Comparing Semi-Structural Methods to Estimate Unobserved Variables: The HPMV and Kalman Filters Approaches

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COMPARING SEMI-STRUCTURAL METHODS TO ESTIMATE UNOBSERVED VARIABLES: THE HPMV AND KALMAN FILTERS APPROACHES

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ABSTRACT/RÉSUMÉ

Economists often seek to estimate unobserved variables, representing "equilibrium" or "expected" values of economic variables, as benchmarks against which observed, realised values of these variables may be evaluated. Such comparisons are often used as economic policy indicators, for example the output gap, as measured by the ratio of actual to potential GDP, is commonly used as a measure of excess demand in assessing inflation pressures. To estimate these unobserved variables, a popular approach is the so-called semi-structural approach which includes: the Hodrick Prescott multivariate filter (developed by Laxton and Tetlow, 1992) and the Kalman filter (see, among others Harvey, 1992 and Cuthberson et al., 1992). This paper shows that the two approaches are closely linked, and specifically, it explains how to reproduce the Hodrick Prescott multivariate filter using the Kalman filter. Being able to do so has at least two possible advantages. First, while the traditional HPMV filter cannot produce margins of uncertainty for the estimated variables, the Kalman filter can. Secondly, it allows wider evaluation of the impact of the different assumptions underlying each technique on the estimates of the unobserved variables. The paper provides a concrete illustration in the context of estimating the structural or non-accelerating inflation rate of unemployment (hereafter the NAIRU) for France and the United States.

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Les économistes cherchent fréquemment à estimer des variables non observables, utilisées comme valeur d'équilibre ou de référence. La différence entre cette valeur estimée et la valeur observée est ensuite un indicateur des tensions économiques : par exemple, l'écart de PIB, mesuré par la différence entre le PIB potentiel et le PIB courant, est souvent utilisé pour évaluer les pressions inflationnistes. Une approche fréquemment utilisée pour estimer des variables inobservées est l'approche dite semi-structurelle, qui englobe notamment le filtre de Hodrick Prescott multivarié (développé par Laxton et Tetlwo 1992) et le filtre de Kalman (voir, entre autres, Harvey 1992 et Cuthberson et al. 1992). Ce document présente le lien entre ces deux filtres et explique comment reproduire le filtre HP multivarié avec un filtre de Kalman. L'intérêt de cette démarche est double. Tout d'abord, alors qu'il n'est pas possible de produire une mesure de confiance d'un estimateur HP multivarié, le filtre de Kalman permet de dériver des intervalles de confiance pour la variable inobservée estimée. Ensuite, cela permet d'évaluer l'impact, sur la variable inobservée, des différentes hypothèses implicites à chaque méthode. Une illustration de ces deux points est présentée, avec une estimation du taux de chômage structurel ou *NAIRU* (non-accelerating inflation rate of unemployment) pour la France et les Etats-Unis.

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Laurence Boone[1]

Introduction

1.   Economists often seek to estimate unobserved variables, representing "equilibrium" or "expected" values of economic variables, as benchmarks against which observed, realised values of these variables may be evaluated. Such comparisons are often used as economic policy indicators, for example the output gap, as measured by the ratio of actual to potential GDP, is commonly used as a measure of excess demand in assessing inflation pressures.

2.   In the literature, three types of approaches are typically utilised for estimating unobserved variables: structural methods which rely on an economic model, filtering methods which apply statistical procedures to the actual variable, or methods which combine both these approaches. For example, the "equilibrium unemployment rate" might be estimated as the underlying solution to a detailed wage-price system; it might also be extracted as the underlying trend of the observed unemployment rate, by means of a simple Hodrick Prescott filter; or it might come from estimating a reduced form Phillips curve, with filtering methods used to apply some constraints on its form, an approach commonly referred to as being "semi-structural".

3.   The purpose of this paper is not to compare such approaches. Rather, it takes stock of current empirical work and focuses on semi-structural approaches.[2] In particular, it looks at two types of semi-structural approaches which are in common use: the Hodrick Prescott multivariate filter (developed by Laxton and Tetlow, 1992) and the Kalman filter (see, among others Harvey, 1992 and Cuthberson et al., 1992). This paper shows that the two approaches are closely linked, and specifically, it explains how to reproduce the Hodrick Prescott multivariate filter (HPMV thereafter) using the Kalman filter. Being able to do so has at least two possible advantages. First, while the traditional HPMV filter cannot produce margins of uncertainty for the estimated variables, the Kalman filter can. Secondly, it allows wider evaluation of the impact of the different assumptions underlying each technique on the estimates of the unobserved variables. It goes on to provide a concrete illustration in the context of estimating the structural or non-accelerating inflation rate of unemployment (hereafter the NAIRU) for France and the United States.

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2.   Over the past five years, there have been a large number of studies estimating potential output and the NAIRU with semi-structural approaches. These include Kuttner (1992), Gordon (1997, 1998), Laxton and Tetlow (1992), Staiger et al. (1997a,b), Smets (1999) and, more recently, Bank of England (1999), Boone and Turner (1999), Irac (1999), Fabiani and Mestre (1999) and Orlandi and Pichelmann (2000).

4.   The organisation of the paper is as follows. A first section shows how the HPMV filter may be rewritten as an unobserved component model, which can be estimated by means of the Kalman filter. Drawing on ongoing empirical work at the OECD, the second section applies both methods to estimating the NAIRU for France and the United States, emphasising and explaining the differences in estimation due to the differences in techniques. A third section focuses on the meaning of the standard errors under both approaches. A final section draws some general conclusions.

