.9); that is, it was necessary to find an x^{**} such that marginal revenue in establishment 1 is equal to marginal cost in establishment 2, so that $\partial r^1(p^1, x^{**})/\partial x = \partial c^2(p^2, x^{**})/\partial x$, and then this common value is the appropriate transfer price w^{**}. This efficient transfer price is an appropriate price for a statistical agency to collect for the traded commodity if it can be identified. From the viewpoint of production theory, the efficient transfer price will have the same standing as the observable external prices p^1 of the establishment in country 1 or the observable external prices p^2 of the establishment in country 2.

19.34 In the following section, the assumption that there is no external market transfer price available is continued.

D. Transfer Pricing When There Are Trade or Profits Taxes and No External Market

D.1 Transfer pricing with profit taxes

19.35 Consider the multinational's profit maximization problem in the case in which there is no external market for the commodity (as in the previous subsection) and there are no trade taxes but there are *differential rates of business income taxation* in the two jurisdictions. Let the rate of business income taxation in country 1 be T_1 and in country 2 be T_2 where the numbers T_i are fractions between 0 and 1. If the multinational chooses the transfer price $w > 0$, then the multinational's global profit maximization problem is now

$$\begin{aligned}&\max_{x,w}\ (1 - T_1)\{r^1(p^1, x) - wx\} + (1 - T_2)\\&\quad \{wx - c^2(p^2, x)\}\\&= \max_{x,w}\ (1 - T_1)r^1(p^1, x) - (1 - T_2)c^2(p^2, x)\\&\quad + (T_1 - T_2)wx. \qquad (19.18)\end{aligned}$$

19.36 Comparing (19.18) with the profit maximization problem (19.8) in the previous section, two differences can be seen:

- The differential rates of business income taxation, T_1 and T_2, lead to a difference between the terms $(1 - T_1)wx$ and $(1 - T_2)wx$ and so the terms involving the transfer price w no longer cancel out as they did in equation (19.8) and
- The multinational is now able to choose the transfer price w as well as the level of international trade in the intermediate input x; that is, instead of just maximizing with respect to x, the firm now maximizes with respect to x and w.

19.37 In order to solve the firm's inter-country joint profit maximization problem, it is necessary to consider two cases, depending on whether the rate of taxation in country 1 is higher than in country 2 or not.[20]

Case 1: Country 1 (the Importing Country) Is the Low Tax Country

19.38 In this case,

$$T_1 < T_2 \quad (19.19)$$

and the importing country is the low business income tax jurisdiction. Looking at the second line of (19.18), it can be seen that the term $(T_1 - T_2)wx$ is negative if $w > 0$ and $x > 0$. Note also that this is the only term where w appears. Hence to maximize overall profits,

[20]If the rates of business income taxation are exactly the same so that $T_1 = T_2$, then equation (19.18) is equivalent to equation (19.8) in Section C.2 and it does not matter what the firm chooses as its transfer price. The efficient transfer price w^{**} is still defined by equation (19.9) in this case.

the multinational will want to choose w to be as *small as possible*. This will make profits in the low tax country (country 1) as big as possible compared to profits in the high tax country (country 2). If there are no constraints on the multinational, the optimal choice of w would be[21]

$$w = 0. \tag{19.20}$$

19.39 However, the tax authorities in country 2 will almost certainly object to the solution $w = 0$. A reasonable hypothesis in the case where losses can be carried forward to offset taxable income in future periods might be that the tax authorities in country 2 insist that the transfer price be high enough so that profits are zero in country 2. This leads to the following constraint on w:[22]

$$wx = c^2(p^2, x). \tag{19.21}$$

19.40 Adding equation (19.21) as a constraint to the multinational's profit maximization problem (19.18) leads to the following *global profit maximization problem*:

$$\begin{aligned}&\max_{x,w}\ \{(1 - T_1)r^1(p^1, x) - (1 - T_2)c^2(p^2, x)\\ &\quad + (T_1 - T_2)wx : wx = c^2(p^2, x)\}\\ &= \max_x\ \{(1 - T_1)r^1(p^1, x) - (1 - T_2)c^2(p^2, x)\\ &\quad + (T_1 - T_2)c^2(p^2, x)\} \quad \text{eliminating } w\\ &= \max_x\ \{(1 - T_1)r^1(p^1, x)\\ &\quad - (1 - T_1)c^2(p^2, x)\} \quad \text{canceling terms}\\ &= (1 - T_1)\max_x\ \{r^1(p^1, x) - c^2(p^2, x)\}.\end{aligned} \tag{19.22}$$

19.41 The last line of equation (19.22) shows that the multinational's global profit maximization problem under the zero profits constraint in the high tax country is *equivalent* to the efficient profit maximization problem defined by equation (19.8) in Section C.2. Hence if the high tax country imposes a zero profits constraint on the transfer price, the multinational will end up making an efficient allocation of resources between the two countries.

19.42 However, although the allocation of resources will be globally efficient in this case, the transfer price w^{***} that the multinational chooses in this case will usually be higher than the efficient transfer price w^{**} defined by equation (19.9) in the previous section. In order to establish this result, it is necessary to assume that when the allocation of resources is efficient and the efficient transfer price w^{**} is used, both establishments make positive profits; that is, assume[23]

$$r^1(p^1, x^{**}) - w^{**}x^{**} > 0;\ w^{**}x^{**} - c^2(p^2, x^{**}) > 0. \tag{19.23}$$

19.43 Now return to equation (19.22) and note that x^{**}, the efficient amount of trade, solves this equation. However, instead of choosing the efficient transfer price w^{**} as in Section C.2, the tax authorities in country 2 now force the multinational to choose the transfer price i^{***}, which satisfies the following equation:

$$w^{***} \equiv c^2(p^2, x^{**})/x^{**}. \tag{19.24}$$

Comparing equation (19.24) with the second equation in (19.23), it can be seen that the profit maximizing transfer price w^{***} will be less than the efficient transfer price w^{**} defined by equation (19.9); that is, it has been shown that

$$w^{***} < w^{**}. \tag{19.25}$$

19.44 The result (equation 19.25) was established under the hypothesis that the tax authorities in country 2 had enough knowledge about establishment 2's costs to be able to impose the zero profits constraint (equation 19.21) on the transfer price. If the tax authorities do not have this knowledge, then there will be an incentive for the multinational to choose an even lower transfer price than w^{***} in order to transfer profits out of the high tax jurisdiction.

19.45 In general, the results for the case in which the rate of income taxation is lower in the importing country than the exporting country can be summed up by stating that the transfer price chosen by the multinational will generally be *lower* than the efficient transfer price and that it will no longer be the case that the chosen transfer price equals marginal cost in the exporting country or marginal revenue in the importing country. Hence the chosen transfer price will no longer represent true opportunity costs in the two countries and hence *is not a suitable price to be collected* if we

[21] In this case, the multinational would choose x to satisfy $(1 - T_1)\partial r^1(p^1, x)/\partial x - (1 - T_2)\partial c^2(p^2, x)/\partial x = 0$, which would not lead to the efficient allocation defined in the previous section.

[22] This method for choosing a transfer price is known as the *cost plus method* in the transfer pricing literature; see Eden (1998, p. 42).

[23] If there are constant returns to scale for establishment 1, then the first inequality in equation (19.23) becomes an equality; if there are constant returns to scale for establishment 2, then the second inequality in equation (19.23) becomes an equality.

are applying the economic approach to index number theory.[24]

19.46 The next case to be considered is case 2.

Case 2: Country 2 (the Exporting Country) Is the Low Tax Country

19.47 In this case,

$$T_1 > T_2 \tag{19.26}$$

and the exporting country (country 2) is the low business income tax jurisdiction. Looking at the second line of equation (19.18), it can be seen that when equation (19.26) holds, the term $(T_1 - T_2)wx$ is positive if $w > 0$ and $x > 0$. As before, note that this is the only term where w appears. Hence to maximize overall profits, the multinational will want to choose w to be as *large as possible*. This will make profits in the low tax country (country 2) as big as possible compared to profits in the high tax country (country 1). If there are no constraints on the multinational, the optimal choice of w would be a very large number.

19.48 However, the tax authorities in country 1 may object to this arbitrarily large solution for w, because it would make taxable income in country 1 arbitrarily negative. A reasonable hypothesis in the case in which losses can be carried forward to offset taxable income in future periods might be that the tax authorities in country 1 insist that the transfer price be low enough so that profits are zero in country 1. This leads to the following constraint on w:[25]

$$wx = r^1(p^1, x). \tag{19.27}$$

19.49 Adding equation (19.27) as a constraint to the multinational's profit maximization problem (19.18) leads to the following *global profit maximization problem*:

$$\begin{aligned}
&\max_{x,w} \{(1 - T_1)r^1(p^1, i) - (1 - T_2)c^2(p^2, x) \\
&\quad + (T_1 - T_2)wx : wx = r^1(p^1, x)\} \\
&= \max_x \{(1 - T_1)r^1(p^1, x) - (1 - T_2)c^2(p^2, x) \\
&\quad + (T_1 - T_2)\, r^1(p^1, x)\} \quad \text{eliminating } w \\
&= \max_x \{(1 - T_2)r^1(p^1, x) - (1 - T_2)c^2(p^2, x)\} \\
&\quad \text{canceling terms} \\
&= (1 - T_2) \max_x \{r^1(p^1, x) - c^2(p^2, x)\}. \qquad (19.28)
\end{aligned}$$

19.50 The last line of equation (19.28) shows that the multinational's global profit maximization problem under the zero profits constraint in the high tax country is *equivalent* to the efficient profit maximization problem defined by equation (19.8) in Section C.2. Hence, if the high tax country imposes a zero profits constraint on the transfer price, the multinational will be induced to make an efficient allocation of resources between the two countries. However, although the allocation of resources will be globally efficient in this case, the transfer price w^{****} that the multinational chooses in this case will usually be *higher* than the efficient transfer price w^{**} defined by equation (19.9) in Section C.2.

19.51 In order to establish this result, it is necessary to assume that when the allocation of resources is efficient and the efficient transfer price w^{**} is used, both establishments make positive profits; that is, assume again that equation (19.23) holds. When solving equation (19.28), instead of choosing the efficient transfer price w^{**}, the multinational now chooses the *profit maximizing transfer price* w^{****}, which is consistent with equation (19.28) when $x = x^{**}$; that is, w^{****} satisfies the following equation:

$$w^{****} \equiv r^1(p^1, x^{**})/x^{**}. \tag{19.29}$$

19.52 Comparing equation (19.29) with the first equation in (19.23), it can be seen that the profit maximizing transfer price w^{****} will be *greater* than the efficient transfer price w^{**} defined by equation (19.9); that is, it has been shown that

$$w^{****} > w^{**}. \tag{19.30}$$

19.53 The result (equation 19.30) was established under the hypothesis that the tax authorities in country 1 had enough knowledge about establishment 1's costs to be able to impose the zero profits constraint (19.27) on the transfer price. If the tax authorities do not have

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[24] In the case where country 2 imposes the zero profits constraint (19.21), the "correct" price to collect from the viewpoint of the economic approach to index number theory is the efficient transfer price w^{**} defined by equation (19.9). In the general case where the business income tax authorities in one or both countries impose the arbitrary transfer price w^b on the multinational, the firm will choose the x^b that solves $\max_x (1 - T_1)r^1(p^1, x) - (1 - T_2)c^2(p^2, x) + (T_1 - T_2)wbx$. In the differentiable case, xb will satisfy the first order condition $(1 - T_1)\partial r^1(p^1, x^b)/\partial x - (1 - T_2)\partial c^2(p^2, xb)/\partial x = (T_1 - T_2)wb$. The economic transfer price that should be collected by the statistical agency in country 1 is the marginal revenue $\partial r^1(p^1, xb)/\partial x$ and the economic transfer price that should be collected by country 2 is the marginal cost $\partial c^2(p^2, xb)/\partial x$.

[25] This method for choosing a transfer price is roughly equivalent to the *resale price method* that is described in the transfer pricing literature as follows: "Under the resale price method, the tax auditor looks for firms at similar trade levels that perform similar distribution functions (i.e., a *functional comparable*). The RP method is best used when the distributor adds relatively little value to the product so that the value of its functions is easier to estimate" (Eden, 1998, p. 40).

this knowledge, then there will be an incentive for the multinational to choose an even higher transfer price than w^{****} in order to transfer profits out of the high tax jurisdiction.

19.54 In general, the results for the case in which the rate of business income taxation is lower in the exporting country than in the importing country can be summed up by stating that the transfer price chosen by the multinational (meeting the constraints imposed by the income tax authorities in both countries) will generally be *higher* than the efficient transfer price and that it will no longer be the case that the chosen transfer price equals marginal cost in the exporting country or marginal revenue in the importing country. Hence the chosen transfer price will no longer represent true opportunity costs in the two countries and hence *is not a suitable price to be collected* if we are applying the economic approach to index number theory.

19.55 The above results rely somewhat on the ability of the tax authorities in the two jurisdictions to be able to determine either the appropriate cost in the exporting country or the appropriate net revenue or markup in the importing country. Needless to say, in actual practice, it is difficult to determine costs or markups accurately. In the cost context, Eden described the situation as follows:

> In order to use the cost plus method, the tax authority or MNE [multinational enterprise] must know the accounting approach adopted by the unrelated parties. For example, what costs are included in the cost base before the mark-up over costs is calculated? Is it *actual cost* or *standard cost* (costs which have been standardized for cyclical fluctuations in production as in the example in Box 1.5)? Are only *manufacturing costs* (cost of goods sold, which includes labour, overhead costs, including depreciation, and material input costs) included or is the cost base the sum of manufacturing costs plus some portion of *operating costs* (i.e., selling, general and administrative (SG&A) expenses and R&D costs)? (1998, pp. 42–43)

19.56 There are additional problems in allocating the cost of capital to various products, including the problem of picking an appropriate benchmark rate of return to the firm's equity capital. Moreover, the problems involved in allocating joint costs over multiple outputs are difficult indeed.

19.57 The main message that has been delivered in this section is this: When there are differential rates of business income taxation in the two countries where two units of a multinational engage in international trade, then the transfer prices that are reported by the multinational are unlikely to represent true opportunity costs. Hence if the statistical agency is using the economic approach to index number theory, these reported transfer prices will generally be biased (and the direction of bias is indicated above).

19.58 In the following subsection, it is assumed that either business income taxation is absent in the two countries or that the rates are equal and instead, the focus is on the distortions induced by trade taxes.

D.2 Transfer pricing with trade taxes and no income taxes

19.59 The multinational's profit maximization problem is now considered in the case where there is no external market for the commodity (as in the previous Sections C.2 and D.1) and there are no business income taxes but there are *trade taxes*. It is assumed that the importing country (country 1) imposes a specific tax or tariff at the rate t_1 and an ad valorem tax at the rate τ_1[26] on each unit of x that is imported. It is also assumed that the exporting country (country 2) imposes a specific tax at the rate t_2 and an ad valorem tax at the rate τ_2[27] on each unit of x that is exported. If the multinational chooses the transfer price $w > 0$, then the multinational's *global profit maximization* problem is now

$$\begin{aligned}&\max_{x,w} \{r^1(p^1, x) - w(1+\tau_1)x - t_1 x\}\\&\quad + \{w(1-\tau_2)x - t_2 x - c^2(p^2, x)\}\\&= \max_{x,w} r^1(p^1, x) - c^2(p^2, x) - (t_1 + t_2)x\\&\quad - w(\tau_1+\tau_2)x. \qquad (19.31)\end{aligned}$$

19.60 Comparing equation (19.31) with the no tax profit maximization problem (19.8) in Section C.2, it can be seen that there are two differences:

- The ad valorem trade tax rates, τ_1 and τ_2, and the specific trade taxes, t_1 and t_2, lead to the terms $(t_1 + t_2)x$ and $w(\tau_1 + \tau_2)x$ in the objective function. In particular, the terms involving the transfer price w no longer cancel out as they did in equation (19.9) and

[26] If the imports are subsidized by country 1, then t_1 and τ_1 are negative or zero.

[27] If the exports are subsidized by country 2, then t_2 and τ_2 are negative or zero. Of course, some of these tax or subsidy rates could be zero.

- The multinational is now able to choose the transfer price w as well as the level of international trade in the intermediate input x; that is, instead of just maximizing with respect to x, the firm now maximizes with respect to x and w.

19.61 In order to solve the firm's global profit maximization problem, it is necessary to consider two cases, depending on whether the ad valorem trade taxes are jointly positive (this is the usual case) or jointly negative.

Case 1: Ad Valorem Trade Taxes Are Jointly Positive

19.62 In this case,

$$\tau_1 + \tau_2 > 0. \quad (19.32)$$

19.63 Looking at the second line of equation (19.31), it can be seen that the term $-w(\tau_1 + \tau_2)x$ is negative in this case. Note also, that this is the only term where w appears. Hence to maximize overall profits, the multinational will want to choose w to be as *small as possible*. If there are no constraints on the multinational, the optimal choice of w would be $w = 0$.[28] However, the trade tax authorities in at least one of the countries would almost certainly object to the solution $w = 0$. It is difficult to specify what transfer price the border tax officials will impose[29]; hence, it will be assumed that it is some positive number, say $w^b > 0$.

19.64 With this exogenous choice w^b for the transfer price w, the multinational's profit maximization problem (19.31) becomes

$$\max_x r^1(p^1, x) - c^2(p^2, x) - (t_1 + t_2)x - w^b(\tau_1 + \tau_2)x. \quad (19.33)$$

19.65 If the revenue and cost functions are differentiable and if x^b solves equation (19.33), then the following first order condition will be satisfied:

$$\partial r^1(p^1, x^b)/\partial x - \partial c^2(p^2, x^b)/\partial x = (t_1 + t_2) + w^b(\tau_1 + \tau_2). \quad (19.34)$$

19.66 If by chance, the sum of the *trade distortion terms* on the right-hand side of equation (19.34) is equal to zero so that

$$(t_1 + t_2) + w^b(\tau_1 + \tau_2) = 0, \quad (19.35)$$

then it can be seen that the solution to equation (19.33) is the efficient solution x^{**} to equation (19.8); that is, under assumption (19.35), it can be shown that $x^b = x^{**}$ and using equation (19.34), it is also the case that

$$\partial r^1(p^1, x^b)/\partial x = \partial c^2(p^2, x^b)/\partial x \equiv w^{**} \quad (19.36)$$

so that marginal revenue and marginal cost in the two establishments will be equal to the efficient transfer price w^{**}.

19.67 However, even in the case where equation (19.35) holds, it will not generally be the case that the imposed transfer price w^b is equal to w^{**}. Hence, in general,

$$\partial r^1(p^1, x^b)/\partial x \neq w^b; \partial c^2(p^2, x^b)/\partial x \neq w^b. \quad (19.37)$$

19.68 In the general case where $(t_1 + t_2) + w^b(\tau_1 + \tau_2) \neq 0$, then it will still be the case that the inequalities in equation (19.37) will hold; that is, in this case, it would only be by chance that marginal revenue or marginal cost in the two establishments equals the border authorities' acceptable transfer price w^b. Because the economic approach to index number theory requires that the transfer price in establishment 1 be set equal to the marginal revenue $\partial r^1(p^1, x^b)/\partial x$ and the transfer price in establishment 2 be set equal to the marginal cost $\partial c^2(p^2, x^b)/\partial x$, it can be seen that *the transfer price that is acceptable to the border tax authorities will not usually be an acceptable one for statistical purposes*.

19.69 Case 2 is now considered.

Case 2: Ad Valorem Trade Taxes Are Jointly Negative

19.70 In this case,

$$\tau_1 + \tau_2 < 0. \quad (19.38)$$

19.71 Looking at the second line of equation (19.31), it can be seen that the term $-w(\tau_1 + \tau_2)x$ is positive in this case. Note, as in the previous case, that this is the only term where w appears. Hence to maximize overall profits, the multinational will want to choose w to be as *large as possible*. If there are no constraints on the multinational, the optimal choice of w would be arbitrarily large.

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[28] In this case, the multinational would choose x to satisfy $\partial r^1(p^1, x)/\partial x - \partial c^2(p^2, x)/\partial x = (t_1 + t_2)$, which would not lead to the efficient allocation defined in Section C.2 unless the sum of the specific taxes were equal to zero; that is, unless $(t_1 + t_2) = 0$.

[29] The border tax authorities will usually not have access to the information possessed by the tax authorities in the two countries and so it will be difficult for them to impose the zero profits constraint on either establishment as in the previous section. The transfer price w^b may not actually be *imposed* by the border trade authorities but it must be *acceptable* to them.

19.72 However, the trade tax authorities in at least one of the countries would almost certainly object to this arbitrarily large transfer price and so again, they will (explicitly or implicitly) impose some acceptable transfer price $w^b > 0$. With this exogenous choice for the transfer price w, the multinational's profit maximization problem (19.31) becomes equation (19.33) and the rest of the analysis proceeds as in the previous case. Thus there is little difference in this case compared to the previous case except in the present case, the multinational will want to choose an acceptable transfer price w^b that is as *large* as possible, whereas in the previous case, the multinational wanted to choose an acceptable transfer price that was as *small* as possible. In either case, it can be seen that *the transfer price that is acceptable to the border tax authorities will not usually be an acceptable one for statistical purposes*.

19.73 The analysis presented in Section D shows that the considerations that go into choosing transfer prices are very complex and are driven mainly by the desire of multinational firms to minimize both trade taxes and business income taxes.[30] On the other hand, governments have strong incentives to force multinationals to choose transfer prices that will enhance their tax revenues. Moreover, often the tax authorities in the same country can have opposing incentives to impose either a high or low transfer price: In order to maximize ad valorem tariff revenue, the customs authorities in the importing country may want to impose a transfer price that is as high as possible while the business income tax authorities in the same country may want to impose a transfer price that is as low as possible. What should the international trade price statistician do under these circumstances? Some suggested answers are presented in the following section.

E. Which Transfer Prices Can Be Usefully Collected by Statistical Agencies?

E.1 General considerations

19.74 As can be seen from the above material, the validity of using transfer prices, either those reported by the respondent or those constructed from outside, is rather suspect. The price associated with a transaction between unaffiliated parties reveals very useful information. In this situation, one party wishes to make the price as small as possible while the other party wishes to make it as large as possible. For the minimizing party (the importer), the price should not exceed the marginal revenue that can be generated by the last unit of the imported commodity. For the maximizing party (the exporter), the price should not be less than the full marginal cost of producing the last unit of the sale. However, the transfer price that is used to value international trades between affiliated establishments in general tells us nothing about marginal costs and marginal revenues: This transfer price will be chosen *strategically* by a profit maximizing multinational in order to maximize its global after-tax profits. Hence, in general, it will not be useful for a statistical agency to collect such a strategically chosen transfer price. Nor would it be easy to construct one. What then should be collected?

19.75 For income taxation and customs valuation purposes, the Organization for Economic Cooperation and Development (OECD) Transfer Pricing Guidelines (OECD, 2001b) recommend that multinational enterprises and national tax authorities follow the *arm's-length standard*, that is, set the transfer price equal to the price that two unrelated parties would negotiate when trading the same or substantially similar products under the same or substantially similar circumstances.

19.76 All OECD countries, and many non-OECD member countries, follow the OECD guidelines by requiring multinationals to report their transfer prices using the *best method* (i.e., the most appropriate method given the facts and circumstances) selected from a set of acceptable transfer pricing methods; see Eden (1998, pp. 549–61) and Feinschreiber (2004). Key to selection of the best method is the concept of *comparability.* Transactions are considered comparable when their "economically relevant characteristics" are the same, or if they differ, the differences have no material impact on the results. The attributes of a transaction that can affect comparability are

- Specific characteristics of the traded product (e.g., weight, quality, product maturity, whether intangibles are bundled with tangibles)
- Functions performed by the parties to the transaction (e.g., manufacturing, distribution, purchasing, marketing)

[30]The analytic framework presented in Sections D.1 and D.2 can be combined to cover situations where there are both trade taxes and business income taxes. This is done in Eden (1998, pp. 302–05) and in Diewert, Alterman, and Eden (2005). However, for present purposes, it is not necessary to present this material; the reader can already see that with both trade and income taxes present, the transfer prices chosen by multinationals are unlikely to be suitable for statistical purposes.

- Contractual terms of the transaction (e.g., warranties, rights, payment and credit terms)
- Economic circumstances of the parties (e.g., wholesale versus retail level, geographic location and relative size of the markets, market competition)
- Business strategies of the parties (e.g., market penetration strategies) (OECD, 2001b, Chap. I, paragraphs 1.19–1.35)

19.77 In practice, because internal and external transactions are unlikely to be exact comparables, the OECD guidelines recommend that material differences be identified, quantified, and adjusted for in determining the arm's-length transfer price. Moreover, because transfer pricing is not an exact science, the guidelines recommend that transfer prices be set inside a range of acceptable arm's-length prices, called the *arm's-length range.*

19.78 The arm's-length transfer price can be measured using either an internal comparable or external comparable to the intra-firm transaction. An *internal or in-house comparable* is a product traded by the multinational on both the internal and external markets, under substantially the same or similar circumstances. For example, a Ford affiliate might buy an auto part from a sister subsidiary and also buy the same part from an arm's-length supplier. An *external comparable* is a transaction, similar to the intra-firm transaction, that occurs between two unrelated firms. For example, the transfer price for an auto part traded between two Ford affiliates could be proxied by an arm's-length transaction between Toyota and General Motors.

19.79 Tax authorities typically view an internal or in-house comparable where the multinational sells (buys) the same product from an unaffiliated firm as it sells (buys) in-house as having a higher degree of comparability, in general, than an external comparable. There is a higher probability that the facts and circumstances are the same or similar (or, alternatively, a lower probability of potential errors and omissions) for in-house comparables, and thus, the "economically relevant characteristics of the transaction" are more likely to be the same.

19.80 There are five acceptable transfer pricing methods for *income tax purposes* in many jurisdictions: comparable uncontrolled price (CUP), the cost plus method, the resale price (minus) method, the transactional net margin method (TNMM),[31] and the profit split method.[32] *Customs authorities* typically require merchandise imports to be priced using one of three methods: transaction value (equivalent to CUP),[33] computed value (similar to cost plus),[34] or deductive value (similar to resale price).[35] Most national tax authorities, but not all (e.g., the United States), rank these methods, with CUP for income tax purposes and transaction value for customs valuation being preferable to the others because they most closely fulfill the conditions required by the arm's-length standard.

19.81 The reasoning developed by income tax and customs valuation authorities should have relevance for statistical agencies. Although their purposes in collecting transfer prices may not be the same, the agencies share a desire for pricing to reflect economically relevant characteristics of the market. This line of reasoning is followed below, in developing recommendations for selecting transfer prices to be used in international price index programs.

19.82 Because an international prices program produces *indices* rather than *actual prices*, the statistical agency's goals are somewhat different from those of income tax and customs authorities. First, in the case of taxes and tariffs, the relevant agencies do want to know the exact price paid or payable in order to determine the applicable tax on the transaction or profits from the transaction. The arm's-length standard is designed to ensure that the multinational sets a transfer price that proxies the price that would be selected by unrelated firms. Second, customs and tax authorities are interested in a particular firm and taxing its transactions or profits.

19.83 On the other hand, for the purpose of calculating national export and import price indices, actual transaction prices are required as the first step in calculating *price changes of product groups (ELIs, entry level items) aggregated across firms.* Thus, *movements* in prices are more important than the *level* of prices, and *representative* firms and transactions are more important than any *individual* firm or transaction. Thus, the appropriate transfer pricing methods for calculating international price indices may differ

[31]In the United States, TNMM is replaced by the comparable profit method. The IRS argues that the two methods are basically equivalent.

Others argue there are recognizable differences. See Eden (1998) and Feinschreiber (2004).

[32]All of these methods are described in more detail in Eden (1998).

[33]This corresponds to the external market price w described in Section C.1.

[34]This corresponds to the transfer price defined by equation (19.24).

[35]This corresponds to the transfer price defined by equation (19.29).

somewhat from those for calculating income taxes and customs duties, even if all three agencies adhere to the arm's-length standard. With these considerations in mind, possible price collection strategies for exports and imports are explored in Sections E.2 and E.3.

E.2 Collecting transfer prices for exports

19.84 For an *exporting establishment*, the following ordering of alternative collection strategies is recommended, in order of their merit, starting with the best method first. The ranked methods are

- Internal comparable,
- Externally referenced comparable,
- External comparable,
- Downstream (or upstream) internal transactions, and
- Declared transfer price.

Each of these alternatives is explained in more detail in the paragraphs below.

19.85 (1) *Internal comparable:* If the same or a sufficiently similar product is sold by the multinational, under the same or substantially similar circumstances, to an unaffiliated third party during the reference period, use that price[36] for the sales of the commodity to affiliated parties rather than the transfer price. Where the same product is exported under the same circumstances to both affiliated and unaffiliated firms, there is an *exact internal comparable* (or *exact internal CUP*)[37] transaction. Where the circumstances are sufficiently similar and differences can be identified, quantified, and adjusted, there is an *inexact internal comparable* (or *inexact internal CUP*). It does not matter whether the unaffiliated sales are domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for. Exact comparables are, of course, preferable to inexact comparables, where they exist.

[36]It may be necessary to make adjustments for transport costs and alternative tax treatments.

[37]For example, suppose Ford-US exports finished cars to an affiliated distributor in Germany and to an arm's-length distributor in France. If wholesale trade between France and Germany in finished cars is unrestricted, it may be possible to use the arm's-length price in France, adjusted for material differences, to proxy for the transfer price to the German affiliate.

19.86 (2) *Externally referenced comparable:* If alternative (2) is not available, but there is a recognized domestic or international exchange (e.g., the London Metal Exchange, the Chicago Mercantile Exchange) that trades in the product, use the corresponding price on the exchange for the reference period, making any necessary adjustments to ensure that the economically relevant characteristics of the transactions are sufficiently similar. It does not matter whether the reference exchange is domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for.[38]

19.87 (3) *External comparable:* If alternatives (1) and (2) are not available, attempt to find a foreign or domestic market price for the product traded between two unaffiliated traders under the same or substantially similar circumstances, and make adjustments for any material differences.[39] It does not matter whether the external transaction is domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for.

19.88 (4) *Downstream (and upstream) internal transactions:* If alternatives (1), (2), and (3) are not available, then attempt to collect the first arm's-length price of a downstream product that uses the intermediate good as a major input. In some cases, the intra-firm transaction will be in unfinished parts or subassemblies that undergo further processing in the foreign affiliate prior to final sale. If the final product is sold through different channels to many downstream buyers, possibly located in different countries, it may be difficult to trace and identify appropriate transactions for comparison purposes. In such cases, it may be possible to trace downstream transactions in the domestic market. The closer the exported product is to final sale, the more likely it should be to obtain downstream internal comparables.[40]

[38]The primary differences are likely to be additional costs for transportation, insurance, and foreign currency transactions.

[39]Such prices may be available from the country's PPI program or from industry sources. Note that this strategy might imply a cooperative collection strategy with other countries.

[40]It may also be possible to determine an arm's-length price by going upstream from the intra-firm transaction. This would be appropriate only in cases where little additional value is added in moving to the downstream stage. This situation resembles the cost plus method, whereby a gross profit margin for arm's-length firms is added to costs to determine the transfer price. For the purposes of an international prices program, the relevant question is whether upstream costs move in a similar fashion to costs of the intra-firm product. For example, related prices collected for a domestic PPI might be a relevant substitute for some intra-firm transactions where there are no external comparables.

19.89 (5) *Declared transfer price:* If none of the above alternatives are available, the international price program should collect the exporting firm's listed transfer price along with a brief description of its type. The data collector should also determine if the transfer price is market-based or cost-based. If the latter, the collector should identify whether a profit component is attached or not (the "plus" in cost plus, as compared to standard or actual cost without any "plus").[41]

E.3 Collecting transfer prices for imports

19.90 For an *importing establishment*, the following ordering of alternative collection strategies is recommended, in order of their merit, starting with the best method first. The ranked methods are

- Internal comparable,
- Externally referenced comparable,
- External comparable,
- Downstream (or upstream) internal transactions, and
- Declared transfer price.

Each of these alternatives is explained in more detail in the paragraphs below.

19.91 (1) *Internal comparable:* If the *same commodity* is purchased from an unaffiliated third party during the reference period, then use that price[42] for the purchases of the commodity from affiliated parties rather than the transfer price. It does not matter whether the unaffiliated purchases are domestic or international for most purposes. Where the same product is imported under the same circumstances from both an affiliated and an unaffiliated firm, there is an *exact internal comparable*. Where the circumstances are sufficiently similar and differences can be identified, quantified, and adjusted, there is an *inexact internal comparable*. It does not matter whether the unaffiliated purchases are domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for. Exact internal comparables are, of course, preferable to inexact internal comparables, where they exist.

19.92 (2) *Externally referenced comparable:* If alternative (1) is not available, but there is a recognized domestic or international exchange (e.g., the London Metal Exchange, the Chicago Mercantile Exchange) that trades in the product, use the price on the exchange for the reference period, making any necessary adjustments to ensure that the economically relevant characteristics of the transactions are sufficiently similar. It does not matter whether the reference exchange is domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for.

19.93 (3) *External comparable:* If alternatives (1) and (2) are not available, attempt to find a foreign or domestic market price for the product traded between two unaffiliated traders under the same or substantially similar circumstances, and make adjustments for any material differences. It does not matter whether the external transaction is domestic or international for most purposes, as long as differences can be identified, quantified, and adjusted for.

19.94 (4) *Downstream (or upstream) internal transactions:* If alternatives (1), (2), and (3) are not available, then attempt to collect the first arm's-length price of a downstream product that uses the intermediate good as a major input. In the simplest case, the imported good will be a finished good and the downstream sale will be to arm's-length distributors.[43] In more complex cases, the intra-firm transaction will be in unfinished parts or subassemblies that undergo further processing in the importing affiliate prior to final sale. It is possible in such cases that the final product will be sold through different channels to many downstream buyers, not only in the United States but also abroad, making it difficult to trace and identify appropriate transactions for comparison purposes. The closer the imported product is to the finished stage, the more likely it should be to obtain downstream internal comparables. Again, similar to the situation with exports, it may be possible to find arm's-length upstream prices that can be used to

[41]Most intermediate transfers are at mandated full costs for cost centers, and at full cost plus a profit markup for profit centers; see Feinschreiber (2004, p. 18). It would also be useful to know whether the affiliate had the responsibility for setting the transfer pricing, either wholly or shared with its trading partner, or whether the transfer price was mandated by the parent firm.

[42]It may be necessary to make adjustments for transport costs and alternative tax treatments.

[43]For example, if Toyota-Japan sells a finished car to Toyota-US, the United States' International Price Program (IPP) collects the selling price from Toyota-US to an unaffiliated Toyota dealer in the United States and adjust for the difference in trade levels. Although this looks similar to the resale price method for tax and customs duty purposes, the key difference is that the import price index is not interested in calculating the gross margin, per se, but rather in whether movements of the downstream price to the arm's-length U.S. distributor is a good proxy for movements in the transfer price of exported finished cars from Toyota-Japan to Toyota-US.

replace the transfer price of the imported product (but this would typically involve cooperation on the part of statistical agencies in the exporting country although an Internet search for prices is a possibility).

19.95 (5) *Declared transfer price:* If none of the above alternatives are available, the international price program should collect the importing firm's listed transfer price along with a brief description of its type. The data collector should also determine if the transfer price is market-based or cost-based. If the latter, the collector should identify whether a profit component is attached or not.

E.4 Practical issues in selecting the best method

19.96 The above two sections have outlined, and ranked, five methods for determining an acceptable transfer price for constructing international price indices: (1) internal comparable, (2) externally referenced comparable, (3) external comparables, (4) downstream (and possibly upstream) internal transactions, and (5) the declared transfer price. In this section, some of the issues of moving from theory to practice are discussed.

19.97 First, note that the transfer pricing methods outlined above bear a close resemblance to those recommended by the OECD Transfer Pricing Guidelines (2001b) in that they stress the importance of comparing economically relevant characteristics, and typically (but not always) prefer internal to external comparables. *In such cases, if the multinational states that its transfer price does follow the same method as the home country statistical agency is using, that transfer price should be collected.* For example, collection strategy (1), internal comparables, is the top preferred method for intra-firm exports. If the multinational states that its transfer price is based on an exact or inexact internal comparable, the transfer price should be collected by the statistical agency.

19.98 Second, in comparing which collection strategy should be selected by the statistical agency, the issues of *feasibility and administrative costs* are important considerations. For example, suppose the multinational respondent states that its declared transfer price follows method (3), an external comparable on a reference exchange. Although method (1), internal comparable, is theoretically preferable, the costs and time involved in collecting this information may outweigh the additional reliability. The statistical agency should, in these situations, collect the declared transfer price that follows method (3), even where in theory method (1) is preferable. Similarly, if the multinational's transfer price is based on method (4), accepting that transfer price may be preferable to the additional time and costs involved in determining the arm's-length price using method (1) or (2).[44]

19.99 Third, it might be thought that the multinational's posted transfer price would be acceptable for statistical purposes, provided that the multinational uses the *same* set of transfer prices for both management and tax purposes. In a recent survey, Ernst and Young (2001, p. 6) reported that 77 percent of multinationals responding to their survey used the same set of transfer prices for both purposes,[45] which seems encouraging at first glance because transfer prices for managerial purposes should approximate economic transfer prices based on opportunity costs. However, the same Ernst and Young survey also reveals that these dual-purpose transfer prices are frequently heavily influenced by tax considerations.[46] Thus, the existence of one—as opposed to two—sets of books is insufficient justification for accepting the multinational's stated transfer price for purposes of calculating XMPIs. Rather, the key issue is whether the multinational uses an economically acceptable transfer pricing methodology.

19.100 Fourth, although in most cases, strategy (1) will be preferred, there are circumstances when this strategy may not be very reliable, such as when the sales to the unaffiliated parties are relatively small and are at prices that are "abnormally" high or low.

19.101 Finally, if there are no sales to unaffiliated parties during the reference period, then as indicated above, it was recommended that methods (2) or (3) be used. However, these (2) and (3) collection strategies can fail under some circumstances:

- The exporting establishment may be shipping a proprietary product to units of a multinational firm in

[44]Moreover, given the reluctance of firms to share transfer pricing information when compliance is voluntary, the statistical agency may fail to collect any price if the firm perceives a request for detailed information as a "fishing expedition" or something that could potentially be used by another agency to collect more income or trade taxes.

[45]"According to the 2001 survey responses, over three quarters of multinational corporations parents (77%) reported using the same set of transfer prices for both tax and management purposes" (Ernst and Young, 2001, p. 6).

[46]"Of those using the same transfer price, about half (52%) use a compromise between satisfying tax requirements and achieving management/operational objectives, while a quarter (26%) and a fifth (21%) base it primarily on tax management/operations respectively" (Ernst and Young, 2001, p. 6).

other countries for further processing and there is no openly traded market for the product anywhere in the world.[47]

- The open market for trades in the product may be small and unrepresentative of the bulk of the trades in the commodity, or price movements in this open market could be very volatile.

F. Conclusion

19.102 The theoretical parts of this chapter attempted to show that in a world where there are taxes on international transactions or where the rates of business income taxation differ across countries, then a multinational enterprise has financial incentives to *strategically choose* a transfer price to reduce the amount of taxation paid in the importing and exporting countries. This strategically chosen transfer price will generally be very different from an economic transfer price (based on opportunity costs) that would be suitable for an import or export price index. Because international trade between affiliated units is somewhere in the neighborhood of 30 to 40 percent of world trade, it can be seen that this problem of determining appropriate transfer prices is a huge one.

19.103 The first best alternative to the firm's listed transfer price is an internal comparable, that is, the average price paid to (for an imported commodity) or received from (for an exported commodity) unaffiliated firms for the same commodity during the reference period, if such unaffiliated purchases or sales exist. If there are no such unaffiliated purchases or sales, then the use of an externally referenced comparable, that is, the price of the commodity on a recognized exchange that trades in the commodity if such an exchange exists, was recommended. If no such exchange exists, then attempting to find an external comparable price based on transactions between unaffiliated traders was recommended. These three methods all focus on the price of the same product traded by different firms. Where this is impossible, the price collector could look at downstream prices, or potentially upstream prices, to see whether an economically acceptable price can be found. Finally, if there are no internal or external comparables, at the same or different levels of the value chain, the international price index should use the multinational's stated transfer price.

19.104 The above recommendations for the collection of transfer prices may appear to be somewhat radical, given that it was recommended that the multinational's listed transfer prices be used only as a last resort. Because the multinational must develop and use transfer prices that meet the arm's-length standard, for both income tax and customs duty valuation purposes, it is possible that the declared transfer price does satisfy one of the preferred recommended methods. However, a multinational that is attempting to maximize its after-tax profits has an incentive to choose relatively extreme transfer prices that will reduce its tax liabilities, suggesting that the reported transfer price may not meet any of the best method tests outlined above. Even in this situation, both time and financial constraints may change the ranking of the acceptable methods or make the multinational's stated transfer price the only practical alternative.

[47]In this case, if the proprietary product is a major component of a product that is traded between unaffiliated parties, then the price of this latter product could be used as a proxy for the transfer price.

20. Exports and Imports from Production and Expenditure Approaches and Associated Price Indices Using a Simplified Example and an Artificial Data Set

A. Introduction

20.1 Chapters 16 to 18 outlined alternative price index number formulas, the factors that determine the nature and extent of differences between their results, and the criteria for choosing among them. The criteria for choosing among the formulas included the fixed-basket, axiomatic, stochastic, and economic theoretic approaches. The first purpose of this chapter is to give the reader some idea of how much the major indices defined in the previous chapters differ using an artificial data set consisting of prices and quantities for six commodities over five periods. The period can be thought of as somewhere between a year and five years. The trends in the data are generally more pronounced than one would see in the course of a year. The six commodities can be thought of as the deliveries to the domestic final demand sector of all industries in the economy.

20.2 Chapter 15 showed how the nominal values of, and thus price indices for, exports and imports fit into the *2008 System of National Accounts* (*2008 SNA*).[1] Particular emphasis was given to the role of price indices as deflators for estimating volume changes in GDP by the expenditure approach. The second purpose of this chapter is to outline how price indices for exports and imports can be defined and reconciled from the expenditure and production approaches to estimating GDP. Indeed, the illustrative data used to outline and demonstrate differences in the results from different index number formulas are applied not only to export and import price indices (XMPIs) but also to price indices for the constituent aggregates of GDP from both the expenditure and production approaches.

[1] In this *Manual,* the *2008 SNA* (Commission of the European Communities and others, 2008) refers to the final draft of Volume 1 (Chapters 1–17) of the updated *System of National Accounts* adopted by the 39th session of the United Nations Statistical Commission, 26–29 February 2008, available at http://unstats.un.org/unsd/sna1993/draftingphase/ChapterList.asp. The *2008 SNA* is an updated version of the 1993 *SNA* (Commission of the European Communities and others, 1993).

20.3 There is a clear relationship in the *2008 SNA* between GDP estimates from these two approaches that derives from the well-known identity between the sources and uses of goods and services as depicted in the *2008 SNA*'s goods and services account. On the left-hand side of the account the total amount of *resources* available to the domestic economy consists of the sum of outputs and imports and this is equal, on the right-hand side, to the total amount *used* for consumption, investment, and exports; that is,

$$O + M + (t - s) = IC + C + I + G + X, \quad (20.1)$$

where O is the value of output of goods and services, M is the value of imports of goods and services, IC is the value of goods and services used in the production process (intermediate consumption), C is final consumption expenditure of households and nonprofit institutions serving households (NPISHs),[2] I is gross capital formation, G is final consumption expenditure of government, X is the value of exports of goods and services, and t and s are taxes and subsidies on products. Goods and services emanate from their original producers, either resident producers or producers abroad, for use by either resident users or users abroad.

20.4 Moving intermediate consumption from the right-hand side of the account to the left, as a negative resource, while moving imports from the left to the right as a negative use, results in both sides now summing to GDP. The left-hand side presents the production approach and the right-hand side presents the expenditure approach.

$$O - IC + (t - s) = C + I + G + (X - M). \quad (20.2)$$

20.5 Exports and imports are explicitly identified in the expenditure approach, but this is not the case

[2] NPISHs are legal entities principally engaged in the production of nonmarket services for households whose main resources are voluntary contributions by households, such as charities and trade unions.

in the production approach. The production account in the *2008 SNA* does not break *O* and *IC* down into output to the domestic market and the rest of the world.

20.6 GDP estimated from the *production approach* is based on the *value added* to the value of goods and services used in the production process (intermediate consumption), *IC*, to generate the value of output, *O*. GDP can be thought of as being equal to the sum of the value added produced by all institutional units resident in the domestic economy. The output is valued at basic prices to exclude taxes and subsidies, *t* and *s*, on products, while intermediate consumption and all other aggregates in the above equations are valued at purchasers' prices to include them. Taxes less subsidies on products need to be added back to value added to ensure that the values of what are supplied and used are equal. GDP is defined from the production approach on the right-hand side of equation (20.2), therefore, as the sum of value added by resident producers plus the value of taxes less subsidies on products.

20.7 The *expenditure approach* involves summing the values of final consumption and gross capital formation (i.e., gross fixed capital formation, changes in inventories, and net acquisition of valuables[3]). These final expenditures do not properly represent all domestic economic activity because they exclude that directed to nonresidents, that is, exports, and include that arising from of nonresidents, that is, imports: Exports and imports are respectively added to and subtracted from final consumption expenditure and capital formation on the right-hand side of equation (20.2) to estimate GDP.

20.8 It was noted above that a second purpose of this chapter is to outline how price indices for exports and imports can be defined and reconciled from the expenditure and production approaches to estimating GDP. Further, the illustrative data used in this chapter to outline and demonstrate differences in the results from different formulas are applied not only to export and import price indices, but also to the constituent aggregates of GDP from both expenditure and production approaches, as indicated in equations (20.1) and (20.2). The representation of the two approaches in equation (20.2) is simplistic because the aggregates are not broken down by commodity detail. Chapter 6 of the *2008 SNA* provides details of the production account but does not elaborate on which industries are actually using the imports or on which industries are actually doing the exporting by commodity. Table 14.12 in the *2008 SNA* is an illustration of the supply and use tables and includes detailed information on output and intermediate consumption. Although Table 14.15 provides details on the amount of imports going to intermediate consumption, final consumption, and capital formation, there is no similar analysis of the amount of output going to exports. Hence, the main additions to the *2008 SNA* Chapters 6 and 14 for *XMPI Manual* purposes are to add tables to the main production accounts that provide industry by commodity detail on exports and imports. With these additional tables on the industry by commodity allocation of exports and imports, the resident's approach to collecting export and import price indices can be embedded in the *SNA* framework.

20.9 The focus of Section B is to outline an expanded production account that includes its constituent commodity detail, in Section B.1, and, in Section B.2 expanded input output tables and a reconciliation of the expanded production and expenditure GDP estimates, with the effects of taxes and subsidies included in Section B.3. *The account given is of a simplified economy for illustration* and Chapter 15 of this *Manual* sets out the framework more formally.

20.10 Section C provides illustrative data for the framework in Section B. The data are used not only to illustrate how XMPIs differ according to the index number formula used, but to embed the price index numbers for exports and imports into an illustrative framework for the deflation of the constituent aggregates of GDP and GDP as a whole from both the expenditure and production frameworks.

20.11 Illustrative data on domestic final demand deliveries are provided for a model of production. There are three industries in the economy and in principle, each industry could produce and use combinations of the six final demand commodities plus an additional imported "pure" intermediate input that is not delivered to the domestic final demand sector. In Section C, the basic industry data are listed in the input output framework that was explained in Section B; that is, there are separate supply and use matrices for domestically produced and used commodities and for internationally traded commodities.

[3]This third category of capital formation, net acquisition (i.e., acquisitions less disposals) of valuables, includes precious stones and metals and paintings used as "stores of value" and not for consumption or production.

20.12 To summarize: Price and quantity data for three industrial sectors of the economy are presented in Section C. This industrial data set is consistent with the domestic final demand data set outlined in Section D. A wide variety of indices are computed in Section D using this final demand data set.

20.13 Section E constructs domestic gross output, export, domestic intermediate input, and import price indices for the aggregate production sector. Only the Laspeyres, Paasche, Fisher, and Törnqvist fixed-base and chained formulas are considered in Section E and subsequent sections because these are the formulas that are likely to be used in practice. The data used in Sections E, F, and G are at producer prices; this means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs.

20.14 In Sections F.1 through F.3, value-added price deflators are constructed for each of the three industries. A national value-added deflator is constructed in Section F.4.

20.15 Section G compares alternative two-stage methods for constructing the national value-added deflator. This deflator can be constructed in a single stage by aggregating the detailed industry data (and this is done in Section F.4) or it can be constructed in two stages by either aggregating the three industry value-added deflators (see Section F.1) or aggregating the gross output, export, intermediate input, and import price indices that were constructed in Section F (see Section G.2). These two-stage national value-added deflators are compared with each other and their single-stage counterpart.

20.16 Finally, in Section H, final demand purchasers' prices are used in order to construct domestic final demand price indices (Section H.1), export price indices (Section H.2), and import price indices (Section H.3). In Section H.4, national GDP price deflators are constructed using final demand prices. Finally, in Section H.5, the national value-added deflator, which is constructed using producer prices, is compared to the national GDP deflator, which is constructed using final demand prices. This section also shows how these two national deflators can be reconciled with each other, provided that detailed industry by commodity data on commodity taxes and subsidies are available.

B. Expanded Production Accounts for the Treatment of International Trade Flows

B.1 Introduction

20.17 In order to set the stage for the economic approaches to the XMPIs from the resident's perspective, it is necessary to provide a set of satellite accounts for the production accounts in the *2008 SNA*. It turns out that the *2008 SNA* treatment of the production accounts is not able to provide an adequate framework for introducing a producer-based economic theory of the XMPIs that would be analogous to the economic producer price indices (PPIs) that were introduced in the *PPI Manual* (ILO and others, 2004b).

20.18 There is an extensive national income accounting literature on how to measure the effects of changes in the terms of trade (the export price index divided by the import price index) on national welfare.[4] However, Kohli (1978 and 1991)[5] observed that most international trade flows through the production sector of the economy and hence a natural starting point, useful for the illustrative needs of this chapter, for developing XMPIs is to embed exports and imports in the production accounts of an economy.

20.19 There are two main differences between the production accounts that are introduced in this chapter and the production accounts that are described in the *2008 SNA:*

- The commodity classification is expanded to distinguish between domestically used and produced goods and services and internationally traded goods and services that flow through the production sector.
- The single supply of products and single use of products matrices (the supply and use matrices) that appear in the *2008 SNA*[6] are in principle replaced by a series of supply and use matrices so that the bilateral transactions of each industry with each one of the remaining industries can be distinguished.[7]

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[4]See Diewert and Morrison (1986) for references to this early literature.

[5]This production theory approach to modeling trade flows was also used by Diewert (1974a, pp. 142–46); Woodland (1982); Diewert and Morrison (1986); Alterman, Diewert, and Feenstra (1999); and Feenstra (2004, pp. 64–98).

[6]See Table 14.12 in *2008 SNA*.

[7]This is what was done in Chapter 18 of the *PPI Manual;* see ILO and others (2004b, pp. 463–507).

Table 20.1. Domestic Supply Matrix in Current Period Values

	Industry *G*	Industry *S*	Industry *T*
G	$p_G^{GS}y_G^{GS} + p_G^{GT}y_G^{GT} + p_G^{GF}y_G^{GF}$	0	0
S	0	$p_S^{SG}y_S^{SG} + p_S^{ST}y_S^{ST} + p_S^{SF}y_S^{SF}$	0
T	0	0	$p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + p_T^{TF}y_T^{TF}$

There is also some discussion of the role of transport in the input output tables because imports and exports of goods necessarily involve some use of transportation services.

B.2 Expanded input output accounts with no commodity taxation

20.20 In this section, a set of production accounts is developed for the production sector of an economy that engages in international trade. In order to simplify the notation, there are only three industries and three commodities in the commodity classification. Industry *G* (the goods producing industry) produces a composite good (commodity *G*), industry *S* produces a composite service that excludes transportation services (commodity *S*), and industry *T* provides transportation services (commodity *T*). In addition to trading goods and services between themselves, the three industries also engage in transactions with two final demand sectors:

- Sector *F*, the domestic final demand sector and
- Sector *R*, the rest of the world sector.

20.21 The three industries deliver goods and services to the domestic final demand sector *F*.[8] They also deliver goods and services to the rest of the world sector *R*[9] and they utilize deliveries from the rest of the world sector as inputs into their production processes.[10]

20.22 The structure of the flows of goods and services between the three production sectors and the two final demand sectors is shown by four value flow matrices in Tables 20.1 through 20.4.

20.23 Table 20.1 shows the value of the *gross output deliveries* to the domestic final demand sector *F* as well as the deliveries of each industry to the remaining two industries: It is the *domestic supply matrix* or *domestic gross output by industry and commodity matrix* for a particular period of time. The industry *G*, *S*, and *T* columns list the sales of goods and services to all domestic demanders for each of the three commodities.

20.24 The value sum in row and column *G*, $p_G^{GS}y_G^{GS} + p_G^{GT}y_G^{GT} + p_G^{GF}y_G^{GF}$, corresponds to the revenues received by the goods producing sector from its sales of good *G* to the service sector, $p_G^{GS}y_G^{GS}$, where p_G^{GS} is the price of sales of good *G* to sector *S* and y_G^{GS} is the corresponding quantity sold,[11] plus the revenues received by the goods producing sector from its sales of good *G* to the transportation sector, $p_G^{GT}y_G^{GT}$, where p_G^{GT} is the price of sales of good *G* to sector *T* and y_G^{GT} is the corresponding quantity sold, plus the revenues received by the goods producing sector from its sales of good *G* to the domestic final demand sector, $p_G^{GF}y_G^{GF}$, where p_G^{GF} is the price of sales of good *G* to sector *F* and y_G^{GF} is the corresponding quantity sold. Similarly, the value sum in row and column *S*, $p_S^{SG}y_S^{SG} + p_S^{ST}y_S^{ST} + p_S^{SF}y_S^{SF}$, corresponds to the revenues received by the service sector from its sales of service *S* to the goods producing sector, the transportation sector, and the domestic final demand sector. Finally, the value sum in row and column *T*, $p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + p_T^{TF}y_T^{TF}$, corresponds to the revenues received by the transportation sector from its sales of transportation services *T* to the goods producing sector, the general services sector, and the domestic final demand sector. It should be mentioned that these transportation prices are margin-type prices; that is, they are the prices for delivering goods from one point to another.[12] Note also that p_G^{GS} will usually not equal p_G^{GT} or p_G^{GF}; that is, for a variety of reasons, the average selling price of the domestic good to the three sectors that demand the good will usually be different and a similar comment applies to the other commodity

[8]These deliveries correspond to the familiar $C + I + G$ final demand sectors.

[9]These deliveries correspond to exports, *X*.

[10]These deliveries correspond to imports, *M*.

[11]Under the assumption that there are no quality differences between units of *G*, the appropriate price will be a unit value and the corresponding quantity will be the total quantity of *G* purchased by sector *S* during the period.

[12]For a more detailed analysis of how transport quantities are tied to shipments of goods from one sector to another, see Diewert (2006).

Table 20.2. Domestic Use Matrix in Current Period Values

	Industry G	Industry S	Industry T
G	0	$p_G^{GS}y_G^{GS}$	$p_G^{GT}y_G^{GT}$
S	$p_S^{SG}y_S^{SG}$	0	$p_S^{ST}y_S^{ST}$
T	$p_T^{TG}y_T^{TG}$	$p_T^{TS}y_T^{TS}$	0

prices.[13] Unfortunately, this means that we cannot use a common price for a commodity across sectors to deflate the value flows in *2008 SNA* Tables A1 and A2 into volume quantity flows by commodity; that is, basic prices that are constant across sectors will usually not exist. This is another reason why it is useful to extend the *2008 SNA* production accounts.

20.25 Table 20.2 shows the value of the purchases of *intermediate inputs* for each industry from domestic suppliers; it is the *domestic use matrix* or *domestic intermediate input by industry and commodity matrix*. Note that the value of purchases of goods from industry G by industry S, $p_G^{GS}y_G^{GS}$, is exactly equal to the value of sales of goods by industry G to industry S and this value appeared in the value of sales of goods G by industry G in Table 20.1. In fact, all of the domestic purchases of intermediate inputs listed in Table 20.2 have their domestic sales counterpart entries in Table 20.1.

20.26 Table 20.3 shows the value of the *gross output deliveries* to the rest of the world (ROW) final demand sector R; it is the *ROW supply matrix* or, more simply, the *export by industry and commodity matrix*.

20.27 The value sum in row and column G, $p_{Gx}^{GR}x_G^{GR}$, corresponds to the revenues received by the goods producing sector from its sales of good G to the ROW sector, where p_{Gx}^{GR} is the price of sales of good G to sector R and x_G^{GR} is the corresponding quantity sold, or more simply, it is the value of exports by the goods producing sector to the rest of the world.[14] Similarly, $p_{Sx}^{SR}x_S^{SR}$ is the value of exports of services produced by the services sector and $p_{Tx}^{TR}x_T^{TR}$ is the value of exports of transportation services produced by the transportation sector. The assumption is made here as a simplification that transportation services are separately invoiced and thus not included in the basic price of the good.[15] Note that not all of these transportation sector export revenues need be associated with the importation of goods into the domestic economy: Some portion of these revenues may be due to the shipment of goods between two or more foreign countries.

Table 20.3. Export or ROW Supply Matrix in Current Period Values

	Industry G	Industry S	Industry T
G	$p_{Gx}^{GR}x_G^{GR}$	0	0
S	0	$p_{Sx}^{SR}x_S^{SR}$	0
T	0	0	$p_{Tx}^{TR}x_T^{TR}$

Note: ROW denotes rest of the world.

Table 20.4. Import or ROW Use Matrix in Current Period Values

	Industry G	Industry S	Industry T
G	$p_{Gm}^{GR}m_G^{GR}$	$p_{Gm}^{SR}m_G^{SR}$	$p_{Gm}^{TR}m_G^{TR}$
S	$p_{Sm}^{GR}m_S^{GR}$	$p_{Sm}^{SR}m_S^{SR}$	$p_{Sm}^{TR}m_S^{TR}$
T	$p_{Tm}^{GR}m_T^{GR}$	$p_{Tm}^{SR}m_T^{SR}$	$p_{Tm}^{TR}m_T^{TR}$

20.28 Table 20.4 shows the value of the purchases of *intermediate inputs* or *imports* from the rest of the world for each industry by commodity; it is the *import* or *ROW use matrix* or *ROW intermediate input by industry and commodity matrix*.

20.29 The value of imports in row and column G, $p_{Gm}^{GR}m_G^{GR}$, corresponds to the payments to the rest of the world by the goods producing sector for its imports of goods, where p_{Gm}^{GR} is the price of imports of good G to industry G and m_G^{GR} is the corresponding quantity purchased, or more simply, it is the cost of imports of goods to the goods producing sector. Similarly, $p_{Sm}^{GR}m_S^{GR}$ is the value of imported services that are used

[13]Even if there is no price discrimination on the part of industry G at any point in time, the price of good G will usually vary over the reference period and hence if the proportion of daily sales varies between the three sectors, the corresponding period average prices for the three sectors will be different. The notation used here is unfortunately much more complicated than the notation that is typically used in explaining input output tables because it is not assumed that each commodity trades across demanders and suppliers at the same price. Thus the above notation distinguishes three superscripts or subscripts instead of the usual two: Two superscripts are required to distinguish the selling and purchasing sectors and one additional subscript is required to distinguish the commodity involved in each transaction. This type of setup was used in Chapter 19 of ILO and others (2004b).

[14]We make the general convention that the last *nontransportation* domestic establishment that handles an exported good is regarded as the sector that exports the good. If we did not make this convention, virtually all exported goods would be credited to the transportation sector. This convention is consistent with our treatment of transportation services as a margin industry.

[15]As outlined in Chapter 4 of this *Manual* and Chapter 14 of the *2008 SNA*.

in the goods producing sector and $p_{Tm}^{GR}m_T^{GR}$ is the value of imported transportation services that are used in the goods producing sector. Note that industry G may purchase and separately invoice transportation services from domestic or foreign suppliers and a similar comment applies to the purchases of transportation services by industries S and T. The imported value flows for industries S and T are similar to the corresponding import values for the goods producing industry.

20.30 The above four matrices are in terms of current-period values. The corresponding *constant* period values or volume *matrices* can readily be derived from the matrices listed in Tables 20.1 through 20.4: Simply drop all of the prices from the above matrices and the resulting matrices, which will have only quantities as entries in each cell, will be the corresponding constant dollar input output matrices. However, note that unless all prices are identical for each entry in each cell of a row, the correct volume entries will *not* be obtained in general by deflating each row of each matrix by a common price deflator. This observation means that statistical agencies that use the common deflator method to obtain volume input output tables from corresponding nominal input output tables may be introducing substantial errors into their estimates of volume value added by sector. In principle, each cell in a nominal use or make matrix will require a separate deflator in order to recover the corresponding correct volume entry.

20.31 The nominal value flow matrices defined by Tables 20.1 through 20.4 and their volume counterparts can be used to derive the traditional supply and use matrices that appear in Table 14.12 of the *2008 SNA:* The conventional supply matrix is the sum of the matrices in Tables 20.1 and 20.3 (the domestic and ROW supply matrices) and the conventional use matrix is the sum of the matrices in Tables 20.2 and 20.4 (the domestic and ROW use matrices). As outlined in Chapters 4 and 15 of this *Manual* and, in more detail, Chapter 14 of the *2008 SNA*, the derivation of supply and use tables in volume terms at the product group level provides a framework that not only facilitates the application of appropriate deflators to the product groups comprising exports and imports, but also enables a reconciliation of the deflators and volume estimates used across all supply and use aggregates at a product group level.

20.32 The matrix that is needed for XMPIs in this illustration can be obtained by adding entries in Tables 20.1 and 20.3 and then subtracting the corresponding entries in Tables 20.2 and 20.4 in order to obtain a *net supply matrix* that gives the value of net commodity supply by commodity and by industry of origin. The net supply matrix can be aggregated in two ways:

- By summing over columns along each row; the resulting value aggregates are *net supplies by commodity*, which are equal to domestic final demands plus exports less imports (*net final demands by commodity*), or
- By summing over rows down each column; the resulting value aggregates are equal to *value added by industry.*

20.33 It will be useful to list the aggregates that result by implementing the above two methods of aggregation using the entries in Tables 20.1 through 20.4. The three commodity final demand aggregates turn out to be the following value aggregates:[16]

$$vf_G \equiv p_G^{GF}y_G^{GF} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Gm}^{SR}m_G^{SR} - p_{Gm}^{TR}m_G^{TR}; \tag{20.3}$$

$$vf_S = p_S^{SF}y_S^{SF} + p_{Sx}^{SR}x_S^{SR} - p_{Sm}^{GR}m_S^{GR} - p_{Sm}^{SR}m_S^{SR} - p_{Sm}^{TR}m_S^{TR}; \tag{20.4}$$

$$vf_T = p_T^{TF}y_T^{TF} + p_{Tx}^{TR}x_T^{TR} - p_{Tm}^{GR}m_T^{GR} - p_{Tm}^{SR}m_T^{SR} - p_{Tm}^{TR}m_T^{TR}. \tag{20.5}$$

20.34 The three industry value-added aggregates are defined as follows:

$$va^G \equiv p_G^{GS}y_G^{GS} + p_G^{GT}y_G^{GT} + p_G^{GF}y_G^{GF} - p_S^{SG}y_S^{SG} - p_T^{TG}y_T^{TG} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Sm}^{GR}m_S^{GR} - p_{Tm}^{GR}m_T^{GR}; \tag{20.6}$$

$$va^S \equiv p_S^{SG}y_S^{SG} + p_S^{ST}y_S^{ST} + p_S^{SF}y_S^{SF} - p_G^{GS}y_G^{GS} - p_T^{TS}y_T^{TS} + p_{Sx}^{SR}x_S^{SR} - p_{Gm}^{SR}m_G^{SR} - p_{Sm}^{SR}m_S^{SR} - p_{Tm}^{SR}m_T^{SR}; \tag{20.7}$$

$$va^T \equiv p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + p_T^{TF}y_T^{TF} - p_G^{GT}y_G^{GT} - p_S^{ST}y_S^{ST} + p_{Tx}^{TR}x_T^{TR} - p_{Gm}^{TR}m_G^{TR} - p_{Sm}^{TR}m_S^{TR} - p_{Tm}^{TR}m_T^{TR}. \tag{20.8}$$

[16]Each of the net supply aggregates defined by (20.3) through (20.5) does not have to be positive; for example, consider the case of an imported intermediate good that is not produced domestically. However, the sum of the net supply aggregates will be substantially positive.

20.35 Note that each commodity final demand value aggregate, vf_G, vf_S, and vf_T, is equal to the value of industry deliveries of each of the three commodities plus export deliveries less imports of the commodity to each of the three industrial sectors. Note also that it is not in general appropriate to set the price of, say, vf_G equal to the value of vf_G divided by the corresponding net deliveries of commodity G to final demand, $y_G^{GS} + y_G^{GT} + y_G^{GF} - y_S^{SG} - y_T^{TG} + x_G^{GR} - m_G^{GR} - m_S^{GR} - m_T^{GR}$, because differences in the prices that are attached to these quantities imply that there are implicit quality differences between these quantities. Thus index number theory should be used to aggregate the value flows on the right-hand sides of equations (20.6) through (20.8). It is clear that index number theory must be used to construct a price and quantity for each of the value-added aggregates, va^G, va^S, and va^T, because by inspecting (20.6) through (20.8), it can be seen that each value-added aggregate is a sum over heterogeneous commodities, some with positive signs associated with their quantities (these are the gross outputs produced by the industry) and some with negative signs (these are the foreign-sourced and domestic intermediate inputs used by the industry).

20.36 The three final demand value aggregates defined by equations (20.6) through (20.8) can be summed and the resulting value aggregate is the GDP generated by the economy's production sector. Alternatively, the value-added aggregates defined by (20.6) through (20.8) can also be summed and this sum will also equal GDP because these two methods of aggregation are simply alternative methods for summing over the elements of the net supply matrix. Thus the following equation must hold:

$$GDP \equiv vf_G + vf_S + vf_T = va^G + va^S + va^T. \qquad (20.9)$$

20.37 It is useful to use equation (20.9), which defines GDP as the sum of the value of final demands, and substitute equations (20.6) through (20.8) into this definition in order to obtain the following expression for GDP after some rearrangement of terms:

$$\begin{aligned} \text{GDP} &= vf_G + vf_S + vf_T \\ &= \left[p_G^{GF} y_G^{GF} + p_S^{SF} y_S^{SF} + p_T^{TF} y_T^{TF}\right] + \left[p_{Gx}^{GR} x_G^{GR} \right. \\ &\quad \left. + p_{Sx}^{SR} x_S^{SR} + p_{Tx}^{TR} x_T^{TR}\right] - \left[p_{Gm}^{GR} m_G^{GR} + p_{Gm}^{SR} m_G^{SR} \right. \\ &\quad + p_{Gm}^{TR} m_G^{TR} + p_{Sm}^{GR} m_S^{GR} + p_{Sm}^{SR} m_S^{SR} + p_{Sm}^{TR} m_S^{TR} \\ &\quad \left. + p_{Tm}^{GR} m_T^{GR} + p_{Tm}^{SR} m_T^{SR} + p_{Tm}^{TR} m_T^{TR}\right] \\ &= [C + I + G] + [X - M]. \end{aligned} \qquad (20.10)$$

20.38 Note that the value aggregate, $p_G^{GF} y_G^{GF} + p_S^{SF} y_S^{SF} + p_T^{TF} y_T^{TF}$, corresponds to the value of domestic final demand, the value aggregate $p_{Gx}^{GR} x_G^{GR} + p_{Sx}^{SR} x_S^{SR} + p_{Tx}^{TR} x_T^{TR}$ corresponds to the value of exports, and the value aggregate $p_{Gm}^{GR} m_G^{GR} + p_{Gm}^{SR} m_G^{SR} + p_{Gm}^{TR} m_G^{TR} + p_{Sm}^{GR} m_S^{GR} + p_{Sm}^{SR} m_S^{SR} + p_{Sm}^{TR} m_S^{TR} + p_{Tm}^{GR} m_T^{GR} + p_{Tm}^{SR} m_T^{SR} + p_{Tm}^{TR} m_T^{TR}$ corresponds to the value of imports. Thus definition (20.10) corresponds to the traditional final demand definition of GDP.[17]

20.39 Equation (20.9) shows that there are two alternative ways that data on transactions between the domestic production sector and the rest of the world could be captured:

- In the *final demand method*, information on the price and quantity for each category of import (export) would be obtained from the foreign supplier (demander). This is the *nonresident point of view.*
- In the *value-added method*, information on the price and quantity of each type of import used by each industry and the price and quantity of each type of export produced by each industry would be obtained from the domestic producer. This is the *resident point of view.*

20.40 It is apparent that the practical compilation of trade price indices can be facilitated by developing the existing PPI methodology:[18] The PPI methodology can be adapted to the XMPI case to expand the commodity classification in order to make the distinction between a domestically sourced intermediate input and a foreign import and make the distinction between an output that is delivered to a domestic demander versus an output that is delivered to a foreign demander, which is an export. Of course, in practice, it may be difficult to make these distinctions. But distinct advantages of building on existing PPI computer routines and data collection and verification methods exist though there the need will remain to extend the sample of establishments and commodities to be representative of buyers and sellers from/to domestic and foreign markets.

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[17]There are some minor complications owing to the fact that small amounts of imports and exports may not pass through the domestic production sector; that is, some tourist expenditures made abroad would not be captured by transactions within the scope of the domestic production sector and a similar comment applies to government expenditures made abroad.

[18]See ILO and others (2004b). Alterman, Diewert, and Feenstra (1999) also used the value-added methodology in their exposition of the economic approach to the XMPIs.

20.41 At this point, it is useful to consider alternative methods for constructing volume measures for GDP originating in the domestic production sector. Thus suppose that data on production sector transactions are available for periods 0 and 1 and that price and quantity information is available for these two periods so that the data in Tables 20.1 through 20.4 are available and hence net supply matrices for the production sector can be calculated for periods 0 and 1. It can be seen that there are *three ways* that a volume or quantity index of net outputs for the production sector of economy could be calculated:

- Change the signs of the nonzero entries in the domestic use matrix defined by Table 20.2 and change signs of the nonzero entries in the ROW use matrix defined by Table 20.4. Look at the nonzero cells in these two three-by-three matrices as well as the cells in the supply matrices defined in Tables 20.1 and 20.3. Collecting all of these nonzero transactions, one can see that there are 27 distinct price times quantity transactions. If there is a negative sign associated with any one of these terms, that negative sign is attached to the quantity. Now apply normal index number theory to these 27 price times quantity components of the aggregate.
- Sum up the value-added aggregates defined by (20.6) through (20.8). The resulting value-added aggregate will have 27 separate price times quantity components. If a value component has a negative sign associated with it, then attach the negative sign to the quantity (so that all prices will always be positive). Now apply normal price index number formulas theory to these 27 price times quantity components of the aggregate.
- Sum up the final demand value aggregates defined by (20.3) through (20.5). The resulting value of final demand aggregate will have 15 separate price times quantity components. If a value component has a negative sign associated with it, then attach the negative sign to the quantity. Now apply normal index number theory to these 15 price times quantity components of the aggregate.

20.42 It is evident that the quantity index or the volume estimate for GDP will be the same using methods 1 and 2 listed above because the two methods generate exactly the same set of 27 separate price times quantity components in the value aggregate. However, it is not evident that volume estimates for GDP based on method 3 will coincide with those generated using methods 1 and 2 because there are 27 price times quantity components to be aggregated when we use methods 1 or 2 compared to only 15 components when we use method 3.

20.43 Denote the 27 dimensional p (price) and q (quantity) vectors that correspond to the first detailed cell and value-added methods for aggregating over commodities listed above as p^{va} and q^{va} respectively and denote the 15 dimensional p and q vectors that correspond to the third aggregation method over final demand components as p^{fd} and q^{fd} respectively.[19] Add a superscript t to denote these vectors evaluated at the data pertaining to period t. Then using equation (20.9), we can see that the inner products of each of these period t price and quantity vectors are equal in the same period because they are each equal to period t nominal GDP:[20]

$$p^{vat} \cdot q^{vat} = p^{fdt} \cdot q^{fdt}; \quad t = 0, 1. \tag{20.11}$$

20.44 What is not immediately obvious is that the inner products of the two sets of price and quantity vectors are also equal if the price vectors are evaluated at the prices of one period and the corresponding quantity vectors are evaluated at the quantities of another period; that is, for periods 0 and 1, the following equalities hold:[21]

$$p^{va1} \cdot q^{va0} = p^{fd1} \cdot q^{fd0}; \tag{20.12}$$

$$p^{va0} \cdot q^{va1} = p^{fd0} \cdot q^{fd1}. \tag{20.13}$$

20.45 Laspeyres and Paasche quantity indices that compare the quantities of period 1 to those of period 0 can be defined as follows:

$$Q_L{}^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1}) \equiv p^{va0} \cdot q^{va1}/p^{va0} \cdot q^{va0};$$

$$Q_L{}^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1}) \equiv p^{fd0} \cdot q^{fd1}/p^{fd0} \cdot q^{fd0}; \tag{20.14}$$

$$Q_P{}^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1}) \equiv p^{va1} \cdot q^{va1}/p^{va1} \cdot q^{va0};$$

$$Q_P{}^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1}) \equiv p^{fd1} \cdot qz^{fd1}/p^{fd1} \cdot q^{fd0}. \tag{20.15}$$

[19] All prices are positive but if a quantity is an input, it is given a negative sign.

[20] Notation: $p \cdot q \equiv \sum_{n=1}^{N} p_n q_n$ where p and q are N-dimensional vectors with components p_n and q_n respectively.

[21] The proof follows using the additivity of the inner products and the exact matching of a domestic intermediate input transaction to a corresponding domestic output transaction. Diewert (2006, pp. 293–94) used this method of proof, drawing on prior discussions on these issues with Kim Zieschang. Moyer, Reinsdorf, and Yuskavage (2006) derived similar results but under the assumption that commodity prices were constant across industries.

20.46 Using equations (20.11) and (20.13), and definitions (20.15), we can see that the two Laspeyres volume indices are equal:

$$Q_L^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1}) = Q_L^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1}). \quad (20.16)$$

20.47 Using equations (20.11) and (20.12) and definitions (20.16), we can see that the two Paasche volume indices are equal:

$$Q_P^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1}) = Q_P^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1}). \quad (20.17)$$

20.48 Because a Fisher ideal quantity index is the square root of the product of a Laspeyres and Paasche quantity index, it can be seen that equations (20.16) and (20.17) imply that all three Fisher quantity indices—constructed by aggregating over input output net supply table cells, by aggregating over industry value-added components (which is equivalent to aggregating over net supply table cells), or by aggregating over final demand components—are equal; that is, we have

$$Q_F^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1}) = Q_F^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1}). \quad (20.18)$$

20.49 The equality between the two methods for constructing volume estimates that is reflected in equations (20.16) through (20.18) could provide a potentially useful check on a statistical agency's methods for constructing aggregate volume GDP measures.

20.50 The above results extend to more complex input output frameworks provided that all transactions between each pair of sectors in the model are accounted for in the model.

20.51 The equality (20.18) between the two methods for constructing an aggregate volume index for GDP using the Fisher quantity index as the index number formula can be extended to the case where the implicit Törnqvist quantity index is used as the index number formula. In this case, the value aggregates are deflated by the Törnqvist price index, and by writing out the formulae, it is straightforward to show that $P_T^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1})$, the Törnqvist price index using the 27 price times quantity components in the value-added aggregate, is equal to $P_T^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1})$, the Törnqvist price index using the 15 price times quantity components in the final demand aggregate.[22]

20.52 It is well known that the Laspeyres and Paasche quantity indices are consistent in aggregation. Thus if Laspeyres indices of volume estimates of value added by industry are constructed in the first stage of aggregation and the resulting industry prices and quantities are used as inputs into a second stage of Laspeyres aggregation, then the resulting two-stage Laspeyres quantity index is equal to the corresponding single-stage index, $Q_L^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1})$. Similarly, if Paasche volume indices of value added by industry are constructed in the first stage of aggregation and the resulting industry prices and quantities are used as inputs into a second stage of Paasche aggregation, then the resulting two-stage Paasche quantity index is equal to the corresponding single-stage index, $Q_P^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1})$.[23] Unfortunately, the corresponding result does not hold for the Fisher index. However, the two-stage Fisher quantity index usually will be quite close to the corresponding single-stage index, $Q_F^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1})$.[24] In the following section, commodity taxes are introduced into the supply and use matrices.

B.3 Input output accounts with commodity taxation and subsidization

20.53 Consider again the production model that corresponds to Tables 20.1 through 20.4 in the previous section but now assume that there is the possibility of a commodity tax (or subsidies) falling on the output of each industry and on the intermediate inputs used by each industry. Assume that the *producing industry* collects these commodity taxes and remits them to the appropriate level of government. These indirect commodity taxes will be introduced into each of the tables listed in the previous section. The counterpart to Table 20.1 is now Table 20.5.

20.54 The quantity of goods delivered to the service sector is y_G^{GS} as before and the service sector pays industry G the price p_G^{GS} for each unit of G that

[22]This observation was made in the *PPI Manual* and was confirmed by numerical computations; see ILO .and others (2004b, pp. 505–06).

[23]A word of warning is in order if two-stage aggregation is used: *The value aggregates in the first stage of aggregation must be of the same sign*. If they are not of the same sign, index number theory will fail. Thus it is not recommended that a first-stage aggregate equal to exports minus imports be constructed, because the value of net exports could be positive in period 0 and negative in period 1. A similar problem arises if one attempts to construct an index of real inventory change because the sign of the value aggregate can change from period to period. Diewert (2005a) provided some examples of index number failure in the inventory change context but his analysis is applicable more generally.

[24]See Diewert (1978, p. 889) and Robert Hill (2006). However, using an artificial data set it will be shown in Section D of this chapter that the two-stage Fisher value-added index is not close to its single-stage counterpart so some caution must be used in aggregating value added across industries in a two-stage aggregation procedure.

Table 20.5. Domestic Supply Matrix in Current Period Values with Commodity Taxes

	Industry G	Industry S	Industry T
G	$(p_G^{GS} - t_G^{GS})y_G^{GS} + (p_G^{GT} - t_G^{GT})y_G^{GT} + (p_G^{GF} - t_G^{GF})y_G^{GF}$	0	0
S	0	$(p_S^{SG} - t_S^{SG})y_S^{SG} + (p_S^{ST} - t_S^{ST})y_S^{ST} + (p_S^{SF} - t_S^{SF})y_S^{SF}$	0
T	0	0	$(p_T^{TG} - t_T^{TG})y_T^{TG} + (p_T^{TS} - t_T^{TS})y_T^{TS} + (p_T^{TF} - t_T^{TF})y_T^{TF}$

Table 20.6. Export or ROW Supply Matrix in Current Period Values with Export Taxes

	Industry G	Industry S	Industry T
G	$(p_{Gx}^{GR} - t_{Gx}^{GR})x_G^{GR}$	0	0
S	0	$(p_{Sx}^{SR} - t_{Sx}^{SR})x_S^{SR}$	0
T	0	0	$(p_{Tx}^{TR} - t_{Tx}^{TR})x_T^{TR}$

was delivered. However, industry G must remit the per unit[25] commodity tax[26] t_G^{GS} of the per unit revenue p_G^{GS} to the government sector and so industry G receives only the revenue $p_G^{GS} - t_G^{GS}$ for each unit of good G sold to industry S. The interpretation of the other prices and commodity taxes that occur in Table 20.5 is similar.

20.55 The domestic use matrix in current-period values is still defined by the entries in Table 20.2. This matrix remains unchanged with the introduction of commodity taxes and subsidies. This is because the domestic taxes and subsidies are assumed to be on the output of the producer. Had they been paid by the domestic purchaser on intermediate consumption they would appear here as part of the purchase price.

20.56 The ROW supply matrix or export by industry and commodity matrix defined earlier by Table 20.3 is now replaced by Table 20.6.

[25]Thus the commodity taxes are modeled as specific taxes rather than ad valorem taxes. This is not a restriction on the analysis because ad valorem taxes can be converted into equivalent specific taxes in each period.

[26]If the sales of commodity G are being subsidized by the government sector, then the tax level per unit t_G^{GS} will be negative instead of positive. It is assumed that the after-tax prices of the form $p_G^{GS} - t_G^{GS}$ are always positive. In a more detailed model, per unit commodity subsidies could be explicitly introduced instead of the present interpretation of t_G^{GS} as a specific *net* (tax less subsidy) commodity tax.

Table 20.7. Import or ROW Use Matrix in Current Period Values with Import Taxes

	Industry G	Industry S	Industry T
G	$(p_{Gm}^{GR} + t_{Gm}^{GR})m_G^{GR}$	$(p_{Gm}^{SR} + t_{Gm}^{SR})m_G^{SR}$	$(p_{Gm}^{TR} + t_{Gm}^{TR})m_G^{TR}$
S	$(p_{Sm}^{GR} + t_{Sm}^{GR})m_S^{GR}$	$(p_{Sm}^{SR} + t_{Sm}^{SR})m_S^{SR}$	$(p_{Sm}^{TR} + t_{Sm}^{TR})m_S^{TR}$
T	$(p_{Tm}^{GR} + t_{Tm}^{GR})m_T^{GR}$	$(p_{Tm}^{SR} + t_{Tm}^{SR})m_T^{SR}$	$(p_{Tm}^{TR} + t_{Tm}^{TR})m_T^{TR}$

20.57 To interpret the entries in Table 20.6, consider the entries for commodity G and industry G. Industry G still gets the revenue $p_{Gx}^{GR}x_G^{GR}$ for its deliveries of goods to foreign purchasers from these purchases but if the government sector imposes a specific export tax equal to t_{Gx}^{GR} per unit of exports, then industry G gets to keep only the amount $p_{Gx}^{GR} - t_{Gx}^{GR}$ per unit sale instead of the full final demander price p_{Gx}^{GR}. If, however, t_{Gx}^{GR} is negative, then the government is subsidizing the export of goods and hence the subsidized price that the producer faces, $p_{Gx}^{GR} - t_{Gx}^{GR}$, is actually higher than the final demander price p_{Gx}^{GR}. The interpretation of the industry S and commodity S and industry T and commodity T entries are similar.

20.58 The ROW use matrix or the import matrix by industry and commodity defined by Table 20.4 in the previous section is now replaced by Table 20.7.

20.59 As in Table 20.4, industry G imports m_G^{GR} units of goods from foreign suppliers and pays these foreign suppliers the amount $p_{Gm}^{GR}m_G^{GR}$. However, if t_{Gm}^{GR} is positive (the usual case), then the government imposes a specific set of tariffs and indirect taxes on each unit imported equal to t_{Gm}^{GR} and hence industry G faces the higher price $p_{Gm}^{GR} + t_{Gm}^{GR}$ for each unit of good G that is imported.[27] The interpretations of the industry S and commodity S and industry T and commodity T entries are similar.

20.60 The *volume industry supply and use matrices* that correspond to the nominal supply matrices defined by Tables 20.5 and 20.6 and nominal use matrices defined by Tables 20.2 and 20.7 can be obtained from their nominal counterparts after deleting all of the price and tax terms. For completeness, these volume supply and use matrices are listed below. These volume allocation of resources matrices apply to both the with and without commodity tax situations.

[27]If t_{Gm}^{GR} is negative, then the government subsidizes the importation of good G for use by industry G.

Table 20.8. Constant Dollar Domestic Supply Matrix

	Industry *G*	Industry *S*	Industry *T*
G	$y_G^{GS} + y_G^{GT} + y_G^{GF}$	0	0
S	0	$y_S^{SG} + y_S^{ST} + y_S^{SF}$	0
T	0	0	$y_T^{TG} + y_T^{TS} + y_T^{TF}$

Table 20.9. Volume Domestic Use Matrix

	Industry *G*	Industry *S*	Industry *T*
G	0	y_G^{GS}	y_G^{GT}
S	y_S^{SG}	0	y_S^{ST}
T	y_T^{TG}	y_T^{TS}	0

Table 20.10. Volume ROW Supply or Export by Industry and Commodity Matrix

	Industry *G*	Industry *S*	Industry *T*
G	x_G^{GR}	0	0
S	0	x_S^{SR}	0
T	0	0	x_T^{TR}

Table 20.11. Volume ROW Use or Import by Industry and Commodity Matrix

	Industry *G*	Industry *S*	Industry *T*
G	m_G^{GR}	m_G^{SR}	m_G^{TR}
S	m_S^{GR}	m_S^{SR}	m_S^{TR}
T	m_T^{GR}	m_T^{SR}	m_T^{TR}

20.61 If we compare the volume allocation of resources matrices defined by Tables 20.8 through 20.11 with their monetary value at producer price counterparts, we can again see that it will generally be impossible to recover the true volume or quantity measures along any row by deflating the nominal values by a single price index for that commodity class; that is, price deflators that are common across industry will generally not exist. Thus the price statistician's task is a rather daunting one: Appropriate specific price deflators or volume extrapolators will in principle be required for *each* nonzero cell in the system of nominal value input output matrices in order to recover the correct volume measures.[28]

[28]Of course, in practice, compromises with the theory will have to be made.