1.   Estimating the HPMV filter using the Kalman filter

1.1   The Kalman filter

5.   The Kalman filter is a popular method which can be used to estimate unobserved variable(s), provided they appear as explanatory variables in a model that can be written in a "state space form". A state space representation is one made up of *measurement* equations, expressing observed or *signal* variables as a function of unobserved or *state* variables, and some *transition* equations, governing the path of the unobserved variables. More formally, a state space model representation is one of the form:

Measurement equation:

$$Y_t = z.A_t + d.X_t + e_t \qquad \text{with } e_t \sim N(0, H) \qquad [1]$$

where Y is a vector of observed variables, X is a vector of exogenous variables, A a vector of unobserved variables. Z and d are vectors of parameters and e represents residuals with variance/covariance matrix H.

Transition equation:

$$A_t = T.A_{t-1} + v_t \qquad \text{with } v_t \sim N(0, Q) \qquad [2]$$

where T is a vector of parameters and v a vector of residuals with variance/covariance matrix Q.

6.   Such a model may be estimated by means of a Kalman filter, a recursive procedure which, combined with a maximum likelihood estimation method, gives optimal estimates of unobserved components (see Technical Annex A for details). This method has been used for a number of applications, such as estimating expectations (Cuthberson *et al.,* 1992), estimating the underlying structural rate of unemployment (among others, Staiger *et al.,* 1997, Gordon, 1997, 1998, Irac, 1999, Orlandi and Pichelman, 2000), or estimating potential output (Smets, 1999, Kichian, 1999).

7.   In principle, with this method all the parameters of the model may be estimated. In practice, there might be a trade-off between the number of parameters being estimated and the convergence of the likelihood function. More specifically, a key variable to the estimation of such models is the relative smoothness of the unobserved variable, which is governed by the relative size of the error variances in [1] and [2]. The higher the ratio of the variance of the transition to the measurement equation residuals, referred to as the "signal-to-noise ratio" (Q/H), the more explanatory power is given to the unobserved variable, and the better the fit of the measurement equation. In the limit, for very large values of Q, the unobserved variable may soak up all the residual variation in the measurement equation. Alternatively if Q is zero, then it will be estimated as a constant. In practice, most studies fix the signal-to-noise ratio so that the estimated unobserved variable is relatively smooth, with fluctuations which are judged to be reasonable

from one period to another, which Gordon (1997) qualifies as "the [unobserved variable] can move around as much as it likes, subject to the qualification that sharp quarter-to-quarter zigzags are ruled out".[3]

1.2   *The HPMV filter*

8.   The Hodrick Prescott MultiVariate (HPMV) filter is an alternative way of estimating unobserved variables, developed at the Bank of Canada (see Laxton and Tetlow, 1992). This method stems from use of the standard Hodrick Prescott (HP thereafter) filter (Hodrick and Prescott, 1981, 1997) augmented by relevant economic information. This is done by minimising the residuals of one or more economic relationships involving the unobserved variable (s). The intuition is that this will help produce an estimate of the unobserved variable (s) which maximises the fit of the estimated economic relationships whilst conforming to the standard properties of an HP filter. The HP and the HPMV filters main features are described in more detail below.

9.   The simple HP filter gives an estimate of the unobserved variable as the solution to the following minimisation problem:

$$\text{Minimise} \qquad \sum (y_t - y*_t)^2 + \lambda_1 (\Delta\Delta y*_t)^2 \qquad\qquad [3]$$

with respect to y*, for a given parameter value of λ_1, where y is the observed variable and y* is the unobserved variable being estimated. The HP filter was originally designed to distinguish between the long-term component of a variable, and its short-run cyclical fluctuations. Hence, y* represents the underlying trend of the variable y, (y-y*) are cyclical fluctuations and $\Delta\Delta y*$ is the change in the growth rate of the underlying trend of the series. Intuitively, the filtered series is then a moving average of the observed series, with λ_1 governing the balance between the smoothness of the trend and the magnitude of the cyclical fluctuations. The relevance of the smoothing constant λ_1 is more easily understood by rewriting the above minimisation problem as:

$$\text{Minimise} \sum \frac{1}{\sigma_0^2}(y_t - y*_t)^2 + \frac{1}{\sigma_1^2}(\Delta\Delta y*_t)^2 \qquad\qquad [4]$$

with respect to y*, where σ_0^2 is the variance of the cyclical component {y-y*} and σ_1^2 is the variance of the growth rate of the trend component, so that $\lambda_1 = \frac{\sigma_0^2}{\sigma_1^2}$. The ratio of the two variances governs the relative weights, in the minimisation problem, given to the smoothness of the filtered series versus the volatility of the cyclical component. Intuitively, the higher σ_0^2 is compared to σ_1^2, i.e. the larger λ_1, the smoother the filtered series.