20.62 As was shown in the previous section, the production sector's nominal value *net supply matrix* that gives the value of net commodity supply by commodity and by industry of origin *at the prices that producers face* can be obtained by adding entries in Tables 20.5 and 20.6 and then subtracting corresponding entries in Tables 20.2 and 20.7. This new net supply matrix gives the value of net commodity supply by commodity and by industry of origin at prices that producers face.

20.63 As shown in the previous section, the net supply matrix can be aggregated by summing over columns along each row (the resulting value aggregates are the values of *net supply by commodity* at producer prices) or by summing over rows down each column (the resulting value aggregates are equal to *value added by industry* at producer prices).

20.64 The three *value of commodity net supply aggregates at producer prices including taxes and subsidies on output* (the counterparts to the aggregates defined by equations (20.6) through (20.8)) turn out to be the following value aggregates:

$$\begin{aligned} vf_G \equiv\ & p_G^{GF} y_G^{GF} + p_{Gx}^{GR} x_G^{GR} - p_{Gm}^{GR} m_G^{GR} - p_{Gm}^{SR} m_G^{SR} \\ & - p_{Gm}^{TR} m_G^{TR} - \big[t_G^{GS} y_G^{GS} + t_G^{GT} y_G^{GT} + t_G^{GF} y_G^{GF} \\ & + t_{Gx}^{GR} x_G^{GR} + t_{Gm}^{GR} m_G^{GR} + t_{Gm}^{SR} m_G^{SR} + t_{Gm}^{TR} m_G^{TR} \big]; \end{aligned} \tag{20.19}$$

$$\begin{aligned} vf_S =\ & p_S^{SF} y_S^{SF} + p_{Sx}^{SR} x_S^{SR} - p_{Sm}^{GR} m_S^{GR} - p_{Sm}^{SR} m_S^{SR} \\ & - p_{Sm}^{TR} m_S^{TR} - \big[t_S^{SG} y_S^{SG} + t_S^{ST} y_S^{ST} + t_S^{SF} y_S^{SF} \\ & + t_{Sx}^{SR} x_S^{SR} + t_{Sm}^{GR} m_S^{GR} + t_{Sm}^{SR} m_S^{SR} + t_{Sm}^{TR} m_S^{TR} \big]; \end{aligned} \tag{20.20}$$

$$\begin{aligned} vf_T =\ & p_T^{TF} y_T^{TF} + p_{Tx}^{TR} x_S^{TR} - p_{Tm}^{GR} m_T^{GR} - p_{Tm}^{SR} m_T^{SR} \\ & - p_{Tm}^{TR} m_T^{TR} - \big[p_T^{TG} y_T^{TG} + p_T^{TS} y_T^{TS} + t_T^{TF} y_T^{TF} \\ & + t_{Tx}^{TR} x_T^{TR} + t_{Tm}^{GR} m_T^{GR} + t_{Tm}^{SR} m_T^{SR} + t_{Tm}^{TR} m_T^{TR} \big]. \end{aligned} \tag{20.21}$$

20.65 Looking at equation (20.19), it can be seen that the net value of production of good *G* at producer prices is equal to $p_G^{GF} y_G^{GF} + p_{Gx}^{GR} x_G^{GR} - p_{Gm}^{GR} m_G^{GR} - p_{Gm}^{SR} m_G^{SR} - p_{Gm}^{TR} m_G^{TR}$, which is the net value of production of commodity *G*, delivered to the domestic final demand and ROW sectors, at final demand prices, less a term in square brackets that represents the net revenue (commodity tax revenue less subsidies for commodity *G*) that the government sector collects by taxing (or subsidizing) transactions that involve commodity G. The interpretations for vf_S and vf_T are similar.

20.66 The three *industry value-added aggregates at producer prices* turn out to be the following value aggregates:

$$\begin{aligned} va^G \equiv & \left(p_G^{GS} - t_G^{GS}\right)y_G^{GS} + \left(p_G^{GT} - t_G^{GT}\right)y_G^{GT} \\ & + \left(p_G^{GF} - t_G^{GF}\right)y_G^{GF} - p_S^{SG}y_S^{SG} - p_T^{TG}y_T^{TG} \\ & + \left(p_{Gx}^{GR} - t_{Gx}^{GR}\right)x_G^{GR} - \left(p_{Gm}^{GR} + t_{Gm}^{GR}\right)m_G^{GR} \\ & - \left(p_{Sm}^{GR} + t_{Sm}^{GR}\right)m_S^{GR} - \left(p_{Tm}^{GR} + t_{Tm}^{GR}\right)m_T^{GR}; \end{aligned} \tag{20.22}$$

$$\begin{aligned} va^S \equiv & \left(p_S^{SG} - t_S^{SG}\right)y_S^{SG} + \left(p_S^{ST} - t_S^{ST}\right)y_S^{ST} \\ & + \left(p_S^{SF} - t_S^{SF}\right)y_S^{SF} - p_G^{GS}y_G^{GS} - p_T^{TS}y_T^{TS} \\ & + \left(p_{Sx}^{SR} - t_{Sx}^{SR}\right)x_S^{SR} - \left(p_{Gm}^{SR} + t_{Gm}^{SR}\right)m_G^{SR} \\ & - \left(p_{Sm}^{SR} + t_{Sm}^{SR}\right)m_S^{SR} - \left(p_{Tm}^{SR} + t_{Tm}^{SR}\right)m_T^{SR}; \end{aligned} \tag{20.23}$$

$$\begin{aligned} va^T \equiv & \left(p_T^{TG} - t_T^{TG}\right)y_T^{TG} + \left(p_T^{TS} - t_T^{TS}\right)y_T^{TS} \\ & + \left(p_T^{TF} - t_T^{TF}\right)y_T^{TF} - p_G^{GT}y_G^{GT} - p_S^{ST}y_S^{ST} \\ & + \left(p_{Tx}^{TR} - t_{Tx}^{TR}\right)x_T^{TR} - \left(p_{Gm}^{TR} + t_{Gm}^{TR}\right)m_G^{TR} \\ & - \left(p_{Sm}^{TR} + t_{Sm}^{TR}\right)m_S^{TR} - \left(p_{Tm}^{TR} + t_{Tm}^{TR}\right)m_T^{TR}. \end{aligned} \tag{20.24}$$

20.67 Looking at equation (20.22), it can be seen that the *value added produced by industry G at producer prices*, va^G, is equal to the value of deliveries of good G to industry S, $(p_G^{GS} - t_G^{GS})y_G^{GS}$,[29] plus the value of deliveries of good G to industry T, $(p_G^{GT} - t_G^{GT})y_G^{GT}$, plus the value of deliveries of finished goods, less payments to industry S for service intermediate inputs, $-p_S^{SG}y_S^{SG}$, less payments to industry T for transportation service intermediate inputs, $-p_T^{TG}y_T^{TG}$, plus the value of exports delivered to the ROW sector, $(p_{Gx}^{GR} - t_{Gx}^{GR})x_G^{GR}$,[30] less payments to the ROW for imports of goods G used by industry G, $-(p_{Gm}^{GR} + t_{Gm}^{GR})m_G^{GR}$, less payments to the ROW for imports of services S used by industry G, $-(p_{Sm}^{GR} + t_{Sm}^{GR})m_S^{GR}$, less payments to the ROW for imports of transportation services T used by industry G, $-(p_{Tm}^{GR} + t_{Tm}^{GR})m_T^{GR}$. The decompositions for the value added produced by industries S and T, va^S and va^T, are similar.

20.68 Looking at equations (20.19) through (20.21), we can see that it is natural to ignore the commodity tax transactions and to sum the remaining transactions involving exports into an aggregate that is the value of exports at final demand prices, $p_{Gx}^{GR}x_G^{GR} + p_{Sx}^{SR}x_S^{SR} + p_{Tx}^{TR}x_S^{TR}$. It is this value aggregate that is equal to the value of X in GDP, valued at final demand prices. However, looking at the industry value-added aggregates defined by equations (20.22) through (20.24), we can see that it is natural to work with the net revenues received by the industries for their exports, which are $(p_{Gx}^{GR} - t_{Gx}^{GR})x_G^{GR}$ for industry G, $(p_{Sx}^{SR} - t_{Sx}^{SR})x_S^{SR}$ for industry S, and $(p_{Tx}^{TR} - t_{Tx}^{TR})x_T^{TR}$ for industry T. Thus from the viewpoint of industry accounts, it is natural to aggregate these export revenues across industries in order to obtain the value of exports aggregate at producer prices, $(p_{Gx}^{GR} - t_{Gx}^{GR})x_G^{GR} + (p_{Sx}^{SR} - t_{Sx}^{SR})x_S^{SR} + (p_{Tx}^{TR} - t_{Tx}^{TR})x_T^{TR}$. Thus for production accounts that are based on the economic approach to index number theory, it is more appropriate to use tax-adjusted producer prices as the pricing concept rather than final demand prices.[31] Similar comments apply to the treatment of imports. Later in this section, it will be shown how final demand based estimates for volume GDP can be reconciled with production-based estimates of volume GDP originating in the production sector at producer prices.

20.69 The three final demand value aggregates defined by equations (20.19) through (20.21) can be summed and the resulting value aggregate is the GDP_P generated by the economy's production sector at producer prices. Note that we have added the subscript P to this GDP concept at producer prices to distinguish it from the more traditional concept of GDP at final demand prices, which we denote by GDP_F. The two GDP concepts will be reconciled later.

20.70 The value-added aggregates at producer prices defined by equations (20.22) through (20.24) can also be summed and this sum will also equal GDP_P because the two methods for forming estimates of GDP_P are simply alternative methods for summing over the

[29] Note that industry S pays industry G the price p_G^{GS} per unit of good G delivered to industry S, but industry G must remit the specific tax t_G^{GS} out of this price to the government sector.

[30] The foreign demanders for the exports of good G by industry G pay industry G the price p_{Gx}^{GR} per unit of the good exported but industry G must pay out of this amount the specific export tax t_{Gx}^{GR} to the government sector for each unit of good G that is exported.

[31] It is also appropriate to use these tax-adjusted producer prices when constructing a PPI that is based on the economic approach to index number theory.

elements of the net supply matrix.[32] Thus the following equation must hold:

$$\mathrm{GDP_P} \equiv vf_G + vf_S + vf_T = va^G + va^S + va^T. \quad (20.25)$$

20.71 It is useful to explicitly write out $\mathrm{GDP_P}$ as the sum of the three final demand aggregates defined in equations (20.22) through (20.24). After some rearrangement of terms the following equation is obtained:

$$\begin{aligned} \mathrm{GDP_P} &= vf_G + vf_S + vf_T \\ &= \mathrm{GDP_F} - T \\ &= [C + I + G] + X - M - T, \end{aligned} \quad (20.26)$$

where T is the *value of commodity tax net revenues* (taxes less subsidies) defined as the sum of the following terms:

$$\begin{aligned} T \equiv\ & t_G^{GS} y_G^{GS} + t_G^{GT} y_G^{GT} + t_G^{GF} y_G^{GF} + t_{Gx}^{GR} x_G^{GR} \\ &+ t_{Gm}^{GR} m_G^{GR} + t_{Gm}^{SR} m_G^{SR} + t_{Gm}^{TR} m_G^{TR} + t_S^{SG} y_S^{SG} \\ &+ t_S^{ST} y_S^{ST} + t_S^{SF} y_S^{SF} + t_{Sx}^{SR} x_S^{SR} + t_{Sm}^{GR} m_S^{GR} + t_{Sm}^{SR} m_S^{SR} \\ &+ t_{Sm}^{TR} m_S^{TR} + p_T^{TG} y_T^{TG} + p_T^{TS} y_T^{TS} + t_T^{TF} y_T^{TF} + t_{Tx}^{TR} x_T^{TR} \\ &+ t_{Tm}^{GR} m_T^{GR} + t_{Tm}^{SR} m_T^{SR} + t_{Tm}^{TR} m_T^{TR}; \end{aligned} \quad (20.27)$$

and $\mathrm{GDP_F}$ is *the value of GDP at final demand prices* defined as the sum of the following components of final demand at final demand prices:

$$\begin{aligned} \mathrm{GDP_F} \equiv\ & \left[p_G^{GF} y_G^{GF} + p_S^{SF} y_S^{SF} + p_T^{TF} y_T^{TF}\right] \\ &+ \left[p_{Gx}^{GR} x_G^{GR} + p_{Sx}^{SR} x_S^{SR} + p_{Tx}^{TR} x_T^{TR}\right] \\ &- \left[p_{Gm}^{GR} m_G^{GR} + p_{Gm}^{SR} m_G^{SR} + p_{Gm}^{TR} m_G^{TR}\right. \\ &+ p_{Sm}^{GR} m_S^{GR} + p_{Sm}^{SR} m_S^{SR} + p_{Sm}^{TR} m_S^{TR} \\ &\left.+ p_{Tm}^{GR} m_T^{GR} + p_{Tm}^{SR} m_T^{SR} + p_{Tm}^{TR} m_T^{TR}\right] \\ =\ & [C + G + I] + [X] - [M]. \end{aligned} \quad (20.28)$$

20.72 Note that the 15 terms that do not involve taxes on the right-hand side of equation (20.26), which define $\mathrm{GDP_F}$, correspond to the 15 terms on the right-hand side of equation (20.10), which provided our initial decomposition of GDP when there were no commodity taxes. However, when there are commodity taxes (and commodity subsidies), the new decomposition of $\mathrm{GDP_P}$ requires that the 21 tax terms defined by equation (20.27) be subtracted from the right-hand side of equation (20.26). Note that using definition (20.28), we can rewrite the identity (20.26) in the following form:

$$\mathrm{GDP_F} = \mathrm{GDP_P} + T. \quad (20.29)$$

20.73 Thus the value of production at final demand prices, $\mathrm{GDP_F}$, is equal to the value of production at producer prices, $\mathrm{GDP_P}$, plus commodity tax revenues less commodity tax subsidies, T, which is a traditional national income accounting identity.

20.74 As was discussed in the previous section, three methods can be used to construct a volume or quantity index of net outputs (at producer prices) produced by the production sector:

- Sum the two supply matrices and subtract the two use matrices and look at the cell entries in the resulting matrix. Collecting all of the nonzero transactions, we can see that there are 48 distinct price times quantity transactions. If a negative sign is associated with any one of these terms, that negative sign is attached to the quantity. Now apply normal index number theory to these 48 price times quantity components of the aggregate.
- Sum up the value-added aggregates defined by equations (20.22) through (20.24). The resulting value-added aggregate will have the same 48 separate price times quantity components that occurred in the first method of aggregation. If a value component has a negative sign associated with it, then attach the negative sign to the quantity (so that all prices will always be positive). Now apply normal index number theory to these 48 price times quantity components of the aggregate. This method will generate the same results as the first method listed above.
- Sum up the final demand value aggregates defined by equations (20.19) through (20.21). The resulting value of final demand aggregate will have 36 separate price times quantity components. If a value component has a negative sign associated with it, then attach the negative sign to the quantity. Now apply normal index number theory to these 36 price times quantity components of the aggregate.

20.75 It is evident that the quantity index or the volume estimate for GDP will be the same using methods 1 and 2 listed above because the two methods generate exactly the same set of 48 separate price times quantity components in the value aggregate. However, it is not evident that volume estimates for GDP based on method 3 will coincide with those generated using methods 1 and 2 because there are 48 price times

[32]The first method sums entries along rows first and then sums down the sum column whereas the second method sums entries down columns first and then sums across the sum row.

quantity components to be aggregated when we use methods 1 or 2 compared to only 36 components when we use method 3. However, equations (20.11) through (20.18) in the previous section (with some obvious changes in notation) continue to hold in this new framework with commodity taxes and subsidies. Thus value-added (at producer prices) Laspeyres, Paasche, and Fisher quantity indices will be equal to their final demand counterparts, where the 21 terms involving taxes are used in the formulas. Note that the specific tax terms play the role of prices in these index number formulas and the associated quantities have negative signs attached to them when calculating these final demand (at producer prices) index numbers.

20.76 The equality (20.18) between the two methods for constructing an aggregate volume index for GDP using the Fisher quantity index as the index number formula can be extended to the case where the implicit Törnqvist quantity index is used as the index number formula. In this case, the value aggregates are deflated by the Törnqvist price index and by writing out the formulas, it is straightforward to show that $P_T^{va}(p^{va0}, p^{va1}, q^{va0}, q^{va1})$, the Törnqvist price index using the 48 price times quantity components in the value-added aggregate, is equal to $P_T^{fd}(p^{fd0}, p^{fd1}, q^{fd0}, q^{fd1})$, the Törnqvist price index using the 36 price times quantity components in the final demand aggregate.[33]

20.77 As noted in the previous section, GDP_P can be calculated using two-stage aggregation where the first stage calculates volume value added (at producer prices) by industry. The two-stage estimates of GDP_P will coincide exactly with their single-stage counterparts if the Laspeyres or Paasche formulas are used and will approximately coincide if the Fisher formula is used. It should be noted that the value added at producer prices approach for the calculation of industry aggregates is suitable for productivity analysis purposes.[34] It should be emphasized that in order to construct accurate productivity statistics for each industry, it generally will be necessary to construct *separate price deflators for each nonzero cell in the augmented input output tables* that have been suggested in this chapter.

20.78 The final topic for this section is how to reconcile volume estimates for GDP at final demand prices, GDP_F, with volume estimates for GDP at producer prices, GDP_P. Recall equation (20.29), which said that GDP_F equals GDP_P plus I. Suppose that data are available for two periods that respect equation (20.29) in each period and a quantity index, GDP_F, is constructed, defined by equation (20.28) with 15 separate price times quantity components. Then noting that GDP_P is defined by the sum of equations (20.22) through (20.24) with 48 price times quantity components,[35] and T is defined by equations (20.27) with 21 price times quantity components, we could combine these transactions and construct an alternative quantity index for this sum of GDP_P and T value aggregate using the same index number formula. Using the same method of proof as was used in the previous section, we can show that the resulting volume estimates for GDP_F and $GDP_P + T$ will coincide if the Laspeyres, Paasche, or Fisher formulas are used. For the $GDP_P + T$ aggregate, two-stage aggregation could be used where the first-stage value aggregates are GDP_P, GDP at producer prices, and T, commodity tax revenue less commodity subsidies. The two-stage estimates will be exactly equal to the corresponding single-stage estimates if the Laspeyres or Paasche formulas are used for the quantity index and will be approximately equal if the Fisher formula is used. This type of decomposition will enable analysts to relate volume growth in final demand GDP_F to volume growth in GDP_P at producer prices plus commodity tax effects. More generally, the identity (20.29) can be used to estimate GDP_F if the statistical agency is able to estimate GDP_P and in addition, the statistical agency can form estimates of the 21 tax times quantity terms on the right-hand side of equation (20.27).[36]

[33]However, in order to obtain this equality for the Törnqvist price index, it is necessary to treat each indirect tax as a separate price component for both the value-added and final demand methods of aggregation; that is, if the terms involving final demand prices and taxes are combined into single producer prices and then fed into the Törnqvist price index formula when using the value-added approach, then the resulting index value is not necessarily equal to the Törnqvist price index that directly aggregates the 36 components of final demand. Thus in Chapter 18, because the second method of aggregation was used, the exact equality of the two Törnqvist price indices does not hold.

[34]See Jorgenson and Griliches (1967 and 1972) for an early exposition of how productivity accounts could be set up. The indirect tax conventions used in this chapter are consistent with the recommendations of Jorgenson and Griliches (1972, p. 85) on the treatment of indirect taxes in a set of productivity accounts.

[35]Alternatively, the tax terms could be combined with the final demand prices and then there would be only 27 price times quantity value transactions in the aggregate.

[36]Conversely, the identity $GDP_P = GDP_F - T$ implies that if the statistical agency is able to estimate GDP_F, and in addition, the statistical agency can make estimates of the 21 tax times quantity terms on the right-hand side of equation (20.27), then estimates of GDP_P can be made. Thus the allocation of commodity taxes and subsidies to the correct cells of the system of supply and use matrices is important. These observations on the importance of commodity taxes in the input output framework are generalizations of observations made by Diewert (2006, pp. 303–04) in the context of a model of a closed economy.

C. The Artificial Data Set

C.1 The artificial data set framework: Real supply and use matrices

20.79 An artificial data set is presented in this section for the supply and use tables outlined in the previous section. It is useful to expand the commodity classification from one good G to four goods, $G1$, $G2$, $G3$, and $G4$, and from one service to two services, $S1$ and $S2$. The four goods are

- $G1$, agricultural products or food good;
- $G2$, crude oil or, more generally, energy products;
- $G3$, an imported pure intermediate good that is used by the domestic goods producing industry, and
- $G4$, a general consumption nonenergy, nonfood good.

The two services are

- $S1$, traditional services, and
- $S2$, high-technology services such as telecommunications and Internet access.

The remaining commodity in the commodity classification is T, transportation services.

20.80 The constant dollar table counterparts to Tables 20.8 through 20.11 are now modified into Tables 20.12 through 20.15. The counterpart to Table 20.8 is Table 20.12. This matrix shows the production by commodity and by industry that is delivered to domestic demanders. Thus y_{G4}^{GS} denotes the quantity of good $G4$ that is delivered by the goods producing industry G to the services industry S, y_{G4}^{GT} denotes the quantity of good $G4$ that is delivered by the goods producing industry G to the transportation industry T, y_{G4}^{GF} denotes the quantity of good $G4$ that is delivered by the goods producing industry G to the domestic final demand sector F, y_{G1}^{SF} denotes the quantity of good $G1$ (food imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{G2}^{SF} denotes the quantity of good $G2$ (energy imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{S1}^{SG} denotes the quantity of traditional services $S1$ that is delivered by the services industry S to the goods producing industry G, y_{S2}^{SG} denotes the quantity of high-tech services $S2$ that is delivered by the services industry S to the goods producing industry G, y_{T}^{TG} denotes the quantity of transportation services T that is delivered by the transportation industry T to the goods producing industry G, and so on.

20.81 Looking at the entries in Table 20.13, we can see that there is no domestic production of goods $G1$ (agricultural products) and $G2$ (crude oil) by industries G and T and no domestic production of $G3$ (the imported intermediate good used by the goods producing industry G) by any of the industries. Industry G produces good $G4$ and delivers y_{G4}^{GS} units of this good to the service industry S to be used as an intermediate input there, delivers y_{G4}^{GT} units of this good to the transportation industry T to be used as an intermediate input there, and delivers y_{G4}^{GF} units of this good to the domestic final demand sector F. Similarly, industry S produces the general service commodity $S1$ and delivers y_{S1}^{SG} units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers y_{S1}^{ST} units of this service to the transportation industry T to be used as an intermediate input there, and delivers y_{S1}^{SF} units of this service to the domestic final demand sector F. Industry S also produces the high-technology service commodity $S2$ and delivers y_{S2}^{SG} units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers y_{S2}^{ST} units of this service to the transportation industry T to be used as an intermediate input there, and delivers y_{S2}^{SF} units

Table 20.12. Real Domestic Supply Matrix

	Industry G	Industry S	Industry T
$G1$	0	y_{G1}^{SF}	0
$G2$	0	y_{G2}^{SF}	0
$G3$	0	0	0
$G4$	$y_{G4}^{GS} + y_{G4}^{GT} + y_{G4}^{GF}$	0	0
$S1$	0	$y_{S1}^{SG} + y_{S1}^{ST} + y_{S1}^{SF}$	0
$S2$	0	$y_{S2}^{SG} + y_{S2}^{ST} + y_{S2}^{SF}$	0
T	0	0	$y_{T}^{TG} + y_{T}^{TS} + y_{T}^{TF}$

Table 20.13. Real Domestic Use Matrix

	Industry *G*	Industry *S*	Industry *T*
*G*1	0	0	0
*G*2	0	0	0
*G*3	0	0	0
*G*4	0	y_{G4}^{GS}	y_{G4}^{GT}
*S*1	y_{S1}^{SG}	0	y_{S1}^{ST}
*S*2	y_{S2}^{SG}	0	y_{S2}^{ST}
T	y_{T}^{TG}	y_{T}^{TS}	0

of this service to the domestic final demand sector *F*. It is also assumed that the service industry imports *G*1 (agricultural produce) and *G*2 (crude oil) and stores and distributes these imports to the household sector; these are the deliveries y_{SG1}^{SF} and y_{SG2}^{SF}.[37] Finally, industry *T* produces the transportation services commodity *T* and delivers y_T^{TG} units of this commodity to the goods producing industry *G* to be used as an intermediate input there, delivers y_T^{TS} units of these transport services to the service industry *S* to be used as an intermediate input there, and delivers y_T^{TF} units of transport services directly to the domestic final demand sector *F*.

20.82 The counterpart to Table 20.9 is now Table 20.13. This matrix lists the industry demands for commodities that originate from domestic sources; that is, it shows the industry by commodity intermediate input demands for commodities that are supplied from domestic sources.

20.83 Because there is no domestic production of goods *G*1 through *G*3, the rows that correspond to these commodities in Table 20.13 all have zero entries. The remainder of the table is the same as Table 20.9. Note that the domestic intersectoral transfers of goods and services in Tables 20.12 and 20.13 match up exactly; that is, the eight nonzero quantities in Table 20.13 are exactly equal to the corresponding entries in Table 20.12. The counterpart to Table 20.10 is now Table 20.14.

20.84 Because there is no exportation of goods *G*1 through *G*3, the rows that correspond to these commodities in Table 20.14 all have zero entries. The remainder of the table is the same as Table 20.10. Thus industry *G* exports x_{G4}^{GR} units of good *G*4, industry *S* exports x_{S1}^{SR} units of traditional services *S*1 and no units of high-tech services, and industry *T* exports x_T^{TR} units of transportation services to the rest of the world.

Table 20.14. Real ROW Supply or Export by Industry and Commodity Matrix

	Industry *G*	Industry *S*	Industry *T*
*G*1	0	0	0
*G*2	0	0	0
*G*3	0	0	0
*G*4	x_{G4}^{GR}	0	0
*S*1	0	x_{S1}^{SR}	0
*S*2	0	0	0
T	0	0	x_{T}^{TR}

Table 20.15. Real ROW Use or Import by Industry and Commodity Matrix

	Industry *G*	Industry *S*	Industry *T*
*G*1	m_{G1}^{GR}	m_{G1}^{SR}	0
*G*2	m_{G2}^{GR}	m_{G2}^{SR}	m_{G2}^{TR}
*G*3	m_{G3}^{GR}	0	0
*G*4	0	0	0
*S*1	m_{S1}^{GR}	m_{S1}^{SR}	0
*S*2	0	0	0
T	0	m_{T}^{SR}	m_{T}^{TR}

20.85 The counterpart to Table 20.11 is now Table 20.15. This matrix lists the industry demands for commodities that originate from foreign sources; that is, it shows the industry by commodity intermediate input demands for intermediate inputs from foreign sources.

20.86 From Table 20.15, it can be seen that the goods producing industry uses m_{G1}^{GR} units of agricultural imports, m_{G2}^{GR} units of crude oil imports, m_{G3}^{GR} units of a pure imported intermediate good, and m_{S1}^{GR} units of imported service inputs. Industry *G* does not import the domestically produced good, *G*4, nor does it import transportation services in this simplified example. Industry *S* imports m_{G1}^{SR} units of agricultural goods (for distribution to domestic households), m_{G2}^{SR} units of crude oil (for distribution to households and own use), m_{S1}^{GR} units of foreign general services, and m_T^{SR} units of foreign transportation services. Industry *T* imports m_{G2}^{TR} units of crude oil and m_T^{TR} units of foreign-sourced transportation services.

[37]This is not the only way the accounts could be set up. Note that the distribution services (in distributing G1 and G2) that the domestic service industry provides in this accounting framework is on a gross basis whereas the treatment of transportation services in industry *T* is on a net basis; that is, the present setup treats transportation services as a margin industry whereas the services associated with the direct distribution of imports to households are not treated in this way. This treatment of imports makes reconciliation of the production accounts with the final demand accounts fairly straightforward.

Table 20.16. Nominal Value Domestic Supply Matrix with Commodity Taxes

	Industry G	Industry S	Industry T
$G1$	0	$(p_{G1}^{SF} - t_{G1}^{SF})y_{G1}^{SF}$	0
$G2$	0	$(p_{G2}^{SF} - t_{G2}^{SF})y_{G2}^{SF}$	0
$G3$	0	0	0
$G4$	$(p_{G4}^{GS} - t_{G4}^{GS})y_{G4}^{GS} + (p_{G4}^{GT} - t_{G4}^{GT})y_{G4}^{GT} + (p_{G4}^{GF} - t_{G4}^{GF})y_{G4}^{GF}$	0	0
$S1$	0	$(p_{S1}^{SG} - t_{S1}^{SG})y_{S1}^{SG} + (p_{S1}^{ST} - t_{S1}^{ST})y_{S1}^{ST} + (p_{S1}^{SF} - t_{S1}^{SF})y_{S1}^{SF}$	0
$S2$	0	$(p_{S2}^{SG} - t_{S2}^{SG})y_{S2}^{SG} + (p_{S2}^{ST} - t_{S2}^{ST})y_{S2}^{ST} + (p_{S2}^{SF} - t_{S2}^{SF})y_{S2}^{SF}$	0
T	0	0	$(p_{T}^{TG} - t_{T}^{TG})y_{T}^{TG} + (p_{T}^{TS} - t_{T}^{TS})y_{T}^{TS} + (p_{T}^{TF} - t_{T}^{TF})y_{T}^{TF}$

C.2 The artificial data set framework: Value supply and use matrices

20.87 The value matrix counterparts to the two supply and two use matrices listed in Section B.1 above are listed in this section. Table 20.16 is the counterpart to Table 20.5.

20.88 All of the prices that begin with the letter p are the prices that *domestic final demanders* pay for a unit of the commodity (except for minor complications with respect to the treatment of export prices). In Table 20.16, these prices correspond to *purchasers' prices* in the *2008 SNA*.[38] However, the industry sellers of these commodities do not generally receive the full final demand price: Commodity taxes less commodity subsidies must be subtracted from these final demand prices in order to obtain the net prices that are listed in Table 20.16. These *net selling prices* are the prices that the industrial producers actually receive for their sales of outputs to domestic demanders. In Table 20.16, these prices correspond to *basic prices* in the *2008 SNA*.[39] The notation used for prices in Table 20.16 matches the notation used for quantities in Table 20.12.

20.89 The reader should note that in this chapter, for convenience, the p prices will be referred to as *final demand prices* and the $p - t$ prices will be referred to as *producer prices*. Conceptually, the *final demand prices* are the prices that domestic final demanders pay per unit for their purchases of commodities delivered to final demand categories. However, for an exported commodity, the final demand price is not the total purchase price (including transportation services provided by foreign establishments, import duties, and other applicable commodity taxes) that the foreign importer pays for the commodity; rather, in this case, the final demand price is only the payment made to the domestic producer by the foreign importer. Conceptually, *producer prices* are the prices that domestic producers receive per unit of output produced that is sold or the prices that domestic producers pay per unit of input that is purchased (including applicable commodity taxes and all transportation margins).[40] Table 20.17 is the counterpart to Table 20.2. It is also the value counterpart to Table 20.13.

20.90 Note that in Table 20.16, industry G receives only the revenue $(p_{G4}^{GS} - t_{G4}^{GS})y_{G4}^{GS}$ for its sales of commodity $G4$ to industry S, whereas in Table 20.17, industry S pays the amount $p_{G4}^{GS}y_{G4}^{GS}$ for these purchases of intermediate inputs from industry G. The difference between

[38]See Chapter 6, Section C, in the *2008 SNA* and Chapter 4 of this *Manual*.

[39]Note that the tax terms in Tables 20.5 through 20.8 are equal to per unit (or specific) commodity taxes less per unit commodity subsidies. These two effects could be distinguished separately at the cost of additional notational complexity. More generally, on basic price valuation, see Chapter 6, Section C, in the *2008 SNA* and Chapter 4 of this *Manual*.

[40]If the production accounts are to be used in order to measure total factor productivity growth using the economic approach suggested by Jorgenson and Griliches (1967 and 1972), it is important to use the prices that producers face in the accounting framework. The treatment of commodity taxes suggested in this *Manual* is consistent with the treatment suggested by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output" (1972 p. 85).

Table 20.17. Nominal Value Domestic Use Matrix

	Industry G	Industry G	Industry T
$G1$	0	0	0
$G2$	0	0	0
$G3$	0	0	0
$G4$	0	$p_{G4}^{GG}y_{G4}^{GG}$	$p_{G4}^{GT}y_{G4}^{GT}$
$G1$	$p_{G1}^{GG}y_{G1}^{GG}$	0	$p_{G1}^{GT}y_{G1}^{GT}$
$G2$	$p_{G2}^{GG}y_{G2}^{GG}$	0	$p_{G2}^{GT}y_{G2}^{GT}$
T	$p_{T}^{TG}y_{T}^{TG}$	$p_{T}^{TG}y_{T}^{TG}$	0

Table 20.18. Value ROW Supply or Export by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
$G1$	0	0	0
$G2$	0	0	0
$G3$	0	0	0
$G4$	$(p_{G4x}^{GR}-t_{G4x}^{GR})x_{G4}^{GR}$	0	0
$S1$	0	$p_{S1x}^{SR}x_{S1}^{SR}$	0
$S2$	0	0	0
T	0	0	$p_{Tx}^{TR}x_{T}^{TR}$

these two intersectoral value flows is $t_{G4}^{GS}y_{G4}^{GS}$, the tax (less subsidy) payments made by industry G to the government on this intersectoral value flow. Thus the values of domestic intersectoral transfers of goods and services in Tables 20.16 and 20.17 do not match up exactly unless the commodity tax less subsidy terms t_{S1}^{SG}, t_{G4}^{GS}, and so on are all zero. The counterpart to Table 20.6 is now Table 20.18, which in turn is the value counterpart to the real Table 20.14.

20.91 Because there is no exportation of goods $G1$ through $G3$, the rows that correspond to these commodities in Table 20.18 all have zero entries. The remainder of the table is straightforward. Thus industry G exports x_{G4}^{GR} units of good $G4$ and the foreign final demander pays the price p_{G4x}^{GR} per unit but the exporting industry gets only the amount $p_{G4x}^{GR} - t_{G4x}^{GR}$ per unit; that is, the government gets the per unit (net) revenue t_{G4x}^{GR} on these sales if it imposes a (net) export tax equal to t_{G4x}^{GR}. Similarly, net export taxes (if applicable) must be subtracted from the final demand prices for the other industries. In the numerical example that follows, it is assumed that the net export tax in industry G is negative (so that exports are subsidized in industry G) and that taxes in industries S and T are zero. The counterpart to Table 20.7 is now Table 20.19, which in turn is the value counterpart to the real Table 20.15.

20.92 It should be straightforward for the reader to interpret the final demand prices (these terms begin with p) and the accompanying import duties, excise duties, and other commodity taxes on imports (these terms begin with t). The quantities of imports (these terms begin with an m) are the same as the quantity terms in the corresponding real table, Table 20.15. From a practical point of view, governments have a tendency to tax imports (so that the tax terms in this table will tend to be positive) and to subsidize exports (so that the tax terms in the previous table will tend to be zero or negative).

C.3 Industry *G* prices and quantities

20.93 All of the price and quantity series used in this chapter are listed in the four nominal value supply and use matrices that are listed in Tables 20.16 through 20.19. The 11 final demand price series that form part of the industry G data in these matrices are listed for five periods in Table 20.20. The commodity that the price refers to is listed in the first row of the table.

Table 20.19. Value ROW Use or Import by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
$G1$	$(p_{G1m}^{GR}+t_{G1m}^{GR})m_{G1}^{GR}$	$(p_{G1m}^{SR}+t_{G1m}^{SR})m_{G1}^{SR}$	0
$G2$	$(p_{G2m}^{GR}+t_{G2m}^{GR})m_{G2}^{GR}$	$(p_{G2m}^{SR}+t_{G2m}^{SR})m_{G2}^{SR}$	$(p_{G2m}^{TR}+t_{G2m}^{TR})m_{G2}^{TR}$
$G3$	$(p_{G3m}^{GR}+t_{G3m}^{GR})m_{G3}^{GR}$	0	0
$G4$	0	0	0
$S1$	$(p_{S1m}^{GR}+t_{S1m}^{GR})m_{S1}^{GR}$	$(p_{S1m}^{SR}+t_{S1m}^{SR})m_{S1}^{SR}$	0
$S2$	0	0	0
T	0	$(p_{Tm}^{SR}+t_{Tm}^{SR})m_{T}^{SR}$	$(p_{Tm}^{TR}+t_{Tm}^{TR})m_{T}^{TR}$

Table 20.20. Industry *G* Final Demand Prices for All Transactions

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
Period	p_{G4}^{GS}	p_{G4}^{GT}	p_{G4}^{GF}	p_{S1}^{SG}	p_{S2}^{SG}	p_{T}^{TG}	p_{G4x}^{GR}	p_{G1m}^{GR}	p_{G2m}^{GR}	p_{G3m}^{GR}	p_{S1m}^{GR}
1	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.2	1.3	1.5	0.8	1.6	1.3	1.4	2.0	0.8	1.4
3	1.5	1.5	1.6	1.8	0.6	1.5	1.6	0.9	1.5	0.6	1.7
4	1.55	1.55	1.65	1.9	0.4	1.3	1.5	1.3	0.9	0.4	1.8
5	1.6	1.6	1.7	2.0	0.2	1.8	1.4	1.5	2.1	0.3	1.9

Table 20.21. Industry *G* Commodity Taxes

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
Period	t_{G4}^{GS}	t_{G4}^{GT}	t_{G4}^{GF}	t_{S1}^{SG}	t_{S2}^{SG}	t_{T}^{TG}	t_{G4x}^{GR}	t_{G1m}^{GR}	t_{G2m}^{GR}	t_{G3m}^{GR}	t_{S1m}^{GR}
1	0.05	0.05	0.10	0	0	0	−0.10	0.11	0.15	0.10	0.05
2	0.07	0.07	0.15	0	0	0	−0.13	0.14	0.20	0.08	0.07
3	0.08	0.08	0.20	0	0	0	−0.16	0.09	0.25	0.06	0.08
4	0.08	0.08	0.22	0	0	0	−0.15	0.13	0.20	0.04	0.09
5	0.09	0.09	0.23	0	0	0	−0.05	0.15	0.25	0.03	0.10

20.94 Some points to note about the price entries in Table 20.20 are as follows. The industry *G* final demand prices that it faces for deliveries of commodity *G*4 to the service industry, p_{G4}^{GS}, to the transportation services industry, p_{G4}^{GT}, and for exports, p_{G4x}^{GR}, are all much the same: Prices trend up fairly rapidly for periods 2 and 3 and then level off for periods 4 and 5. However, the final demand price for deliveries of *G*4 to the domestic final demand sector *F*, p_{G4}^{GF}, is somewhat higher than the corresponding prices for deliveries of *G*4 to the service sector *S*, p_{G4}^{GS}, and to the transportation sector *T*, p_{G4}^{GT}, owing to higher commodity taxes on deliveries to sector *F*. The price of traditional domestic services used as an intermediate input by industry *G*, p_{S1}^{SG}, also increases rapidly initially and then levels off for the last two periods. However, the price of high-tech domestic services used as an intermediate input by industry *G*, p_{S2}^{SG}, drops rapidly throughout the sample period. The price of transportation services used as an intermediate input by industry *G*, p_{T}^{TG}, increases dramatically in period 2 owing to the increase of the price of imported oil and then decreases for the next two periods as the price of oil drops before increasing again in period 5. The price of agricultural imports into industry *G*, p_{G1m}^{GR}, fluctuates considerably from period to period but overall, agricultural import prices do not increase as rapidly as do many other prices. The price of oil imports into industry *G*, p_{G2m}^{GR}, fluctuates violently, doubling in period 2, then falling so that by period 4, the price is below the period 1 price but then the price more than doubles for period 5. The price of the imported intermediate good, p_{G3m}^{GR}, steadily drops at a rapid pace over the five periods.[41] Finally, the price of the imported services commodity, p_{Sm}^{GR}, increases rapidly over periods 2 and 3 but then the rate of price increase slows down. Over the entire period, the price of services tends to increase somewhat more rapidly than the price of manufactured output, *G*4.

20.95 The 11 commodity tax series that form part of the industry *G* taxes listed in Tables 20.16 through 20.19 are listed for five periods in Table 20.21. Recall that by convention, the selling industry pays all commodity taxes so the taxes on industry *G*'s purchases of intermediate inputs from industries *S* and *T*, t_{S1}^{SG}, t_{S2}^{SG} and t_{T}^{TG}, are all identically equal to zero. However, in Table 20.21, these tax rates are listed (with zero entries) so that Table 20.21 is dimensionally comparable to Table 20.20.

[41]Think of this pure imported intermediate as being high-tech equipment, which has been dropping in price owing to the computer chip revolution.

Table 20.22. Industry *G* Quantities of Outputs and Intermediate Inputs

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
Period	y_{G4}^{GS}	y_{G4}^{GT}	y_{G4}^{GF}	y_{S1}^{SG}	y_{S2}^{SG}	y_{T}^{TG}	x_{G4x}^{GR}	m_{G1m}^{GR}	m_{G2m}^{GR}	m_{G3m}^{GR}	m_{S1m}^{GR}
1	5	2	35	4	2	3	25	5	10	10	2
2	6	2.5	40	5	4	3.5	28	6	12	13	2
3	7	3	45	6	8	4	32	7	15	19	3
4	7	3.5	49	8	14	5	40	7.5	18	25	4
5	8	4	55	10	20	6	54	8	15	35	6

Table 20.23. Industry *S* Final Demand Prices

	G1	G2	S1	S1	S1	S2	S2	S2
Period	p_{G1}^{SF}	p_{G2}^{SF}	p_{S1}^{SG}	p_{S1}^{ST}	p_{S1}^{SF}	p_{S2}^{SG}	p_{S2}^{ST}	p_{S2}^{SF}
1	1.2	1.4	1.0	1.0	1.3	1.0	1.0	1.15
2	1.5	2.8	1.5	1.4	1.8	0.8	0.8	0.94
3	1.2	2.2	1.8	1.7	2.2	0.6	0.6	0.72
4	1.6	1.5	1.9	1.8	2.4	0.4	0.4	0.45
5	1.7	3.0	2.0	1.9	2.6	0.2	0.2	0.23

	G4	T	S1	G1	G2	S1	T
Period	p_{G4}^{GS}	p_{T}^{TS}	p_{S1x}^{SR}	p_{G1m}^{SR}	p_{G2m}^{SR}	p_{S1m}^{SR}	p_{Tm}^{SR}
1	0.9	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.6	1.3	1.3	2.1	1.3	1.6
3	1.5	1.5	1.6	1.0	1.6	1.6	1.5
4	1.55	1.3	1.5	1.4	1.1	1.7	1.3
5	1.6	1.8	1.4	1.5	2.2	1.7	1.8

20.96 Note that the taxes listed above are all positive or zero except that the exports of good $G4$ by industry G are subsidized so the taxes t_{G4x}^{GR} have a negative sign attached to them instead of the usual positive sign.

20.97 The 11 quantity series that form part of the industry G data in Tables 20.16 through 20.19 are listed for five periods in Table 20.22.

20.98 The quantities of good $G4$ produced by industry G, y_{G4}^{GS}, y_{G4}^{GT}, y_{G4}^{GF}, which are deliveries to the domestic services industry, the domestic transportation industry, and the domestic final demand sector respectively, all grow at roughly the same rate. However, the quantities of $G4$ exported by industry G, x_{G4x}^{GR}, grow a bit more rapidly, particularly during the final two periods. The quantity of traditional domestic services used as an intermediate input by industry G, y_{S1}^{SG}, more than doubles over the five periods but the quantity of high-tech services used as an intermediate input, y_{S1}^{SG}, grows tenfold owing to the rapid price drop in this commodity. The quantity of domestic transportation services used as an intermediate input by industry G, y_{T}^{TG}, exactly doubles over the five periods. The quantity of agricultural imports used by industry G, m_{G1m}^{GR}, increases steadily from 5 units to 8 units while the quantity of oil imports increases from 10 in period 1 to 15 in period 3 but then the growth rate slows over the final two periods. Imports of the high-technology pure intermediate imported good, m_{G3m}^{GR}, increase rapidly from 10 to 35 units, reflecting the real-world tendency towards globalization and international outsourcing. Finally, imports of service inputs into industry G increase rapidly, growing from 2 units in period 1 to 6 units in period 5.

C.4 Industry *S* prices and quantities

20.99 The 15 final demand price series that form part of the industry S data in Tables 20.16 through 20.19 are listed for five periods in Table 20.23.

20.100 Some points to note about the price entries in Table 20.23 are as follows. The prices of service sector deliveries to industry G, p_{S1}^{SG} and p_{S2}^{SG}, and the prices of deliveries of good $G4$ from industry G to industry S, p_{G4}^{GS}, are exactly the same as in Tables 20.20 and 20.23. This reflects the bilateral nature of transactions between sectors. Industry S sells commodities $S1$ and $S2$ to industries G and T (these are the prices p_{S1}^{SG} and p_{S2}^{SG} for sales to industry G and p_{S1}^{ST} and p_{S2}^{ST} for sales to industry T) and it sells commodities $S1$ and $S2$ to the domestic final demand sector F at prices p_{S1}^{SF} and p_{S2}^{SF} and it sells $S1$ to the rest of the world R as an export at the price p_{S1x}^{SR}. The industry S final demand selling prices are much the same over these four destinations, except that export price for $S1$ falls off somewhat and the selling prices to the domestic final demand sector for the high-technology service $S2$ are somewhat higher, reflecting a higher level of final demand taxation. Industry S also imports $G1$ (agricultural or food imports for resale to domestic households) and $G2$ (oil imports for resale to domestic households) and it also imports some foreign general services $S1$ and some foreign transportation services T. These import prices are p_{G1m}^{SR}, p_{G2m}^{SR}, p_{S1m}^{SR}, and p_{Tm}^{SR}, respectively. The import prices for these first three classes of imports are much the same as the corresponding import prices that applied to the imports of these commodities by industry G. The price of imported transportation services, p_{Tm}^{SR}, is the same as the price of domestic transportation services provided to industry S, p_{T}^{TS}. Note that the service sector selling prices of goods $G1$ and $G2$ to the domestic final demand sector, p_{G1}^{SF} and p_{G2}^{SF}, are somewhat higher than the corresponding import purchase prices for these goods, p_{G1m}^{SR} and p_{G2}^{SF}, but this is natural: The service sector must make a positive margin on its trading in these commodities in order to cover the costs of storage and distribution.

20.101 The service industry obviously contains elements of the traditional storage, wholesaling, and retailing industries. The treatment of these industries that is followed in the artificial data example is a *gross output treatment* as opposed to a *margin industry treatment*. In the gross output treatment, goods for resale are purchased and the full purchase price times the amount purchased appears as an intermediate input cost and then the goods are sold subsequently at a higher price and this selling price times the amount sold appears as a contribution to gross output. In the margin treatment, it is assumed that the amount sold during the accounting period is at least roughly equal to the amount purchased, and the difference between the selling price and the purchase price (the margin) is multiplied by the amount bought and sold and is treated as a gross output with no corresponding intermediate input cost. Thus for the case of an imported good, if the margin treatment of wholesaling/retailing/storage (WRS) output is used, the margin would be credited to this WRS industry and the full import price plus the margin would appear as an intermediate input by the purchasing industry (or final demand sector). Thus the margin treatment of the WRS industry would be similar to the margin treatment that has been accorded to the transportation industry. However, there is a difference between the WRS industry and the transportation industry: For the transportation industry, one can be fairly certain that the goods "purchased" by the transport industry are equal to the goods "sold" by the industry and the margin treatment is perfectly justified. This is not necessarily the case for the WRS industry: Sales are not necessarily equal to purchases in any given accounting period. Thus it seems preferable to use the gross output treatment for these distributive industries over the margin approach, although individual countries may feel that sales are sufficiently close to purchases so that the margin approach is a reasonable approximation to the actual situation and hence can be used in their national accounts.[42]

20.102 The 15 commodity tax series that form part of the industry S taxes listed in Tables 20.16 through 20.19 are listed for five periods in Table 20.24.

20.103 Note that the tax rates on domestic intermediate inputs used by industry S are all set equal to zero under the convention used in this chapter that the selling industry pays any applicable commodity taxes.[43]

20.104 The 15 quantity series that form part of the industry S data in Tables 20.16 through 20.19 are listed for five periods in Table 20.25.

20.105 The quantities of industry S deliveries to industry G, y_{S1}^{SG} and y_{S2}^{SG}, and the quantities of deliveries of good $G4$ from industry G to industry S, y_{G4}^{GS}, are exactly the same in Tables 20.22 and 20.25.

20.106 Note that y_{G1}^{SF}, the quantity of imported food $G1$ sold by industry S to domestic final demanders F, is

[42]For theoretical treatments of the accounting problems associated with measuring the contribution of inventories to retailing and wholesaling production, see Paragraphs 6.57 through 6.79 of the *2008 SNA*, Diewert and Smith (1994), Ehemann (2005), Diewert (2005a), and Peter Hill (2005).

[43]The selling industry also receives any applicable commodity subsidies.

Table 20.24. Industry *S* Commodity Taxes

Commodity	G1	G2	S1	S1	S1	S2	S2	S2
Period	t_{G1}^{SF}	t_{G2}^{SF}	t_{S1}^{SG}	t_{S1}^{ST}	t_{S1}^{SF}	t_{S2}^{SG}	t_{S2}^{ST}	t_{S2}^{SF}
1	0.02	0.17	0.01	0.01	0.10	0.15	0.15	0.30
2	0.05	0.23	0.02	0.02	0.15	0.11	0.11	0.25
3	0.02	0.19	0.03	0.03	0.18	0.08	0.08	0.20
4	0.06	0.17	0.03	0.03	0.19	0.05	0.05	0.10
5	0.07	0.24	0.03	0.02	0.20	0.02	0.02	0.05
	G4	T	S1	G1	G2	S1	T	
Period	t_{G4}^{GS}	t_{T}^{TS}	t_{S1x}^{SR}	t_{G1m}^{SR}	t_{G2m}^{SR}	t_{S1m}^{SR}	t_{Tm}^{SR}	
1	0	0	0	0.02	0.15	0.05	0.03	
2	0	0	0	0.03	0.20	0.06	0.04	
3	0	0	0	0.04	0.25	0.09	0.04	
4	0	0	0	0.04	0.20	0.09	0.03	
5	0	0	0	0.04	0.25	0.10	0.03	

Table 20.25. Industry *S* Quantities of Outputs and Inputs

	G1	G2	S1	S1	S1	S2	S2	S2
Period	y_{G1}^{SF}	y_{G2}^{SF}	y_{S1}^{SG}	y_{S1}^{ST}	y_{S1}^{SF}	y_{S2}^{SG}	y_{S2}^{ST}	y_{S2}^{SF}
1	10	8	4	2.0	15	2	1.1	3.0
2	11	9	5	2.5	20	4	1.5	4.3
3	12	9	6	3.0	25	8	2.1	6.5
4	13	10	8	3.5	33	14	3.5	10.5
5	14	11	10	3.5	40	20	5.0	15.0
	G4	T	S1	G1	G2	S1	T	
Period	y_{G4}^{GS}	y_{T}^{TS}	x_{S1x}^{SR}	m_{G1m}^{SR}	m_{G2m}^{SR}	m_{S1m}^{SR}	m_{Tm}^{SR}	
1	5	1.0	14	10	10	3	1.0	
2	6	1.1	19	11	11	4	1.5	
3	7	1.2	24	12	11	6	1.7	
4	7	1.3	31	13	12	9	1.9	
5	8	1.3	42	14	13	13	2.0	

exactly equal to m_{G1m}^{SR}, imports of food into industry *S*. However, y_{G2}^{SF}, the quantity of imported energy products *G*2 sold by industry *S* to domestic final demanders, is less than the quantity of energy imported by industry *S*, m_{G2m}^{SR}. The reason for this difference is that industry *S* uses some of the imported energy for heat and other purposes as it supplies services to other sectors of the economy. Sales by industry *S* of traditional services *S*1 to industry *G*, industry *T*, domestic final demand *F*, and the rest of the world *R*, y_{S1}^{SG}, y_{S1}^{ST}, y_{S1}^{SF}, and x_{S1x}^{SR}, respectively, all increase quite rapidly, doubling or tripling over the five periods. Imports of traditional services *S*1 into industry *S*, m_{S1m}^{SR}, increase even more rapidly, growing from 3 to 13 over the five periods. The sales of high-tech services by industry *S* to industries *G* and *T* and to the domestic final demand sector *F*, y_{S2}^{SG}, y_{S2}^{ST}, and y_{S2}^{SF}, respectively, all increase very rapidly, growing between 5- and 10-fold over the five periods.

The quantities of domestic intermediate inputs of good $G4$, y_{G4}^{GS}, and of the transportation service, y_T^{TS}, used by industry S grew fairly slowly over the five periods. Imported transportation services, m_{Tm}^{SR}, associated with the importation of $G1$ and $G2$ by industry S, doubled over the five periods.

C.5 Industry *T* prices and quantities

20.107 The nine final demand price series that form part of the industry T data in Tables 20.16 through 20.19 are listed for five periods in Table 20.26.

20.108 The entries for p_T^{TS}, p_{S1}^{ST}, and p_{S2}^{ST} in Tables 20.26 and 20.23 are the same series as are the entries for p_T^{TG} and p_{G4}^{GT} in Tables 20.26 and 20.20. Again, this reflects the fact that the sellers and purchasers of domestic intermediate inputs pay and receive the same amounts of money for their cross-industry purchases and sales.

20.109 The industry selling prices for transportation services show much the same trends across all destinations. The selling prices of transportation services to domestic final demand, p_T^{TF}, are higher than the other selling prices owing to higher commodity taxation for deliveries to final demand.

20.110 The commodity tax series that form part of the industry T taxes listed in Tables 20.16 through 20.19 are listed for five periods in Table 20.27.

20.111 The commodity taxes on deliveries of transportation services to industries G and S, t_T^{TG} and t_T^{TS} respectively, are small but the taxes on deliveries to the final demand sector, t_T^{TF}, are fairly substantial, as are the taxes on the transportation sector's imports of energy products, t_{G2m}^{TR}. By convention, any taxes on industry T's use of domestic intermediate inputs are paid by the selling industry so t_{G4}^{GT}, t_{S1}^{ST}, and t_{S2}^{ST} are all zero. There are no taxes on the export of transportation services in this economy so that t_{Tx}^{TR} is zero as well. There are small taxes on industry T's importation of transport services, t_{Tm}^{TR}.

20.112 The nine final demand quantity series that form part of the industry T data in Tables 20.16 through 20.19 are listed for five periods in Table 20.28.

20.113 The entries for y_T^{TS}, y_{S1}^{ST}, and y_{S2}^{ST} in Tables 20.28 and 20.25 are the same series as are the entries for y_T^{TG} and y_{G4}^{GT} in Tables 20.28 and 20.22. All transportation service inputs and outputs grow relatively smoothly and roughly double over the five periods.

Table 20.26. Industry *T* Final Demand Prices

	T	*T*	*T*	*G*4	*S*1	*S*2	*T*	*G*2	*T*
Period	p_T^{TG}	p_T^{TS}	p_T^{TF}	p_{G4}^{GT}	p_{S1}^{ST}	p_{S2}^{ST}	p_{Tx}^{TR}	p_{G2m}^{TR}	p_{Tm}^{TR}
1	1.0	1.0	1.2	0.9	1.0	1.0	1.1	1.0	1.0
2	1.6	1.6	1.8	1.2	1.4	0.8	1.7	2.1	1.6
3	1.5	1.5	1.7	1.5	1.7	0.6	1.5	1.6	1.4
4	1.3	1.3	1.6	1.55	1.8	0.4	1.3	1.1	1.2
5	1.8	1.8	2.2	1.6	1.9	0.2	1.8	2.2	1.8

Table 20.27. Industry *T* Commodity Taxes

	T	*T*	*T*	*G*4	*S*1	*S*2	*T*	*G*	*T*
Period	t_T^{TG}	t_T^{TS}	t_T^{TF}	t_{G4}^{GT}	t_{S1}^{ST}	t_{S2}^{ST}	t_{Tx}^{TR}	t_{G2m}^{TR}	t_{Tm}^{TR}
1	0.01	0.01	0.10	0	0	0	0	0.15	0.03
2	0.02	0.02	0.15	0	0	0	0	0.20	0.04
3	0.03	0.03	0.18	0	0	0	0	0.25	0.04
4	0.03	0.03	0.19	0	0	0	0	0.20	0.03
5	0.03	0.03	0.20	0	0	0	0	0.25	0.03

Table 20.28. Industry *T* Quantities of Outputs and Inputs

	T	*T*	*T*	*G*4	*S*1	*S*2	*T*	*G*2	*T*
Period	y_T^{TG}	y_T^{TS}	y_T^{TF}	y_{G4}^{GT}	y_{S1}^{ST}	y_{S2}^{ST}	x_{Tx}^{TR}	m_{G2m}^{TR}	m_{Tm}^{TR}
1	3	1.0	5	2	2.0	1.1	3	3	1.5
2	3.5	1.1	5	2.5	2.5	1.5	4	3	1.7
3	4	1.2	6	3	3.0	2.1	5	3.5	2.2
4	5	1.3	7	3.5	3.5	3.5	5.5	4	2.4
5	6	1.3	7	4	3.5	5.0	6	4.5	2.5

20.114 This completes the listing of the basic price, tax, and quantity data that are used in subsequent sections of this chapter in order to illustrate how various index number formulas differ and how consistent sets of producer price indices can be formed in a set of production accounts that are roughly equivalent to the production accounts that are described in Chapter 15 of the *2008 SNA*.

D. The Artificial Data Set for Domestic Final Demand

D.1 The final demand data set

20.115 In order to illustrate what kind of differences can result from the choice of different index number formulas, the price and quantity data that correspond to domestic deliveries to final demand that were listed in the previous section are used as a test data set in this section. The six final demand price series are listed in Table 20.29 and the corresponding quantity series are listed in Table 20.30.

20.116 The prices p_1^t, p_2^t, p_3^t, p_4^t, p_5^t, and p_6^t in Table 20.30 correspond to the final demand prices p_{G1}^{SF}, p_{G2}^{SF}, p_{G4}^{GF}, p_{S1}^{SF}, p_{S2}^{SF}, and p_T^{TF}, respectively, which are listed in Tables 20.20, 20.23, and 20.26.

20.117 The quantities q_1^t, q_2^t, q_3^t, q_4^t, q_5^t, and q_6^t in Table 20.30 correspond to the final demand quantities y_{G1}^{SF}, y_{G2}^{SF}, y_{G4}^{GF}, y_{S1}^{SF}, y_{S2}^{SF}, and y_T^{TF}, respectively, which are listed in Tables 20.22, 20.25, and 20.28.

Table 20.29. Prices for Six Domestic Final Demand Commodities

	*G*1	*G*2	*G*4	*S*1	*S*2	*T*
	Food	Energy	Goods	Services	High-tech services	Transport
Period *t*	p_1^t	p_2^t	p_3^t	p_4^t	p_5^t	p_6^t
1	1.2	1.4	1.0	1.3	1.15	1.2
2	1.5	2.8	1.3	1.8	0.94	1.8
3	1.2	2.2	1.6	2.2	0.72	1.7
4	1.6	1.5	1.65	2.4	0.45	1.6
5	1.7	3.0	1.7	2.6	0.23	2.2

Table 20.30. Quantities for Six Domestic Final Demand Commodities

	*G*1	*G*2	*G*4	*S*1	*S*2	*T*
	Food	Energy	Goods	Services	High-tech Services	Transport
Period *t*	q_1^t	q_2^t	q_3^t	q_4^t	q_5^t	q_6^t
1	10	8	35	15	3.0	5
2	11	9	40	20	4.3	5
3	12	9	45	25	6.5	6
4	13	10	49	33	10.5	7
5	14	11	55	40	15.0	7

Table 20.31. Total Expenditures and Expenditure Shares for Six Domestic Final Demand Commodities

	Expenditures	Food	Energy	Goods	Services	High-Tech Services	Transport Services
Period t	$p^t \cdot q^t$	s_1^t	s_2^t	s_3^t	s_4^t	s_5^t	s_6^t
1	87.150	0.1377	0.1285	0.4016	0.2238	0.0396	0.0688
2	142.742	0.1156	0.1765	0.3643	0.2522	0.0283	0.0631
3	176.080	0.0818	0.1124	0.4089	0.3124	0.0266	0.0579
4	211.775	0.0982	0.0708	0.3818	0.3740	0.0223	0.0529
5	273.150	0.0871	0.1208	0.3423	0.3807	0.0126	0.0564

20.118 It is useful to also list the period T expenditures on all six domestic finally demanded commodities, $p^t \cdot q^t$, along with the corresponding expenditure shares, $s_1^t, \ldots, s_6^t$; see Table 20.31.

20.119 The expenditure shares for food, goods, and high-tech services decrease markedly over the five periods, the share for transport services decreases somewhat, the share of energy stays roughly constant but with substantial period-to-period fluctuations, and the share of general services increases substantially.

20.120 Note that the price of food and energy fluctuates considerably from period to period but the quantities demanded tend to trend upwards at a fairly smooth rate, reflecting the low elasticity of price demand for these products. The fluctuations in energy prices tend to produce similar fluctuations in the price of domestic transportation services but the fluctuations in price are more damped in the case of transport services. The price of goods trends up fairly rapidly in periods 2 and 3 but then the rate of increase falls off. The corresponding quantity trends upwards fairly steadily. The price of traditional services, p_4^t, increases rapidly in periods 2 and 3 and then increases more slowly. Overall, the price of traditional services increases more rapidly than the price of goods but the quantity of services demanded, q_4^t, increases more rapidly than the quantity of goods, q_3^t. The price of high-technology services, p_5^t, decreases rapidly over the five periods, falling to about one-fifth of the initial price level. The corresponding quantities demanded, q_5^t, increase rapidly, growing five-fold over the sample period. Thus overall, the data set exhibits a rather wide variety of trends in prices and quantities but these trends are not unrealistic in today's world economy. The movements of prices and quantities in this artificial data set are more pronounced than the year-to-year movements that would be encountered in a typical country but they do illustrate the problem that is facing compilers of producer and consumer price indices: Namely, *year-to-year price and quantity movements are far from being proportional across commodities so the choice of index number formula will matter.*

D.2 Some familiar index number formulas

20.121 Every price statistician is familiar with the *Laspeyres index*, P_L, and the *Paasche index*, P_P, defined in Chapter 15. These indices are listed in Table 20.32 along with the two main unweighted indices that were considered in Chapters 10, 17, and 21: the *Carli index* and the *Jevons index*. The indices in Table 20.31 compare the prices in period t with the prices in period 1; that is, they are *fixed-base indices*. Thus the period t entry for the Carli index, P_C, is simply the arithmetic mean of the eight price relatives, $\sum_{i=1}^{6}(1/6)(p_i^t/p_i^1)$, while the period t entry for the Jevons index, P_J, is the geometric mean of the six long-term price relatives, $\prod_{i=1}^{6}(p_i^t/p_i^1)0^{1/6}$.

20.122 Note that by period 5, the spread between the fixed-base Laspeyres and Paasche price indices is not negligible: P_L is equal to 1.7348 while P_P is 1.6570, *a spread of about 4.7 percent*. Because both of these indices have exactly the same *theoretical* justification, it can be seen that the choice of index number formula matters. There is also a substantial spread between the two unweighted indices by period 5: The fixed-base Carli index is equal to 1.5488, while the fixed-base Jevons index is 1.2483, *a spread of about* 24 *percent*.

Table 20.32. Fixed-Base Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P_L^t	P_P^t	P_C^t	P_J^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4832	1.4775	1.3177	1.2478
4	1.5043	1.4916	1.2709	1.1464
5	1.7348	1.6570	1.5488	1.2483

Table 20.33. Chained Laspeyres, Paasche, Carli, and Jevons Indices

Period t	P^t_L	P^t_P	P^t_C	P^t_J
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4931	1.4952	1.3178	1.2478
4	1.5219	1.5219	1.2527	1.1464
5	1.7176	1.7065	1.4745	1.2483

However, more troublesome than this spread is the fact that *the unweighted indices are well below both the Paasche and Laspeyres indices* by period 5. Thus when there are divergent trends in both prices and quantities, it will usually be the case that unweighted price indices will give very different answers than their weighted counterparts. Because none of the index number theories considered in previous chapters supported the use of unweighted indices, their use is not recommended for aggregation at the "higher level," that is, when data on weights are available. However, in Chapter 21 aggregation at the "lower level" is considered for when weights are unavailable, and the use of unweighted index number formulas will be revisited. Finally, note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[44]

20.123 It is of interest to recalculate the four indices listed in Table 20.32 using *the chain principle* rather than the *fixed-base principle*. Our expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chained indices are listed in Table 20.33.

20.124 It can be seen comparing Tables 20.32 and 20.33 that chaining eliminated most of the spread between the fixed-base Paasche and Laspeyres indices for period 5; that is, the spread between the chained Laspeyres and Paasche indices has dropped from 4.7 percent to 0.6 percent. Note that chaining did not affect the Jevons index. This is an advantage of the index but the lack of weighting is a fatal flaw. The "truth" would be expected to lie between the Paasche and Laspeyres indices but from Tables 20.32 and 20.33, the unweighted Jevons index is far below this acceptable range. The fixed-base and chained Carli indices also lie outside this acceptable range.

D.3 Asymmetrically weighted index number formulas

20.125 A systematic comparison of all of *the asymmetrically weighted price indices* is now undertaken. The *fixed-base indices* are listed in Table 20.34. The fixed-base *Laspeyres* and *Paasche indices*, P_L and P_P, are the same as those indices listed in Table 20.32. The *Palgrave index*, P_{PAL}, is defined by equation (16.55) in Chapter 16. The indices denoted by P_{GL} and P_{GP} are *the geometric Laspeyres and geometric Paasche indices*,[45] which were defined in Chapter 16. For *the geometric Laspeyres index*, P_{GL}, the weights for the price relatives are the *base-period expenditure shares*, s_i^1. This index should be considered an alternative to the fixed-base Laspeyres index because each of these indices makes use of the same information set. For *the geometric Paasche index*, P_{GP}, the weights for the price relatives are the *current-period expenditure shares*, s_i^t. Finally, the index P_{HL} *is the harmonic Laspeyres index* that was defined by equation (16.59).

20.126 By looking at the period 5 entries in Table 20.34, we can see that the spread between all of these fixed-base asymmetrically weighted indices has increased to be much larger than our earlier spread of 4.7 percent between the fixed-base Paasche and Laspeyres indices. In Table 20.34, the period 5

[44]This is the Theorem of the Arithmetic and Geometric Mean; see Hardy, Littlewood, and Pólya (1934) and Chapter 20.

[45]Vartia (1978, p. 272) used the terms *logarithmic Laspeyres* and *logarithmic Paasche*, respectively.

Table 20.34. Asymmetrically Weighted Fixed-Base Indices

Period t	P^t_{PAL}	P^t_{GP}	P^t_L	P^t_{GL}	P^t_P	P^t_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.5400	1.5145	1.4832	1.4477	1.4775	1.3995
4	1.6064	1.5650	1.5043	1.4469	1.4916	1.3502
5	1.8316	1.7893	1.7348	1.6358	1.6570	1.3499

Palgrave index is about 1.36 times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, *this illustrates the point that owing to the nonproportional growth of prices and quantities in most economies today, the choice of index number formula is very important*.[46]

20.127 It is possible to explain why certain of the indices in Table 20.34 are bigger than others. When all weights are positive, it can be shown that a *weighted arithmetic mean* of N numbers is equal to or greater than the corresponding *weighted geometric mean* of the same N numbers which in turn is equal to or greater than the corresponding *weighted harmonic mean* of the same N numbers.[47] It can be seen that the three indices P_{PAL}, P_{GP}, and P_P all use the current-period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1) but P_{PAL} is a weighted *arithmetic* mean of these price relatives, P_{GP} is a weighted *geometric* mean of these price relatives, and P_P is a weighted *harmonic* mean of these price relatives. Thus because there are no negative components in final demand, by Schlömilch's (1858) inequality,[48]

$$P_{PAL} \geq P_{GP} \geq P_P. \tag{20.30}$$

20.128 Viewing Table 20.34, we can see that the inequalities (20.30) hold for all periods. It can also be verified that the three indices P_L, P_{GL}, and P_{HL} all use the base-period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1) but P_L is a weighted *arithmetic* mean of these price relatives, P_{GL} is a weighted *geometric* mean of these price relatives, and P_{HL} is a weighted *harmonic* mean of these price relatives. Because all of the expenditure shares are positive, then by Schlömilch's inequality,[49]

$$P_L \geq P_{GL} \geq P_{HL}. \tag{20.31}$$

Viewing Table 20.34, we can see that the inequalities (20.31) hold for all periods.

20.129 Now continue with the systematic comparison of all of *the asymmetrically weighted price indices*. These indices using the *chain principle* are listed in Table 20.35.

Table 20.35. Asymmetrically Weighted Chained Indices

Period t	P^t_{PAL}	P^t_{GP}	P^t_{L}	P^t_{GL}	P^t_{P}	P^t_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.6019	1.5488	1.4931	1.4400	1.4952	1.3870
4	1.6734	1.5987	1.5219	1.4461	1.5219	1.3690
5	1.9802	1.8375	1.7176	1.5954	1.7065	1.4788

20.130 Viewing Table 20.35, we can see that the use of the chain principle only marginally reduced the spread between all of the asymmetrically weighted indices compared to their fixed-base counterparts in Table 20.34. For period 5, the spread between the smallest and largest asymmetrically weighted fixed-base index was 35.7 percent but for the period 5 chained indices, this spread was marginally reduced to 33.9 percent.

D.4 Symmetrically weighted index number formulas

20.131 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory; that is, as was seen in Chapter 18, a superlative index is exact for a representation of the producer's production function or the corresponding unit revenue function that can provide a second order approximation to arbitrary technologies that satisfy certain regularity conditions. In Chapter 18 four primary superlative indices were considered:

- The *Fisher ideal price index*, P_F, defined by equation (16.12);
- The *Walsh price index*, P_W, defined by equation (16.19) (this price index also corresponds to the quantity index Q^1 defined by equation (18.60));
- The *Törnqvist Theil price index*, P_T, defined by equation (18.11); and
- The *implicit Walsh price index*, P_{IW}, that corresponds to the Walsh quantity index Q_W defined by equation (17.34) (this is also the index P^1 defined by equation (18.60)).

[46]Allen and Diewert (1981) showed that the Paasche, Laspeyres, and Fisher indices will all be equal if either prices or quantities move in a proportional manner over time. Thus in order to get a spread between the Paasche and Laspeyres indices, it is required that *both* prices and quantities move in a nonproportional manner.

[47]This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood, and Pólya (1934, chapter 11).

[48]These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[49]These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

Table 20.36. Symmetrically Weighted Fixed-Base Indices

Period t	P^t_T	P^t_{IW}	P^t_W	P^t_F	P^t_D	P^t_{ME}
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.39347	1.39312	1.39307	1.39297	1.39298	1.39267
3	1.48073	1.48219	1.48129	1.48034	1.48034	1.47990
4	1.50481	1.50627	1.50216	1.49796	1.49797	1.49645
5	1.71081	1.72041	1.70612	1.69545	1.69589	1.68389

Table 20.37. Symmetrically Weighted Chained Indices

Period t	P^t_T	P^t_{IW}	P^t_W	P^t_F	P^t_D	P^t_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3935	1.3931	1.3931	1.3930	1.3930	1.3927
3	1.4934	1.4941	1.4945	1.4941	1.4941	1.4942
4	1.5205	1.5219	1.5224	1.5219	1.5219	1.5218
5	1.7122	1.7122	1.7127	1.7120	1.7121	1.7116

20.132 These four symmetrically weighted superlative price indices are listed in Table 20.36 using the fixed-base principle. Also listed in this table are two symmetrically weighted price indices:[50]

- The Marshall Edgeworth price index, P_{ME}, defined by equation (15.18), and
- The Drobisch price index, P_D, the arithmetic average of the Paasche and Laspeyres price indices.

20.133 Note that the Drobisch index P_D is always equal to or greater than the corresponding Fisher index P_F. This follows from the fact that the Fisher index is the geometric mean of the Paasche and Laspeyres indices while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices and an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed-base asymmetrically weighted indices, Table 20.34, with the symmetrically weighted indices, Table 20.36, *we can see that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices.* The spread was $1.8316/1.3499 = 1.357$ for the asymmetrically weighted indices but only $1.72041/1.68389 = 1.022$ for the symmetrically weighted indices. If the analysis is restricted to the superlative indices listed for period 5 in Table 20.19, then this spread is further reduced to $1.72041/1.69545 = 1.015$; that is, the spread between the fixed-base superlative indices is only 1.5 percent compared to the fixed-base spread between the Palgrave and harmonic Laspeyres indices of 35.7 percent ($1.8316/1.3499 = 1.357$). The spread between the superlative indices can be expected to be further reduced if the chain principle is used.

20.134 The symmetrically weighted indices are recomputed using the chain principle. The results may be found in Table 20.37. A quick glance at Table 20.20 shows that *the combined effect of using both the chain principle and symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles.* The spread between all of the symmetrically weighted indices in period 5 is only $1.7127/1.7116 = 1.0006$ or 0.06 percent, which is negligible.

20.135 The results listed in Table 20.37 reinforce the numerical results tabled in Robert Hill (2006) and Diewert (1978, p. 894): *The most commonly used chained superlative indices will generally give approximately the same numerical results.*[51] This numerical approximation property holds in spite of the erratic nature of the fluctuations in the data in Tables 20.29 through 20.31. In particular, the chained Fisher, Törnqvist, and Walsh indices generally approximate each other very closely.

[50]Diewert (1978, p. 897) showed that the Drobisch Sidgwick Bowley price index approximates any superlative index to the second order around an equal price and quantity point; that is, P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall Edgeworth index P_{ME} is also pseudo-superlative.

[51]More precisely, the superlative quadratic mean of order r price indices P^r defined by equation (17.84) and the implicit quadratic mean of order r price indices P^{r*} defined by equation (17.81) will generally closely approximate each other provided that r is in the interval $0 \leq r \leq 2$.

D.5 Superlative indices and two-stage aggregation

20.136 Attention is now turned to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see Section E.6 of Chapter 18 for a discussion of the issues and a listing of the formulas used. In the artificial data set for domestic final demand, the first three commodities are aggregated into a *goods aggregate* and the final three commodities are aggregated into a *services aggregate*. In the second stage of aggregation, the good and services components will be aggregated into a domestic final demand price index.

20.137 The results of single-stage and two-stage aggregation are reported in Table 20.38 using period 1 as the *fixed base* for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indices, P_W and P_{IW}.

20.138 Viewing Table 20.38, it can be seen that the fixed-base single-stage superlative indices generally approximate their fixed-base two-stage counterparts fairly closely. The divergence between the single-stage Törnqvist index P_T and its two-stage counterpart P_{T2S} in period 5 is 1.7108/1.7007 = 1.006 or 0.6 percent. The other divergences are even less.

20.139 The results reported in Table 20.39 use *chained* versions of these indices for the two-stage aggregation procedure. Again, the single-stage and their two-stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T, and the Walsh and implicit Walsh indices, P_W and P_{IW}.

20.140 Viewing Table 20.39, we can see that the chained single-stage superlative indices generally approximate their fixed-base two-stage counterparts quite closely. The divergence between the chained Törnqvist index P_T and its two-stage counterpart P_{T2S} in period 5 is 1.7136/1.7122 = 1.0008 or 0.08 percent. The other divergences are all less than this. Given the large dispersion in period-to-period price movements, these two-stage aggregation errors are not large. However, the important point that emerges from Table 20.39 is that *the use of the chain principle has reduced the spread between all eight single-stage and two-stage superlative indices* compared to their fixed-base counterparts in Table 20.38. The maximum spread for the period 5 chained index values is 0.09 percent while the maximum spread for the period 5 fixed-base index values is 1.5 percent.

20.141 The final formulas that is illustrated using the artificial final expenditures data set are the *additive percentage change decompositions* for the Fisher ideal index that were discussed in Section C.8 of Chapter 17. The *chain links* for the Fisher price index will first be decomposed using the Diewert (2002a) decomposition formulas (16.41) to (16.43). The results of the decomposition are listed in Table 20.40. Thus $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t-1$ to t and *the decomposition factor* $v_{Fi}\Delta p_i = v_{Fi}\,(p_i^t - p_i^{t-1})$ is the contribution to the total

Table 20.38. Single-Stage and Two-Stage Fixed-Base Superlative Indices

Period t	P^t_F	P^t_{F2S}	P^t_T	P^t_{T2S}	P^t_W	P^t_{W2S}	P^t_{IW}	P^t_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4803	1.4808	1.4807	1.4800	1.4813	1.4813	1.4822	1.4821
4	1.4980	1.4998	1.5048	1.5003	1.5022	1.5021	1.5063	1.5051
5	1.6954	1.7012	1.7108	1.7007	1.7061	1.7063	1.7204	1.7176

Table 20.39. Single-Stage and Two-Stage Chained Superlative Indices

Period t	P^t_F	P^t_{F2S}	P^t_T	P^t_{T2S}	P^t_W	P^t_{W2S}	P^t_{IW}	P^t_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4941	1.4943	1.4934	1.4942	1.4945	1.4944	1.4941	1.4945
4	1.5219	1.5221	1.5205	1.5218	1.5224	1.5223	1.5219	1.5226
5	1.7120	1.7125	1.7122	1.7136	1.7127	1.7127	1.7122	1.7132

Table 20.40. Diewert Additive Percentage Change Decomposition of the Fisher Index

Period t	$P_F^t - 1$	$v_{F1}^t \Delta p_1^t$	$v_{F2}^t \Delta p_2^t$	$v_{F3}^t \Delta p_3^t$	$v_{F4}^t \Delta p_4^t$	$v_{F5}^t \Delta p_5^t$	$v_{F6}^t \Delta p_6^t$
2	0.3930	0.0331	0.1253	0.1185	0.0928	−0.0082	0.0314
3	0.0726	−0.0225	−0.0353	0.0831	0.0586	−0.0077	−0.0036
4	0.0186	0.0261	−0.0347	0.0123	0.0301	−0.0118	−0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0321	−0.0123	0.0185

percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1, 2, \ldots, 6$.

20.142 Viewing Table 20.40, we can see that the price index going from period 1 to 2 grew 39.30 percent and the contributors to this change were the increases in the price of commodity 1, finally demanded agricultural products (3.31 percentage points); commodity 2, finally demanded energy (12.53 percentage points); commodity 3, finally demanded goods (11.85 percentage points); commodity 4, traditional services (9.28 percentage points); and commodity 6, transportation services (3.14 percentage points). High-technology services, commodity 5, decreased in price and this fall in prices subtracted 0.82 percentage points from the overall Fisher price index going from period 1 to 2. The sum of the last six entries for period 2 in Table 20.40 is equal to 0.3930, the percentage increase in the Fisher price index going from period 1 to 2. It can be seen that a big price change in a particular component i combined with a big expenditure share in the two periods under consideration will lead to a big decomposition factor, $v_{Fi}\Delta p_i$.

20.143 Our final set of computations illustrates the *additive percentage change decomposition* for the Fisher ideal index that is due to Van Ijzeren (1987, p. 6) that was mentioned in Section C.8 of Chapter 17.[52] First, the Fisher price index going from period $t-1$ to t is written in the following form:

$$P_F(p^{t-1}, p^t, q^{t-1}, q^t) = \frac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^t}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}, \qquad (20.32)$$

where the reference quantities need to be defined somehow. Van Ijzeren (1987, p. 6) showed that the following reference weights provided an *exact additive representation for the Fisher ideal price index*:

$$q_{Fi}{}^{*} \equiv (1/2)q_i^{t-1} + [(1/2)Q_F(p^{t-1}, p^t, q^{t-1}, q^t)]; \quad i = 1, 2, \ldots, 6, \qquad (20.33)$$

where Q_F is the overall Fisher quantity index. Thus using the Van Ijzeren quantity weights $q_{Fi}{}^{*}$, we obtain the *following Van Ijzeren additive percentage change decomposition for the Fisher price index*:

$$P_F(p^0, p^1, q^0, q^1) - 1 = \frac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^t}{\sum_{i=1}^{6} q_{Fi}^{t*} p_i^{t-1}} - 1$$

$$= \sum_{i=1}^{6} v_{Fi}^{t*} (p_i^t - p_i^{t-1}), \qquad (20.34)$$

where the *Van Ijzeren weight* for commodity i, v_{Fi}^{t*}, is defined as

$$v_{Fi}^{t*} \equiv \frac{\sum_{i=1}^{6} q_{Fi}^{*}}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}; \quad i = 1, 2, \ldots, 6. \qquad (20.35)$$

20.144 The *chain links* for the Fisher price index are decomposed using formulas (20.32) to (20.35). The results of the decomposition are listed in Table 20.41. Thus $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t-1$ to t, and *the Van Ijzeren decomposition factor* $v_{Fi}^{t*}\Delta p_i^t$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1, 2, \ldots, 6$.

20.145 Comparing the entries in Tables 20.40 and 20.41, we can see that the differences between the Diewert and Van Ijzeren decompositions of the Fisher price index are *very small*.[53] This is somewhat surprising given the very different nature of the two decompositions.[54] As was mentioned in Section C.8

[52]It was also independently derived by Dikhanov (1997) and used by Ehemann, Katz, and Moulton (2002).

[53]The maximum difference between the two tables occurs in period 2 for the p_4 contribution factor, which is 0.0928 in Table 20.40 and 0.0917 in Table 20.41.

[54]The terms in Diewert's decomposition can be given economic interpretations whereas the terms in the other decomposition are more difficult to interpret from the economic perspective. However, Reinsdorf, Diewert, and Ehemann (2002) showed that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and where the two quantity vectors are equal.

Table 20.41. Van Ijzeren Additive Percentage Change Decomposition of the Fisher Index

Period t	$P_F^t - 1$	$v_{F1}^{t^*}\Delta p_1^t$	$v_{F2}^{t^*}\Delta p_2^t$	$v_{F3}^{t^*}\Delta p_3^t$	$v_{F4}^{t^*}\Delta p_4^t$	$v_{F5}^{t^*}\Delta p_5^t$	$v_{F6}^{t^*}\Delta p_6^t$
2	0.3930	0.0333	0.1256	0.1186	0.0917	−0.0080	0.0318
3	0.0726	−0.0226	−0.0354	0.0833	0.0586	−0.0077	−0.0036
4	0.0186	0.0261	−0.0347	0.0123	0.0301	−0.0118	−0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0320	−0.0122	0.0185

of Chapter 17, the Van Ijzeren decomposition of the chain Fisher *quantity* index is used by the Bureau of Economic Analysis in the United States.[55]

E. National Producer Price Indices

E.1 The national gross domestic output price index at producer prices

20.146 In this subsection and the following three subsections, national domestic gross output, export, domestic intermediate input, and import price indices at producer prices (i.e., at basic prices for outputs and purchaser's prices for intermediate inputs) will be calculated using the data for each of the three industrial sectors listed in Section B. Only fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist indices will be computed because these are the ones most likely to be used in practice.

20.147 It should be noted that the price indices computed in this section are appropriate ones to use for the calculation of business sector labor or multifactor productivity purposes.

20.148 The data listed in Tables 20.20 through 20.28 for industries *G*, *S*, and *T* are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for domestic outputs (at producer prices or basic prices in this case) for periods t equal 1 to 5, P_L^t, P_P^t, P_F^t, and P_T^t, respectively. Producer prices (as opposed to final demand prices) are used in these computations. There are three domestic output deliveries from industry *G*, eight domestic output deliveries from industry *S*, and three domestic output deliveries from industry *T* so that each index is an aggregate of 14 separate series. The fixed-base results are listed in Table 20.42.

[55]Details of its use can be found in Ehemann, Katz, and Moulton (2002).

20.149 By period 5, the spread between the fixed-base Laspeyres and Paasche national domestic output price indices is 1.7017/1.5424 = 1.103 or 10.3 percent and the spread between the Fisher and Törnqvist indices is 1.6581/1.6201 = 1.023 or 2.3 percent. In Table 20.43, the four indices are recomputed using the chain principle. It is expected that the use of the chain principle will narrow the spreads between the various indices.

20.150 An examination of the entries in Table 20.43 shows that chaining did indeed reduce the spread between the various index numbers. In period 5, the spread between the chained Laspeyres and Paasche national domestic output price indices is 1.6644/1.6328 = 1.019 or 1.9 percent and the spread between the chained Fisher and Törnqvist indices is 1.6500/1.6485 = 1.0009 or 0.09 percent, which is negligible considering the variation in the underlying data.

Table 20.42. Fixed-Base National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4762	1.4459	1.4610	1.4650
4	1.4826	1.4203	1.4511	1.4683
5	1.7017	1.5424	1.6201	1.6581

Table 20.43. Chained National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4832	1.4728	1.4780	1.4783
4	1.4919	1.4759	1.4839	1.4839
5	1.6644	1.6328	1.6485	1.6500

Table 20.44. National Fixed-Base Export Price Indices at Producer Prices

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191
3	1.5826	1.5799	1.5812	1.5813
4	1.4766	1.4762	1.4764	1.4763
5	1.3672	1.3694	1.3683	1.3682

Table 20.45. National Chained Export Price Indices at Producer Prices

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191
3	1.5786	1.5788	1.5787	1.5786
4	1.4717	1.4729	1.4723	1.4723
5	1.3690	1.3624	1.3657	1.3654

E.2 The national export price index at producer prices

20.151 The data listed in Tables 20.20 through 20.28 for industries G, S, and T are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for all exported outputs (at producer prices or basic prices in this case), P^t_L, P^t_P, P^t_F, and P^t_T, respectively. There is one exported good from each of the three industries so that each export price index is an aggregate of three separate series. The fixed-base results are listed in Table 20.44.

20.152 There is very little difference in any of the fixed-base series listed in Table 20.44. The corresponding chained indices in Table 20.45 are also very close to each other.

E.3 The national domestic intermediate input price index at producer prices

20.153 The data listed in Tables 20.20 through 20.28 for industries G, S, and T are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for all domestic intermediate inputs (at producer prices or purchase prices in this case), P^t_L, P^t_P, P^t_F, and P^t_T, respectively. There are three domestic intermediate inputs used in each of industries G, S, and T so that each domestic intermediate input price index is an aggregate of eight separate series. The fixed-base results are listed in Table 20.46.

Table 20.46. Fixed-Base National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.3443	1.3053	1.3247	1.3265
3	1.4928	1.3441	1.4165	1.4324
4	1.4686	1.1836	1.3184	1.3619
5	1.5887	1.1306	1.3402	1.4268

Table 20.47. Chained National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.3443	1.3053	1.3247	1.3265
3	1.4765	1.4045	1.4400	1.4435
4	1.4217	1.3272	1.3736	1.3782
5	1.4573	1.3398	1.3973	1.4015

20.154 The spread between the Laspeyres and Paasche fixed-base indices is very large by period 5, equaling 1.5887/1.1306 = 1.405 or 40.5 percent. The spread between the Fisher and Törnqvist fixed-base indices is not negligible either, equaling 1.4268/1.3402 = 1.065 or 6.5 percent in period 5. These relatively large spreads are due to the fact that the price of high-tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.47.

20.155 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4573/1.3398 = 1.088 or 8.8 percent and between the Fisher and Törnqvist to 1.4015/1.3973 = 1.003 or 0.3 percent, which is an acceptable degree of divergence considering the volatility of the underlying data.

E.4 The national import price index at producer prices

20.156 The data listed in Tables 20.20 through 20.28 for industries G, S, and T are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices for all imported intermediate inputs (at producer prices or purchase prices in this case), P^t_L, P^t_P, P^t_F, and P^t_T, respectively. There are four imported intermediate inputs used in industry G, four imported intermediate inputs used in

Table 20.48. Fixed-Base National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2426	1.2037	1.2230	1.2241
4	1.0844	1.0370	1.0604	1.0669
5	1.5776	1.3596	1.4645	1.4736

industry S, and two imported intermediate inputs used in industry T so that each import input price index is an aggregate of 10 separate series. The fixed-base results are listed in Table 20.48.

20.157 The spread between the Laspeyres and Paasche fixed-base import price indices is fairly large by period 5, equaling 1.5776/1.3596 = 1.160 or 16.0 percent. The spread between the Fisher and Törnqvist fixed-base indices is much smaller, equaling 1.4736/1.4645 = 1.006 or 0.6 percent in period 5. Note that each import price index has relatively large period-to-period fluctuations owing to the large fluctuations in the price of imported energy. As usual, we expect the fixed-base spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.49.

20.158 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.5128/1.4236 = 1.063 or 6.3 percent and between the Fisher and Törnqvist to 1.4680/1.4675 = 1.0003 or 0.03 percent, a negligible amount.

20.159 The domestic output price index and the domestic export index can be regarded as subindices of an overall gross output price index of the type that was described in the *PPI Manual* (ILO and others, 2004b). Similarly, the domestic intermediate input price index and the import price index can be regarded as subindices of the overall intermediate input price index that was described in the *PPI Manual*. All of these subindices can be thought of as aggregations of the same commodity (or group of commodities) across industries. At a second stage of aggregation, it is possible to aggregate over the domestic output price index and the export price index and to also aggregate over the domestic intermediate input price index and the import price index (with quantities indexed with negative signs) in order to form an economy-wide *value-added price index*. In the following section, the first stage of aggregation is across commodities within an industry; that is, in the following section, industry value-added price indices are constructed. A national value-added price index is also constructed in Section F. In Section F, the industry value-added deflators constructed in Section F are aggregated in order to form a two-stage economy-wide value-added price index. This two-stage aggregate value-added deflator will be compared with the two-stage aggregation method that aggregates over the domestic output price index, the export price index, the domestic intermediate input price index, and the import price index. These two methods of two-stage aggregation are compared in Section F along with the corresponding single-stage national value-added deflator.

Table 20.49. Chained National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2438	1.2384	1.2411	1.2415
4	1.0810	1.0723	1.0766	1.0773
5	1.5128	1.4236	1.4675	1.4680

F. Value-Added Price Deflators

F.1 Value-added price deflators for the goods producing industry

20.160 The data listed in Tables 20.20 through 20.22 for industry G are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added price indices or deflators at producer prices. This means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs. The quantities of domestic intermediate inputs and imports are indexed with negative signs. Fixed-base and chained value-added Laspeyres, Paasche, Fisher, and Törnqvist price indices, P_L^t, P_P^t, P_F^t, and P_T^t, respectively, will be constructed. There are three domestic outputs and one export produced by industry G, and three domestic intermediate inputs and four imported commodities used as inputs by industry G, so that each value-added price index is an aggregate of 11 separate series. The fixed-base results are listed in Table 20.50.

20.161 The spread between the Laspeyres and Paasche fixed-base value-added price indices is enormous by period 5, equaling 5.7905/1.7605 = 3.289 or 328.9 percent. The spread between the Fisher and Törnqvist fixed-base indices is large as well, equaling 3.1928/2.1276 = 1.501 or 50.1 percent in period 5.

Table 20.50. Fixed-Base Value-Added Price Deflators for Industry *G*

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.2260	3.5528	2.8122	2.5489
4	2.4403	8.0774	4.4398	3.0649
5	1.7605	5.7905	3.1928	2.1276

Table 20.51. Chained Value-Added Price Deflators for Industry *G*

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.4490	3.2741	2.8317	2.7527
4	2.8776	4.0277	3.4044	3.3096
5	1.8066	2.9594	2.3122	2.2720

These very large spreads are due to the fact that the price of high-tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As well, because quantities have positive and negative weights in value-added price indices, the divergences between various index number formulas can become very large. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.51.

20.162 Chaining reduces the period 5 spread between the Laspeyres and Paasche to 2.9594/1.8066 = 1.638 or 63.8 percent and between the Fisher and Törnqvist to 2.3122/2.2720 = 1.018 or 1.8 percent, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value-added price indices for this industry will give rise to estimates of price change that are very far from the corresponding superlative index estimates. *Thus the corresponding Laspeyres or Paasche estimates of real value added may be rather inaccurate, giving rise to inaccurate estimates of industry productivity growth.*

F.2 Value-added price deflators for the services industry

20.163 The data listed in Tables 20.23 through 20.25 for industry *S* are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added price indices at producer prices. There are eight domestic outputs and one export produced by industry *S*, and two domestic intermediate inputs and four imported commodities used as inputs by industry *S*, so that each value-added price index is an aggregate of 15 separate series. The fixed-base results are listed in Table 20.52. Producer prices are used in these computations.

20.164 The spread between the Laspeyres and Paasche fixed-base value-added price indices for industry *S* is 1.4913/1.2797 = 1.165 or 16.5 percent, which is a substantial gap. The spread between the Fisher and Törnqvist fixed-base indices is fairly small, equaling 1.3942/1.3814 = 1.009 or 0.9 percent in period 5. Note that the gap between the fixed-base Paasche and Laspeyres value-added price indices for the services industry is very much less than the corresponding gap for the fixed-base Paasche and Laspeyres value-added price indices for the goods producing industry. An explanation for this narrowing of the Paasche and Laspeyres gap is that while the services industry was subject to some very large fluctuations in the prices it faced, because most of the big fluctuations occurred for the food and energy imports, which are margin goods for the industry, these fluctuations were passed on to final demanders, leaving industry distribution margins largely intact. Thus the fluctuations in the value-added price indices for industry *S* turned out to be less severe than for industry *G*. As usual, the spreads between the Paasche and Laspeyres price indices should narrow when the chain principle is used; see Table 20.53.

Table 20.52. Fixed-Base Value-Added Price Deflators for Industry *S*

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4876	1.4160	1.4514	1.4537
4	1.5035	1.3531	1.4264	1.4380
5	1.4913	1.2797	1.3814	1.3942

Table 20.53. Chained Value-Added Price Deflators for Industry *S*

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4700	1.4411	1.4555	1.4579
4	1.4620	1.4201	1.4409	1.4432
5	1.4363	1.3863	1.4111	1.4145

20.165 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4363/1.3863 = 1.036 or 3.6 percent in period 5 and between the Fisher and Törnqvist to 1.4145/1.4111 = 1.002, or 0.2 percent, which is negligible.

F.3 Value-added price deflators for the transportation industry

20.166 The data listed in Tables 20.26 through 20.28 for industry *T* are used to calculate fixed-base Laspeyres, Paasche, Fisher, and Törnqvist value-added price indices at producer prices. There are three domestic outputs and one export produced by industry *T*, and three domestic intermediate inputs and two imported commodities used as inputs by industry *T*, so that each value-added price index is an aggregate of nine separate series. The fixed-base results are listed in Table 20.54.

20.167 The spread between the Laspeyres and Paasche fixed-base value-added price indices is enormous by period 5, equaling 4.8128/1.8028 = 2.670 or 267.0 percent. The spread between the Fisher and Törnqvist fixed-base indices is fairly large as well, equaling 2.9456/2.2114 = 1.332 or 33.2 percent in period 5. These very large spreads are due to the fact that the price of high-tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.55.

Table 20.54. Fixed-Base Value-Added Price Deflators for Industry *T*

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.1204	1.1913	1.1553	1.1173
4	1.0977	1.3541	1.2192	1.0679
5	1.8028	4.8128	2.9456	2.2114

Table 20.55. Chained Value-Added Price Deflators for Industry *T*

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.0374	1.1271	1.0813	1.0509
4	0.9428	1.0563	0.9979	0.9667
5	1.9916	2.4248	2.1975	2.2389

20.168 Chaining reduces the period 5 spread between Laspeyres and Paasche to 2.4248/1.9916 = 1.218 or 21.8 percent and between the Fisher and Törnqvist to 2.2389/2.1975 = 1.019 or 1.9 percent, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value-added price indices for this industry will give rise to estimates of price change that are fairly far from the corresponding chained superlative index estimates, a situation that is similar to what occurred for the industry *G* data. Thus whenever possible, it seems preferable to use chained superlative indices when constructing annual industry value-added deflators as opposed to using fixed-base or chained Paasche or Laspeyres indices. In the following section, all of the industry data are aggregated to form a national value-added deflator.

F.4 The national value-added price deflator

20.169 The data listed in Tables 20.20 through 20.28 for industries *G*, *S*, and *T* are used to calculate national Laspeyres, Paasche, Fisher, and Törnqvist value-added price indices at producer prices; that is, in this subsection, the national value-added deflator is constructed. Fixed-base and chained value-added Laspeyres, Paasche, Fisher, and Törnqvist price indices will be constructed, P^t_L, P^t_P, P^t_F, and P^t_T, respectively. There are 14 domestic outputs, 3 exported commodities, 8 domestic intermediate inputs, and 10 imported commodities so that each national value-added deflator is an aggregate of 35 separate series. The fixed-base results are listed in Table 20.56.

20.170 The spread between the national Laspeyres and Paasche fixed-base value-added price indices is fairly large by period 5, equaling 1.7555/1.6176 = 1.085 or 8.5 percent. The spread between the Fisher and Törnqvist fixed-base indices is small, equaling 1.6970/1.6851 = 1.007 or 0.7 percent in period 5. The corresponding chained indices are listed in Table 20.57.

Table 20.56. Fixed-Base National Value-Added Deflators

Period t	P^t_L	P^t_P	P^t_F	P^t_T
1	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261
3	1.7776	1.8533	1.8151	1.8173
4	1.8743	1.9822	1.9275	1.9455
5	1.6176	1.7555	1.6851	1.6970

Table 20.57. Chained National Value-Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261
3	1.7711	1.8336	1.8021	1.8098
4	1.8855	1.9530	1.9190	1.9315
5	1.6380	1.7612	1.6985	1.7156

20.171 The spread in period 5 between the national Laspeyres and Paasche chained value-added price indices equals 1.7612/1.6380 = 1.075 or 7.5 percent, which is slightly smaller than the corresponding 8.5 percent spread for the fixed-base Laspeyres and Paasche indices. The spread between the Fisher and Törnqvist chained indices in period 5 is 1.7156/1.6985 = 1.010 or 1.0 percent, which is slightly larger than the corresponding fixed-base spread of 0.7 percent. At the national level, the fixed-base and chained Fisher and Törnqvist indices all give much the same answer.

G. Two-Stage Value-Added Price Deflators

G.1 Two-stage national value-added price deflators: Aggregation over industries

20.172 In Section E.6 of Chapter 18, methods for constructing a price index by aggregating in two stages are discussed. It is pointed out that if a Laspeyres index is constructed in two stages of aggregation and the Laspeyres formula is used in each stage of aggregation, then the two-stage index will necessarily coincide with the corresponding single-stage index. A similar consistency in aggregation property holds if the Paasche formula is used at each stage of aggregation. Unfortunately, this consistency in aggregation property does not hold for superlative indices but it is pointed out in Chapter 18 that superlative indices should be approximately consistent in aggregation. In this section, the artificial data set is used in order to evaluate this approximate consistency in aggregation property of the Fisher and Törnqvist indices.

20.173 In the present context, there are two natural ways of aggregating in two stages. In method 1, the first stage of aggregation is the construction of a value-added deflator for each industry (along with the corresponding quantity indices) and in the second stage, the three industry value-added deflators are aggregated into a national value-added deflator. In method 2, the first stage of aggregation is the construction of national domestic output, domestic intermediate input, and export and import price indices (along with the corresponding quantity indices) and in the second stage, these four price indices are aggregated into a national value-added deflator.[56] The results for method 1 are listed in this subsection while the results for method 2 are listed in Section F.2.

20.174 In Table 20.58, the fixed-base single-stage Laspeyres, Paasche, Fisher, and Törnqvist indices are listed in the first four columns of the table[57] and the corresponding method 1 fixed-base two-stage indices are listed in the last four columns of the table.

20.175 As is expected from the theory in Chapter 18, the single-stage Laspeyres and Paasche indices coincide exactly with their two-stage counterparts. What is not expected is how far the two-stage Fisher index, P_{F2S}^t, is from its single-stage counterpart, P_F^t, for periods 3 through 5. Obviously, the period-to-period changes in the Fisher industry value-added indices are so large that the two-stage approximation results discussed in Chapter 18 break down for this artificial data set. The spread between the fixed-base single-stage Fisher and Törnqvist indices in period 5 is 1.6970/1.6851 = 1.007 or 0.7 percent but the spread between the two-stage Fisher and Törnqvist indices in period 5 is 1.9488/1.6579 = 1.175 or 17.5 percent, a rather large deviation.

20.176 In Table 20.59, the chained single-stage Laspeyres, Paasche, Fisher, and Törnqvist indices are listed in the first four columns of the table[58] and the corresponding method 1 chained two-stage indices are listed in the last four columns of the table.

20.177 It can be seen that chaining has reduced the spread between the two-stage superlative indices. The spread between the chained single-stage Fisher and Törnqvist indices in period 5 is 1.7156/1.6985 = 1.007 or 1.0 percent and the spread between the chained two-stage Fisher and Törnqvist indices in period 5 is 1.7270/1.7137 = 1.008 or 0.8 percent, a rather modest deviation. As is expected from the theory in Chapter 18, the single-stage chained Laspeyres and Paasche indices coincide exactly with their two-stage counterparts.

[56]The domestic output and export quantities are positive numbers in this second stage of aggregation but the domestic intermediate input and import quantities are negative numbers in the second stage of aggregation.

[57]These indices are the same as those listed in Table 20.56.

[58]These indices are the same as those listed in Table 20.57.

Table 20.58. Fixed-Base Single-Stage and Two-Stage National Value-Added Deflators: Aggregation over Industries Method

Period t	P^t_L	P^t_P	P^t_F	P^t_T	P^t_{L2S}	P^t_{P2S}	P^t_{F2S}	P^t_{T2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8915	1.8110
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	2.2616	1.9254
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.9488	1.6579

Table 20.59. Chained Single-Stage and Two-Stage National Value-Added Deflators: Aggregation over Industries Method

Period t	P^t_L	P^t_P	P^t_F	P^t_T	P^t_{L2S}	P^t_{P2S}	P^t_{F2S}	P^t_{T2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8365	1.8124
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9587	1.9326
5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7270	1.7137

Table 20.60. Fixed-Base Single-Stage and Two-Stage National Value-Added Deflators: Aggregation over Commodities Method

Period t	P^t_L	P^t_P	P^t_F	P^t_T	P^t_{L2S}	P^t_{P2S}	P^t_{F2S}	P^t_{T2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8067	1.8094
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	1.9066	1.9269
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.6641	1.6822

G.2 Two-stage national value-added price deflators: Aggregation over commodities

20.178 In this subsection, the national value-added price index is formed by an alternative two-stage aggregation procedure. In the first-stage aggregation, national domestic output, export, domestic intermediate input, and import price indices are calculated along with the corresponding quantity indices as was done in Section F. In the second stage of aggregation, the sign of the quantity indices that correspond to the domestic intermediate input and import indices is changed from positive to negative and the four price and quantity series are aggregated together to form an estimate for the national value-added deflator. The resulting two-stage fixed-base Laspeyres, Paasche, Fisher, and Törnqvist price indices, P^t_{L2S}, P^t_{P2S}, P^t_{F2S}, and P^t_{T2S}, are listed in the last four columns of Table 20.60 along with their fixed base single-stage counterparts, P^t_L, P^t_P, P^t_F, and P^t_T.

20.179 Note that the single-stage fixed-base indices, P^t_L, P^t_P, P^t_F, and P^t_T, listed in Table 20.60 coincide with the single-stage fixed-base indices P^t_L, P^t_P, P^t_F, and P^t_T listed in Table 20.58. Note also that the single-stage Paasche and Laspeyres indices coincide with their two-stage counterparts in Table 20.60 as is expected from index number theory. Finally, note that the two-stage superlative indices, P^t_{F2S} and P^t_{T2S}, are reasonably close to their single-stage counterparts, P^t_F and P^t_T. The spread between the four superlative indices is 1.6970/1.6641 = 1.054 or 5.4 percent. It seems that the method 2 (aggregation over commodities method) two-stage aggregation procedure works more smoothly than the method 1 (aggregation over industry value-added method) two-stage aggregation procedure, leading to a reasonably close

Table 20.61. Chained Single-Stage and Two-Stage National Value-Added Deflators: Aggregation over Commodities Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8037	1.8150
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9202	1.9318
5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7069	1.7186

Table 20.62. Fixed-Base and Chained Domestic Final Demand Deflators

	Fixed-Base Indices				Chained Indices			
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3930	1.3935	1.3967	1.3893	1.3930	1.3935
3	1.4832	1.4775	1.4803	1.4807	1.4931	1.4952	1.4941	1.4934
4	1.5043	1.4916	1.4980	1.5048	1.5219	1.5219	1.5219	1.5205
5	1.7348	1.6570	1.6954	1.7108	1.7176	1.7065	1.7120	1.7122

approximation between the single-stage and two-stage estimators for the national value-added deflator in the case of method 2.

20.180 In Table 20.61, the method 2 two-stage chained Laspeyres, Paasche, Fisher, and Törnqvist price indices, P_{L2S}^t, P_{P2S}^t, P_{F2S}^t, and P_{T2S}^t, are listed in the last four columns along with their fixed-base single-stage counterparts, P_L^t, P_P^t, P_F^t, and P_T^t.

20.181 As expected, chaining reduces the spread between the superlative indices. The spread between the four superlative indices is now 1.7186/1.6985 = 1.012 or 1.2 percent. Note also that the single-stage Paasche and Laspeyres chained indices coincide with their two-stage counterparts in Table 20.61. In the following section, the focus shifts from industry price indices to final demand price indices.

H. Final Demand Price Indices

H.1 Domestic final demand price indices

20.182 In this section, the standard fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist price indices are listed for deliveries of commodities to the domestic final demand sector; see Table 20.62. Each index is an aggregate of six separate final demand series.

20.183 The indices listed in Table 20.62 have already been listed in various tables in Section C above but for convenience, they are tabled again. Because these indices are discussed in Section C, the discussion is not repeated here.

H.2 Export price indices at final demand prices

20.184 In this subsection, the standard fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist price indices are calculated for the three export series that are listed in Section B above. Final demand prices are used when calculating the indices listed in Table 20.63.

20.185 Because the three export price and quantity series have fairly smooth trends that are roughly proportional to each other, all of the indices listed above in Table 20.63 are quite close to each other.

H.3 Import price indices at final demand prices

20.186 In this subsection, the standard fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist price indices are calculated for the 10 import series that are listed in Section B above. Final demand prices are used when calculating the indices listed in Table 20.64.

Table 20.63. Fixed-Base and Chained Export Price Indices at Final Demand Prices

	Fixed-Base Indices				Chained Indices			
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3191	1.321	1.3201	1.3202	1.3191	1.3210	1.3201	1.3202
3	1.5816	1.5789	1.5802	1.5803	1.5775	1.5777	1.5776	1.5776
4	1.4752	1.4750	1.4751	1.4750	1.4703	1.4716	1.4709	1.4709
5	1.4184	1.4152	1.4168	1.4167	1.4140	1.4076	1.4108	1.4105

Table 20.64. Fixed-Base and Chained Import Price Indices at Final Demand Prices

	Fixed-Base Indices				Chained Indices			
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.5495	1.5279	1.5387	1.5369	1.5495	1.5279	1.5387	1.5369
3	1.2270	1.1907	1.2087	1.2099	1.2293	1.2261	1.2277	1.2285
4	1.0739	1.0289	1.0512	1.0580	1.0709	1.0642	1.0676	1.0682
5	1.5946	1.3726	1.4794	1.4873	1.5257	1.4321	1.4782	1.4785

20.187 Because price and quantity trends for imports are far from being proportional, there are substantial differences between the Paasche and Laspeyres price indices. The spread between the fixed-base Paasche and Laspeyres is 1.5946/1.3726 = 1.162 or 16.2 percent while the spread between the chained Paasche and Laspeyres is 1.5257/1.4321 = 1.065 or 6.5 percent so that as usual, chaining reduces the spread. All of the superlative indices are close to each other.

H.4 GDP deflators

20.188 In this subsection, various GDP deflators are calculated; that is, the standard fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist price indices are calculated for the 19 final demand series that are listed in Section B above. Final demand prices are used when calculating the indices listed in Table 20.65.

20.189 The spread between the Paasche and Laspeyres fixed-base GDP deflators in period 5 is 1.7044/1.6591 = 1.027 or 2.7 percent while the spread between the Paasche and Laspeyres chained GDP deflators in period 5 is 1.7044/1.6591 = 1.048 or 4.8 percent. Thus in this case, chaining did not reduce the spread between the Paasche and Laspeyres indices. The superlative indices are all rather close to each other; in period 5, the spread between the four superlative indices was 1.7099/1.6816 = 1.017 or 1.7 percent and the spread between the two chained superlative indices was only 1.7099/1.6981 = 1.007 or 0.7 percent.

H.5 The reconciliation of the GDP deflator with the value-added deflator

20.190 The final set of tables for this chapter draws on the theory developed in Section B.3 of this Chapter.

Table 20.65. Fixed-Base and Chained GDP Deflators

	Fixed-Base Indices				Chained Indices			
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2417
3	1.7317	1.7696	1.7506	1.7546	1.7252	1.7632	1.7441	1.7499
4	1.8107	1.8476	1.8291	1.8488	1.8139	1.8507	1.8322	1.8420
5	1.6591	1.7044	1.6816	1.6995	1.6581	1.7391	1.6981	1.7099

In that section, it was shown how volume estimates for GDP at final demand prices, GDP_F, could be reconciled with volume estimates for GDP at producer prices, GDP_P, using equation (20.26). Equation (20.26) said that GDP_F equals GDP_P plus a sum of tax terms, T. It was also shown that two-stage price and quantity indices for GDP_F could be constructed by aggregating over the 35 separate price and quantity series that are used to construct price and quantity indices for GDP_P plus aggregating over all of the tax series that make up the T aggregate. It was shown that the resulting price and volume estimates for GDP_F and $GDP_P + T$ will coincide if the Laspeyres, Paasche, or Fisher formula is used. This methodology is tested out on the artificial data set for both fixed-base and chained Laspeyres, Paasche, Fisher, and Törnqvist price indices in Tables 20.66 (fixed base indices) and 20.67 (chained indices). The P^t_L, P^t_P, P^t_F, and P^t_T indices reported in Table 20.66 are the fixed-base single-stage GDP deflators (for GDP_F) that were listed in the first four columns of Table 20.65 while the P^t_{L2S}, P^t_{P2S}, P^t_{F2S}, and P^t_{T2S} indices reported in Table 20.66 are the two-stage fixed-base price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in Section B above and make up the tax aggregate *T*.

20.191 As predicted by the theory presented in Chapter 18, the Laspeyres, Paasche, and Fisher single-stage estimates for the GDP deflator (the first three columns in Table 20.66) coincide exactly with the corresponding two-stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single-stage Törnqvist GDP deflator, P^t_T, does not coincide with its two-stage counterpart, P^t_{T2S}, but the correspondence is fairly close.

20.192 The P^t_L, P^t_P, P^t_F, and P^t_T indices reported in Table 20.67 are the chained single-stage GDP deflators (for GDP_F) that were listed in the last four columns of Table 20.65 while the P^t_{L2S}, P^t_{P2S}, P^t_{F2S}, and P^t_{T2S} indices reported in Table 20.67 are the two-stage chained price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in Section B above and make up the tax aggregate *T*.

20.193 Again as predicted by the theory presented in Chapter 18, the Laspeyres, Paasche, and Fisher single-stage estimates for the GDP deflator (the first three columns in Table 20.67) coincide exactly with the corresponding two-stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single-stage Törnqvist GDP deflator, P^t_T, does not coincide with its two-stage counterpart, P^t_{T2S}, but again, the correspondence is fairly close.

Table 20.66. Fixed-Base GDP Deflators Calculated in Two Stages

Period *t*	P^t_L	P^t_P	P^t_F	P^t_T	P^t_{L2S}	P^t_{P2S}	P^t_{F2S}	P^t_{T2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7317	1.7696	1.7506	1.7546	1.7317	1.7696	1.7506	1.7538
4	1.8107	1.8476	1.8291	1.8488	1.8107	1.8476	1.8291	1.8470
5	1.6591	1.7044	1.6816	1.6995	1.6591	1.7044	1.6816	1.7020

Table 20.67. Chained GDP Deflators Calculated in Two Stages

Period *t*	P^t_L	P^t_P	P^t_F	P^t_T	P^t_{L2S}	P^t_{P2S}	P^t_{F2S}	P^t_{T2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7252	1.7632	1.7441	1.7499	1.7252	1.7632	1.7441	1.7488
4	1.8139	1.8507	1.8322	1.8420	1.8139	1.8507	1.8322	1.8405
5	1.6581	1.7391	1.6981	1.7099	1.6581	1.7391	1.6981	1.7120

20.194 The equality of the single-stage and two-stage Laspeyres, Paasche, and Fisher GDP_F deflators in Tables 20.66 and 20.67 provides a very good check on the correctness of all of the various index number calculations that are associated with PPI programs and the production of GDP volume estimates.

I. Conclusion

20.195 Some tentative conclusions that can be drawn from the various indices that have been computed using the artificial data set are as follows:

- It is risky to use fixed-base Paasche or Laspeyres indices in the sense that they can be rather far from the theoretically preferred superlative indices.
- Chained indices seem preferable to the use of fixed-base indices in the sense that chaining generally reduces the spread between the Paasche and Laspeyres indices.
- Chained Paasche and Laspeyres indices can be close to the theoretically preferred superlative indices, except in the value-added context; that is, chained Paasche and Laspeyres indices are often fairly close to each other (and the corresponding chained superlative indices) when constructing output, export, intermediate input, and import price indices. However, when constructing value-added indices, it seems preferable to use chained superlative indices.

21. Elementary Indices

A. Introduction

21.1 The subject of this chapter is the appropriate formula(s) to use when the aggregation of price changes does not benefit from information on weights. The absence of information on weights is invariably at the lower, elementary level of aggregation. The resulting indices from these elementary aggregates are referred to as *elementary aggregate indices* or, more simply, *elementary indices*. At the next stage of aggregation weights are applied to the elementary indices, and weights are again applied to the resulting indices at higher stages of aggregation, until an overall index is derived.

21.2 The main concern of this chapter is with the choice of the most appropriate unweighted index number formula for the elementary indices. It is stressed that the choice of the most appropriate elementary index formula is a second-best solution. The optimum strategy is to attempt to obtain information on the values of goods purchased as imports, or sold as exports, and apply weights at all stages of aggregation.

21.3 Data on prices may be unit values from the records on foreign trade transactions maintained by national customs authorities, or from the records of transactions by a surveyed establishment. A unit value for a specified period of time is obtained, for a commodity classification or specified commodity, by dividing the value traded by the corresponding quantity. A unit value elementary *index* is derived by dividing, for the same commodity classification or commodity, the unit value in the current period by the unit value in the reference period. The calculation of a unit value at the elementary aggregate level thus makes implicit use of information on quantities; there is some form of weighting at this elementary level. In particular, it is shown below in Section B that as the prices and quantities of the same, very narrowly defined commodity vary within a reporting period, say a month, a unit value index, as a surrogate measure of price changes, weights the price changes according to their corresponding quantities: It solves the *time aggregation problem*.

21.4 There is a history to the use of unit value *indices* derived from customs data as the principal method for compiling trade price indices. The unit values for such indices are surrogates for prices in each of the two periods. Each unit value index, derived for a detailed commodity classification, has a weight attached to it for aggregation to a higher level of classification. However, the commodity composition in customs data, for which the unit values are compared over time, is generally not homogeneous; the product mix and its quality can vary over time. As such, unit value indices from customs data may not just reflect price changes. They are prone to bias and should be used only in circumstances where the product mix and quality of items compared over time can be reliably taken to be unchanging. Unit value indices and their properties were considered in detail in Chapter 2. Unit values from customs data are used as proxies for prices and unit value indices are used as proxies for price changes. The concern of this chapter is the formula to use when calculating an index of establishment survey prices for which there is no information on quantities or values.

21.5 Price data collected from price surveys of establishments should relate to specified commodities whose quality characteristics are well specified so that changes in product mix and quality are not reflected in the price index. The "prices" recorded may be unit values for a batch of sales or purchases, but they will be defined for tightly specified commodities/transactions selected from detailed commodity categories, from establishments. Each establishment also should have available information on the traded values associated with these prices for the selected commodities within each commodity category. The use of explicit weights at this elementary level of aggregation can but benefit the index. It is common in consumer price index (CPI) compilation that the aggregation of prices across different outlets of the prices of relatively homogeneous items is undertaken using unweighted aggregation formulas. For example, for a geometric mean of price changes, each outlet's price change has an equal weight ascribed to it, irrespective of the importance of the relative sales of the outlet. For export and import price indices (XMPIs) and producer price indices (PPIs), the direct contact with the responding establishment may allow highly detailed data on prices and quantities/values to be made available, possibly electronically, in a manner that is not feasible for price collectors visiting outlets for CPI data collection. Where possible, the first stage of aggregation for XMPIs should include weighting information. The issue of which index number formula to use for the aggregation of weighted price changes was the subject of Chapters 16 to 18.

21.6 It should be noted that even if information on prices and values is collected, the estimation of the weights to use at the lower level should take account of the sample design used in the selection of commodities/establishments. Consider, for example, the selection of establishments of which the single largest establishment, say, responsible for an export value 8,000 of exports, for a category, was selected using cutoff sampling. Consider further the selection of, say, 10 establishments at random from the remaining 20 establishments on the sampling frame, each of which is found, for simplicity, to be responsible for the same export value of 100. The weight for the single large establishment would be

$$8{,}000/[8{,}000/1.0 + (10 \times 100)/0.5] = 0.8$$

and, for *each* of the 10 small ones selected:

$$100/0.5[8{,}000/1.0 + (10 \times 100)/0.5] = 0.02.$$

The weights for the establishments are adjusted to take into account the probability of selecting the establishment, as determined by the sample design.

21.7 More generally, information on weights may not be available to be directly incorporated into the aggregation formula, but such information may be implicit in the sample design. Unweighted commodity price changes from establishments selected at random with, say, probability proportional to expenditure shares in the base period can be considered to be sample estimators of a base-period expenditure weighted population index number formula. Such considerations are examined below in Section G.

21.8 The principal concern of this chapter is with the choice of formula when no data on weights are available, neither explicitly nor implicitly, by way of the sample design, nor by construction as unit values for a homogeneous commodity. Alternative formulas for such unweighted aggregation are considered by recourse to the axiomatic, economic, and sampling approaches to elementary indices in Sections E, F, and G below.

21.9 If the compilation of XMPIs at the lower level does not benefit from the availability of information on weights, then there are two distinct stages to the index number compilation. In the first stage of calculation, *elementary price indices* are estimated for the *elementary aggregates* of the trade price index. In the second and higher stages of aggregation, these elementary price indices are combined to obtain higher-level indices using information on the trade values on each elementary aggregate as weights. Elementary aggregate indices by definition do not use a weighted index number formula. The scope of the elementary aggregates would be relatively homogeneous sets of commodities defined within the industrial classification used in the XMPIs. Samples of prices would be collected within each elementary aggregate, so that elementary aggregates serve as strata for sampling purposes.

21.10 Data on the traded values, or quantities, of different goods and services are thus not available within an elementary aggregate. Because there are no quantity or value weights, most of the index number theory outlined from Chapter 16 to 18 is not directly applicable. As was noted in Chapter 1, an elementary price index is a more primitive concept that often relies on price data only.

21.11 The question of which is the most appropriate formula to use to estimate an elementary price index is considered in this chapter. For commodity groups in which weights are unavailable at this elementary level, the quality of XMPIs depends heavily on the quality of the elementary indices, which are the basic building blocks from which the XMPIs are constructed.

21.12 As was explained in Chapter 6, compilers have to select *representative commodities* within an elementary aggregate and then collect a sample of prices for each of the representative commodities, usually from a sample of different establishments. The individual commodities whose prices actually are collected are described as the *sampled commodities*. Their prices are collected over a succession of time periods. An elementary price index is therefore typically calculated from two sets of matched price observations. It is assumed in this chapter that there are no missing observations and no changes in the quality of the commodities sampled, so that the two sets of prices are perfectly matched. The treatment of new and disappearing commodities, and of quality change, is a separate and complex issue that was discussed in detail in Chapters 8 and 9, and is continued in Chapter 22 of this *Manual*.

21.13 Even though quantity or traded value weights are usually not available to weight the individual elementary price quotes, it is useful to consider an *ideal framework* where such information is available. This is done in Section B. The problems involved in aggregating narrowly defined price quotes over *time* also are discussed in this section. The discussion in Section B provides a theoretical target for practical elementary price indices for homogeneous items shown to be a unit value index. This ideal framework and its findings remain important as a benchmark against which elementary index number formulas can be considered. Indeed one feature of the idealized measure is its requirement of commodity homogeneity.

21.14 Section C introduces the main elementary index formulas used in practice and Section D develops some numerical relationships between the various indices. Chapters 15 to 17 developed the various approaches to index number theory when information on both prices and quantities is available. It also is possible to develop axiomatic, economic, or sampling approaches to elementary indices and these three approaches are discussed below in Sections E, F, and G. Section H develops a simple statistical approach to elementary indices that resembles a highly simplified hedonic regression model. Section I concludes with an overview of the various results.[1]

B. Ideal Elementary Indices

21.15 The aggregates covered by XMPIs, a CPI, or a PPI usually are arranged in the form of a treelike hierarchy, such as the Harmonized Commodity Description and Coding System, the Classification of Individual Consumption by Purpose, or the General Industrial Classification of Economic Activities within the European Communities. An *aggregate* is a set of economic transactions pertaining to a set of commodities over a specified time period. Every economic transaction relates to the change of ownership of a specific, well-defined commodity (good or service) at a particular place and date, and comes with a quantity and a price. The price index for an aggregate is calculated as a weighted average of the price indices for the subaggregates, the weights and type of average being determined by the index formula. One can descend in such a hierarchy as far as available information allows the weights to be decomposed. The lowest level aggregates are called *elementary* aggregates. They are basically of two types:

(1) Those for which all detailed price and quantity information is available, and

(2) Those for which the statistician, considering the operational cost and the response burden of getting detailed price and quantity information about all the transactions, decides to make use of a representative sample of commodities or respondents.

The practical relevance of studying this topic is great. Because the elementary aggregates form the building blocks of XMPIs, the choice of an inappropriate formula at this level can have a tremendous impact on the overall index.

21.16 In this section, it is assumed that detailed price and quantity information are available for all transactions pertaining to the elementary aggregate for the two time periods under consideration. This assumption allows us to define an *ideal elementary aggregate*. Subsequent sections relax this strong assumption about the availability of detailed price and quantity data on transactions, but it is necessary to have a theoretically ideal target for the practical elementary index.

21.17 The detailed price and quantity data, although perhaps not available to the statistician, are, in principle, available in the outside world. It is frequently the case that at the respondent level (i.e., at the firm level), some aggregation of the individual transactions information has been executed, usually in a form that suits the respondent's financial or management information system. This respondent-determined level of information could be called the *basic information level*. This is, however, not necessarily the finest level of information that could be made available to the price statistician. One could always ask the respondent to provide more disaggregated information. For instance, instead of

[1] This chapter draws heavily on the recent contributions of Dalén (1992), Balk (1994, 1998b, and 2005), Diewert (1995a, 2002a, and 2002b), and Silver and Heravi (2007).

monthly data, one could ask for weekly data; or, when appropriate, one could ask for regional instead of global data; or one could ask for data according to a finer commodity classification. The only natural barrier to further disaggregation is the individual transaction level.[2]

21.18 It is now necessary to discuss a problem that arises when detailed data on *individual transactions* are available. This may occur at the individual establishment level, or even for individual production runs. Recall that in Chapter 16, the price and quantity indices, $P(p^0, p^1, q^0, q^1)$ and $Q(p^0, p^1, q^0, q^1)$, were introduced. These (bilateral) price and quantity indices decomposed the value ratio V^1/V^0 into a price change part $P(p^0, p^1, q^0, q^1)$ and a quantity change part $Q(p^0, p^1, q^0, q^1)$. In this framework, it was taken for granted that the period t price and quantity for commodity i, p_i^t, and q_i^t, were well defined. However, these definitions are not straightforward, because individual importers may buy the *same* commodity during period *t at different prices*. Similarly, consider the exports of a particular establishment; *the same commodity may sell at very different prices during the course of the period*. Hence before a traditional bilateral price index of the form $P(p^0, p^1, q^0, q^1)$ considered in previous chapters of this *Manual* can be applied, there is a nontrivial *time aggregation problem* to obtain the basic prices p_i^t and q_i^t that are the components of the price vectors p^0 and p^1 and the quantity vectors q^0 and q^1. Walsh[3] (1901 and 1921a) and Davies (1924 and 1932) suggested a solution in a CPI context to this time aggregation problem: The appropriate quantity at this very first stage of aggregation is the *total quantity purchased* of the narrowly defined commodity, and the corresponding price is the value of purchases of this commodity divided by the total amount purchased, which is a *narrowly defined unit value*. The appropriate unit value for an import price index (MPI) or export price index (XPI) context is the value of imports or exports divided by the total amount sold. In more recent times, other researchers have adopted the Walsh and Davies solution to the time aggregation problem.[4] Note that this solution to the time aggregation problem has the following advantages:

(1) The quantity aggregate is intuitively plausible, being the total quantity of the narrowly defined commodities traded during the time period under consideration, and

(2) The price times quantity of the commodity equals the total value traded during the time period under consideration.

This solution will be adopted for the time aggregation problem as a valid concept for the price and quantity at this first stage of aggregation.

21.19 The *2008 System of National Accounts (2008 SNA)* (Paragraph 15.69) advocated the use of unit value indices for homogeneous goods and services:

> When there is price variation for the same quality of good or service, the price relatives used for index number calculation should be defined as the ratio of the weighted average price of that good or service in the two periods, the weights being the relative quantities sold at each price. Suppose, for example, that a certain quantity of a particular good or service is sold at a lower price to a particular category of purchaser without any difference whatsoever in the nature of the good or service offered, location, timing or conditions of sale, or other factors. A subsequent decrease in the proportion sold at the lower price raises the average price paid by purchasers for quantities of a good or service whose quality is the same and remains unchanged, by assumption. It also raises the average price received by the seller without any change in quality. This must be recorded as a price and not a volume increase.

21.20 Having decided on an appropriate theoretical definition of price and quantity for a commodity at the very lowest level of aggregation (i.e., a narrowly defined unit value and the total quantity traded), we must now consider how to aggregate these narrowly defined elementary prices and quantities into an overall elementary aggregate. Suppose that there are M lowest level items, or specific commodities, in this chosen elementary category. Denote the period t quantity of commodity m by q_m^t and the corresponding time aggregated unit value by p_m^t for $t = 0, 1$ and for commodities $m = 1, 2, \ldots, M$. Define the period t quantity and

[2]See Balk (1994) for a similar approach.

[3]Walsh explained his reasoning as follows: "Of all the prices reported of the same kind of article, the average to be drawn is the arithmetic; and the prices should be weighted according to the relative mass quantities that were sold at them" (1901, p. 96). "Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principal market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities" (1921a, p. 88).

[4]See, for example, Szulc (1987, p. 13), Dalén (1992, p. 135), Reinsdorf (1994), Diewert (1995a, pp. 20–21), Reinsdorf and Moulton (1997), and Balk (2005).

price vectors as $q^t \equiv [q_1^t, q_2^t, \ldots, q_M^t]$ and $p^t \equiv [p_1^t, p_2^t, \ldots, p_M^t]$ for $t = 0, 1$. It is now necessary to choose a theoretically ideal index number formula $P(p^0, p^1, q^0, q^1)$ that will aggregate the individual commodity prices into an overall aggregate price relative for the M commodities in the chosen elementary aggregate. However, this problem of choosing a functional form for $P(p^0, p^1, q^0, q^1)$ is identical to the overall index number problem that was addressed in Chapters 16 to 18. In these chapters, four different approaches to index number theory were studied that led to specific index number formulas as being best from each perspective. From the viewpoint of *fixed-basket approaches,* it was found that the Fisher (1922) and Walsh (1901) price indices, P_F and P_W, appeared to be best. From the viewpoint of the *test approach,* the Fisher index appeared to be best. From the viewpoint of the *stochastic approach* to index number theory, the Törnqvist Theil (Törnqvist, 1936; and Theil, 1967) index number formula P_T emerged as being best. Finally, from the viewpoint of the *economic approach* to index number theory, the Walsh price index P_W, the Fisher ideal index P_F, and the Törnqvist Theil index number formula P_T were all regarded as being equally desirable. It also was shown that the same three index number formulas numerically approximate each other very closely, so it will not matter very much which of these alternative indices is chosen.[5] Hence, the *theoretically ideal elementary index number formula* is taken to be one of the three formulas, $P_F(p^0, p^1, q^0, q^1)$, $P_W(p^0, p^1, q^0, q^1)$, or $P_T(p^0, p^1, q^0, q^1)$, where the period t quantity of commodity m, $q_m{}^t$, is the total quantity of that narrowly defined commodity produced by the establishment during period t, and the corresponding price for commodity m is $p_m{}^t$, the time aggregated unit value for $t = 0, 1$, and for commodities $m = 1, \ldots, M$.

21.21 Various practical elementary price indices are defined in the following sections. These indices do not have quantity weights and thus are functions only of the price vectors p^0 and p^1, which contain time aggregated unit values for the M commodities in the elementary aggregate for periods 0 and 1. Thus, when a practical elementary index number formula, say $P_E(p^0, p^1)$, is compared with an ideal elementary price index, say the Fisher price index, $P_F(p^0, p^1, q^0, q^1)$, then obviously P_E will differ from P_F because the prices are not weighted according to their economic importance in the practical elementary formula. Call this difference between the two index number formulas *formula approximation error*.

21.22 Practical elementary indices are subject to two other types of error:

(1) The statistical agency may not be able to collect information on all M prices in the elementary aggregate; that is, only a *sample* of the M prices may be collected. Call the resulting divergence between the incomplete elementary aggregate and the theoretically ideal elementary index the *sampling error*.

(2) Even if a price for a narrowly defined commodity is collected by the statistical agency, it may not be equal to the theoretically appropriate time aggregated unit value price. This use of an inappropriate price at the very lowest level of aggregation gives rise to *time aggregation error*.

The role of unit values, as outlined above, is as a theoretical concept of price, for aggregating transaction prices of the same commodity from the same establishment over a specified time period. The unit values serve as basic data input on prices at the lowest level. They are the basic prices p_i^t and have associated quantities q_i^t that are the components of the price vectors p^0 and p^1 and the quantity vectors q^0 and q^1 for index number formulas. However, unit values are also used in XMPIs in a second respect: as unit value *indices,* that is, as a price index number formula, derived as a ratio of unit values in two time periods. As a price index, there is a particular functional form to the aggregator used whose properties require consideration. The formula for a unit value index is outlined, and evaluated, in terms of some principal axiomatic tests and a sampling approach, in Section G.

21.23 Because unit value indices are the appropriate target for homogeneous items and superlative indices the appropriate target for heterogeneous items, and because such indices can give very different results, there is a question as to the appropriate index for items that are broadly similar. While the subject is still under study, Dálen (2001), De Haan (2004 and 2007), and Silver (2008) argued for quality-adjusted unit value indices that remove the effect on prices of product heterogeneity. They thus generalize the application of unit value indices to situations of items for which meaningful quality adjustments can be made, that is, broadly comparable items.

[5]Theorem 5 in Diewert (1978, p. 888) showed that P_F, P_T, and P_W will approximate each other to the second order around an equal price and quantity point; see Diewert (1978, p. 894), Robert Hill (2006), and Chapter 20, Section D, for some empirical results.

21.24 In Section C, the five main elementary index number formulas are defined, and in Section D, various numerical relationships between these five indices are developed. Sections E and F develop the axiomatic and economic approaches to elementary indices, and the five main elementary formulas used in practice are evaluated in light of these approaches. In Section G, a sampling framework for the collection of prices that can reduce the above three types of error is discussed.

C. Elementary Indices Used in Practice

21.25 Suppose that there are M lowest level commodities or specific commodities in a chosen elementary category. Denote the period t price of commodity m by p_m^t for $t = 0, 1$ and for commodities $m = 1, 2, \ldots, M$. Define the period t price vector as $p^t \equiv [p_1^t, p_2^t, \ldots, p_M^t]$ for $t = 0, 1$.

21.26 The first widely used elementary index number formula is from the French economist Dutot (1738):

$$P_D(p^0, p^1) \equiv \left[\sum_{m=1}^{M} \frac{1}{M}\left(p_m^1\right)\right] \Big/ \left[\sum_{m=1}^{M} \frac{1}{M}\left(p_m^0\right)\right]$$

$$= \left[\sum_{i=1}^{M} \left(p_m^1\right)\right] \Big/ \left[\sum_{i=1}^{M} \left(p_m^0\right)\right]. \qquad (21.1)$$

Thus the Dutot elementary price index is equal to the arithmetic average of the M period 1 prices divided by the arithmetic average of the M period 0 prices.

21.27 The second widely used elementary index number formula is from the Italian economist Carli (1764):

$$P_C(p^0, p^1) \equiv \sum_{m=1}^{M} \frac{1}{M}\left(p_m^1/p_m^0\right). \qquad (21.2)$$

Thus the Carli elementary price index is equal to the *arithmetic* average of the M commodity price ratios or price relatives, p_m^1/p_m^0.

21.28 The third widely used elementary index number formula is from the English economist Jevons (1863):

$$P_J(p^0, p^1) \equiv \prod_{m=1}^{M} \left(p_m^1/p_m^0\right)^{1/M}. \qquad (21.3)$$

Thus the Jevons elementary price index is equal to the *geometric* average of the M commodity price ratios or price relatives, p_m^1/p_m^0.

21.29 The fourth elementary index number formula P_H is the *harmonic* average of the M commodity price relatives, and it was first suggested in passing as an index number formula by Jevons (1865, p. 121) and Coggeshall (1886):

$$P_H(p^0, p^1) \equiv \left[\sum_{m=1}^{M} \frac{1}{M}\left(p_m^1/p_m^0\right)^{-1}\right]^{-1}. \qquad (21.4)$$

21.30 Finally, the fifth elementary index number formula is the geometric average of the Carli and harmonic formulas; that is, it is *the geometric mean of the arithmetic and harmonic means of the M price relatives:*

$$P_{CSW}(p^0, p^1) \equiv \sqrt{P_C(p^0, p^1)\, P_H(p^0, p^1)}. \qquad (21.5)$$

This index number formula was first suggested by Fisher (1922, p. 472) as his formula 101. Fisher also observed that, empirically for his data set, P_{CSW} was very close to the Jevons index P_J, and these two indices were his best unweighted index number formulas. In more recent times, Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992, p. 140) also proposed P_{CSW} as an elementary index number formula.

21.31 Now that the most commonly used elementary formulas have been defined, the question arises: Which formula is best? Obviously, this question cannot be answered until desirable properties for elementary indices are developed. This is done in a systematic manner in Section E, but in the present section, one desirable property for an elementary index is noted: the *time reversal test,* noted in Chapter 17. In the present context, this test for the elementary index $P(p^0, p^1)$ becomes

$$P(p^0, p^1)\, P(p^1, p^0) = 1. \qquad (21.6)$$

21.32 This test says that if the prices in period 2 revert to the initial prices of period 0, then the product of the price change going from period 0 to 1, $P(p^0, p^1)$, times the price change going from period 1 to 2, $P(p^1, p^0)$, should equal unity; that is, under the stated conditions, the index should end up where it started. It can be verified that the Dutot, Jevons, and Carruthers, Sellwood, and Ward indices, P_D, P_J, and P_{CSW}, all satisfy the time reversal test, but the Carli and harmonic indices, P_C and P_H, fail this test. In fact, these last two indices fail the test in the following *biased* manner:

$$P_C(p^0, p^1)\, P_C(p^1, p^0) \geq 1, \qquad (21.7)$$

$$P_H(p^0, p^1)\, P_H(p^1, p^0) \leq 1, \qquad (21.8)$$

with strict inequalities holding in equations (21.7) and (21.8), provided that the period 1 price vector p^1 is not proportional to the period 0 price vector p^0.[6] Thus the Carli index will generally have an upward bias whereas the harmonic index will generally have a downward bias. Fisher (1922, pp. 66 and 383) seems to have been the first to establish the upward bias of the Carli index,[7] and he made the following observations on its use by statistical agencies:

> In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose. (Irving Fisher, 1922, pp. 29–30)

21.33 In the following section, some numerical relationships between the five elementary indices defined in this section are established. Then, in the subsequent section, a more comprehensive list of desirable properties for elementary indices is developed, and the five elementary formulas are evaluated in light of these properties or tests.

D. Numerical Relationships Between the Frequently Used Elementary Indices

21.34 It can be shown[8] that the Carli, Jevons, and harmonic elementary price indices satisfy the following inequalities:

$$P_H(p^0, p^1) \le P_J(p^0, p^1) \le P_C(p^0, p^1); \qquad (21.9)$$

that is, the harmonic index is always equal to or less than the Jevons index, which in turn is always equal to or less than the Carli index. In fact, the strict inequalities in formula (21.9) will hold, provided that the period 0 vector of prices, p^0, is not proportional to the period 1 vector of prices, p^1.

21.35 The inequalities in formula (21.9) do not tell us by how much the Carli index will exceed the Jevons index and by how much the Jevons index will exceed the harmonic index. Hence, in the remainder of this section, some approximate relationships among the five indices defined in the previous section are developed to provide some practical guidance on the relative magnitudes of each of the indices.

21.36 The first approximate relationship derived is between the Carli index P_C and the Dutot index P_D. For each period t, define the *arithmetic mean of the M prices* pertaining to that period as follows:

$$p^{t*} \equiv \sum_{m=1}^{M} \frac{1}{M}\left(p_m^t\right); \; t = 0, 1. \qquad (21.10)$$

Now define the *multiplicative deviation of the* m*th price in period* t *relative to the mean price in that period,* e_m^t, as follows:

$$p_m^t = p^{t*}\left(1 + e_m^t\right); \; m = 1, \ldots, M; \; t = 0, 1. \qquad (21.11)$$

Note that formula (21.10) and formula (21.11) imply that the deviations e_m^t sum to zero in each period; that is,

$$\sum_{m=1}^{M} \frac{1}{M}\left(e_m^t\right) = 0; \; t = 0, 1. \qquad (21.12)$$

Note that the Dutot index can be written as the ratio of the mean prices, p^{1*}/p^{0*}; that is,

$$P_D(p^0, p^1) = p^{1*}/p^{0*}. \qquad (21.13)$$

Now substitute formula (21.11) into the definition of the Jevons index, formula (21.3):

$$\begin{aligned} P_J(p^0, p^1) &= \prod_{m=1}^{M}\left[p^{1*}\left(1 + e_m^1\right)/p^{0*}\left(1 + e_m^0\right)\right]^{1/M} \\ &= \left(p^{1*}/p^{0*}\right)\prod_{m=1}^{M}\left[\left(1 + e_m^1\right)/\left(1 + e_m^0\right)\right]^{1/M} \\ &= P_D(p^0, p^1)\, f(e^0, e^1), \text{ using formula (21.13)}, \end{aligned} \qquad (21.14)$$

where $e^t \equiv \left[e_1^t, \ldots, e_m^t\right]$ for $t = 0$ and 1 and the function f is defined as follows:

$$f(e^0, e^1) \equiv \prod_{m=1}^{M}\left[\left(1 + e_m^1\right)/\left(1 + e_m^0\right)\right]^{1/M}. \qquad (21.15)$$

Expand $f(e^0, e^1)$ by a second-order Taylor series approximation around $e^0 = 0_M$ and $e^1 = 0_M$. Using formula (21.12), it can be verified[9] that the following second

[6]These inequalities follow from the fact that a harmonic mean of M positive numbers is always equal to or less than the corresponding arithmetic mean; see Walsh (1901, p. 517) or Fisher (1922, pp. 383–84). This inequality is a special case of Schlömilch's Inequality; see Hardy, Littlewood, and Pólya (1934, p. 26).

[7]See also Pigou (1924, pp. 59 and 70), Szulc (1987, p. 12), and Dalén (1992, p. 139). Dalén (1994, pp. 150–51) provided some nice intuitive explanations for the upward bias of the Carli index.

[8]Each of the three indices P_H, P_J, and P_C is a mean of order r where r equals −1, 0, and 1, respectively, and so the inequalities follow from Schlömilch's inequality; see Hardy, Littlewood, and Pólya (1934, p. 26).

[9]This approximate relationship was first obtained by Carruthers, Sellwood, and Ward (1980, p. 25).

order approximate relationship between P_J and P_D results:

$$P_J(p^0, p^1) \approx P_D(p^0, p^1)\left[1 + \left(\tfrac{1}{2}M\right) e^0 e^0 - \left(\tfrac{1}{2}M\right) e^1 e^1\right]$$
$$= P_D(p^0, p^1)\left[1 + \left(\tfrac{1}{2}\right)\text{var}(e^0) - \left(\tfrac{1}{2}\right)\text{var}(e^1)\right], \quad (21.16)$$

where var(e^t) is the variance of the period t multiplicative deviations; that is, for $t = 0, 1$:

$$\text{var}(e^t) \equiv (1/M)\sum_{m=1}^{M} \left(e_m^t - e^{t*}\right)^2$$
$$= (1/M)\sum_{m=1}^{M} \left(e_m^t\right)^2 \text{ because } e^{t*} = 0 \text{ using (12)}$$
$$= (1/M)\, e^t e^t. \quad (21.17)$$

21.37 Under normal conditions,[10] the variance of the deviations of the prices from their means in each period is likely to be approximately constant, and so under these conditions, the Jevons price index will approximate the Dutot price index to the second order. With the exception of the Dutot formula, the remaining four elementary indices defined in Section C are functions of the relative prices of the M commodities being aggregated. This fact is used to derive some approximate relationships between these four elementary indices. Thus define the m*th price relative* as

$$r_m \equiv p_m^1/p_m^0;\ m = 1, \ldots, M. \quad (21.18)$$

Define the arithmetic mean of the m price relatives as

$$r^* \equiv (1/M)\sum_{m=1}^{M} (r_m) = P_C(p^0, p^1), \quad (21.19)$$

where the last equality follows from the definition of formula (21.2) of the Carli index. Finally, define the *deviation* e_m *of the* m*th price relative* r_m *from the arithmetic average of the* M *price relatives* r^* as follows:

$$r_m = r^*(1 + e_m);\ m = 1, \ldots, M. \quad (21.20)$$

21.38 Note that formula (21.19) and formula (21.20) imply that the deviations e_m sum to zero; that is,

$$\sum_{m=1}^{M} (e_m) = 0. \quad (21.21)$$

Now substitute formula (21.20) into the definitions of P_C, P_J, P_H, and P_{CSW}, formulas (21.2) to (21.5), to obtain the following representations for these indices in terms of the vector of deviations, $e \equiv [e_1, \ldots, e_M]$:

$$P_C(p^0, p^1) = \sum_{m=1}^{M} (1/M\,(r_m)) = r^*\ 1 \equiv r^* f_C(e); \quad (21.22)$$

$$P_J(p^0, p^1) = \prod_{m=1}^{M} (r_m)^{1/M} = r^* \prod_{m=1}^{M} (1 + e_m)^{1/M} \equiv r^* f_J(e); \quad (21.23)$$

$$P_H(p^0, p^1) = \left[\sum_{m=1}^{M} (1/M\,(r_m))^{-1}\right]^{-1}$$
$$= r^*\left[\sum_{m=1}^{M} (1/M\,(1 + e_m))^{-1}\right]^{-1}$$
$$\equiv r^* f_H(e); \quad (21.24)$$

$$P_{CSW}(p^0, p^1) = \sqrt{P_C(p^0, p^1) \cdot P_H(p^0, p^1)}$$
$$= r^* \sqrt{f_C(e) \cdot f_H(e)} \equiv r^* f_{CSW}(e), \quad (21.25)$$

where the last equation in (21.22) to (21.25) serves to define the deviation functions, $f_C(e)$, $f_J(e)$, $f_H(e)$, and $f_{CSW}(e)$. The second-order Taylor series approximations to each of these functions around the point $e = 0_M$ are

$$f_C(e) \approx 1; \quad (21.26)$$

$$f_J(e) \approx 1 - \left(\tfrac{1}{2}M\right) e \cdot e = 1 - \left(\tfrac{1}{2}\right)\text{var}(e); \quad (21.27)$$

$$f_H(e) \approx 1 - (1/M) e \cdot e = 1 - \text{var}(e); \quad (21.28)$$

$$f_{CSW}(e) \approx 1 - \left(\tfrac{1}{2}M\right) e \cdot e = 1 - \left(\tfrac{1}{2}\right)\text{var}(e); \quad (21.29)$$

where repeated use is made of formula (21.21) in deriving the above approximations.[11] Thus to the second order, the Carli index P_C will *exceed* the Jevons and Carruthers, Sellwood, and Ward indices, P_J and P_{CSW}, by $(1/2)r^*\text{var}(e)$, which is one-half of the variance of the M price relatives p_m^1/p_m^0. Much like the second order, the harmonic index P_H will *lie below* the Jevons and Carruthers, Sellwood, and Ward indices, P_J and P_{CSW}, by one-half of the variance of the M price relatives p_m^1/p_m^0.

[10]If there are significant changes in the overall inflation rate, some studies indicate that the variance of deviations of prices from their means also can change. Also, if M is small, there will be sampling fluctuations in the variances of the prices from period to period.

[11]These second-order approximations are from Dalén (1992, p. 143) for the case $r^* = 1$ and from Diewert (1995a, p. 29) for the case of a general r^*.

21.39 Thus, empirically, it is expected that the Jevons and Carruthers, Sellwood, and Ward indices will be very close to each other. Using the previous approximation result formula (21.16), it is expected that the Dutot index P_D also will be fairly close to P_J and P_{CSW}, with some fluctuations over time because of changing variances of the period 0 and 1 deviation vectors e^0 and e^1. Thus, it is expected that these three elementary indices will give similar numerical answers in empirical applications. On the other hand, the Carli index can be expected to be substantially *above* these three indices, with the degree of divergence growing as the variance of the M price relatives grows. Similarly, the harmonic index can be expected to be substantially *below* the three middle indices, with the degree of divergence growing as the variance of the M price relatives grows.

E. The Axiomatic Approach to Elementary Indices

21.40 Recall that in Chapter 17, the axiomatic approach to bilateral price indices, $P(p^0, p^1, q^0, q^1)$, was developed. In the present chapter, the elementary price index $P(p^0, p^1)$ depends only on the period 0 and 1 price vectors, p^0 and p^1, not on the period 0 and 1 quantity vectors, q^0 and q^1. One approach to obtaining new tests (T) or axioms for an elementary index is to look at the 20 or so axioms listed in Chapter 17 for bilateral price indices $P(p^0, p^1, q^0, q^1)$, and adapt those axioms to the present context; that is, use the old bilateral tests for $P(p^0, p^1, q^0, q^1)$ that do not depend on the quantity vectors q^0 and q^1 as tests for an elementary index $P(p^0, p^1)$.[12]

21.41 The first eight tests or axioms are reasonably straightforward and uncontroversial:

T1: *Continuity*: $P(p^0, p^1)$ is a continuous function of the M positive period 0 prices $p^0 \equiv [p_1^0, \ldots, p_M^0]$ and the M positive period 1 prices $p^1 \equiv [p_1^1, \ldots, p_M^1]$.

T2: *Identity*: $P(p, p) = 1$; that is, if the period 0 price vector equals the period 1 price vector, then the index is equal to unity.

T3: *Monotonicity in Current-Period Prices*: $P(p^0, p^1) < P(p^0, p)$ if $p^1 < p$; that is, if any period 1 price increases, then the price index increases.

[12]This was the approach used by Diewert (1995a, pp. 5–17), who drew on the earlier work of Eichhorn (1978, pp. 152–60) and Dalén (1992).

T4: *Monotonicity in Base-Period Prices*: $P(p^0, p^1) > P(p, p^1)$ if $p^0 < p$; that is, if any period 0 price increases, then the price index decreases.

T5: *Proportionality in Current-Period Prices*: $P(p^0, \lambda p^1) = \lambda P(p^0, p^1)$ if $\lambda > 0$; that is, if all period 1 prices are multiplied by the positive number λ, then the initial price index is also multiplied by λ.

T6: *Inverse Proportionality in Base-Period Prices*: $P(\lambda p^0, p^1) = \lambda^{-1} P(p^0, p^1)$ if $\lambda > 0$; that is, if all period 0 prices are multiplied by the positive number λ, then the initial price index is multiplied by $1/\lambda$.

T7: *Mean Value Test*: $\min_m \{p_m^1/p_m^0: m = 1, \ldots, M\} \le P(p^0, p^1) \le \max_m \{p_m^1/p_m^0: m = 1, \ldots, M\}$; that is, the price index lies between the smallest and largest price relatives.

T8: *Symmetric Treatment of Establishments/Commodities*: $P(p^0, p^1) = P(p^{0*}, p^{1*})$, where p^{0*} and p^{1*} denote the *same* permutation of the components of p^0 and p^1; that is, if there is a change in ordering of the establishments from which the price quotations (or commodities within establishments) are obtained for the two periods, then the elementary index remains unchanged.

21.42 Eichhorn (1978, p. 155) showed that tests T1, T2, T3, and T5 imply T7, so that not all of the above tests are logically independent. The following tests are more controversial and are not necessarily accepted by all price statisticians.

T9: *The Price Bouncing Test*: $P(p^0, p^1) = P(p^{0*}, p^{1**})$ where p^{0*} and p^{1**} denote possibly *different* permutations of the components of p^0 and p^1; that is, if the ordering of the price quotes for both periods is changed in possibly different ways, then the elementary index remains unchanged.

21.43 Obviously, test T8 is a special case of test T9 where in test T8 the two permutations of the initial ordering of the prices are restricted to be the same. Thus test T9 implies test T8. Test T9 was developed by Dalén (1992, p. 138) who justified this test by suggesting that the price index should remain unchanged if outlet (for CPIs) prices "bounce" in such a manner that the outlets are just exchanging prices with each other over the two periods. Although this test has some intuitive appeal, it is not consistent with the idea that outlet prices should be matched to each other in a one-to-one manner across the two periods. If elementary aggregates contain thousands of individual commodities that differ not only by outlet, there is less reason to maintain this test.

21.44 The following test was also proposed by Dalén (1992) in the elementary index context:

T10: *Time Reversal*: $P(p^1, p^0) = 1/P(p^0, p^1)$; that is, if the data for periods 0 and 1 are interchanged, then the resulting price index should equal the reciprocal of the original price index.

21.45 Because many price statisticians approve of the Laspeyres price index in the bilateral index context, and this index does not satisfy the time reversal test, it is obvious that not all price statisticians would regard the time reversal test in the elementary index context as being a fundamental test that must be satisfied. Nevertheless, many other price statisticians do regard this test as fundamental, because it is difficult to accept an index that gives a different answer if the ordering of time is reversed.

T11: *Circularity*: $P(p^0, p^1)P(p^1, p^2) = P(p^0, p^2)$; that is, the price index going from period 0 to 1, times the price index going from period 1 to 2, equals the price index going from period 0 to 2 directly.

21.46 The circularity and identity tests imply the time reversal test (just set $p^2 = p^0$). Thus, the circularity test is essentially a strengthening of the time reversal test, so price statisticians who did not accept the time reversal test are unlikely to accept the circularity test. However, if there are no obvious drawbacks to accepting the circularity test, it would seem to be a very desirable property: It is a generalization of a property that holds for a single price relative.

T12: *Commensurability*: $P(\lambda_1 p_1{}^0, \ldots, \lambda_M p_M{}^0; \lambda_1 p_1{}^1, \ldots, \lambda_M p_M{}^1) = P(p_1{}^0, \ldots, p_M{}^0; p_1{}^1, \ldots, p_M{}^1) = P(p^0, p^1)$ for all $\lambda_1 > 0, \ldots, \lambda_M > 0$; that is, if the units of measurement for each commodity in each establishment are changed, then the elementary index remains unchanged.

21.47 In the bilateral index context, virtually every price statistician accepts the validity of this test. However, in the elementary context, this test is more controversial. If the M commodities in the elementary aggregate are homogeneous, then it makes sense to measure all of the commodities in the same units. The very essence of homogeneity is that quantities can be added up in an economically meaningful way. Hence, if the unit of measurement is changed, then test T12 should restrict all of the λ_m to be the same number (say λ) and test T12 becomes

$$P(\lambda p^0, \lambda p^1) = P(p^0, p^1); \; \lambda > 0. \qquad (21.30)$$

This modified test T12 will be satisfied if tests T5 and T6 are satisfied. Thus, if the commodities in the elementary aggregate are very homogeneous, then there is no need for test T12.

21.48 However, in actual practice, there usually will be thousands of individual commodities in each elementary aggregate, and the hypothesis of commodity homogeneity is not warranted. Under these circumstances, it is important that the elementary index satisfy the commensurability test, because the units of measurement of the heterogeneous commodities in the elementary aggregate are arbitrary and hence *the price statistician can change the index simply by changing the units of measurement for some of the commodities.*

21.49 This completes the listing of the tests for an elementary index. There remains the task of evaluating how many tests each of the five elementary indices defined in Section C passed.

21.50 The Jevons elementary index, P_J, satisfies *all* of the tests, and hence emerges as being best from the viewpoint of the axiomatic approach to elementary indices.

21.51 The Dutot index, P_D, satisfies all of the tests with the important exception of the Commensurability Test T12, which it fails. Heterogeneous commodities in the elementary aggregate constitute a rather serious failure, and price statisticians should be careful using this index under these conditions.

21.52 The geometric mean of the Carli and harmonic elementary indices, P_{CSW}, fails only the price bouncing test T9 and the circularity test T11. The failure of these two tests is probably not a fatal failure, so this index could be used by price statisticians if, for some reason, they decided not to use the Jevons formula. It particularly would be suited to those who favor the test approach for guidance in choosing an index formula. As observed in Section D, numerically, P_{CSW} will be very close to P_J.

21.53 The Carli and harmonic elementary indices, P_C and P_H, fail the price bouncing test T9, the time reversal test T10, and the circularity test T11 and pass the other tests. The failure of tests T9 and T11 is not a fatal failure, but the failure of the time reversal test T10 is rather serious, so price statisticians should be cautious in using these indices.

F. The Economic Approach to Elementary Indices

21.54 Recall the notation and discussion in Section B. First, it is necessary to recall some of the basics of the economic approach from Chapter 18. This approach allowed the aggregator functions representing the producing technology and the behavioral assumptions of the economic agents implicit in different formulas to be identified. The more realistic these were, the more support was given to the corresponding index number formula. The economic approach helps identify what the target index should be.

21.55 Consider the economic theory relating to an XPI. Suppose, for example, that each establishment producing commodities in the elementary aggregate, for export only, has a set of inputs, and the linearly homogeneous aggregator function $f(q)$ describes what (export) output vector $q \equiv [q_1, \ldots, q_M]$ can be produced from the inputs. Further assume that each establishment engages in (export) revenue maximizing behavior in each period. Then, as was seen in Chapter 18, it can be shown that certain specific functional forms for the aggregator $f(q)$ or its dual unit revenue function $R(p)$[13] lead to specific functional forms for the price index, $P(p^0, p^1, q^0, q^1)$, with

$$P(p^0, p^1, q^0, q^1) = R(p^1)/R(p^0). \qquad (21.31)$$

21.56 Suppose that the establishments have aggregator functions f defined as follows:[14]

$$f(q_1, \ldots, q_M) \equiv \max_m \{q_m/\alpha_m : m = 1, \ldots, M\}, \qquad (21.32)$$

where the α_m are positive constants. Then under these assumptions, it can be shown that equation (21.31) becomes[15]

$$R(p^1)/R(p^0) = p^1 q^0/p^0 q^0 = p^1 q^1/p^0 q^1, \qquad (21.33)$$

and the quantity vector of commodities produced during the two periods must be proportional; that is,

$$q^1 = \lambda q^0 \text{ for some } \lambda > 0. \qquad (21.34)$$

21.57 From the first equation in formula (21.33), it can be seen that the true output price index, $R(p^1)/R(p^0)$, under assumptions of formula (21.32) about the aggregator function f, is equal to the Laspeyres price index, $P_L(p^0, p^1, q^0, q^1) \equiv p^1{\cdot}q^0/p^0{\cdot}q^0$. The Paasche formula $P_P(p^0, p^1, q^0, q^1) \equiv p^1q^1/p^0q^1$ is equally justified under formula (21.34).

21.58 Formula (21.32) on f thus justifies the Laspeyres and Paasche indices as being the "true" elementary aggregate from the economic approach to elementary indices. Yet this is a restrictive assumption, at least from an economic viewpoint, that relative quantities produced do not vary with relative prices. Other less restrictive assumptions on technology can be made. For example, as shown in Sections E and F, Chapter 18, certain assumptions on technology justify the Törnqvist price index, P_T, whose logarithm is defined as

$$\ln P_T(p^0, p^1, q^0, q^1) \equiv \sum_{i=1}^{M} \frac{\left(s_i^0 + s_i^1\right)}{2} \ln\left(\frac{p_i^1}{p_i^0}\right). \qquad (21.35)$$

21.59 Suppose now that commodity revenues are proportional for each commodity over the two periods so that

$$p_m^1 q_m^1 = \lambda\, p_m^0 q_m^0 \text{ for } m = 1, \ldots, M \text{ and for some } \lambda > 0. \qquad (21.36)$$

Under these conditions, the base-period revenue shares $s_m{}^0$ will equal the corresponding period 1 revenue shares $s_m{}^1$, as well as the corresponding $\beta(m)$; that is, formula (21.36) implies

$$s_m^0 = s_m^1 \equiv \beta(m)\ ;\ m = 1, \ldots, M. \qquad (21.37)$$

Under these conditions, the Törnqvist index reduces to the following weighted Jevons index:

$$P_J(p^0, p^1, \beta(1), \ldots, \beta(M)) = \prod_{m=1}^{M} \left(p_m^1/p_m^0\right)^{\beta(m)}. \qquad (21.38)$$

21.60 Thus, if the relative prices of commodities in a Jevons index are weighted using weights proportional to base (which equals current) period revenue shares in the commodity class, then the Jevons index defined by

[13] The unit revenue function is defined as $R(p) \equiv \max_q \{p{\cdot}q : f(q) = 1\}$.

[14] The preferences that correspond to this f are known as Leontief (1936) or no substitution preferences.

[15] This is demonstrated in Pollak (1983).

formula (21.38) is equal to the following approximation to the Törnqvist index:

$$P_J(p^0, p^1, s^0) \equiv \prod_{m=1}^{M} \left(p_m^1/p_m^0\right)^{s_m^0}. \qquad (21.39)$$

Of course at the elementary aggregate level there is no information on quantities and revenues, but at least there is an understanding of the assumptions required for the Jevons index to approximate the Törnqvist index.

21.61 In Section G, the sampling approach shows how, under various sample designs, elementary index number formulas have implicit weighting systems. Of particular interest are sample designs where commodities are sampled with probabilities proportionate to quantity or revenue shares in either period. Under such circumstances, quantity weights are implicitly introduced, so that the sample elementary index is an estimate of a population-weighted index. The economic approach then provides a basis for deciding whether the economic assumptions underlying the resulting population estimates are reasonable. For example, the above results show that the sample Jevons elementary index can be justified as an approximation to an underlying Törnqvist price index for a homogeneous elementary aggregate *under a price sampling scheme with probabilities of selection proportionate to base (which equals current) period revenue shares.*

21.62 Two assumptions have been outlined here: the assumption that the quantity vectors pertaining to the two periods under consideration are proportional (see formula (21.34)) and the assumption that revenues are proportional over the two periods (see formula (21.36)).

21.63 The choice between formulas depends not only on the sample design used but also on the relative merits of the proportional quantities versus proportional revenues assumption. These considerations apply to all index number formulas for it is assumed that underlying each formula are not prices and quantities that are independent of each other, but prices and quantities that are interdependent. The economic theory of MPIs (from a resident producer's perspective), CPIs, or intermediate input PPIs is similar insofar as the aggregator function describes the preferences of a cost-minimizing purchaser. C*ost-minimizing purchasers* will purchase fewer sampled commodities with above-average price increases; the quantities can be expected to fall rather than remain constant. Such a decrease in quantities combined with the increase in price makes the assumption of constant expenditures more tenable. In this context, index number theorists have debated the relative merits of the proportional quantities versus proportional expenditures assumption for a long time. Authors who thought that the proportional expenditures assumption was more likely empirically include Jevons (1865, p. 295) and Ferger (1931, p. 39; and 1936, p. 271). These early authors did not have the economic approach to index number theory at their disposal, but they intuitively understood, along with Pierson (1895, p. 332), that substitution effects occurred and, hence, the proportional expenditures assumption was more plausible than the proportional quantities assumption. However, this is for the economic theory of agents who act as purchasers. In Chapter 18 the economic theory of XPIs, as is the case with output PPIs, predicted that *revenue-maximizing establishments* will produce *more* sampled commodities with above-average price increases, making assumptions of constant revenues less tenable. However, the theory presented in Chapter 18 also indicated that technical progress was a complicating factor largely absent in the consumer context.

21.64 If quantities supplied move proportionally over time, then this is consistent with a Leontief technology, and the use of a Laspeyres index is perfectly consistent with the economic approach to the output price index. On the other hand, if the probabilities used for sampling of prices for the Jevons index are taken to be the arithmetic average of the period 0 and 1 commodity revenue shares, and narrowly defined unit values are used as the price concept, then the weighted Jevons index becomes an ideal type of elementary index discussed in Section B. In general, the biases introduced by the use of an unweighted formula cannot be assessed accurately unless information on weights for the two periods is somehow obtained.

G. Sampling Approach to Elementary Indices

21.65 It can now be shown how various elementary formulas can estimate the Laspeyres formula under alternative assumptions about the sampling of prices.

21.66 To justify the use of the Dutot elementary formula, consider the expected value of the Dutot index when sampling with *base-period commodity inclusion probabilities* equal to the sales quantities of commodity m in the base period relative to total sales quantities of all commodities in the commodity class in the base period. Assume *that these definitions require that all commodities in the commodity class have the same units*.[16] The discussion is in terms of commodities sold to the export market, but can be applied to purchases of imported commodities.

21.67 The expected value of the sample Dutot index is[17]

$$\left(\frac{\sum_{m=1}^{M} p_m^1 q_m^0}{\sum_{m=1}^{M} q_m^0}\right) \Bigg/ \left(\frac{\sum_{m=1}^{M} p_m^0 q_m^0}{\sum_{m=1}^{M} q_m^0}\right), \qquad (21.40)$$

which is the familiar Laspeyres index,

$$\sum_{m=1}^{M} p_m^1 q_m^0 \Big/ \sum_{m=1}^{M} p_m^0 q_m^0 \equiv P_L(p^0, p^1, q^0, q^1). \qquad (21.41)$$

21.68 Now it is easy to see how this sample design could be turned into a rigorous sampling framework for sampling prices in the particular commodity class under consideration. If commodity prices in the commodity class were sampled proportionally to their base-period probabilities, then the Laspeyres index formula (21.41) could be estimated by a probability weighted Dutot index where the probabilities are defined by their base-period quantity shares. Of course, a sample design that uses relative quantities of a good can be meaningfully applied only to homogeneous goods. In general, with an appropriate sampling scheme, the use of the Dutot formula at the elementary level of aggregation *for homogeneous commodities* can be perfectly consistent with a Laspeyres index concept. Put otherwise, under this sampling design, the expectation of the sample Dutot is equal to the population Laspeyres. This need not be a virtue of the Dutot unless the Laspeyres index is the target index. However, for homogeneous items, as noted above, a unit value index is the appropriate target.

21.69 The Dutot formula also can be consistent with a Paasche index concept at the elementary level of aggregation. If sampling is with *period 1 item inclusion probabilities,* the expectation of the sample Dutot is equal to

$$\left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1}\right) \Bigg/ \left(\frac{\sum_{m=1}^{M} p_m^0 q_m^1}{\sum_{m=1}^{M} q_m^1,}\right) \qquad (21.42)$$

which is the familiar Paasche formula,

$$\sum_{m=1}^{M} p_m^1 q_m^1 \Big/ \sum_{m=1}^{M} p_m^0 q_m^1 \equiv P_P(p^0, p^1, q^0, q^1). \qquad (21.43)$$

21.70 Put otherwise, under this sampling design, the expectation of the sample Dutot is equal to the population Paasche index. Again, it is easy to see how this sample design could be turned into a rigorous sampling framework for sampling prices in the particular commodity class under consideration. If commodity prices in the commodity class were sampled proportionally to their period 1 probabilities, then the Paasche index formula (21.43) could be estimated by the probability-weighted Dutot index. In general, with an appropriate sampling scheme, the use of the Dutot formula at the elementary level of aggregation (*for a homogeneous elementary aggregate*) can be perfectly consistent with a Paasche index concept.[18]

21.71 Rather than use the fixed-basket representations for the Laspeyres and Paasche indices, the value share representations for the Laspeyres and Paasche indices could be used along with the value shares s_m^0 or s_m^1 as probability weights for price relatives. Under sampling proportional to base-period value shares, the expectation of the Carli index is

$$P_C(p^0, p^1, s^0) \equiv \sum_{m=1}^{M} s_m^0 \left(p_m^1/p_m^0\right), \qquad (21.44)$$

which is the population Laspeyres index. Of course, formula (21.44) does not require the assumption of homogeneous commodities as did formula (21.40) and formula (21.42) above. On the other hand, one can show analogously that under sampling proportional to period 1 revenue shares, the expectation of the reciprocal of the sample harmonic index is equal to the reciprocal of the population Paasche index, and thus that the expectation of the sample harmonic index,

$$P_H(p^0, p^1, s^1) \equiv \left[\sum_{m=1}^{M} s_m^1 \left(p_m^1/p_m^0\right)^{-1}\right]^{-1}, \qquad (21.45)$$

will be equal to the Paasche index.

21.72 The above results show that the sample Dutot elementary index can be justified as an approximation to an underlying population Laspeyres or Paasche price index for a homogeneous elementary aggregate *under appropriate price sampling schemes*. The above

[16]The inclusion probabilities are meaningless unless the products are homogeneous.

[17]There is a technical bias because $E(x/y)$ is approximated by $E(x)/E(y)$, but this will approach zero as m gets larger.

[18]Of course, the Dutot index as an estimate of a population Paasche index will differ from the Dutot index as an estimate of a population Laspeyres index because of representativity or substitution bias.

results also show that the sample Carli and harmonic elementary indices can be justified as approximations to an underlying population Laspeyres or Paasche price index for a heterogeneous elementary aggregate *under appropriate price sampling schemes.*

21.73 Thus if the relative prices of commodities in the commodity class under consideration are sampled using weights that are proportional to the arithmetic average of the base- and current-period revenue shares in the commodity class, then the expectation of the sample Jevons index is equal to the population Törnqvist index formula (21.35).

21.74 *Sample elementary indices* sampled under appropriate probability designs were capable of approximating various population economic elementary indices, with the approximation becoming exact as the sampling approached complete coverage. Conversely, it can be seen that, in general, it will be impossible for a sample *elementary price index,* of the type defined in Section C, to provide an unbiased estimate of the theoretically population ideal elementary price index defined in Section B, even if all commodity prices in the elementary aggregate were sampled. Hence, rather than just sampling prices, it will be necessary for the price statistician to collect information on the *transaction values* (or quantities) associated with the sampled prices to form sample elementary aggregates that will approach the target ideal elementary aggregate as the sample size becomes large. Thus instead of just collecting a sample of prices, it will be necessary to collect corresponding sample quantities (or values) so that a sample Fisher, Törnqvist, or Walsh price index can be constructed. This sample-based superlative elementary price index will approach the population ideal elementary index as the sample size becomes large. This approach to the construction of elementary indices in a sampling context was recommended by Pigou (1924, pp. 66–67), Fisher (1922, p. 380), Diewert (1995a, p. 25), and Balk (2005).[19] In particular, Pigou (1924, p. 67) suggested that the sample-based Fisher ideal price index be used to deflate the value ratio for the aggregate under consideration to obtain an estimate of the quantity ratio for the aggregate under consideration.

21.75 Until fairly recently, it was not possible to determine how close an unweighted elementary index, defined in Section C, was to an ideal elementary aggregate. However, with the availability of *electronic transaction data* (i.e., of detailed data on the prices and quantities of individual products that are sold in retail outlets), it has been possible to compute ideal elementary aggregates for some product strata and compare the results with statistical agency estimates of price change for the same class of products. Of course, the statistical agency estimates of price change usually are based on the use of the Dutot, Jevons, or Carli formulas. These studies relate to CPIs, the data collected from the bar-code readers of retail outlets. But the concern here is with the discrepancy between unweighted and weighted indices used at this elementary aggregate level, and the discrepancies are sufficiently large to merit highlighting in the context of trade price indices. The following quotation (Diewert, 2002c, pp. 609–10) summarizes many of these scanner data studies using the following citations:

> A second major recent development is the willingness of statistical agencies to experiment with scanner data, which are the electronic data generated at the point of sale by the retail outlet and generally include transactions prices, quantities, location, date and time of purchase and the product described by brand, make or model. Such detailed data may prove especially useful for constructing better indexes at the elementary level. Recent studies that use scanner data in this way include Silver (1995), Reinsdorf (1996), Bradley, Cook, Leaver and Moulton (1997), Dalén (1997), de Haan and Opperdoes (1997) and Hawkes (1997). Some estimates of elementary index bias (on an annual basis) that emerged from these studies were: 1.1 percentage points for television sets in the United Kingdom; 4.5 percentage points for coffee in the United States; 1.5 percentage points for ketchup, toilet tissue, milk and tuna in the United States; 1 percentage point for fats, detergents, breakfast cereals and frozen fish in Sweden; 1 percentage point for coffee in the Netherlands and 3 percentage points for coffee in the United States respectively. These bias estimates incorporate both elementary and outlet substitution biases and are significantly higher than our earlier ballpark estimates of .255 and .41 percentage points. On the other hand, it is unclear to what extent these large bias estimates can be generalized to other commodities. (Diewert, 1998a, pp. 54–55).