10.   The HPMV filter seeks to estimate the unobserved variable as the solution to the following minimisation problem:

$$\text{Minimise} \qquad \sum (y_t - y*_t)^2 + \lambda_1 (\Delta\Delta y*_t)^2 + \lambda_2 \xi_t^2 \qquad\qquad [5]$$

with respect to y*, for given λ_1 and λ_2. This is a basic HP filter, augmented with the residuals (ξ) from an estimated economic relationship:

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3.   See Bank of England (1998) for a survey. Some exceptions are Apel and Jansson (1998, 1999) for Sweden, Kichian (1999) for Canada. These are countries specific studies, using quite sophisticated models.

$$y'_t = \beta y*_t + d_2 . X_t + \xi_t \qquad \text{with } \xi_t \sim N(0, H_2) \qquad [6]$$

where y' another explanatory variable can be explained by the unobserved variable y*, with a coefficient β; X represents other exogenous variables, affected by the parameters d_2. The residuals ξ are normal with a variance/covariance matrix H_2.

11.   This method has been used, for example, by the Bank of Canada to estimate potential output in the QPM model (see Butler, 1996), by the Reserve Bank of New Zealand to estimate potential output, (see Conway and Hunt, 1997) and by the OECD (1999) to estimate the NAIRU. As for the simple HP filter, the smoothing constants λ_1 and λ_2 reflect the weights attached to different elements of the minimisation problem as between cyclical fluctuations, the growth rate of the trend and the residuals of the economic relationship. This is best seen by rewriting [5] as:

$$\text{Mininimise } \sum \frac{1}{\sigma_0^2}(y_t - y*_t)^2 + \frac{1}{\sigma_1^2}(\Delta\Delta y*_t)^2 + \frac{1}{\sigma_2^2}\xi^2 \qquad [7]$$

with $\lambda_1 = \dfrac{\sigma^2}{\sigma_1^2}$ and $\lambda_2 = \dfrac{\sigma_0^2}{\sigma_2^2}$

Intuitively, the estimated unobserved variable is not only a simple moving average going through the observed series, but is also modelled to give a better fit to the economic relationship. Hence, the smaller the variance of the residuals σ_2^2, i.e. the higher λ_2, the more important will be the information added by the economic relationship.

1.3   *Comparing the HPMV and the Kalman filter*

12.   At first sight, the HPMV and Kalman filter approaches might quite seem different, but it is possible to rewrite the former as a state space model and impose some specific restrictions on its elements so that it can be reproduced with the latter. Harvey (1985) explains how to reproduce the simple HP filter with the Kalman filter and this section draws on his work, extending it to reproduce an HPMV filter with the Kalman filter. This is done in two steps: first, the minimisation problem is written as a state space model. Secondly, restrictions are imposed on the variances of the equations of the state space model, to reproduce the balance between the elements of the minimisation programme, as represented by the smoothing constants λ_i.

13.   Following Harvey (1985), the HP filter can be written in a state space form as follows:

Measurement equation:

$$y_t = y*_t + e_t \qquad \text{with } e_t \sim N(0, H) \qquad [8]$$

Equation [8] is directly analogous to the first term in expression [3] for the HP filter and e_t is the difference between the extracted trend and the observed variable.[4]

Transition equation:

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4 .   Equation [8] implies that the "business cycle" in the series y_t is simply determined as a residual.

$$\Delta y *_t = \Delta y *_{t-1} + v_t^2 \qquad\qquad \text{where } v_t^2 \sim N(0, Q_2) \qquad\qquad [9]$$

Equation [9] is then directly analogous to the second term in expression [3] for the HP filter.[5] v_t^2 is the change in the growth rate of the filtered series or trend. In other words, the change in the trend follows a random walk.

This system is fully equivalent to the system presented in section 1.1:

Measurement equation [1]:

$$Y_t = z.A_t + d.X_t + e_t$$

where the unobserved variable A is explained by the transition equation [2]:

$$A_t = T.A_{t-1} + v_t$$

with the following restrictions:

$$Y_t = y_t, \; A_t = \begin{bmatrix} y *_t \\ g_t \end{bmatrix}, \; z = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; d = 0, \; T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{and } v_t = \begin{bmatrix} v_t^1 \\ v_t^2 \end{bmatrix} = \begin{bmatrix} 0 \\ v_t^2 \end{bmatrix} \qquad [10]$$

Hence the HP filter as represented by equations [8]-[9] is written as a state space model and therefore may be estimated by means of the Kalman filter.

14.   The second step then consists of defining the balance between the smoothness of the trend and the magnitude of the cyclical fluctuations. As H is the variance of the cyclical fluctuations (y - y*), and Q of the growth rate of the trend ($\Delta\Delta y*$), this can be done by simply choosing values for H and Q such that:

$$\text{H/Q} = \lambda_1 = \frac{\sigma_0^2}{\sigma_1^2} .$$

15.   The same line of reasoning may be used to reproduce an HPMV filtered series with the Kalman filter. In this case, the minimisation problem is written as the following state space representation.

There are two measurement equations. The first one is similar to that of the HP filter

$$y_t = y *_t + e_t \qquad\qquad \text{with } e_t \sim N(0, H) \qquad\qquad [8]$$

and corresponds to the first term in the minimisation problem [7].

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5.   Formally, the transitions equations are written as follows:

$$y *_t = y *_{t-1} + g_{t-1} + v_t^1 \qquad\qquad [9a]$$

$$g_t = g_{t-1} + v_t^2 \qquad\qquad [9b]$$

with $\begin{pmatrix} v_t^1 \\ v_t^2 \end{pmatrix} \sim N(0, Q)$ and $\quad Q = \begin{pmatrix} 0 & 0 \\ 0 & Q_2 \end{pmatrix}$

Combining [9a] and [9b] gives [9] in the main text.