Before considering the results it is worth commenting on some general findings from scanner data. It is stressed that the results here are for an experiment in which the same data were used to compare different methods. The results for the U.K. Retail Prices Index can not be fairly compared since they are based on quite different practices and data, their data being collected by price collectors and having strengths as well as weaknesses (Fenwick, Ball, Silver and Morgan (2002)). Yet it is worth following up on Diewert's (2002c) comment on the U.K. Retail Prices Index electrical appliances section, which includes a wide variety of appliances, such as irons, toasters, refrigerators, etc. which went from 98.6 to 98.0, a drop of 0.6 percentage

[19] Balk (2005) provided the details for this sampling framework.

points from January 1998 to December 1998. He compares these results with those for washing machines and notes that ". . . it may be that the non washing machine components of the electrical appliances index increased in price enough over this period to cancel out the large apparent drop in the price of washing machines but I think that this is somewhat unlikely." A number of studies on similar such products have been conducted using scanner data for this period. Chained Fishers [*sic*] indices have been calculated from the scanner data, (the RPI (within year) indices are fixed base Laspeyres ones), and have been found to fall by about 12% for televisions (Silver and Heravi, 2001), 10% for washing machines, 7.5% for dishwashers, 15% for cameras and 5% for vacuum cleaners (Silver and Heravi, [now published as 2004]). These results are quite different from those for the RPI section and suggest that the washing machine disparity, as Diewert notes, may not be an anomaly. Traditional methods and data sources seem to be giving much higher rates for the CPI than those from scanner data, though the reasons for these discrepancies were not the subject of this study. (Silver and Heravi, 2001b, p. 25).

21.76 These studies indicate that when detailed price and quantity data are used to compute superlative indices or hedonic indices for an expenditure category, the resulting measures of price change are often below the corresponding official statistical agency estimates of price change for that category. Sometimes the measures of price change based on the use of scanner data are *considerably below* the corresponding official measures.[20] These results indicate that there may be large gains in the precision of elementary indices if a *weighted* sampling framework is adopted.

21.77 Is there a simple intuitive explanation for the above empirical results? The empirical work is on CPIs, and the behavioral assumptions relate to such indices, though they equally apply to MPIs. Furthermore, the analysis can be undertaken readily based on the behavioral assumptions underlying XPIs, its principles being more important. A partial explanation may be possible by looking at the dynamics of product demand. In any market economy, firms and outlets sell products that are either declining or increasing in price. Usually, the products that decline in price experience an increase in sales. Thus, the expenditure shares associated with products declining in price usually increase, and the reverse is true for products increasing in price. Unfortunately, elementary indices cannot pick up the effects of this negative correlation between price changes and the induced changes in expenditure shares, because elementary indices depend only on prices and not on expenditure shares.

21.78 An example can illustrate this point. Suppose, that for an MPI, there are only three commodities in the elementary aggregate, and that in period 0, the price of each commodity is $p_m^0 = 1$, and the expenditure share for each commodity is equal, so that $s_m^0 = 1/3$ for $m = 1, 2, 3$. Suppose that in period 1, the price of commodity 1 increases to $p_1^1 = 1 + i$, the price of commodity 2 remains constant at $p_2^1 = 1$, and the price of commodity 3 decreases to $p_3^1 = (1 + i)^{-1}$, where the commodity 1 rate of increase in price is $i > 0$. Suppose further that the expenditure share of commodity 1 decreases to $s_1^1 = (1/3) - \sigma$ where σ is a small number between 0 and 1/3, and the expenditure share of commodity 3 increases to $s_3^1 = (1/3) + \sigma$. The expenditure share of commodity 2 remains constant at $s_2^1 = 1/3$. The five elementary indices, defined in Section C, all can be written as functions of the commodity 1 inflation rate i (which is also the commodity 3 deflation rate) as follows:

$$P_J(p^0, p^1) = \left[(1 + i)(1 + i)^{-1}\right]^{\frac{1}{3}} = 1 \equiv f_J(i); \tag{21.46}$$

$$P_C(p^0, p^1) = \frac{1}{3}(1 + i) + \frac{1}{3} + \frac{1}{3}(1 + i)^{-1} \equiv f_C(i); \tag{21.47}$$

$$P_H(p^0, p^1) = \frac{1}{3}(1 + i)^{-1} + \frac{1}{3} + \frac{1}{3}(1 + i)^{-1} \equiv f_H(i); \tag{21.48}$$

$$P_{CSW}(p^0, p^1) = \sqrt{P_C(p^0, \mathrm{p}^1)P_H(p^0, p^1)} \equiv f_{CSW}(i); \tag{21.49}$$

$$P_D(p^0, p^1) = \frac{1}{3}(1 + i) + \frac{1}{3} + \frac{1}{3}(1 + i)^{-1} \equiv f_D(i). \tag{21.50}$$

21.79 Note that in this particular example, the Dutot index $f_D(i)$ turns out to equal the Carli index $f_C(i)$. The second-order Taylor series approximations to the five elementary indices formulas (21.46) to (21.50) are given by formulas (21.51) to (21.55) below:

$$f_J(i) = 1; \tag{21.51}$$

$$f_C(i) \approx 1 + \frac{1}{3}i^2; \tag{21.52}$$

[20] However, scanner data studies do not always show large potential biases in official CPIs. Masato Okamoto (2001) of the National Statistics Center in Japan informed us that a large-scale internal study was undertaken. Using scanner data for about 250 categories of processed food and daily necessities collected over the period 1997 to 2000, researchers found that the indices based on scanner data averaged only about 0.2 percentage points below the corresponding official indices per year. Japan uses the Dutot formula at the elementary level in its official CPI.

$$f_H(i) \approx 1 - \frac{1}{3}i^2; \tag{21.53}$$

$$f_{CSW}(i) \approx 1; \tag{21.54}$$

$$f_D(i) \approx 1 + \frac{1}{3}i^2. \tag{21.55}$$

Thus for small i, the Carli and Dutot indices will be slightly greater than 1,[21] the Jevons and Carruthers, Sellwood, and Ward indices will be approximately equal to 1, and the harmonic index will be slightly less than 1. Note that the first order Taylor series approximation to all five indices is 1; that is, to the accuracy of a first order approximation, all five indices equal unity.

21.80 Now calculate the Laspeyres, Paasche, and Fisher indices for the elementary aggregate:

$$P_L = \frac{1}{3}(1 + i) + \frac{1}{3} + \frac{1}{3}(1 + i)^{-1} \equiv f_L(i); \tag{21.56}$$

$$P_P = \left[\left(\frac{1}{3} - \sigma\right)(1 + i) + \frac{1}{3} + \left(\frac{1}{3} + \sigma\right)(1 + i)^{-1}\right]^{-1}$$
$$\equiv f_P(i); \tag{21.57}$$

$$P_F = \sqrt{P_L \times P_P} \equiv f_F(i). \tag{21.58}$$

First order Taylor series approximations to the above indices formulas (21.56) to (21.58) around $i = 0$ are given by formulas (21.59) through (21.61):

$$f_L(i) \approx 1; \tag{21.59}$$

$$f_P(i) \approx 1 - 2\sigma i; \tag{21.60}$$

$$f_F(i) \approx 1 - \sigma i. \tag{21.61}$$

An ideal elementary index for the three commodities is the Fisher ideal index $f_F(i)$. The approximations in formulas (21.51) to (21.55) and formula (21.61) show that the Fisher index will lie below all five elementary indices by the amount σi using first order approximations to all six indices. *Thus all five elementary indices will have an approximate upward bias equal to* σi *compared with an ideal elementary aggregate.*

21.81 Suppose that the annual commodity inflation rate for the commodity rising in price is equal to 10 percent, so that $i = 0.10$ (and, hence, the rate of price decrease for the commodity decreasing in price is approximately 10 percent as well). If the expenditure share of the increasing price commodity declines by 5 percentage points, then $\sigma = 0.05$, and the annual approximate upward bias in all five elementary indices is $\sigma i = 0.05 \times 0.10 = 0.005$ or one-half of a percentage point. If i increases to 20 percent and σ increases to 10 percent, then the approximate bias increases to $\sigma i = 0.10 \times 0.20 = 0.02$, or 2 percent.

21.82 The above example is highly simplified, but more sophisticated versions of it are capable of explaining at least some of the discrepancy between official elementary indices and superlative indices calculated by using scanner data for an expenditure class. Basically, elementary indices defined without using associated quantity or value weights are incapable of picking up shifts in expenditure shares induced by fluctuations in commodity prices.[22] To eliminate this problem, it will be necessary to sample values along with prices in both the base and comparison periods.

21.83 There is an approach to considering the numerical difference between the Dutot and Jevons index that utilizes the sampling approach and has a bearing on the test approach. Silver and Heravi (2007a) derived an analytical framework to examine the difference between the Dutot and Jevons formulas. The approach benefits from being able to distinguish calculated indices based on sample data as estimators of their population counterparts. The difference between the two formulas is shown to depend on the change over time in price dispersion, which is consistent with the findings of Section D above. The axiomatic approach in Section E above found that the Dutot index should not be used for heterogeneous item groups. Thus some of the price dispersion, and thus difference between the formulas, will be due to product heterogeneity. There is then the question as to how much of the difference between the results of the two indices can be reasonably attributed to the Dutot index's failure of the commensurability test. Silver and Heravi's (2007a) analytical framework used hedonic regressions to control for price dispersion arising from product heterogeneity to further explain that part of the difference between the Jevons and Dutot indices is due to product heterogeneity. In the empirical work they found that this reduction in price dispersion accounted for a large part of the difference between the Jevons and Dutot indices.

21.84 In the following section, a simple regression-based approach to the construction of elementary indices

[21]Recall the approximate relationship in formula (21.16) in Section C between the Dutot and Jevons indices. In the example, $\mathrm{var}(e^0) = 0$, whereas $\mathrm{var}(I^1) > 0$. This explains why the Dutot index is not approximately equal to the Jevons index in the example.

[22]Put another way, elementary indices are subject to substitution or representativity bias.

is outlined and, again, the importance of weighting the price quotes will emerge from the analysis.

H. A Simple Stochastic Approach to Elementary Indices

21.85 Recall the notation used in Section B. Suppose the prices of the M commodities for period 0 and 1 are equal to the right-hand sides of formulas (21.62) and (21.63) below:

$$p_m^0 = \beta_m;\ m = 1, \ldots, M; \tag{21.62}$$

$$p_m^1 = \alpha\beta_m;\ m = 1, \ldots, M, \tag{21.63}$$

where α and β_m are positive parameters. Note that there are $2M$ prices on the left-hand sides of equations (21.62) and (21.63) but only $M + 1$ parameters on the right-hand sides of these equations. The basic hypothesis in equations (21.62) and (21.63) is that the two price vectors p^0 and p^1 are proportional (with $p^1 = \alpha p^0$, so that α is the factor of proportionality) except for random multiplicative errors and, hence, α represents the underlying elementary price aggregate. If logarithms are taken of both sides of equations (21.62) and (21.63) and some random errors e_m^0 and e_m^1 are added to the right-hand sides of the resulting equations, the following *linear regression model* results:

$$\ln p_m^0 = \delta_m + e_m^0;\ m = 1, \ldots, M; \tag{21.64}$$

$$\ln p_m^1 = \gamma + \delta_m + e_m^1;\ m = 1, \ldots, M, \tag{21.65}$$

where

$$\gamma \equiv \ln \alpha \text{ and } \delta_m \equiv \ln \beta_m;\ m = 1, \ldots, M. \tag{21.66}$$

21.86 Note that equations (21.64) and (21.65) can be interpreted as a highly simplified *hedonic regression model*.[23] The only characteristic of each commodity is the commodity itself. This model is also a special case of the *country product dummy method* for making international comparisons between the prices of different countries.[24] A major advantage of this regression method for constructing an elementary price index is that *standard errors* for the index number α can be obtained. This advantage of the stochastic approach to index number theory was stressed by Selvanathan and Rao (1994).

[23]See Chapters 7, 8, and 21 for material on hedonic regression models.

[24]See Summers (1973). In our special case, there are only two "countries," which are the two observations on the prices of the elementary aggregate for two periods.

21.87 It can be verified that the least squares estimator for γ is

$$\gamma^* \equiv \sum_{m=1}^{M} 1/M \ln\left(p^1/p^0\right). \tag{21.67}$$

If γ^* is exponentiated, then the following estimator for the elementary aggregate α is obtained:

$$\alpha^* \equiv \prod_{m=1}^{M} \left(p_m^1/p_m^0\right)^{1/M} \equiv P_J(p^0, p^1), \tag{21.68}$$

where $P_J(p^0, p^1)$ is the *Jevons elementary price index* defined in Section C above. Thus, there is a regression model-based justification for the use of the Jevons elementary index.

21.88 Consider the following unweighted *least squares model:*

$$\min_{\gamma,\, \delta\text{'s}} \sum_{m=1}^{M} \left(\ln p_m^1 - \delta_m\right)^2 + \sum_{m=1}^{M} \left(\ln p_m^0 - \gamma - \delta_m\right)^2. \tag{21.69}$$

It can be verified that the γ solution to the unconstrained minimization problem (21.69) is the γ^* defined by (21.67).

21.89 There is a problem with the unweighted least squares model defined by formula (21.69): The logarithm of each price quote is given exactly the *same weight* in the model, no matter what the revenue on that commodity was in each period. This is obviously unsatisfactory, because a price that has very little economic importance is given the same weight in the regression model compared with a very important commodity. The economic importance of a commodity for an XPI is given by its revenue share in each period, and for an MPI, by its share of purchases. The remainder of the section is outlined in terms of an XPI, but the arguments apply equally to an MPI. Because commodities have different weights, it is useful to consider the following *weighted least squares model*:

$$\min_{\gamma,\, \delta\text{'s}} \sum_{m=1}^{M} s_m^0 \left(\ln p_m^0 - \delta_m\right)^2 + \sum_{m=1}^{M} s_m^1 \left(\ln p_m^1 - \gamma - \delta_m\right)^2, \tag{21.70}$$

where the period t revenue share on commodity m is defined in the usual manner as

$$s_m^t \equiv \frac{p_m^t q_m^t}{\sum_{m=1}^{M} p_m^t q_m^t};\ t = 0, 1;\ m = 1, \ldots, M. \tag{21.71}$$

Thus in the model (21.70), the logarithm of each commodity price quotation in each period is weighted by its revenue share in that period.

21.90 The γ solution to (21.70) is

$$\gamma^{**} = \sum_{m=1}^{M} h\left(s_m^0, s_m^1\right) \ln\left(p_m^1/p_m^0\right), \qquad (21.72)$$

where

$$h(a, b) \equiv \left[\frac{1}{2}a^{-1} + \frac{1}{2}b^{-1}\right]^{-1} = 2ab/[a + b], \qquad (21.73)$$

and $h(a, b)$ is the *harmonic mean* of the numbers a and b. Thus γ^{**} is a share-weighted average of the logarithms of the price ratios $p_m{}^1/p_m{}^0$. If γ^{**} is exponentiated, then an estimator α^{**} for the elementary aggregate α is obtained.

21.91 How does α^{**} compare with the three ideal elementary price indices defined in Section B? It can be shown[25] that α^{**} approximates those three indices to the second order around an equal price and quantity point; that is, for most data sets, α^{**} will be very close to the Fisher, Törnqvist, and Walsh elementary indices.

21.92 The results in this section provide some weak support for the use of the Jevons elementary index, but they provide much stronger support for the use of weighted elementary indices of the type defined in Section B above. The results in this section also provide support for the use of value or quantity weights in hedonic regressions.

I. Conclusion

21.93 The main results in this chapter can be summarized as follows:

(1) To define a "best" elementary index number formula, it is necessary to have a target index number concept. In Section B, it is suggested that normal bilateral index number theory applies at the elementary level as well as at higher levels, and hence the target concept should be one of the Fisher, Törnqvist, or Walsh formulas.

(2) When the prices of the same narrowly defined commodity within a period are aggregated, the narrowly defined unit value is a reasonable target price concept. If the unit value is not narrowly defined, it is subject to bias, the nature of which was considered in Chapter 2.

(3) The axiomatic approach to traditional elementary indices (i.e., no quantity or value weights are available) supports the use of the Jevons formula under all circumstances. If the commodities in the elementary aggregate are very homogeneous (i.e., they have the same unit of measurement), then the Dutot formula can be used. In the case of a heterogeneous elementary aggregate (the usual case), the Carruthers, Sellwood, and Ward formula can be used as an alternative to the Jevons formula, but both will give much the same numerical answers.

(4) The Carli index has an upward bias and the harmonic index has a downward bias.

(5) All five unweighted elementary indices are not really satisfactory. A much more satisfactory approach would be to collect quantity or value information along with price information and form sample superlative indices as the preferred elementary indices.

(6) A simple hedonic regression approach to elementary indices supports the use of the Jevons formula. However, a more satisfactory approach is to use a weighted hedonic regression model. The resulting index will closely approximate the ideal indices defined in Section B.

[25] Use the techniques discussed in Diewert (1978).

22. Quality Change and Hedonics

A. New and Disappearing Items and Quality Change: Introduction

22.1 Chapters 16 to 18 and 21 cover theoretical issues relating to the choice of index number formulas and are based on a simplifying assumption: that the aggregation was over the same matched $i = 1, \ldots, n$ items in the two periods being compared. This meets the needs of the discussion of alternative index number formulas, because a measure of price change between two periods requires the quality of each item to remain the same. The practical compilation of export and import price indices (XMPIs) involves defining the *price basis* (quality specification and terms of sale) of a sample of items in an initial period and monitoring the prices of this matched sample over time, so that only pure price changes, not price changes tainted by changes in quality, are measured. In practice this matching becomes imperfect. The quality of what is produced *does* change and, furthermore, new goods (and services) appear on the market that the matched sampling ignores. The relative price changes of these new goods may differ from those of the existing ones, leading to bias in the index if they are excluded. In this chapter, a theoretical framework is outlined that extends the definition of items to include their quality characteristics. The focus of the chapter is on the *economic* theory of the market for quality characteristics and its practical manifestation in hedonic regression outlined in Chapter 8, Section E.4. This chapter provides a *background* for the more practical issues relating to quality adjustments in Chapter 8 and item substitution in Chapter 9.

22.2 The assumption in the previous chapters was that the same set of items was being compared in each period. Such a set can be considered as a sample from all the matched items available in periods 0 and t—the *intersection universe,* which includes only matched items.[1] Yet for many commodity markets old items disappear and new items appear. Constraining the sample to be drawn from this intersection universe is unrealistic. Establishments may produce an item in period 0, but it may not be sold in subsequent periods t.[2] New items may be introduced after period 0 that cannot be compared with a corresponding item in period 0. These items may be variants of the old existing one, or provide totally new services that cannot be directly compared with anything that previously existed. This universe of all items in periods 0 and t is the dynamic *double universe.*

22.3 There is a third universe from which prices might be sampled: a *replacement* universe. The prices reported by establishments are those for an agreed *price basis*—a detailed description of the item being sold and the terms of the transaction. The price basis for items in period 0 are first determined, and then their prices are monitored in subsequent periods. If the item is discontinued and there are no longer prices to record for a particular price basis, prices of a comparable replacement item may be used to continue the series of prices. This universe is a *replacement universe* that starts with the base-period universe, but it also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

22.4 When a comparable replacement is unavailable, a noncomparable one may be selected. In this case, an explicit adjustment has to be made to the price of either the old or the replacement item for the quality difference. Because the replacement is of a different quality than the old item, it is likely to have a different price basis. Alternatively, assumptions may be made so that the price change of the old item (had it continued to exist) follows those of other items, keeping to the matched universe. In this second case, an implicit

[1]The terminology is credited to Dalén (1998); see also Appendix 8.1.

[2]Its absence may be temporary, because it is a seasonal item, and specific issues and methods for such temporarily unavailable items are considered in Chapter 9. The concern here is with items that disappear permanently.

adjustment is being made for quality changes, so that the difference in price changes for the group and the old item (had it continued to exist) is equivalent to their quality differences.[3] What is stressed here is that the problem of missing items is the problem of adjusting prices for quality differences.

22.5 Three practical problems emerge. First is the problem of explicit quality adjustment between a replacement and old item. The item is no longer produced, a replacement is found that is not strictly comparable in quality, the differences in quality are identified, and a price has to be put on these differences if the series of prices for the new replacement item are to be used to continue those of the old series.

22.6 Second, in markets where the turnover of items is high, the sample space selected from the matched universe is going to become increasingly unrepresentative of the dynamic universe, as argued in detail in Chapters 8 and 9. Even the replacement universe may be inappropriate, as it will be made of series carrying with them quality adjustments in each period whose overall accuracy, given the rapidly changing technology, may be tenuous. In such cases, it may be that prices are no longer collected from a matched sample but from a sample of the main items available in each period even though they are of a different quality. A comparison between the average prices of such items would be biased if, say, the quality of the items was improving. The need for, and details of, mechanisms to remove the effects of such changes from the average price comparisons were discussed in some detail in Chapter 8, Section G.

22.7 Finally, there is the problem of new and disappearing goods and services—when the new item is not a variant of the old but provides a completely new service. It is not possible to use it as a replacement for an old item by adjusting a price for the quality differential because what it provides is, by definition, something new.

22.8 There are a number of approaches to quality adjustment, and these are considered in Chapter 8. One of the approaches is to make explicit adjustments to prices for the quality difference between the old and replacement item using the coefficients from hedonic regression equations. *Hedonic regressions* are regressions of the prices of individual models of a product on their characteristics—for example, the prices of television sets are regressed on screen size, stereo sound, and text retrieval. The coefficients on such variables provide estimates of the monetary values of different quantifiable characteristics of the product. They can be used to adjust the price of a noncomparable replacement item for quality differences compared with the old item—for example, the replacement television set may have text-retrieval facilities that the previous version did not. Yet, it is important that a clear understanding exists of the meaning of such estimated coefficients if they are to be used for quality adjustment, especially given that their use is being promoted.[4] To understand what these estimated parameters mean, it is first necessary to conceive of products as aggregates of their characteristics because, unlike items, characteristics have no separate prices attached to them. The price of the item is the price of a "tied" bundle of characteristics. One must also consider what determines the prices of these characteristics. Economic theory points toward examining demand and supply factors (Sections B.2 and B.3) and the interaction of the two to determine an equilibrium price (Section B.4). Having developed the analytical framework for such prices, it is then necessary to see what interpretation the economic theoretic framework allows us to put on these calculated coefficients (Section B.5). It will be seen that unless there is uniformity of buyers' tastes or technologies, an identification problem prevents an unambiguous supply or demand interpretation. Based on a framework by Diewert (2002d), a demand-side interpretation that assumes firms are competitive price takers is provided, which, under this user-value approach, shows the assumptions required to generate such meaningful coefficients (Section B.6). All of the aforementioned analysis assumes competitive behavior, an assumption which is relaxed in Section B.7.

22.9 Chapter 8, Section G, recommends two main approaches for handling industries with rapid turnover of items. If the sample in period 0 is soon outdated, the matched universe, with even one-on-one replacements, can become increasingly unrepresentative of the double universe, and repeated sampling from the double universe is required. In this case either chained indices are advised, as in Chapter 8, Section G.3, or one of a number of *hedonic indices,* described in Chapter 8, Section G.2. Such indices differ from the use of hedonic regression equations for adjusting prices for quality differences for a missing item. These indices use hedonic regressions, say, by including a dummy

[3]Such methods and their assumptions are outlined in detail in Chapter 8.

[4]See Boskin (1996), Boskin and others (1998), and Schultze and Mackie (2002) on this point.

variable for time on the right-hand side of the equation, to estimate the quality-adjusted price change, as outlined below in Section C and in Chapter 8. An understanding of hedonic regression equations requires that the economic theory of output price indices, outlined in Chapter 18, be developed to include goods that can be defined in terms of tied bundles of their characteristics. *Theoretical output price indices* are defined that include changes in the prices of characteristics. Yet, as with the output price indices for goods considered in Chapter 18, there are many formulations that hedonic indices can take, and analogous issues and formulas arise here when discussing alternative approaches in Sections C.3 through C.6.

22.10 The estimation of hedonic regressions and the testing of their statistical properties is facilitated by the availability of user-friendly, yet powerful, statistical and econometric software. There are many standard issues in the estimation of regression equations, which can be examined by the diagnostics tests available in such software, as discussed in Kennedy (2003) and Maddala (1988). However, there are issues regarding functional form, the use of weighted least squares estimators, and specifications that are quite specific to the estimation of hedonic equations. While many of these are taken up in Chapter 8, where an illustration is provided, Appendix 22.1 considers some of the theoretical issues.

22.11 Finally, in Section D, economic theory is used to advise on the problem of new and disappearing goods and services. This problem arises where differences between existing goods and services and the new goods and services are substantive and cannot be meaningfully compared with an old item, even with a quality adjustment. The economic theory of reservation prices is considered and some issues about its practical implementation are discussed.

B. Hedonic Prices and Implicit Markets

B.1 Items as tied bundles of characteristics

22.12 A *hedonic regression* is a regression equation that relates the prices of items, p, to the quantities of characteristics, given by the vector $z = (z_1, z_2, \ldots, z_n)$, that is,

$$p(z) = p(z_1, z_2, \ldots, z_n), \qquad (22.1)$$

where the items are defined in terms of varying amounts of their characteristics. In practice, what will be observed for each item or variant of the commodity is its price, a set of its characteristics, and possibly the quantity and thus the value sold. Empirical work in this area has been concerned with two issues: estimating how the price of an item changes as a result of unit changes in each characteristic—that is, the estimated coefficients of equation (22.1)—and estimating the demand and supply functions for each characteristic. The depiction of an item as a basket of characteristics, each characteristic having its own implicit (shadow) price, requires in turn the specification of a market for such characteristics, because prices result from the workings of markets. Houthakker (1952), Becker (1965), Lancaster (1966), and Muth (1966) have identified the demand for items in terms of their characteristics. The sale of an item is the sale of a tied bundle of characteristics to consumers, whose economic behavior in choosing between items is depicted as one of choosing between bundles of characteristics.[5] However, Rosen (1974) further developed the analysis by providing a structural market framework in terms of both producers and consumers. There are two sides: demand and supply. How much of each characteristic is supplied and consumed is determined by the interaction of the demand for characteristics by consumers and the supply of characteristics by producers. These are considered in turn.

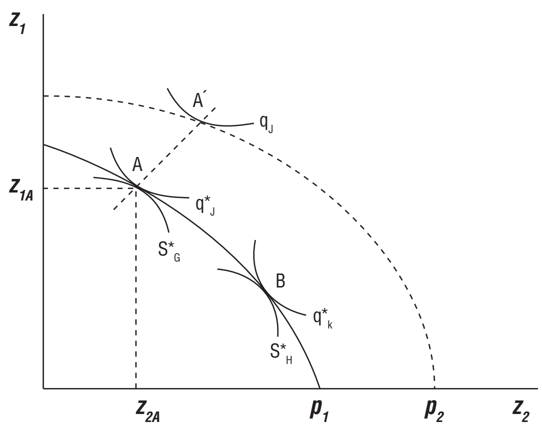
B.2 Consumer or demand side

22.13 Figure 8.1 in Triplett (1987, p. 634) presents a simplified version of the characteristic space between two characteristics. This figure is reproduced here as Figure 22.1. The hedonic surfaces denoted by p_1 and p_2 in that figure trace out all the combinations of the two characteristics z_1 and z_2 that can be purchased at prices p_1 and p_2. An indifference curve q_j* maps the combinations of z_1 and z_2 that the consumer is indifferent to purchasing; that is, the consumer will derive the same utility from any point on the curve. The tangency of q_j* with p_1 at A is the solution to the utility maximization problem for a given budget (price p_1) and tastes (reflected in q_j*).

22.14 The slope of the hedonic surface is the marginal cost of acquiring the combination of characteristics,

[5]Consumers are typically assumed to have preferences over alternative combinations of characteristics that give rise to continuously differentiable price functions. However, for some models, the price functions are piecewise linear and hence continuous but not differentiable; for example, see Lancaster (1971) or Gorman (1980).

Figure 22.1. Consumption and Production Decisions for Combinations of Characteristics



and the slope of the utility function is the marginal utility gained from their purchase. The tangency at A is the utility-maximizing combination of characteristics to be purchased at that price. If consumers purchased any other combination of characteristics in the space of Figure 22.1, it would either cost them more to do so or lead to a lower level of utility. Position A', for example, has more of both z_1 and z_2, and the consumer receives a higher level of utility being on q_j, but the consumer also has to have a higher budget and pays p_2 for being there. Note that the hedonic surface depicted here is nonlinear, so that relative characteristic prices are not fixed. The consumer with tastes $q_k{}^*$ chooses characteristic set B at p_1. Thus, the data observed in the market depend on the set of tastes. Triplett (2004) has argued that if tastes were all the same, then only one model of a personal computer would be purchased. But in the real world more than one model does exist, reflecting heterogeneous tastes and income levels. Rosen (1974) showed that of all the characteristic combinations and prices at which they may be offered, the hedonic surface traces out an envelope[6] of tangencies including those on $q_j{}^*$ and $q_k{}^*$ on p_1 in Figure 22.1. This envelope is simply a description of the locus of the points chosen. Because rational consumers who optimize are assumed, these are the points that will be observed in the market and are thus used to estimate the hedonic regression. Alternative z points on the same indifference curve will allow the relative price of z_1 to z_2 to be determined. However, observed data are likely to result from a locus of points on an expansion path such as $A\ A'$. There may be expansion paths for consumers with different income levels and tastes, such as B, and this may give rise to conflicting valuations, so that the overall parameter estimates determined by the regression from transactions observed in the market are an amalgam of such data. And of course this would just be a reflection of the reality of economic life. What arises from this exposition is the fact that the form of the hedonic function is determined in part by the distribution of buyers and their tastes in the market.

[6]An envelope is more formally defined by letting $f(x, y, k) = 0$ be an implicit function of x and y. The form of the function is assumed to depend on k, the tastes in this case. A different curve corresponds to each value of k in the xy plane. The envelope of this family of curves is itself a curve with the property that it is tangent to each member of the family. The equation of the envelope is obtained by taking the partial derivative of $f(x, y, k)$ with respect to k and eliminating k from the two equations $f(x, y, k) = 0$ and $fk(x, y, k) = 0$. (See Osgood, 1925.)

22.15 The exposition is now formalized to include parameters for tastes and a numeraire commodity[7] against which combinations of other aggregates are selected following Rosen (1974). The hedonic function $p(z)$ describes variation in the market price of the items in terms of their characteristics. The consumer purchase decision is assumed to be based on utility maximization behavior, the utility function being given by $U(z, x; \alpha)$, where x is a numeraire commodity, the maximization of utility being subject to a budget constraint given by income y measured as $y = x + p(z)$ (the amount spent on the numeraire commodity and the hedonic commodities), and α is a vector of the features of the individual consumer that describe his tastes. Consumers maximize their utility by selecting a combination of quantities of x and characteristics z subject to a budget constraint. The market is assumed to be competitive and consumers are described as price takers; they purchase only the one item, so their purchase decision does not influence the market price. The price they pay for a combination of characteristics, vector z, is given by $p(z)$. Because they are optimizing consumers the combination chosen is such that

$$[\partial U(z, y - p(z); \alpha)/\partial z_i]/[\partial U(z, y - p(z); \alpha)/\partial x] = \partial p(z)/\partial z_i \equiv p_i(z), \qquad (22.2)$$

where $\partial p(z)/\partial z_i$ is the first derivative of the hedonic function in equation (22.1) with respect to each z

[7]The numeraire commodity represents all other goods and services consumed—it represents the normal nonhedonic commodities. The price of x is set equal to unity; $p(z)$ and income are measured in these units.

characteristic. The coefficients of the hedonic function are equal to their shadow price p_i, which measures the utility derived from that characteristic relative to the numeraire good for given budgets and tastes.

22.16 A *value function* θ can be defined as the value of expenditure a consumer with tastes α is willing to pay for alternative values of z at a given utility u and income y represented by $\theta(z;\ u,\ y,\ \alpha)$. It defines a family of indifference curves relating the z_i to forgone x, "money." For individual characteristics z_i, θ is the marginal rate of substitution between z_i and money, or the implicit marginal valuation the consumer with tastes α puts on z_i at a given utility level and income. It is an indication of the reservation demand price[8] for additional units of z_i.[9] The price in the market is $p(z)$, and utility is maximized when $\theta(z;\ u,\ y,\ \alpha) = p(z)$, that is, the purchase takes place where the surface of the indifference curve θ is tangent to the hedonic price surface. If different buyers have different value functions (tastes), some will buy more of a characteristic than others for a given price function, as illustrated in Figure 22.1.

22.17 The joint distribution function of tastes and income sets out a family of value functions, each of which, when tangential to the price function, depicts a purchase and simultaneously defines the price function whose envelope is the market hedonic price function. The points of purchase traced out by the hedonic function thus depend on the budget of the individual and the tastes of the individual consumer purchasing an individual set of characteristics. If demand functions are to be traced out, the joint probability distribution of consumers with particular budgets and tastes occurring in the market needs to be specified, that is, $F(y, \alpha)$. This function, along with equation (22.1), allows the demand equations to be represented for each characteristic.

B.3 Producer or supply side

22.18 Triplett's (1987) Figure 8.1 also shows the production side. In Chapter 18, Section D.1, a revenue-maximizing producer was considered whose revenue maximization problem was given by equation (18.1):[10]

$$R(p, v) \equiv \max_q \left[\sum_{n=1}^{N} p_n q_n : q \text{ belongs to } S(v)\right], \quad (22.3)$$

where $R(p, v)$ is the maximum value of output, $\sum_{n=1}^{N} p_n q_n$, that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v is available for use, using the period t technology. Figure 18.1 illustrated in goods-space how the producer would choose between different combinations of outputs, q_1 and q_2. In Figure 22.1, the characteristics-space problem is analogous to the goods-space one with producers choosing here between combinations of z_1 and z_2 to produce for a particular level of technology and inputs $S(v)$. For a particular producer with level of inputs and technology S^*_G facing a price surface p_1, the optimal production combination is at A. However, a different producer with technology and inputs S^*_H facing a price surface p_1 would produce at B. At these points, the marginal cost of z_1 with respect to z_2 is equal to its marginal price from the hedonic surface as depicted by the tangency of the point. Production under these circumstances at any other combination would not be optimal. The envelope of tangencies such as S^*_G and S^*_H trace out the production decisions that would be observed in the market from optimizing, price-taking producers and be used as data for estimating the hedonic regressions. The hedonic function can be seen to be determined, in part, by the distribution of technologies of producers, including their output scale.

22.19 Rosen (1974) formalized the producer side, whereby price-taking producers are assumed to have cost functions described by $C(M, z; \tau)$[11] where $Q = Q(z)$ is the output scale, that is, the number of units produced by an establishment offering specifications of an item with characteristics z. They have to decide which items to produce, that is, which package of z to produce. The solution for each producer is to choose the output that minimizes costs given its own technology: the output combinations each producer can produce with given input costs using its factors of production and factor prices and technology. The cost function includes τ, equivalent to $S(v)$ above, a vector of the technology and inputs of each producer. It is the variation in τ across producers that distinguishes producer A's decision about which combination of z to produce from that of producer B in Figure 22.1. Producers are optimizers who seek to maximize profits given by

$$Q\ p(z) - C(Q, z; \tau), \quad (22.4)$$

[8]This is the hypothetical price that makes the demand for the characteristic equal to zero, that is, it is the price that, when inserted into the demand function, sets demand to zero.

[9]The utility function is assumed to be strictly concave so that θ is concave in z, and the value function is increasing in z_i at a decreasing rate.

[10]The time superscripts are not relevant in this context.

[11]The cost function is assumed to be convex with no indivisibilities. The marginal cost of producing one more item of a given combination of characteristics is assumed to be positive and increasing and, similarly, the marginal cost of increasing production of each component characteristic is positive and nondecreasing.

by selecting Q and z optimally. The supplying market is assumed to be competitive, and producers are price takers so the producers cannot influence price by their production decision. Their decision about how much to produce of each z is determined by the price of z, assuming that the producer can vary Q and z in the short run.[12] Dividing equation (22.4) by Q and setting the resulting expression equal to zero, the first-order profit-maximizing conditions are given by

$$\frac{\partial p}{\partial z_i} = p_i = \frac{C_{zi}\,(Q, z; \tau)}{Q}, \qquad (22.5)$$

where $p = p(z_1, z_2, \ldots, z_n)$ as in equation (22.1).

22.20 The *marginal unit revenue* from producing characteristic z_i is given by its shadow price in the price function and its marginal cost of production. In the producer case, a knowledge of the probability distribution of the technologies of firms, $G(\tau)$, is necessary if the overall quantity supplied of items with given characteristic sets is to be revealed. Because it is a profit maximization problem to select the optimal combination of characteristics to produce, marginal revenue from the additional attributes must equal their marginal cost of production per unit sold. Quantities are produced up to the point where unit revenues $p(z)$ equal marginal production costs, evaluated at the optimum bundle of characteristics supplied.

22.21 Whereas for consumers a *value function* was considered, producers require an *offer function* $\phi(z; \pi, \tau)$. The offer price is the price the seller is willing to accept for various designs at constant profit level π, when quantities produced are optimally chosen, while $p(z)$ is the maximum price obtainable from those models in the market. Producer equilibrium is characterized by a tangency between two surfaces: the profit characteristics indifference surface and the market characteristics price surface, where $p_i(z_i) = \phi_{zi}(z; \pi, \tau)$ and $p(z) = \phi_z; \pi, \tau)$. Because there is a distribution of technologies $G(\tau)$, the producer equilibrium is characterized by a family of offer functions that envelop the market hedonic price function. The varying τ will depend on different factor prices for items produced in different countries, multiproduct firms with economies of scale, and differences in the technology, whether these differences have to do with quality of capital, labor, or intermediate inputs and their organization. Different values of τ will define a family of production surfaces.

B.4 Equilibrium

22.22 The theoretical framework first defined each item as a point on a plane of several dimensions made up by the $z_1, z_2, \ldots, z_n$ quality characteristics; each item was a combination of values $z_1, z_2, \ldots, z_n$. If only two characteristics defined the item, then each point in the positive space of Figure 22.1 would define an item. The characteristics were not bought individually but as bundles of characteristics tied together to make up an item. It was assumed that the markets were differentiated so that there was a wide range of choices to be made.[13] The market was also assumed to be perfectly competitive with consumers and producers as price takers undertaking optimizing behavior to decide which items (tied sets of characteristics) to buy and sell. Competitive markets in characteristics and optimizing behavior are assumed so that the quantity demanded of characteristics z must equal the quantity supplied. It has been shown that consumers' and producers' choices or "locations" on the plane will be dictated by consumer tastes and producer technology. Tauchen and Witte (2001, p. 4) showed that the hedonic price function will differ across markets in accordance with the means and variances (and in some cases also higher moments) of the distributions of household and firm characteristics.

22.23 Rosen (1974, p. 44) noted that a buyer and seller are perfectly matched when their respective value and offer functions are tangential. The common gradient at that point is given by the gradient of the market-clearing implicit price function $p(z)$. The consumption and production decisions were seen in the value and offer functions to be jointly determined, for given $p(z)$, by $F(y, \alpha)$ and $G(\tau)$. In competitive markets there is a simultaneity in the determination of the hedonic equation, because the distribution of $F(y, \alpha)$ and $G(\tau)$ helps determine the quantities demanded and supplied and also the slope of the function. Although the decisions are made by consumers and producers as price takers, the prices taken are those from the hedonic function. There is a sense in which the hedonic function and its shadow prices emerge from the

[12]Rosen (1974) considered two other supply characterizations: the short run in which only Q is variable, and a long run in which plants can be added and retired. The determination of equilibrium supply and demand is not straightforward. A function $p(z)$ is required such that market demand for all z will equate to market supply and clear the market. But demand and supply depend on the whole $p(z)$, because any adjustment to prices to equate demand and supply for one combination of items will induce substitutions and changes for others. Rosen (1974, pp. 44–48) discussed this in some detail.

[13]In order to ensure that choices among combinations of z are continuous, assume further that $p(z)$ possesses continuous first order derivatives.

operations of the market. The product markets implicitly reveal the hedonic function. Because consumers and producers are optimizers in competitive markets, the hedonic function, in principle, gives the minimum price of any bundle of characteristics. Given all of this, Rosen (1974, p. 44) asked: What do hedonic prices mean?

B.5 What do hedonic prices mean?

22.24 It would be convenient if, for import price index construction, the estimated coefficients from hedonic regressions were estimates of the marginal production cost or producer value of a characteristic or, for export price index construction, they were estimates of the marginal nonresident user value from a characteristic. But theory tells us that this is not the case and that the interpretation is not clear.

22.25 There was an erroneous perception in the 1960s that the coefficients from hedonic methods represented user values as opposed to resource costs. Rosen (1974), as has been shown, found that hedonic coefficients generally reflect both user values and resource costs in both supply and demand situations. The ratios of these coefficients may reflect consumers' marginal rates of substitution or producers' marginal rates of substitution (transformation) for characteristics. There is what is referred to in econometrics as an "identification" problem in which the observed prices and quantities are jointly determined by supply and demand considerations, and their underlying effects cannot be separated. The data collected on prices jointly arise from variations in demand by different consumers with different tastes and preferences, and from variations in supply by producers with different technologies.

22.26 First, it is necessary to come to terms with this simultaneity problem. Hedonic regressions are an increasingly important analytical tool, one implicitly promoted by the attention given to it in this *Manual* and by publications of the International Labour Organization (ILO) and others (2004a, 2004b), but also promoted in separate manuals by organizations such as the Organization for Economic Co-operation and Development (see Triplett, 2004) and Eurostat (2001),[14] and widely used by the U.S. Bureau of Labor Statistics (Kokoski, Waehrer, and Rozaklis, 2001; Moulton, 2001). So how do economists writing on the subject shrug their intellectual shoulders in light of these findings?

22.27 Rosen (1974, p. 43) referred to the hedonic function as "a joint envelope of a family of value functions and another family of offer functions. An envelope function by itself reveals nothing about the underlying members that generate it; and they in turn constitute the generating structure of the observations."

22.28 Griliches noted the following:

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions *per se,* though it can also be useful for these purposes. . . . [W]hat is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances. (1988, p. 120)

22.29 Triplett (1987) stated:

> It is well-established—but still not widely understood—that the form of $h(\cdot)$ [the hedonic function] cannot be derived from the form of $Q(\cdot)$ and $t(\cdot)$ [utility and production functions], nor does $h(\cdot)$ represent a "reduced form" of supply and demand functions derived from $Q(\cdot)$ and $t(\cdot)$. (1987, p. 631).

22.30 Diewert, with his focus on the consumer side, said:

> Thus, I am following Muellbauer's (1974, p. 977) example where he says that his "approach is unashamedly one-sided; only the demand side is treated. . . . Its subject matter is therefore rather different from that of the recent paper by Sherwin Rosen. The supply side and simultaneity problems which may arise are ignored." (2003, p. 320)

Diewert (2002e) has also considered the theoretical producer price indices with a focus on the producer side. He based the optimizing problem the establishments face when deciding on which combinations of characteristics to produce, however, on the consumer's valuations, giving them precedence. There are many industries in which firms are effective price takers, and the prices taken are dictated by the consumer side rather than by cost and technological considerations. Section B.6 outlines this framework, which allows a

[14] Notwithstanding some objections, "the Commission (Eurostat) leaves open the possibility of agreeing with Member States that such methods may be preferable to many of the current practices" (Eurostat, 2001, p. 70).

more straightforward development of the theory of hedonic index numbers for XMPIs.

22.31 Second, Rosen's theoretical framework allows the consideration of the conditions under which the hedonic coefficients are determined by only demand-side or supply-side factors—the circumstances under which clear explanations would be valid. The problem is that because the coefficients of a hedonic function are the outcome of the interaction of consumer and producer optimizing conditions, it is not possible to interpret the function only in terms of, say, producer marginal costs or consumer marginal values. However, suppose the *production technology* τ was the same for each producing establishment. Buyers differ but sellers are identical. Then, instead of a confusing family of offer functions, there is a unique offer function with the hedonic function describing the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. The offer function becomes $p(z)$, because there is no distribution of τ to confuse it. There are different tastes on the consumer side, and so what appears in the market is the result of firms trying to satisfy consumer preferences all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. In Figure 22.1 only the expansion path traced out by, say, $S_H{}^*$ akin to $A\ A'$ would be revealed. Now, suppose sellers differ, but *buyers' tastes* α are identical. Here the family of *value functions* collapses to be revealed as the hedonic function $p(z)$, which identifies the structure of demand, such as $A\ A'$ in Figure 22.1.[15] Diewert's (2003) approach follows a representative consumer, rather than consumers with different tastes, so that the demand side alone can be identified. Triplett (1987, p. 632) noted that of these possibilities, uniformity of technologies is the most likely, especially when access to technology is unrestricted in the long run, while uniformity of tastes is unlikely. There may, of course, be segmented markets where tastes are more uniform to which specific sets of items are tailored and for which hedonic equations can be estimated for individual segments.[16] In some industries there may be a prior expectation of uniformity of tastes against uniformity of technologies and interpretation of coefficients will accordingly follow. In many cases, however, the interpretation may be more problematic. The pure producer approach requires assumptions of uniformity of technology and input prices that cannot of course be generally assumed. But the key assumption that will not generally be satisfied in the producer context is that each *producer is able to produce the entire array of hedonic models* whereas, in the consumer context, it is quite plausible that each consumer has the possibility of purchasing and consuming each model.

22.32 Third, issues relating to the estimation of the underlying supply and demand functions for characteristics have implications for the estimation of hedonic functions. In Appendix 22.2, identification and estimation issues are considered in this light. Finally, the subsequent concern with new products in Section D of this chapter refers to demand functions. However, attention is now turned to hedonic *indices*. In the next section, these are noted to have a quite different application than that for the quality adjustment of noncomparable replacement items.

B.6 An alternative hedonic theoretical formulation

22.33 This section is based on a formulation by Diewert (2002d). It assumes competitive price-taking behavior on the part of firms. In this approach, the user's valuations of the various models that could be produced flow to producers via the hedonic function in the same way that output prices are taken, as given in the usual theory of the output price index. It is necessary to set up the establishment's revenue maximization problem assuming that it produces a single output, but in each period, the establishment has a choice of which type of model it could produce. Let the model be identified by a K dimensional vector of characteristics, $z \equiv [z_1, \ldots, z_K]$. Before the establishment's revenue maximization problem is tackled, it is necessary to characterize the set of output prices that the establishment faces in period t as a function of the characteristics of the model that the establishment might produce. It is assumed that in period t, the demanders of the output of the establishment

[15]Correspondingly, if the supply curves were perfectly inelastic, so that a change in price would not affect the supply of any of the differentiated products, then the variation in prices underlying the data and feeding the hedonic estimates would be determined by demand factors. The coefficients would provide estimates of user values. Similarly, if the supplying market were perfectly competitive, the estimates would be of resource costs. None of the price differences between differentiated items would be due to, say, novel configurations of characteristics, and no temporary monopoly profit would be achieved as a reward for this, or as a result of the exercise of market power. See Berndt (1983).

[16]Berry, Levinsohn, and Pakes (1995) provided a detailed and interesting example for automobiles in which makes are used as market segments, while Tauchen and Witte (2001) provided a systematic theoretical study of estimation issues for supply, demand, and hedonic functions where consumers and producers and their transactions are indexed across communities.

have a cardinal utility function, $f^t(z)$, that enables each demander to determine that the value of a model with the vector of characteristics $z^1 \equiv [z_1{}^1, \ldots, z_K{}^1]$ compared with a model with characteristics vector $z^2 \equiv [z_1{}^2, \ldots, z_K{}^2]$ is $f^t(z^1)/f^t(z^2)$. Thus, in period t, demanders are *willing to pay* the amount of money $p^t(z)$ for a model with the vector of characteristics z where

$$\Pi^t(z) \equiv \rho^t f^t(z), \quad t = 0, 1. \tag{22.6}$$

The scalar ρ^t is inserted into the willingness-to-pay function because, under certain restrictions, ρ^t can be interpreted as a period t price for the entire family of hedonic models that might be produced in period t. These restrictions are

$$f^0 = f^1, \tag{22.7}$$

that is, the *model relative utility functions* f^t are identical for the two periods under consideration. We make use of the specific assumption in equation (22.7) later.

22.34 In what follows, it is assumed that econometric estimates for the period 0 and 1 *hedonic model price functions,* Π^0 and Π^1, are available, although we also consider the case where only an estimate for Π^0 is available.[17]

22.35 Now, consider an establishment that produces a single model in each period in the marketplace that is characterized by the hedonic model price functions, $\Pi^t(z)$, for periods $t = 0, 1$. Suppose that in period t, the establishment has the *production function* F^t, where

$$q = F^t(z, v) \tag{22.8}$$

is the number of models, each with vector of characteristics z, that can be produced if the vector of inputs v is available for use by the establishment in period t. As is usual in the economic approach to index numbers, we assume a competitive model, where each establishment takes output prices as fixed parameters beyond its control. In this case, there is an entire schedule of model prices that the establishment takes as given instead of just a single price in each period. Thus, it is assumed that if the establishment decides to produce a model with the vector of characteristics z, then it can sell any number of units of this model in period t at the price $\Pi^t(z) = \rho^t f^t(z)$. Note that the establishment is allowed to choose which model type to produce in each period.

22.36 Now, define the establishment's *revenue function, R*, assuming the establishment is facing the period s hedonic price function $\Pi^s = \rho^s f^s$ and is using the vector of inputs v and has access to the period t production function F^t:

$$R(\rho^s f^s, F^t, Z^t, v) \equiv \max_{q,z} \{\rho^s f^s(z) q : q = F^t(z, v);\ z \text{ belongs to } Z^t\} = \max_z \{\rho^s f^s(z) F^t(z, v) : z \text{ belongs to } Z^t\}, \tag{22.9}$$

where Z^t is a *technologically feasible set of model characteristics* that can be produced in period t. The second line follows from the line above by substituting the production-function constraint into the objective function.

22.37 The actual period t revenue maximization problem that the establishment faces is defined by the revenue function equation (22.9), except that we replace the period s hedonic price function $\rho^s f^s$ by the period t hedonic price function $\rho^t f^t$, and the generic input quantity vector v is replaced by the observed period t input quantity vector used by the establishment, v^t. Further, assume that the establishment produces q^t units of a single model with characteristics vector z^t and that $[q^t, z^t]$ solves the period t revenue maximization problem—that is, $[q^t, z^t]$ is a solution to[18]

$$R(\rho^t f^t, F^t, Z^t, v^t) \equiv \max_{q,z} \{\rho^t f^t(z) q : q = F^t(z, v^t);\ z \text{ belongs to } Z^t\}; \quad t = 0, 1 \tag{22.10}$$
$$= \rho^t f^t(z^t) q^t$$

where the period t establishment output q^t is equal to

$$q^t = F^t(z^t, v^t); \quad t = 0, 1. \tag{22.11}$$

Now, a family of *Konüs-type hedonic output price indices* P between periods 0 and 1 can be defined as follows:

$$P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v) \equiv R(\rho^1 f^1, F^t, Z^t, v) / R(\rho^0 f^0, F^t, Z^t, v). \tag{22.12}$$

[17]We will need some identifying restrictions in order to identify the parameters of f^0 and f^1 along with ρ^0 and ρ^1. One common model sets $\rho^0 = 1$ and $f^0 = f^1$. A more general model sets $\rho^0 = 1$ and $f^0(z^*) = f^1(z^*)$ for a reference characteristics vector, $z^* \equiv [z_1{}^*, \ldots, z_K{}^*]$.

[18]If the establishment is competitively optimizing with respect to its choice of inputs as well, then the period t input vector v^t, along with q^t and z^t, is a solution to the following period t profit maximization problem for the establishment: $\max_{q,z,v} \{\rho^t f^t(z) q - w^t \cdot v : q = F^t(z, v);\ z \text{ belongs to } Z^t\}$, where w^t is a vector of input prices that the establishment faces in period t and $w^t \cdot v$ denotes the inner product of the vectors w^t and v. It is possible to rework our analysis presented below, conditioning on an input price vector rather than on an input quantity vector.

22.38 Thus, a particular member of the above family of indices is equal to the establishment's revenue ratio, where the revenue in the numerator of equation (22.12) uses the hedonic model price function for period 1, and the revenue in the denominator of equation (22.12) uses the hedonic model price function for period 0. For both revenues, however, the technology of period t is used (i.e., F^t and Z^t are used in both revenue maximization problems), and the same input quantity vector v is used. This is the usual definition for an economic output price index, except that instead of a single price facing the producer in each period, we have a whole family of model prices facing the establishment in each period. Note that the only variables that are different in the numerator and denominator of equation (22.12) are the two hedonic model price functions facing the establishment in periods 0 and 1.

22.39 The right-hand side of equation (22.12) looks a bit complex. However, if the assumption in equation (22.7) holds (i.e., the period 0 and 1 hedonic model price functions are identical except for the multiplicative scalars ρ^0 and ρ^1), then equation (22.12) reduces to the very simple ratio, ρ^1/ρ^0. To see this, use equations (22.12) and (22.10) as follows:

$$\begin{aligned} P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v) &\equiv R(\rho^1 f^1, F^t, Z^t, v)/R(\rho^0 f^0, F^t, Z^t, v) \\ &= \max_z \{\rho^1 f^1(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\} \\ &= \max_z \{\rho^1 f^0(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\} \quad (22.13) \end{aligned}$$

using equation (22.7)

$$= [\rho^1/\rho^0]\max_z \{\rho^0 f^0(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z, v^t);\ z \text{ belongs to } Z^t\}$$

assuming ρ^0 and ρ^1 are positive and canceling terms

$$= \rho^1/\rho^0.$$

This is a very useful result because many hedonic regression models have been successfully estimated using equation (22.7). Under this assumption, *all* the theoretical hedonic establishment output price indices reduce to the observable ratio, ρ^1/ρ^0.

22.40 We return to the general case where the assumption in equation (22.7) is not made. As usual, it is always of interest to specialize equation (22.12) to the special cases where the conditioning variables that are held constant in the numerator and denominator of equation (22.12), F^t, Z^t, and v, are equal to the period 0 and 1 values for these variables, namely, F^0, Z^0, and v^0, and F^1, Z^1, and v^1. Thus, define the *Laspeyres-type hedonic output price index* between periods 0 and 1 for our establishment as follows:

$$\begin{aligned} P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0) &\equiv R(\rho^1 f^1, F^0, Z^0, v^0)/R(\rho^0 f^0, F^0, Z^0, v^0) \\ &= R(\rho^1 f^1, F^0, Z^0, v^0)/\rho^0 f^0(z^0)q^0 \quad \text{using equation (22.10) for } t = 0 \\ &= \max_z\{\rho^1 f^1(z)F^0(z, v^0);\ z \text{ belongs to } Z^0\}/\rho^0 f^0(z^0)q^0 \quad \text{using equation (22.9)} \\ &\geq \rho^1 f^1(z^0)F^0(z^0, v^0)/\rho^0 f^0(z^0)q^0 \quad \text{because } z^0 \text{ is feasible for the maximization problem} \\ &= \rho^1 f^1(z^0)q^0/\rho^0 f^0(z^0)q^0 \quad \text{using equation (22.11) for } t = 0 \\ &= \rho^1 f^1(z^0)/\rho^0 f^0(z^0) \\ &\equiv P_{HL} \end{aligned} \quad (22.14)$$

where the *observable hedonic Laspeyres output price index* P_{HL} is defined as

$$P_{HL} \equiv \rho^1 f^1(z^0)/\rho^0 f^0(z^0). \quad (22.15)$$

Thus, the inequality in equation (22.14) says that the unobservable theoretical Laspeyres-type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ is bounded from below by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0, and f^1) hedonic Laspeyres output price index P_{HL}. The inequality in equation (22.14) is the hedonic counterpart to a standard Laspeyres-type inequality for a theoretical output price index.

22.41 It is of modest interest to rewrite P_{HL} in terms of the observable model prices for the establishment in periods 0 and 1. Denote these prices by P^0 and P^1, respectively. Using equation (22.6),

$$P^0 = \rho^0 f^0(z^0) \text{ and } P^1 = \rho^1 f^1(z^1). \quad (22.16)$$

Now, rewriting equation (22.15) as follows:

$$\begin{aligned} P_{HL} &\equiv \rho^1 f^1(z^0)/\rho^0 f^0(z^0) \quad (22.17) \\ &= \rho^1 f^1(z^1)[f^1(z^0)/f^1(z^1)]/\rho^0 f^0(z^0) \\ &= P^1[f^1(z^0)/f^1(z^1)]/P^0 \quad \text{using equation (22.16)} \\ &= [P^1/f^1(z^1)]/[P^0/f^1(z^0)]. \end{aligned}$$

The prices $P^1/f^1(z^1)$ and $P^0/f^1(z^0)$ can be interpreted as *quality-adjusted model prices* for the establishment in periods 1 and 0, respectively, using the hedonic regression pertaining to period 1 to do the quality adjustment.

22.42 In the theoretical hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ defined by equation (22.14) above, we conditioned on F^0 (the base-period production function), Z^0 (the base-period set of models that were technologically feasible in period 0), and v^0 (the establishment's base-period input vector). We now define a companion period 1 theoretical hedonic output price that conditions on the period 1 variables, F^1, Z^1, v^1. Thus, define the *Paasche-type hedonic output price index* between periods 0 and 1 for an establishment as follows:[19]

$$\begin{aligned} P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1) &\equiv R(\rho^1 f^1, F^1, Z^1, v^1)/R(\rho^0 f^0, F^1, Z^1, v^1) \\ &= \rho^1 f^1(z^1)q^1/R(\rho^0 f^0, F^1, Z^1, v^1) \quad \text{using equation (22.10) for } t = 1 \\ &= \rho^1 f^1(z^1)q^1/\max_z \{\rho^0 f^0(z)F^1(z, v^1);\ z \text{ belongs to } Z^1\} \quad \text{using equation (22.9)} \\ &\leq \rho^1 f^1(z^1)q^1/\rho^0 f^0(z^1)F^1(z^1, v^1) \quad \text{because } z^1 \text{ is feasible for the maximization problem} \\ &= \rho^1 f^1(z^1)q^1/\rho^0 f^0(z^1)q^1 \quad \text{using equation (22.11) for } t = 1 \\ &= \rho^1 f^1(z^1)/\rho^0 f^0(z^1) \\ &\equiv P_{HP}, \end{aligned} \tag{22.18}$$

where the *observable hedonic Paasche output price index* P_{HP} is defined as

$$P_{HP} \equiv \rho^1 f^1(z^1)/\rho^0 f^0(z^1). \tag{22.19}$$

Thus, the inequality in equation (22.18) says that the unobservable theoretical Paasche-type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$ is bounded from above by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0, and f^1) hedonic Paasche output price index P_{HP}. The inequality in equation (22.18) is the hedonic counterpart to a standard Paasche-type inequality for a theoretical output price index.

22.43 Again, it is of interest to rewrite P_{HP} in terms of the observable model prices for the establishment in periods 0 and 1. Rewrite equation (22.19) as follows:

$$\begin{aligned} P_{HP} &\equiv \rho^1 f^1(z^1)/\rho^0 f^0(z^1) \\ &= \rho^1 f^1(z^1)/\{\rho^0 f^0(z^0)[f^0(z^1)/f^0(z^0)]\} \\ &= P^1/\{P^0[f^0(z^1)/f^0(z^0)]\} \quad \text{using equation (22.16)} \\ &= [P^1/f^0(z^1)]/[P^0/f^0(z^0)]. \end{aligned} \tag{22.20}$$

The prices $P^1/f^0(z^1)$ and $P^0/f^0(z^0)$ can be interpreted as *quality-adjusted model prices* for the establishment in periods 1 and 0, respectively, using the hedonic regression pertaining to period 0 to do the quality adjustment.

[19]Assume that all ρ^t, $f^t(z)$, and $F^t(z, v^t)$ are positive for $t = 0, 1$.

22.44 It is possible to adapt a technique originally credited to Konüs (1924) and obtain a theoretical hedonic output price index that lies between the observable Laspeyres and Paasche bounding indices, P_{HL} and P_{HP}, defined above. Recall the definition of the revenue function, $R(\rho^s f^s, F^t, Z^t, v)$, from equation (22.9) above. Instead of using either F^0, Z^0, v^0 or F^1, Z^1, v^1 as reference production functions, feasible characteristics sets, and input vectors for the establishment in equation (22.12), use a *convex combination* or *weighted average* of these variables in our definition of a theoretical hedonic output price index. Thus, for each scalar λ between 0 and 1, define the theoretical hedonic output price index between periods 0 and 1, $P(\lambda)$, as follows:

$$\begin{aligned} P(\lambda) &\equiv R(\rho^1 f^1, (1-\lambda)F^0 + \lambda F^1, (1-\lambda)Z^0 + \lambda Z^1, (1-\lambda)v^0 + \lambda v^1)/R(\rho^0 f^0, (1-\lambda)F^0 + \lambda F^1, (1-\lambda)Z^0 + \lambda Z^1, (1-\lambda)v^0 + \lambda v^1) \\ &= \max_z\{\rho^1 f^1(z)[(1-\lambda)F^0(z, (1-\lambda)v^0 + \lambda v^1) + \lambda F^1(z, (1-\lambda)v^0 + \lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0 + \lambda Z^1\}/ \\ &\quad \max_z\{\rho^0 f^0(z)[(1-\lambda)F^0(z, (1-\lambda)v^0 + \lambda v^1) + \lambda F^1(z, (1-\lambda)v^0 + \lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0 + \lambda Z^1\}. \end{aligned} \tag{22.21}$$

When $\lambda = 0$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$, the Laspeyres-type hedonic output price index defined by equation (22.14) above. Thus, using the inequality in equation (22.14), we have

$$P(0) \geq P_{HL}, \tag{22.22}$$

where P_{HL} is equal to $\rho^1 f^1(z^0)/\rho^0 f^0(z^0)$, the observable Laspeyres hedonic output price index defined by equation (22.15) above. When $\lambda = 1$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$, the Paasche-type hedonic output price index defined by equation (22.18) above. Thus, using the inequality in equation (22.18), we have

$$P(1) \leq P_{HP}\, P_{HL}, \tag{22.23}$$

where P_{HP} is equal to $\rho^1 f^1(z^1)/\rho^0 f^0(z^1)$, the observable Paasche hedonic output price index defined by equation (22.20) above.

22.45 If $P(\lambda)$ is a continuous function of λ between 0 and 1, then we can adapt the proof of Diewert (1983a, pp. 1060–61), which in turn is based on a technique of proof by Konüs (1924), and show that there exists a λ^* such that $0 \leq \lambda^* \leq 1$ and either

$$P_{HL} \leq P(\lambda^*) \leq P_{HP} \text{ or } P_{HP} \leq P(\lambda^*) \leq P_{HL}, \tag{22.24}$$

that is, there exists a theoretical hedonic output price index between periods 0 and 1 using a technology that is intermediate to the technology of the establishment between periods 0 and 1, $P(\lambda^*)$, that lies *between* the observable[20] Laspeyres and Paasche hedonic output price indices, P_{HL} and P_{HP}. However, to obtain this result, we need conditions on the hedonic model price functions, $\rho^0 f^0(z)$ and $\rho^1 f^1(z)$, on the production functions, $F^0(z, v)$ and $F^1(z, v)$, and on the feasible characteristics sets, Z^0 and Z^1, that will ensure that the maximum functions in the numerator and denominator in the last equality of equation (22.21) are continuous in λ. Sufficient conditions to guarantee continuity are as follows:[21]

- The production functions $F^0(z, v)$ and $F^1(z, v)$ are positive and jointly continuous in z, v,
- The hedonic model price functions $f^0(z)$ and $f^1(z)$ are positive and continuous in z,
- ρ^0 and ρ^1 are positive, and
- The sets of feasible characteristics Z^0 and Z^1 are convex, closed, and bounded.

22.46 A theoretical output price index has been defined that is bounded by two observable indices. It is natural to take a symmetric mean of the bounds to obtain a best single number that will approximate the theoretical index. Thus, let $m(a, b)$ be a symmetric homogeneous mean of the two positive numbers a and b. We want to find a best $m(P_{HL}, P_{HP})$. If we want the resulting index, $m(P_{HL}, P_{HP})$, to satisfy the time reversal test, then we can adapt the argument of Diewert (1997, p. 138) and show that the resulting $m(a, b)$ must be the geometric mean, $a^{1/2} b^{1/2}$. Thus, a good candidate to best approximate a theoretical hedonic output price index is the following observable *Fisher hedonic output price index:*

$$\begin{aligned} P_{HF} &\equiv [P_{HL} P_{HP}]^{1/2} \\ &= [\rho^1 f^1(z^0)/\rho^0 f^0(z^0)]^{1/2} [\rho^1 f^1(z^1)/\rho^0 f^0(z^1)]^{1/2} \\ &\qquad \text{using equations (22.15) and (22.21)} \\ &= [\rho^1/\rho^0][f^1(z^0)/f^0(z^0)]^{1/2}[f^1(z^1)/f^0(z^1)]^{1/2}. \end{aligned} \tag{22.25}$$

Note that P_{HF} reduces to ρ^1/ρ^0 if $f^0 = f^1$, that is, if the hedonic model price functions are identical for each of the two periods under consideration, except for the proportional factors, ρ^1 and ρ^0.

22.47 Instead of using equations (22.15) and (22.17) in the first line of equation (22.7), equations (22.17) and (22.20) can be used. The resulting formula for the Fisher hedonic output price index is

$$\begin{aligned} P_{HF} &\equiv [P_{HL} P_{HP}]^{1/2} \\ &= \{[P^1/f^1(z^1)]/[P^0/f^1(z^0)]\}^{1/2} \{[P^1/f^0(z^1)]/ \\ &\qquad [P^0/f^0(z^0)]\}^{1/2}. \end{aligned} \tag{22.26}$$

Equation (22.26) is preferred. It is the geometric mean of two sets of quality-adjusted model price ratios, using the hedonic regression in each of the two periods to do one of the quality adjustments.

22.48 The above theory, for the quality adjustment of establishment output prices, is not perfect. It has two weak parts:

- Using a convex combination of the two reference period technologies may not appeal to everyone, and
- Our technique for converting the bounds to a single number is only one method out of many.

22.49 The initial Laspeyres-type bounds and Paasche-type bounds formalize the bounds outlined in Section C.5 below and referred to in Section C.2. The quality adjustments in equations (22.13) and (22.14) are seen from this approach and are to be made using the user's model valuation functions, $f^0(z)$ and $f^1(z)$. Producers' costs or production functions enter into the quality adjustment only to determine z^0 and z^1, that is, only to determine which models the establishment will produce. Hence, establishments that have different technologies or primary inputs or face different input prices will in general choose to produce different models in the same period. The choice problem has been modeled here facing only a single establishment, although the generalization should be straightforward.

B.7 Markups and imperfect competition

22.50 Section B.5 showed there was some ambiguity in the interpretation of hedonic coefficients. A user-value or resource-cost interpretation was possible if there was uniformity in buyer's tastes or suppliers' technologies, respectively. In Section B.6 an assumption of price-taking behavior on the part of firms was introduced and a formal setting given to a user-value

[20] We need estimates of the hedonic model price functions for both periods to implement these "observable" indices.

[21] The result follows using Debreu's (1952, pp. 889–90; and 1959, p. 19) maximum theorem.

interpretation, albeit involving some restrictive assumptions. Yet the approaches in Sections B.5 and B.6 both assume perfectly competitive behavior, and the discussion extends now to the effects of markups in imperfect competition. Feenstra (1995) noted that in imperfect competition, when pricing is above marginal cost, the hedonic function should include a term for the price-cost markup.

22.51 Pakes (2003) has developed the argument focusing on the study of new products as the result of prior investments in product development and marketing. A competitive marginal cost-pricing assumption would require that either (1) products with identical characteristics are developed from such investments, so that the law of one price for these identical products will eliminate any margin, or (2) all products lose their investment (markup) in the new products. Neither of these is reasonable. Indeed, varying markups are a feature of differentiated products (see Feenstra and Levinsohn, 1995, for example). Pakes (2003) argued that markups should change over time. When new products are introduced, the improvements and associated markups are directed to characteristics where markups have previously been high. The markups on existing products with these characteristics will fall, and hedonic coefficients will thus change over time. Pakes (2003) also argued that there may be an ambiguity as to the signs of the coefficients—that there is no economic reason to expect a positive relationship between price and a desirable characteristic. Such a conclusion would be at odds with a resource-cost or user-value approach. If the characteristics being compared are *vertical*—that is, they are characteristics that everyone would like more of—then we can expect the sign to be positive. However, Pakes (2003) has argued that the sign on *horizontal* characteristics—that is, for which the ordering of the desirable amounts of characteristics is not the same for all consumers—can be negative. The entry of new products aimed at some segments of the market may drive down the markup on products with more desirable attributes. For example, some consumers may have a preference for television sets with smaller screen sizes and be willing to pay a premium price. Indeed, the required technology for the production of these sets may have required increased investment and thus increased expected markups. It may be that the quality of the picture on these sets is such that it drives down the price of large-sized sets, resulting in an inverse relationship between price and screen size, where the latter is taken as one variable over the full range of screen sizes. Prior (to the modeling) information on the two markets would allow the regression equation to be appropriately specified, with dummy slope and intercepts for the ranges of screen sizes with new and old technologies.

22.52 Pakes (2003) took the view that no meaning can be attributed to estimated coefficients and predicted values should be used for price comparisons of models of different quality attributes, rather than the individual coefficients. There are many good reasons for this, as discussed in Chapter 8, Section E.4.3 and Section G.2.2, and Appendix 22.1 to this chapter. Yet, it must be stressed that for vertical characteristics the coefficients may be quite meaningful, and even for horizontal characteristics or new characteristics, embodied with the latest research and development, some sense can be made by recourse to the above considerations. But again, theory does not support any easy answer to the interpretation of the coefficients from hedonic regressions. Their relevance is that they emanate from market data, from the often complex interaction of demand and supply and strategic pricing decisions. That theory warns us not to give simplistic interpretations to such coefficients, and allows an understanding of the factors underlying them, is a strength. Yet the coefficients remain and are generally regarded (Schultze and Mackie, 2002) as the most promising objective basis for estimating the marginal value of quality dimensions of products, even though a purist interpretation is beyond their capability.[22]

C. Hedonic Indices

C.1 The need for such indices

22.53 In Section A it was noted that hedonic functions are required for two purposes with regard to a quality adjustment. The first is when an item is no longer produced and the replacement item, whose price is used to continue the series, is of a quality different from that of the item used for the original price basis. The differences in quality can be established in terms of different values of a subset of the z price-determining variables. The coefficients from the hedonic regressions, as estimates of the monetary value of additional units of each quality component z, can then be used to adjust the price of the old item so that it is comparable with the

[22]Diewert (2002f) went further in suggesting positive sign restrictions should be imposed on the coefficients in the econometric estimation, particularly when the hedonic regression is being used to adjust the price of a replacement item in order to make it comparable with the price of an item that has disappeared.

price of the new[23]—so that, again, like is compared with like. This process could be described as "patching," in that an adjustment is needed to the price of the old (or new replacement) series for the quality differences, to enable the new series to be patched onto the old. A second use of hedonic functions referred to in Section A is for estimating *hedonic indices*. These are suitable when the pace and scale of replacements of items is substantial and an extensive use of patching might (1) lead to extensive errors if there were some error or bias in the quality-adjustment process and (2) lead to sampling from a biased replacement universe as outlined in Section A. Hedonic indices use data in each period from a sample of items that should include those with substantial share of sales revenue—sampling in each period from the double universe. There is no need to establish a price basis and for respondents to keep quoting prices from that basis. What is required are samples of items to be redrawn in each month along with information on their prices, characteristics z_i, and, possibly, quantities or values. The identification of multiple characteristics in the hedonic regressions controls for quality differences, as opposed to the matching of price quotes on the same price basis by the respondents. A number of procedures for estimating hedonic indices are briefly considered below.

C.2 Theoretical characteristics price indices

22.54 In Chapter 18 theoretical output price indices were defined and practical index number formulas considered as estimates of these indices. Theoretical output index numbers are defined here not just on the goods produced, but also on their characteristics. $R(p, S(v))$ was defined in Chapter 18 as the maximum value of output that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v (using technology S) is available for use. The establishment's *output price index* P between any two periods, say period 0 and period 1, was defined as

$$P(p^0, p^1, v) = R(p^1, S(v))/R(p^0, S(v)), \qquad (22.27)$$

where p^0 and p^1 are the vectors of output prices that the establishment faces in periods 0 and 1, respectively, and $S(v)$ is a constant reference vector of technology using v intermediate and primary inputs.[24] For theoretical indices in characteristic space, the revenue functions are also defined over goods made up of tied bundles of characteristics, hedonic commodities, represented by the hedonic function[25]

$$P(p^0, p^1, v, z^0, z^1) = \frac{R(p^1, p(z^1), S(v))}{R(p^0, p(z^0), S(v))}. \qquad (22.28)$$

Note that the establishment faces prices p^1 for regular outputs and, from equation (22.6), the entire hedonic schedule of prices, $p^1(z) = \rho^1 f^1(z)$ for the hedonic commodity, and similarly for period 0. This schedule is a user valuation schedule and hence is exogenous to the establishment. The establishment then decides which model to produce in light of this schedule. Extending the framework in Section B.6 to include regular and hedonic commodities (equation 22.28) is an extension of definition (22.12), where the extension is that the period $t = 0$, 1 establishment production function is now $q^t = F^t(q, z, v)$, in place of the old equation (22.11): $q^t = F^t(z, v)$, where q^t is the hedonic commodity, q is a vector of "regular" commodities, z is the vector of characteristics for the hedonic commodity, v is a vector of inputs, and F^t is the production function.

22.55 The output price index defined by equation (22.28) is a ratio of hypothetical revenues that the establishment could realize, with a given technology and vector of inputs v to work with. Equation (22.28) incorporates substitution effects: If the prices of some characteristics increase more than others, then the revenue-maximizing establishment can switch its output mix of characteristics in favor of such characteristics. The numerator in equation (22.28) is the maximum revenue that the establishment could attain if it

[23] Various mechanisms for such adjustments are varied, as outlined in Chapter 8, Section E.4.3, and Triplett (2004). They include using the coefficients from the salient set of characteristics or using the predicted values from the regression as a whole and, in either case, making the adjustment to the old for comparison with the new, or to the new for comparison with the old, or some effective average of the two.

[24] This concept of the output price index (or a closely related variant) was defined by F.M. Fisher and Shell (1972, pp. 56–58), Samuelson and Swamy (1974, pp. 588–92), Archibald (1977, pp. 60–61), Diewert (1980, pp. 460–61; 1983a, p. 1055), and Balk (1998b, pp. 83–89). Readers who are familiar with the theory of the true cost-of-living index will note that the output price index defined by equation (17.2) is analogous to the true cost-of-living index, which is a ratio of cost functions, say $C(u, p1)/C(u, p0)$, where u is a reference utility level: R replaces C, and the reference utility level u is replaced by the vector of reference variables $S(v)$. For references to the theory of the true cost-of-living index, see Konüs (1924), Pollak (1983), or ILO and others (2004a), which is the consumer price index counterpart to this *Manual*.

[25] Triplett (1987) and Diewert (2002d), following Pollak (1975), consider a two-stage budgeting process whereby that portion of utility concerned with items defined as characteristics has its theoretical index defined in terms of a cost-minimizing selection of characteristics, conditioned on an optimum output level for composite and hedonic commodities. These quantities are then fed back into the second-stage overall revenue maximization.

faced the output prices and implicit hedonic shadow prices of period 1, p^1 and $p(z^1)$, while the denominator in equation (22.28) is the maximum revenue that the establishment could attain if it faced the output and characteristic's prices of period 0, p^0 and $p(z^0)$. Note that all the variables in the numerator and denominator functions are exactly the same, except that the output price and characteristics price vectors differ. This is a defining characteristic of an output price index: The technology and inputs are held constant. As with the economic indices in Chapter 16, there is an entire *family* of indices depending on which reference technology and reference input vector v is chosen. In Section C.5 some explicit formulations are considered, including a base-period 0 reference technology and inputs and a current-period 1 reference technology and inputs analogous to the derivation of Laspeyres and Paasche in Chapter 18, Section D.1. Before considering such hedonic indices in Section C.5, two simpler formulations are first considered in Sections C.3 and C.4: hedonic regressions using dummy variables on time and period-on-period hedonic indices. They are simpler and widely used because they require no information on quantities or weights. Yet, their interpretation from economic theory is therefore more limited. However, as is shown, weighted formulations are possible using a weighted least squares (WLS) estimator, although they are first considered in their unweighted form.

C.3 Hedonic regressions and dummy variables on time

22.56 Let there be K characteristics of a product, and let model or item i of the product in period t have the vector of characteristics $z_i^t \equiv [z_{i1}{}^t, \ldots, z_{iK}{}^t]$ for $i = 1, \ldots, K$ and $t = 1, \ldots, T$. Denote the price of model i in period t by $p_i{}^t$. A hedonic regression of the price of model i in period t on its characteristics set $z_i{}^t$ is given by

$$\ln p_i^t = \gamma_1 + \sum_{t=2}^{T} \gamma_t D_t + \sum_{k=1}^{K} \beta_k z_{ik}^t + \varepsilon_i^t, \qquad (22.29)$$

where D_t are dummy variables for the time periods, D_2 being 1 in period $t = 2$, zero otherwise; D_3 being 1 in period $t = 3$, zero otherwise, and so on. The coefficients γ_t are estimates of quality-adjusted price changes, having controlled for the effects of variation in quality (via $\sum_{k=2}^{K} \beta_k z_{ik}^t$)—although see Goldberger (1968) and Teekens and Koerts (1972) for the adjustment for estimation bias.

22.57 The above approach uses the dummy variables on time to compare prices in period 1 with prices in each subsequent period. In doing so, the γ parameters are constrained to be constant over the period $t = 1, \ldots, T$. Such an approach is fine retrospectively, but in real time the index may be estimated as a fixed-base or chained-base formulation. The *fixed-base* formulation would estimate the index for period 1 and 2, $I_{1,2}$, using equation (22.29) for $t = 1, 2$; the index for period 3, $I_{1,3}$, would use equation (22.29) for $t = 1, 3$; for period 4, $I_{1,4}$, using equation (22.29) for $t = 1, 4$; and so forth. In each case the index constrains the parameters to be the same over the current and base period. A fixed-base, bilateral comparison using equation (22.29) makes use of the constrained parameter estimates over the two periods of the price comparison. A *chained* formulation would estimate $I_{1,4}$, for example, as the product of a series of links: $I_{1,4} = I_{1,2} \times I_{2,3} \times I_{3,4}$.[26] Each successive binary comparison or link is combined by successive multiplication. The index for each link is estimated using equation (22.24). Because the periods of time being compared are close, it is generally more likely that the constraining of parameters required by chained-time dummy hedonic indices is considered to be less severe than that required of their fixed-base counterparts.

22.58 There is no explicit weighting in these formulations, and this is a serious disadvantage. In practice, cutoff sampling might be employed to include only the most important items. If sales data are available, a WLS (weighted by relative sales shares—see Appendix 22.1 and Diewert (2005b)) estimator should be used instead of an ordinary least squares (OLS) estimator.[27] A WLS estimator is equivalent to replicating the sample in proportion to the weights and applying an OLS estimator.

C.4 Period-on-period hedonic indices

22.59 An alternative approach to comparing prices in period 0 and 1 is to estimate a hedonic regression for period 1 and insert the values of the characteristics of each model existing in period 0 into the period 1 regression to predict, for each item, its price $\hat{p}_i{}^1(z_i{}^0)$. This would generate predictions of the price of items existing in period 0, at period 1 shadow prices, $\hat{p}_i{}^t(z_i{}^1)$, $i = 1, \ldots, N$. These prices (or an average) can be compared with (the average of) the actual prices of models $i = 1, \ldots, N$ models in period 0. The averages may

[26]Chapter 16, Section F, contains a detailed account of chained indices.

[27]Ioannidis and Silver (1999) and Bodé and van Dalen (2001) compared the results from these different estimators, finding notable differences, but not in all cases (see also Heravi and Silver, 2007).

be arithmetic, as in a Dutot index, or geometric, as in a Jevons index. The arithmetic formulation is defined as follows:

$$\frac{\sum_{i=1}^{N}(1/N)\hat{p}_i^1\left(z_i^0\right)}{\sum_{i=1}^{N}(1/N)p_i^0\left(z_i^0\right)}. \tag{22.30a}$$

22.60 Alternatively, the characteristics of models existing in period 1 can be inserted into a regression for period 0. Predicted prices of period 1 items generated at period 0 shadow prices (or an average) can be compared with (the average of) the actual prices in period t:

$$\frac{\sum_{i=1}^{N}(1/N)p_i^1\left(z_i^1\right)}{\sum_{i=1}^{N}(1/N)\hat{p}_i^0\left(z_i^1\right)}. \tag{22.30b}$$

22.61 For a fixed-base bilateral comparison using either equation (22.30a) or (22.30b), the hedonic equation need be estimated only for one period. The denominator in equation (22.30a) is the average observed price in period 0, which should be equal to the average price that a hedonic regression based on period 0 data will predict using period 0 characteristics. The numerator, however, requires an estimated hedonic regression to predict period 0 characteristics at period 1 hedonic prices. Similarly, in equation (22.30b), a hedonic regression is required only for the denominator. For reasons analogous to those explained in Chapters 16, 17, and 18, a symmetric average of these indices should have some theoretical support.

22.62 Note that all the indices described in Sections C.1 and C.2 use all the data available in each period. If there is a new item, for example, in period 4, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old items drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure, unlike using matched data and hedonic adjustments on noncomparable replacements when items are no longer produced.

22.63 As with the dummy variable approach, there is no need for matched data. Yet there is also no explicit weighting in these formulations and this is a serious disadvantage. Were data on quantities or values available, it is immediately apparent that such weights could be attached to the individual $i = 1, \ldots, N$ prices or their estimates. This is considered in the next section.

C.5 Superlative and exact hedonic indices

22.64 In Chapter 18, Laspeyres and Paasche bounds were defined on a theoretical basis, as were superlative indices, which treat both periods symmetrically. These superlative formulas, in particular the Fisher index, were also seen in Chapter 16 to have desirable axiomatic properties. Furthermore, the Fisher index was supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds and was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index seemed to be best from the stochastic viewpoint and also did not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices were found to correspond to (be *exact* for) underlying (Leontief) aggregator functions with no substitution possibilities while superlative indices were exact for flexible functional forms including the quadratic and translog forms for the Fisher and Törnqvist indices, respectively. If data on prices, characteristics, *and quantities* are available, analogous approaches and findings arise for hedonic indices (see Fixler and Zieschang, 1992, and Feenstra, 1995). Exact bounds on such an index were defined by Feenstra (1995). Consider the theoretical index in equation (22.28), but now defined only over items in terms of their characteristics. The prices are still of items, but they are wholly defined through $p(z)$. An arithmetic aggregation for a linear hedonic equation finds a Laspeyres lower bound (as quantities supplied are *increased* with increasing relative prices) is given by

$$\frac{R(p(z^1), S(v^0))}{R(p(z^0), S(v^0))} \geq \frac{\sum_{i=1}^{N} x_i^0\, \hat{p}_i^1}{\sum_{i=1}^{N} x_i^0\, p_i^0} = \sum_{i=1}^{N} s_i^0 \left(\frac{\hat{p}_i^1}{p_i^0}\right), \tag{22.31a}$$

where $R(.)$ denotes the revenue at a set of output prices, input quantities, v, and technology, S, following the fixed input-output price index model. The price comparison is evaluated at a fixed level of period 0 technology and inputs. s_i^0 are the shares in total value of output of product i in period 0, where $s_i^0 = x_i^0 p_i^0 / \sum_{i=1}^{N} x_i^0 p_i^0$, and

$$\hat{p}_i^1 \equiv p_i^1 - \sum_{k=1}^{K} \beta_k^1\left(z_{ik}^1 - z_{ik}^0\right), \tag{22.31b}$$

are prices in period 1 adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. As noted in Appendix 22.1, β_k^1 may be estimated using a WLS estimator where the weights are the sales quantities. The summation is over the same i in both periods, because replacements are included when items are missing and equation (22.31b) adjusts their prices for quality differences.

22.65 A Paasche upper bound is estimated as

$$\frac{R(p_1, S(v^1))}{R(p_0, S(v^1))} \leq \frac{\sum_{i=1}^{N} x_i^1 p_i^1}{\sum_{i=1}^{N} x_i^1 p_i^0} = \left[\sum_{i=1}^{N} s_i^1 \left(\frac{p_i^1}{\hat{p}_i^0}\right)^{-1}\right]^{-1} \tag{22.32a}$$

where $s_i^1 = x_i^1 p_i^1 / \sum_{i=1}^{N} x_i^1 p_i^1$, and

$$\hat{p}_i^0 \equiv p_i^0 + \sum_{k=1}^{K} \beta_k^0 (z_{ik}^1 - z_{ik}^0), \tag{22.32b}$$

which are prices in period 0 adjusted for the sum of the changes in each quality characteristic weighted by its respective coefficients derived from a linear hedonic regression.

22.66 These inequalities follow from the inequalities derived in Chapter 18, where the Laspeyres P_L and Paasche P_P form bounds on their true, economic theoretic indices:

$$P_L \leq P(p^0, p^1, \alpha) \leq P_P \text{ or } P_L \leq P(p^0, p^1, \alpha) \geq P_P. \tag{22.33}$$

22.67 The superlative and exact hedonic index approach thus first applies the coefficients from hedonic regressions to changes in the characteristics to adjust observed prices for quality changes equations (22.31b and 22.32b). Second, it incorporates a weighting system using data on the value of output of each model and its characteristics, rather than treating each model as equally important equations (22.31a and 22.32a). Finally, it has a direct correspondence to formulation defined from economic theory.