The second measurement equation corresponds to the additional *third* term in the minimisation problem [7], based on the following economic relationship:

$$y'_t = \beta y*_t + d_2.X_t + \xi_t \qquad \text{with } \xi_t \sim N(0, H_2) \qquad [6]$$

where y'_t is an observed variable, β and d_2 are parameters.

As for the HP filter, the transition equation is:

$$\Delta y*_t = \Delta y*_{t-1} + v_t^2 \qquad \text{where } v_t^2 \sim N(0, Q_2) \qquad [9]$$

On this basis, the HPMV filter is re-written as a state space model where the measurement equation is equation [1]:

$$Y_t = z.A_t + d.X_t + e_t$$

and the transition equation is equation [2]:

$$A_t = T.A_{t-1} + v_t$$

with the following restrictions :

$$Y_t = \begin{bmatrix} y_t \\ y'_t \end{bmatrix} A_t = \begin{bmatrix} y*_t \\ g_t \end{bmatrix}, \quad z = \begin{bmatrix} 1 & 0 \\ \beta & 0 \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0 \\ d_2 \end{bmatrix} \text{ is a vector of parameters, } T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{and}$$

$$v_t = \begin{bmatrix} v_t^1 \\ v_t^2 \end{bmatrix} = \begin{bmatrix} 0 \\ v_t^2 \end{bmatrix}. \qquad [11]$$

Two points are worth highlighting here. Firstly, in this form, the HPMV filter can be estimated using the Kalman filter. Secondly, the only difference between it and a simple HP filter is the addition of the second measurement equation.

16.   As for the HP filter, the subsequent steps consist of defining the balance between the smoothness of the series, the magnitude of the cyclical fluctuations and the fit of the economic relationship. As before, H is the variance of the cyclical fluctuations (y – y*), and Q is the variance of the growth rate of the trend ($\Delta\Delta$y*). The novelty is the term H_2 the variance of the residuals of the economic relationship [6]. The balance between the HP filter and the economic information that is embodied in the HPMV filter can be defined by following a similar reasoning to paragraph 14, by choosing values for H, H_2 and Q such that:

$$\text{H/Q} = \lambda_1 = \frac{\sigma_0^2}{\sigma_1^2} \text{ and H/H}_2 = \lambda_2 = \frac{\sigma_0^2}{\sigma_2^2}.$$

These ratios show how the HPMV filter strikes a balance between the relative smoothness of the trend, and the magnitude of the fluctuations with the fit of the economic relationship. The higher the smoothing constant λ_1, the more weight is given to minimising cyclical fluctuations: hence the unobserved tends towards the observed variable. At the same time, a high λ_2, corresponds to higher fit of the economic relationship, and an unobserved variable that can depart significantly from the observed variable.

17.   There are several advantages to being able to reproduce an HPMV filter with the Kalman filter. Firstly, the estimation procedure using the Kalman filter is more straightforward. The economic relationship(s) and the unobserved variable are estimated simultaneously, using maximum likelihood. By contrast, the HPMV filter as originally described by Laxton and Tetlow is more complex to estimate in that it requires at least a two-step procedure. This consists of, first, estimating or calibrating the economic relationship [6] with OLS, using a proxy for the unobserved variable. The residuals of this regression are then plugged into the minimisation problem [7], which gives a new estimate for the unobserved variable y*. OECD (1999) uses this procedure with further iterations. The new estimate is, in turn, put back into the economic relationship [6] and estimated with OLS to get new residuals, which are again plugged into the minimisation problem [7]. This procedure is carried until convergence is reached. Secondly, by using a maximum likelihood procedure, the Kalman filter may produce standard errors for the unobserved variable, something not available when using the two-steps or iterative HPMV estimation method.

2.   An empirical comparison of the HPMV and standard Kalman filter estimates

18.   The preceding analysis would be of purely technical interest if the estimated series were always identical under the two alternative methods of estimation. However, in practice, researchers tend to use these alternative methods under assumptions which may seem "reasonable" for certain series, but may have different interpretations within the above state space framework. As a result, different researchers can obtain different results for relatively arbitrary reasons. In this section, the possible differences between results obtained with the two approaches are identified in more details, using a concrete illustration.

19.   The chosen example seeks to estimate the NAIRU for France and the United States.[6] The NAIRU is an unobserved variable defined as the rate of unemployment at which inflation stabilises, in the absence of any temporary supply shocks. If unemployment falls below the NAIRU, inflation will rise until unemployment returns to the NAIRU, at which time inflation will stabilise at a permanently higher level. This is not to say that there cannot be a lower level of the NAIRU compatible with stable inflation. Rather, it means that when markets adjust slowly to macroeconomic shocks, a lower (or higher) actual unemployment rate may be related to stable inflation in the long term, but this unemployment rate may not be reached within a year, without triggering movement in inflation.

20.   The simplest theoretical framework incorporating the NAIRU concept in a transparent fashion is the expectation-augmented Phillips curve, which is also consistent with a variety of alternative structural models:

$$\Delta \pi_t = \alpha(L)\Delta \pi_{t-1} - \beta(y_t - y^*_{t-1}) + \delta z_t + \xi_t \qquad [12]$$

where y* is the NAIRU, y is the current unemployment rate, π stands for inflation so that $\Delta\pi$ is its growth rate, $\alpha(L)$ is a polynomial in the lag operator, β is defined as before, δ is a vector of parameters, z a vector of temporary supply shocks and ξ the residuals.

21.   Recently, an important number of studies have been published, producing estimates of the NAIRU, based on the Phillips curve, using either the HPMV or the Kalman filter. For the former, the residuals of the Phillips curve are included in the minimisation problem [7], as the third term (ξ). Standard values for the smoothing constant λ_1 are generally used: 400 for semi-annual data, as for the HP filter.[7] For

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6.   Common references to this type of model analysis is Gordon (1997, 1998) for the United States, Irac (1999) for France, Fabiani and Mestre (1999) for Euro-zone, Orlandi and Pichelman (2000) for the European Commission.

7.   100 for annual data and 1600 for quarterly data. Hodrick and Prescott (1981, 1997) used 1600 for estimating trend US GDP, as a value producing "a trend that most closely correspond to the line that students would fit through GDP by hand and eye".

the additional smoothing constant λ_2, $H_2 = \sigma_2^2$, two values are generally used: $\lambda_2 = 1$, or $\lambda_2 = 16$. For the Kalman filter, the Phillips curve is used directly as the only measurement equation. The model of the NAIRU is different from that of the HPMV filter (equation [9]). It generally is a simple random walk:

$$y*_t = y*_{t-1} + \nu_t \qquad\qquad \text{with } \nu_t \sim N(0,Q) \qquad\qquad [13]$$

but may also take the form of an auto regressive process, to reflect possible persistence in the NAIRU series (this might be relevant, for example, to the structural unemployment rate in Continental Europe which has been trended upwards for the past twenty years):

$$\Delta y*_t = \phi\Delta y*_{t-1} + \nu_t \qquad\qquad \text{with } \nu_t \sim N(0,Q) \qquad\qquad [14]$$

where the auto regressive parameter ϕ lies between zero and 1. As mentioned above, the smoothness of the NAIRU relative to the Phillips curve is governed by the signal-to-noise ratio Q/H_2 (which is equivalent to the inverse of λ_2). Standard values in the literature (referred to in Bank of England, 1998) range between 0.1 and 0.5. Table 1 below summarises the features of the estimation procedure under each methodology, when estimating the NAIRU using the information provided by a Phillips curve relationship.

Table 1. **The HPMV and standard Kalman filter in a state space framework**

	HPMV	Standard Kalman filter
Measurement equation(s)	$y_t = y*_t + e_t$ [8] Phillips curve [12]	Phillips curve [12]
Transition equation(s)[a]	$\Delta y*_t = \Delta y*_{t-1} + \nu_t$ [9]	United States: $y*_t = y*_{t-1} + \nu_t$ [13] France: $\Delta y*_t = \phi\Delta y*_{t-1} + \nu_t$ [14]
Variances[b]	$e_t \sim N(0, \sigma_0^2 = 1)$ $\xi_t \sim N(0, \sigma_2^2 = 1/16)$ $\nu_t \sim N(0, \sigma_1 = 1/400)$	$\xi_t \sim N(0, H_2)$ $\nu_t \sim N(0,Q)$
Estimated parameters	Phillips curve coefficients	Phillips curve coefficients Phillips curve residual variance Auto regressive parameter ϕ

a) Here it is assumed that for a stationary unemployment rate, such as the United States, a random walk is a reasonable representation for the NAIRU. When a trend appears, as for France, an auto regressive representation seems a better model.

b) The values for the HPMV variances presented here are the standard values used in the literature, normalised on the variance of the first measurement equation. The ratio $Q/H2$ is fixed between 0.1 and 0.5 (see paragraph 21).

22.   The main difference between the two approaches is the inclusion, with the HPMV, of measurement equation [8], which ties the unobserved variable to the observed series. By excluding this term, the standard Kalman filter gives more weight to the fit of the economic relationship. Furthermore, while the HPMV filter typically uses only one representation for the unobserved variable [a random walk with stochastic drift, equation 9], alternative representations are often used with the standard Kalman filter, such as the random walk when the unobserved appears stationary (for example, for the United States), or an auto regressive process to reflect persistence (for example, for France).

23.   Although the standard Kalman filter permits, in principle, the estimation of the smoothing parameters (or equivalently the variances Q and H_2), in practice researchers tend to use fixed values as described in paragraph 21. In both cases, however, the ratio(s) of the variances rather than their absolute values (provided these are reasonable of course) are the important variables.

24.   Figures B and C illustrate that, under the commonly adopted parameters cited above, the HPMV NAIRU estimate may be quite different from the standard Kalman filter estimate, but very close to the HP filter of actual unemployment.[8] However, it is also possible to see that the HPMV departs from the HP filter at specific time periods. For the United States, this is especially true at the beginning of the eighties. For France, the difference between both the HP and the HPMV filter, and the HPMV and the Kalman filter is more striking between 1980 and 1987.