22.68 Semilogarithmic hedonic regressions would supply a set of β coefficients suitable for use with these base-period and current-period geometric bounds:

$$\prod_{i=1}^{N} \left(\frac{p_i^1}{\hat{p}_i^0}\right)^{s_i^1} \geq \frac{R(p(z^1), q, T)}{R(p(z^0), q, T)} \geq \prod_{i=1}^{N} \left(\frac{\hat{p}_i^1}{p_i^0}\right)^{s_i^0} \tag{22.34a}$$

$$\hat{p}_i^0 \equiv p_i^0 \exp\left[\sum_{k=1}^{K} \beta_k^0 (z_{ik}^1 - z_{ik}^0)\right] \text{ and}$$

$$\hat{p}_i^1 \equiv p_i^1 \exp\left[-\sum_{k=1}^{K} \beta_k^1 (z_{ik}^1 - z_{ik}^0)\right]. \tag{22.34b}$$

22.69 In equation (22.34a) the two bounds on their respective theoretical indices have been shown to be brought together. The calculation of such indices is no small task. For examples see Silver and Heravi (2001a and 2003) and Chapter 8, Section G.2, for comparisons over time; Kokoski, Moulton, and Zieschang (1999) for price comparisons across areas of a country; and Heravi, Heston, and Silver (2003) for comparisons across countries.

22.70 The methods outlined above show how practical hedonic indices relate to theoretical counterparts. There are many more variants of such practical formulas, some of which are outlined in Chapter 8. Their nature depends on the approach adopted, time dummy or period-on-period indices, whether the indices are fixed base or chained, whether geometric, arithmetic, or harmonic aggregators are used, and whether base-period, current-period, or some average of the two period's weights are used. Heravi and Silver (2007b) explored such differences in a meta analysis of the results of a whole variety of such measures.

C.6 The difference between the period-on-period and time dummy approaches

22.71 The dummy variable method outlined in Section C.3 and the period-on-period hedonic indices, outlined in Sections C.4 and C.5—also referred to as "hedonic imputation indexes" by Diewert, Heravi and Silver (2009) and as "characteristic price index numbers" by Triplett (2004)—not only correct price changes for changes in the quality of items purchased, but also allow the indices to incorporate matched and unmatched models. They provide a means by which price changes can be measured in product markets where there is a rapid turnover of differentiated models. However, they can yield quite different results. Diewert, Heravi, and Silver (2009) provided a formal exposition of the factors underlying such differences and the implications for choice of method. This was undertaken for the Törnqvist index, but the analysis can be readily extended to other formulas. They found that differences between the two approaches may arise from both parameter instability over the two periods compared and changes over the two periods compared in the characteristics of the models sold, and that such differences are compounded when both such changes occur. They further showed that similarities between the two approaches resulted if there was little difference in either component change.

22.72 The above in Section C has illustrated how weighted index number formulas might be constructed using data on prices, quantities, and characteristics for an item when the data are not matched. But for analytical purposes it is useful to decompose price changes into those due to matched price changes, those due to unmatched new models introduced, and those due to unmatched old models that are retired. The analysis is useful for determining the bias in just using matched models.

C.7 Decomposing price changes into matched and unmatched components

22.73 Following Silver and Heravi (2005) the hedonic formulation in equation (22.29) is used to derive the basic matched-model result for hedonic time dummy indices over two periods, originally developed by Triplett and McDonald (1977). However, we reformulate equation (22.29) as

$$\ln p_m^t = \sum_{t=1}^{T} \alpha^t D^t + \sum_{k=1}^{K} z_{mk}^t \beta_k + \varepsilon_m^t; \quad m \in S(t);\ t = 1, 2, \ldots, T, \qquad (22.35)$$

where $S(t)$ is the set of models available in period t, p_{tm} is the period t price of model m, D_t is a time dummy variable that is 1 if the left-hand side observation is the log of a period t price and is 0 otherwise, z_{mk}^t is the amount of characteristic k model m in period t possesses, and ε_m^t is an error term. Let the number of models available in period t be $N(t)$; that is, there are $N(t)$ models in the set $S(t)$ for each t. The coefficients α^t and β_k are typically estimated using least squares. It should be mentioned that there is no constant term in equation (22.35); rather, there is a time dummy for every period. It is straightforward to show that this specification is equivalent to the usual hedonic model with time dummies that has a constant term. α_t is an estimate of the (logarithm of the) average price of models in period t having controlled for the z_{mk}^t characteristics (though see Teekens and Koerts (1972) for an adjustment). Consider a special case of the general equation (22.35) in which there are only two periods so $T = 2$ and assume that the models are matched in each of the two periods so that $S(1) = S(2)$ and $N(1) = N(2) \equiv M$ so that the same M models are available in each period. Hence the model characteristics are the same in each; that is, we have

$$z_{mk}^t = z_{mk} \text{ say, for } t = 1, 2,\ m = 1, \ldots, M \text{ and } k = 1, \ldots, K. \qquad (22.36)$$

With these restrictions the least squares estimates for the unknown parameters in equation (22.35) are denoted by α^{1*} and α^{2*} and β_k^* for $k = 1, \ldots, K$.

22.74 Define *price levels* for periods 1 and 2, P^1 and P^2 respectively, in terms of the least squares estimates for α^1 and α^2 as follows:

$$\ln P^1 \equiv \alpha^{1*};\ \ln P^2 \equiv \alpha^{2*}. \qquad (22.37)$$

Hence the logarithm of the *price index going from period 1 to 2* is defined as

$$\ln P^2/P^1 \equiv \alpha^{2*} - \alpha^{1*}. \qquad (22.38)$$

22.75 A property of least squares regression estimates is that the column vector of least squares residuals is orthogonal to each column vector of exogenous variables (this follows a technique of proof used by Diewert (2001a)). Using this property for the first two columns of exogenous variables corresponding to the time dummy variables leads to the following two equations (using equation (22.36) as well):

$$\sum_{m=1}^{M} \ln p_m^1 = M\,\alpha^{1*} + \sum_{m=1}^{M}\sum_{k=1}^{K} z_{mk}\,\beta_k^*; \qquad (22.39)$$

$$\sum_{m=1}^{M} \ln p_m^2 = M\,\alpha^{2*} + \sum_{m=1}^{M}\sum_{k=1}^{K} z_{mk}\,\beta_k^*. \qquad (22.40)$$

Divide both sides of equations (22.39) and (22.40) by M and solve the resulting equations for the least squares estimates, α^{1*} and α^{2*}. Substituting these expressions for α^{1*} and α^{2*} into equation (22.38) leads to the following formula for the log of the *hedonic price index:*

$$\ln P^2/P^1 = \alpha^{2*} - \alpha^{1*} = (1/M) \sum_{m=1}^{M} \ln\left[p_m^2/p_m^1\right]. \qquad (22.41)$$

Exponentiating both sides of equation (22.41) shows that the hedonic model price index going from period 1 to 2 under the above matched-model conditions is equal to *the equally weighted geometric mean of the* M *model price relatives,* which would be a conventional matched-model statistical agency estimate of the price index for this elementary group of commodities.

22.76 Now let us relax the matched-model restriction, but still assume that $T = 2$, that is, that there are only two periods in the hedonic regression model defined by equation (22.35). Some additional notation is required in order to model this case. Define the following sets of models:

$$S(1 \cap 2) \equiv S(1) \cap S(2); \quad (22.42)$$

$$S(1 \neg 2) \equiv S(1) \neg S(2); \quad (22.43)$$

$$S(2 \neg 1) \equiv S(2) \neg S(1). \quad (22.44)$$

Thus $S(1 \cap 2)$ is the set of models that are present in both periods 1 and 2, $S(1 \neg 2)$ is the set of models that are present in period 1 but not period 2, and $S(2 \neg 1)$ is the set of models that are present in period 2 but not period 1. Let the number of models in the sets $S(1 \cap 2)$, $S(1 \neg 2)$, and $S(2 \neg 1)$ be denoted by $N(1 \cap 2)$, $N(1 \neg 2)$, and $N(2 \neg 1)$, respectively. Relating our new notation to the total number of models in periods 1 and 2, $N(1)$ and $N(2)$, respectively, it can be seen that

$$N(1) = N(1 \cap 2) + N(1 \neg 2) \text{ and} \quad (22.45)$$

$$N(2) = N(1 \cap 2) + N(2 \neg 1). \quad (22.46)$$

22.77 The least squares estimates for the equation defined by equation (22.35) when $T = 2$ can now be obtained. Again recalling that the column vector of least squares residuals is orthogonal to each column vector of exogenous variables, we obtain the following two equations, where this orthogonality property was used for the first two columns of the exogenous variables corresponding to the time dummy variables:

$$\sum_{m\in S(1\cap 2)} \ln p_m^1 + \sum_{m\in S(1\neg 2)} \ln p_m^1 = N(1 \cap 2)\,\alpha^{1*} + N(1 \neg 2)\alpha^{1*} + \sum_{m\in S(1\cap 2)} \sum_{k=1}^{K} z_{mk}^1 \beta_k^* + \sum_{m\in S(1\neg 2)} \sum_{k=1}^{K} z_{mk}^1 \beta_k^*; \quad (22.47)$$

$$\sum_{m\in S(1\cap 2)} \ln p_m^2 + \sum_{m\in S(2\neg 1)} \ln p_m^2 = N(1 \cap 2)\alpha^{2*} + N(2 \neg 1)\alpha^{2*} + \sum_{m\in S(1\cap 2)} \sum_{k=1}^{K} z_{mk}^2 \beta_k^* + \sum_{m\in S(2\neg 1)} \sum_{k=1}^{K} z_{mk}^2 \beta_k^*. \quad (22.48)$$

22.78 If equations (22.47) and (22.48) are divided by the number of common models in the two periods, $N(1 \cap 2)$, expressions for α^{1*} and α^{2*} can be obtained. Substituting these expressions into (22.38) and using $z_{mk}^1 = z_{mk}^2$ for the common models $m \in S(1 \cap 2)$ leads to the following formula for the log of the *hedonic price index*:

$$\ln P^2/P^1 = \alpha^{2*} - \alpha^{1*} - \frac{1}{N(1 \cap 2)} \sum_{m\in S(1\cap 2)} \ln\left[p_m^2/p_m^1\right] + \frac{1}{N(1 \cap 2)} \sum_{m\in S(2\neg 1)} \left[\ln p_m^2 - \sum_{k=1}^{K} z_{mk}^2 \beta_k^* - \alpha^{2*}\right] - \frac{1}{N(1 \cap 2)} \sum_{m\in S(1\neg 2)} \left[\ln p_m^1 - \sum_{k=1}^{K} z_{mk}^1 \beta_k^* - \alpha^{1*}\right]. \quad (22.49)$$

22.79 The first set of terms on the right-hand side of equation (22.49) is the *matched-model contribution* to the overall index, $\ln P^2/P^1$. The next two set of terms are respectively the change in price owing to unmatched models existing in period 2, but not in 1, and unmatched models existing in period 1 but not in 2. These expressions are not captured in a matched-models index. If the second set of terms, $\frac{1}{N(1 \cap 2)}\sum_{m\in S(2\neg 1)} \left[\ln p_m^2 - \sum_{k=1}^{K} z_{mk}^2 \beta_k^* - \alpha^{2*}\right]$, is positive, then the matched-model price index is too low and must be adjusted upward. Consider a new model m introduced in period 2. If (the logarithm of) its price, $\ln p_m^2$, is above that predicted from a period 2 hedonic regression $\left(\sum_{k=1}^{K} z_{mk}^2 \beta_k^* - \alpha^{2*}\right)$, then this will raise the overall price index and a matched-model index would be too low if it ignored such new models (Triplett and McDonald (1977) have a similar interpretation). Similarly, consider the last set of terms in equation (22.49) and an unmatched old model, introduced in period 1 but no longer available in period 2. If it was priced in period 1 above its period 1 predicted price then the matched-model price index would be too high (note the negative sign). The extent and nature of the bias depend on the pricing strategy of new and old models.

22.80 The extent of any difference depends, in this unweighted formulation, on the proportions of old and new items leaving and entering the sample and on the price changes of old and new items relative to those of matched items. If the market for commodities is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes (see Silver and Heravi, 2005; and Berndt, Ling, and Kyle, 2003, for examples). Different market behavior will lead to different forms of bias. The above expression is for unweighted price changes, but the principles extend to similar findings for weighted price changes and, by association, weighted index numbers, as shown in Silver and Heravi (2005). As noted in the appendix to this chapter, and argued in Diewert (2005b), different weighting systems in a WLS hedonic regression correspond to different index number formulas.

D. New Goods and Services

22.81 This section briefly highlights issues relating to the incorporation of new goods into the index. Practical issues were outlined in Chapter 9, Section D.3. The term "new goods" includes services and is used here to refer to those that provide a substantial and substantive change in what is provided. This is as opposed to more of a currently available set of service flows, such as a new model of an automobile that has a bigger engine. In this instance, there is a continuation of a service and production flow, and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of new goods as opposed to quality changes is that the former cannot be easily linked to existing items as a continuation of an existing resource base and service flow, because of the very nature of their "newness." There are alternative definitions; Oi (1997) directed the problem of defining new goods to that of defining a monopoly. If there is no close substitute, the good is new. A monopoly supplier may be able to supply an item with new combinations of the hedonic z characteristics because of a new technology and have a monopoly power in doing so, but in practice the new good can be linked via the hedonic characteristics set to the existing ones. In this practical sense, such goods are not considered new for the purposes of the *Manual*.

22.82 Merkel took a similar practical line in devising a classification scheme that will meet the practical needs of XMPI compilation. He considered *evolutionary* and *revolutionary* goods. The former are defined as

> extensions of existing goods. From a production inputs standpoint, evolutionary goods are similar to pre-existing goods. They are typically produced on the same production line and/or use largely the same production inputs and processes as pre-existing goods. Consequently, in theory at least, it should be possible to quality adjust for any differences between a pre-existing good and an evolutionary good. (2000, p. 6)

22.83 In contrast, revolutionary goods are goods that are substantially different from preexisting goods. They are generally produced on entirely new production lines or with substantially new production inputs and processes in comparison with those used to produce preexisting goods. These differences make it virtually impossible, from both a theoretical and practical standpoint, to quality adjust between a revolutionary good and any preexisting good. The main concern regarding the incorporation of new goods into the XMPIs is the decision on the need and timing for their inclusion. Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new products may lead to errors in the measurement of price changes if the unusual price movements at critical stages in the product life cycles are ignored. There are practical approaches to the early adoption of both evolutionary and revolutionary goods. These are outlined in Chapter 9, Section D.3. For evolutionary goods, such strategies include the rebasing of the index, resampling of items, and introduction of new goods as directed *sample substitutions* (Merkel, 2000). Also of use are hedonic quality adjustments and indices outlined in Chapter 8, Section E.4, and Section C above that facilitate the incorporation of such evolutionary goods, because they possess a characteristics set similar to existing ones but deliver different quantities of these characteristics. The modified short-run or chained framework outlined in Chapter 8, Sections H through G, may also be more appropriate for product areas with high turnover of items. These approaches can incorporate the price change of new goods into the index as soon as prices are available for two successive periods, although issues relating to the proper weighting of such changes may remain.

22.84 However, for revolutionary goods, substitution may not be appropriate. First, they may not be able to be defined within the existing classification systems. Second, they may be primarily produced by a new establishment, which will require extending the sample to such establishments. Third, there will be no previous items to match against and make a quality adjustment to prices, because by definition, they are substantially different from preexisting goods. And, finally, there is no weight to attach to the new establishment or item(s). *Sample augmentation* is appropriate for revolutionary goods, whereas sample substitution is appropriate for evolutionary goods. It is necessary to bring the new revolutionary goods into the sample in addition to what exists. This may involve extending the classification, the sample of establishments, and item list within new or existing establishments (Merkel, 2000).

Appendix 22.1 Some Econometric Issues

22.85 Hedonic regression estimates have been seen in Chapter 8 to have potential use for the quality adjustment of prices. There are a number of issues that arise from the specification and estimation of hedonic regressions, the use of diagnostic statistics, and courses of action when the standard ordinary least squares (OLS) assumptions are seen to break down. Many of these issues are standard econometric ones and not the subject of this *Manual*. This is not to say they are unimportant. The use of hedonic regressions will require some econometric or statistical expertise, but suitable texts are generally available. See Berndt (1991)—particularly the chapter on hedonic regressions—and Maddala (1988) and Kennedy (2003), among many others. Modern statistical and econometric software has adequate diagnostic tests for testing when OLS assumptions break down. There remain, however, some specific issues that merit attention, although it must be stressed that these points are over and above, and should not be taken to diminish, the important standard econometric issues found in econometric texts.

Identification and appropriate estimators

22.86 Wooldridge (1996, pp. 400–01) has shown on standard econometric grounds that the estimation of supply and demand functions by OLS is biased *and this bias carries over to the estimation of the hedonic function*. It is first useful to consider estimation issues in the supply and demand functions. These functions are rarely estimated in practice. The more common approach is to estimate offer functions, with the marginal price offered by the firm dependent upon chosen attributes (product characteristics) and firm characteristics, and to estimate *bid* or value functions, with the marginal prices paid by a consumer dependent on chosen attributes and consumer characteristics.[28] As noted earlier, the observed prices and quantities are the result of the interaction of structural demand and supply equations and the distributions of producer technologies and consumer tastes and cannot reveal the parameters of these offer and value functions. Rosen (1974, pp. 50–51) suggested a procedure for determining these parameters. Because these estimates are conditioned on tastes (α) and technologies (τ), the estimation procedure needs to include empirical measures or "proxy variables" of α and τ. For the tastes α of consumers, the empirical counterparts may be sociodemographic and economic variables, which may include age, income, prices, and quantities of nonhedonic commodities demanded by households,[29] education, and geographical region. For technologies τ, variables may include technologies and factor prices. First, the hedonic equation is estimated without these variables in the normal manner using the best-fitting functional form. This is to represent the price function consumers and producers face when making their decisions. Then, an implicit marginal price function is computed for each characteristic as $\partial p(z)/\partial z_i = \hat{p}_i(z)$, where $\hat{p}(z)$ is the estimated hedonic equation. Bear in mind that in normal demand and supply studies for *products,* the prices are observed in the market. For *characteristics* they are unobserved, and this first stage must be to estimate the parameters from the hedonic regression. The actual values of each z_i bought and sold are then inserted into each implicit marginal price function to yield a numerical value for each characteristic. These marginal values are used in the second stage[30] of estimation as endogenous variables for the estimation of the demand side:

$$\hat{p}_i(z) = G(z1, \ldots, zK, \alpha^*), \qquad \text{(A22.1)}$$

where α^* are the proxy variables for tastes.

22.87 The supply side estimating equations might look like

$$\hat{p}_i(z) = G^*(z1, \ldots, zK, \tau^*), \qquad \text{(A22.2)}$$

where τ^* are the proxy variables for technologies.

22.88 The variables τ^* drop out when there is no variation in technologies and $\hat{p}_i(z)$ is an estimate of the offer function. Similarly, the variables α^* drop out when sellers differ and buyers are identical and cross-section estimates trace out compensated demand functions.

22.89 Epple (1987) has argued that Rosen's modeling strategy is likely to give rise to inappropriate estimation procedures of the demand and supply parameters. The hedonic approach to estimating the demand for characteristics has a difficulty arising from the fact that marginal prices are likely to be endogenous—they depend on the amount of each characteristic consumed and must be estimated from the hedonic function rather than observed directly. There are two resulting problems. First, there is an identification problem (see Epple, 1987) because both the marginal price of a characteristic and the inverse bid depend on the levels of characteristics consumed.

[28]These are equivalent to inverse demand (supply) functions, with the prices dependent upon the quantities demanded (supplied) and the individual consumer (producer) characteristics.

[29]The consumer theory approach used by Diewert (2003) to derive the hedonic function rested on rather strong separability assumptions on consumer preferences. Once these separability assumptions are relaxed, the demand for nonhedonic commodities will provide a means for identification of the hedonic preferences.

[30]This two-stage approach is common in the literature, though Wooldridge (1996) discussed the joint estimation of the hedonic and demand and supply side functions as a system.

Second, if important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients on measured characteristics will be biased. This applies to all econometric models, but it is particularly relevant to hedonic models; on this point see Wooldridge (1996, pp. 400–01). The equilibrium conditions for characteristic prices imply functional relationships among the characteristics of demanders, suppliers, and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with the included variables of the model (see also Bartik, 1988, on this point). The bias arises because buyers are differentiated by characteristics (y, α) and sellers by technologies τ. The type of item buyers will purchase is related to (y, α) and the type sellers provide to τ. On the plane of combinations of z transacted, the equilibrium ones chosen may be systematically related; the characteristics of buyers are related to those of sellers. Epple (1987) uses the example of stereo equipment: The higher income of some buyers leads to purchases of high-quality equipment and the technical competence of sellers leads them to provide it. The consumer and producer characteristics may be correlated.

22.90 Wooldridge (1996, pp. 400–01) suggested that individual consumer and firm characteristics such as income, education, and input prices should be used as instruments in estimating hedonic functions. In addition, variables other than a good's characteristics should be included as instruments if they are price determining, such as geographical location—say proximity to ports, good road systems, climate, and so on. Communities of economic agents are assumed, within which consumers consume and producers produce for each other at prices that vary across communities for identical goods. Variables on the characteristics of the communities will not in themselves enter the demand and supply equation but are price determining for observed prices recorded across communities. Tauchen and Witte (2001) provided a systematic investigation of the conditions under which consumer and producer and community characteristics will affect the hedonic parameter estimates for a single-regression equation estimated across all communities. A key concern is whether the hedonic price function error term represents factors that are unobserved by both the economic agents and the researcher, or by the researcher only. In the latter case the error term may be correlated with the product attributes and instrumental variable estimation is required. If the error term is *not* correlated with the product characteristics—preferences are quasi-linear—then a properly specified hedonic regression, including community-specific characteristics or appropriate slope dummies, can be estimated using OLS. In other cases, depending on the correlation between consumer and producer characteristics, assumptions about the error term, and the method of incorporating community characteristics into the regression, instrumental variables, including consumer or producer or community dummy or characteristics, may need to be used.

Functional form

22.91 Triplett (1987 and 2004) argued that neither classical utility theory nor production theory can specify the functional form of the hedonic function.[31] This point dates back to Rosen (1974) who described the observations as being "a joint-envelope function and cannot by themselves identify the structure of consumer preferences and producer technologies that generate them" (p. 54). A priori judgments about what the form should look like may be based on ideas about how consumers and production technologies respond to price changes. These judgments are difficult to make when the observations are jointly determined by demand and supply factors but impossible only in rare instances. However, it is complicated when pricing is with a markup, the extent of which may vary over the life cycle of a product. Some tied combinations of characteristics will have higher markups than others. New item introductions are likely to be attracted to these combinations, and this will have the effect of increasing supply and thus lowering the markup and price (Cockburn and Anis, 1998; Feenstra, 1995, p. 647; and Triplett, 1987, p. 38). This again must be taken into account in any a priori reasoning—not an easy or straightforward matter.

22.92 It may be that in some cases the hedonic function's functional form will be very straightforward. For example, prices on the websites for options for products are often additive. The underlying cost and utility structure is unlikely to jointly generate such linear functions, but the producer or consumer is also paying for the convenience of selling in this way and is willing to bear losses or make gains if the cost or utility at higher values of z is priced lower/worth more than the price set. But, in general, the data should convey what the functional form should look like, and imposing artificial structures simply leads to specification bias. For examples of econometric testing of hedonic functional form, see Cassel and Mendelsohn (1985);

[31] Arguea, Hsiao, and Taylor (1994) proposed a linear form on the basis of arbitrage for characteristics, held to be likely in competitive markets, although Triplett (2004) argued that this is unlikely to be a realistic scenario in most commodity markets.

Cropper, Deck, and McConnell (1988); Rasmussen and Zuehlke (1990); Bodé and van Dalen (2001); and Curry, Morgan, and Silver (2001).

22.93 The three forms prevalent in the literature are linear, semilogarithmic, and double-logarithmic (log-log). A number of studies have used econometric tests, in the absence of a clear theoretical statement, to choose between them. There have been a large number of hedonic studies and, as illustrated in Curry, Morgan, and Silver (2001), in many of these the quite simple forms do well, at least in terms of the $\bar{R}^2$ presented, and the parameters accord with a priori reasoning, usually on the consumer side. Of the three popular forms some are favored in testing. For example, Murray and Sarantis (1999) favored the semilogarithmic form, while in others—for example, Hoffmann (1998)—the three functional forms were found to scarcely differ in terms of their explanatory power. That the parameters from these simple forms accord with a priori reasoning, usually from the consumer side, is promising, but researchers should be aware that such matters are not assured. Of the three forms, the semilogarithmic form has much to commend it. The interpretation of its coefficients is quite straightforward—the coefficients represent proportionate changes in prices arising from a unit change in the value of the characteristic.[32] This is a useful formulation because quality adjustments are usually undertaken by making multiplicative instead of additive adjustments (see Chapter 8, Section C.3). The semilogarithmic form, unlike the log-log model, can also incorporate dummy variables for characteristics that are either present, $z_i = 1$, or not, $z_i = 0$.[33]

22.94 More complicated forms are possible. Simple forms have the virtue of parsimony and allow more efficient estimates to be made for a given sample. However, parsimony is not something to be achieved at the cost of misspecification bias. First, if the hedonic function is estimated across multiple independent markets, then interaction terms are required (see Mendelsohn, 1984, for fishing sites). Excluding them is tantamount to omitting variables and inappropriately constraining the estimated coefficients of the regression. Tauchen and Witte (2001) have outlined the particular biases that can arise from such omitted variables in hedonic studies. Second, it may be argued that the functional form should correspond to the aggregator for the index—linear for a Laspeyres index, logarithmic for a geometric Laspeyres index, translog for a Törnqvist index, and quadratic for a Fisher index (see Chapter 18). However, as Triplett (2004) noted, the purpose of estimating hedonic regressions is to adjust prices for quality differences, and imposing a functional form on the data that is inconsistent with the data might create an error in the quality adjustment procedure. Yet, as Diewert (2003) noted, flexible functional forms encompass these simple forms. The log-log form is a special case of the translog form as in equation (18.12), and the semi-log form is a special case of the semi-log quadratic form as in equation (18.38). If there are a priori reasons to expect interaction terms for specific characteristics, as illustrated in the example in Chapter 8, Section E.4, then these more general forms allow this, and the theory of hedonic functions neither dictates the form of the hedonic form nor restricts it.

Changing tastes and technologies

22.95 The estimates of the coefficients may change over time. Some of this can be attributed to sampling error, especially if multicollinearity is present, as discussed below. But, in other cases, it may be a genuine reflection of changes in tastes and technologies. If a subset of the estimated coefficients from a hedonic regression is to be used to quality adjust a noncomparable replacement price, then the use of estimated out-of-date coefficients from some previous period to adjust the prices of the new replacement model would be inappropriate. There would be a need to update the indices as regularly as the changes demanded.[34] For estimating hedonic indices, the matter is more complicated. The coefficients in a simple dummy time-period model as in Section C.3 of this chapter now have different estimates of the parameters in each period. Silver (1999), using a simple example, showed how the estimate of quality-adjusted price change from such a dummy variable

[32] It is noted that the anti-log of the OLS-estimated coefficients is not unbiased—the estimation of semilogarithmic functions as transformed linear regressions requires an adjustment to provide minimum-variance unbiased estimates of parameters of the conditional mean. A standard adjustment is to add one-half of the coefficient's squared standard error to the estimated coefficient (Goldberger, 1968; and Teekens and Koerts, 1972).

[33] Diewert (2002f) argued against the linear form on the grounds that, while the hedonic model is linear, the estimation required is of a nonlinear *regression* model, and the semi-log and log-log models are linear *regression* models. He also noted that the semi-log form has the disadvantage against the log-log of not being able to impose constraints of constant returns to scale. Diewert (2003) also argued for the use of nonparametric functional forms and the estimation of linear generalized dummy variable hedonic regression models. This has been taken up in Curry, Morgan, and Silver (2001), who used neural networks that have been shown to work well, although the variable set required for their estimation has to be relatively small.

[34] In Chapter 16, Section C.3, the issue of adjusting the base versus the current period's price is discussed, because there are different data demands.

model requires a reference basket of characteristics. This is apparent for the hedonic imputation indices where separate indices using base- and current-period characteristics are estimated. A symmetric average of such indices is considered appropriate. A hedonic index based on a time dummy variable implicitly constrains the estimated coefficients from the base and current periods to be the same. Diewert (2003) formalized the problem of choosing the reference characteristics when comparing prices over time when the parameters of the hedonic function may themselves be changing over time. He found the results of hedonic indices to *not* be invariant to the choice of reference-period characteristic vector set z. The use of a sales (quantity) weighted average vector of characteristics proposed by Silver (1999) was considered, but Diewert noted that over long time periods this may become unrepresentative.[35] Of course, if the dummy variable approach is used in a chained formulation as outlined in Section C.3, the weighted averages of characteristics remain reasonably up to date, though chaining has its own pros and cons (see Chapter 16). A fixed-base alternative noted by Diewert (2003) is to use a Laspeyres-type comparison with the base-period parameter set, and a Paasche-type current-period index with the current-period parameter set, and take the geometric mean of the two indices for reasons similar to those given in Chapter 18, Section E.3. The resulting Fisher-type index is similar to that given in by a geometric mean of the Laspeyres and Paasche indices in equations (22.31a) and 22.32a), proposed by Feenstra (1995).[36] A feature of the time dummy approach is that it implicitly takes a symmetric average of the coefficients by constraining them to be the same. But what if, as is more likely the case, only base-period hedonic regression coefficients are available? Because hedonic indices based on a symmetric average of the coefficients are desirable, the spread or difference between estimates based on either a current- or a reference-period characteristics set is an indication of potential bias, and estimates of such spread may be undertaken retrospectively. If the spread is large, estimates based on the use of a single period's characteristics set, say the current period, should be treated with caution. More regular updating of the hedonic regressions is likely to reduce spread because the periods being compared will be closer and the characteristics of the items in the periods compared more similar.

Weighting

22.96 OLS estimators implicitly treat each item as being of equal importance, although some items will have quite substantial sales, whereas for others sales will be minimal. It is axiomatic that an item with sales of more than 5,000 in a month should not be given the same influence in the regression estimator as one with a few transactions. Commodities with very low sales may be at the end of their life cycles or be custom made. Either way, their (quality-adjusted) prices and price changes may be unusual.[37] Such observations with unusual prices should not be allowed to unduly influence the index.[38] The estimation of hedonic regression equations by a weighted least squares (WLS) estimator is preferable. This estimator minimizes the sum of *weighted* squared deviations between the actual prices and the predicted prices from the regression equation, as opposed to OLS estimation, which uses an equal weight for each observation. There is a question as to whether to use quantity (volume) or expenditure weights. The use of quantity weights can be supported by considering the nature of their equivalent "price." Such prices are the average (usually the same) price over a number of transactions. The underlying sampling unit is the individual transaction, so there is a sense that the data may be replicated as being composed of, say, 12 individual observations using an OLS estimator, as opposed to a single observation with a weight of 12 using a WLS estimator. Both would yield the same result. Inefficient estimates arise if the variance of the errors, $V(u_i)$, is not constant—that is, they are heteroscedastic. WLS is equivalent to assuming that the error variances are related to the weights in a multiplicative manner, say $V(u_i) = \sigma^2 w_i^2$.[39] A priori notions as to whether a hedonic regression model predicts better/worse at different levels of quantities or expenditures may help in identifying which weights are appropriate; however, statistical tests or plots of heteroscedasticity may be more useful.

[35]Other averages may be proposed—for example, the needs of an index representative of the "typical" establishment would be better met by a trimmed mean or median.

[36]Diewert (2002c) also suggested matching items where possible and using hedonic regressions to impute the prices of the missing old and new ones. Different forms of weighting systems, including superlative ones, can be applied to this set of price data in each period for both matched and unmatched data.

[37]Such observations have higher variances of their error terms, leading to imprecise parameter estimates. This would argue for the use of WLS estimators with quantity sold as the weight. This is one of the standard treatments for heteroscedastic errors (see Berndt, 1991).

[38]See Berndt, Ling, and Kyle (2003), Cockburn and Anis (1998), and Silver and Heravi (2005) for examples. Silver and Heravi (2005) showed that old items have above-average leverage effects and below-average residuals. Not only are they different, but they exert undue influence for their size (number of observations).

[39]Estimating an equation for which each variable is divided by the square root of the weight using OLS is an equivalent procedure.

22.97 The sole use of statistical criteria for deciding on which weighing system to use has rightfully come under some criticism. Diewert (2002c and 2005b) and Silver (2002) have argued that what matters is whether the estimates are representative of the target index in mind. Conventional target index numbers, such as those of Laspeyres, Paasche, Fisher, and Törnqvist, weight price changes by expenditure shares, and the latter two formulas have received support from the axiomatic, stochastic, fixed-base, and economic theoretic approaches, as shown in Chapters 16 through 18. Thus, value weights are preferred to quantity weights: "The problem with quantity weighting is this: it will tend to give too little weight to cheap models that have low amounts of useful characteristics" (Diewert, 2002c, p. 8). Diewert continued to argue that for a WLS estimator of hedonic time dummy variable indices, expenditure *share* weights should be used, as opposed to the *value* of expenditure, to avoid inflation-increasing period 1 value weights, resulting in possible heteroscedastic residuals. Furthermore, for a semilogarithmic hedonic function when models are present in both periods, the average expenditure shares in periods 0 and 1 for m items, $\frac{1}{2}(s_{m0} + s_{m1})$, should be used as weights in the WLS estimator. If only matched models exist in the data, then such an estimator may be equivalent to the Törnqvist index. If an observation m is available in only one of the periods, its weight should be s_{m0} or s_{m1} accordingly, and the WLS estimator provides a *generalization* of the Törnqvist index.

22.98 Silver (2002) has shown that a WLS estimator using value weights will not necessarily give each observation a weight equal to its relative value. The estimator will give more weight to those observations with high leverage effects and residuals. Observations with values of characteristics with large deviations from their means—say, very old or new models—have relatively high leverage. New and old models are likely to be priced at quite different prices than those predicted from the hedonic regression, even after taking into account their different characteristics. Such prices result, for example, from a pricing strategy designed to skim segments of the market willing to pay a premium for a new model, or from a strategy to charge relatively low prices for an old model to dump it to make way for a new one. In such cases the influence these models have on deriving the estimated coefficients will be over and above that attributable to their value weights. Silver (2002) suggested that leverage effects should be calculated for each observation, and those with high leverage and low weights should be deleted, and the regression re-run. Thus, although quantity or value weights are preferable to no weights (i.e., OLS), value weights are more appropriate than quantity ones and, even so, account should be taken of observations with undue influence.

22.99 Diewert (2002f) has also considered the issue of weighting with respect to the time dummy hedonic indices outlined in Section C.6. The use of WLS by value involves weights being applied to observations in both periods. However, if, for example, there is high inflation, then the sales values for a model in the current period will generally be larger than those of the corresponding model in the base period, and the assumption of homoskedastic residuals is unlikely to be met. Diewert (2002f and 2005b) suggested the use of expenditure *shares* in each period, as opposed to values, as weights for WLS for time dummy hedonic indices. He also suggested that an average of expenditure shares in the periods being compared be used for matched models.

22.100 Data on sales are not always available for weights, but the major selling items can generally be identified. In such cases, it is important to restrict the number of observations of items with relatively low sales, the extent of the restriction depending on the number of observations and the skewness of the sales distribution. In some cases, items with few sales provide the variability necessary for efficient estimates of the regression equation. In other cases, their low sales may be due to factors that make them unrepresentative of the hedonic surface, their residuals being unusually high. An example is low-selling models about to be dumped to make way for new models. Unweighted regressions may thus suffer from a sampling problem—even if the prices are perfectly quality adjusted, the index can be biased because it is unduly influenced by low-selling items with unrepresentative price–characteristic relationships. In the absence of weights, regression diagnostics have a role to play in helping to determine whether the undue variance in some observations belongs to such unusually low-selling items.[40]

[40]A less formal procedure is to take the standardized residuals from the regression and plot them against model characteristics that may denote low sales, such as certain brands (makes) or vintages (if not directly incorporated) or some technical feature that makes it unlikely that the item is being bought in quantity. Higher variances may be apparent from the scatter plot. If certain features are expected to have, on average, low sales, but seem to have high variances, leverages, and residuals (see Silver and Heravi, 2005), a case exists for at least downplaying their influence. Bodé and van Dalen (2001) used formal statistical criteria to decide between different weighting systems and compare the results of OLS and WLS, finding, as with Ioannidis and Silver (1999), that different results can arise.

22.101 There is a situation in which an unweighted OLS estimator is preferred. This is when markets are in perfect hedonic equilibrium. Observations with unusual characteristics, say old or new models, would take values that were particularly dispersed from their means and thus increase the variation of the sample for the same underlying model. Such increased variation leads to an increase in the efficiency of the estimates. However, theory and empirical observation (see Silver and Heravi, 2005) find that such outliers do not have the same structural relationships as do other models. If the sales shares of these new and old models are low relative to the number of models they represent in the market, then an OLS regression would give them undue weight.

Multicollinearity

22.102 There are a priori reasons to expect for some commodities that the variation in the values of one characteristic will not be independent of one or a linear combination of other *z* characteristics. As a result, parameter estimates will be unbiased yet imprecise. To illustrate this, a plot of the confidence interval for one parameter estimate against another collinear one is often described as elliptical, because the combinations of possible values they may take can easily drift from, say, high values of β_1 and low β_2 to higher values of β_2 and lower values of β_1. Because the sample size for the estimates is effectively reduced, relatively small additions to and deletions from the sample may affect the parameter estimates more than would be expected. These are standard statistical issues, and the reader is referred to Maddala (1988) and Kennedy (2003). In a hedonic regression, multicollinearity might be expected because some characteristics may be technologically tied to others. Producers including one characteristic may need to include others for it all to work, whereas for the consumer side, purchasers buying, for example, an up-market brand may expect a certain bundle of features to come with it. Triplett (2004) argued strongly for the researcher to be aware of the features of the product and consumer market. There are standard, though not completely reliable, indicators of multicollinearity (such as variance inflation factors), but an exploration of its nature is greatly aided by an understanding of the market along with exploration of the effects of including and excluding individual variables on the signs and coefficients and on other diagnostic test statistics (see Maddala, 1988).[41]

22.103 If a subset of the estimated coefficients from a hedonic regression is to be used to quality adjust a noncomparable replacement price, and if there is multicollinearity *between* variables in this subset *and* other independent variables, then the estimates of the coefficients to be used for the adjustment will be imprecise. The multicollinearity effectively reduces the sample size, and some of the effects of the variables in the subset may be wrongly ascribed to the other independent variables. The extent of this error will be determined by the strength of the multiple-correlation coefficient between all such "independent" variables (the multicollinearity), the standard error or "fit" of the regression, the dispersion of the independent variable concerned, and the sample size. These all affect the precision of the estimates, because they are components in the standard error of the *t*-statistics. Even if multicollinearity is expected to be quite high, large sample sizes and a well-fitting model may reduce the standard errors on the *t*-statistics to acceptable levels. If multicollinearity is expected to be severe, the predicted value for an item's price may be computed using the whole regression and an adjustment made using the predicted value, as explained in Chapter 8, Section E.4, because there is a sense in which it would not matter whether the variation was wrongly attributed to either β_1 or β_2. If dummy variable hedonic *indices* are being calculated (Section B.3 above), the time trend will be collinear with an included variable if a new feature appears in a new month for the vast majority of the items, so that the data are not rich enough to allow the separate effects of the coefficient on the time dummy to be precisely identified. The extent of the imprecision of the coefficient on the time dummy will be determined by the aforementioned factors. A similar argument holds for omitted variable bias.

Omitted variable bias

22.104 The exclusion of tastes and technology and community characteristics has already been discussed. The concern here is with product characteristics. Consider again the use of a subset of the estimated coefficients from a hedonic regression to quality adjust a noncomparable replacement price. It is well established that multicollinearity of omitted variables with included variables leads to bias in the estimates of the coefficients of included ones. If omitted variables are *independent* of the included variables, then the estimates of the coefficients on the included variables are unbiased. This is acceptable in this instance; the only caveat is that the quality adjustment for the replacement item may also require an adjustment for these

[41]Triplett (2004) stressed the point that $\bar{R}^2$ alone is insufficient for this purpose.

omitted variables, and this adjustment, as noted by Triplett (2004), has to be undertaken using a separate method and data. But what if the omitted variable is multicollinear with a subset of included ones, and these included ones are to be used to quality adjust a noncomparable item? In this case, the coefficient on the subset of the included variables may be wrongly picking up some of the omitted variables' effects. The coefficients will be used to quality adjust prices for items that differ only with regard to this subset of included variables, and the price comparison will be biased if the characteristics of both included and omitted variables have different price changes. For hedonic *indices* using a dummy time trend, the estimates of quality-adjusted price changes will suffer from a similar bias if omitted variables multicollinear with the time change are excluded from the regression. What are picked up as quality-adjusted price changes over time may, in part, be changes due to the prices of these excluded variables. This requires that the prices on the omitted characteristics follow a different trend. Such effects are most likely when there are gradual improvements in the quality of items, such as the reliability and safety of consumer durables,[42] which are difficult to measure, at least for the sample of items in real time. The quality-adjusted price changes will thus overstate price changes in such instances.

[42]There are some commodity areas, such as airline comfort, that have been argued to have overall patterns of decreasing quality.

23. Treatment of Seasonal Products

A. Problem of Seasonal Products

23.1 The existence of seasonal products poses some significant challenges for price statisticians. *Seasonal commodities* are products that are either (1) not available in the marketplace during certain seasons of the year or (2) available throughout the year but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[1] A commodity that satisfies (1) is termed a *strongly seasonal commodity,* whereas a commodity that satisfies (2) is called a *weakly seasonal commodity.* Strongly seasonal products create the biggest problems for price statisticians in the context of producing monthly or quarterly export and import price indices (XMPIs). If a product price is available in only one of the two months (or quarters) being compared, then it is not possible to calculate a relative price for the product, and traditional bilateral index number theory breaks down. In other words, if a product is present in one month but not the next, how can the month-to-month amount of price change for that product be computed?[2] In this chapter, a solution to this problem is presented that works even if the products produced are entirely different for each month of the year.[3]

23.2 There are two main sources of seasonal fluctuations in prices and quantities: (1) climate and (2) custom.[4] In the first category, fluctuations in temperature, precipitation, and hours of daylight cause fluctuations in the demand or supply for many products; for example, think of summer versus winter clothing, the demand for light and heat, vacations, and so on. With respect to custom and convention as a cause of seasonal fluctuations, consider the following quotation:

> Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law. . . . Many of the conventional seasons have considerable effects on economic behaviour. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the first of July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase in bankruptcies in January, and so on. (Mitchell, 1927, p. 237)

23.3 Examples of important seasonal products are the following: many food items; alcoholic beverages; many clothing and footwear items; water, heating oil, and electricity; flowers and garden supplies; vehicle purchases, vehicle operation; many entertainment and recreation expenditures; books; insurance expenditures; wedding expenditures; recreational equipment; toys and games; software; air travel; and tourism purchases. For a typical country, seasonal purchases will often amount to one-fifth to one-third of all consumer purchases.[5]

23.4 In the context of producing monthly or quarterly XMPIs, it must be recognized that there is no completely satisfactory way of dealing with strongly seasonal products. If a product is present in one month but missing in the next month, then none of the index number theories that were considered in Chapters 16

[1]This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a, p. 7; 1980b, p. 110; and 1980c, p. 68). Diewert (1998b, p. 457) used the terms "type 1" and "type 2" seasonality.

[2]Zarnowitz was perhaps the first to note the importance of this problem: "But the main problem introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis" (1961, p. 238).

[3]However, the same products must reappear each year for each separate month!

[4]This classification dates back to Mitchell at least: "Two types of seasons produce annually recurring variations in economic activity—those which are due to climates and those which are due to conventions" (1927, p. 236).

[5]Alterman, Diewert, and Feenstra (1999, p. 151) found that over the 40 months between September 1993 and December 1996, somewhere between 23 and 40 percent of U.S. imports and exports exhibited seasonal variations in quantities, whereas only about 5 percent of U.S. export and import prices exhibited seasonal fluctuations.

through 21 can be applied because all of these theories assumed that the dimensionality of the product space was constant for the two periods being compared. However, if seasonal products are present in the market during each season, then, in theory, traditional index number theory can be applied in order to construct month-to-month or quarter-to-quarter price indices. This traditional approach to the treatment of seasonal products is followed in Sections H, I, and J of this chapter. The reason why this straightforward approach is deferred to the end of the chapter is twofold:

- The approach that restricts the index to products that are present in every period often does not work well in the sense that systematic *biases* can occur; and
- The approach is not fully *representative;* that is, it does not make use of information on products that are not present in every month or quarter.

23.5 In Section B, a modified version of Turvey's (1979) artificial data set is introduced. This data set is used to numerically evaluate all of the index number formula that are suggested in this chapter. It will be seen in Section G that large seasonal fluctuations in volumes combined with systematic seasonal changes in price can make month-to-month or quarter-to-quarter price indices behave rather poorly.

23.6 Even though existing index number theory cannot deal satisfactorily with seasonal products in the context of constructing month-to-month indices of consumer and producer prices, it can deal satisfactorily with seasonal products if the focus is changed from month-to-month XMPIs to XMPIs that compare the prices of one month with the prices of the *same* month in a previous year. Thus, in Section C, *year-over-year monthly XMPIs* are studied. Turvey's seasonal data set is used to evaluate the performance of these indices, and they are found to perform quite well.

23.7 In Section D, the year-over-year monthly indices defined in Section C are aggregated into an *annual index* that compares all of the monthly prices in a given calendar year with the corresponding monthly prices in a base year. In Section E, this idea of comparing the prices of a current calendar year with the corresponding prices in a base year is extended to annual indices that compare the prices of the last 12 months with the corresponding prices in the 12 months of a base year. The resulting *rolling-year indices* can be regarded as seasonally adjusted price indices. The modified Turvey data set is used to test out these year-over-year indices and they are found to work very well on this data set.

23.8 The rolling-year indices can provide an accurate gauge of the movement of prices in the current rolling year compared to the base year. However, this measure of price inflation can be regarded as a measure of inflation for a year that is centered around a month that is six months prior to the last month in the current rolling year. As a result, for some policy purposes, this type of index is not as useful as an index that compares the prices of the current month to the previous month, so that more up-to-date information on the movement of prices can be obtained. However, in Section F, it is shown that under certain conditions, the current-month year-over-year monthly index, along with last month's year-over-year monthly index, can successfully *predict* or *forecast* a rolling-year index that is centered around the current month.

23.9 The year-over-year indices defined in Section C and their annual averages studied in Sections D and E offer a theoretically satisfactory method for dealing with *strongly seasonal products,* that is, products that are available only during certain seasons of the year. However, these methods rely on the year-over-year comparison of prices; therefore, these methods cannot be used in the month-to-month or quarter-to-quarter type of index, which is typically the main focus of a consumer and a producer price program. Thus, there is a need for another type of index, one that may not have strong theoretical foundations but can deal with seasonal products in the context of producing a *month-to-month index*. In Section G, such an index is introduced, and it is implemented using the artificial data set for the products that are available during each month of the year. Unfortunately, owing to the seasonality in both prices and quantities in the always available products, this type of index can be systematically biased. This bias is apparent in the modified Turvey data set.

23.10 Because many XMPIs are month-to-month indices that use *annual basket quantity weights,* this type of index is studied in Section H. For months when the product is not available in the marketplace, the last available price is carried forward and used in the index. In Section I, an annual quantity basket is again used but instead of carrying forward the prices of seasonally unavailable items, an imputation method is used to fill in the missing prices. The annual basket-type indices defined in Sections H and I are implemented using the artificial data set. Unfortunately, the empirical results are not satisfactory because the indices show tremendous seasonal fluctuations in prices. This volatility makes them unsuitable for users who want up-to-date information on *trends* in general inflation.

23.11 In Section J, the artificial data set is used in order to evaluate another type of month-to-month index that is frequently suggested in the literature on how to deal with seasonal products: namely the *Bean and Stine* (1924) *Type C* or *Rothwell* (1958) index. Again, this index does not get rid of the tremendous seasonal fluctuations that are present in the modified Turvey data set.

23.12 Sections H and I showed that the annual basket-type indices with carryforward of missing prices (Section H) or imputation of missing prices (Section I) do not get rid of seasonal fluctuations in prices. However, in Section K, it is shown how seasonally adjusted versions of these annual basket indices can be used to successfully *forecast* rolling-year indices that are centered in the current month. In addition, the results in Section K show how these annual basket-type indices can be seasonally adjusted (using information obtained from rolling-year indices from prior periods or by using traditional seasonal adjustment procedures). Hence, these seasonally adjusted annual basket indices could be used as successful indicators of general inflation on a timely basis.

23.13 Section L concludes with several suggestions for dealing with seasonal products.

B. A Seasonal Product Data Set

23.14 It will be useful to illustrate the index number formulas that are defined in subsequent sections by computing them for an actual data set. Turvey (1979) constructed an artificial data set for five seasonal products (apples, peaches, grapes, strawberries, and oranges) for four years by month, so that there are 5 times 4 times 12 observations, equal to 240 observations in all. At certain times of the year, peaches and strawberries (products 2 and 4) are unavailable, so in Tables 23.1 and 23.2, the prices and quantities for these products are entered as zeros.[6] The data in Tables 23.1 and 23.2 are essentially equal to that constructed by Turvey except that a number of adjustments were made in order to illustrate various points. The two most important adjustments were as follows:

- The data for product 3 (grapes) were adjusted, so that the annual Laspeyres and Paasche indices (which are defined in Section D below) would differ more than in the original data set;[7] and
- After the aforementioned adjustments were made, each price in the last year of data was escalated by the monthly inflation factor 1.008, so that month-to-month inflation for the last year of data would be at an approximate monthly rate of 1.6 percent per month, compared with about 0.8 percent per month for the first three years of data.[8]

23.15 Turvey sent his artificial data set to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indices. About 20 countries replied; Turvey summarized the responses as follows:

> It will be seen that the monthly indices display very large differences, for example, a range of 129.12–169.50 in June, while the range of simple annual means is much smaller. It will also be seen that the indices vary as to the peak month or year. (Turvey, 1979, p. 13)

The (modified) data below are used to test out various index number formulas in subsequent sections.

C. Year-over-Year Monthly Indices

23.16 It can be seen that the existence of seasonal products that are present in the marketplace in one month but absent the next causes the accuracy of a month-to-month index to fall.[9] A way of dealing with

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[6]The corresponding prices are not necessarily equal to zero (the commodities may be offered for sale at certain prices but there are no purchasers at those prices), but they are entered as zeros for convenience in programming the various indices.

[7]After the first year, the price data for grapes were adjusted downward by 30 percent each year and the corresponding volume was adjusted upward by 40 percent each year. In addition, the quantity of oranges (product 5) for November 1971 was changed from 3,548 to 8,548 so that the seasonal pattern of change for this product would be similar to that of other years. For similar reasons, the price of oranges in December 1970 was changed from 1.31 to1.41 and in January 1971 from 1.35 to 1.45.

[8]Pierre Duguay of the Bank of Canada, while commenting on a preliminary version of this chapter, observed that rolling-year indices would not be able to detect the *magnitude* of systematic changes in the month-to-month inflation rate. The original Turvey data set was roughly consistent with a month-to-month inflation rate of 0.8 percent per month; that is, prices grew roughly at the rate 1.008 each month over the four years of data. Hence this second major adjustment of the Turvey data was introduced to illustrate Duguay's observation, which is quite correct: The centered rolling-year indices pick up the correct magnitude of the new inflation rate only after a lag of half a year or so. However, they do quickly pick up the direction of change in the inflation rate.

[9]In the limit, if each product appeared in only one month of the year, then a month-to-month index would break down completely.

Table 23.1. Artificial Seasonal Data Set: Prices

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1970	1	1.14	0	2.48	0	1.30
	2	1.17	0	2.75	0	1.25
	3	1.17	0	5.07	0	1.21
	4	1.40	0	5.00	0	1.22
	5	1.64	0	4.98	5.13	1.28
	6	1.75	3.15	4.78	3.48	1.33
	7	1.83	2.53	3.48	3.27	1.45
	8	1.92	1.76	2.01	0	1.54
	9	1.38	1.73	1.42	0	1.57
	10	1.10	1.94	1.39	0	1.61
	11	1.09	0	1.75	0	1.59
	12	1.10	0	2.02	0	1.41
1971	1	1.25	0	2.15	0	1.45
	2	1.36	0	2.55	0	1.36
	3	1.38	0	4.22	0	1.37
	4	1.57	0	4.36	0	1.44
	5	1.77	0	4.18	5.68	1.51
	6	1.86	3.77	4.08	3.72	1.56
	7	1.94	2.85	2.61	3.78	1.66
	8	2.02	1.98	1.79	0	1.74
	9	1.55	1.80	1.28	0	1.76
	10	1.34	1.95	1.26	0	1.77
	11	1.33	0	1.62	0	1.76
	12	1.30	0	1.81	0	1.50
1972	1	1.43	0	1.89	0	1.56
	2	1.53	0	2.38	0	1.53
	3	1.59	0	3.59	0	1.55
	4	1.73	0	3.90	0	1.62
	5	1.89	0	3.56	6.21	1.70
	6	1.98	4.69	3.51	3.98	1.78
	7	2.07	3.32	2.73	4.30	1.89
	8	2.12	2.29	1.65	0	1.91
	9	1.73	1.90	1.15	0	1.92
	10	1.56	1.97	1.15	0	1.95
	11	1.56	0	1.46	0	1.94
	12	1.49	0	1.73	0	1.64
1973	1	1.68	0	1.62	0	1.69
	2	1.82	0	2.16	0	1.69
	3	1.89	0	3.02	0	1.74
	4	2.00	0	3.45	0	1.91
	5	2.14	0	3.08	7.17	2.03
	6	2.23	6.40	3.07	4.53	2.13
	7	2.35	4.31	2.41	5.19	2.22
	8	2.40	2.98	1.49	0	2.26
	9	2.09	2.21	1.08	0	2.22
	10	2.03	2.18	1.08	0	2.31
	11	2.05	0	1.36	0	2.34
	12	1.90	0	1.57	0	1.97

Table 23.2. Artificial Seasonal Data Set: Quantities

Year *t*	Month *m*	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
1970	1	3,086	0	82	0	10,266
	2	3,765	0	35	0	9,656
	3	4,363	0	9	0	7,940
	4	4,842	0	8	0	5,110
	5	4,439	0	26	700	4,089
	6	5,323	91	75	2,709	3,362
	7	4,165	498	82	1,970	3,396
	8	3,224	6,504	1,490	0	2,406
	9	4,025	4,923	2,937	0	2,486
	10	5,784	865	2,826	0	3,222
	11	6,949	0	1,290	0	6,958
	12	3,924	0	338	0	9,762
1971	1	3,415	0	119	0	10,888
	2	4,127	0	45	0	10,314
	3	4,771	0	14	0	8,797
	4	5,290	0	11	0	5,590
	5	4,986	0	74	806	4,377
	6	5,869	98	112	3,166	3,681
	7	4,671	548	132	2,153	3,748
	8	3,534	6,964	2,216	0	2,649
	9	4,509	5,370	4,229	0	2,726
	10	6,299	932	4,178	0	3,477
	11	7,753	0	1,831	0	8,548
	12	4,285	0	496	0	10,727
1972	1	3,742	0	172	0	11,569
	2	4,518	0	67	0	10,993
	3	5,134	0	22	0	9,621
	4	5,738	0	16	0	6,063
	5	5,498	0	137	931	4,625
	6	6,420	104	171	3,642	3,970
	7	5,157	604	202	2,533	4,078
	8	3,881	7,378	3,269	0	2,883
	9	4,917	5,839	6,111	0	2,957
	10	6,872	1,006	5,964	0	3,759
	11	8,490	0	2,824	0	8,238
	12	5,211	0	731	0	11,827
1973	1	4,051	0	250	0	12,206
	2	4,909	0	102	0	11,698
	3	5,567	0	30	0	10,438
	4	6,253	0	25	0	6,593
	5	6,101	0	220	1,033	4,926
	6	7,023	111	252	4,085	4,307
	7	5,671	653	266	2,877	4,418
	8	4,187	7,856	4,813	0	3,165
	9	5,446	6,291	8,803	0	3,211
	10	7,377	1,073	8,778	0	4,007
	11	9,283	0	4,517	0	8,833
	12	4,955	0	1,073	0	12,558

these strongly seasonal products is to change the focus from short-term month-to-month price indices to year-over-year price comparisons for each month of the year. In the latter type of comparison, there is a good chance that seasonal products that appear in February, for example, will also appear in subsequent Februarys, so that the overlap of products will be maximized in these year-over-year monthly indices.

23.17 For more than a century, it has been recognized that making year-over-year comparisons[10] provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations:

> In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes. (Jevons, 1863 reprinted 1884, p. 3)

23.18 The economist Flux and the statistician Yule also endorsed the idea of making year-over-year comparisons to minimize the effects of seasonal fluctuations:

> Each month the average price change compared with the corresponding month of the previous year is to be computed. . . . The determination of the proper seasonal variations of weights, especially in view of the liability of seasons to vary from year to year, is a task from which, I imagine, most of us would be tempted to recoil. (Flux, 1921, pp. 184–85)

> My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures. (Yule, 1921, p. 199)

In more recent times, Zarnowitz also endorsed the use of year-over-year monthly indices:

> There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit "season", and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons. (Zarnowitz, 1961, p. 266)

23.19 In the remainder of this section, it is shown how year-over-year Fisher indices and approximations to them can be constructed.[11] For each month $m = 1, 2, \ldots, 12$, let $S(m)$ denote the set of products that are available for purchase in each year $t = 0, 1, \ldots, T$. For $t = 0, 1, \ldots, T$ and $m = 1, 2, \ldots, 12$, let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is available in month m of year t for n belongs to $S(m)$. Let $p^{t,m}$ and $q^{t,m}$ denote the month m and year t price and quantity vectors, respectively. Then *the year-over-year monthly Laspeyres, Paasche, and Fisher indices* going from month m of year t to month m of year $t + 1$ can be defined as follows:

$$P_L(p^{t,m}, p^{t+1,m}, q^{t,m}) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t,m}}; \quad m = 1, 2, \ldots, 12; \quad (23.1)$$

$$P_P(p^{t,m}, p^{t+1,m}, q^{t+1,m}) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t+1,m}}; \quad m = 1, 2, \ldots, 12; \quad (23.2)$$

$$P_F(p^{t,m}, p^{t+1,m}, q^{t,m}, q^{t+1,m}) \equiv \sqrt{P_L(p^{t,m}, p^{t+1,m}, q^{t,m}) P_P(p^{t,m}, p^{t+1,m}, q^{t+1,m})}; \quad m = 1, 2, \ldots, 12. \quad (23.3)$$

23.20 The above formulas can be rewritten in price relative and monthly value share form as follows:

$$P_L(p^{t,m}, p^{t+1,m}, s^{t,m}) = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right); \quad m = 1, 2, \ldots, 12; \quad (23.4)$$

$$P_P(p^{t,m}, p^{t+1,m}, s^{t+1,m}) = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}; \quad m = 1, 2, \ldots, 12; \quad (23.5)$$

[10]In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924, p. 31) Type D index.

[11]Diewert (1996b, pp. 17–19; 1999a, p. 50) noted various separability restrictions on purchaser preferences that would justify these year-over-year monthly indices from the viewpoint of the economic approach to index number theory.

$$P_F(p^{t,m}, p^{t+1,m}, s^{t,m}, s^{t+1,m})$$
$$\equiv \sqrt{P_L(p^{t,m}, p^{t+1,m}, s^{t,m})P_P(p^{t,m}, p^{t+1,m}, s^{t+1,m})};$$
$$m = 1, 2, \ldots, 12 \quad (23.6)$$
$$= \sqrt{\sum_{n \in S(m)} s_n^{t,m} \left(p_n^{t+1,m}/p_n^{t,m}\right)}$$
$$\times \sqrt{\left[\sum_{n \in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}}$$

where the monthly value share for product $n \in S(m)$ for month m in year t is defined as

$$s_n^{t,m} = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i \in S(m)} p_i^{t,m} q_i^{t,m}}\ ; m = 1, 2, \ldots, 12; n \in S(m);$$
$$t = 0, 1, \ldots, T; \quad (23.7)$$

and $s^{t,m}$ denotes the vector of month m value shares in year t, $[s_n^{t,m}]$ for $n \in S(m)$.

23.21 Current-period value shares $s_n^{t+1,m}$ are not likely to be available. As a consequence, it will be necessary to approximate these shares using the corresponding value shares from a base year 0.

23.22 Use the base-period monthly value share vectors $s^{0,m}$ in place of the vector of month m and year t value shares $s^{t,m}$ in equation (23.4) and use the base-period monthly value share vectors $s^{0,m}$ in place of the vector of month m and year $t + 1$ value shares $s^{t+1,m}$ in equation (23.5). Similarly, replace the share vectors $s^{t,m}$ and $s^{t+1,m}$ in equation (23.6) with the base-period value share vector for month m, $s^{0,m}$. The resulting *approximate year-over-year monthly Laspeyres, Paasche, and Fisher indices* are defined by equations (23.8) through (23.10):[12]

$$P_{AL}(p^{t,m}, p^{t+1,m}, s^{0,m}) = \sum_{n \in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$
$$m = 1, 2, \ldots, 12; \quad (23.8)$$

$$P_{AP}(p^{t,m}, p^{t+1,m}, s^{0,m}) = \left[\sum_{n \in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1};$$
$$m = 1, 2, \ldots, 12; \quad (23.9)$$

$$P_{AF}(p^{t,m}, p^{t+1,m}, s^{0,m}, s^{0,m})$$
$$\equiv \sqrt{P_{AL}(p^{t,m}, p^{t+1,m}, s^{0,m})P_P(p^{t,m}, p^{t+1,m}, s^{0,m})};$$
$$m = 1, 2, \ldots, 12$$
$$= \sqrt{\sum_{n \in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)\left[\sum_{n \in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}}.$$
$$(23.10)$$

23.23 The approximate Fisher year-over-year monthly indices defined by equation (23.10) will provide adequate approximations to their true Fisher counterparts defined by equation (23.6) only if the monthly value shares for the base year 0 are not too different from their current-year t and $t + 1$ counterparts. Thus, it will be useful to construct the true Fisher indices on a delayed basis in order to check the adequacy of the approximate Fisher indices defined by equation (23.10).

23.24 The year-over-year monthly approximate Fisher indices defined by equation (23.10) will normally have a certain amount of upward bias, because these indices cannot reflect long-term substitution toward products that are becoming relatively cheaper over time. This reinforces the case for computing true year-over-year monthly Fisher indices defined by equation (23.6) on a delayed basis, so that this substitution bias can be estimated.

23.25 Note that the approximate year-over-year monthly Laspeyres and Paasche indices, P_{AL} and P_{AP}, defined by equations (23.8) and (23.9), satisfy the following inequalities:

$$P_{AL}(p^{t,m}, p^{t+1,m}, s^{0,m})\ P_{AL}(p^{t+1,m}, p^{t,m}, s^{0,m}) \geq 1;$$
$$m = 1, 2, \ldots, 12; \quad (23.11)$$

$$P_{AP}(p^{t,m}, p^{t+1,m}, s^{0,m})P_{AP}(p^{t+1,m}, p^{t,m}, s^{0,m}) \leq 1;$$
$$m = 1, 2, \ldots, 12; \quad (23.12)$$

with strict inequalities if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other.[13] Equation (23.11) says that the approximate year-over-year monthly Laspeyres index *fails the time reversal test* with an upward bias while equation (23.12) says that the approximate year-over-year monthly Paasche index *fails*

[12]If the monthly revenue shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by (23.10) reduces to Fisher's (1922, p. 472) formula 101. Fisher (1922, p. 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992, p. 143) and Diewert (1995a, p. 29) showed analytically that these two indices approximated each other to the second order. The equally weighted version of equation (23.10) was recommended as an elementary index by Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992, p. 140).

[13]For reasons given in Hardy, Littlewood, and Pólya (1934), p. 26.

the time reversal test with a downward bias. As a result, the fixed-weights approximate Laspeyres index P_{AL} has a built-in upward bias whereas the fixed-weights approximate Paasche index P_{AP} has a built-in downward bias. *Statistical agencies should avoid the use of these formulas.* However, they can be combined, as in the approximate Fisher formula in equation (23.10). The resulting index should be free from any systematic formula bias, although some substitution bias could still exist.

23.26 The year-over-year monthly indices defined in this section are illustrated using the artificial data set tabled in Section B. Although fixed-base indices were not formally defined in this section, these indices have formulas similar to those of the year-over-year indices that were defined, with the exception that the variable-base year t is replaced by the fixed-base year 0. The resulting 12 year-over-year monthly fixed-base Laspeyres, Paasche, and Fisher indices are listed in Tables 23.3 to 23.5.

23.27 Comparing the entries in Tables 23.3 and 23.4, one can see that the year-over-year monthly fixed-base Laspeyres and Paasche price indices do not differ substantially for the early months of the year. However, there are substantial differences between the indices for the last five months of the year by the time the year 1973 is reached. The largest percentage difference between the Laspeyres and Paasche indices is 12.5 percent for month 10 in 1973 (1.4060/1.2496 = 1.125). However, all of the year-over-year monthly series show a nice smooth year-over-year trend.

23.28 Approximate fixed-base year-over-year Laspeyres, Paasche, and Fisher indices can be constructed by replacing current-month revenue shares for the five products with the corresponding base-year monthly revenue shares for the same five products. The resulting approximate Laspeyres indices are equal to the original fixed-base Laspeyres, so there is no need to table the approximate Laspeyres indices. However, the approximate year-over-year Paasche and Fisher indices do differ from the fixed-base Paasche and Fisher indices found in Tables 23.4 and 23.5, so these new approximate indices are listed in Tables 23.6 and 23.7.

23.29 Comparing the entries in Table 23.4 with the corresponding entries in Table 23.6, it can be seen that

Table 23.3. Year-over-Year Monthly Fixed-Base Laspeyres Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2060	1.2442	1.3062	1.2783	1.2184	1.1734	1.2364	1.1827	1.1049	1.1809	1.2550	1.1960
1973	1.3281	1.4028	1.4968	1.4917	1.4105	1.3461	1.4559	1.4290	1.2636	1.4060	1.5449	1.4505

Table 23.4. Year-over-Year Monthly Fixed-Base Paasche Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2023	1.2436	1.3038	1.2773	1.2024	1.1657	1.2307	1.1455	1.0695	1.1274	1.2218	1.1901
1973	1.3190	1.4009	1.4912	1.4882	1.3715	1.3266	1.4433	1.3122	1.1664	1.2496	1.4296	1.4152

Table 23.5. Year-over-Year Monthly Fixed-Base Fisher Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2041	1.2439	1.3050	1.2778	1.2104	1.1695	1.2336	1.1640	1.0870	1.1538	1.2383	1.1930
1973	1.3235	1.4019	1.4940	1.4900	1.3909	1.3363	1.4496	1.3694	1.2140	1.3255	1.4861	1.4327

Table 23.6. Year-over-Year Approximate Monthly Fixed-Base Paasche Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2025	1.2421	1.3036	1.2757	1.2110	1.1640	1.2267	1.1567	1.0788	1.1309	1.2244	1.1862
1973	1.3165	1.3947	1.4880	1.4858	1.3926	1.3223	1.4297	1.3315	1.1920	1.2604	1.4461	1.4184

Table 23.7. Year-over-Year Approximate Monthly Fixed-Base Fisher Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2043	1.2432	1.3049	1.2770	1.2147	1.1687	1.2316	1.1696	1.0918	1.1557	1.2396	1.1911
1973	1.3223	1.3987	1.4924	1.4888	1.4015	1.3341	1.4428	1.3794	1.2273	1.3312	1.4947	1.4344

with few exceptions, the entries correspond fairly well. One of the bigger differences is the 1973 entry for the fixed-base Paasche index for month 9, which is 1.1664, while the corresponding entry for the approximate fixed-base Paasche index is 1.1920 for a 2.2 percent difference (1.1920/1.1664 = 1.022). In general, the approximate fixed-base Paasche indices are a bit bigger than the true fixed-base Paasche indices, as one might expect because the approximate indices have some substitution bias built in. This is due to the fact that their revenue shares are held fixed at the 1970 levels.

23.30 Turning now to the chained year-over-year monthly indices using the artificial data set, the resultant 12 year-over-year monthly chained Laspeyres, Paasche, and Fisher indices, P_L, P_P, and P_F, where the month-to-month links are defined by equations (23.4) through (23.6), are listed in Tables 23.8 to 23.10.

Table 23.8. Year-over-Year Monthly Chained Laspeyres Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2058	1.2440	1.3058	1.2782	1.2154	1.1720	1.2357	1.1753	1.0975	1.1690	1.2491	1.1943
1973	1.3274	1.4030	1.4951	1.4911	1.4002	1.3410	1.4522	1.3927	1.2347	1.3593	1.5177	1.4432

Table 23.9. Year-over-Year Monthly Chained Paasche Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2039	1.2437	1.3047	1.2777	1.2074	1.1682	1.2328	1.1569	1.0798	1.1421	1.2321	1.1908
1973	1.3243	1.4024	1.4934	1.4901	1.3872	1.3346	1.4478	1.3531	1.2018	1.3059	1.4781	1.4305

Table 23.10. Year-over-Year Monthly Chained Fisher Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2048	1.2438	1.3052	1.2780	1.2114	1.1701	1.2343	1.1660	1.0886	1.1555	1.2405	1.1926
1973	1.3258	1.4027	1.4942	1.4906	1.3937	1.3378	1.4500	1.3728	1.2181	1.3323	1.4978	1.4368

23.31 Comparing the entries in Tables 23.8 and 23.9, it can be seen that the year-over-year monthly chained Laspeyres and Paasche price indices have smaller differences than the corresponding fixed-base Laspeyres and Paasche price indices in Tables 23.3 and 23.4. This is a typical pattern that was found in Chapter 20: *The use of chained indices tends to reduce the spread between Paasche and Laspeyres indices compared to their fixed-base counterparts.* The largest percentage difference between corresponding entries for the chained Laspeyres and Paasche indices in Tables 23.8 and 23.9 is 4.1 percent for month 10 in 1973 (1.3593/1.3059 = 1.041). Recall that the fixed-base Laspeyres and Paasche indices differed by 12.5 percent for the same month so that *chaining does tend to reduce the spread between these two equally plausible indices.*

23.32 The chained year-over-year Fisher indices listed in Table 23.10 are regarded as the best estimates of year-over-year inflation using the artificial data set.

23.33 The year-over-year chained Laspeyres, Paasche, and Fisher indices listed in Tables 23.8 to 23.10 can be approximated by replacing current-period product revenue shares for each month with the corresponding base-year monthly revenue shares. The resultant 12 year-over-year monthly approximate chained Laspeyres, Paasche, and Fisher indices (P_{AL}, P_{AP}, and P_{AF}), where the monthly links are defined by equations (23.8) through (23.10), are listed in Tables 23.11 through 23.13.

23.34 The year-over-year chained indices listed in Tables 23.11 through 23.13 approximate their true chained counterparts listed in Tables 23.8 through 23.10 closely. For 1973, the largest discrepancies are for the Paasche and Fisher indices for month 9: The chained Paasche is 1.2018, while the corresponding approximate chained Paasche is 1.2183, for a difference of 1.4 percent. The chained Fisher is 1.2181, while the corresponding approximate chained Fisher is 1.2305, for a difference of 1.0 percent. It can be seen that for the modified Turvey data set, the approximate year-over-year monthly Fisher indices listed in Table 23.13 approximate the theoretically preferred (but practically unfeasible) Fisher chained indices listed in Table 23.10

Table 23.11. Year-over-Year Monthly Approximate Chained Laspeyres Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2056	1.2440	1.3057	1.2778	1.2168	1.1712	1.2346	1.1770	1.0989	1.1692	1.2482	1.1939
1973	1.3255	1.4007	1.4945	1.4902	1.4054	1.3390	1.4491	1.4021	1.2429	1.3611	1.5173	1.4417

Table 23.12. Year-over-Year Monthly Approximate Chained Paasche Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2033	1.2424	1.3043	1.2764	1.2130	1.1664	1.2287	1.1638	1.0858	1.1438	1.2328	1.1886
1973	1.3206	1.3971	1.4914	1.4880	1.3993	1.3309	1.4386	1.3674	1.2183	1.3111	1.4839	1.4300

Table 23.13. Year-over-Year Monthly Approximate Chained Fisher Indices

	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2044	1.2432	1.3050	1.2771	1.2149	1.1688	1.2317	1.1704	1.0923	1.1565	1.2405	1.1912
1973	1.3231	1.3989	1.4929	1.4891	1.4024	1.3349	1.4438	1.3847	1.2305	1.3358	1.5005	1.4358

quite satisfactorily. Because the approximate Fisher indices are just as easy to compute as the approximate Laspeyres and Paasche indices, it may be useful to ask statistical agencies to make available to the public these approximate Fisher indices, along with the approximate Laspeyres and Paasche indices.

D. Year-over-Year Annual Indices

23.35 Assuming that each product in each season of the year is a separate annual product is the simplest and theoretically most satisfactory method for dealing with seasonal products when the goal is to construct annual price and quantity indices. This idea can be traced back to Mudgett in the consumer price context and to Stone in the producer price context:

> The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years. (Mudgett, 1955, p. 97)

> The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities. (Stone, 1956, p. 74–75)

23.36 Using the notation introduced in the previous section, the *Laspeyres, Paasche, and Fisher annual (chain link) indices* comparing the prices of year t with those of year $t+1$ can be defined as follows:

$$P_L(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t,1},\ldots,q^{t,12}) \equiv \frac{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t+1,m}\, q_n^{t,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t,m}\, q_n^{t,m}}; \tag{23.13}$$

$$P_P(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t+1,1},\ldots,q^{t+1,12}) \equiv \frac{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t+1,m}\, q_n^{t+1,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t,m}\, q_n^{t+1,m}}; \tag{23.14}$$

$$P_F(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t,1},\ldots,q^{t,12};q^{t+1,1},\ldots,q^{t+1,12}) \equiv \sqrt{P_L(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t,1},\ldots,q^{t,12})} \times \sqrt{P_P(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t+1,1},\ldots,q^{t+1,12})}. \tag{23.15}$$

23.37 The above formulas can be rewritten in price relative and monthly value share form as follows:

$$P_L(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};\sigma_1^t s^{t,1},\ldots,\sigma_{12}^t s^{t,12}) \equiv \sum_{m=1}^{12}\sum_{n\in S(m)} \sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right); \tag{23.16}$$

$$\begin{aligned} &P_P(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};\sigma_1^{t+1}s^{t+1,1},\ldots,\sigma_{12}^{t+1}s^{t+1,12}) \\ &\equiv \left[\sum_{m=1}^{12}\sum_{n\in S(m)} \sigma_m^{t+1} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1} \\ &= \left[\sum_{m=1}^{12}\sigma_m^{t+1}\sum_{n\in s(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1} \\ &= \left[\sum_{m=1}^{12}\sigma_m^{t+1}\left[P_P\left(p^{t,m},p^{t+1,m},s^{t+1,m}\right)\right]^{-1}\right]^{-1}; \end{aligned} \tag{23.17}$$

$$\begin{aligned} &P_F\Big(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};\sigma_1^t s^{t,1},\ldots,\sigma_{12}^t s^{t,12};\sigma_1^{t+1}s^{t+1,1},\ldots,\sigma_{12}^{t+1}s^{t+1,12}\Big) \\ &\equiv \sqrt{\sum_{m=1}^{12}\sum_{n\in S(m)} \sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)} \\ &\quad\times \sqrt{\left[\sum_{m=1}^{12}\sum_{n\in S(m)} \sigma_m^{t+1} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}} \\ &= \sqrt{\sum_{m=1}^{12}\sigma_m^t\left[P_L\left(p^{t,m},p^{t+1,m},s^{t,m}\right)\right]} \\ &\quad\times \sqrt{\left[\sum_{m=1}^{12}\sigma_m^{t+1}\left[P_P\left(p^{t,m},p^{t+1,m},s^{t+1,m}\right)\right]^{-1}\right]^{-1}}, \end{aligned} \tag{23.18}$$

where *the value share* for month m in year t is defined as

$$\sigma_m^t \equiv \frac{\sum_{n\in S(m)} p_n^{t,m} q_n^{t,m}}{\sum_{i=1}^{12}\sum_{j\in S(i)} p_j^{t,i}\, q_j^{t,i}}; \quad m=1,2,\ldots,12; \quad t=0,1,\ldots,T; \tag{23.19}$$

and the year-over-year monthly Laspeyres and Paasche (chain link) price indices $P_L(p^{t,m}, p^{t+1,m}, s^{t,m})$ and $P_P(p^{t,m}, p^{t+1,m}, s^{t+1,m})$ were defined in the previous section by equations (23.4) and (23.5), respectively. As usual, the annual chain link Fisher index P_F defined by equation (23.18), which compares the prices in every month of year t with the corresponding prices in year $t+1$, is the geometric mean of the annual chain link Laspeyres and Paasche indices, P_L and P_P, defined by equations (23.16) and (23.17). The last equation in equations (23.16), (23.17), and (23.18) shows that these annual indices can be defined as (monthly) share-weighted averages of the year-over-year monthly chain link Laspeyres and Paasche indices, $P_L(p^{t,m}, p^{t+1,m}, s^{t,m})$ and $P_P(p^{t,m}, p^{t+1,m}, s^{t+1,m})$, defined earlier by equations (23.4) and (23.5). Hence, once the year-over-year monthly indices defined in the previous section have been numerically calculated, it is easy to calculate the corresponding annual indices.

23.38 Fixed-base counterparts to the formulas defined by equations (23.16) through (23.18) can readily be defined: Simply replace the data pertaining to period t with the corresponding data pertaining to the base period 0.

23.39 Using the data from the artificial data set in Table 23.1 of Section B, the annual fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 23.14. Table 23.14 shows that by 1973, the annual fixed-base Laspeyres index exceeds its Paasche counterpart by 4.5 percent. Note that each series increases steadily.

23.40 The annual fixed-base Laspeyres, Paasche, and Fisher indices can be approximated by replacing any current shares with the corresponding base-year shares. The resulting annual approximate fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 23.15. Also listed in the last column of Table 23.15 is the fixed-base geometric Laspeyres annual index, P_{GL}. It is the weighted geometric mean counterpart to the fixed-base Laspeyres index, which is equal to a base-period weighted arithmetic average of the long-term price relative (see Chapter 20). It can be shown that P_{GL} approximates the approximate fixed-base Fisher index P_{AF} to the second order around a point where all of the long-term price relatives are equal to unity.[14] It is evident that the entries for the Laspeyres price indices are exactly the same in Tables 23.14 and 23.15. This is as it should be because the fixed-base Laspeyres price index uses only revenue shares from the base year 1970; consequently, the approximate fixed-base Laspeyres index is equal to the true fixed-base Laspeyres index. Comparing the columns labeled P_P and P_F in Table 23.14 and P_{AP} and P_{AF} in Table 23.15 shows that the approximate Paasche and approximate Fisher indices are quite close to the corresponding annual Paasche and Fisher indices. Thus, for the artificial data set, *the true annual fixed-base Fisher can be closely approximated by the corresponding approximate Fisher index* P_{AF} *(or the geometric Laspeyres index* P_{GL}*), which can be computed using the same information set that is normally available to statistical agencies.*

Table 23.14. Annual Fixed-Base Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2091	1.1884	1.1987
1973	1.4144	1.3536	1.3837

Table 23.15. Annual Approximate Fixed-Base Laspeyres, Paasche, Fisher, and Geometric Laspeyres Indices

Year	P_{AL}	P_{AP}	P_{AF}	P_{GL}
1970	1.0000	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982	1.0983
1972	1.2091	1.1903	1.1996	1.2003
1973	1.4144	1.3596	1.3867	1.3898

Table 23.16. Annual Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2052	1.1949	1.2001
1973	1.3994	1.3791	1.3892

23.41 Using the data from the artificial data set in Table 23.1 of Section B, the annual chained Laspeyres, Paasche, and Fisher indices can readily be calculated using the equations (23.16) through (23.18) for the chain links. The resulting indices are listed in Table 23.16. That table shows that the use of chained indices has substantially narrowed the gap between the Paasche and Laspeyres indices. The difference between the chained annual Laspeyres and Paasche indices in 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas in Table 23.14, the difference between the fixed-base annual Laspeyres and Paasche indices in 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained annual indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices.* Comparing Tables 23.14 and 23.16, one can see that for this particular artificial data set, the annual fixed-base Fisher indices are very close to their annual chained Fisher counterparts. However, the annual chained Fisher indices should normally be regarded as the more desirable target index to approximate, because this index will

[14]See footnote 12.

Table 23.17. Annual Approximate Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_{AL}	P_{AP}	P_{AF}
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982
1972	1.2051	1.1952	1.2002
1973	1.3995	1.3794	1.3894

normally give better results if prices and revenue shares are changing substantially over time.[15]

23.42 The current-year weights, $s_n^{t,m}$ and σ_m^t and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link equations (23.16) through (23.18), can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and σ_m^0. This leads to the annual approximate chained Laspeyres, Paasche, and Fisher indices listed in Table 23.17.

23.43 Comparing the entries in Tables 23.16 and 23.17 shows that the approximate chained annual Laspeyres, Paasche, and Fisher indices are extremely close to the corresponding true chained annual Laspeyres, Paasche, and Fisher indices. Therefore, for the artificial data set, the true annual chained Fisher can be closely approximated by the corresponding approximate Fisher index, which can be computed using the same information set that is normally available to statistical agencies.

23.44 The approach to computing annual indices outlined in this section, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indices, should be contrasted with the approach that simply takes the arithmetic mean of the 12 monthly indices. The problem with the latter approach is that months in which revenues are below the average (e.g., February) are given the same weight in the unweighted annual average as are months in which revenues are above the average (e.g., December).

E. Rolling-Year Annual Indices

23.45 In the previous section, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. However, there is no need to restrict attention to calendar-year comparisons; any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the noncalendar year are compared to the January data of the base year, the February data of the noncalendar year are compared to the February data of the base year, and so on.[16] Alterman, Diewert, and Feenstra (1999, p. 70) called the resulting indices *rolling-year* or *moving-year* indices.[17]

23.46 In order to theoretically justify the rolling-year indices from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996b, pp. 32–34; and 1999a, pp. 56–61).

23.47 The problems involved in constructing rolling-year indices for the artificial data set that was introduced in Section B are now considered. For both fixed-base and chained rolling-year indices, the first 13 index number calculations are the same. For the year that ends with the data for December 1970, the index is set equal to 1 for the Laspeyres, Paasche, and Fisher moving-year indices. The base-year data are the 44 nonzero price and quantity observations for the calendar year 1970. When the data for January 1971 become available, the three nonzero price and quantity entries for January of calendar year 1970 are dropped and replaced with the corresponding entries for January 1971. The data for the remaining months of the comparison year remain the same; that is, for February through December of the comparison year, the data for the rolling year are set equal to the corresponding entries for February through December 1970. Thus, the Laspeyres, Paasche, or Fisher rolling-year index value for January 1971 compares the prices and quantities of January 1971 with the corresponding prices and quantities of January 1970, and for the remaining months of this first moving year, the prices and quantities of February through December 1970 are simply compared with the exact same prices and quantities of February through December 1970. When the data for February 1971 become available, the three nonzero price and quantity entries for February for the last rolling year (which are equal to the three nonzero price and quantity entries for February 1970) are dropped and replaced with the corresponding entries for February 1971. The resulting data become the price and quantity data for the second rolling year. The Laspeyres, Paasche, or Fisher rolling-year index value

[15]"Better" in the sense that the gap between the Laspeyres and Paasche indices will normally be reduced using chained indices under these circumstances. Of course, if there are no substantial trends in prices so that prices are just randomly changing, then it will generally be preferable to use the fixed-base Fisher index.

[16]Diewert (1983b) suggested this type of comparison and termed the resulting index a "split year" comparison.

[17]Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term "rolling year" seems to be well established in the business literature in the United Kingdom.

for February 1971 compares the prices and quantities of January and February 1971 with the corresponding prices and quantities of January and February 1970. For the remaining months of this first moving year, the prices and quantities of March through December 1971 are compared with the exact same prices and quantities of March through December 1970. This process of exchanging the price and quantity data of the current month in 1971 with the corresponding data of the same month in the base year 1970 in order to form the price and quantity data for the latest rolling year continues until December 1971 is reached, when the current rolling year becomes the calendar year 1971. Thus, the Laspeyres, Paasche, and Fisher rolling-year indices for December 1971 are equal to the corresponding fixed-base (or chained) annual Laspeyres, Paasche, and Fisher indices for 1971 listed in Tables 23.14 or 23.16 above.

23.48 Once the first 13 entries for the rolling-year indices have been defined as indicated, the remaining fixed-base rolling-year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and rearranging them so that the January data in the rolling year are compared to the January data in the base year, the February data in the rolling year are compared to the February data in the base year, and so on. The resulting fixed-base rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in Table 23.18.