25.   An interpretation of the differences in Kalman filter and HPMV results is related to the weight given to explanation relying on inflationary developments. For the United States, restrictive monetary policy implied a fall in inflation of 10 percentage points between 1980 and 1983. This is typical of a restrictive demand policy that might not normally affect the NAIRU, but rather opens the unemployment gap. For France, such an interpretation is more difficult as, at the same time as a restrictive monetary policy was implemented, (resulting in a fall of the CPI growth rate from 12 per cent in 1982 to 2.7 per cent in 1988), a restrictive fiscal policy was also being implemented. Measures on the labour market were also taken, whose effects can be difficult to disentangle: increase in minimum wage, reduction of the workweek hours, but also wage freezes. Hence, the NAIRU must have risen, but possibly more slowly than the HPMV filter suggests.[9]

26.   These differences are the direct product of the underlying assumptions of the two approaches. The standard Kalman filter estimate depends heavily on the fit of the economic relationship, i.e., on the importance given to the NAIRU explanatory power of inflation evolution. Hence, the relative strength of this approach depends on the degree of confidence in the economic relationship that is adopted. By contrast, the HPMV estimate is largely an HP filter estimate, marginally amended by the use of the economic relationship.

27.   The hypothesis implicit in the HPMV filter that the unemployment gap is white noise prevents the estimated variable from wandering away from the observed variable, even if there are good economic reasons for doing so. More intuitively, the NAIRU is anchored on the observed unemployment rate. This hypothesis may be tested, within the more general framework provided by the Kalman filter, by using alternative assumptions, for example that the gap is persistent and might be better represented as an auto regressive process, as follows.[10]

$$y_t - y{*}_t = \psi(y_{t-1} - y{*}_{t-1}) + \varepsilon_t \quad \text{with } \varepsilon_t \sim N(0, H_3) \text{ and } 0 < \psi < 1 \qquad [10]$$

Such an alternative has been examined for both countries, under equivalent specifications (to the HPMV filter) for the variances.[11] The overall results (represented in Figure D) suggest that modelling the gap as an

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8.   This result is all the more striking that the specification applied here increases the weight given to the Phillips curve by a factor of 16 compare to the standard HPMV, where the smoothing constant $\lambda_2 = 1$.

9.   Further, a rising NAIRU might also reflect (partial) hysteresis phenomenon, whereby the level of the unemployment rate reflects (at least partially) the cumulative effect of all past shocks to the economy, including those to demand.

10. This assumption for the gap was tested by Laubach (1999) for the G7 and found to improve the estimated NAIRU for a number of countries.

11. See technical Annex B for a computation of equivalent variances for the residuals under this new gap specification

auto regressive process may make more sense conceptually, but does not yield any significant differences in the estimated variables.[12]

28.   The above exercise suggests that imposing a restriction on the gap (whether white noise or auto regressive), in other words imposing an anchor on the unobserved variable, is most important in explaining the differences between the HPMV and the Kalman filters. The choice between the two techniques for estimating an unobserved variable is then made according the economic priors of the researcher, depending on whether the unobserved variable is believed to be tightly linked to the observed variable, or whether economic shocks should allow a gap to open for persistent periods.

3.   *Estimating standard errors for the unobserved variables*

29.   An important feature of the Kalman filter is that it uses a maximum likelihood estimation method, which also provides estimates of standard errors for the unobserved variables. The size of these standard errors can be shown to be directly related to the variance Q, which controls the sample volatility of the NAIRU, and the variance H, which reflects the uncertainty linked to the model itself. In other words, the size of the standard errors is the joint product of the magnitude of the variances, and the identification restrictions. The following paragraphs construct a simulation example, which illustrates the influence of these assumptions on the standard errors.

30.   Formally, the standard errors of the unobserved variable are given by the square root of the time-varying covariance matrix of the estimation error, P_t, which may be expressed as a function of H and Q [see Technical Annex C for the algebra]. [13]

$$P_t = P_{t-} + Q - \frac{\beta \ [P_{t-} + Q]}{\beta \ [P_{t-} + Q] + H}$$

where β is the coefficient associated with the unobserved variable, for example the parameter of the unemployment gap in the Phillips curve relationship.

31.   To assess the influence of the respective variances Q and H, the above expression can be simulated for given values of P_0. In the following example, the number of observations is assumed to be N = 50, which corresponds to around 25 years of semi-annual data, β takes a value 0.2 (within the range found for the Phillips curve for the United States and France), and 3 initial values for P_0 were tested. Table 2 below shows the corresponding results (averaging the standard errors over the different initial values):

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12 . Some of the cyclical swings in the HPMV NAIRU are somewhat smoothed with an autoregressive gap, but the point difference is small.

13.   The implicit assumption of H and Q being scalars is made here, for ease of exposition. This does not affect the results.

Table 2. **Influence of Q and H on the size of the average standard errors**

Q \ H	0.0025	0.1	0.5	1
1/256	0.13 (0.64)	0.17 (25.6)	0.29 (128)	0.3 (256)
1/16	0.29 (0.04)	0.39 (1.6)	0.86 (8)	0.92 (16)
¼	0.42 (0.01)	0.58 (0.4)	1.22 (2)	1.42 (4)
½	0.49 (0.005)	0.68 (0.2)	1.47 (1)	1.73 (2)
1	0.55 (0.0025)	0.78 (0.1)	1.75 (0.5)	2.1 (1)

Note: the figures between brackets represent the ratio Q/H.

32.   The ratio Q/H may be thought of as one of the smoothing constant in the HPMV filter, or more directly the signal to noise ratio of the standard Kalman filter.[14] Common smoothing constants for the HPMV filter use values for H and Q which are fixed close to the first column of Table 2, whereas typical values for H and Q in the standard Kalman filter are values closer to the bottom of the second column. The shaded area highlights values of both the standard errors and the smoothing constant that would appear "reasonable". Table 2 emphasises that the common values for Q/H in the standard Kalman filter tend to generate unobserved variables with associated standard errors that are higher than those given by the standard smoothing constants in the HPMV filter.[15] It also illustrates the existing link between the size of the smoothing constants and the magnitude of the standard errors. The more volatile the unobserved variable (i.e., the higher Q/H), the lower the estimated standard errors.

33.   More generally, the standard errors produced by any filter embody the assumptions that all the restrictions on the variances are correct and the model is known with certainty. However, any uncertainty attached to the model affects the estimated unobserved variable and its margin of errors. When the variances are estimated together with the parameters of the measurement equation, as in the standard Kalman filter, a Monte Carlo analysis may be undertaken to take into account the increment in uncertainty due to parameters (coefficients and variances) estimation (Hamilton 1986, 1994). Producing such standard errors with an HPMV filter is possible when it is rewritten as a state space model.

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14. It is not exactly the HPMV smoothing constant as the HPMV filter requires a multivariate state space model, whereas the one assumed here is, like in the standard Kalman filter, univariate. This does not affect the intuition being given.

15. Simulations to assess the influence of the measurement equations under the two filters on the standard errors were run. These consisted of choosing equivalent smoothing restrictions under both methods. The size of the resulting standard errors appear similar with the two filters, under a range of identical smoothing restrictions. This exercise suggests that the differences in the size of the standard errors with the alternative approaches is the result of different assumptions on the smoothing constants. Results are available from the author.

4. Conclusion

34.   This paper explains how to rewrite the HPMV filter in a state space form, which can be estimated with maximum likelihood techniques, by means of the Kalman filter. This has several implications. Firstly, it provides a better framework for understanding the mechanics of each filter and the role of the underlying specific assumptions. Their impact on the estimated variables may be assessed, with respect to other models estimated with the Kalman filter. An empirical application to the estimation of the NAIRU for the United States and France provided a simple illustration of these points. Two important issues were emphasised. Firstly, the importance of the underlying relationship, whether economic (such as the Phillips curve) or statistical (such as in the HPMV). Although the latter appears "ad hoc", it could seem useful when the underlying economic relationship is not very robust. However, a better alternative is probably to use another economic relationships. For example, an Okun's law has often been used as a complement of the Phillips curve to estimate the output gap (as in Smets, 1999, Apel and Janssen, 1999, Kichian, 1999).

35.   Secondly, these examples also provide a basis for an assessment of the impact of the "standard" restrictions on the smoothness of the unobserved variable on the uncertainty surrounding the estimates. The magnitude of the standard errors was shown to vary with the size of the smoothness constant(s). An important remaining issue is, therefore, to provide a rule to fix the choice of these smoothness parameters, whenever they cannot be estimated. Here again, a road for further research might be to combine several economic relationships: it is likely that a very narrow range (or a unique value) for the signal-to-noise ratio provides an unobserved variable that maximises the goodness-of-fit of several economic relationships.

TECHNICAL ANNEX

A.   Using the Kalman filter to estimate a time-varying NAIRU

36.   The Kalman filter is a convenient way of working out the likelihood function for complex models that may be re-written in a state space form. That is a system composed of measurement equations explaining observed variables and transition equations governing the evolution of state variables [Cuthbertson, Hall and Taylor (1992), Harvey (1992) and Hamilton (1994) are standard references].

37.   The state space representation as used in the text is made of :

A measurement equation :

$$Y_t = zA_t + d.X_t + e_t \qquad\qquad [A1]$$

with the same notations and

A transition equation :

$$A_t = TA_{t-} + v_t \qquad\qquad [A2]$$

where e_t and v_t are iid, normally distributed with a mean zero and variances H and Q respectively. The ratio Q_t/H is called the signal-to-noise ratio. T = [1] in the case of a random walk, $\begin{pmatrix} \varphi & 1 \\ 0 & 1 \end{pmatrix}$ for an auto regressive variable, with $\varphi = 1$ for a random walk with drift, and between 0 and 1 for an AR(2) process.

38.   The Kalman filter is a tool enabling one to estimate the parameters of such a model as well as the state variable A_t using maximum likelihood. It is made of two stages: first a filtering procedure, second a smoothing procedure. The filtering procedure builds up the estimates as new information about the observed variable becomes available. The smoothing procedure allows to smooth the estimate, taking the information available from the whole sample of observation. These two steps are described below.

1. Filtering

1.1 Prediction equations:

39. Let us call a_t the optimal estimate of the state variable A_t and P_t the optimal estimate of the associated variance/covariance matrix of errors attached to this estimate. Then, given a_{t-1} and P_{t-1}, the prediction equations are:[16]

$$a_{t|t-1} = Ta_{t-1} \qquad\qquad\qquad [A3]$$

and

$$P_{t|t-1} = TP_{t-1}T + Q_t \text{ ' for all t.} \qquad\qquad\qquad [A4]$$

1.2 Updating equations

40. The Kalman filter stems from recursive methods. Once a new observation becomes available, it is possible to update the prediction equations according to:

$$a_t = a_{t|t-1} + P_{t|t-1}ZF_t^{-1}(y_t - Za_{t|t-1} - R.D_t) \qquad\qquad\qquad [A5]$$

and $P_t = P_{t|t-1} - ZF_t^{-1}ZP_{t|t-1}$ [A6]

where $F_t = ZP_{t|t-1}Z + H_t$ for all t = 1,...,T.   [A7]

The variance/covariance matrices H and Q, and the state variable are called the hyper-parameters. The variable a_t is the Minimum Mean Square Estimate of the state variable. From the variance/covariance matrix P_t, one can derive standard errors for it.