23.49 Once the first 13 entries for the fixed-base rolling-year indices have been defined as indicated above, the remaining *chained* rolling-year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and comparing them to the corresponding data of the rolling year of the 12 months preceding the current rolling year. The resulting chained rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in the last three columns of Table 23.18. Note that the first 13 entries of the fixed-base Laspeyres, Paasche, and Fisher indices are equal to the corresponding entries for the chained Laspeyres, Paasche, and Fisher indices. Also the entries for December (month 12) of 1970, 1971, 1972, and 1973 for the fixed-base rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding fixed-base annual Laspeyres, Paasche, and Fisher indices listed in Table 23.14. Similarly, the entries in Table 23.18 for December (month 12) 1970, 1971, 1972, and 1973 for the chained rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding chained annual Laspeyres, Paasche, and Fisher indices listed in Table 23.16.

23.50 In Table 23.18, the rolling-year indices are smooth and free from seasonal fluctuations. For the fixed-base indices, each entry can be viewed as a *seasonally adjusted annual XMPI* that compares the data of the 12 consecutive months that end with the year and month indicated with the corresponding price and quantity data of the 12 months in the base year, 1970. Thus, rolling-year indices offer statistical agencies an *objective* and *reproducible* method of seasonal adjustment that can compete with existing time-series methods of seasonal adjustment.[18]

23.51 Table 23.18 shows that the use of chained indices has substantially narrowed the gap between the fixed-base moving-year Paasche and Laspeyres indices. The difference between the rolling-year chained Laspeyres and Paasche indices in December 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas the difference between the rolling-year fixed-base Laspeyres and Paasche indices in December 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices.* As in the previous section, the chained Fisher rolling-year index is regarded as the *target seasonally adjusted annual index* when seasonal products are in the scope of the consumer price index. This type of index is also a suitable index for central banks to use for inflation targeting purposes.[19] The six series in Table 23.18 are charted in Figure 23.1. The fixed-base Laspeyres index is the highest one, followed by the chained Laspeyres, the two Fisher indices (which are virtually indistinguishable), the chained Paasche, and, finally, the fixed-base Paasche. An increase in the slope of each graph can clearly be seen for the last 8 months, reflecting the increase in the month-to-month inflation rates that was built into the data for the last 12 months of the data set.[20]

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[18]For discussions on the merits of econometric or time-series methods versus index number methods of seasonal adjustment, see Diewert (1999a, pp. 61–68) and Alterman, Diewert, and Feenstra (1999, pp. 78–110). The basic problem with time-series methods of seasonal adjustment is that the target seasonally adjusted index is difficult to specify in an unambiguous way; that is, there are an infinite number of possible target indices. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Thus, different econometricians will tend to generate different seasonally adjusted series, which leads to a lack of reproducibility.

[19]See Diewert (2002c) for a discussion of the measurement issues involved in choosing an index for inflation targeting purposes.

[20]The arithmetic average of the 36 month-over-month inflation rates for the rolling-year fixed-base Fisher indices is 1.0091; the average of these rates for the first 24 months is 1.0076; for the last 12 months it is 1.0120; and for the last 2 months it is 1.0156. Thus, the increased month-to-month inflation rates for the last year are not *fully* reflected in the rolling-year indices until a full 12 months have passed. However, the fact that inflation has *increased* for the last 12 months of data compared to the earlier months is picked up almost immediately.

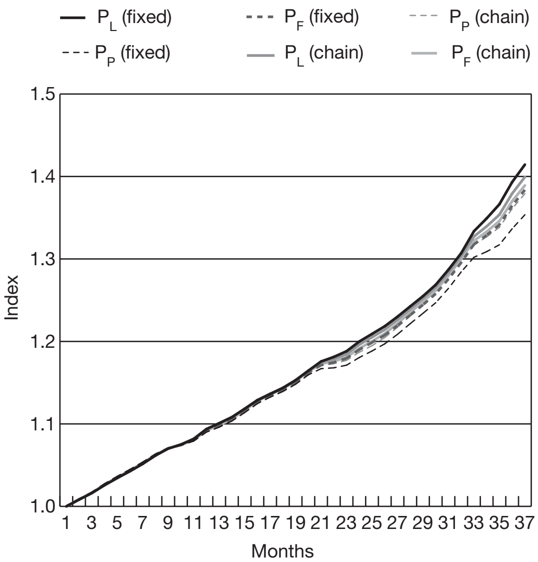
Table 23.18. Rolling-Year Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L (fixed)	P_P (fixed)	P_F (fixed)	P_L (chained)	P_P (chained)	P_F (chained)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0087	1.0085	1.0082	1.0087	1.0085
	2	1.0161	1.0170	1.0165	1.0161	1.0170	1.0165
	3	1.0257	1.0274	1.0265	1.0257	1.0274	1.0265
	4	1.0344	1.0364	1.0354	1.0344	1.0364	1.0354
	5	1.0427	1.0448	1.0438	1.0427	1.0448	1.0438
	6	1.0516	1.0537	1.0527	1.0516	1.0537	1.0527
	7	1.0617	1.0635	1.0626	1.0617	1.0635	1.0626
	8	1.0701	1.0706	1.0704	1.0701	1.0706	1.0704
	9	1.0750	1.0740	1.0745	1.0750	1.0740	1.0745
	10	1.0818	1.0792	1.0805	1.0818	1.0792	1.0805
	11	1.0937	1.0901	1.0919	1.0937	1.0901	1.0919
	12	1.1008	1.0961	1.0984	1.1008	1.0961	1.0984
1972	1	1.1082	1.1035	1.1058	1.1081	1.1040	1.1061
	2	1.1183	1.1137	1.1160	1.1183	1.1147	1.1165
	3	1.1287	1.1246	1.1266	1.1290	1.1260	1.1275
	4	1.1362	1.1324	1.1343	1.1366	1.1342	1.1354
	5	1.1436	1.1393	1.1414	1.1437	1.1415	1.1426
	6	1.1530	1.1481	1.1505	1.1528	1.1505	1.1517
	7	1.1645	1.1595	1.1620	1.1644	1.1622	1.1633
	8	1.1757	1.1670	1.1713	1.1747	1.1709	1.1728
	9	1.1812	1.1680	1.1746	1.1787	1.1730	1.1758
	10	1.1881	1.1712	1.1796	1.1845	1.1771	1.1808
	11	1.1999	1.1805	1.1901	1.1962	1.1869	1.1915
	12	1.2091	1.1884	1.1987	1.2052	1.1949	1.2001
1973	1	1.2184	1.1971	1.2077	1.2143	1.2047	1.2095
	2	1.2300	1.2086	1.2193	1.2263	1.2172	1.2218
	3	1.2425	1.2216	1.2320	1.2393	1.2310	1.2352
	4	1.2549	1.2341	1.2444	1.2520	1.2442	1.2481
	5	1.2687	1.2469	1.2578	1.2656	1.2579	1.2617
	6	1.2870	1.2643	1.2756	1.2835	1.2758	1.2797
	7	1.3070	1.2843	1.2956	1.3038	1.2961	1.3000
	8	1.3336	1.3020	1.3177	1.3273	1.3169	1.3221
	9	1.3492	1.3089	1.3289	1.3395	1.3268	1.3331
	10	1.3663	1.3172	1.3415	1.3537	1.3384	1.3460
	11	1.3932	1.3366	1.3646	1.3793	1.3609	1.3700
	12	1.4144	1.3536	1.3837	1.3994	1.3791	1.3892

23.52 As in the previous section, the current-year weights, $s_n^{t,m}$ and σ_m^t and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link equations (23.16) through (23.18) or in the corresponding fixed-base formulas, can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and σ_m^0. This leads to the annual approximate fixed-base and chained rolling-year Laspeyres, Paasche, and Fisher indices listed in Table 23.19.

23.53 Comparing the indices in Tables 23.18 and 23.19, one can see that the approximate rolling-year fixed-base and chained Laspeyres, Paasche, and

Figure 23.1. Rolling-Year Fixed-Base and Chained Laspeyres, Paasche, and Fisher Indices



Fisher indices listed in Table 23.19 are very close to their true rolling-year counterparts listed in Table 23.18. In particular, the approximate chain rolling-year Fisher index (which can be computed using just base-year expenditure share information along with current information on prices) is very close to the preferred target index, the rolling-year chained Fisher index. In December 1973, these two indices differ by only 0.014 percent (1.3894/1.3892 = 1.00014). The indices in Table 23.19 are charted in Figure 23.2. Figures 23.1 and 23.2 are similar; in particular, the Fisher fixed-base and chained indices are virtually identical in both figures.

23.54 These tables demonstrate that year-over-year monthly indices and their generalizations to rolling-year indices perform very well using the modified Turvey data set; that is, like is compared to like and the existence of seasonal products does *not* lead to erratic fluctuations in the indices. The only drawback to the use of these indices is that it seems that they cannot give any information on *short-term month-to-month fluctuations in prices. This is most evident if seasonal baskets are completely different for each month, because in this case there is no possibility of comparing prices on a month-to-month basis.* However, the following section shows that a current period year-over-year monthly index *can* be used to predict a rolling-year index that is centered at the current month.

F. Predicting Rolling-Year Index Using Current-Period Year-over-Year Monthly Index

23.55 In a regime where the long-run trend in prices is smooth, changes in the year-over-year inflation rate for this month compared to last month could theoretically give valuable information about the long-run trend in price inflation. For the modified Turvey data set, this conjecture turns out to be true, as is seen below.

23.56 The basic idea is illustrated using the fixed-base Laspeyres rolling-year indices that are listed in Table 23.18 and the year-over-year monthly fixed-base Laspeyres indices listed in Table 23.3. In Table 23.18, the fixed-base Laspeyres rolling-year entry for December 1971 compares the 12 months of price and quantity data pertaining to 1971 with the corresponding prices and quantities pertaining to 1970. This index number is the first entry in the first column of Table 23.20 and is labeled as P_L. Thus, in the first column of Table 23.20, the fixed-base rolling-year Laspeyres index, P_{LRY} taken from Table 23.18, is tabled starting at December 1971 and carrying through to December 1973, 24 observations in all. The first entry of this column shows that the index is a weighted average of year-over-year price relatives over all 12 months in 1970 and 1971. Thus, this index is an average of year-over-year monthly price changes, centered between June and July of the two years whose prices are being compared. As a result, an *approximation* to this annual index could be obtained by taking the arithmetic average of the June and July year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 6 and 7 for the year 1971 in Table 23.3, 1.0844 and 1.1103).[21] For the next rolling-year fixed-base Laspeyres index corresponding to the January 1972 entry in Table 23.18, *an approximation to this rolling-year index,* P_{ARY}, could be derived by taking the arithmetic average of the July and August year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 7 and 8

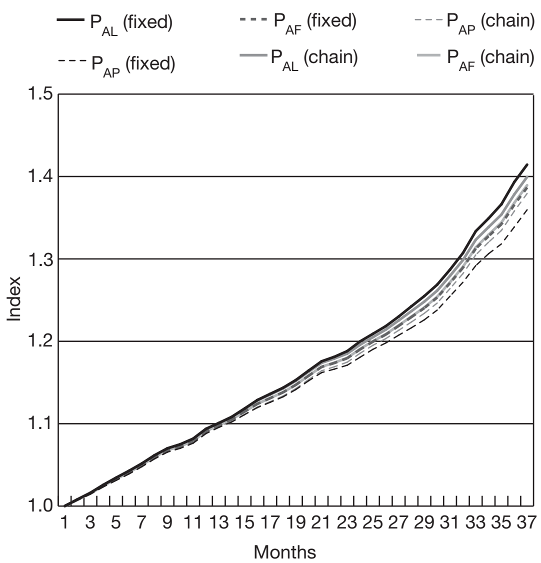
[21] If an average of the year-over-year monthly indices for May, June, July, and August were taken, a better approximation to the annual index could be obtained, and if an average of the year-over-year monthly indices for April, May, June, July, August, and September were taken, an even better approximation to the annual index could be obtained, and so on.

Table 23.19. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_{AL} (fixed)	P_{AP} (fixed)	P_{AF} (fixed)	P_{AL} (chained)	P_{AP} (chained)	P_{AF} (chained)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0074	1.0078	1.0082	1.0074	1.0078
	2	1.0161	1.0146	1.0153	1.0161	1.0146	1.0153
	3	1.0257	1.0233	1.0245	1.0257	1.0233	1.0245
	4	1.0344	1.0312	1.0328	1.0344	1.0312	1.0328
	5	1.0427	1.0390	1.0409	1.0427	1.0390	1.0409
	6	1.0516	1.0478	1.0497	1.0516	1.0478	1.0497
	7	1.0617	1.0574	1.0596	1.0617	1.0574	1.0596
	8	1.0701	1.0656	1.0679	1.0701	1.0656	1.0679
	9	1.0750	1.0702	1.0726	1.0750	1.0702	1.0726
	10	1.0818	1.0764	1.0791	1.0818	1.0764	1.0791
	11	1.0937	1.0881	1.0909	1.0937	1.0881	1.0909
	12	1.1008	1.0956	1.0982	1.1008	1.0956	1.0982
1972	1	1.1082	1.1021	1.1051	1.1083	1.1021	1.1052
	2	1.1183	1.1110	1.1147	1.1182	1.1112	1.1147
	3	1.1287	1.1196	1.1241	1.1281	1.1202	1.1241
	4	1.1362	1.1260	1.1310	1.1354	1.1268	1.1311
	5	1.1436	1.1326	1.1381	1.1427	1.1336	1.1381
	6	1.1530	1.1415	1.1472	1.1520	1.1427	1.1473
	7	1.1645	1.1522	1.1583	1.1632	1.1537	1.1584
	8	1.1757	1.1620	1.1689	1.1739	1.1642	1.1691
	9	1.1812	1.1663	1.1737	1.1791	1.1691	1.1741
	10	1.1881	1.1710	1.1795	1.1851	1.1747	1.1799
	11	1.1999	1.1807	1.1902	1.1959	1.1855	1.1907
	12	1.2091	1.1903	1.1996	1.2051	1.1952	1.2002
1973	1	1.2184	1.1980	1.2082	1.2142	1.2033	1.2087
	2	1.2300	1.2074	1.2187	1.2253	1.2133	1.2193
	3	1.2425	1.2165	1.2295	1.2367	1.2235	1.2301
	4	1.2549	1.2261	1.2404	1.2482	1.2340	1.2411
	5	1.2687	1.2379	1.2532	1.2615	1.2464	1.2540
	6	1.2870	1.2548	1.2708	1.2795	1.2640	1.2717
	7	1.3070	1.2716	1.2892	1.2985	1.2821	1.2903
	8	1.3336	1.2918	1.3125	1.3232	1.3048	1.3139
	9	1.3492	1.3063	1.3276	1.3386	1.3203	1.3294
	10	1.3663	1.3182	1.3421	1.3538	1.3345	1.3441
	11	1.3932	1.3387	1.3657	1.3782	1.3579	1.3680
	12	1.4144	1.3596	1.3867	1.3995	1.3794	1.3894

for the year 1971 in Table 23.3, 1.1103 and 1.0783, respectively). These arithmetic averages of the two year-over-year monthly indices that are in the middle of the corresponding rolling year are listed in the third column of Table 23.20. Table 23.20 shows that column 3, P_{ARY}, does not approximate column 1 particularly well, because the approximate indices in column 3 have some pronounced seasonal fluctuations, whereas the rolling-year indices in column 1, P_{LRY}, are free from seasonal fluctuations.

Figure 23.2. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices



23.57 In the fourth column of Table 23.20, some *seasonal adjustment factors* are listed. For the first 12 observations, the entries in column 4 are simply the ratios of the entries in column 1 divided by the corresponding entries in column 3; that is, for the first 12 observations, the seasonal adjustment factors, *SAF,* are simply the ratio of the rolling-year indices starting at December 1971 divided by the arithmetic average of the two year-over-year monthly indices that are in the middle of the corresponding rolling year.[22] The initial 12 seasonal adjustment factors are then just repeated for the remaining entries for column 4.

23.58 Once the seasonal adjustment factors have been defined, the approximate rolling-year index P_{ARY} can be multiplied by the corresponding seasonal adjustment factor, *SAF*, to form a *seasonally adjusted approximate rolling-year index,* P_{SAARY}, which is listed in column 2 of Table 23.20.

Table 23.20. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices

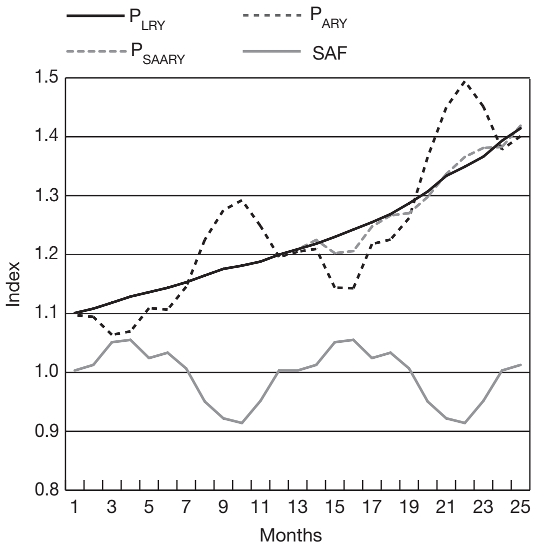
Year	Month	P_{LRY}	P_{SAARY}	P_{ARY}	*SAF*
1971	12	1.1008	1.1008	1.0973	1.0032
1972	1	1.1082	1.1082	1.0943	1.0127
	2	1.1183	1.1183	1.0638	1.0512
	3	1.1287	1.1287	1.0696	1.0552
	4	1.1362	1.1362	1.1092	1.0243
	5	1.1436	1.1436	1.1066	1.0334
	6	1.1530	1.1530	1.1454	1.0066
	7	1.1645	1.1645	1.2251	0.9505
	8	1.1757	1.1757	1.2752	0.9220
	9	1.1812	1.1812	1.2923	0.9141
	10	1.1881	1.1881	1.2484	0.9517
	11	1.1999	1.1999	1.1959	1.0033
	12	1.2091	1.2087	1.2049	1.0032
1973	1	1.2184	1.2249	1.2096	1.0127
	2	1.2300	1.2024	1.1438	1.0512
	3	1.2425	1.2060	1.1429	1.0552
	4	1.2549	1.2475	1.2179	1.0243
	5	1.2687	1.2664	1.2255	1.0334
	6	1.2870	1.2704	1.2620	1.0066
	7	1.3070	1.2979	1.3655	0.9505
	8	1.3336	1.3367	1.4498	0.9220
	9	1.3492	1.3658	1.4943	0.9141
	10	1.3663	1.3811	1.4511	0.9517
	11	1.3932	1.3827	1.3783	1.0032
	12	1.4144	1.4188	1.4010	1.0127

23.59 Compare columns 1 and 2 in Table 23.20: The rolling-year fixed-base Laspeyres index P_{LRY} and the seasonally adjusted approximate rolling-year index P_{SAARY} are identical for the first 12 observations, which follows by construction because P_{SAARY} equals the approximate rolling-year index P_{ARY} multiplied by the seasonal adjustment factor *SAF,* which in turn is equal to the rolling-year Laspeyres index P_{LRY} divided by P_{ARY}. However, starting at December 1972, the rolling-year index P_{LRY} differs from the corresponding seasonally adjusted approximate rolling-year index P_{SAARY}. It is apparent that for these last 13 months, P_{SAARY} is surprisingly close to P_{LRY}.[23] P_{LRY}, P_{SAARY}, and P_{ARY} are

[22]Thus, if *SAF* is *greater than one,* this means that the two months in the middle of the corresponding rolling-year have year-over-year rates of price increase that average out to a number *below* the overall average of the year-over-year rates of price increase for the entire rolling-year. The opposite is true if *SAF* is less than one.

[23]The means for the last 13 observations in columns 1 and 2 of Table 23.20 are1.2980 and 1.2930. A regression of P_L on P_{SAARY} leads to an R^2 of 0.9662 with an estimated variance of the residual of 0.000214.

Figure 23.3. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices



graphed in Figure 23.3. Owing to the acceleration in the monthly inflation rate for the last year of data, it can be seen that the seasonally adjusted approximate rolling-year series, P_{SAARY}, does not pick up this accelerated inflation rate for the first few months of the last year (it lies well below P_{LRY} for February and March 1973), but in general, it predicts the corresponding centered year quite well.

23.60 The above results for the modified Turvey data set are quite encouraging. If these results can be replicated for other data sets, *statistical agencies will be able to use the latest information on year-over-year monthly inflation to predict reasonably well the (seasonally adjusted) rolling-year inflation rate for a rolling year that is centered around the last two months*. Thus, policymakers and other interested users of the XMPI could obtain a reasonably accurate forecast of trend inflation (centered around the current month) some six months in advance of the final estimates.

23.61 The method of seasonal adjustment used in this section is rather crude compared with some of the sophisticated econometric or statistical methods that are available. These more sophisticated methods could be used to improve the forecasts of trend inflation. However, it should be noted that if improved forecasting methods are used, it will be useful to use the rolling-year indices as *targets* for the forecasts rather than using a statistical package that simultaneously seasonally adjusts current data and calculates a trend rate of inflation. What is being suggested here is that the rolling-year concept can be used to make reproducible the estimates of trend inflation that are generated by existing statistical methods of seasonal adjustment.[24]

23.62 In this section and the previous sections, all of the suggested indices have been based on year-over-year monthly indices and their averages. In the subsequent sections of this chapter, attention is turned to more traditional price indices that attempt to compare the prices in the current month with the prices in a previous month.

G. Maximum Overlap Month-to-Month Price Indices

23.63 A reasonable method for dealing with seasonal products in the context of picking a target index for a month-to-month XMPI is the following:[25]

- Identify products that are produced in both months; and
- For this maximum overlap set of products, calculate one of the three indices recommended in previous chapters, that is, the Fisher, Walsh, or Törnqvist Theil index.[26]

Thus, the bilateral index number formula is applied only to the subset of products that are present in both periods.[27]

23.64 The question now arises: Should the comparison month and the base month be adjacent months (thus

[24]The operator of a statistical seasonal adjustment package has to make somewhat arbitrary decisions on many factors; for example, are the seasonal factors additive or multiplicative? How long should the moving average be and what type? Thus, different operators of the seasonal adjustment package will tend to produce different estimates of the trend and the seasonal factors.

[25]For more on the economic approach and the assumptions on consumer preferences that can justify month-to-month maximum overlap indices, see Diewert (1999a, p. 51–56).

[26]In order to reduce the number of equations, definitions, and tables, only the Fisher index is considered in detail in this chapter.

[27]Keynes (1930, p. 95) called this the highest common factor method for making bilateral index number comparisons. This target index drops those strongly seasonal products that are not present in the marketplace during one of the two months being compared. Thus, the index number comparison is not completely comprehensive. Mudgett (1951, p. 46) called the error in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the "homogeneity error."

leading to chained indices) or should the base month be fixed (leading to fixed-base indices)? It seems reasonable to prefer chained indices over fixed-base indices for two reasons:

- The set of seasonal products that overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed-base month (such as January of a base year). The comparisons made using chained indices, therefore, will be more comprehensive and accurate than those made using a fixed base; and
- In many economies, on average 2 or 3 percent of price quotes disappear each month owing to the introduction of new products and the disappearance of older ones. This rapid sample attrition means that fixed-base indices rapidly become unrepresentative; as a consequence, it seems preferable to use chained indices, which can more closely follow market developments.[28]

23.65 It will be useful to review the notation at this point and define some new notation. Let there be N products that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is in the marketplace[29] in month m of year t (if the product is unavailable, define $p_n^{t,m}$ and $q_n^{t,m}$ to be 0). Let $p^{t,m} \equiv [p_1^{t,m}, p_2^{t,m}, \ldots, p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m}, q_2^{t,m}, \ldots, q_N^{t,m}]$ be the month m and year t price and quantity vectors, respectively. Let $S(t, m)$ be the set of products that is present in month m of year *t and the following month.* Then the maximum overlap Laspeyres, Paasche, and Fisher indices going from month m of year t to the following month can be defined as follows:[30]

$$P_L(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m)) = \frac{\sum_{n \in S(t,m)} p_n^{t,m+1} q_n^{t,m}}{\sum_{n \in S(t,m)} p_n^{t,m} q_n^{t,m}}; \; m = 1, 2, \ldots, 11; \tag{23.20}$$

$$P_P(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m)) = \frac{\sum_{n \in S(t,m)} p_n^{t,m+1} q_n^{t,m+1}}{\sum_{n \in S(t,m)} p_n^{t,m} q_n^{t,m+1}}; \; m = 1, 2, \ldots, 11; \tag{23.21}$$

$$P_F(p^{t,m}, p^{t,m+1}, q^{t,m}, q^{t,m+1}, S(t,m)) \equiv \sqrt{P_L(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m))} \times \sqrt{P_P(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m))}; \quad m = 1, 2, \ldots, 11. \tag{23.22}$$

Note that P_L, P_P, and P_F depend on the two (complete) price and quantity vectors pertaining to months m and $m + 1$ of year t, $p^{t,m}$, $p^{t,m+1}$, $q^{t,m}$, $q^{t,m+1}$, but they also depend on the set $S(t, m)$, which is the set of products that are present in both months. Thus, the product indices n that are in the summations on the right-hand sides of equations (23.20) through (23.22) include indices n that correspond to products that are present in *both* months, which is the meaning of $n \in S(t, m)$; that is, n belongs to the set $S(t, m)$.

23.66 To rewrite equations (23.20) through (23.22) in value share and price relative form, some additional notation is required. Define the value shares of product n in month m and $m + 1$ of year t, using the set of products that are present in month m of year t and the subsequent month, as follows:

$$s_n^{t,m}(t,m) = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i \in S(t,m)} p_i^{t,m} q_i^{t,m}}; \; n \in S(t,m); \; m = 1, 2, \ldots, 11; \tag{23.23}$$

$$s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1} q_n^{t,m+1}}{\sum_{i \in S(t,m)} p_i^{t,m+1} q_i^{t,m+1}}; \quad n \in S(t,m); \; m = 1, 2, \ldots, 11. \tag{23.24}$$

The notation in equations (23.23) and (23.24) is rather messy because $s_n^{t,m+1}(t, m)$ has to be distinguished from $s_n^{t,m+1}(t, m + 1)$. The value share $s_n^{t,m+1}(t, m)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month m of year t and the subsequent month, whereas $s_n^{t,m+1}(t, m + 1)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month $m + 1$ of year t and the subsequent month. Thus, the set of superscripts, t, $m + 1$ in $s_n^{t,m+1}(t, m)$, indicates that the value share is calculated using the price and quantity data of month

[28] This rapid sample degradation essentially forces some form of chaining at the elementary level in any case.

[29] As was seen in Chapter 20, it is necessary to have a target concept for the individual prices and quantities $p_n^{t,m}$ and $q_n^{t,m}$ at the finest level of aggregation. In most circumstances, these target concepts can be taken to be unit values for prices and total revenues for the quantities purchased.

[30] The equations are slightly different for the indices that go from December to January of the following year. In order to simplify the exposition, these equations are left for the reader.

$m + 1$ of year t and (t, m) indicates that the set of admissible products is restricted to the set of products that are present in both month m and the subsequent month.

23.67 Now define vectors of value shares. If product n is present in month m of year t and the following month, define $s_n^{t,m}(t, m)$ using equation (23.23); if this is not the case, define $s_n^{t,m}(t, m) = 0$. Similarly, if product n is present in month m of year t and the following month, define $s_n^{t,m+1}(t, m)$ using equation (23.24); if this is not the case, define $s_n^{t,m+1}(t, m) = 0$. Now define the N dimensional vectors:

$$s_n^{t,m}(t, m) \equiv \left[s_1^{t,m}(t, m), s_2^{t,m}(t, m), \ldots, s_N^{t,m}(t, m)\right]$$

and

$$s^{t,m+1}(t, m) \equiv \left[s_1^{t,m+1}(t, m), s_2^{t,m+1}(t, m), \ldots, s_N^{t,m+1}(t, m)\right].$$

Using these share definitions, one can rewrite the month-to-month Laspeyres, Paasche, and Fisher equations (23.20) through (23.22) in value share and price form as follows:

$$P_L(p^{t,m}, p^{t,m+1}, s^{t,m}(t, m)) \equiv \sum_{n \in S(t,m)} s_n^{t,m}(t, m)\left(p_n^{t,m+1}/p_n^{t,m}\right);\ m = 1, 2, \ldots, 11; \quad (23.25)$$

$$P_P(p^{t,m}, p^{t,m+1}, s^{t,m+1}(t, m)) \equiv \left[\sum_{n \in S(t,m)} s_n^{t,m+1}(t, m)\left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1};\ m = 1, 2, \ldots, 11; \quad (23.26)$$

$$P_F(p^{t,m}, p^{t,m+1}, s^{t,m}(t, m), s^{t,m+1}(t, m)) \equiv \sqrt{\sum_{n \in S(t,m)} s_n^{t,m}(t, m)\left(p_n^{t,m+1}/p_n^{t,m}\right)} \times \sqrt{\left[\sum_{n \in S(t,m)} s_n^{t,m+1}(t, m)\left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1}};\ m = 1, 2, \ldots, 11. \quad (23.27)$$

23.68 It is important to recognize that the value shares $s_n^{t,m}(t, m)$ that appear in the maximum overlap month-to-month Laspeyres index defined by equation (23.25) are *not* the value shares that could be taken from an establishment production survey for month m of year t; instead, they are the shares that result from trade in seasonal products that are present in month m of year t and are present in the following month. Similarly, the value shares $s_n^{t,m+1}(t, m)$ that appear in the maximum overlap month-to-month Paasche index defined by equation (23.26) are *not* the value shares that could be taken from an establishment production survey for month $m + 1$ of year t; instead, they are the shares that result from trade in seasonal products that are present in month $m + 1$ of year t but are also present in the preceding month.[31] The maximum overlap month-to-month Fisher index defined by equation (23.27) is the geometric mean of the Laspeyres and Paasche indices defined by equations (23.25) and (23.26).

23.69 Table 23.21 lists the maximum overlap chained month-to-month Laspeyres, Paasche, and Fisher price indices for the data listed in Section B above. These indices are defined by equations (23.25), (23.26), and (23.27).

23.70 The chained maximum overlap Laspeyres, Paasche, and Fisher indices for December 1973 are 1.0504, 0.1204, and 0.3556, respectively. Comparing these results to the year-over-year results listed in Tables 23.3, 23.4, and 23.5 indicates that the results in Table 23.21 are not at all realistic. These hugely different direct indices compared with the last row of Table 23.21 indicate that *the maximum overlap indices suffer from a significant downward bias for the artificial data set.*

23.71 What are the factors that can explain this downward bias? It is evident that part of the problem has to do with the seasonal pattern of prices for peaches and strawberries (products 2 and 4). These products are not present in the market for each month of the year. For the first month of the year when they become available, they have relatively high prices; in subsequent months, their prices drop substantially. The effects of these initially high prices (compared to the relatively low prices that prevailed in the last month that the products were available in the previous year) are not captured by the maximum overlap month-to-month indices, so the resulting indices build up a tremendous downward bias. The downward bias is most pronounced in the Paasche indices, which use the quantities or volumes of the current month. These volumes are relatively large compared to those of the initial month when the products become available, reflecting the effects of lower prices as the quantity made available in the market increases.

[31]It is important that the revenue value that are used in an index number formula add up to unity. The use of unadjusted expenditure shares from an establishment survey would lead to a systematic bias in the index number formula.

Table 23.21. Month-to-Month Maximum Overlap Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L	P_P	P_F
1970	1	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777
	3	0.9587	0.9594	0.9590
	4	1.0290	1.0534	1.0411
	5	1.1447	1.1752	1.1598
	6	1.1118	1.0146	1.0621
	7	1.1167	1.0102	1.0621
	8	1.1307	0.7924	0.9465
	9	1.0033	0.6717	0.8209
	10	0.9996	0.6212	0.7880
	11	1.0574	0.6289	0.8155
	12	1.0151	0.5787	0.7665
1971	1	1.0705	0.6075	0.8064
	2	1.0412	0.5938	0.7863
	3	1.0549	0.6005	0.7959
	4	1.1409	0.6564	0.8654
	5	1.2416	0.7150	0.9422
	6	1.1854	0.6006	0.8438
	7	1.2167	0.6049	0.8579
	8	1.2230	0.4838	0.7692
	9	1.0575	0.4055	0.6548
	10	1.0497	0.3837	0.6346
	11	1.1240	0.3905	0.6626
	12	1.0404	0.3471	0.6009
1972	1	1.0976	0.3655	0.6334
	2	1.1027	0.3679	0.6369
	3	1.1291	0.3765	0.6520
	4	1.1974	0.4014	0.6933
	5	1.2818	0.4290	0.7415
	6	1.2182	0.3553	0.6579
	7	1.2838	0.3637	0.6833
	8	1.2531	0.2794	0.5916
	9	1.0445	0.2283	0.4883
	10	1.0335	0.2203	0.4771
	11	1.1087	0.2256	0.5001
	12	1.0321	0.1995	0.4538
1973	1	1.0866	0.2097	0.4774
	2	1.1140	0.2152	0.4897
	3	1.1532	0.2225	0.5065
	4	1.2493	0.2398	0.5474
	5	1.3315	0.2544	0.5821
	6	1.2594	0.2085	0.5124
	7	1.3585	0.2160	0.5416
	8	1.3251	0.1656	0.4684
	9	1.0632	0.1330	0.3760
	10	1.0574	0.1326	0.3744
	11	1.1429	0.1377	0.3967
	12	1.0504	0.1204	0.3556

23.72 Table 23.22 lists the results using chained Laspeyres, Paasche, and Fisher indices for the artificial data set where the strongly seasonal products 2 and 4 are dropped from each comparison of prices. Thus, the indices in Table 23.22 are the usual chained Laspeyres, Paasche, and Fisher indices restricted to products 1, 3, and 5, which are available in each season. These indices are labeled as $P_L(3)$, $P_P(3)$, and $P_F(3)$.

23.73 The chained Laspeyres, Paasche, and Fisher indices (using only the three year-round products) for January 1973 are 1.2038, 0.5424, and 0.8081, respectively. From Tables 23.8, 23.9, and 23.10, the chained year-over-year Laspeyres, Paasche, and Fisher indices for January 1973 are 1.3274, 1.3243, and 1.3258 respectively. Thus, the chained indices using the year-round products, which are listed in Table 23.22, evidently *suffer from substantial downward biases.*

23.74 The data in Tables 23.1 and 23.2 demonstrate that the quantity of grapes (product 3) available in the market varies tremendously over the course of a year, with substantial increases in price for the months when grapes are almost out of season. Thus, the price of grapes decreases substantially as the quantity increases during the last half of each year, but the annual substantial increase in the price of grapes takes place in the first half of the year, when quantities in the market are small. This pattern of seasonal price and quantity changes will cause the overall index to take on a downward bias.[32] To verify that this conjecture is true, see the last three columns of Table 23.22, where chained Laspeyres, Paasche, and Fisher indices are calculated using only products 1 and 5. These indices are labeled $P_L(2)$, $P_P(2)$, and $P_F(2)$, respectively, and for January 1973, they are equal to 1.0033, 0.9408, and 0.9715, respectively. These estimates based on two year-round products are much closer to the chained year-over-year Laspeyres, Paasche, and Fisher indices for January 1973, which were 1.3274, 1.3243, and 1.3258, respectively, than were the estimates based on the three year-round products. However, it is clear that *the chained Laspeyres, Paasche, and Fisher indices restricted to products 1 and 5 still have substantial downward biases for the artificial data set.* Basically, the problems are caused by the high volumes associated with low or declining prices and the low volumes are caused by high or rising prices. These weight effects make the seasonal price

[32]Baldwin used the Turvey data to illustrate various treatments of seasonal products. He has a good discussion of what causes various month-to-month indices to behave badly. “It is a sad fact that for some seasonal product groups, monthly price changes are not meaningful, whatever the choice of formula” (1990, p. 264).

Table 23.22. Month-to-Month Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	$P_L(3)$	$P_P(3)$	$P_F(3)$	$P_L(2)$	$P_P(2)$	$P_F(2)$
1970	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777	0.9751	0.9780	0.9765
	3	0.9587	0.9594	0.9590	0.9522	0.9574	0.9548
	4	1.0290	1.0534	1.0411	1.0223	1.0515	1.0368
	5	1.1447	1.1752	1.1598	1.1377	1.1745	1.1559
	6	1.2070	1.2399	1.2233	1.2006	1.2424	1.2214
	7	1.2694	1.3044	1.2868	1.2729	1.3204	1.2964
	8	1.3248	1.1537	1.2363	1.3419	1.3916	1.3665
	9	1.0630	0.9005	0.9784	1.1156	1.1389	1.1272
	10	0.9759	0.8173	0.8931	0.9944	1.0087	1.0015
	11	1.0324	0.8274	0.9242	0.9839	0.9975	0.9907
	12	0.9911	0.7614	0.8687	0.9214	0.9110	0.9162
1971	1	1.0452	0.7993	0.9140	0.9713	0.9562	0.9637
	2	1.0165	0.7813	0.8912	0.9420	0.9336	0.9378
	3	1.0300	0.7900	0.9020	0.9509	0.9429	0.9469
	4	1.1139	0.8636	0.9808	1.0286	1.0309	1.0298
	5	1.2122	0.9407	1.0679	1.1198	1.1260	1.1229
	6	1.2631	0.9809	1.1131	1.1682	1.1763	1.1723
	7	1.3127	1.0170	1.1554	1.2269	1.2369	1.2319
	8	1.3602	0.9380	1.1296	1.2810	1.2913	1.2861
	9	1.1232	0.7532	0.9198	1.1057	1.0988	1.1022
	10	1.0576	0.7045	0.8632	1.0194	1.0097	1.0145
	11	1.1325	0.7171	0.9012	1.0126	1.0032	1.0079
	12	1.0482	0.6373	0.8174	0.9145	0.8841	0.8992
1972	1	1.1059	0.6711	0.8615	0.9652	0.9311	0.9480
	2	1.1111	0.6755	0.8663	0.9664	0.9359	0.9510
	3	1.1377	0.6912	0.8868	0.9863	0.9567	0.9714
	4	1.2064	0.7371	0.9430	1.0459	1.0201	1.0329
	5	1.2915	0.7876	1.0086	1.1202	1.0951	1.1075
	6	1.3507	0.8235	1.0546	1.1732	1.1470	1.1600
	7	1.4091	0.8577	1.0993	1.2334	1.2069	1.2201
	8	1.4181	0.7322	1.0190	1.2562	1.2294	1.2427
	9	1.1868	0.5938	0.8395	1.1204	1.0850	1.1026
	10	1.1450	0.5696	0.8076	1.0614	1.0251	1.0431
	11	1.2283	0.5835	0.8466	1.0592	1.0222	1.0405
	12	1.1435	0.5161	0.7682	0.9480	0.8935	0.9204
1973	1	1.2038	0.5424	0.8081	1.0033	0.9408	0.9715
	2	1.2342	0.5567	0.8289	1.0240	0.9639	0.9935
	3	1.2776	0.5755	0.8574	1.0571	0.9955	1.0259
	4	1.3841	0.6203	0.9266	1.1451	1.0728	1.1084
	5	1.4752	0.6581	0.9853	1.2211	1.1446	1.1822
	6	1.5398	0.6865	1.0281	1.2763	1.1957	1.2354
	7	1.6038	0.7136	1.0698	1.3395	1.2542	1.2962
	8	1.6183	0.6110	0.9944	1.3662	1.2792	1.3220
	9	1.3927	0.5119	0.8443	1.2530	1.1649	1.2081
	10	1.3908	0.5106	0.8427	1.2505	1.1609	1.2049
	11	1.5033	0.5305	0.8930	1.2643	1.1743	1.2184
	12	1.3816	0.4637	0.8004	1.1159	1.0142	1.0638

declines bigger than the seasonal price increases using month-to-month index number formulas with variable weights.[33]

23.75 In addition to the downward biases that show up in Tables 23.21 and 23.22, all of these month-to-month chained indices show substantial seasonal fluctuations in prices over the course of a year. Therefore, these month-to-month indices are of little use to policymakers who are interested in short-term inflationary trends. *If the purpose of the month-to-month XMPIs is to indicate changes in general inflation, then statistical agencies should be cautious about including products that show strong seasonal fluctuations in prices in the month-to-month index.*[34] If seasonal products are included in a month-to-month index that is meant to indicate general inflation, then a seasonal adjustment procedure should be used to remove these strong seasonal fluctuations. Some simple types of seasonal adjustment procedures are considered in Section K.

23.76 The rather poor performance of the month-to-month indices listed in the last two tables does not always occur in the context of seasonal products. In the context of calculating XMPIs using quarterly data for the United States, Alterman, Diewert, and Feenstra (1999) found that maximum overlap month-to-month indices worked reasonably well.[35] However, statistical agencies should check that their month-to-month indices are at least approximately consistent with the corresponding year-over-year indices.

23.77 The various Paasche and Fisher indices computed in this section could be approximated by indices that replaced all current-period value shares with the corresponding value shares from the base year. These approximate Paasche and Fisher indices will not be reproduced here because they resemble their real counterparts and are themselves subject to tremendous downward bias.

H. Annual Basket Indices with Carryforward of Unavailable Prices

23.78 Recall that the Lowe (1823) index defined in earlier chapters had two reference periods:[36]

- The vector of quantity weights and
- The base-period prices.

The *Lowe index* for month m was defined by the following equation:

$$P_{LO}(p^0, p^m, q) = \frac{\sum_{n=1}^{N} p_n^m q_n}{\sum_{n=1}^{N} p_n^0 q_n}; \tag{23.28}$$

where $p^0 \equiv [p_1^0, \ldots, p_N^0]$ is the price reference period price vector, $p^m \equiv [p_1^m, \ldots, p_N^m]$ is the current month m price vector, and $q \equiv [q_1, \ldots, q_N]$ is the weight reference year quantity vector. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year is 1970, and the resulting reference year quantity vector turns out to be

$$q \equiv [q_1, \ldots, q_5] = [53889, 12881, 9198, 5379, 68653]. \tag{23.29}$$

The price reference period for the prices is December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Lowe index with carryforward of missing prices using the modified Turvey data set can be found in column 1 of Table 23.23.

[33]This remark has an application to Chapter 20 on elementary indices where it was noted that irregular sales during the course of a year could induce a similar downward bias in a month-to-month index that used monthly weights. Another problem with month-to-month chained indices is that purchases and sales of individual products can become irregular as the time period becomes shorter and shorter and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003, p. 125) find an *upward* bias for their chained *weekly* indices for canned tuna compared to a fixed-base index; their bias was caused by variable weight effects owing to the timing of advertising expenditures. In general, these drift effects of chained indices can be reduced by lengthening the time period, so that the *trends* in the data become more prominent than the *high-frequency fluctuations*.

[34]However, if the purpose of the index is to compare the prices that resident households and producers *actually receive* and nonresident households and producers actually *pay* in two consecutive months, ignoring the possibility that the purchasers may regard a seasonal good as being qualitatively different in the two months, then the production of month-to-month XMPIs that have large seasonal fluctuations can be justified.

[35]They checked the validity of their month-to-month indices by cumulating them for four quarters and comparing them to the corresponding year-over-year indices. They found only relatively small differences. However, note that irregular high-frequency fluctuations tend to be smaller for quarters than for months. For this reason chained quarterly indices can be expected to perform better than chained monthly or weekly indices.

[36]In the context of seasonal price indices, this type of index corresponds to Bean and Stine's (1924, p. 31) Type A index.

Table 23.23. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carryforward Prices

Year	Month	P_{LO}	P_Y	P_{GL}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0554	1.0609	1.0595	1.0091
	2	1.0711	1.0806	1.0730	1.0179
	3	1.1500	1.1452	1.1187	1.0242
	4	1.2251	1.2273	1.1942	1.0298
	5	1.3489	1.3652	1.3249	1.0388
	6	1.4428	1.4487	1.4068	1.0478
	7	1.3789	1.4058	1.3819	1.0547
	8	1.3378	1.3797	1.3409	1.0631
	9	1.1952	1.2187	1.1956	1.0729
	10	1.1543	1.1662	1.1507	1.0814
	11	1.1639	1.1723	1.1648	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.1370	1.1523	1.1465	1.1065
	2	1.1731	1.1897	1.1810	1.1174
	3	1.2455	1.2539	1.2363	1.1254
	4	1.3155	1.3266	1.3018	1.1313
	5	1.4262	1.4508	1.4183	1.1402
	6	1.5790	1.5860	1.5446	1.1502
	7	1.5297	1.5550	1.5349	1.1591
	8	1.4416	1.4851	1.4456	1.1690
	9	1.3038	1.3342	1.2974	1.1806
	10	1.2752	1.2960	1.2668	1.1924
	11	1.2852	1.3034	1.2846	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.2427	1.2710	1.2518	1.2386
	2	1.3003	1.3308	1.3103	1.2608
	3	1.3699	1.3951	1.3735	1.2809
	4	1.4691	1.4924	1.4675	1.2966
	5	1.5972	1.6329	1.5962	1.3176
	6	1.8480	1.8541	1.7904	1.3406
	7	1.7706	1.8010	1.7711	0.0000
	8	1.6779	1.7265	1.6745	0.0000
	9	1.5253	1.5676	1.5072	0.0000
	10	1.5371	1.5746	1.5155	0.0000
	11	1.5634	1.5987	1.5525	0.0000
	12	1.4181	1.4521	1.4236	0.0000

23.79 Baldwin's comments on this type of annual basket (AB) index are worth quoting at length:

> For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256–257) calls it an index of "a hybrid sort." Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is "What would have [sic] the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices nonetheless retained their own seasonal behaviour?" It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences. (1990, p. 258)

In spite of Baldwin's somewhat negative comments on the Lowe index, it is the index that is preferred by many statistical agencies, so it is necessary to study its properties in the context of strongly seasonal data.

23.80 Recall that the *Young* (1812) *index* was defined in Chapters 1 and 16 as follows:

$$P_Y(p^0, p^m, s) = \sum_{n=1}^{N} s_n \left(p_n^m / p_n^0\right), \tag{23.30}$$

where $s \equiv [s_1, \ldots, s_N]$ is the weight reference year vector of value shares. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year is 1970 and the resulting value share vector turns out to be

$$s \equiv [s_1, \ldots, s_5] = [0.3284, 0.1029, 0.0674, 0.0863, 0.4149]. \tag{23.31}$$

Again, the base period for the prices is December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Young index with carryforward of missing prices using the modified Turvey data set can be found in column 2 of Table 23.23.

23.81 The *geometric Laspeyres index* was defined in Chapter 20 as follows:

$$P_{GL}(p^0, p^m, s) \equiv \prod_{n=1}^{N} \left(p_n^m / p_n^0\right)^{s_n}. \tag{23.32}$$

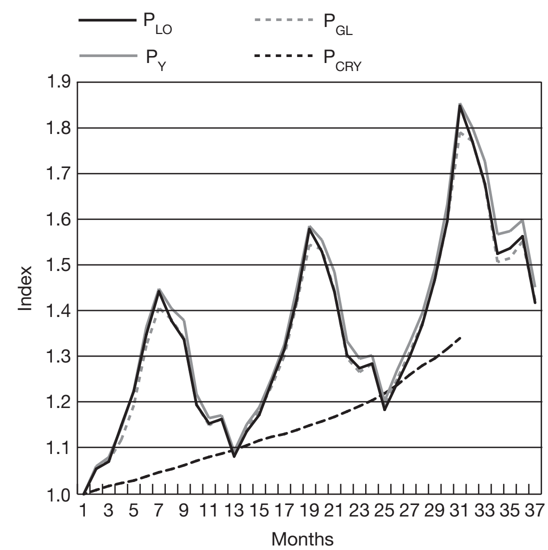
Thus, the geometric Laspeyres index makes use of the same information as the Young index, except that a geometric average of the price relatives is taken instead of an arithmetic one. Again, the weight reference year is 1970, the price reference period is December 1970, and the index is illustrated using the modified Turvey data set with carryforward of missing prices. See column 3 of Table 23.23.

23.82 It is interesting to compare the above three indices that use annual baskets to the fixed-base Laspeyres rolling-year indices computed earlier. However, the rolling-year index that ends in the current month is centered five and a half months backward. Thus, the above three annual basket-type indices will be compared with an arithmetic average of two rolling-year indices that have their last month five and six months forward. This latter centered rolling-year index is labeled P_{CRY} and is listed in the last column of Table 23.23.[37] Note that zeros are entered for the last six rows of this column, because the data set does not extend six months into 1974. As a result, the centered rolling-year indices cannot be calculated for these last six months.

23.83 It can be seen that the Lowe, Young, and geometric Laspeyres indices have a considerable amount of seasonality in them and do not at all approximate their rolling-year counterparts listed in the last column of Table 23.23.[38] Therefore, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[39] The four series, P_{LO}, P_Y, P_{GL}, and P_{CRY}, listed in Table 23.23 are also plotted in Figure 23.4. The Young price index is generally the highest, followed by the Lowe index and the geometric Laspeyres. The centered rolling-year Laspeyres counterpart index, P_{CRY}, is generally below the other three indices (and does not have the strong seasonal movements of the other three series), but it moves in a fashion roughly parallel to the other three indices.[40] Note that the seasonal movements of P_{LO}, P_Y, and P_{GL} are quite regular. This regularity is exploited in Section K in order to use these month-to-month indices to predict their rolling-year counterparts.

23.84 Part of the problem may be the fact that the prices of strongly seasonal goods have been carried forward for the months when the products are not available. This will tend to add to the amount of seasonal movements in the indices, particularly when there is high general inflation. For this reason, the Lowe, Young, and geometric Laspeyres indices are recomputed in the following section, using an imputation method for the missing prices rather than simply carrying forward the last available price.

Figure 23.4. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carryforward Prices



I. Annual Basket Indices with Imputation of Unavailable Prices

23.85 Instead of simply carrying forward the last available price of a seasonal product that is not sold during a particular month, it is possible to use an *imputation method* to fill in the missing prices. Alternative imputation methods are discussed by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2000), but the basic idea is to take the last available price and *impute* prices for the missing periods that trend with another index. This other index could be an index of available prices for the general category of product or higher-level components of the XMPIs. For the purposes of this section, the imputation index is taken to be a price index that grows at the multiplicative rate of 1.008, because the fixed-base rolling-year Laspeyres indices for the modified Turvey

[37]This series was normalized to equal 1 in December 1970 so that it would be comparable to the other month-to-month indices.

[38]The sample means of the four indices are 1.2935 (Lowe), 1.3110 (Young), 1.2877 (geometric Laspeyres), and 1.1282 (rolling year). The geometric Laspeyres indices will always be equal to or less than their Young counterparts, because a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

[39]In Section K, the Lowe, Young, and geometric Laspeyres indices are seasonally adjusted.

[40]In Figure 23.4, P_{CRY} stops at the June 1973 value for the index, which is the last month that the centered index can be constructed from the available data.

data set grow at approximately 0.8 percent per month.[41] Using this imputation method to fill in the missing prices, the Lowe, Young, and geometric Laspeyres indices defined in the previous section can be recomputed. The resulting indices are listed in Table 23.24, along with the centered rolling-year index P_{CRY} for comparison purposes.

23.86 As could be expected, the Lowe, Young, and geometric Laspeyres indices that used imputed prices are on average a bit *higher* than their counterparts that used carryforward prices, but the variability of the imputed indices is generally a bit *lower*.[42] The series that are listed in Table 23.24 are also plotted in Figure 23.5. It is apparent that the Lowe, Young, and geometric Laspeyres indices that use imputed prices still have a huge amount of seasonality in them and do not closely approximate their rolling-year counterparts listed in the last column of Table 23.24.[43] Consequently, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices using imputed prices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[44] As these indices stand, they are not suitable as measures of general inflation going from month to month.

J. Bean and Stine Type C or Rothwell Indices

23.87 The final month-to-month index[45] considered in this chapter is the *Bean and Stine Type C* (1924, p. 31) or *Rothwell* (1958, p. 72) index.[46] This index makes use of *seasonal baskets* in the base year, denoted

Table 23.24. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices

Year	Month	P_{LOI}	P_{YI}	P_{GLI}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0568	1.0624	1.0611	1.0091
	2	1.0742	1.0836	1.0762	1.0179
	3	1.1545	1.1498	1.1238	1.0242
	4	1.2312	1.2334	1.2014	1.0298
	5	1.3524	1.3682	1.3295	1.0388
	6	1.4405	1.4464	1.4047	1.0478
	7	1.3768	1.4038	1.3798	1.0547
	8	1.3364	1.3789	1.3398	1.0631
	9	1.1949	1.2187	1.1955	1.0729
	10	1.1548	1.1670	1.1514	1.0814
	11	1.1661	1.1747	1.1672	1.0885
	12	1.0863	1.0972	1.0939	1.0965
1972	1	1.1426	1.1580	1.1523	1.1065
	2	1.1803	1.1971	1.1888	1.1174
	3	1.2544	1.2630	1.2463	1.1254
	4	1.3260	1.3374	1.3143	1.1313
	5	1.4306	1.4545	1.4244	1.1402
	6	1.5765	1.5831	1.5423	1.1502
	7	1.5273	1.5527	1.5326	1.1591
	8	1.4402	1.4841	1.4444	1.1690
	9	1.3034	1.3343	1.2972	1.1806
	10	1.2758	1.2970	1.2675	1.1924
	11	1.2875	1.3062	1.2873	1.2049
	12	1.1888	1.2078	1.1981	1.2203
1973	1	1.2506	1.2791	1.2601	1.2386
	2	1.3119	1.3426	1.3230	1.2608
	3	1.3852	1.4106	1.3909	1.2809
	4	1.4881	1.5115	1.4907	1.2966
	5	1.6064	1.6410	1.6095	1.3176
	6	1.8451	1.8505	1.7877	1.3406
	7	1.7679	1.7981	1.7684	0.0000
	8	1.6773	1.7263	1.6743	0.0000
	9	1.5271	1.5700	1.5090	0.0000
	10	1.5410	1.5792	1.5195	0.0000
	11	1.5715	1.6075	1.5613	0.0000
	12	1.4307	1.4651	1.4359	0.0000

as the vectors $q^{0,m}$ for the months $m = 1, 2, \ldots, 12$. The index also makes use of a vector of *base-year unit value prices*, $p^0 \equiv [p_1^0, \ldots, p_5^0]$, where the *n*th price in this vector is defined as

[41]For the last year of data, the imputation index is escalated by an additional monthly growth rate of 1.008.

[42]For the Lowe indices, the mean for the first 31 observations increases (with imputed prices) from 1.3009 to 1.3047 but the standard deviation decreases from 0.18356 to 0.18319; for the Young indices, the mean for the first 31 observations increases from 1.3186 to 1.3224, but the standard deviation decreases from 0.18781 to 0.18730; and for the geometric Laspeyres indices, the mean for the first 31 observations increases from 1.2949 to 1.2994, and the standard deviation also increases slightly from 0.17582 to 0.17599. The imputed indices are preferred to the carryforward indices on general methodological grounds: In high-inflation environments, the carryforward indices will be subject to sudden jumps when previously unavailable products become available.

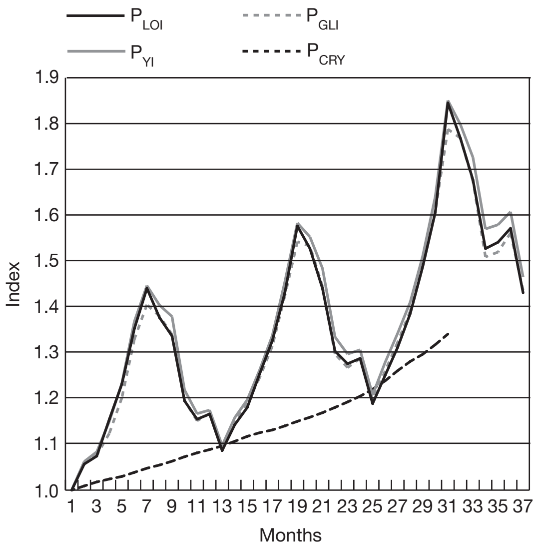
[43]Note also that Figures 23.4 and 23.5 are similar.

[44]In Section K, the Lowe, Young, and geometric Laspeyres indices using imputed prices are seasonally adjusted.

[45]For other suggested month-to-month indices in the seasonal context, see Balk (1980a, 1980b, 1980c, and 1981).

[46]This is the index favored by Baldwin (1990, p. 271) and many other price statisticians in the context of seasonal products.

Figure 23.5. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices



$$p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}}; \; n = 1, \ldots, 5. \quad (23.33)$$

The *Rothwell price index for month* m *in year* t can now be defined as follows:

$$P_R(p^0, p^{t,m}, q^{0,m}) \equiv \frac{\sum_{n=1}^{5} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{5} p_n^{0} q_n^{0,m}};$$

$$m = 1, \ldots, 12. \quad (23.34)$$

Thus, as the month changes, the quantity weights for the index change. The month-to-month movements in this index, therefore, are a mixture of price and quantity changes.[47]

23.88 Using the modified Turvey data set, the base year is chosen to be 1970 as usual, and the index is started off at December 1970. The Rothwell index P_R is compared

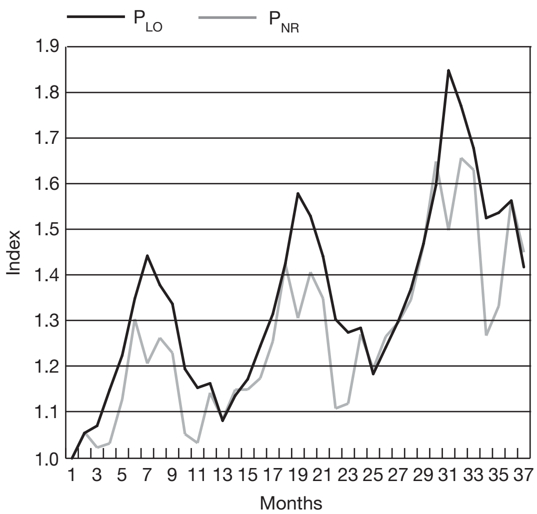
Table 23.25. Lowe with Carryforward Prices, Normalized Rothwell, and Rothwell Indices

Year	Month	P_{LO}	P_{NR}	P_R
1970	12	1.0000	1.0000	0.9750
1971	1	1.0554	1.0571	1.0306
	2	1.0711	1.0234	0.9978
	3	1.1500	1.0326	1.0068
	4	1.2251	1.1288	1.1006
	5	1.3489	1.3046	1.2720
	6	1.4428	1.2073	1.1771
	7	1.3789	1.2635	1.2319
	8	1.3378	1.2305	1.1997
	9	1.1952	1.0531	1.0268
	10	1.1543	1.0335	1.0077
	11	1.1639	1.1432	1.1146
	12	1.0824	1.0849	1.0577
1972	1	1.1370	1.1500	1.1212
	2	1.1731	1.1504	1.1216
	3	1.2455	1.1752	1.1459
	4	1.3155	1.2561	1.2247
	5	1.4262	1.4245	1.3889
	6	1.5790	1.3064	1.2737
	7	1.5297	1.4071	1.3719
	8	1.4416	1.3495	1.3158
	9	1.3038	1.1090	1.0813
	10	1.2752	1.1197	1.0917
	11	1.2852	1.2714	1.2396
	12	1.1844	1.1960	1.1661
1973	1	1.2427	1.2664	1.2348
	2	1.3003	1.2971	1.2647
	3	1.3699	1.3467	1.3130
	4	1.4691	1.4658	1.4292
	5	1.5972	1.6491	1.6078
	6	1.8480	1.4987	1.4612
	7	1.7706	1.6569	1.6155
	8	1.6779	1.6306	1.5898
	9	1.5253	1.2683	1.2366
	10	1.5371	1.3331	1.2998
	11	1.5634	1.5652	1.5261
	12	1.4181	1.4505	1.4143

to the Lowe index with carryforward of missing prices P_{LO} in Table 23.25. To make the series a bit more comparable, the *normalized Rothwell index* P_{NR} is also listed in Table 23.25; this index is simply equal to the original Rothwell index divided by its first observation.

[47]Rothwell (1958, p. 72) showed that the month-to-month movements in the index have the form of an expenditure ratio divided by a quantity index.

Figure 23.6. Lowe and Normalized Rothwell Indices



23.89 Figure 23.6, which plots the Lowe index with the carryforward of the last price and the normalized Rothwell index, makes it clear that the Rothwell index has smaller seasonal movements than the Lowe index and is less volatile in general.[48] However, it is evident that there still are large seasonal movements in the Rothwell index and it may not be a suitable index for measuring general inflation without some sort of seasonal adjustment.

23.90 In the following section, the annual basket-type indices (with and without imputation) defined earlier in Sections H and I are seasonally adjusted using essentially the same method that was used in Section F and compared with a standard seasonal adjustment using say the Census X-11 or X-12 software.

K. Forecasting Rolling-Year Indices Using Month-to-Month Annual Basket Indices

23.91 Recall that Table 23.23 in Section H presented the Lowe, Young, geometric Laspeyres (using carryforward prices) and centered rolling-year indices for the 37 observations running from December 1970 to December 1973 (P_{LO}, P_Y, P_{GL}, and P_{CRY}, respectively). For each of the first three series, define a seasonal adjustment factor, SAF, as the centered rolling-year index P_{CRY} divided by P_{LO}, P_Y, and P_{GL}, respectively, for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for observations 13 through 24 and then repeat them for the remaining observations. These operations will create three SAF series for all 37 observations (label them SAF_{LO}, SAF_Y, and SAF_{GL}, respectively), but only the first 12 observations in the P_{LO}, P_Y, P_{GL}, and P_{CRY} series are used to create the three SAF series. Finally, define *seasonally adjusted Lowe, Young, and geometric Laspeyres indices* by multiplying each unadjusted index by the appropriate seasonal adjustment factor:

$$P_{LOSA} \equiv P_{LO}\, SAF_{LO};\ P_{YSA} \equiv P_Y\, SAF_Y;$$
$$P_{GLSA} \equiv P_{GL}\, SAF_{GL}. \qquad (23.35)$$

These three seasonally adjusted annual basket-type indices are listed in Table 23.26, along with the target index, the centered rolling-year index, P_{CRY}. In addition, one could seasonally adjust the original Lowe, Young, and geometric Laspeyres indices using a standard seasonal adjustment procedure such as X-11. Table 23.26 also contains Lowe, Young, and geometric Laspeyres series that have been seasonally adjusted using the X-11 multiplicative model with default settings.[49] The series have been normalized to set December 1970 =1.0. They are labeled P_{LOX11}, P_{YX11}, and P_{GLX11}, respectively.

23.92 The first four series in Table 23.26 coincide for their first 12 observations, which follows from the way the seasonally adjusted series were defined. Also, the last six observations are missing for the centered rolling-year series, P_{CRY}, because data for the first six months of 1974 would be required to calculate all of these index values. Note that from December 1971 to December 1973, the three seasonally adjusted annual basket-type indices (P_{LOSA}, P_{YSA}, and P_{GLSA}) can be used to *predict* the corresponding centered rolling-year entries; see Figure 23.7.A for plots of these predictions. What is remarkable in Table 23.26 and Figure 23.7.A is that *the predicted values of these seasonally adjusted series are fairly close to the corresponding target index*

[48]For all 37 observations in Table 23.25, the Lowe index has a mean of 1.3465 and a standard deviation of 0.20313, while the normalized Rothwell has a mean of 1.2677 and a standard deviation of 0.18271.

[49]Many statistical offices have access to moving average seasonal adjustment programs such as the X-11 system developed by the U.S. Census Bureau and Statistics Canada. The seasonal adjustment performed here simply ran the data through the multiplicative version of X-11, leaving all options set at their default levels.

Table 23.26. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Carryforward Prices and Centered Rolling-Year Index

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0077	1.0088	1.0088
	2	1.0179	1.0179	1.0179	1.0179	1.0009	1.0044	0.9986
	3	1.0242	1.0242	1.0242	1.0242	1.0208	1.0205	1.0029
	4	1.0298	1.0298	1.0298	1.0298	1.0314	1.0364	1.0157
	5	1.0388	1.0388	1.0388	1.0388	1.0604	1.0666	1.0490
	6	1.0478	1.0478	1.0478	1.0478	1.0302	1.0402	1.0258
	7	1.0547	1.0547	1.0547	1.0547	1.0237	1.0409	1.0213
	8	1.0631	1.0631	1.0631	1.0631	1.0572	1.0758	1.0561
	9	1.0729	1.0729	1.0729	1.0729	1.0558	1.0665	1.0626
	10	1.0814	1.0814	1.0814	1.0814	1.0500	1.0598	1.0573
	11	1.0885	1.0885	1.0885	1.0885	1.0598	1.0714	1.0666
	12	1.0824	1.0932	1.0900	1.0965	1.0828	1.0931	1.0901
1972	1	1.0871	1.0960	1.0919	1.1065	1.0856	1.0957	1.0916
	2	1.1148	1.1207	1.1204	1.1174	1.0963	1.1059	1.0992
	3	1.1093	1.1214	1.1318	1.1254	1.1056	1.1173	1.1083
	4	1.1057	1.1132	1.1226	1.1313	1.1076	1.1203	1.1072
	5	1.0983	1.1039	1.1120	1.1402	1.1211	1.1334	1.1229
	6	1.1467	1.1471	1.1505	1.1502	1.1276	1.1387	1.1264
	7	1.1701	1.1667	1.1715	1.1591	1.1361	1.1514	1.1343
	8	1.1456	1.1443	1.1461	1.1690	1.1393	1.1580	1.1385
	9	1.1703	1.1746	1.1642	1.1806	1.1517	1.1676	1.1531
	10	1.1946	1.2017	1.1905	1.1924	1.1599	1.1777	1.1640
	11	1.2019	1.2102	1.2005	1.2049	1.1703	1.1912	1.1762
	12	1.1844	1.2032	1.1938	1.2203	1.1848	1.2031	1.1938
1973	1	1.1882	1.2089	1.1922	1.2386	1.1940	1.2163	1.1998
	2	1.2357	1.2536	1.2431	1.2608	1.2260	1.2480	1.2314
	3	1.2201	1.2477	1.2575	1.2809	1.2296	1.2569	1.2469
	4	1.2349	1.2523	1.2656	1.2966	1.2529	1.2764	1.2678
	5	1.2299	1.2425	1.2514	1.3176	1.2628	1.2820	1.2743
	6	1.3421	1.3410	1.3335	1.3406	1.3175	1.3285	1.3035
	7	1.3543	1.3512	1.3518	0.0000	1.3123	1.3313	1.3069
	8	1.3334	1.3302	1.3276	0.0000	1.3254	1.3460	1.3186
	9	1.3692	1.3800	1.3524	0.0000	1.3489	1.3739	1.3411
	10	1.4400	1.4601	1.4242	0.0000	1.4016	1.4351	1.3962
	11	1.4621	1.4844	1.4508	0.0000	1.4308	1.4691	1.4296
	12	1.4181	1.4521	1.4236	0.0000	1.4332	1.4668	1.4374

Figure 23.7.A. Seasonally Adjusted Lowe, Young, Geometric Laspeyres, and Centered Rolling Indices

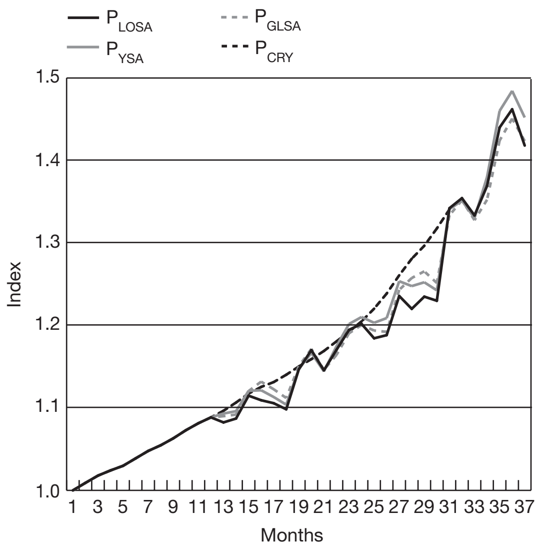
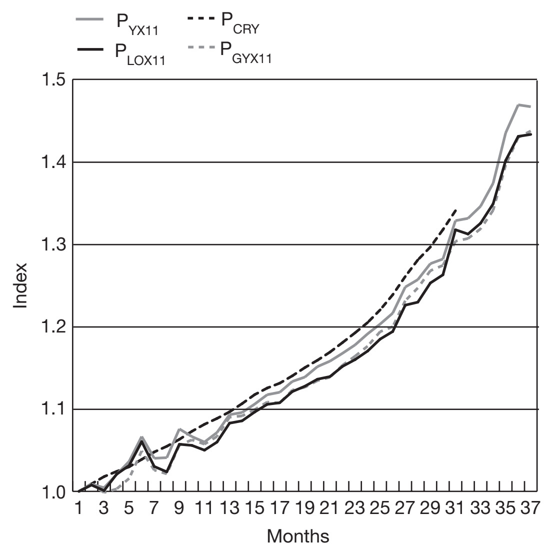


Figure 23.7.B. Lowe, Young, Geometric Laspeyres, and Centered Rolling Indices Using X-11 Seasonal Adjustment



values.[50] This result is somewhat unexpected because the annual basket indices use price information for only two consecutive months, whereas the corresponding centered rolling-year index uses price information for some 25 months.[51] It should also be noted that the seasonally adjusted geometric Laspeyres index is generally the best predictor of the corresponding rolling-year index for this data set. Figure 23.7.A shows that, for the first few months of 1973, the three month-to-month indices underestimate the centered rolling-year inflation rate, but by the middle of 1973, the month-to-month indices are right on target.[52]

23.93 The last three series in Table 23.26 reflect the seasonal adjustment of the Lowe, Young, and geometric Laspeyres using the X-11 program. The seasonally adjusted series (P_{LOX11}, P_{YX11}, and P_{GLX11}) are normalized to December 1970, so that they may easily be compared with the centered rolling-year index, P_{CRY}. Again, these seasonally adjusted series compare rather well with the trend of P_{CRY} and appear to predict the corresponding target values. Figure 23.7.B shows a graph of these series, and the X-11 seasonal adjustment appears to provide a somewhat smoother series than those for the first three series in Table 23.26. This occurs because the X-11 program estimates seasonal factors over the whole data series but requires a minimum of three years of monthly data. The seasonal factors (SAF) for the first three series are based on the 12 estimated monthly factors for 1971 that are simply repeated for subsequent years.[53] Although the trends of X-11 series and the target index (P_{CRY}) are similar, the X-11 series are consistently lower than the target series owing to the normalization of the X-11 series. December is a month that has a larger

[50]For observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8816 is obtained; for the seasonally adjusted Young index, an R^2 of 0.9212 is derived and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9423 is derived. These fits are not as good as the fit obtained in Section F above where the seasonally adjusted approximate rolling-year index was used to predict the fixed-base Laspeyres rolling-year index. This R^2 was 0.9662; recall the discussion around Table 23.20.

[51]However, for seasonal data sets that are not as regular as the modified Turvey data set, the predictive power of the seasonally adjusted annual basket-type indices may be considerably less; that is, if there are abrupt changes in the seasonal pattern of prices, one could not expect these month-to-month indices to accurately predict a rolling-year index.

[52]Recall that the last six months of P_{CRY} are missing; six months of data for 1974 would be required to evaluate these centered rolling-year index values, and these data are not available.

[53]Again, for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the X-11 seasonally adjusted Lowe index, an R^2 of 0.9873 is derived; for the X-11 seasonally adjusted Young index, an R^2 of 0.9947 is derived; and for the X-11 seasonally adjusted geometric Laspeyres index, an R^2 of 0.9952 is derived. These fits are better than those obtained above and in Section F. However, the X-11 seasonal adjustment procedure uses the entire data set to do the adjusting, whereas the index number methods of seasonal adjustment used only the first 12 months of data.

Table 23.27. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices; Seasonally Adjusted Rothwell; and Centered Rolling-Year Indices

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0091	1.0125	1.0131	1.0133
	2	1.0179	1.0179	1.0179	1.0179	1.0179	1.0083	1.0109	1.0057
	3	1.0242	1.0242	1.0242	1.0242	1.0242	1.0300	1.0288	1.0121
	4	1.0298	1.0298	1.0298	1.0298	1.0298	1.0418	1.0460	1.0267
	5	1.0388	1.0388	1.0388	1.0388	1.0388	1.0680	1.0753	1.0574
	6	1.0478	1.0478	1.0478	1.0478	1.0478	1.0367	1.0485	1.0362
	7	1.0547	1.0547	1.0547	1.0547	1.0547	1.0300	1.0450	1.0251
	8	1.0631	1.0631	1.0631	1.0631	1.0631	1.0637	1.0807	1.0615
	9	1.0729	1.0729	1.0729	1.0729	1.0729	1.0607	1.0713	1.0685
	10	1.0814	1.0814	1.0814	1.0814	1.0814	1.0536	1.0634	1.0615
	11	1.0885	1.0885	1.0885	1.0885	1.0885	1.0631	1.0741	1.0704
	12	1.0863	1.0972	1.0939	1.0849	1.0965	1.0867	1.0973	1.0940
1972	1	1.0909	1.0999	1.0958	1.0978	1.1065	1.0948	1.1043	1.1004
	2	1.1185	1.1245	1.1244	1.1442	1.1174	1.1079	1.1168	1.1109
	3	1.1129	1.1250	1.1359	1.1657	1.1254	1.1191	1.1300	1.1224
	4	1.1091	1.1167	1.1266	1.1460	1.1313	1.1220	1.1341	1.1233
	5	1.0988	1.1043	1.1129	1.1342	1.1402	1.1298	1.1431	1.1328
	6	1.1467	1.1469	1.1505	1.1339	1.1502	1.1345	1.1476	1.1377
	7	1.1701	1.1666	1.1715	1.1746	1.1591	1.1427	1.1559	1.1386
	8	1.1457	1.1442	1.1461	1.1659	1.1690	1.1464	1.1632	1.1444
	9	1.1703	1.1746	1.1642	1.1298	1.1806	1.1570	1.1729	1.1594
	10	1.1947	1.2019	1.1905	1.1715	1.1924	1.1639	1.1818	1.1685
	11	1.2019	1.2103	1.2005	1.2106	1.2049	1.1737	1.1943	1.1805
	12	1.1888	1.2078	1.1981	1.1960	1.2203	1.1892	1.2079	1.1983
1973	1	1.1941	1.2149	1.1983	1.2089	1.2386	1.1906	1.2118	1.1954
	2	1.2431	1.2611	1.2513	1.2901	1.2608	1.2205	1.2415	1.2244
	3	1.2289	1.2565	1.2677	1.3358	1.2809	1.2221	1.2483	1.2370
	4	1.2447	1.2621	1.2778	1.3373	1.2966	1.2431	1.2656	1.2542
	5	1.2338	1.2459	1.2576	1.3131	1.3176	1.2613	1.2833	1.2694
	6	1.3421	1.3406	1.3335	1.3007	1.3406	1.3298	1.3440	1.3208
	7	1.3543	1.3510	1.3518	1.3831	0.0000	1.3246	1.3407	1.3158
	8	1.3343	1.3309	1.3285	1.4087	0.0000	1.3355	1.3531	1.3266
	9	1.3712	1.3821	1.3543	1.2921	0.0000	1.3539	1.3780	1.3470
	10	1.4430	1.4634	1.4271	1.3949	0.0000	1.4023	1.4346	1.3971
	11	1.4669	1.4895	1.4560	1.4903	0.0000	1.4252	1.4617	1.4237
	12	1.4307	1.4651	1.4359	1.4505	0.0000	1.4205	1.4540	1.4250

seasonal component in the X-11 adjustment than that for the series using the rolling average. Normalizing the X-11 adjusted series for December results in the first few months of the series showing relatively little growth.

23.94 The manipulations above can be repeated, replacing the *carryforward* annual basket indices with their *imputed* counterparts, that is, using the information in Table 23.24 in Section I (instead of Table 23.23 in Section H) and replacing Table 23.26 with Table 23.27.

Figure 23.8.A. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices; Seasonally Adjusted Rothwell and Centered Rolling-Year Indices

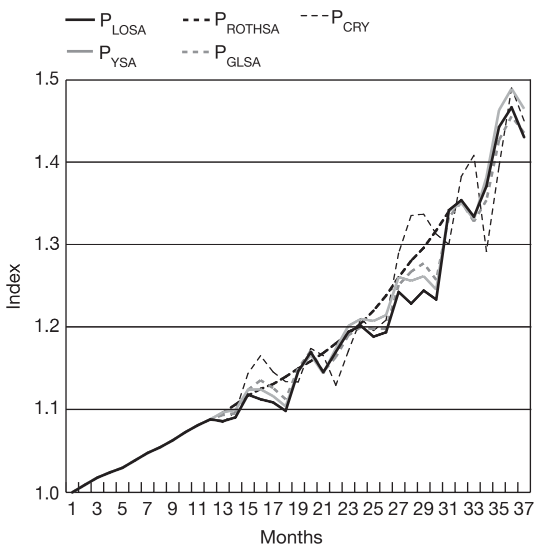
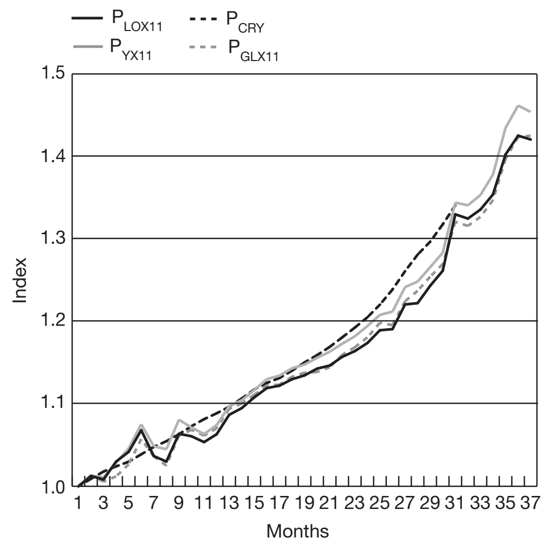


Figure 23.8.B. Lowe, Young, and Geometric Laspeyres Indices Using X-11 Seasonal Adjustment with Imputed Prices, and Centered Rolling-Year Indices



A seasonally adjusted version of the Rothwell index presented in the previous section may also be found in Table 23.27.[54] The eight series in Table 23.27 are also graphed in Figures 23.8.A and 23.8.B.

23.95 Again, the seasonally adjusted annual basket-type indices listed in the first three data columns of Table 23.27 (using imputations for the missing prices) are reasonably close to the corresponding centered rolling-year index listed in the fifth data column of Table 23.27.[55] The seasonally adjusted geometric Laspeyres index is the closest to the centered rolling-year index, and the seasonally adjusted Rothwell index is the furthest away. The three seasonally adjusted month-to-month indices that use annual weights—P_{LOSA}, P_{YSA}, and P_{GLSA}—dip below the corresponding centered rolling-year index, P_{CRY}, for the first few months of 1973 when the rate of month-to-month inflation suddenly increases. But by the middle of 1973, all four indices are fairly close to each other. The seasonally adjusted Rothwell does not do a very good job of approximating P_{CRY} for this particular data set although this could be a function of the rather simple method of seasonal adjustment that was used. The series adjusted using X-11 again are smoother than the other series and show trends very similar to those of the target index.

23.96 In comparing the results in Tables 23.26 and 23.7, one can see that it did not make a great deal of difference for the modified Turvey data set whether missing prices are carried forward or imputed; the seasonal adjustment factors picked up the lumpiness in the unadjusted indices that happens when the carryforward method is used. However, the three month-to-month indices that used annual weights and imputed prices did predict the corresponding centered rolling-year indices somewhat better than did the three indices that used carryforward prices. Therefore, the use of imputed prices over carryforward prices is recommended.

[54]The same seasonal adjustment technique that was defined by equation (23.35) was used.

[55]Again for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8994 is derived; for the seasonally adjusted Young index, an R^2 of 0.9294 is derived; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9495 is derived. For the seasonally adjusted Rothwell index, an R^2 of 0.8704 is derived, which is lower than the other three fits. For the X-11 seasonally adjusted series, the R^2 values are 0.9644 for the Lowe, 0.9801 for the Young, and 0.9829 for the geometric Laspeyres. All of the Lowe, Young, and geometric Laspeyres indices, using imputed prices, have higher R^2 values than those obtained using carryforward prices.

23.97 The conclusions that emerge from this section are rather encouraging for statistical agencies that wish to use an annual basket-type index as their flagship index.[56] It appears that for product groups that have strong seasonality, an annual basket-type index for this group can be seasonally adjusted,[57] and the resultant seasonally adjusted index value can be used as a price relative for the group at higher stages of aggregation. The preferred type of annual basket type index appears to be the geometric Laspeyres index, rather than the Lowe index, but the differences between the two were not large for this data set.

L. Conclusions

23.98 A number of tentative conclusions can be drawn from the results of the sections in this chapter:

- The inclusion of seasonal products in maximum overlap month-to-month indices will frequently lead to substantial biases. Therefore, unless the maximum overlap month-to-month indices using seasonal products cumulated for a year are close to their year-over-year counterparts, the seasonal products should be excluded from the month-to-month index or the seasonal adjustment procedures suggested in Section K should be used;
- Year-over-year monthly indices can always be constructed even if there are strongly seasonal products.[58] Many users will be interested in these indices; moreover, these indices are the building blocks for annual indices and for rolling-year indices. As a result, statistical agencies should compute these indices. They can be labeled analytic series in order to prevent user confusion with the primary month-to-month XMPI;
- Rolling-year indices should also be made available as analytic series. These indices will give the most reliable indicator of annual inflation at a monthly frequency. This type of index can be regarded as a seasonally adjusted XMPI. It is the most natural index to use as a central bank inflation target. It has the disadvantage of measuring year-over-year inflation with a lag of six months; thus, it cannot be used as a short-run indicator of month-to-month inflation. However, the techniques suggested in Sections F and K could be used so that timely forecasts of these rolling-year indices can be made using current price information;
- Annual basket indices can also be successfully used in the context of seasonal commodities. However, many users of XMPIs will want to use seasonally adjusted versions of these annual basket-type indices. The seasonal adjustment can be done using the index number methods explained in Section K or traditional statistical agency seasonal adjustment procedures;[59]
- From an a priori point of view, when making a price comparison between any two periods, the Paasche and Laspeyres indices are of equal importance. Under normal circumstances, the spread between the Laspeyres and Paasche indices will be reduced by using chained indices rather than fixed-base indices. As a result, when constructing year-over-year monthly or annual indices, choose the chained Fisher index (or the chained Törnqvist Theil index, which closely approximates the chained Fisher) as the target index that a statistical agency should aim to approximate. However, when month-to-month indices are constructed, chained indices should always be compared to their year-over-year counterparts to check for chain drift. If substantial drift is found, the chained month-to-month indices must be replaced with fixed-base indices or seasonally adjusted annual basket-type indices;[60]
- If current-period revenue shares are not all that different from base-year revenue shares, approximate chained Fisher indices will normally provide

[56]In light of results described in previous chapters, the use of the annual basket Young index is not encouraged because of its failure of the time reversal test and the resultant upward bias.

[57]It is not necessary to use rolling-year indices in the seasonal adjustment process, but the use of rolling-year indices is recommended because they will increase the objectivity and reproducibility of the seasonally adjusted indices.

[58]There can be problems with the year-over-year indices if shifting holidays or abnormal weather changes normal seasonal patterns. In general, choosing a longer time period will mitigate these types of problems; that is, quarterly seasonal patterns will be more stable than monthly patterns, which in turn will be more stable than weekly patterns.

[59]However, there is a problem with using traditional X-11-type seasonal adjustment procedures for adjusting XMPIs because final seasonal adjustment factors are generally not available until an additional two or three years' data have been collected. If the XMPIs cannot be revised, this may preclude using X-11-type seasonal adjustment procedures. Note that the index number method of seasonal adjustment explained in this chapter does not suffer from this problem. It does, however, require the use of seasonal factors derived from a single year of data, so that the year used should reflect a normal seasonal pattern. If the seasonal patterns are irregular, it may be necessary to use the average of two or more years of past adjustment factors. If the seasonal patterns are regular but slowly changing, then it may be preferable to update the index number seasonal adjustment factors on a regular basis.

[60]Alternatively, some sort of multilateral index number formula could be used; for example, see Caves, Christensen, and Diewert (1982a) or Feenstra and Shapiro (2003).

a close practical approximation to the chained Fisher target indices. Approximate Laspeyres, Paasche, and Fisher indices use base-period expenditure shares whenever they occur in the index number formula in place of current-period (or lagged current-period) revenue shares. Approximate Laspeyres, Paasche, and Fisher indices can be computed by statistical agencies using their normal information sets; and

- The geometric Laspeyres index is an alternative to the approximate Fisher index that uses the same information. It will normally be close to the approximate Fisher index.

It is evident that more research needs to be done on the problems associated with the index number treatment of seasonal products. A consensus on what is best practice in this area has not yet formed.

24. Measuring the Effects of Changes in the Terms of Trade

A. Introduction

A.1 Chapter overview

24.1 A terms of trade index is generally defined as an economy's index of export prices divided by an index of import prices. The implementation of this definition would not warrant a chapter in a textbook or in a manual: It is more or less straightforward. However, many economists over the years have observed that an improvement in an economy's terms of trade has effects that are very similar to an improvement in total factor productivity or multifactor productivity.[1] Economists have also been interested in quantifying the effects of changing international prices on the real income generated by an economy. Once the discussion of changes in the terms of trade is broadened to include these topics, the original simplicity of the terms of trade index vanishes. Thus the purpose of this chapter is to address the effects of changing international prices on the real income of an economy or a sector of an economy. In order to narrow the topic, only an approach to these measurement problems that is based on economic approaches to producer and consumer theory is considered.

24.2 In Section A.2 of this chapter, a technical introduction to the effects of changing international prices on the growth of an economy's real income is undertaken. A production theory framework is laid out and some preliminary definitions are made.[2]

24.3 Section B considers the effects of a change in the real export price facing the economy on the real income generated by the market-oriented production sector of the economy and Section C considers the effects of a change in the real import price. Various theoretical definitions for these effects are considered and empirical approximations to these theoretical indices are defined and analyzed. In Section D, the combined effects of changes in real import and export prices on the real income generated by the production sector are considered. These combined effects indices are then related to the partial indices defined in Sections B and C.

24.4 Some goods and services are imported directly into the household sector. An important example of such expenditures is tourism expenditures abroad. The production theory approach developed in Sections B through D is not applicable for these classes of household imported goods and services so in Section E, a consumer theory approach is developed. It turns out that the structure of the producer theory methodology can readily be adapted to deal with this situation with a few key changes.

24.5 There are also certain goods and services that are directly exported by households. For example, self-employed consultants can directly export business services to customers around the world. Also, small-scale household manufacturers of clothing and other goods can advertise on the Internet and sell their products abroad rather easily. Thus there is a need to model household exports, as well as goods and services that are directly imported by households. However, in principle, household exports can be treated using the production theory methodology developed in Sections B through D: All that needs to be done is to create a set of household production accounts.[3] Thus these household production units will use various capital inputs (machines, parts of the structures that they inhabit), intermediate inputs, and their own labor in order to produce commodities for sale in their

[1]For materials on these productivity concepts, see the pioneering articles by Jorgenson and Griliches (1967 and 1972) and the excellent Organization for Economic Cooperation and Development manuals, OECD (2001) and Schreyer (2007).

[2]Background material on producer theory approaches to production theory can be found in Caves, Christensen, and Diewert (1982b); Diewert (1983a); Balk (1998a); Alterman, Diewert, and Feenstra (1999); and Chapter 18 of the present *Manual*.

[3]In practice, this is not an easy task.

domestic and foreign markets. This household production sector is much the same as "regular" incorporated production units except that it will usually be difficult to get accurate measures of the capital employed and the labor used by these household production units. However, as the reader will note, when the producer theory approach to exports and imports is developed in Sections B through D, it is not necessary to know what inputs of labor and capital are actually used by the production units in order to implement the terms of trade adjustment factors that are developed in these sections. Thus there is no need to develop a separate theory for directly exported goods and services by households. Section E concludes.

A.2 Technical introduction

24.6 Let P_X^t be the price index for exports in an economy in period t and let P_M^t be the corresponding import price index. Then the period t *terms of trade index*, T^t, is defined as an export price index divided by an import price index:

$$T^t \equiv P_X^t / P_M^t; \quad t = 0, 1. \tag{24.1}$$

24.7 A country's terms of trade is said to have *improved* going from period 0 to 1 if T^1/T^0 is greater than one and to have *deteriorated* if T^1/T^0 is less than one. For an improvement, the export price index has increased more rapidly than the import price index.

24.8 Thus the definition of a terms of trade index is very straightforward and relatively easy to implement: Only the exact form of the export and import price index needs to be determined. Presumably, preliminary versions of a terms of trade index would use Laspeyres-type indices whereas a retrospective, historical version, compiled when current-period weights become available, would use a superlative index. However, the definition of a terms of trade index is not the end of the story as is explained below.

24.9 It has been well known for a long time that an improvement in a country's terms of trade is beneficial for a country and has effects that are similar to an improvement in the country's total factor productivity or multifactor productivity.[4] However, determining how to measure *precisely* the degree of improvement owing to a change in a country's terms of trade has proven to be a difficult question.

[4]See Diewert and Morrison (1986), Morrison and Diewert (1990), and Kohli (1990).

24.10 The measurement question addressed in this chapter is the following one: Can the effects of changes in the price of exports and imports on the growth of *real income* in the economy be determined? Thus at the outset, the focus is on the measurement of the real income generated by the economy, and then the effects of changes in international prices on the chosen real income measure are considered.

24.11 To begin the analysis, consider the following definition for the *net domestic product* of a country in period t, NDP^t, as the sum of the usual macroeconomic aggregates:

$$NDP^t = P_C^t C^t + P_I^t I^t + P_G^t G^t + P_X^t X^t - P_M^t M^t; \quad t = 0, 1, \tag{24.2}$$

where NDP^t is the net domestic product produced by the economy in period t; C^t, I^t, G^t, X^t, and M^t are the period t quantities of consumption, net investment,[5] government final consumption, exports, and imports, respectively; and P_C^t, P_I^t, P_G^t, P_X^t, and P_M^t are the corresponding period t final demand prices. Using the usual circular flow arguments employed by national income accountants, net domestic product is produced by the production sector in the economy and the value of this production generates a flow of income received by primary inputs used in the economy. Growth in this flow of income (which is also equal to NDP^t) is analyzed in this chapter.[6]

24.12 The rate of growth in the flow of nominal net product going from period 0 to 1, NDP^1/NDP^0 (or more accurately, one plus this rate of growth), is of limited interest to policy analysts and the public as an indicator of welfare growth because it includes the effects of general inflation. Thus it is necessary to deflate the nominal net domestic product in period t, NDP^t, by a "reasonable" period t deflator or price index, say P_D^t.

[5]Note that when the focus is on income flows generated by an economy, it is necessary to deduct depreciation of capital from gross investment because depreciation is not a sustainable income flow. Thus in this chapter, the target macroeconomic aggregate is (deflated) net domestic product rather than gross domestic product.

[6]Note that the flow of income of concern here is the income received by primary inputs used in the market sector of the economy and thus excludes the difference between real primary incomes and current transfers receivable and payable from abroad. Indeed the framework used by the Commission of the European Communities and others (2008), *2008 System of National Accounts (2008 SNA)* and outlined in Silver and Mahdavy (1989) defines real net disposable national income as the volume of GDP, plus the trading gain or loss resulting from changes in the terms of trade, plus difference between real primary incomes and current transfers receivable and payable from abroad. The formulas for the terms of trade effect given in the *2008 SNA* are, unlike the formal framework outlined here, heuristic in nature.

The first problem that needs to be addressed is: What is a "reasonable" deflator?

24.13 Three choices have been suggested in the literature:

- The price of consumption, P_C^t;
- The price of domestic goods or the price of absorption, P_A^t (an aggregate of P_C^t, P_I^t, and P_G^t); or
- The net domestic product deflator, P_N^t (an aggregate of P_C^t, P_I^t, P_G^t, P_X^t, and P_M^t, where P_M^t has negative weights).

24.14 The consumption price deflator, P_C^t, and the absorption deflator, P_A^t, can be justified. Diewert and Lawrence (2006) and Diewert (2008) preferred the first deflator while Kohli (2006) preferred the second one. However, these authors do not recommend the use of either the GDP deflator or the net domestic product (NDP) deflator in the present context because they maintain that because virtually all internationally traded goods are intermediate goods and hence are not directly consumed by households, the prices of these goods are not needed to deflate nominal income flows into real income flows.[7] The case for using the price of consumption as a deflator for the nominal income that is generated by the production side of the economy is very simple: The deflated amount, NDP^t/P_C^t, is the potential amount of consumption that could be purchased by the owners of primary inputs in period t if they chose to buy zero units of net investment and government outputs. If the price of domestic absorption is used as the deflator, then NDP^t/P_A^t is the number of units of a (constant utility) aggregate of C, I, and G that could be purchased by the suppliers of primary inputs to the production sector of the economy in period t.

24.15 Suppose that a choice of the nominal income deflator, P_D^t, has been made. It is now desirable to look at the *growth of the real income generated by the production sector in the economy*, that is, look at the growth of NDP^t/P_D^t,

$$\begin{aligned} NDP^t/P_D^t &= \left[P_C^tC^t + P_I^tI^t + P_G^tG^t + P_X^tX^t - P_M^tM^t\right]/P_D^t; \quad t = 0, 1 \\ &= p_C^tC^t + p_I^tI^t + p_G^tG^t + p_X^tX^t - p_M^tM^t \end{aligned} \tag{24.3}$$

where the *real prices* of consumption, net investment, government consumption, exports, and imports are defined as the nominal prices divided by the chosen income deflator P_D^t:[8]

$$p_C^t \equiv P_C^t/P_D^t;\ p_I^t \equiv P_I^t/P_D^t;\ p_G^t \equiv P_G^t/P_D^t;\ p_X^t \equiv P_X^t/P_D^t;\ p_M^t \equiv P_M^t/P_D^t. \tag{24.4}$$

24.16 Using equations (24.3) and definitions (24.4), (one plus) *the rate of growth of real income* over the two periods under consideration can be defined as follows:

$$\begin{aligned} &\left[NDP^1/P_D^1\right]/\left[NDP^0/P_D^0\right] \\ &= \left[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1\right]/\left[p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0\right]. \end{aligned} \tag{24.5}$$

24.17 Looking at equation (24.5), it can be seen that, holding all else constant, an increase in the period 1 real price of exports p_X^1 will *increase* real income growth generated by the production sector of the economy. Conversely, an increase in the period 1 real price of imports p_M^1 will *decrease* real income growth.

24.18 Equation (24.5) indicates the complexity of trying to determine the effects of changes in real import and export prices on the growth of real income: p_X and p_M change but so do the real prices of consumption, net investment, and government consumption. In addition, the quantities of C, I, G, X, and M are changing and in the background, there are also changes in the amount of labor L and capital K that is being utilized by the economy's production sector. It is evident that some measure of the effect on real income growth of the changes in the real prices of exports and imports is desired, holding constant the rest of the economic environment. But if export and import prices change, producers will be induced to change the composition of their exports and imports. Thus a careful specification of what is exogenous and what is endogenous is needed in order to isolate the effects of changes in real export and import prices.

24.19 In the following section, a production theory framework is used in order to specify more precisely

[7]There are other reasons for not using the GDP or NDP deflators as measures of general inflation; see Kohli (1982, p. 211; and 1983, p. 142), Hill (1996, p. 95), and Diewert (2002**c**, pp. 556–60) for additional discussion.

[8]If it is desired to explain nominal income growth generated by the production sector, then it is not necessary to deflate the period t data by P_D^t. In this case, it can be assumed that P_D^t equals one so that $P_C^0 = p_C^0$, and so forth.

exactly what is being held fixed and what is being allowed to vary as real export and import prices change. Other approaches to modeling the effects on production and welfare of changes in the prices of exports and imports are reviewed in Diewert and Morrison (1986), Silver and Mahdavy (1989), and Kohli (2006).[9]

B. The Effects of Changes in the Real Price of Exports

B.1 Theoretical measures of the effects of changes in the real price of exports

24.20 Kohli (1978 and 1991) has long argued that because most internationally traded goods are intermediate products and services, it is natural to model the effects of international trade using production theory.[10] Kohli's example is followed in this section and in subsequent sections and a production theory framework is used with exports as outputs of the production sector and imports as intermediate inputs into the production sector.

24.21 For simplicity, it is assumed that C, I, G, and X (consumption, net investment, government consumption, and exports) are outputs of the production sector and M, L, and K (imports, labor, and capital) are inputs into the production sector.[11] In period t, there is a feasible set of (C, I, G, X, M, L, K) outputs and inputs, which is denoted by the set S^t for periods t equal to 0 and 1. It will prove useful to define the economy's period t *real net domestic product function*, $n^t(p_C, p_I, p_X, p_M, L, K)$ for $t = 0, 1$:

$$n^t(p_C, p_I, p_G, p_X, p_M, L, K) \equiv \max_{C, I, G, X, M} \{p_C C + p_I I + p_G G + p_X X - p_M M: (C, I, G, X, M, L, K) \text{ belongs to } S^t\}. \quad (24.6)$$

24.22 Thus the real net product $n^t(p_C, p_I, p_G, p_X, p_M, L, K)$ is the maximum amount of (net) real value added that the economy can produce if producers face the real price p_C for consumption, the real price p_I for net investment, the real price p_G for government consumption, the real price p_X for exports, and the real price p_M for imports and given that producers have at their disposal the period t production possibilities set S^t as well as the amount L of labor services and the amount K of capital (waiting) services.[12]

24.23 It is reasonable to assume that the actual period t amounts of outputs produced and inputs used in period t, C^t, I^t, G^t, X^t, M^t, L^t, K^t, belong to the corresponding period t production possibilities set, S^t, for $t = 0, 1$. It is a stronger assumption to assume that producers are competitively profit maximizing in periods 0 and 1 so that the following equalities are valid:

$$n^t\left(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t\right) = p_C^t C^t + p_I^t I^t + p_G^t G^t + p_X^t X^t - p_M^t M^t; \ t = 0, 1, \quad (24.7)$$

where p_C^t, p_I^t, p_G^t, p_X^t, p_M^t are the real prices for consumption, net investment, government consumption, exports, and imports that producers face in period t[13] and L^t and K^t are the amounts of labor and capital used by producers in period t. In what follows, it is assumed that equations (24.7) hold. Basically, these equations rest on the assumption that producers in the economy are competitively maximizing net domestic product in periods 0 and 1 subject to the technological constraints on the economy for each period.

24.24 In a first attempt to measure the effects of changing real export prices over the two periods under consideration, a hypothetical net domestic product maximization problem is considered where producers have at their disposal the period 0 technology set S^0, and the period 0 actual labor and capital inputs, L^0 and K^0, respectively, and they face the period 0 real prices for consumption, net investment, government consumption, and imports, p_C^0, p_I^0, p_G^0,

[9]Ulrich Kohli, the chief economist for the Swiss National Bank, has long had an interest in adjusting income measures for changes in a country's terms of trade using production theory; see Kohli (1990, 2003, 2004a, 2004b, and 2006) and Fox and Kohli (1998). Kohli's methodology is compared with the Diewert and Lawrence methodology that is used in this chapter and in Diewert (2008).

[10]The recent textbook by Feenstra (2004) also takes this point of view.

[11]These scalar quantities could be replaced by vectors but this extension is left to the reader.

[12]Depreciation has been subtracted from gross investment so the user cost of capital in the present model excludes depreciation so the price of capital services is basically Rymes' (1968 and 1983) waiting services; see also Cas and Rymes (1991).

[13]Producers actually face the prices P_C^t, P_I^t, P_G^t, P_X^t, P_M^t rather than the deflated (by P_D^t) prices p_C^t, p_I^t, p_G^t, p_X^t, p_M^t. However, if producers maximize net product facing the prices P_C^t, P_I^t, P_X^t, P_M^t, they will also maximize net product facing the real prices p_C^t, p_I^t, p_G^t, p_X^t, p_M^t. There is one additional difficulty: The prices that producers face are different than the prices that consumers and other final demanders face because of commodity taxes. Thus strictly speaking, the theory that is developed in this section and subsequent sections that relies on production theory applies to producer prices (or basic prices) rather than final demand prices.

and p_M^0 respectively, but they face the period 1 real export price, p_X^1. The solution to this hypothetical net product maximization problem is $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0)$. Using this hypothetical net product or net income, *a theoretical Laspeyres type measure* α_{LX} *of the effects on real income growth of changes in real export prices* from the period 0 level, p_X^0, to the period 1 level, p_X^1, can be defined as the ratio of the hypothetical net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0)$ to the actual period 0 net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0)$:[14]

$$\alpha_{LX} \equiv n^0\left(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0\right) / n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right). \tag{24.8}$$

24.25 The index α_{LX} of the effects of the change in the real price of exports is termed a *Laspeyres type index* because it holds constant all exogenous prices and quantities at their period 0 levels except for the two real export prices, p_X^0 and p_X^1, and the index also holds technology constant at the base-period level.

24.26 Using assumption (24.7) for $t = 0$, the denominator on the right-hand side of (24.8) is equal to period 0 observed real net product, $p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0$. Using definition (24.6), it can be seen that C^0, I^0, G^0, X^0, and M^0 is a feasible solution for the net product maximization problem defined by the numerator on the right-hand side of (24.8), $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0)$. These facts mean that there is the following *observable lower bound* to the theoretical index α_{LX} defined by (24.8):[15]

$$\alpha_{LX} \geq \left[p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^1X^0 - p_M^0M^0\right] / \left[p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0\right] \equiv P_{LX}, \tag{24.9}$$

where P_{LX} is an *observable Laspeyres type index* of the effects on real income of a change in real export prices going from period 0 to 1. P_{LX} generally understates the hypothetical change in the real income generated by the economy which is defined by the theoretical index α_{LX} owing to *substitution bias*; that is, the change in the real price of exports will induce producers to substitute away from their base-period production decisions in order to take advantage of the change in real export prices from p_X^0 to p_X^1. Note that the numerator and denominator on the right-hand side of (24.9) are identical except that p_X^1 appears in the numerator and p_X^0 appears in the denominator.

24.27 It is possible to show that the Laspeyres type observable index P_{LX} is a first order Taylor series approximation to the theoretical Laspeyres type index α_{LX} (see below). A first order Taylor series approximation to the hypothetical net real income defined by $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0)$ is given by the first line of equation (24.10):[16]

$$\begin{aligned} & n^0\left(p_C^0, p_I^0, p_G^0, p_X^1, p_M^0, L^0, K^0\right) \\ & \approx n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right) \\ & \quad + \left[\partial n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right)/\partial p_X\right]\left[p_X^1 - p_X^0\right] \\ & = n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right) + X^0\left[p_X^1 - p_X^0\right] \\ & \qquad \text{using Hotelling's Lemma}^{17} \\ & = p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0 \\ & \quad + X^0\left[p_X^1 - p_X^0\right] \qquad \text{using (24.7) for } t = 0 \\ & = p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^1X^0 - p_M^0M^0, \end{aligned} \tag{24.10}$$

where $p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^1X^0 - p_M^0M^0$ is the numerator on the right-hand side of equation (24.9). Because the denominator on the right-hand side of equation (24.9) is equal to $p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0$, which in turn is equal to $n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0)$, it can be seen that P_{LX} is indeed a first order approximation to the theoretical index α_{LX} defined by (24.8).[18]

24.28 In a second attempt to measure the effects of changing real export prices over the two periods under consideration, a hypothetical net domestic product maximization problem is considered where

[14]Definition (24.8) is similar to the Laspeyres output price effect defined by Diewert and Morrison (1986, p. 666) except that they used a GDP function instead of a net product function and they did not deflate their aggregate by a price index. Diewert, Mizobuchi, and Nomura (2005, pp. 19–20) and Diewert and Lawrence (2006, pp. 12–17) developed much of the theory used in this chapter.

[15]The inequality (24.9) rests on a feasibility argument and this type of argument was first used by Konüs (1924) in the consumer price context.

[16]This type of approximation was used by Diewert (1983a, pp. 1095–96) and Morrison and Diewert (1990, pp. 211–12) in the producer theory context but the basic technique (in the consumer theory context) is from Hicks (1942, pp. 127–34; and 1946, p. 331).

[17]Hotelling's (1932, p. 594) Lemma says that the first order partial derivatives of the net product function $n^t(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t)$ with respect to the prices p_C, p_I, p_G, p_X, p_M are equal to C^t, I^t, G^t, X^t and $-M^t$ respectively for $t = 0, 1$.

[18]This result was established by Diewert and Lawrence (2006, p. 16).

producers have at their disposal the period 1 technology set S^1, and the period 1 actual labor and capital inputs, L^1 and K^1 respectively, and they face the period 1 real prices for consumption, net investment, government consumption, and imports, p_C^1, p_I^1, p_G^1, and p_M^1 respectively, but they face the period 0 real export price, p_X^0. The solution to this hypothetical (real) net product maximization problem is $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$. Using this hypothetical net product or net income, *a theoretical Paasche type measure* α_{PX} *of the effects on real income growth of changes in real export prices* from the period 0 level, p_X^0, to the period 1 level, p_X^1, can be defined as the ratio of the actual period 1 net real income $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)$ to the hypothetical net real income $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$:[19]

$$\alpha_{PX} \equiv n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)/ n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1). \quad (24.11)$$

24.29 The index α_{PX} of the effects of the change in the real price of exports is termed a *Paasche type index* because it holds constant all exogenous prices and quantities at their period 1 levels except for the two real export prices, p_X^0 and p_X^1, and the index also holds technology constant at the period 1 level.

24.30 Using assumption (24.7) for $t = 1$, the numerator on the right-hand side of equation (24.11) is equal to period 1 observed real net product, $p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1$. Using definition (24.6), it can be seen that C^1, I^1, G^1, X^1, and M^1 is a feasible solution for the net product maximization problem defined by the denominator on the right-hand side of equation (24.11), $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$. These facts mean that there is the following *observable upper bound* to the theoretical index α_{LX} defined by equation (24.11):

$$\alpha_{PX} \le [p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1]/ [p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1] \equiv P_{PX}, \quad (24.12)$$

where P_{PX} is an *observable Paasche type index* of the effects on real income of a change in real export prices going from period 0 to 1. P_{PX} generally overstates the hypothetical change in the real income generated by the economy which is defined by the theoretical index α_{PX} owing to *substitution bias*; that is, the change in the real price of exports from p_X^1 to p_X^0 will induce producers to substitute away from their period 1 production decisions so that $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$ will generally be greater than $[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1]$ so that $1/n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$ will generally be less than $1/[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1]$ and the inequality in equation (24.12) follows. Note that the numerator and denominator on the right-hand side of equation (24.12) are identical except that p_X^1 appears in the numerator and p_X^0 appears in the denominator.

24.31 It is possible to show that the Paasche type observable index P_{PX} is a first order Taylor series approximation to the theoretical Paasche type index α_{PX} as is shown below. The proof is entirely analogous to the derivation of equation (24.10). A first order Taylor series approximation to the hypothetical net real income defined by $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1)$ is given by the first line of equation (24.13) below:

$$\begin{aligned} &n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^1, L^1, K^1) \\ &\approx n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1) \\ &\quad + [\partial n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)/\partial p_X][p_X^0 - p_X^1] \\ &= n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1) + X^1[p_X^0 - p_X^1] \\ &\qquad \text{using Hotelling's Lemma} \\ &= p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1 \\ &\quad + X^1[p_X^0 - p_X^1] \qquad \text{using (24.7) for } t = 1 \\ &= p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1, \qquad (24.13) \end{aligned}$$

where $p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1$ is the denominator on the right-hand side of equation (24.12). Because the numerator on the right-hand side of equation (24.12) is equal to $p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1$, which in turn is equal to $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)$, it can be seen that P_{PX} is indeed a first order approximation to the theoretical index α_{PX} defined by equation (24.11).[20]

24.32 Note that both the Laspeyres and Paasche theoretical indices of the effects on real income generated by the production sector of a change in the (real) price of exports are equally plausible and there is no reason

[19]Definition (24.11) is analogous to the Paasche output price effect defined by Diewert and Morrison (1986, p. 666) in the nominal GDP context. Diewert, Mizobuchi, and Nomura (2005, p. 19) and Diewert and Lawrence (2006, p. 13) used definitions (24.8), (24.11), and (24.14).

[20]This result was established by Diewert and Lawrence (2006, p. 16) and is closely related to similar results derived by Morrison and Diewert (1990, pp. 211–13).

to use one or the other of these two indices. Thus if it is desired to have a single theoretical measure of the effects of a change in real export prices, α_{LX} and α_{PX} should be averaged in a symmetric fashion to form a single target index that would summarize the effects on real income growth of a change in real export prices. Two obvious choices for the symmetric average are the arithmetic or geometric means of α_{LX} and α_{PX}. Following Diewert (1997) and Chapter 16, it seems preferable to use the geometric mean of α_{LX} and α_{PX} as the "best" single theoretical estimator of the effects of a change in real export prices on real income growth, because the resulting Fisher-like (1922) theoretical index satisfies the time reversal test so that if the ordering of the two periods is switched, the resulting index is the reciprocal of the original index.[21] Thus define the *theoretical Fisher type measure* α_{FX} *of the effects on real income growth of changes in real export prices* as the geometric mean of the Laspeyres and Paasche type theoretical measures:

$$\alpha_{FX} \equiv [\alpha_{LX}\alpha_{PX}]^{1/2}. \tag{24.14}$$

With the target index defined by equation (24.14) in mind, in the following section we consider the problem of finding empirical approximations to this theoretical index.

B.2 Empirical measures of the effects of changes in the real price of exports on the growth of real income generated by the production sector

24.33 Two empirical indices that provide estimates of the effects on the growth of real income of a change in real export prices have already been defined in Section B.1: the Laspeyres type index P_{LX} defined on the right-hand side of equation (24.9) and the Paasche type index P_{PX} defined on the right-hand side of equation (24.12). It was noted that P_{LX} was a lower bound to the theoretical index α_{LX} and P_{PX} was an upper bound to the theoretical index α_{PX}. Thus P_{LX} will generally have a downward bias compared to its theoretical counterpart whereas P_{PX} will generally have an upward bias compared to its theoretical counterpart. These inequalities suggest that the geometric mean of P_{LX} and P_{PX} is likely to be a reasonably good approximation to the target Fisher type index α_{FX} defined as the geometric mean of α_{LX} and α_{PX}. Thus define the *Diewert Lawrence index of the effects on real income of a change in real export prices going from period 0 to 1* as follows:[22]

$$P_{DLX} \equiv [P_{LX}P_{PX}]^{1/2}. \tag{24.15}$$

It will be useful to develop some alternative expressions for the indices P_{LX}, P_{PX}, and P_{DLX}.

24.34 As a preliminary step in developing these alternative expressions, recall definitions (24.7) which defined the production sector's period t real net product, $n^t(p^t_C, p^t_I, p^t_G, p^t_X, p^t_M, L^t, K^t)$ for $t = 0, 1$, which will be abbreviated to n^t. The period t shares of net product of C, I, G, X, and M are defined in the usual way as follows:

$$s^t_C \equiv p^t_C C^t/n^t;\ s^t_I \equiv p^t_I I^t/n^t;\ s^t_G \equiv p^t_G G^t/n^t;$$
$$s^t_X \equiv p^t_X X^t/n^t;\ s^t_M \equiv -p^t_M M^t/n^t;\ t = 0, 1. \tag{24.16}$$

24.35 It can be seen that the shares defined by equation (24.16) sum up to unity for each period t but note that the period t "share" for imports, s^t_M, is negative whereas the other shares are positive.

24.36 Now consider the definition of P_{LX} which occurred in equation (24.9) and subtract 1 from this expression:

$$\begin{aligned} &P_{LX} - 1 \\ &= \left[p^0_C C^0 + p^0_I I^0 + p^0_G G^0 + p^1_X X^0 - p^0_M M^0\right]/ \\ &\quad \left[p^0_C C^0 + p^0_I I^0 + p^0_G G^0 + p^0_X X^0 - p^0_M M^0\right] - 1 \\ &= \left[p^1_X - p^0_X\right] X^0/ \\ &\quad \left[p^0_C C^0 + p^0_I I^0 + p^0_G G^0 + p^0_X X^0 - p^0_M M^0\right] \\ &= \left[\left(p^1_X/p^0_X\right) - 1\right]\left[p^0_X X^0\right]/ \\ &\quad \left[p^0_C C^0 + p^0_I I^0 + p^0_G G^0 + p^0_X X^0 - p^0_M M^0\right] \\ &= s^0_X\,[r_X - 1], \end{aligned} \tag{24.17}$$

where r_X is (one plus) the rate of growth in the real price of exports going from 0 to 1; that is,

$$r_X \equiv p^1_X/p^0_X. \tag{24.18}$$

24.37 Thus P_{LX} depends on only $(r_X - 1)$, the growth rate in the price of real exports going from period 0

[21]The arithmetic average of the Laspeyres and Paasche theoretical indices does not satisfy this time reversal test.

[22]Diewert and Lawrence (2006, pp. 14–17) seem to have been the first to define and empirically estimate the indices defined by P_{LX}, P_{PX}, and (24.15) but the closely related work of Morrison and Diewert (1990, pp. 211–12) should also be noted.

to 1, and s_X^0, the share of exports in period 0 real net product; that is,

$$P_{LX} = 1 + s_X^0 (r_X - 1). \tag{24.19}$$

24.38 Using similar techniques, it can be shown that P_{PX} depends only on s_X^1, the share of exports in period 1, and the real export price relative, r_X, defined by equation (24.18):

$$P_{PX} = \left[1 + s_X^1\left(r_X^{-1} - 1\right)\right]^{-1}. \tag{24.20}$$

Comparing equations (24.19) and (24.20), it can be seen that both P_{LX} and P_{PX} are increasing functions of r_X so that as the real price of exports increases, both indices of growth in real income also increase as expected. It can also be seen that P_{LX} is increasing (decreasing) in s_X^0 and P_{PX} is increasing (decreasing) in s_X^1 if r_X is more (less) than one. These properties are also intuitively sensible. Substituting expressions (24.19) and (24.20) into (24.15) leads to the following expression for the Diewert Lawrence export index:

$$P_{DLX} = \left\{\left[1 + s_X^0(r_X - 1)\right]\Big/\left[1 + s_X^1\left(r_X^{-1} - 1\right)\right]\right\}^{1/2}. \tag{24.21}$$

As indicated above, the Diewert Lawrence index P_{DLX} defined by equation (24.21) is likely to be closer to the target Fisher index α_{FX} defined by equation (24.14) than the Laspeyres and Paasche type indices P_{LX} and P_{PX} defined by equations (24.19) and (24.20).

24.39 There is one additional empirically defined index that attempts to measure the effects of a change in real export prices on the growth of real income generated by the production sector and that is based on the work of Diewert and Morrison (1986). With the same notation that is used in equation (24.21), the logarithm of the *Diewert Morrison index*, P_{DMX}, *of the effects on real income of a change in real export prices going from period 0 to 1* is defined as follows:[23]

$$\ln P_{DMX} \equiv (1/2)\left(s_X^0 + s_X^1\right)\ln r_X. \tag{24.22}$$

24.40 It can be verified that P_{DMX} satisfies the time reversal property that was mentioned earlier; that is, if the two time periods are switched, then the new P_{DMX} index is equal to the reciprocal of the original P_{DMX} index.

24.41 The interest in the Diewert Morrison index stems from the fact that it has a very direct connection with production theory; in fact this index is *exactly* equal to the target index α_{FX} provided that the technology of the production sector can be represented by a general translog functional form in each period. This sentence is explained in more detail below.

24.42 In order to explain the above result, it is necessary to establish a general mathematical result. Thus let $x \equiv [x_1, \ldots, x_N]$ and $y \equiv [y_1, \ldots, y_M]$ be N and M dimensional vectors respectively and let f^0 and f^1 be two general quadratic functions defined as follows:

$$\begin{aligned} f^0(x, y) \equiv a_0^0 &+ \sum_{n=1}^{N} a_n^0 x_n + \sum_{m=1}^{M} b_m^0 y_m \\ &+ (1/2)\sum_{n=1}^{N}\sum_{j=1}^{N} a_{nj}^0 x_n x_j + \\ &+ (1/2)\sum_{m=1}^{M}\sum_{k=1}^{M} b_{mk}^0 y_m y_k \\ &+ \sum_{n=1}^{N}\sum_{j=1}^{N} c_{nm}^0 x_n y_m; \end{aligned} \tag{24.23}$$

$$\begin{aligned} f^1(x, y) \equiv a_0^1 &+ \sum_{n=1}^{N} a_n^1 x_n + \sum_{m=1}^{M} b_m^1 y_m \\ &+ (1/2)\sum_{n=1}^{N}\sum_{j=1}^{N} a_{nj}^1 x_n x_j + \\ &+ (1/2)\sum_{m=1}^{M}\sum_{k=1}^{M} b_{mk}^1 y_m y_k \\ &+ \sum_{n=1}^{N}\sum_{j=1}^{N} c_{nm}^1 x_n y_m; \end{aligned} \tag{24.24}$$

where the parameters a_{nj}^t satisfy the symmetry restrictions $a_{nj}^t = a_{jn}^t$ for $n, j = 1, \ldots, N$ and $t = 0, 1$ and the parameters b_{mk}^t satisfy the symmetry restrictions $b_{mk}^t = b_{km}^t$ for $m, k = 1, \ldots, M$ and $t = 0, 1$. It can be shown that if

$$a_{nj}^0 = a_{nj}^1 \text{ for } n, j = 1, \ldots, N, \tag{24.25}$$

then the following equation holds for all vectors x^0, x^1, y^0, and y^1:

$$\begin{aligned} &f^0(x^1, y^0) - f^0(x^0, y^0) + f^1(x^1, y^1) - f^0(x^0, y^1) \\ &= \sum_{n=1}^{N}\left[\partial f^0(x^0, y^0)/\partial x_n + \partial f^1(x^1, y^1)/\partial x_n\right]\left[x_n^1 - x_n^0\right]. \end{aligned} \tag{24.26}$$

[23]Strictly speaking, Diewert and Morrison (1986, p. 666) defined their index in the context of a GDP function rather than a net product function and did not deflate prices by a price index. The first applications of formula (24.22) were made by Diewert, Mizobuchi, and Nomura (2005) and Diewert and Lawrence (2006), but the basic methodology is from Diewert and Morrison. Kohli (1990) independently developed the same methodology as that of Diewert and Morrison.

24.43 The proof of the above proposition is very simple: Just use definitions (24.23) and (24.24), do the differentiation on the right-hand side of equation (24.26), and the result will emerge. The above result is a generalization of Diewert's (1976, p. 118) *quadratic identity*. A logarithmic version of the above identity corresponds to the *translog identity* which was established in the appendix to Caves, Christensen, and Diewert (1982b, pp. 1412–13).

24.44 Recall the definition of the period t real net product function $n^t(p_C, p_I, p_G, p_X, p_M, L, K)$ defined by equation (24.6). The notation will now be changed a bit. Let $p \equiv [p_1, \ldots, p_5]$ denote the vector of real output prices $[p_C, p_I, p_G, p_X, p_M]$ and let $z \equiv [z_1, z_2]$ denote the vector of primary input quantities $[L, K]$. The example of Diewert and Morrison (1986, p. 663) is now followed and it is assumed that the log of the period t real net product function, $n^t(p, z)$, has the following translog functional form:[24]

$$\begin{aligned}\ln n^t(p, z) &\equiv a_0^t + \sum_{n=1}^{5} a_n^t \ln p_n \\ &+ (1/2)\sum_{n=1}^{5}\sum_{j=1}^{5} a_{nj} \ln p_n \ln p_j + \sum_{m=1}^{2} b_m^t \ln z_m \\ &+ (1/2)\sum_{m=1}^{2}\sum_{k=1}^{2} b_{mk}^t \ln z_m \ln z_k \\ &+ \sum_{n=1}^{5}\sum_{m=1}^{2} c_{nm}^t \ln p_n \ln z_m; \quad t = 0, 1. \end{aligned} \tag{24.27}$$

24.45 Note that the coefficients for the quadratic terms in the logarithms of prices are assumed to be constant over time; that is, it is assumed that $a_{nj}^0 = a_{nj}^1 = a_{nj}$. The coefficients must satisfy the following restrictions in order for n^t to satisfy the linear homogeneity properties that are consistent with a constant returns to scale technology:[25]

$$\sum_{n=1}^{5} a_n^t = 1 \text{ for } t = 0, 1; \tag{24.28}$$

$$\sum_{m=1}^{2} b_m^t = 1 \text{ for } t = 0, 1; \tag{24.29}$$

$$a_{nj} = a_{jn} \text{ for all } n, j; \tag{24.30}$$

$$b_{mk}^t = b_{km}^t \text{ for all } m, k \text{ and } t = 0, 1. \tag{24.31}$$

$$\sum_{k=1}^{M} a_{mk} = 0 \text{ for } m = 1, 2; \tag{24.32}$$

$$\sum_{j=1}^{N} b_{nj}^t = 0 \text{ for } n = 1, \ldots, 5 \text{ and } i = 0, 1; \tag{24.33}$$

$$\sum_{n=1}^{N} c_{mn}^t = 0 \text{ for } m = 1, 2 \text{ and } t = 0, 1; \tag{24.34}$$

$$\sum_{m=1}^{M} c_{mn}^t = 0 \text{ for } n = 1, \ldots, 5 \text{ and } t = 0, 1. \tag{24.35}$$

24.46 Note that using Hotelling's Lemma, the logarithmic derivatives of $n^t(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t)$ with respect to the logarithm of the export price are equal to the following expressions for $t = 0, 1$:

$$\begin{aligned}&\partial \ln n^t\big(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t\big)/\partial \ln p_X \\ &\quad = \big[p_X^t/n^t\big]\, \partial n^t\big(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t\big)/\partial p_X \\ &\quad = \big[p_X^t/n^t\big]\, X^t \\ &\quad = s_X^t \qquad \text{using equation (24.16).}\end{aligned} \tag{24.36}$$

24.47 Because assumptions (24.27) imply that the logarithms of the net product functions are quadratic in the logarithms of prices and quantities, the result given by equation (24.26) can be applied to definitions (24.7), (24.8), (24.11), and (24.14) to imply the following result:

$$\begin{aligned}2 \ln \alpha_{FX} &= \ln \alpha_{LX} + \ln \alpha_{PX} \\ &= \big[\partial \ln n^0\big(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\big)/\partial \ln p_X \\ &\quad + \partial \ln n^1\big(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1\big)/ \\ &\quad \partial \ln p_X\big] \times \big[\ln p_X^1 - \ln p_X^0\big] \\ &= \big[s_X^0 + s_X^1\big] \ln\big(p_X^1/p_X^0\big) \\ &\qquad \text{using equation (24.36).}\end{aligned} \tag{24.37}$$

24.48 Thus using equations (24.22) and (24.37), one can see that under the assumptions made on the technology, the following exact equality holds:[26]

$$\alpha_{FX} = P_{DMX}. \tag{24.38}$$

[24]This functional form was first suggested by Diewert (1974a, p. 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson, and Lau (1971). Diewert (1974a, p. 139) indicated that this functional form was flexible. Flexible functional forms can approximate arbitrary functions to the second order at any given point and hence it is desirable to assume that the technological production possibilities can be represented by a flexible functional form in each period. Flexible functional forms are discussed in more detail in Diewert (1974a).

[25]There are additional restrictions on the parameters which are necessary to ensure that $n^t(p, z)$ is convex in p and concave in z. The restrictions (24.29), (24.33), and (24.34) are not required for the results in this chapter. However, they impose constant returns to scale on the technology which is useful if a complete decomposition of real income growth into explanatory factors is attempted as in Diewert and Lawrence (2006).

[26]This result is a straightforward adaptation of the results of Diewert and Morrison (1986, p. 666).

Thus the Diewert Morrison index P_{DMX} defined by equation (24.22) is exactly equal to the target theoretical index, α_{FX}, under very weak assumptions on the technology.

24.49 Although the Diewert Morrison index gets a strong endorsement from the above result, the Diewert Lawrence index also had a reasonably strong justification and so the question arises: Which index should be used in empirical applications? Section B.3 shows that numerically these two indices will be quite close and so empirically, it will usually not matter which of these two alternative indices is chosen.

B.3 The numerical equivalence of the Diewert Lawrence and Diewert Morrison measures of the effects of changes in the real price of exports

24.50 Let $p \equiv [p_1, \ldots, p_5]$ denote the vector of real output prices $[p_C, p_I, p_G, p_X, p_M]$ and let $q \equiv [q_1, \ldots, qp_5]$ denote the corresponding vector of quantities $[C, I, G, X, -M]$. Thus the data pertaining to period t can be denoted by the vectors $p^t \equiv [p^t_C, p^t_I, p^t_G, p^t_X, p^t_M]$ and $q^t \equiv [C^t, I^t, G^t, X^t, -M^t]$ for $t = 0, 1$. Note that each of the four empirical indices P_{LX}, P_{PX}, P_{DLX}, and P_{DMX} defined in the previous section can be regarded as functions of the data pertaining to the two periods under consideration. Thus P_{LX} should be more precisely be written as the function $P_{LX}(p^0, p^1, q^0, q^1)$, P_{PX} should be written as $P_{PX}(p^0, p^1, q^0, q^1)$, and so on. In this section, it is desired to compare the numerical properties of the four indices P_{LX}, P_{PX}, P_{DLX}, and P_{DMX}.

24.51 Diewert (1978) undertook a similar comparison of all superlative indices that were known at that time. He showed that all known superlative indices approximated each other to the second order around when the derivatives were evaluated at a point where the period 0 price vector p^0 was equal to the period 1 price vector p^1 and where the period 0 quantity vector was equal to the period 1 quantity vector.[27]

24.52 A somewhat similar result holds in the present context; that is, it can be shown that the following equalities hold for the four indices P_{LX}, P_{PX}, P_{DLX}, and P_{DMX}:[28]

$$P_{LX}(p, p, q, q) = P_{PX}(p, p, q, q) = P_{DLX}(p, p, q, q) = P_{DMX}(p, p, q, q) = 1; \qquad (24.39)$$

$$\nabla P_{LX}(p, p, q, q) = \nabla P_{PX}(p, p, q, q) = \nabla P_{DLX}(p, p, q, q) = \nabla P_{DMX}(p, p, q, q), \qquad (24.40)$$

where $\nabla P_{LX}(p, p, q, q)$ is the 20-dimensional vector of first order partial derivatives of $P_{LX}(p^0, p^1, q^0, q^1)$ with respect to the components of p^0, p^1, q^0, and q^1 but evaluated at a point where $p^0 = p^1 \equiv p$ and $q^0 = q^1 \equiv q$. The meaning of equations (24.39) and (24.40) is that the four indices approximate each other to the accuracy of a first order Taylor series approximation around a data point where the real prices are equal in each period and the net output quantities are also equal to each other across periods.

24.53 The second order derivatives of the Laspeyres and Paasche type indices, P_{LX} and P_{PX}, are *not* equal to each other when evaluated at an equal price and quantity point; that is,

$$\nabla^2 P_{LX}(p, p, q, q) \neq \nabla^2 P_{PX}(p, p, q, q), \qquad (24.41)$$

where $\nabla^2 P_{LX}(p, p, q, q)$ is the 20 by 20 dimensional matrix of second order partial derivatives of $P_{LX}(p^0, p^1, q^0, q^1)$ with respect to the components of p^0, p^1, q^0, and q^1 but evaluated at a point where $p^0 = p^1 \equiv p$ and $q^0 = q^1 \equiv q$. Thus as might be expected, P_{LX} and P_{PX} do not approximate each other to the accuracy of a second order Taylor series approximation around an equal price and quantity point.

24.54 However, the second order derivatives of the Diewert Lawrence and Diewert Morrison indices, P_{DLX} and P_{DMX}, are equal to each other when evaluated at an equal price and quantity point; that is,[29]

$$\nabla^2 P_{DLX}(p, p, q, q) = \nabla^2 P_{DMX}(p, p, q, q). \qquad (24.42)$$

24.55 Thus P_{DLX} and P_{DMX} approximate each other to the accuracy of a second order Taylor series approximation around a data point where the real prices are equal in each period and the net output quantities are also equal to each other across periods. The practical significance of this result is that for normal time series data where adjacent periods are compared, the Diewert Lawrence and Diewert Morrison indices will give virtually identical results.[30]

[27] Subsequent research by Robert Hill (2006) has shown that Diewert's approximation results break down for the quadratic mean of order r superlative indices as r becomes large in magnitude.

[28] The proof is a series of straightforward computations.

[29] Again, a long series of routine computations establishes this result. Note that these second derivative matrices are not equal to $\nabla^2 P_{LX}(p, p, q, q)$ or to $\nabla^2 P_{PX}(p, p, q, q)$.

[30] See Tables 5 and 9 in Diewert and Lawrence (2006), which establish the approximate equality of these indices (to four significant figures) using Australian data in a gross product framework and Tables 12 and 14 which establish the approximate equality of these indices in a net product framework for Australia.

B.4 Real-time approximations to the preferred measures

24.56 The Diewert Lawrence index of the effects on real income growth of a change in the real export price, P_{DLX} defined by equation (24.21), depends on the real export price relative, r_X, the period 0 real export share in net product, s_X^0, and the corresponding period 1 real export share, s_X^1. Our other preferred measure of the effects of a change in the real export price, P_{DMX} defined by equation (24.22), also depends on these same three variables, r_X, s_X^0, and s_X^1. However, the current-period export share s_X^1 is unlikely to be available to analysts until some time later than the current period. Thus the question arises: How can approximations be formed to the preferred indices defined by equations (24.21) and (24.22)? An answer to this question is as follows:

24.57 Suppose that it is suspected that quantities are relatively unresponsive to changes in relative prices so that the period 1 quantity vector $[C^1, I^1, G^1, X^1, -M^1]$ will be approximately proportional to the corresponding period 0 quantity vector $[C^0, I^0, G^0, X^0, -M^0]$. Under these conditions, α_{LX} will be close to the Laspeyres type index defined by (24.19), which is $P_{LX} = 1 + s_X^0(r_X - 1)$, and a close approximation to α_{PX} can be obtained by using the formula $[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1]/[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^0X^1 - p_M^1M^1]$. Now multiply this last formula by P_{LX} and take the positive square root in order to obtain a good approximation to the theoretical export price effects index α_{FX}. Suppose that the share of exports in net product in period 1, s_X^1, is expected to be approximately equal to the corresponding period 0 share, s_X^0. Then simply use formula (24.22) with s_X^1 set equal to s_X^0. If neither of the above conditions is expected to hold for the period 1 data, simply make an approximate forecast for the period 1 export share s_X^1 and use equation (24.22).

C. The Effects of Changes in the Real Price of Imports

24.58 The theory that was outlined in Section B can be repeated in the present section in order to measure the effects on real income generated by the production sector of a change in real import prices. Basically, all that needs to be done is to replace p_X by p_M and note that the import shares s_M^t defined in equation (24.16) are negative whereas the export shares s_X^t used in Section B were positive. Some of the definitions are listed here without much explanation. The reader should be able to work out the analogies with the export indices.

24.59 *A theoretical Laspeyres type measure* α_{LM} *of the effects on real income growth of changes in real import prices* from the period 0 level, p_M^0, to the period 1 level, p_M^1, can be defined as the ratio of the hypothetical net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^1, L^0, K^0)$ to the actual period 0 net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0)$:

$$\alpha_{LM} \equiv n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^1, L^0, K^0)/ n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0). \qquad (24.43)$$

The index α_{LM} of the effects of the change in the real price of imports is termed a *Laspeyres type index* because it holds constant all exogenous prices and quantities at their period 0 levels except for the two real import prices, p_M^0 and p_M^1, and the index also holds technology constant at the base-period level.

24.60 There is the following *observable lower bound* to the theoretical index α_{LM} defined by equation (24.43):

$$\begin{aligned}\alpha_{LM} &\geq \left[p_C^0 C^0 + p_I^0 I^0 + p_G^0 G^0 + p_X^0 X^0 - p_M^1 M^0\right]/\\ &\quad \left[p_C^0 C^0 + p_I^0 I^0 + p_G^0 G^0 + p_X^0 X^0 - p_M^0 M^0\right]\\ &\equiv P_{LM}, \end{aligned} \qquad (24.44)$$

where P_{LM} is an *observable Laspeyres type index* of the effects on real income of a change in real import prices going from period 0 to 1. Note that the numerator and denominator on the right-hand side of equation (24.44) are identical except that p_M^1 appears in the numerator and p_M^0 appears in the denominator.

24.61 It is possible to show that the Laspeyres type observable index P_{MX} is a first order Taylor series approximation to the theoretical Laspeyres type index α_{MX}; that is, it is possible to derive a counterpart to the approximation (24.10).

24.62 *A theoretical Paasche type measure* α_{PM} *of the effects on real income growth of changes in real import prices* from the period 0 level, p_M^0, to the period 1 level, p_M^1, can be defined as the ratio of the actual period 1 net real income $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)$ to the hypothetical net real income $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^0, L^1, K^1)$:

$$\alpha_{PM} \equiv \left(n^1 p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1\right)/ n^1\left(p_C^1, p_I^1, p_G^1, p_X^1, p_M^0, L^1, K^1\right). \qquad (24.45)$$

The index α_{PM} of the effects of the change in the real price of imports is termed a *Paasche type index* because it holds constant all exogenous prices and quantities at their period 1 levels except for the two real import prices, p_M^0 and p_M^1, and the index also holds technology constant at the period 1 level.

24.63 Using assumption (24.7) for $t = 1$, the numerator on the right-hand side of equation (24.45) is equal to period 1 observed real net product, $p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1$. Using definition (24.6), it can be seen that C^1, I^1, G^1, X^1, and M^1 is a feasible solution for the net product maximization problem defined by the denominator on the right-hand side of equation (24.45), $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^0, L^1, K^1)$. These facts mean that there is the following *observable upper bound* to the theoretical index α_{LM} defined by equation (24.45):

$$\alpha_{PM} \leq \left[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^1M^1\right] / \left[p_C^1C^1 + p_I^1I^1 + p_G^1G^1 + p_X^1X^1 - p_M^0M^1\right] \equiv P_{PM}, \quad (24.46)$$

where P_{PM} is an *observable Paasche type index* of the effects on real income of a change in real import prices going from period 0 to 1. Note that the numerator and denominator on the right-hand side of equation (24.46) are identical except that p_M^1 appears in the numerator and p_M^0 appears in the denominator.

24.64 It is possible to show that the Paasche type observable index P_{PM} is a first order Taylor series approximation to the theoretical Paasche type index α_{PM}; that is, a counterpart to the approximation (24.13) can be derived.

24.65 Note that both the Laspeyres and Paasche theoretical indices of the effects on real income generated by the production sector of a change in the (real) price of imports are equally plausible and there is no reason to use one or the other of these two indices. Thus if it is desired to have a single theoretical measure of the effects of a change in real import prices, α_{LM} and α_{PM} should be geometrically averaged. Thus define the *theoretical Fisher type measure* α_{FM} *of the effects on real income growth of changes in real import prices* as the geometric mean of the Laspeyres and Paasche type theoretical measures:

$$\alpha_{FM} \equiv [\alpha_{LM}\alpha_{PM}]^{1/2}. \quad (24.47)$$

Now that the target import index defined by equation (24.47) is in hand, the problem of finding empirical approximations to this theoretical index is now considered.

24.66 Two empirical indices that provide estimates of the effects on the growth of real income of a change in real import prices have already been defined above: the Laspeyres type index P_{LM} defined on the right-hand side of equation (24.44) and the Paasche type index P_{PM} defined on the right-hand side of equation (24.46). It was noted that P_{LM} was a lower bound to the theoretical index α_{LM} and P_{PM} was an upper bound to the theoretical index α_{PM}. Thus P_{LM} will generally have a downward bias compared to its theoretical counterpart whereas P_{PM} will generally have an upward bias compared to its theoretical counterpart. These inequalities suggest that the geometric mean of P_{LM} and P_{PM} is likely to be a reasonably good approximation to the target Fisher type index α_{FM} defined as the geometric mean of α_{LM} and α_{PM}. Thus define the *Diewert Lawrence index of the effects on real income of a change in real import prices going from period 0 to 1* as follows:[31]

$$P_{DLM} \equiv [P_{LM}\,P_{PM}]^{1/2}. \quad (24.48)$$

24.67 As in Section B, it will be useful to develop some alternative expressions for the indices P_{LM}, P_{PM}, and P_{DLM}. Define the *price relative* r_M *for real import prices* as

$$r_M \equiv p_M^1/p_M^0. \quad (24.49)$$

24.68 With the techniques described in Section B, the following alternative formulas for P_{LM}, P_{PM}, and P_{DLM} can be derived:[32]

$$P_{LM} = 1 + s_M{}^0\,(r_M - 1); \quad (24.50)$$

$$P_{PM} = \left[1 + s_M{}^1(r_M{}^{-1} - 1)\right]^{-1}. \quad (24.51)$$

24.69 Noting that s_M^0 and s_M^1 are negative, one can see that both P_{LM} and P_{PM} are decreasing functions of r_M so that as the real price of imports increases, both indices of growth in real income also decrease as expected.

24.70 Substituting expressions (24.50) and (24.51) into (24.48) leads to the following expression for the Diewert Lawrence import index:

$$P_{DLM} = \left\{\left[1 + s_M^0\,(r_M - 1)\right] / \left[1 + s_M^1\left(r_M^{-1} - 1\right)\right]\right\}^{1/2}. \quad (24.52)$$

[31]Diewert and Lawrence (2006, pp. 14–17) seem to have been the first to define and empirically estimate the indices defined by P_{LM}, P_{PM}, and equation (24.48).

[32]Remember that the period 0 "share" of imports in net real product, s_M^0, is negative whereas the period 0 share of exports which appeared in the counterpart result (24.19), s_X^0, was positive.

24.71 The Diewert Lawrence index P_{DLM} defined by equation (24.52) is likely to be closer to the target Fisher index α_{FM} defined by equation (24.47) than the Laspeyres and Paasche type indices P_{LM} and P_{PM} defined by equations (24.50) and (24.51).

24.72 Using the same notation that is used in equation (24.52), the logarithm of the *Diewert Morrison index*, P_{DMM}, *of the effects on real income of a change in real import prices going from period 0 to 1* is defined as follows:[33]

$$\ln P_{DMM} \equiv (1/2)\left(s_M^0 + s_M^1\right)\ln r_M. \qquad (24.53)$$

24.73 It can be verified that P_{DMM} satisfies the time reversal; that is, if the two time periods are switched, then the new P_{DMM} index is equal to the reciprocal of the original P_{DMM} index.

24.74 As in Section B, the interest in the Diewert Morrison index stems from the fact that it has a very direct connection with production theory; in fact, this index is *exactly* equal to the target index α_{FM} provided that the technology of the production sector can be represented by a general translog functional form in each period. Again make the translog assumptions (24.27) through (24.35).

24.75 Using Hotelling's Lemma, the logarithmic derivatives of $n^t(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t)$ with respect to the logarithm of the import price are equal to the following expressions for $t = 0, 1$:

$$\begin{aligned}
&\partial\ln\left(n^t p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t\right)/\partial\ln p_M \\
&= \left[p_M^t/n^t\right] \partial n^t\left(p_C^t, p_I^t, p_G^t, p_X^t, p_M^t, L^t, K^t\right)/\partial p_M \\
&= \left[p_M^t/n^t\right][-M^t] \\
&= s_M^t \qquad \text{using (24.16).} \qquad (24.54)
\end{aligned}$$

24.76 Noting that assumptions (24.27) imply that the logarithms of the net product functions are quadratic in the logarithms of prices and quantities, one can apply the result given by (24.26) to definitions (24.43), (24.45), and (24.47) to imply the following result:

$$\begin{aligned}
2\ln\alpha_{FM} &= \ln\alpha_{LM} + \ln\alpha_{PM} \\
&= \left[\partial\ln n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right)/\partial\ln p_M \right.\\
&\quad + \partial\ln n^1\left(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1\right)/ \\
&\quad \left.\partial\ln p_M\right]\left[\ln p_M^1 - \ln p_M^0\right] \\
&= \left[s_M^0 + s_M^1\right]\ln\left(p_M^1/p_M^0\right) \qquad \text{using (24.54).} \qquad (24.55)
\end{aligned}$$

24.77 Thus using (24.53) and (24.55), one can see that under the assumptions made on the technology, the following exact equality holds:[34]

$$\alpha_{FM} = P_{DMM}. \qquad (24.56)$$

Thus the Diewert Morrison import price effects on real income growth index P_{DMM} defined by equation (24.53) are *exactly* equal to the target theoretical index, α_{FM}, under very weak assumptions on the technology.

24.78 It can be shown that the following equalities hold for the four empirical indices P_{LM}, P_{PM}, P_{DLM}, and P_{DMM}:[35]

$$\begin{aligned}
&P_{LM}(p, p, q, q) = P_{PM}(p, p, q, q) = P_{DLM}(p, p, q, q) \\
&= P_{DMM}(p, p, q, q) = 1; \qquad (24.57)
\end{aligned}$$

$$\begin{aligned}
&\nabla P_{LM}(p, p, q, q) = \nabla P_{PM}(p, p, q, q) \\
&= \nabla P_{DLM}(p, p, q, q) = \nabla P_{DMM}(p, p, q, q), \qquad (24.58)
\end{aligned}$$

where $\nabla P_{LM}(p, p, q, q)$ is the 20 dimensional vector of first order partial derivatives of $P_{LM}(p^0, p^1, q^0, q^1)$ with respect to the components of p^0, p^1, q^0, and q^1 but evaluated at a point where $p^0 = p^1 \equiv p$ and $q^0 = q^1 \equiv q$. As usual, the meaning of equations (24.57) and (24.58) is that the four indices approximate each other to the accuracy of a first order Taylor series approximation around an equal prices and quantities data point.

24.79 The second order derivatives of the Laspeyres and Paasche type indices, P_{LM} and P_{PM}, are *not* equal to each other when evaluated at an equal price and quantity point; that is,

$$\nabla^2 P_{LM}(p, p, q, q) \neq \nabla^2 P_{PM}(p, p, q, q), \qquad (24.59)$$

where $\nabla^2 P_{LM}(p, p, q, q)$ is the 20 by 20 dimensional matrix of second order partial derivatives of $P_{LM}(p^0, p^1, q^0, q^1)$ with respect to the components of p^0, p^1, q^0, and

[33]Strictly speaking, Diewert and Morrison (1986; p. 666) defined their index in the context of a GDP function rather than a net product function and did not deflate prices by a price index. The first applications of formula (24.22) were made by Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

[34]This result is a straightforward adaptation of the results of Diewert and Morrison (1986, p. 666).

[35]The proof is a series of straightforward computations.

q^1 but evaluated at a point where $p^0 = p^1 \equiv p$ and $q^0 = q^1 \equiv q$. Thus as might be expected, P_{LM} and P_{PM} do *not* approximate each other to the accuracy of a second order Taylor series approximation around an equal price and quantity point.

24.80 However, the second order derivatives of the Diewert Lawrence and Diewert Morrison import indices, P_{DLM} and P_{DMM}, are equal to each other when evaluated at an equal price and quantity point; that is,[36]

$$\nabla^2 P_{DLM}(p, p, q, q) = \nabla^2 P_{DMM}(p, p, q, q). \quad (24.60)$$

24.81 Thus P_{DLM} and P_{DMM} approximate each other to the accuracy of a second order Taylor series approximation around a data point where the real prices are equal in each period and the net output quantities are also equal to each other across periods. The practical significance of this result is that for normal time series data where adjacent periods are compared, the Diewert Lawrence and Diewert Morrison indices give virtually identical results.[37]

D. The Combined Effects of Changes in the Real Prices of Exports and Imports

24.82 In this section, instead of *separately* considering the effects of a change in real export or real import prices on the real income generated by the production sector, the effects of a *combined* change in real export and import prices are considered. It turns out that the same type of analysis that was used in the previous two sections can be used in the present section.

24.83 In a first attempt to measure the effects of changing real import and export prices over the two periods under consideration, a hypothetical period 1 net domestic product maximization problem is considered where producers have at their disposal the period 0 technology set S^0 and the period 0 actual labor and capital inputs, L^0 and K^0 respectively, and they face the period 0 real prices for consumption, net investment, and government consumption, p_C^0, p_I^0, and p_G^0 respectively, but they face the period 1 real export and import prices, p_X^1 and p_M^1. The solution to this hypothetical net product maximization problem is $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^1, L^0, K^0)$. A *theoretical Laspeyres type measure* α_{LXM} *of the effects on real income growth of the combined changes in real export and import prices* from their period 0 levels, p_X^0 and p_M^0, to their period 1 levels, p_X^1 and p_M^1, can be defined as the ratio of the hypothetical net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^1, p_M^1, L^0, K^0)$ to the actual period 0 net real income $n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0)$:

$$\alpha_{LXM} \equiv n^0\left(p_C^0, p_I^0, p_G^0, p_X^1, p_M^1, L^0, K^0\right) / n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right). \quad (24.61)$$

24.84 The index α_{LXM} of the effects of the change in the real prices of exports and imports is termed a *Laspeyres type index* because it holds constant all exogenous prices and quantities at their period 0 levels except for the four real export and import prices, p_X^0, p_X^1, p_M^0, and p_M^1, and the index also holds technology constant at the base-period level.

24.85 As usual, a feasibility argument leads to the following *observable lower bound* to the theoretical index α_{LXM} defined by equation (24.61):

$$\alpha_{LXM} \geq \left[p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^1X^0 - p_M^1M^0\right] / \left[p_C^0C^0 + p_I^0I^0 + p_G^0G^0 + p_X^0X^0 - p_M^0M^0\right] \equiv P_{LXM}, \quad (24.62)$$

where P_{LXM} is an *observable Laspeyres type index* of the effects on real income of a change in real export and import prices going from period 0 to 1. Note that the numerator and denominator on the right-hand side of equation (24.62) are identical except that p_X^1 and p_M^1 appear in the numerator while p_X^0 and p_M^0 appear in the denominator. It is possible to show that the Laspeyres type observable index P_{LMX} is a first order Taylor series approximation to the theoretical Laspeyres type index α_{LMX}; that is, it is possible to derive a counterpart to the approximation (24.10).

24.86 *A theoretical Paasche type measure* α_{PM} *of the effects on real income growth of changes in real export and import prices* from the period 0 levels, p_X^0 and p_M^0, to the period 1 levels, p_X^1 and p_M^1, can be defined as the ratio of the actual period 1 net real income $n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)$ to the hypothetical net real income defined by $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^0, L^1, K^1)$:

$$\alpha_{PXM} \equiv n^1\left(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1\right) / n^1\left(p_C^1, p_I^1, p_G^1, p_X^0, p_M^0, L^1, K^1\right). \quad (24.63)$$

[36] Again a long series of routine computations establishes this result.

[37] See Tables 5, 9, 12, and 14 in Diewert and Lawrence (2006) for a numerical illustration of this result.

The index α_{PXM} of the effects of the changes in the real prices of exports and imports is termed a *Paasche type index* because it holds constant all exogenous prices and quantities at their period 1 levels except for the real export and import prices, p_X^0, p_X^1, p_M^0, and p_M^1, and the index also holds technology constant at the period 1 level.

24.87 Using assumption (24.7) for $t = 1$, the numerator on the right-hand side of equation (24.63) is equal to the period 1 observed real net product, $p_C^1 C^1 + p_I^1 I^1 + p_G^1 G^1 + p_X^1 X^1 - p_M^1 M^1$. Using definition (24.6), one can see that C^1, I^1, G^1, X^1, and M^1 are a feasible solution for the net product maximization problem defined by the denominator on the right-hand side of equation (24.63), $n^1(p_C^1, p_I^1, p_G^1, p_X^0, p_M^0, L^1, K^1)$. These facts mean that there is the following *observable upper bound* to the theoretical index α_{LXM} defined by equation (24.63):

$$\alpha_{PXM} \leq \left[p_C^1 C^1 + p_I^1 I^1 + p_G^1 G^1 + p_X^1 X^1 - p_M^1 M^1\right] / \left[p_C^1 C^1 + p_I^1 I^1 + p_G^1 G^1 + p_X^0 X^1 - p_M^0 M^1\right] \equiv P_{PXM}, \quad (24.64)$$

where P_{PXM} is an *observable Paasche type index* of the effects on real income of a change in real export and import prices going from period 0 to 1. Note that the numerator and denominator on the right-hand side of equation (24.64) are identical except that p_X^1 and p_M^1 appear in the numerator while p_X^0 and p_M^0 appear in the denominator.

24.88 As usual, it is possible to show that the Paasche type observable index P_{PXM} is a first order Taylor series approximation to the theoretical Paasche type index α_{PXM}; that is, a counterpart to the approximation (24.13) can be derived.

24.89 Note that both the Laspeyres and Paasche theoretical indices of the effects on real income generated by the production sector of a change in the (real) prices of exports and imports are equally plausible and there is no reason to use one or the other of these two indices. Thus, as usual, α_{LXM} and α_{PXM} should be geometrically averaged. Hence define the *theoretical Fisher type measure* α_{FXM} *of the effects on real income growth of changes in real export and import prices* as the geometric mean of the Laspeyres and Paasche type theoretical measures:

$$\alpha_{FXM} \equiv [\alpha_{LXM}\alpha_{PXM}]^{1/2}. \quad (24.65)$$

Now that a target export and import index has been defined by (24.65), the problem of finding empirical approximations to this theoretical index is considered.

24.90 Two empirical indices that provide estimates of the effects on the growth of real income of a change in real export prices have already been defined above: the Laspeyres type index P_{LXM} defined on the right-hand side of equation (24.44) and the Paasche type index P_{PXM} defined on the right-hand side of equation (24.46). It was noted that P_{LXM} was a lower bound to the theoretical index α_{LXM} and P_{PXM} was an upper bound to the theoretical index α_{PXM}. Thus P_{LXM} will generally have a downward bias compared to its theoretical counterpart whereas P_{PXM} will generally have a upward bias compared to its theoretical counterpart These inequalities suggest that the geometric mean of P_{LXM} and P_{PXM} is likely to be a reasonably good approximation to the target Fisher type index α_{FXM} defined as the geometric mean of α_{LXM} and α_{PXM}. Thus define the *Diewert Lawrence index of the effects on real income of a change in real export and import prices going from period 0 to 1* as follows:[38]

$$P_{DLXM} \equiv [P_{LXM}\, P_{PXM}]^{1/2}. \quad (24.66)$$

24.91 As in Section B, it will be useful to develop some alternative expressions for the indices P_{LXM}, P_{PXM}, and P_{DLXM}. Using the techniques described in Section B, one can derive the following alternative formulas for P_{LXM}, P_{PXM}, and P_{DLXM}:

$$P_{LXM} = 1 + s_X^0 (r_X - 1) + s_M^0 (r_M - 1); \quad (24.67)$$

$$P_{PXM} = \left[1 + s_X^1\left(r_X^{-1} - 1\right) + s_M^1\left(r_M^{-1} - 1\right)\right]^{-1}, \quad (24.68)$$

where the export shares s_X^t (positive) and import shares s_M^t (negative) are defined in (24.16), $r_X \equiv p_X^1/p_X^0$ is the real export price relative, and $r_M \equiv p_M^1/p_M^0$ is the real import price relative. It can be seen that both P_{LXM} and P_{PXM} are increasing functions of r_X and decreasing functions of r_M, because the s_M^t are negative as defined in equation (24.16), so that as the real price of exports increases, both indices of growth in real income increase and as the real price of imports increases, both indices of growth in real income decrease.

[38]Diewert and Lawrence (2006, p. 15) did not actually define the index on the right-hand side of equation (24.66); instead they defined a counterpart index that looked at the effects on real income growth of a change in all real output prices (rather than the effects of a change in just real export and import prices). However, the basic idea behind equation (24.48) is from Diewert and Lawrence.

24.92 Substituting expressions (24.67) and (24.68) into (24.48) leads to the following expression for the Diewert Lawrence export and import index:

$$P_{DLXM} = \left\{\left[1 + s_X^0 (r_X - 1) + s_M^0 (r_M - 1)\right] \Big/ \left[1 + s_X^1(r_X^{-1} - 1) + s_M^1(r_M^{-1} - 1)\right]\right\}^{1/2}. \quad (24.69)$$

The Diewert Lawrence index P_{DLXM} defined by equation (24.69) is likely to be closer to the target Fisher index α_{FXM} defined by equation (24.65) than the Laspeyres and Paasche type indices P_{LXM} and P_{PXM} defined by equations (24.67) and (24.68).

24.93 As in Sections B and C, there is an alternative to the Diewert Lawrence index P_{DLXM} defined by equation (24.69), namely the Diewert Morrison index P_{DMMX}. Using the same notation that is used in equation (24.69) above, the logarithm of the *Diewert Morrison index*, P_{DMXM}, *of the effects on real income of changes in real export and import prices going from period 0 to 1* is defined as follows:[39]

$$\ln P_{DMXM} \equiv (1/2)\left(s_X^0 + s_X^1\right)\ln r_X + (1/2)\left(s_M^0 + s_M^1\right)\ln r_M. \quad (24.70)$$

24.94 It can be verified that P_{DMXM} satisfies the time reversal; that is, if the two time periods are switched, then the new P_{DMXM} index is equal to the reciprocal of the original P_{DMXM} index.

24.95 As in Section B, the Diewert Morrison index is *exactly* equal to the target index α_{FXM} provided that the technology of the production sector can be represented by a general translog functional form in each period. Thus again make the translog assumptions (24.27) through (24.35). Using the Hotelling's Lemma results (24.36) and (24.54) and noting that assumptions (24.27) imply that the logarithms of the net product functions are quadratic in the logarithms of prices and quantities, one can apply the result given by equation (24.26) to definitions (24.61), (24.63), and (24.65) to imply the following result:

$$\begin{aligned}
2 \ln \alpha_{FXM} &= \ln \alpha_{LXM} + \ln \alpha_{PXM} \\
&= \left[\partial \ln n^0\left(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0\right)/\partial \ln p_X + \partial \ln n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)/\partial \ln p_X\right] \\
&\quad \times \left[\ln p_X^1 - \ln p_X^0\right] \\
&\quad + \left[\partial \ln n^0(p_C^0, p_I^0, p_G^0, p_X^0, p_M^0, L^0, K^0)/\partial \ln p_M + \partial \ln n^1(p_C^1, p_I^1, p_G^1, p_X^1, p_M^1, L^1, K^1)/\partial \ln p_M\right] \\
&\quad \times \left[\ln p_M^1 - \ln p_M^0\right] \\
&= \left[s_X^0 + s_X^1\right] \ln\left(p_X^1/p_X^0\right) + \left[s_M^0 + s_M^1\right] \ln\left(p_M^1/p_M^0\right).
\end{aligned} \quad (24.71)$$

24.96 Thus using equations (24.70) and (24.71), one can see that under the assumptions made on the technology, the following exact equality holds:[40]

$$\alpha_{FM} = P_{DMM}. \quad (24.72)$$

24.97 Thus the Diewert Morrison combined export and import price effects on real income growth index P_{DMXM} defined by equation (24.70) is *exactly* equal to the target theoretical index α_{FXM} defined by equation (24.65) under very weak assumptions on the technology.

24.98 As in Sections B and C above, it can be shown that the four empirical indices P_{LXM}, P_{PXM}, P_{DLXM}, and P_{DMXM} numerically approximate each other to the first order around an equal price and quantity point:

$$P_{LXM}(p, p, q, q) = P_{PXM}(p, p, q, q) = P_{DLXM}(p, p, q, q) = P_{DMXM}(p, p, q, q) = 1; \quad (24.73)$$

$$\nabla P_{LXM}(p, p, q, q) = \nabla P_{PXM}(p, p, q, q) = \nabla P_{DLXM}(p, p, q, q) = \nabla P_{DMXM}(p, p, q, q), \quad (24.74)$$

where $\nabla P_{LXM}(p, p, q, q)$ is the 20 dimensional vector of first order partial derivatives of $P_{LXM}(p^0, p^1, q^0, q^1)$ with respect to the components of p^0, p^1, q^0, and q^1 but evaluated at a point where $p^0 = p^1 \equiv p$ and $q^0 = q^1 \equiv q$.

24.99 The second order derivatives of the Laspeyres and Paasche type indices, P_{LXM} and P_{PxM}, are *not* equal to each other when evaluated at an equal price and quantity point; that is,

$$\nabla^2 P_{LXM}(p, p, q, q) \neq \nabla^2 P_{PXM}(p, p, q, q). \quad (24.75)$$

However, the second order derivatives of the Diewert Lawrence and Diewert Morrison combined export and import indices, P_{DLXM} and P_{DMXM}, are equal to each

[39]As noted above, strictly speaking, Diewert and Morrison (1986, p. 666) defined their index in the context of a GDP function rather than a net product function and did not deflate prices by a price index. The first applications of formula (24.70) were made by Diewert, Mizobuchi, and Nomura (2005) and Diewert and Lawrence (2006).

[40]This result is again a straightforward adaptation of the results of Diewert and Morrison (1986, p. 666).

other when evaluated at an equal price and quantity point; that is,[41]

$$\nabla^2 P_{DLXM}(p, p, q, q) = \nabla^2 P_{DMXM}(p, p, q, q). \quad (24.76)$$

24.100 Thus P_{DLXM} and P_{DMXM} approximate each other to the accuracy of a second order Taylor series approximation around a data point where the real prices are equal in each period and the net output quantities are also equal to each other across periods. The practical significance of this result is that for normal time series data where adjacent periods are compared, the Diewert Lawrence and Diewert Morrison combined effects indices will give virtually identical results.

24.101 The above material is very similar to the results derived in Sections B and C. But at this point, some new results can be derived. In Section B, measures of the effects on real income growth of a change in real export prices were derived; in Section C, measures of the effects on real income growth of a change in real import prices were derived; and finally, in this section, measures of the combined effects on real income growth of a change in both real export prices and real import prices were derived. A natural question to ask at this point is: How do the partial measures considered in Sections B and C compare to the combined effects measures considered in the present section?

24.102 Using the Diewert Morrison measures, the answer to the above question is very simple. Recalling the expressions (24.22), (24.53), and (24.70), which defined the Diewert Morrison index of the effects on real income growth of a change in real export prices P_{DMX}, in real import prices P_{DMM}, and in the combined effects of changes in real export and import prices P_{DMXM}, respectively, one can see the following *simple multiplicative relationship* between these three indices:

$$P_{DMXM} = P_{DMX}\, P_{DMM}; \quad (24.77)$$

that is, the combined price effects index P_{DMXM} is exactly equal to the product of the export price effect index P_{DMX} and the import price effect index P_{DMM}.[42] Thus when the Diewert Morrison indices are used, the product of the partial effects is equal to the combined effect.[43]

[41]Again a series of routine computations establishes this result.

[42]This is a counterpart to a result obtained by Diewert and Morrison (1986, p. 666) and Kohli (1990).

[43]This result generalizes to the case where there is a finer classification of exports and imports. In this case, the Diewert Morrison disaggregated effect indices can be multiplied together to obtain the overall effect of changes in all real export and import prices.

24.103 The exact decomposition given by equation (24.77) for the Diewert Morrison indices translates into the following approximate decomposition for the Diewert Lawrence indices:

$$P_{DLXM} \approx P_{DLX}\, P_{DLM}. \quad (24.78)$$

24.104 The meaning of the approximate equality is this: From the approximation results derived in this section and the previous sections, it is known that the Diewert Morrison combined effects index P_{DMXM} approximates the Diewert Lawrence combined effects index P_{DLXM} to the second order around an equal price and quantity point and the Diewert Morrison separate effects indices P_{DMX} and P_{DMM} similarly approximate the corresponding Diewert Lawrence separate effects indices P_{DMX} and P_{DMM}. Using these approximation results and the exact identity (24.77) means that the right-hand side of equation (24.78) will approximate the left-hand side of equation (24.78) to the second order around an equal price and quantity point.

24.105 The decomposition results derived above should be useful when dealing with disaggregated export and import data. The reader should be able to use the techniques explained in this chapter to extend the analysis to the case where there are a large number of export and import categories. The corresponding analogues to equations (24.77) and (24.78) will enable the analyst to decompose the overall effects on real income growth owing to changes in the prices of internationally traded goods into separate effects in each category. These separate effects multiply together to give the overall effect on real income growth of changes in the real prices of exports and imports.

E. The Effects on Household Cost-of-Living Indices of Changes in the Prices of Directly Imported Goods and Services

E.1 The case of a single household: Basic framework

24.106 As was mentioned in the introduction to this chapter, households frequently directly import consumer goods and services from abroad without these goods and services passing through the production sector of the economy. Examples of such commodities are tourist expenditures abroad and the direct importation of automobiles. Thus it would be useful to have a framework for modeling the effects of changes in the prices of these directly imported products on household welfare.

24.107 The case of a single household that imports a product is considered here. Let C^t and M^t denote the quantities of a domestic and foreign commodity consumed by the household in period t and let P_C^t and P_M^t denote the corresponding period t nominal prices for $t = 0, 1$.[44] The *period* t *household nominal expenditure on all goods and services* or period t household *"income,"* Y^t, is defined as the total value of consumer expenditures on consumption products provided by the domestic production sector and by directly imported products:

$$Y^t \equiv P_C^t C^t + P_M^t M^t; \quad t = 0, 1. \tag{24.79}$$

24.108 As opposed to the generation of real income approach taken in previous sections in this chapter, in this section, a more traditional *cost-of-living approach* to changes in the prices of directly imported goods and services is taken. Thus the objective of the present section is to derive measures of the effects on the household's cost-of-living index of a change in import prices from the period 0 nominal level, P_M^0, to the period 1 nominal level, P_M^1.

24.109 At this point, household preferences over different combinations of C and M are brought into the picture. It is assumed that in period $t = 0, 1$, the household's preferences are defined by the period t utility function, $U^t(C, M)$, where the function U^t is increasing, continuous, and quasiconcave in its two variables C and M. It will prove useful to define the household's period t *expenditure function,* $e^t(P_C, P_M, u)$ for periods $t = 0, 1$, positive (nominal) prices P_C and P_M, and utility level u belonging to the range of U^t:

$$e^t(P_C, P_M, u) \equiv \min_{C,M} \{P_C C + P_M M : U^t(C, M) \geq u\}; \quad t = 0, 1. \tag{24.80}$$

Thus $e^t(P_C, P_M, u)$ is the minimum income that the household needs in period t in order to attain the utility level u, given that it faces the prices P_C and P_M for domestically and foreign supplied goods and services respectively.[45]

24.110 In what follows, it is assumed that the household minimizes the cost of achieving its utility level $u^t \equiv U^t(C^t, M^t)$ in each period t so that the following equalities hold:

$$e^t(P_C^t, P_M^t, u^t) = P_C^t C^t + P_M^t M^t; \quad t = 0, 1. \tag{24.81}$$

Assumptions (24.81) are the household counterparts to the producer equalities (24.7). The household expenditure functions defined in this subsection play a key role in the remainder of Section E.

E.2 Theoretical measures of the effects on income of changes in household import prices

24.111 Note that $e^0(P_C{}^0, P_M{}^1, u^0)$ is the amount of income that the household would need, using the household preferences of period 0, to be able to attain the same level of utility that it attained in period 0 (which is u^0) if it faced the period 0 domestic consumption price P_C^0 but the period 0 household import price was changed from P_M^0 to the period 1 import price P_M^1. This hypothetical amount of expenditure could be compared to the period 0 actual expenditure level, $e^0(P_C^0, P_M^0, u^0)$. Thus a *theoretical Konüs (1924) Laspeyres partial cost-of-living index that measures the effects of changes in the price of imports that the household faces* going from the period 0 level, P_M^0, to the period 1 level, P_M^1, can be defined as the ratio of the hypothetical expenditure $e^0(P_C^0, P_M^1, u^0)$ to the actual period 0 expenditure:

$$\kappa_{LM} \equiv e^0\left(P_C^0, P_M^1, u^0\right)/e^0\left(P_C^0, P_M^0, u^0\right). \tag{24.82}$$

24.112 The index κ_{LM} of the effects of the change in the price of imports is termed a (partial) *Laspeyres type index* because it holds constant all exogenous prices and utility levels at their period 0 levels except for the import price, P_M^1.[46]

24.113 Note that as P_M^1 *increases,* κ_{LM} defined by equation (24.82) also *increases*. This is quite different from the properties of the corresponding producer index α_{LM} defined by equation (24.43) where α_{LM} *decreased* as the (real) price of imports *increased*. But there is a difference in perspective between the previous sections and the present one: In the previous sections, growth of real income

[44]These price and quantity scalars can be replaced by vectors but for simplicity, only the scalar case is considered here. Note that the C^t in the present section matches up with the C^t that appeared in previous sections but that the M^t in this section is not equal to the production theoretic M^t that appeared in previous sections.

[45]The expenditure function e^t can be used to represent consumer preferences under some assumptions on the utility function U^t; see Diewert (1974b) for the properties of e^t and references to the literature. An important property for present purposes is that $e^t(P_C, P_M, u)$ is increasing in P_C and P_M.

[46]The index defined by equation (24.82) is a partial price change counterpart to a cost-of-living index concept defined by Balk (1989, p. 159) in the context of changing consumer preferences. Balk used the idea of holding tastes constant to work out the effects of price changes. The measure defined by equation (24.84) is also related to various *price compensating variations* defined by Hicks (1945–46, p. 68; and 1946, pp. 331–32) except that Hicks used differences rather than ratios.

owing to changes in international prices was positive for the providers of primary input services whereas in the present section, growth in cost owing to changes in international prices is negative for households.

24.114 There is the following *observable upper bound* to the theoretical index κ_{LM} defined by equation (24.82):[47]

$$\kappa_{LM} \leq \left[P_C^0 C^0 + P_M^1 M^0\right] / \left[P_C^0 C^0 + P_M^0 M^0\right] \equiv P_{LM}^* \qquad (24.83)$$

where $P_{LM}{}^*$ is an *observable Laspeyres partial import price index* of the effects on the cost-of-living of a change in import prices going from period 0 to 1, holding the price of consumption constant at the period 0 level. Note that the numerator and denominator on the right-hand side of (24.83) are identical except that P_M^1 appears in the numerator and P_M^0 appears in the denominator.

24.115 It is possible to show that the Laspeyres type observable index P_{LM}^* is a first order Taylor series approximation to the theoretical index κ_{LM}; that is, it is possible to derive a counterpart to the approximation (24.10).[48]

24.116 The above theoretical measure of the effects of a change in the price of imports used the period 0 preferences for the consumer. It is possible to develop a parallel measure of price change using the consumer's period 1 preferences. Note that $e^1(P_C^1, P_M^0, u^1)$ is the amount of income that the household would need, using the household preferences of period 1, to be able to attain the same level of utility that it attained in period 1 (which is u^1) if it faced the period 1 domestic consumption price P_C^1 and the period 0 import price P_M^0. This hypothetical amount of expenditure could be compared to the period 1 actual expenditure level, $e^1(P_C^1, P_M^1, u^1)$. Thus a *theoretical Konüs Paasche partial cost-of-living index that measures the effects of changes in the price of imports that the household faces* going from the period 0 level, P_M^0, to the period 1 level, P_M^1, can be defined as the ratio of the actual period 1 expenditure $e^1(P_C^1, P_M^1, u^1)$ to the hypothetical expenditure $e^1(P_C^1, P_M^0, u^1)$:

$$\kappa_{PM} \equiv e^1\left(P_C^1, P_M^1, u^1\right) / e^1\left(P_C^1, P_M^0, u^1\right). \qquad (24.84)$$

The index κ_{PM} of the effects of the change in the price of imports is termed a (partial) *Paasche type index* because it holds constant all exogenous prices and utility levels at their period 1 levels except for the two import prices, P_M^0 and P_M^1.[49]

24.117 There is the following *observable lower bound* to the theoretical index κ_{PM} defined by equation (24.84):

$$\kappa_{PM} \geq \left[P_C^1 C^1 + P_M^1 M^1\right] / \left[P_C^1 C^1 + P_M^0 M^1\right] \equiv P_{PM}^*, \qquad (24.85)$$

where $P_{PM}{}^*$ is an *observable Paasche partial import price index* of the effects on the cost of living of a change in real import prices going from period 0 to 1, holding the price of consumption constant at the period 1 level. Note that as usual, the numerator and denominator on the right-hand side of equation (24.85) are identical except that P_M^1 appears in the numerator and P_M^0 appears in the denominator.

24.118 It is possible to show that the Paasche type observable index $P_{PM}{}^*$ is a first order Taylor series approximation to the theoretical index κ_{LM}; that is, it is possible to derive a counterpart to the approximation (24.13).

24.119 Note that both the Konüs Laspeyres and Paasche theoretical partial cost-of-living indices of the effects generated by a change in the price of imports are equally plausible and there is no reason to use one or the other of these two indices. Thus if it is desired to have a single theoretical measure of the effects of a change in import prices on the household's cost of living, κ_{LM} and κ_{PM} should be geometrically averaged. Hence define the *theoretical Fisher type partial cost-of-living index* κ_{FM} *of the effects of changes in household import prices* as the geometric mean of the Konüs Laspeyres and Paasche theoretical measures:

$$\kappa_{FM} \equiv [\kappa_{LM}\, \kappa_{PM}]^{1/2}. \qquad (24.86)$$

With the target import index (24.86) defined, the problem of finding empirical approximations to this theoretical index will now be considered.

[47] Note that assumption (24.81) for $t = 0$ implies that $e^0(P_C^0, P_M^0, u^0) = P_C^0 C^0 + P_M^0 M^0$. Because $[C^0, M^0]$ is a feasible solution for the cost minimization problem defined by $e(P_C^0, P_M^1, u^0)$, it must be the case that $e^0(P_C^0, P_M^1, u^0)$ is equal to or less than $P_C^0\, C^0 + P_M^1\, M^0$, which establishes the inequality in equation (24.83).

[48] A first order Taylor series approximation to $e^0(P_C^0, P_M^1, u^0)$ is $e^0(P_C^0, P_M^0, u^0) + [\partial e^0(P_C^0, P_M^0, u^0)/\partial P_M][P_M^1 - P_M^0] = [p_C^0\, C^0 + p_M^0\, M^0] + M^0[P_M^1 - P_M^0] = P_C^0\, C^0 + P_M^1\, M^0$ using $M^0 = \partial_e^0(P_C^0, P_M^0, u^0)/\partial P_M$ which is implied by Shephard's (1953, p. 11) Lemma. This type of approximation is from Hicks (1942 and 1946, p. 331).

[49] The index defined by equation (24.85) is also a partial price change counterpart to a cost-of-living index concept defined by Balk (1989, p. 159) in the context of changing consumer preferences.

E.3 Empirical measures of the effects on income of changes in household import prices

24.120 Two empirical indices that provide estimates of the effects on the cost of living of a household have been defined above: the Laspeyres partial import price index P^*_{LM} defined on the right-hand side of equation (24.83) and the Paasche partial import price index P^*_{PM} defined on the right-hand side of equation (24.85). It was noted that P^*_{LM} was an upper bound to the theoretical index κ_{LM} and P^*_{LM} was a lower bound to the theoretical index κ_{PM}. Thus P^*_{LM} will generally have a upward bias compared to its theoretical counterpart while P^*_{PM} will generally have a downward bias compared to its theoretical counterpart These inequalities suggest that the geometric mean of P^*_{LM} and P^*_{PM} is likely to be a reasonably good approximation to the target Fisher type index κ_{FM} defined as the geometric mean of κ_{LM} and κ_{PM}. Thus define the *Diewert Lawrence partial cost-of-living index of the effects of a change in import prices going from period 0 to 1* as follows:[50]

$$P^*_{DLM} \equiv \left[P^*_{LM} P^*_{PM}\right]^{1/2}. \tag{24.87}$$

24.121 As in Section B, it will be useful to develop some alternative expressions for the indices P^*_{LM}, P^*_{PM}, and P^*_{DLM}. Define the *household's period* t *share of directly imported commodities* S^t_M for $t = 0,1$ and the *price relative for nominal import prices* R_M as follows:

$$S^t_M \equiv P^t_M M^t / \left[P^t_C C^t + P^t_M M^t\right], \; t = 0, 1; \; R_M \equiv P^1_M / P^0_M. \tag{24.88}$$

24.122 Using the techniques described in Section B, we can derive the following alternative formulas for P^*_{LM} , P^*_{PM} , and P^*_{DLM} :

$$P^*_{LM} = 1 + S^0_M (R_M - 1); \tag{24.89}$$

$$P^*_{PM} = \left[1 + S^1_M (R^{-1}_M - 1)\right]^{-1}. \tag{24.90}$$

24.123 Because S^0_M and S^1_M are positive, it can be seen that both P^*_{LM} and P^*_{PM} are increasing functions of R_M so that as the nominal price of imports increases, both partial cost-of-living indices increase as expected.

[50]Diewert and Lawrence did not actually suggest this index in the consumer context but it is the consumer theory counterpart to their producer context index discussed earlier.

24.124 Substituting expressions (24.89) and (24.90) into (24.87) leads to the following expression for the Diewert Lawrence import index:

$$P^*_{DLM} = \left\{\left[1 + S^0_M (R_M - 1)\right] / \left[1 + S^1_M \left(R^{-1}_M - 1\right)\right]\right\}^{1/2}. \tag{24.91}$$

The Diewert Lawrence index P^*_{DLM} defined by equation (24.91) is likely to be closer to the target Fisher index κ_{FM} defined by equation (24.86) than the Laspeyres and Paasche type indices P^*_{LM} and P^*_{PM} defined by equations (24.89) and (24.90).

24.125 Using the same notation that is defined in equation (24.88), the logarithm of the *Diewert Morrison partial cost-of-living index*, P^*_{DMM}, *of the effects of a change in import prices going from period 0 to 1* is defined as follows:[51]

$$\ln P^*_{DMM} \equiv (1/2) \left(S^0_M + S^1_M\right) \ln R_M. \tag{24.92}$$

It can be verified that P^*_{DMM} satisfies the time reversal; that is, if the two time periods are switched, then the new P^*_{DMM} index is equal to the reciprocal of the original P^*_{DMM} index.[52]

24.126 As in Section B, the interest in the Diewert Morrison index stems from the fact that it has a very direct connection with consumer theory; in fact this index is *exactly* equal to the target index κ_{FM} provided that the preferences of the consumer are translog in each period with certain quadratic coefficients equal to each other. The assumptions made on the consumer's expenditure functions e^t for each period are the following general translog counterparts to the translog assumptions (24.27) through (24.35) that were made in earlier sections, letting $p \equiv [P_C, P_M]$:

$$\begin{aligned} \ln e^t(p, u) \equiv{}& a^t_0 + \sum_{n=1}^{2} a^t_n \ln p_n \\ &+ (1/2) \sum_{n=1}^{2} \sum_{j=1}^{2} a_{nj} \ln p_n \ln p_j + b^t_1 \ln u \\ &+ (1/2)\, b^t_{11} (\ln u)^2 + \sum_{n=1}^{2} c^t_n \ln p_n \ln u; \\ & t = 0, 1. \end{aligned} \tag{24.93}$$

[51]Strictly speaking, Diewert and Morrison did not define this index in the consumer context but it obviously has the same structure as their partial index which was defined in the producer context.

[52]The Diewert Lawrence index P^*_{DLM} also satisfies this time reversal property.

24.127 Note that as before, the coefficients for the quadratic terms in the logarithms of prices are assumed to be constant over time; that is, it is assumed that $a_{nj}^0 = a_{nj}^1 = a_{nj}$. The coefficients must satisfy the following restrictions in order for e^t to be linearly homogeneous in the prices p:

$$\sum_{n=1}^{2} a_n^t = 1 \quad \text{for } t = 0, 1; \tag{24.94}$$

$$a_{nj} = a_{jn} \quad \text{for all } n, j; \tag{24.95}$$

$$\sum_{k=1}^{2} a_{nk} = 0 \quad \text{for } n = 1, 2; \tag{24.96}$$

$$\sum_{n=1}^{5} c_n^t = 0 \quad \text{for } t = 0, 1. \tag{24.97}$$

24.128 Note that using Shephard's Lemma, the logarithmic derivatives of $e^t(P_C^t, P_M^t, u^t)$ with respect to the logarithm of the import price are equal to the following expressions:

$$\begin{aligned} &\partial \ln e^t\left(P_C^t, P_M^t, u^t\right)/\partial \ln P_M \\ &= \left[P_M^t/e^t\right] \partial e^t\left(P_C^t, P_M^t, u^t\right)/\partial P_M; \quad t = 0, 1 \\ &= \left[P_M^t/e^t\right] M^t \\ &= S_M^t \qquad \text{using equation (24.88).} \end{aligned} \tag{24.98}$$

24.129 Noting that assumptions (24.93) imply that the logarithms of the expenditure functions are quadratic in the logarithms of prices and utility, one can apply the result given by equation (24.26) to definitions (24.82), (24.84), and (24.86) to imply the following result:

$$\begin{aligned} 2\ln \kappa_{FM} &= \ln \kappa_{LM} + \ln \kappa_{PM} \\ &= \left[\partial \ln e^0\left(P_C^0, P_M^0, u^0\right)/\partial \ln P_M + \partial \ln e^1\left(P_C^1, P_M^1, u^1\right)/\partial \ln P_M\right]\left[\ln P_M^1 - \ln P_M^0\right] \\ &= \left[S_M^0 + S_M^1\right] \ln\left(P_M^1/P_M^0\right) \qquad \text{using equation (24.98).} \end{aligned} \tag{24.99}$$

Thus using equations (24.92) and (24.99), it can be seen that under the assumptions made on the technology, the following exact equality holds:[53]

$$\kappa_{FM} = P_{DMM}^*. \tag{24.100}$$

Thus the Diewert Morrison partial cost-of-living P_{DMM}^* defined by equation (24.92) is *exactly* equal to the target theoretical index, κ_{FM}, under very weak assumptions on the technology.

24.130 It can be shown that counterparts to the equalities (24.57), (24.58), and (24.60) hold for the four empirical partial cost-of-living indices P_{LM}^*, P_{PM}^*, P_{DLM}^*, and P_{DMM}^*.[54] Thus P_{DLM}^* and P_{DMM}^* approximate each other to the accuracy of a second order Taylor series approximation around a data point where the prices are equal in each period and the consumer demands are also equal to each other across periods. The practical significance of this result is that for normal time series data where adjacent periods are compared, the Diewert Lawrence and Diewert Morrison indices will give virtually identical results.

24.131 Obviously, the above analysis can be repeated to develop theoretical and empirical partial indices that measure the effects on the cost of living of a change in domestic consumer prices P_C^t. Thus define the period t share of consumer expenditures on domestic goods as S_C^t and the consumption price relative R_C as follows:

$$S_C^t \equiv P_C^t C^t/\left[P_C^t C^t + P_M^t M^t\right],\ t = 0,1\ ;\ R_C \equiv P_C^1/P_C^0. \tag{24.101}$$

24.132 Using this notation, we can define the logarithm of the *Diewert Morrison partial cost-of-living index*, P_{DMC}^*, *of the effects of a change in consumption prices going from period 0 to 1* as follows:

$$\ln P_{DMC}^* \equiv (1/2)\left(S_C^0 + S_C^1\right) \ln R_C. \tag{24.102}$$

24.133 The Diewert Morrison complete cost-of-living index can also be defined and this index, P_{DMCM}^*, is simply the usual Törnqvist Theil price index which is defined as follows:

$$\begin{aligned} \ln P_{DMCM}^* \equiv\ &(1/2)\left(S_C^0 + S_C^1\right) \ln R_C \\ &+ (1/2)\left(S_M^0 + S_M^1\right)\ln R_M. \end{aligned} \tag{24.103}$$

24.134 Using equations (24.92), (24.102), and (24.103), we have the following counterpart to the earlier production theory multiplicative result (24.77):

$$P_{DMCM}^* = P_{DMC}^* P_{DMM}^*; \tag{24.104}$$

[53]This result is a partial counterpart to results obtained by Caves, Christensen, and Diewert (1982b, p. 1410); and Balk (1989, pp. 165–66).

[54]As usual, the proof is a series of straightforward computations.

that is, the overall cost-of-living index P^*_{DMCM} is exactly equal to the product of the partial domestic consumption price cost-of-living index P^*_{DMC} and the partial import price cost-of-living index P^*_{DMM}. Thus when the Diewert Morrison indices are used, the product of the partial effects is equal to the combined effect.[55]

24.135 The exact decomposition given by equation (24.104) for the Diewert Morrison indices translates into the following approximate decomposition for the counterpart Diewert Lawrence indices:

$$P^*_{DLCM} \approx P^*_{DLC}\, P^*_{DLM}. \qquad (24.105)$$

F. Conclusion

24.136 There are a large number of approaches that have been suggested over the years that attempt to determine the welfare effects of changes in the prices of exports and imports. The approach taken in this chapter is rather narrow in scope in that only approaches to the measurement of the effects on real income of changes in international prices that are based on producer theory (Sections B–D) or consumer theory (Section E) have been considered. However, the approaches outlined in this chapter should prove to be useful in an environment where large fluctuations in food and energy prices are taking place.

24.137 The production theory approach outlined in this chapter can be extended to provide a more complete description of the factors that determine the growth in the real income generated by a production sector. In addition to changes in the real prices of exports and imports, other determinants include changes in real domestic prices, changes in the utilization of primary inputs, and changes in productivity.[56] For additional materials on how these additional explanatory factors can be added to the export and import price change factors, see Diewert, Mizobuchi, and Nomura (2005); Diewert and Lawrence (2006); and Kohli (2006).[57]

[55]This result generalizes to the case where there is a finer classification of domestic consumption and directly imported household imports.

[56]Another factor which is important in explaining real income growth is tax and tariff policy; see Diewert (2001) and Feenstra, Reinsdorf, and Slaughter (2008) on this topic.

[57]Diewert (2008) provides a reconciliation of the approaches of these authors.

Glossary of Terms

Accrual accounting

The recording of the value of a purchase or other transaction at the time the obligation to pay is incurred, as distinct from the time payment is made. For an XPI and an MPI, this means that the prices of goods are generally recorded at the time of shipment and the prices of services at the time of delivery.

Additivity

At current prices, the value of an aggregate is equal to the sum of its components. Additivity requires this identity to be preserved for the extrapolated values of the aggregate and its components when their current values in some period are extrapolated using a set of interrelated quantity indices; or, alternatively, when the current values of the aggregate and its components in some period are deflated using a set of interrelated price indices. See also "consistency in aggregation."

Aggregate

A set of transactions relating to a specified flow of goods and services, such as the total exports produced by resident establishments in a given period or the total imports of intermediate inputs made by resident establishments in a given period. The term "aggregate" is also used to mean the value of the specified set of transactions.

Aggregation

The process of combining different sets of transactions to obtain a larger set of transactions. The larger set is described as having a higher *level* of aggregation than the sets of which it is composed. The term "aggregation" includes the process of adding the values of lower-level aggregates to obtain higher-level aggregates. In the case of price indices, it means the process by which price indices for lower-level aggregates are averaged, or otherwise combined, to obtain price indices for higher-level aggregates.

ASYCUDA

Automated System for Customs Data. A computer system designed by UNCTAD (United Nations Conference on Trade and Development) for the immediate verification of customs declarations. It ensures that declarations contain all information required, including quantity information, in order to receive customs clearance. The system also validates customs values by matching the unit values on the declarations against those for a predetermined list of commodity prices.

Asymmetric index

An index that does not treat the two periods being compared in a symmetric or balanced way by attaching equal importance to the price and value data in both periods. For example, the Laspeyres price index, the Paasche price index, and the Palgrave price index are asymmetrically weighted price indices: the Laspeyres because it uses the value shares of the base period, the Paasche and Palgrave because they use the value shares of the current period.

Axiomatic approach

The approach to index number theory that determines the choice of index number formula on the basis of its mathematical properties. A list of tests is drawn up, each test requiring an index to possess a certain property or satisfy a certain axiom. The choice of index is made on the basis of the number of tests satisfied. Not all tests may be considered equally important, and the failure to satisfy certain key tests may be considered sufficient grounds for rejecting an index. An important feature of the axiomatic approach is that prices and quantities are considered as separate variables and no account is taken of possible links between them. Also known as the "test approach."

Base period

The base period is generally understood to be the period with which other periods are compared. However, the term is not a precise one and may be used to mean rather different things. Three types of base period may be distinguished:

- The *price reference period*—the period that provides the prices to which the prices in other periods are compared. The prices of the price reference period appear in the denominators of the price relatives used to calculate the index.
- The *weight reference period*—the period, usually one or more years, whose values serve as weights for the index. When the values are hybrid (that is, the quantities of one period are valued at the prices of some other period), the weight reference period is the period to which the quantities refer.
- The *index reference period*—the period for which the index is set equal to 100.

The three reference periods may coincide but frequently do not.

Base-weighted index

See “Laspeyres price index.”

Basic price

The amount received by the producer from the purchaser for a unit of good or service produced as output. It *includes* subsidies on products and other taxes on production. It *excludes* taxes on products, other subsidies on production, supplier’s retail and wholesale margins, and separately invoiced transport and insurance charges. Basic prices are the prices most relevant for decision making by suppliers.

Basket

A specified set of quantities of goods and services for which prices are collected for the purpose of compiling an index.

Basket price index

See “Lowe index” and equation (G.1) in the Appendix.

BEC

Classification by Broad Economic Categories. An internationally agreed classification that groups commodities according to their main end use: consumption goods, intermediate goods, capital goods, dual-purpose goods (motor spirit, passenger motor cars), and military and other goods not elsewhere classified. The categories and subcategories of the classification are defined in terms of SITC Rev.3 basic headings.

Bias

A systematic tendency of the calculated index to diverge from some ideal or preferred index. Bias can arise for a number of reasons, including the design of the sample selected, the price measurement procedures followed, the data processing methods used, and/or the index number formula employed.

Bilateral price index

The value ratio decomposition approach to index numbers involves breaking down the value ratio between two periods into two numbers: one representing the price change between the two periods and the other representing the quantity change between the two periods. The resulting price index is called a “bilateral price index.” “Bilateral” refers to the assumption that the price index depends only on price and quantity data pertaining to the two periods being compared.

Book price

See “list price.”

Bouncing

The fluctuation or oscillation of prices up and down in a persistent pattern.

BPM6

Balance of Payments and International Investment Position Manual, Sixth Edition. An internationally agreed set of accounts summarizing the economic relationships between residents of an economy and the rest of the world. The accounts provide an integrated framework for recording an economy’s international current, capital, and financial account transactions, and the financial assets and liabilities of its residents vis-à-vis nonresidents. See also “EBOPS.”

Business register

A register or list of enterprises or establishments involved in productive activities that is maintained by countries, often by their national statistical institutes, to provide the frame for carrying out their economic censuses and surveys. Such registers usually contain information on location, economic activity, size (employment, payroll, annual production, or turnover), contact persons, links with tax and other administrative registers, etc.

Carli price index

An elementary price index defined as the simple, or unweighted, arithmetic average of the current to base period price relatives.

$$P_C \equiv \frac{1}{n}\sum\left(\frac{p^t}{p^0}\right)$$

Carry forward

The situation in which a missing price for a product in the current period is imputed as being equal to the last price observed for that product.

Chain index

An index number series for a long sequence of periods obtained by linking together index numbers spanning shorter sequences of periods, each with their own weights. The linking may be made as frequently as the weights change and the data permit, or at specified intervals, such as every five or ten periods. Each link in the chain consists of an index comparing each period with the previous period. See also equation (G.6) in the Appendix.

Chain linking

Splicing together two consecutive sequences of price indices that overlap in one or more periods. When the two sequences overlap by a single period, the usual procedure is simply to rescale one or another sequence so that the value in the overlap period is the same in both sequences and the spliced sequences form one continuous series. More complex methods may be used to link indices that overlap by more than one period. Also known as "chaining" or "linking."

Chain linking bias

See "drift."

Characteristics

The physical and economic attributes of a good or service that serve to identify it and enable it to be classified.

C.i.f. price

Cost, insurance, and freight price. The price of a good delivered at the customs frontier of the importing country or the price of a service delivered to a resident. It *includes* any insurance and freight charges incurred to that point. It *excludes* any import duties or other taxes on imports and trade and transport margins within the importing country.

Circularity

An index number property such that the algebraic product of the price index comparing period i with period j and the price index comparing period k with period j is equal to the price index that compares period k directly with period i. The property is also known as "transitivity." When the axiomatic approach is used, a price index number may be required to satisfy the "circularity test."

COLI

Cost of living index. An index that measures the change between two periods in the minimum expenditures that would be incurred by a utility-maximizing consumer, whose preferences or tastes remain unchanged, in order to maintaining a given level of utility (or standard of living or welfare). Because consumers may be expected to change the quantities they consume in response to changes in relative prices (the "substitution effect"), the COLI is not a fixed-basket index. COLIs cannot be directly calculated but may be approximated by superlative indices. A *conditional cost of living index* is one that assumes that all the factors that may influence the consumer's utility or welfare *other than prices* (such as the physical environment) do not change.

Commensurability test

See "invariance to changes in the units of measurement test."

Commodity

A generic term for a good or a service, interchangeable with the term "product." Commodities or products are the result of production. They are exchanged and used for various purposes: as inputs in the production of other goods and services, as final consumption, or as investment. Individual sampled commodities or products are often described as "items."

Commodity reversal test

A test under the axiomatic approach that requires that, for a given set of products, the price index should remain unchanged when the ordering of the products is changed.

Comparison period

See "current period."

Compensation of employees

The total remuneration, in cash or kind, payable by enterprises, to employees in return for work done by the latter during the accounting period. Includes employers' actual and imputed social contributions.

Consistency in aggregation

An index is said to be consistent in aggregation when the index for some aggregate has the same value whether it is calculated directly in a single operation, without distinguishing its components, or it is calculated in two or more steps by first calculating separate indices, or subindices, for its components, or subcomponents, and then aggregating them, the same formula being used at each step. See also "additivity."

Constant elasticity of substitution index

A family of price indices that allows for substitution between products. The Lloyd-Moulton index is such an index.

Constant prices test

See "identity test."

Constant quantities test

See "fixed-basket test."

Consumption of fixed capital

The reduction in the value of the fixed assets used in production during the accounting period resulting from physical deterioration, normal obsolescence, or normal accidental damage.

Continuity

The property whereby the price index is a continuous function of its price and quantity vectors.

Contract escalation

See "indexation of contracts."

Contract price

A general term referring to a written sales instrument that specifies both the price and shipment terms. A contract may include arrangements for a single shipment or multiple shipments. Usually it covers a period of time in excess of one month. Contracts are often unique in that all the price-determining characteristics in one contract are not repeated exactly in any other contract.

Coverage

The set of goods and services whose prices are actually included in the index. For practical reasons, coverage may have to be less than the scope or domain of the index. In other words, the set of goods and services covered by the index may be narrower than the set of goods and services that the compliers of the index would prefer to include if it were feasible.

CPA

Statistical Classification of Products by Activity within the European Economic Community. The classification of goods and services by originating activity favored by the European Union. Originating activities are those defined by NACE.

CPC

Central Product Classification. An internationally agreed classification of goods and services based on the physical characteristics of goods or on the nature of the services rendered. Each type of good or service distinguished in the CPC is defined in such a way that it is normally produced by only one activity as defined in ISIC.

CPI

Consumer price index. A monthly or quarterly price index compiled and published by an official statistical agency that measures the changes in the prices of consumption goods and services acquired or used by households. Its exact definition may vary from country to country.

CSWD index

Carruthers, Sellwood, Ward, Dalén index. A geometric average of the Carli index and the harmonic mean of price relatives index.

$$P_{CSWD} \equiv \sqrt{P_C \times P_{HR}}$$

Current period

In principle, the current period should refer to the most recent period for which an index has been computed or is being computed. However, the term is widely used to refer to any period that is compared with the base period or index reference period. It is also widely used simply to mean the latter of the two periods being compared. The exact meaning is usually clear in the context. Also referred to as the "comparison period."

Current-weighted index

See "Paasche price index."

Cutoff sampling

A sampling procedure in which a predetermined threshold is established with all units in the universe at or above the threshold being included in the sample and all units below the threshold being excluded. The threshold is usually specified in terms of the size of some known variable, the largest sampling units being included and the rest given a zero chance of inclusion. For XMPIs, size is usually defined in terms of export or import values or shares.

Deflation

The division of the current value of some aggregate by a price index—described as a "deflator"—in order to revalue its quantities at the prices of the price reference period or to revalue the aggregate at the general price level of the price reference period.

Discount

A deduction from the list or advertised price of a good or service that is available to specific customers under specific conditions. Examples include cash discounts, prompt payment discounts, volume discounts, trade discounts, and advertising discounts.

Divisia approach

A price or quantity index that treats both prices and quantities as continuous functions of time. By differentiation with respect to time, the rate of change in the value of the aggregate in question is partitioned into two components, one of which is the price index and the other the quantity index. In practice, the indices cannot be calculated directly, but it may be possible to approximate them by chain indices in which indices measuring changes between consecutive discrete periods are linked together.

Domain

An alternative term for the scope of an index. See "scope" and "coverage."

Drift

A chain index is said to drift if it does not return to unity when prices in the current period return to their levels in the base period. Chain indices are liable to drift when prices fluctuate or "bounce" over the periods they cover. Also known as "chain linking bias."

Drobisch price index

A price index defined as the arithmetic average of the Laspeyres price index and the Paasche price index. It is a symmetric index and a pseudo-superlative index.

$$P_{DR} \equiv \frac{1}{2}(P_L + P_P)$$

Dutot index

An elementary price index defined as the ratio of the unweighted arithmetic average of the prices in the current period to the unweighted arithmetic average of the prices in the base period.

$$P_D = \frac{\frac{1}{n}\sum p^t}{\frac{1}{n}\sum p^0}$$

EBOPS

Extended Balance of Payments Services Classification. An extension to the BPM6 classification of international trade in services. It is primarily a product classification and as such is defined in terms of the CPC. However, in line with the BPM6, it includes some classes that are not compatible with the CPC or with other international product classifications such as the CPA. It also includes various supplementary items and alternative groupings that are intended to provide additional information on the transactions recorded in the system.

Economic approach

The approach to index number theory that assumes that the quantities are functions of the prices and not independent variables. The observed price and quantity data are assumed to be generated as solutions to various economic optimization problems. Also known as the "microeconomic approach."

Economic territory (of a country)

The economic territory of a country consists of the geographical territory administered by a government within which persons, goods, and capital circulate freely. It includes (1) the air space, territorial waters, and continental shelf lying in international waters over which the country enjoys exclusive rights or over which it has, or claims to have, jurisdiction with respect to the right to fish or to exploit fuels or minerals below the sea bed; (2) territorial enclaves in the rest of the world (clearly demarcated areas of land that are located in other countries and that are used by the government for diplomatic, military, scientific, or other purposes with the formal political agreement of the government of the country in which they are physically located); and (3) any free zones, or bonded warehouses or factories operated by offshore enterprises under customs control (these form part of the economic territory of the country in which they are physically located).

Economically significant prices

Prices that have a significant influence on the amounts producers are willing to supply and on the amounts purchasers wish to buy.

Editing

The process of scrutinizing and checking the prices reported or collected for the price index. Checks may be carried out by computers using statistical programs written for the purpose. See "input editing" and "output editing."

Elementary aggregate

The lowest level of aggregation for which value data are available and used in the calculation of the price index. The values of elementary aggregates are used to weight the elementary price indices associated with them to obtain indices for higher-level aggregates. Elementary aggregates usually cover a relatively narrow range of goods or services. They also serve as strata for the sample of products selected for pricing.

Elementary item

One of the individual goods or services that make up an elementary aggregate. An individual good or service selected for pricing from among the various goods or services comprising an elementary aggregate. Often referred to as an "item."

Elementary price index

A price index for an elementary aggregate. Value weights cannot usually be assigned to the price relatives for sample products within an elementary aggregate, although other kinds of weighting may be explicitly or implicitly introduced into the calculation of elementary indices. Three examples of elementary index number formulas are the Carli, the Dutot, and the Jevons indices.

Enterprise

An institutional unit in its capacity as a producer of goods and services consisting of one or more establishments. An enterprise may be a corporation, a quasi-corporation, a nonprofit institution, or an unincorporated enterprise.

Error

The difference between the observed value of an index and its "true" value. Errors may be random or systematic. Random errors are generally referred to as "errors." Systematic errors are called "biases."

Establishment

An enterprise, or part of an enterprise, that is situated in a single location and in which a single nonancillary productive activity is carried out or in which the principal productive activity accounts for most of the value added. Also referred to as "LKAU" or "local kind of activity unit." An establishment should be capable of providing basic accounting information on the prices and quantities it produces and the inputs it uses during an accounting period. This is often not the case in practice, particularly when an establishment is one of a number of production units belonging to an enterprise. In these circumstances, an establishment is defined either as the enterprise itself or in terms of the smallest subsets of production units for which the enterprise (or the subsets themselves) is able to supply accounting information on inputs and outputs for the time period under consideration.

Evolutionary goods

Goods that are similar to or extensions of existing goods. They are typically produced on the same production line using production inputs and processes that are largely the same as those used to produce existing goods. It is possible, at least in theory, to adjust for any quality differences between an evolutionary good and an existing good.

Explicit quality adjustment

A direct estimate of how much of the change in price of a product is attributable to changes in its physical or economic characteristics. It requires an evaluation of the contributions of the differences in particular characteristics to the differences in the observed prices of two products. It includes quality adjustments based on hedonic methods. See also "quality adjustment" and "implicit quality adjustment."

Factor reversal test

Suppose the prices and quantities in a price index are interchanged to yield a quantity index of exactly the same functional form as the price index. Under the axiomatic approach, the factor reversal test requires that the product of this quantity index and the original price index should be identical with the proportionate change in the value of the aggregate in question. Also known as the "product test."

Factory gate price

A basic price with the "factory gate" as the pricing point.

FEPI

Final expenditure price index. A measure of the changes in prices paid by consumers, businesses, and government for final purchases of goods and services. Intermediate purchases are excluded.

FIOPI

Fixed-input output (export) price index. The theoretical model for an XPI for resident producers based on the assumption of fixed technology and inputs. It requires the index to reflect changes in revenue resulting from the export of the same products—although not necessarily the same mix of products—produced under the same circumstances and sold under the same terms. In other words, changes in the index arise solely from changes in export prices and are not influenced by changes in inputs. Revenue maximizing behavior is assumed on the part of the exporting producer.

Fisher price index

A price index defined as the geometric average of the Laspeyres price index and the Paasche price index. It is a symmetric and superlative index.

$$P_F \equiv \sqrt{P_L \times P_P}$$

Fixed-basket price index

The traditional conceptualization of a price index. The index measures the change in value of a fixed set of quantities—commonly described as a "fixed basket of good and services"—between two periods. Because the quantities or weights remain fixed, any change in the index is due to price changes only. In principle, there is no restriction on the quantities that make up the basket. They may be those of one of the two periods being compared, they may refer to the quantities in some third period, or they may constitute a hypothetical basket such as an average of the quantities in the two periods. Moreover, the quantities may refer to a much longer period of time than the periods of the index: for example, quantities exported or imported over a period of a year or more may be used for a monthly or quarterly XPI or MPI. A fixed-basket index is sometimes described as a "pure price index." Also known as a "fixed-weight price index."

Fixed-basket test

A test under the axiomatic approach that requires that if all the quantities remain unchanged (that is, the sets of quantities in both periods are identical), the price index should equal the proportionate change in the value of the aggregate. Also known as the "constant quantities test."

F.o.b. price

Free on board price. The price of a good delivered at the customs frontier of the exporting country or the price of a service delivered to a nonresident. It *includes* the freight and insurance charges incurred to that point and any export duties or other taxes on exports levied by the exporting country.

FOIPI

Fixed-output input (import) price index. The theoretical model for an MPI for resident producers based on the assumption of fixed technology and outputs. It requires the index to reflect changes in costs resulting from the import of the same inputs—although not necessarily the same mix of inputs—purchased under the same terms in order to produce the same output with the same technology. In other words, changes in the index arise solely from changes in import prices of inputs and are not influenced by changes in outputs. Cost minimizing behavior is assumed on the part of the importing producer.

Formula bias

Bias arising from the index number formula used to calculate the index. See "bias" and "substitution bias."

GDDS

General Data Dissemination System. A system designed by the IMF to promote the development and dissemination of comprehensive macroeconomic, financial, and socio-demographic data sets in line with international methodology and good practice. It enables official statistical agencies to identify and prioritize improvements to their statistics in a thorough and systematic way.

Geometric Laspeyres price index

A price index defined as the weighted geometric average of the current to base period price relatives using the value shares of the base period as weights. Also known as the "logarithmic Laspeyres price index."

$$P_{GL} \equiv \prod \left(\frac{p^t}{p^0}\right)^{s^0} \text{ where } s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}$$

Geometric Paasche price index

A price index defined as the weighted geometric average of the current to base period price relatives using the value shares of the current period as weights. Also known as the "logarithmic Paasche price index."

$$P_{GP} \equiv \prod \left(\frac{p^t}{p^0}\right)^{s^t} \text{ where } s^t \equiv \frac{p^t q^t}{\sum p^t q^t}$$

Goods

Physical objects for which a demand exists, over which ownership rights can be established, and whose ownership can be transferred from one institutional unit to another by engaging in transactions on the market. They are in demand because they may be used to satisfy the needs or wants of households or the community or used to produce other goods or services.

Gross value added

See "value added."

Harmonic mean of price relatives

An elementary index that constitutes the harmonic average counterpart to the Carli index.

$$P_{HR} \equiv \frac{1}{\frac{1}{n}\sum\left(\frac{p^0}{p^t}\right)}$$

Harmonic means price index

See "ratio of harmonic means of prices."

Hedonic method

A regression technique in which observed prices of different qualities or models of the same generic good or service are expressed as a function of the characteristics of the goods or services in question. It is based on the hypothesis that products can be treated as bundles of characteristics and that prices can be attached to the characteristics. The characteristics may be non-numerical attributes that are represented by dummy variables. The regression coefficients are treated as estimates of the contributions of the characteristics to the overall prices. The estimates may be used to predict the price of a new quality or model whose mix of characteristics is different from that of any product already on the market. The hedonic method can therefore be used to estimate the effects of quality changes on prices.

Higher-level index

A term used to describe an index for an aggregate at a higher level of aggregation than an elementary aggregate. An index constructed from elementary or lower-level indices. Weights are used to combine them. Also referred to as an "upper-level index."

Homogeneity tests

Four tests under the axiomatic approach, namely: the proportionality in current prices test, the inverse proportionality in base prices test, the invariance to proportional changes in current quantities test, and the invariance to proportional changes in base quantities test.

HS

Harmonized Commodity Description and Coding System or *Harmonized System*. An internationally agreed commodity classification providing the basis for customs tariffs and for the compilation and dissemination of statistics on international merchandized trade. Goods are classified primarily according to the component material or the type of product, degree of processing, function, and economic activity. The HS was introduced in 1988 as a replacement for the SITC. It is updated every four to six years to take account of changing conditions of international trade. It was last revised in 2007 (HS07). There is full correspondence between the six-digit subheadings of HS07 and the five-digit basic headings of SITC Rev.4.

Hybrid (export or import) index

A price index that combines price indices for some of the elementary aggregates covered by the index with unit value indices for the other elementary aggregates covered by the index. Price indices are used for heterogeneous elementary aggregates with unacceptable levels of unit value bias. Unit value indices are used for homogeneous elementary aggregates with acceptable levels of unit value bias. See "price dispersion test" and "quantity proportionality test."

Hybrid values

Hypothetical values in which quantities are valued at a different set of prices from those at which they were actually bought or sold. For example, when the quantities purchased in an earlier period are valued at prices prevailing in a later period.

ICP

International Comparison Programme. A worldwide statistical program that compares the price and volume levels of GDP and its component expenditures across participating countries. Comparisons are made every five or so years. Before such comparisons can be made, it is first necessary to express the GDPs of participating countries—which are in national currencies and valued at national price levels—in a common currency at a uniform price level. Purchasing power parities (PPPs) are used to do this. The PPPs are calculated with price and expenditure data collected from participating countries. The price and expenditure data collected cover the whole range of final goods and services included in GDP.

Identity test

A test under the axiomatic approach that requires that if the prices remain unchanged between the two periods (that is, the sets of prices are identical), the price index should equal unity. Also known as the "constant prices test."

Implicit quality adjustment

Inferring indirectly the change in the quality of a product whose characteristics change over time by estimating, or assuming, the pure price change that has occurred. For example, if the pure price change is assumed to be equal to the average for some other group of products, the implied change in quality is equal to the actual observed price change divided by the assumed pure price change. If the whole of the observed price change is assumed to be pure price change, there is assumed to be no change in quality. See also "quality adjustment" and "explicit quality adjustment."

Imputed price

The price assigned to a product for which the price is missing in a particular period. The term "imputed price" may also refer to the price assigned to a product that is not sold on the market, such as a good or service produced for own consumption, including housing services produced by owner-occupiers, or one received as payment in kind or as a free transfer from a government or nonprofit institution.

Index number problem

How to combine the relative changes in the prices and quantities of various products into (1) a single measure of the relative change of the overall price level and (2) a single measure of the relative change of the overall quantity level. Or, conversely, how a value ratio pertaining to two periods can be decomposed into a component that measures the overall change in prices between the two periods—that is, the price index—and a component that measures the overall change in quantities between the two periods—that is, the quantity index.

Index reference period

The period for which the value of the index is set at 100. See also "base period."

Indexation

The periodic adjustment of the money values of some regular scheduled payments based on the movement of a nominated price index such as the CPI or another similar price index. The payments may be wages or salaries, social security or other pensions, other social security benefits, rents, interest payments, etc. The purpose of indexation is to protect the recipients of the payments against inflation. Also known as "index linking."

Indexation of contracts

A procedure whereby a long-term contract for the provision of goods or services includes a periodic adjustment to the prices paid for the goods or services based on the increase or decrease in the level of a nominated price index, such as the PPI, the XPI, or the MPI. The purpose of the indexation is to take inflationary risk out of the contract. Also known as "contract escalation."

Industry

A general term to describe a group of establishments engaged in the same, or similar, kinds of production activity. Also a specific term used to describe establishments engaged in mining and quarrying, manufacturing, electricity, gas, and water (Sections C, D, and E of ISIC, Rev. 3).

Input editing

The process of analyzing the prices reported by an individual respondent and querying price changes that are above a specified level or are inconsistent across product lines. An important objective of the process is to ensure that actual transaction prices are reported and to detect any changes in the specifications.

Institutional unit

A national accounts concept defined as an economic entity that is capable, in its own right, of owning assets, incurring liabilities, and engaging in economic activities and transactions with other entities. Enterprises are institutional units. Other kinds of units include households and governments.

Intermediate basket

A basket derived as the average of the baskets of two time periods, usually the base and current periods. The average can be arithmetic, as in the Marshall-Edgeworth price index, or geometric, as in the Walsh price index.

Intermediate consumption

The value of goods and services used or consumed as intermediate inputs by a process of production.

Intermediate inputs

Goods and services, other than primary inputs and fixed assets, used as inputs into the production process of an establishment, which are produced elsewhere in the economy or are imported. They may be either transformed or used up by the production process. Also called "intermediate products."

Intra-company transfer price

The value assigned on a per unit or per shipment basis to goods transferred from one establishment of an enterprise to another. It may or may not be economically significant. However, it is not a market price because ownership of the good does not change hands. See "transfer price."

Invariance or symmetry tests

Five tests under the axiomatic approach, namely: the commodity reversal test, the commensurability test, the time reversal test, the quantity reversal test, and the price reversal test.

Invariance to changes in the units of measurement test

A test under the axiomatic approach that requires that the price index does not change when the units of quantity to which the prices refer are changed: for example, when the price of some drink is quoted per liter rather than per pint. Also known as the "commensurability test."

Invariance to proportional change in current or base quantities test

A test under the axiomatic approach that requires that the price index remains unchanged when all the base period quantities, or all the current period quantities, are multiplied by a positive scalar.

Inverse proportionality in base prices test

A test under the axiomatic approach that requires that if all base period prices are multiplied by the positive scalar λ, the new price index is $1/\lambda$ times the old price index.

ISIC

International Standard Industrial Classification of All Economic Activities. An internationally agreed classification that allows enterprises and establishments to be classified according to economic activity based on the class of goods produced or services rendered.

Item

An individual good or service in the sample of products selected for pricing. Also referred to as an "elementary item."

Item or product rotation

The deliberate replacement of a sampled item, or product, for which prices are being collected, by another product *before* the replaced product has disappeared from the market or individual establishment. It is designed to keep the sample of products up to date and reduce the need for forced replacements caused by the disappearance of products. See "replacement product."

Item or product substitution

The deliberate or forced replacement of a sampled item, or product, for which prices are being collected, by a new product because it is about to disappear or has disappeared from the market or specific establishment. See "replacement product."

ITRS

International Transactions Reporting System. An ITRS measures individual balance of payments cash transactions passing through the domestic banks and foreign bank accounts of enterprises, noncash transactions, and stock positions. Statistics are compiled from forms submitted by domestic banks and from forms submitted by enterprisers to the compiler.

Jevons price index

An elementary price index defined as the unweighted geometric average of the current to base period price relatives also equal to the ratio of geometric means of the current to base period prices.

$$P_J \equiv \prod \left(\frac{p^t}{p^0}\right)^{1/n} = \frac{\prod (p^t)^{1/n}}{\prod (p^0)^{1/n}}$$

Laspeyres price index

A price index defined as a fixed-basket, or fixed-weight, index that uses the basket of goods and services of the base period. The base period serves as both the weight reference period and the price reference period. It is identical to a weighted arithmetic average of the current to base period price relatives using the value shares of the base period as weights. Also called a "base-weighted index."

$$P_L \equiv \frac{\sum p^t q^0}{\sum p^0 q^0} = \sum s^0 \left(\frac{p^t}{p^0}\right) \text{ where } s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}$$

List price

The price of a product as quoted in the seller's price list, catalogue, Internet site, etc. The gross price exclusive of all discounts, surcharges, rebates, and the like that apply to an actual transaction. Also known as the "book price."

LKAU

Local kind of activity unit. See "establishment."

Lloyd-Moulton price index

A particular case of a constant elasticity of substitution price index. In its unweighted form, the Lloyd-Moulton formula is

$$P_{LM} \equiv \left[\sum \frac{1}{n}\left(\frac{p^t}{p^0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

In its weighted form, the Lloyd-Moulton formula is

$$P_{LM} \equiv \left[\sum s^0 \left(\frac{p^t}{p^0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \text{ where } s^0 \equiv \frac{p^0 q^0}{\sum p^0 q^0}.$$

Lowe price index

Although called a "Lowe price index" after the index number pioneer who first proposed this general type of index, it is not a single index but a family of basket price indices, each of which measures the proportionate change between periods 0 and t in the total value of a specified basket of goods and services.

$$P_{LO} \equiv \frac{\sum p^t q_n}{\sum p^0 q_n} \text{ where the terms } q_n \text{ are the specified quantities.}$$

The family of Lowe indices includes the Laspeyres index, the Paasche index, the Marshall-Edgeworth index, and the Walsh index. See equation (G.1) in the Appendix.

In practice, official statistical agencies frequently use a Lowe price index with a quantity basket of period b, where b denotes some period before 0, and hybrid value shares valued at prices in period 0, the price reference period. The share-weighted Lowe index is

$$P_{LO} \equiv \sum s^{b0} \left(\frac{p^t}{p^0}\right) \text{ where } s^{b0} \equiv \frac{p^0 q^b}{\sum p^0 q^b}.$$

Lower-level index

See "elementary price index."

Market price

The amount of money a willing buyer pays to acquire a good or service from a willing seller. The actual price for a transaction agreed by the transactors. The net price inclusive of all discounts, surcharges, and rebates applied to the transaction. From the seller's point of view, the market price is the basic price; from the buyer's point of view, the market price is the purchaser's price. Also referred to as "transaction price."

Marshall-Edgeworth price index

A price index defined as the weighted arithmetic average of the current to base period price relatives that uses the quantities of an intermediate basket as weights. The quantities of the intermediate basket are arithmetic averages of the quantities of the base and current periods. It is a symmetric index and a pseudo-superlative index.

$$P_{ME} \equiv \frac{\sum p^t (q^t + q^0)/2}{\sum p^0 (q^t + q^0)/2}$$

Matched models or products method

The practice of pricing exactly the same model or product in two or more consecutive periods. It is designed to ensure that the observed price changes are not affected by quality change. The change in price between two perfectly matched models or products is described as a "pure" price change.

Mean value tests

Three tests under the axiomatic approach, namely: the mean value test for prices, the mean value test for quantities, and the Paasche and Laspeyres bounding test.

Mean value test for prices

A test under the axiomatic approach that requires that the price index lies between the smallest price relative and the largest price relative.

Mean value test for quantities

A test under the axiomatic approach that requires that the implicit quantity index lies between the minimum and maximum rates of growth of the individual quantities.

Merchanting

The purchase of goods by a resident from a nonresident combined with the subsequent resale of the goods to another nonresident without the resident taking physical possession of the goods. In other words, the goods do not pass through the economic territory in which the resident resides.

Microeconomic approach

See "economic approach."

Mid-period or midyear price index

A price index that utilizes *either* the quantity or value weights from an intermediate period that lies between the base period and the current period when the number of periods between them is odd, *or* the average of the quantity or value weights for two consecutive intermediate periods that lie between the base period and the current period when the number of periods between them is even.

Mirror price index

An XPI for a country constructed as the weighted average of the corresponding MPIs of the countries to which it exports. An MPI for a country constructed as the weighted average of the corresponding XPIs of the countries from which it imports.

"Modified," "short-term change," or "two-stage" Laspeyres price index

These often-used descriptions of the Laspeyres index have at least three meanings:

As a short-run modified Laspeyres index. This is an index with weight reference period b and price changes between periods 0 and t where the latter are decomposed into price changes between period 0 and $t-1$ and period $t-1$ and t.

$$P_{MLAS} \equiv \sum s^b \left(\frac{p^{t-1}}{p^0}\right)\left(\frac{p^t}{p^{t-1}}\right) \text{ where } s^b \equiv \frac{p^b q^b}{\sum p^b q^b}$$

The decomposition helps deal with changes in the sampled products. In the absence of changes in the sample, P_{MLAS} reduces to a Young index between t and 0 with weight reference period b.

$$P_{MLAS} \equiv \sum s^b \left(\frac{p^t}{p^0}\right) = P_Y$$

As a price-updated version of a Young index. This is a fixed-weight index in which the quantities are those of the weight reference period b, but the price reference period is a later period 0 preceding the current period t. The implicit expenditure weights are obtained by revaluing the quantities of the weight reference period b at the prices of the price reference period 0, a procedure described as "price updating." This modified Laspeyres index between periods 0 and t can be interpreted as a weighted average of the price relatives between 0 and t, using the price-updated expenditure weights. Its definition is

$$\sum \left(\frac{s^b(p^0/p^b)}{\sum s^b(p^0/p^b)}\right)\left(\frac{p^t}{p^0}\right)$$

and corresponds to a Lowe price index between periods 0 and t with weight reference period b. See also "price updating" and "Lowe price index."

As a two-stage Laspeyres index. The two-stage procedure decomposes a Laspeyres price index between b and t into a Laspeyres price index between b and 0 and a Lowe price index between 0 and t.

$$\frac{\sum p^t q^b}{\sum p^b q^b} = \frac{\sum p^0 q^b}{\sum p^b q^b}\frac{\sum p^t q^b}{\sum p^0 q^b}$$

See also equation (G.3) in the Appendix.

Monotonicity in prices

The property whereby if any current period price increases or any base period price decreases, the price index increases.

Monotonicity in quantities

The property whereby if any current period quantity increases or any base period quantity decreases, the implicit quantity index that corresponds to the price index increases.

Monotonicity tests

Four tests under the axiomatic approach that test for monotonicity in current prices, in base prices, in current quantities, and in base quantities.

MPI

Import price index. See “XMPIs.”

Multi-factor productivity

See “total factor productivity.”

NACE

The acronym from the French for *General Industrial Classification of Economic Activities within the European Communities*. The classification is basically a more detailed version of ISIC appropriate to European circumstances.

Net exports

The value of exports of goods and services *less* the value of imports of goods and services. Also known as the “balance of exports and imports.”

Net value added

See “value added.”

New good problem

Difficulty in comparing prices between two periods when a product enters the basket only in period 2, so that a price for the product does not exist in period 1.

New goods

See “evolutionary goods” and “revolutionary goods.”

Non-probability sampling

Selection in which establishments or products do not have a known non-zero probability of selection. Also known as “non-random sampling.” It includes the deliberate selection of a sample of establishments and products on the basis of the knowledge or judgment of the person responsible. Also known as “purposive sampling” and “judgmental sampling.”

Nonresident

An institutional unit whose center of economic interest is not in the economic territory of the country.

Non-response bias

The bias that arises when those who do not respond have different price experiences than those who do respond.

Non-sampling error

An error in sample estimates that cannot be attributed to sample variation. It can arise from any number of different sources, including defects in the sampling frame and/or in the selection of sample units, faulty demarcation of sample units, mistakes in the collection of data owing to misunderstandings or dishonesty on the part of the enumerator and/or the respondent, and mistakes during the processing of data.

Observation

The price collected or reported for a sampled product or item.

Order price

The price quoted at the time the order is placed by the purchaser.

Other subsidies on production

The subsidies that resident enterprises may receive as a consequence of engaging in production; for example, subsidies on payroll or workforce, or subsidies to reduce pollution. They do not include subsidies on products.

Other taxes on production

The taxes that resident enterprises may pay as a consequence of engaging in production. They mainly consist of current taxes on the labor or capital employed in the enterprise, such as payroll taxes or current taxes on vehicles or buildings. They do not include taxes on products.

Outlier

A term that is generally used to describe any extreme value in a set of survey data. In an XMPI context, it is used for an extreme value of price or price relative that requires further investigation or that has been verified as being correct.

Output

The goods or services that are produced within an establishment that become available for use outside that establishment, plus any goods and services produced for own final use.

Output editing

The process of comparing the price levels and price movements of similar products between different respondents and querying any outliers.

Paasche and Laspeyres bounding test

A test under the axiomatic approach that requires that the price index lies between the Laspeyres price index and the Paasche price index.

Paasche price index

A price index defined as a fixed-basket, or fixed-weight, index that uses the basket of goods and services of the current period. The current period serves as the weight reference period and the base period as the price reference period. It is identical to a weighted harmonic average of the current to base period price relatives using the value shares of the current period as weights. Also called a "current weighted index."

$$P_P \equiv \frac{\sum p^t q^t}{\sum p^0 q^t} = \left[\sum s^t \left(\frac{p^t}{p^0}\right)^{-1}\right]^{-1} \text{ where } s^t \equiv \frac{p^t q^t}{\sum p^t q^t}$$

Palgrave price index

A price index defined as the weighted arithmetic average of the current to base period price relatives using the current period value shares as weights.

$$P_{Pal} \equiv \sum s^t \left(\frac{p^t}{p^0}\right) \text{ where } s^t \equiv \frac{p^t q^t}{\sum p^t q^t}$$

Point-in-time prices

Transaction prices prevailing on a particular day of the month.

Positivity

The property whereby the price index and its constituent vectors of prices and quantities are positive—that is, greater than zero.

PPI

Producer price index. A measure of the change in the prices of goods and services either as they leave their place of production or as they enter the production process. A measure of the change in the prices received by domestic producers for their outputs or of the change in the prices paid by domestic producers for their intermediate inputs.

PPPs

Purchasing power parities. Spatial deflators and currency converters that eliminate the effects of the differences in price levels between countries, thus allowing volume-level comparisons and price-level comparisons of GDP and its components.

PPS

Probability proportional to size. A sampling procedure whereby each unit in the universe has a probability of selection proportional to the size of some known variable. In the case of XMPIs, size is usually defined in terms of export or import values or shares.

Price adjustment escalator

The price index nominated to determine the periodic adjustments to regular scheduled payments or to prices of long-term contracts required under indexation. See "indexation" and "indexation of contracts."

Price dispersion test

A test to determine whether the individual unit values for an elementary aggregate (defined as the lowest level of the customs commodity classification by destination or source country) are sufficiently uniform for the overall unit value to be used as a surrogate price in a hybrid XPI or MPI. The test involves computing unit values for every transaction covered by the elementary aggregate for a given month or quarter (depending on the frequency of the index) and measuring their statistical variance. A low variance measurement indicates that the overall unit value for the elementary aggregate could be included in the index. Before doing so, the test is repeated for a number of months or quarters to verify that the low variance measurement persists over time. See also "quantity proportionality test."

Price index

A measure reflecting the average of the proportionate changes in the prices of the specified set of goods and services between two periods of time. Usually a price index is assigned a value of 100 in some selected base period, and the values of the index for other periods are intended to indicate the average percentage change in prices compared with the base period.

Price reference period

The period with whose prices the prices in the current period are compared. The period whose prices appear in the denominators of the price relatives. See also "base period."

Price relative

The ratio of the price of an individual good or service in one period to the price of that same good or service in some other period. Also referred to as a "price ratio."

Price reversal test

A test under the axiomatic approach that requires that the quantity index remains unchanged after the price vectors for the two periods being compared are interchanged.

Price updating

A procedure whereby the quantities of an earlier period are revalued at the prices of a later period. The resulting values are hybrid. In practice, the price-updated values may be obtained by multiplying the original values by price relatives or price indices. Also known as "value updating."

Pricing point

The point in the production or distribution process to which the price refers. For an exported good, it is the customs frontier of the exporting country; for an imported good, it is the customs frontier of the importing country. For an exported service, it is the place of delivery to a nonresident; for an imported service, it is the place of delivery to a resident.

Pricing to constant quality

See "specification pricing."

Primary inputs

Land, labor, and capital.

Probability sampling

The random selection of a sample of units, such as establishments or commodities, in such a way that each unit has a known non-zero probability of selection. It ensures that the units are selected in an impartial and objective fashion and permits the measurement of the quality of survey results through estimates of the variance or sampling error. Also known as "random sampling" or "scientific sampling."

PRODCOM

The acronym from *Production Communautaire,* the name in French for the European Union's system for the collection and dissemination of statistics on the production of manufactured goods. Manufactured goods are those produced by the mining, quarrying, and manufacturing industries. The system is based on a product classification—the Prodcom List—with about 4,500 headings relating to manufactured products.

Producer's index

An index constructed from price data supplied by producers—that is, by exporters in the case of XMPIs.

Producer's price

The amount received by the producer from the purchaser for a unit of good or service produced as output. It *excludes* any VAT (or similar deductible tax on products) invoiced to the purchaser. It also *excludes* supplier's retail and wholesale margins and separately invoiced transport and insurance charges. (A producer's price for a product is the basic price *plus* any nondeductible tax on products paid by the producer *less* any subsidies on products received by the producer.)

Product

See "commodity."

Product line

A group, class, or category of products that is relatively homogeneous in use and price behavior.

Product line specification

A statement of the characteristics of the range of products included in a product line. Its purpose is to provide the frame within which individual products may be selected as part of the sample for pricing. It may also describe the products included in a subindex. See "SPD."

Product specification

A detailed list of the characteristics that identify an individual sampled product. Its purpose is to ensure that a consistent price is collected from period to period relating to a consistent product with the same terms of sale in each period. Hence, the characteristics listed cover both the product (name, serial number, description, etc.) and the transaction (class of customer, size of shipment, discounts, payment terms, delivery details, etc.).

Product test

See "factor reversal test."

Production

An activity that transforms or combines material inputs into other material outputs—as in agricultural, mining, manufacturing, or construction activities—or transports materials from one location to another. Production also includes storage activities—which in effect transport materials in the same location from one time period to another—and the creation of services of all types.

Proportionality in current prices test

A test under the axiomatic approach that requires that if all current period prices are multiplied by the positive scalar λ, the new price index is λ times the old price index.

Pseudo-superlative index

An index that approximates any superlative index to the second order around an equal price and quantity point.

Purchaser's index

An index constructed from price data supplied by purchasers—that is, by importers in the case of XMPIs.

Purchaser's price

The amount paid by the purchaser in order to take delivery of a unit of a good or service at the time and place required by the purchaser. It *excludes* any VAT (or similar deductible tax on products) that the purchaser can deduct from his own VAT liability with respect to VAT invoiced to his customers. It *includes* supplier's retail and wholesale margins, separately invoiced transport and insurance charges, and any VAT (or similar deductible tax on products) that the purchaser cannot deduct from his own VAT liability. (A purchaser's price for a product is the producer's price *plus* supplier's retail and wholesale margins, separately invoiced transport and insurance charges, and nondeductible taxes on products payable by the purchaser.) Purchasers' prices are the prices most relevant for decision making by buyers.

"Pure" price change

The change in the price of a good or service whose characteristics are unchanged; or the change in the price after adjusting for any change in quality.

"Pure" price index

A price index that measures "pure" price change. A price index that is based on pricing a fixed basket of products and product quantities at the prices of the base period and at the prices of the current period. Because the products and their quantities remain constant, any change in the index is due to price changes only. Also called an "unequivocal price index."

"Pure" quantity change

The change in the quantity of a good or service whose characteristics are unchanged.

"Pure" quantity index

A quantity index that measures "pure" quantity change. A quantity index that is based on a fixed basket of products where the quantities of the base period and the quantities of the current period are both valued with the same set of prices. Because the products and their prices remain constant, any change in the index is due to changes in quantities only. Also called an "unequivocal quantity index."

Purposive sampling

See "non-probability sampling."

Quality adjustment

An adjustment to the change in the price of a product whose characteristics change over time that is designed to remove the contribution of the change in the characteristics to the observed price. In a price index context, an adjustment is needed when the price of a replacement product has to be compared with the price of the product it replaces. In practice, the required adjustment can only be estimated. Different methods of estimation, including hedonic methods, may be used in different circumstances. See "explicit quality adjustment" and "implicit quality adjustment."

Quality change bias

Systematic error arising from the application of inappropriate or no quality adjustments.

Quantity index

A measure reflecting the average of the proportionate changes in the quantities of a specified set of goods and services between two periods of time. Usually a quantity index is assigned a value of 100 in some selected base period, and the values of the index for other periods are intended to indicate the average percentage change in quantities compared with the base period. See "volume index."

Quantity proportionality test

An elementary aggregate (defined as the lowest level of the customs commodity classification by destination or source country) may fail the price dispersion test, yet its overall unit value may still be suitable for inclusion as a surrogate price in a hybrid XPI or MPI if the quantities of the various transactions covered by the elementary aggregate remain in fixed proportions over time. The quantity proportionality test is used to verify whether or not this is the case. It involves computing the average quantities transacted across quantiles of the transaction values for a number of months or quarters (depending on the frequency of the index) and comparing the relatives of the average quantity transacted between months or quarters for each quantile. If the quantity relatives are roughly the same across quantiles, this adds support to the proposition that quantity proportions have not changed over time. See also "price dispersion test."

Quantity relative

The ratio of the quantity of a specific good or service in one period to the quantity of the same good or service in some other period.

Quantity reversal test

A test under the axiomatic approach that requires that the price index remains unchanged after the quantity vectors for the two periods being compared are interchanged.

Quantity weights

Weights defined in terms of physical quantities, such as the number or total weight of goods or the number of services. Quantity weights are feasible only at the detailed product level because meaningful aggregation of product weights requires them to be commensurate. Values rather than quantities act as weights for price relatives. See "value weights."

Random sampling

See "probability sampling."

Ratio of harmonic means of prices

An elementary index that constitutes the harmonic average counterpart to the Dutot index. Also known as the "harmonic means price index."

$$P_{RH} \equiv \frac{\sum n/p^0}{\sum n/p^t}$$

Real GDI

Real gross domestic income. A real income measure defined as the volume of GDP plus the trading gain or loss resulting from changes in the terms of trade.

Rebasing

Rebasing may have different meanings in different contexts. It may mean

- changing the weights used for a series of indices,
- changing the price reference period used for a series of indices, or
- changing the index reference period for a series of indices.

The weights, the price reference period, and the index reference period may be changed separately or at the same time.

Rebate

A discount paid to the customer after the transaction has occurred.

Replacement product

The product chosen to replace a product previously sampled, either because it has disappeared completely from the market or because its market share has fallen either in a specific establishment or in the market as a whole.

Representative product

A product, or product line, that accounts for a significant proportion of the total trade within an elementary aggregate, and/or for which the average price change is expected to be close to the average for all products within the aggregate.

Representativity bias

Bias in a basket index that results from the use of products that are not representative of those in the two periods being compared; that is, the bias that arises from the use of products whose aggregate price change systematically diverges from the average price change in the two periods.

Resident

An institutional unit whose center of economic interest is in the economic territory of the country.

Rest of the world

All nonresident institutional units that enter into transactions with resident units or have economic links with resident units.

Revenue

The value of output sold. The value of invoiced sales of goods or services supplied to third parties during the reference period.

Revenue weights

Weights or shares based on the value of output sold in the weight reference period. See "value weights."

Revolutionary goods

Goods that are significantly different from existing goods. They are generally produced on entirely new production lines using production inputs and processes that are considerably newer than those used to produce existing goods. It is virtually impossible, both in theory and in practice, to adjust for any quality differences between a revolutionary good and any existing good.

Reweighting

Replacing the weights used in an index with a new set of weights.

Rolling year indices

A monthly series of indices that compares successive non-calendar years, each of 12 consecutive months, with a base calendar year. The monthly indices are constructed by comparing each month in the non-calendar year with the corresponding month in the base calendar year. The resulting indices can be regarded as seasonally adjusted price indices. For example, if the base calendar year is 2000, the index for January 2001 will refer to the non-calendar year February 2000 to January 2001: February 2000 will be compared with February 2000, March 2000 with March 2000, ..., December 2000 with December 2000, and January 2001 with January 2000. The index for February 2001 will refer to the non-calendar year March 2000 to February 2001: March 2000 will be compared with March 2000, April 2000 with April 2000, ..., January 2001 with January 2000, and February 2001 with February 2000, and so on for each month in the series.

Sample augmentation

Maintaining and adding to the sample of establishments in the survey panel to ensure that it continues to be representative of the population of establishments. A fixed sample of establishments tends to become depleted as establishments cease producing or stop responding. Recruiting new establishments also facilitates the inclusion of new commodities in an XPI or an MPI.

Sample rotation

Limiting the length of time that establishments and/or commodities are included in price surveys by dropping a proportion of them, or possibly all of them, after a certain period of time and selecting a new sample of establishments and/or commodities. Rotation is designed to keep the sample up to date. It also helps to alleviate the problems caused by sample depletion.

Sampling error

A measure of the chance of a difference between the results obtained from the units sampled and the results that would have been obtained from a complete enumeration of all units in the universe.

Sampling frame

The list of the units in the universe from which a sample of units is to be selected. It provides for each unit the details, such as the unit's location, size, value, and type of exports and/or imports, required to pick the sample. Also referred to as the "survey frame."

Sauerbeck price index

A price index defined as the weighted arithmetic average of the current to previous period price relatives using the values of the base period as weights. The price reference period is the previous period (that is, the period immediately before the current period), and the weight reference period is some other fixed period before the previous period. A time-series index is derived by chaining, which, because the weight reference period remains fixed, can result in a serious upward drift in the index when price changes are large and erratic.

Scope

The set of goods and services or transactions for which the index is intended to measure the price changes. In practice, certain goods and services may have to be excluded because it is too difficult, time-consuming, or costly to collect the required data on values or prices. The coverage of an index denotes the actual set of goods and services included, as distinct from the intended scope of the index. Also referred to as "domain."

Seasonal products

Products that either (1) are not available on the market during certain seasons or periods of the year or (2) are available throughout the year but with regular fluctuations in their quantities and prices that are linked to the season or time of year. A product that satisfies (1) is said to be "strongly seasonal." A product that satisfies (2) is said to be "weakly seasonal."

Sector

A general term used to describe a group of establishments engaged in similar kinds of economic activity. A sector can be a subgroup of an economic activity—as in "coal mining sector"—or a group of economic activities—as in "service sector"—or a cross-section of a group of economic activities—as in "informal sector." Also a specific term used in the SNA to denote one of the five mutually exclusive institutional sectors that group together institutional units on the basis of their principal functions, behavior, and objectives, namely: nonfinancial corporations, financial corporations, general government, nonprofit institutions serving households, and households.

Services

Outputs produced to order and that cannot be traded separately from their production. Ownership rights cannot be established over services, and by the time their production is completed they must have been provided to the consumers. However, as an exception to this rule, there is a group of industries, generally classified as service industries, some of whose outputs have characteristics of goods. These are the industries concerned with the provision, storage, communication, and dissemination of information, advice, and entertainment in the broadest sense of those terms. The products of these industries, where ownership rights can be established, may be classified as either goods or services depending on the medium by which these outputs are supplied.

Shipment price

The price at the time the order is delivered to the purchaser.

SITC

Standard International Trade Classification. An internationally agreed commodity classification for the compilation and dissemination of statistics on international merchandised trade. SITC commodity groupings reflect (1) the nature of the product and the materials used in its production, (2) the stage of processing, (3) marketing practices and the uses of the product, (4) the importance of the product in world trade, and (5) technological changes. The original SITC was published in 1951 and has been revised periodically. It was last revised in 2006 (SITC Rev.4). There is full correspondence between the five-digit basic headings of SITC Rev.4 and the six-digit subheadings of HS07.

SNA 2008

System of National Accounts, 2008. A coherent, consistent, and integrated set of macroeconomic accounts, balance sheets, and tables based on a set of internationally agreed concepts, definitions, classifications, and accounting rules. The system first appeared in 1953 and since then has evolved, going through major revisions in 1968 and 1993. *SNA 2008* is an update of *SNA 1993*, the version currently adhered to by most countries.

SPDs

Structured product descriptions. The generic product descriptions used to define the product specifications to be priced for the ICP. An SPD refers to a product cluster. It lists the technical and economic characteristics that products constituting the cluster can possess. A product cluster usually covers a narrow range of homogeneous products, but variation in their common set of characteristics is to be expected. A product can therefore be distinguished from others in the cluster by identifying its specific subset of characteristics. The subset provides the basis for a product specification should the product be selected for pricing. See "product line specification."

Specification

A description or list of the characteristics that can be used to identify an individual sampled product to be priced. A tight specification is a fairly precise description of an item intended to narrow the range of items from which a price collector might choose, possibly reducing it to a unique item, such as a particular brand of television set identified by a specific code number. A loose definition is a generic description of a range of items that allows the price collector some discretion as to which particular item or model to select for pricing, such as color televisions sets of a particular screen size.

Specification pricing

The pricing methodology whereby a manageable sample of precisely specified products is selected, in consultation with each reporting establishment, for repeat pricing. Products are fully defined in terms of all characteristics that influence their transaction prices. The objective is to price to constant quality in order to produce an index showing pure price change.

Splicing

Introducing a replacement item and attributing any price change between the replacement item in the period it is introduced and the replaced item in the period prior to the introduction to the change in quality.

Spot market price

A generic term referring to any short-term sales agreement. Generally it refers to single-shipment orders with delivery expected in less than one month. Goods sold on this basis are usually off-the-shelf and not subject to customization. Spot market prices are subject to discounting and directly reflect current market conditions.

Stage of processing

The classification of goods and services according to their position in the chain of production. However, unlike the classification by stage of production, a product is allocated to only one stage even though it may occur in several stages. Goods and services are classified as either primary products, intermediate products, or finished products.

Stage of production

The classification of goods and services according to their position in the chain of production, but allowing for the multi-function nature of products. Unlike the classification by stage of processing, a product is allocated to each stage to which it contributes and not assigned solely to one stage. Goods and services are classified as primary products, intermediate products, and/or finished products.

Stochastic approach

The approach to index number theory that treats the observed price relatives as if they were a random sample drawn from a defined universe for which the mean can be interpreted as the general rate of inflation. The sample mean provides an estimate of the rate of inflation.

Subsidies on products

The subsidies on goods or services produced as the outputs of resident enterprises that become payable as the result of the production of those goods or services. They are payable per unit of good or service produced.

Subsidized prices

Prices that differ from market prices in that some significant portion of variable and/or fixed costs are covered by a revenue source other than the selling price.

Substitution bias

The bias that results when a fixed basket index is used. By definition, the quantities of the goods and services in the basket to be priced are fixed, and so the index cannot take into account the effects of substitutions made by exporters or importers in response to changes in relative prices. Exporters seeking to maximize revenue may shift to exporting items with above-average relative price increases and, as a result, an XPI with base period quantities will systematically understate average price increases. Importers seeking to minimize costs may shift to importing items with below-average relative price increases and, as a result, an MPI with base period quantities will systematically overstate average price increases. The direction of the bias will be reversed if current period quantities are used—that is, the XPI will systematically overstate average price increases and the MPI will systematically understate them. See “FIOPI,” “FOIPI,” and “true index.”

Superlative index

An index that is “exact” for a “flexible aggregator.” A flexible aggregator is a second-order approximation to an arbitrary cost, production, utility, or distance function. Exactness implies that a particular index number can be derived directly from a specific flexible aggregator. The Fisher price index, the Törnqvist price index, and the Walsh price index are superlative indices. Superlative indices are generally symmetric indices.

Surcharge

An addition to the list price of a good or a service. It is generally of short duration, reflecting unusual cost pressures affecting the producer; for example, a fuel surcharge for transport operators.

Survey frame

See “sampling frame.”

SUTs

Supply and use tables. SUTs are in the form of matrices that record how supplies of different kinds of goods and services originate from domestic industries and imports and how those supplies are allocated between various intermediate and final uses, including exports.

Symmetric index

An index that treats the two periods being compared symmetrically by attaching equal importance to the price and value data in both periods. The price and value data for both periods are entered into the index number formula in a symmetric or balanced way. The superlative price indices of Fisher, Törnqvist, and Walsh are symmetric indices, as are the pseudo-superlative price indices of Drobisch and Marshall-Edgeworth.

Target index

The theoretical index that compilers would choose to calculate in the ideal hypothetical situation in which they have complete information about prices and quantities in the two periods being compared. In practice, the target index is likely to be an economic index as approximated by a superlative price index (such as a Fisher, Törnqvist, or Walsh) or a basket or Lowe index (usually a Laspeyres) that has a clear meaning and can be easily explained to users.

Taxes on products

The taxes on goods or services produced as the outputs of resident enterprises that become payable as the result of the production of those goods or services. They are payable per unit of good or service produced.

Terms of trade index

The ratio of the export price index to the import price index: XPI/MPI. A terms of trade index shows the relationship between the prices at which a country sells its exports and the prices it pays for its imports. If the prices of a country's exports rise relative to the prices of its imports, its terms of trade are said to have moved in a favorable direction, because, in effect, it now receives more imports for each unit it exports.

Test approach

See "axiomatic approach."

Time aggregation problem

How to define and obtain the basic price and quantity for a precisely specified product included in a price index given that, during the time period under consideration, individual buyers may buy the same product at different prices and an individual seller may sell the same product at different prices.

Time reversal test

A test under the axiomatic approach that requires that if the prices and quantities in the two periods being compared are interchanged, the resulting price index is the reciprocal of the original price index. When an index satisfies this test, the same result is obtained whether the direction of change is measured forward in time from the first to the second period or backward from the second to the first period.

Törnqvist price index

A price index defined as the weighted geometric average of the current to base period price relatives in which the weights are the simple unweighted arithmetic averages of the value shares in the two periods. It is a symmetric index and a superlative index. Also known as the "Törnqvist-Theil price index."

$$\ln P_T \equiv \sum \frac{1}{2}\,(s^0 + s^t)\,\ln\left(\frac{p^t}{p^0}\right) \text{ where } s^j \equiv \frac{p^j q^j}{\sum p^j q^j};\quad j = t, 0.$$

Also written as $P_T \equiv \prod\left(\frac{p^t}{p^0}\right)^{\frac{(s^0+s^t)}{2}}$.

Total factor productivity

Relates a measure of output to a measure of combined primary inputs. Rates of change of multi-factor productivity are typically measured residually as that change in output that cannot be accounted for by the change in combined inputs.

Transaction

The buying and selling of a good or service on terms mutually agreed by the buyer and seller.

Transaction price

See "market price."

Transfer price

A price adopted for bookkeeping purposes that is used to value transactions between affiliated enterprises integrated under the same management at artificially high or low levels in order to effect an unspecified income payment or capital transfer between those enterprises. See "intra-company transfer price."

Transitivity

See "circularity."

"True" index

A theoretically defined index that lies between the Laspeyres price index and the Paasche price index. For a theoretical XPI, the Laspeyres price index is the lower bound and the Paasche price index is the upper bound. For a theoretical MPI, the situation is reversed: the Paasche price index is the lower bound and the

Laspeyres price index is the upper bound. See "FIOPI," "FOIPI," and "substitution bias."

Unequivocal price index

See "'pure' price index."

Unequivocal quantity index

See "'pure' quantity index."

Unilateral price index

One approach to index numbers involves decomposing a value aggregate for one period into two numbers: one representing the price level in the period and the other representing the quantity level in the period. The resulting price index is called a "unilateral price index." "Unilateral" refers to the assumption that the price index depends only on price and quantity data of the period under consideration. It is also called an "absolute index number."

Unique product

A product that is manufactured only once to the specification of an individual customer.

Unit value bias

The ratio of a unit value index to a price index calculated with an acceptable index number formula such as the Fisher price index. The term "bias" assumes a systematic divergence. If the divergence is not systematic, then the term "error" is more appropriate. See "bias."

Unit value index

An index that measures the change in the average unit value of items comprising an individual commodity class. The items may not be homogeneous, and the unit value index may therefore be affected by changes in the mix of items (compositional quality and quantity changes) as well as by changes in their prices. Unit value indices are not price indices, but serve as surrogates for price indices. A unit value elementary index, P_U, is given for a price comparison between the current period 1 and reference period 0 over $m = 1, \ldots, M$ items in period 1 and over $n = 1, \ldots, N$ items in period 0 by:

$$P_U \equiv \left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1} \right) \Bigg/ \left(\frac{\sum_{n=1}^{N} p_n^0 q_n^0}{\sum_{n=1}^{N} q_n^0} \right)$$ where prices and

quantities are given respectively as p_m^1 and q_m^1 for period 1, and p_n^0 and q_n^0 for period 0.

Unit value "mix" problem

That the change in the value of a unit value index, thereby implying a "price change," arises from a change in the relative quantities of the items covered without any change in their prices.

Universe

For an XPI, the population of exporters and exports to be sampled. For an MPI, the population of importers and imports to be sampled.

Upper-level index

See "higher-level index."

Value

At the level of a single homogeneous good or service, value is equal to the price per unit of quantity multiplied by the number of quantity units of that good or service. Unlike price, value is independent of the choice of quantity unit. Values are expressed in terms of a common unit of currency and are commensurate and additive across different products. Quantities, on the other hand, are not commensurate and additive across different products even when measured in the same kind of physical units.

Value added

Gross value added is the value of output less the value of intermediate consumption. It is a measure of the contribution to GDP made by an individual producer, industry, or sector. It is the source from which the primary incomes of the SNA are generated. *Net value added* is the value of output less the values of both intermediate consumption and consumption of fixed capital.

Value updating

See "price updating."

Value weights

The measures of the relative importance of products in the index. The weight reference period values or shares of the various elementary aggregates covered by the index. Being commensurate and additive across different products, value weights can be used at aggregation levels above the detailed product level. See "quantity weights."

VAT

Value-added tax. A wide-ranging tax usually designed to cover most or all goods and services. It is collected in stages by enterprises that are obliged to pay to government only the difference between the VAT on their sales and the VAT on their purchases for intermediate consumption or capital formation. VAT is not usually charged on exports.

Virtual corporation

A partnership among several enterprises sharing complementary expertise created expressly to produce a product with a short–prospective life span, with the production of the product being controlled through a computerized network. The corporation is disbanded upon the conclusion of the product's life span.

Volume index

The weighted average of the proportionate changes in the quantities of a specified set of goods and services between two periods of time. The quantities compared must be homogeneous, while the changes for the different goods and services must be weighted by their economic importance as measured by their values in one or both periods.

Walsh price index

A price index defined as the change in the value of a fixed intermediate basket of quantities. The quantities of the intermediate basket are geometric averages of the quantities of the base and current periods. It is a symmetric index and a superlative index.

$$P_W \equiv \frac{\sum p^t \, (q^t q^0)^{\frac{1}{2}}}{\sum p^0 \, (q^t q^0)^{\frac{1}{2}}}$$

Weight reference period

The period of which the value shares serve as weights for a Young index, or of which the quantities make up the basket for a Lowe index. It does not have to have the same duration as the periods for which the index is calculated, and in the case of XMPIs is typically longer, a year or more, rather than a month or quarter. There may be no weight reference period when the value shares for the two periods are averaged, as in a Törnqvist index, or when the quantities are averaged, as in a Marshall-Edgeworth index and a Walsh index. See also "base period."

Weights

A set of numbers summing to unity that are used to calculate averages. Value shares sum to unity by definition and are used to weight price relatives or elementary price indices when these are averaged to obtain price indices or higher-level indices. Although quantities are frequently described as weights, they cannot serve as weights for the prices of different types of products whose quantities are not commensurate and that use different units of quantity that are not additive. The term "quantity weights" is generally used loosely to refer to the quantities that make up the basket of goods and services covered by an index and included in the value weights. See "quantity weights" and "value weights."

Wholesale price index

A measure that reflects changes in the prices paid for goods at various stages of distribution up to the point of retail. It can include prices of raw materials for intermediate and final consumption, prices of intermediate or unfinished goods, and prices of finished goods. The goods are usually valued at purchasers' prices. For historical reasons, some countries call their PPI a "wholesale price index," even though the index no longer measures changes in wholesale prices.

XMPIs

Export and import price indices. The *export price index* (XPI) measures the average change over time in the prices of the goods and services supplied by residents of the economic territory of a country and used by nonresidents (the rest of the world). The *import price index* (MPI) measures the average change over time in the prices of goods and services supplied by nonresidents (the rest of the world) and used by residents of the economic territory of a country.

XPI

Export price index. See "XMPIs."

Year-over-year indices

Year-over-year monthly indices: Indices in which a given month in the current period is compared with the corresponding month in the base period. For example, if the current period is 2001 and the base period is 2000, January 2001 would be compared with January 2000, February 2001 with February 2000, March 2001 with March 2000, etc. There are in effect 12 separate series of indices, one for each month of the year. Year-over-year monthly indices are a means of dealing with seasonal variation. It is assumed that the patterns of seasonal variation remain the same month by month, year after year.

Year-on-year annual indices: The weighted average of the 12 year-over-year monthly indices. The weights are monthly value (revenue) shares of the base year or the current year or the average of the two, depending on the index number formulation.

Young index

This manual specifically refers to the Young price index as a weighted arithmetic average of price ratios between the current period t and the price reference period 0, where the weights are value shares in a (usually) preceding period b.

$$P_{YO} \equiv \sum s^b \left(\frac{p^t}{p^0}\right).$$

More generally, a Young index can be defined as a weighted arithmetic average of price ratios between the current period t and the price reference period, where the weights are value shares (s_n) that sum to 1. The Young index is thus defined as

$$P_{YO} \equiv \sum s_n \left(\frac{p^t}{p^0}\right).$$

Special cases include the Laspeyres index when $s_n = s^0 = \frac{p^0 q^0}{\sum p^0 q^0}$ and the Paasche index when s_n are hybrid weights using period t quantities valued at period 0 prices, that is, $s_n = s^{0t} \equiv \frac{p^0 q^t}{\sum p^0 q^t}$.

Appendix: Some basic index number formulas and terminology

1. A *basket* price index (called here a *Lowe* price index after the index number pioneer who first proposed this general type of index) is an index of the form[1]

$$\frac{\sum p_n^t q_n}{\sum p_n^0 q_n}, \tag{G.1}$$

which compares the prices of period t with those of (an earlier) period 0, using a certain specified quantity basket. The family of Lowe indices includes some well-known indices as special cases:

- When $q_n = q_n^0$, we get the Laspeyres index;
- When $q_n = q_n^t$, we get the Paasche index;
- When $q_n = \left(q_n^0 + q_n^1\right)/2$, we get the Marshall-Edgeworth index; and
- When $q_n = \left(q_n^0\, q_n^t\right)^{1/2}$, we get the Walsh index.

In practice, official statistical agencies frequently work with a Lowe index in which $q_n = q_n^b$, where b denotes some period before 0.

2. A useful feature of a basket price index for period t relative to period 0 is that it can be decomposed, or factored, into the product of two or more indices of the same type: for instance, as the product of an index for period t – 1 relative to period 0 and an index for period t relative to period t – 1. Formally,

$$\frac{\sum p_n^t q_n}{\sum p_n^0 q_n} = \frac{\sum p_n^{t-1} q_n}{\sum p_n^0 q_n} \frac{\sum p_n^t q_n}{\sum p_n^{t-1} q_n}. \tag{G.2}$$

The index on the right side of equation (G.2) is described as a "two-stage index." It is identical to the single-basket index that compares period t directly with period 0, provided the same set of prices is available and used in all three periods, 0, t – 1, and t.

In particular, when $q_n = q_n^0$, equation (G.2) turns into

$$\frac{\sum p_n^t q_n^0}{\sum p_n^0 q_n^0} = \frac{\sum p_n^{t-1} q_n^0}{\sum p_n^0 q_n^0} \frac{\sum p_n^t q_n^0}{\sum p_n^{t-1} q_n^0}. \tag{G.3}$$

The left side of equation (G.3) is a direct Laspeyres index. Note that only the first of the indices that make up the "two-stage Laspeyres" index on the right side is itself a Laspeyres index, the second being a Lowe index for period t relative to period t – 1 that uses the quantity basket of period 0 (not t – 1). Some official statistical agencies describe the two-stage Laspeyres index given in equation (G.3) as a "modified Laspeyres" index.

3. In a time-series context, say when t runs from 1 to T, the series

$$\frac{\sum p_n^1 q_n}{\sum p_n^0 q_n}, \frac{\sum p_n^2 q_n}{\sum p_n^0 q_n}, \ldots\ldots, \frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \tag{G.4}$$

is termed a series of *fixed*-basket price indices. In particular, when $q_n = q_n^0$, we get a series of Laspeyres indices.

[1]The sums are understood to be running over all items n.

4. At period T, one could change to a new quantity basket q' and calculate from this period onward

$$\frac{\sum p_n^{T+1} q_n}{\sum p_n^T q_n}, \frac{\sum p_n^{T+2} q_n}{\sum p_n^T q_n}, \frac{\sum p_n^{T+3} q_n}{\sum p_n^T q_n}, \ldots\ldots \quad (G.5)$$

To relate the prices of periods $T+1$, $T+2$, $T+3, \ldots.$ to those of period 0, *chain linking* can be used to transform (G.5) into a series of the form

$$\frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+1} q_n}{\sum p_n^T q_n}, \frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+2} q_n}{\sum p_n^T q_n},$$
$$\frac{\sum p_n^T q_n}{\sum p_n^0 q_n} \frac{\sum p_n^{T+3} q_n}{\sum p_n^T q_n}, \ldots\ldots \quad (G.6)$$

This could be termed a series of *chain-linked* fixed-basket price indices. In particular, when $q_n = q_n^0$ and, $q_n = q_n^T$ we get a series of chain-linked Laspeyres indices. Because the basket was changed at period T, the adjective "fixed" applies literally over only a certain number of time intervals. The basket was fixed from period 1 to period T and is fixed again from period $T+1$ onward. When the time intervals during which the basket is kept fixed are of the same length, such as one, two, or five years, this feature can be made explicit by describing the index as an annual, biannual, or five-yearly chain-linked fixed-basket price index.

5. A *weighted arithmetic*[2]*-average(-type)* price index (called here a *Young* price index after another index number pioneer) is an index of the form

$$\sum w_n \left(p_n^t / p_n^0\right) \quad (G.7)$$

which compares the prices of period t with those of period 0, using a certain set of weights adding up to 1. Note that any basket price index in the form of equation (G.1) can be expressed in the form of equation (G.7), because

$$\frac{\sum p_n^t q_n}{\sum p_n^0 q_n} = \sum \frac{p_n^0 q_n}{\sum p_n^0 q_n} \frac{p_n^t}{p_n^0}. \quad (G.8)$$

In particular, when

(i) $w_n = s_n^0 \equiv p_n^0 q_n^0 / \sum p_n^0 q_n^0,$ (G.9)

that is, period 0 value shares, equation (G.7) turns into the Laspeyres index. When

(ii) $w_n = p_n^0 q_n^t / \sum p_n^0 q_n^t,$ (G.10)

that is, hybrid period (0, t) value shares, we get the Paasche index. One could also think of setting

(iii) $w_n = s_n^b \equiv p_n^b q_n^b / \sum p_n^b q_n^b,$ (G.11)

that is, period b value shares.[3]

Instead of working with equation (G.11), one frequently works with

(iv) $w_n = s_n^b \left(p_n^0 / p_n^b\right) / \sum s_n^b \left(p_n^0 / p_n^b\right) = p_n^0 q_n^b / \sum p_n^0 q_n^b,$ (G.12)

that is, *price-updated* period b value shares.

Note that hybrid value shares, such as equation (G.10) or (G.12), typically are not constructed by multiplying sums of prices of one period times quantities of another period. They must be constructed using price relatives and actual expenditure shares, as in the first part of equation (G.12).

6. In a time-series context, when t runs from 1 to T, the series

$$\sum w_n \left(p_n^1 / p_n^0\right), \sum w_n \left(p_n^2 / p_n^0\right), \ldots\ldots, \sum w_n \left(p_n^T / p_n^0\right) \quad (G.13)$$

is termed a series of *fixed* weighted arithmetic-average price indices. In particular, when the weights are equal to the period 0 expenditure shares, we get a series of Laspeyres indices, and when the weights are equal to the price-updated period b expenditure shares, we get a series of Lowe indices in which the quantities in the basket are those of period b.

7. In period T, one could change to a new set of weights w', and calculate from this period onward

$$\sum w'_n \left(p_n^{T+1} / p_n^T\right), \sum w'_n \left(p_n^{T+2} / p_n^T\right), \sum w'_n \left(p_n^{T+3} / p_n^T\right), \ldots.., \quad (G.14)$$

or, using linking to relate the prices of periods $T+1$, $T+2$, $T+3, \ldots.$ to those of period 0,

$$\sum w_n \left(p_n^T / p_n^0\right) \sum w'_n \left(p_n^{T+1} / p_n^T\right),$$
$$\sum w_n \left(p_n^T / p_n^0\right) \sum w'_n \left(p_n^{T+2} / p_n^T\right), \ldots\ldots \quad (G.15)$$

This could be termed a series of *linked* fixed-weighted arithmetic-average price indices. In particular, when $w_n = s_n^0$ and $w_n = s_n^T$, we get a

[2]To distinguish from geometric or other kinds of average.

[3]This manual specifically refers to this as a Young index.

series of chain-linked Laspeyres indices. When $w_n = s_n^b(p_n^0/p_n^b)/\sum s_n^b(p_n^0/p_n^b)$ and $w_n = s_n^b(p_n^T/p_n^b)/\sum s_n^b(p_n^T/p_n^b)$ for some later period *b'*, we get a series of chain-linked Lowe indices.

8. Again, because the weights were changed at period *T*, the adjective "fixed" applies literally over only certain time intervals. The weights were fixed from period 1 to period *T* and are again fixed from period *T* + 1 onward. When the time intervals during which the weights are kept fixed are of the same length, this feature can be made explicit by adding a temporal adjective, such as biannual or five-yearly. When the intervals are annual, the term "chain-linking" is used.

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