41. These equations allow to compute the **prediction errors** v_t for period t as:

$$v_t = y_t - Za_{t|t-1} - d.X_t \qquad\qquad\qquad [A8]$$

The prediction errors are then included into the likelihood function:

$$l_t = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log|F_t| - \frac{1}{2}v_t F_t^{-1}v_t \qquad\qquad\qquad [A9]$$

which can be maximised with the usual algorithms, for different computations of the prediction errors, and t = 1,...T. This gives the optimal series of the state variable $\{A_t\}$.

---

16. The initial values for a_0 and P_0 are important for the optimisation process to converge. The starting values may cause real trouble if the user of the Kalman filter has no prior information about it: as with all maximisation procedure, if the starting values are too far away from the true values the system will not converge. There is no standard or theoretical procedure to overcome this problem. When it is possible, a practical solution is to realise an OLS estimation first that will give an idea about the value of the parameter in the vector A. Yet, this does not help with the initial value for the variance/covariance matrix. The usual "trick" is to give this matrix an extremely high value so as to go away from the initial values of the parameters very quickly.

2.   *Smoothing*

42.   The aim of filtering is to produce an estimate of the state variable at time t, taking into account the information available at this time. More formally, filtering produces the estimate $E(A_t | y_t)$. The aim of smoothing is to take account of all the information available after time t, i.e., over the whole sample. Hence the smoothed estimate of a_t is:

$$a_{t|T} = \underset{T}{E}(A_t) = E(A_t | Y_T)$$ [A10]

and the covariance matrix of A_t conditional on all T observations is then given by :

$$P_{t|T} = \underset{T}{E}\left[(A_t - a_{t|T})(A_t - a_{t|T})'\right]$$ [A11]

43.   More explicitly, the smoothing procedure is a backward recursion which starts at time T and produces the smoothed estimates in the order T,...,1, following the equations :

$$a_{t|T} = a_t + P_t^*(a_{t+1|T} - T_{t+1}a_t)$$ A12]

$$P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*'}$$ [A13]

$$P_t^* = P_t T_{t+1}' P_{t+1|t}^{-1}$$ [A14]

with $a_{T|T} = a_T$ and $P_{T|T} = P_T$.

B.   Link between auto regressive and random walk representations for the variances

44.   Consider the random walk process:

$$x_t = x_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2)$$ [B1]

It is easily seen that:

$$Var(x_t - x_{t-1}) = Var(\varepsilon_t) = \sigma^2$$ [B2]

Now consider the auto regressive process:

$$x_t - x_{t-1} = \varphi(x_{t-1} - x_{t-2}) + v_t \text{ with } v_t \sim N(0, \sigma_{ar}^2)$$ [B3]

To retrieve the above expression, B3 needs to be rewritten so that:

$$x_t - x_{t-1} - \varphi(x_{t-1} - x_{t-2}) = v_t$$ [B4]

then the variance of the error terms is given by:

$$Var[1 - \varphi L(x_t - x_{t-1})] = Var(v_t)$$ [B5]

that is:

$$Var(v_t) = (1 + \varphi^2).Var(\varepsilon_t) = (1 + \varphi^2).\sigma^2$$ [B6]

C.   About the standard errors

45.   An easy way of getting an intuition of the impact of Q and H on the standard errors is to use the equations describing the evolution of the variance/covariance matrix of the errors:

$$P_t = P_{t|t-1} - P_{t|t-1} Z F_t^{-1} Z P_{t|t-1}$$ [A6]

where $F_t = Z P_{t|t-1} Z + H_t$ for all t = 1,...,T. [A7]

46.   For ease of computation (which does not affect the intuition), assuming one measurement error only, a random walk specification for the unobserved variable (T = [1]) and Z = β permits to rewrite equations [A.6] and [A.7] as:

$$P_t = P_{t|t-1} - \beta^2 P_{t|t-1} F_t^{-1} P_{t|t-1}$$ [C1]

$$F_t = \beta^2 P_{t|t-1} + H$$ [C2]

Replacing $P_{t|t-1}$ by its expression in equation [A4] gives:

$$P_{t|t-1} = P_{t-1} + Q$$ [C3]

which may be included in equations [C1] and [C2]

$$P_t = P_{t-1} + Q - \beta^2 (P_{t-1} + Q) F_t^{-1} (P_{t-1} + Q)$$ [C4]

$$F_t = \beta^2 (P_{t-1} + Q) + H$$ [C5]

Combining the two equations [C4] and [C5] gives P_t as a function of its past values and Q and H:[17]

$$P_t = P_{t-1} + Q - \frac{\beta^2 [P_{t-1} + Q]^2}{\beta^2 [P_{t-1} + Q] + H}$$

Then, for a given value of P_0 and the parameter β, it is possible to assess the impact of Q and H on P. Initial values for P_0 were chosen within the range of "observed" unemployment rate in the 1970s for the United States and France. In this case, the parameter β took a value close to the estimated β coefficient in the Phillips curve for these two countries.

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17.   The implicit assumption of H and Q being scalars is made here, for ease of exposition. This does not affect the intuition.

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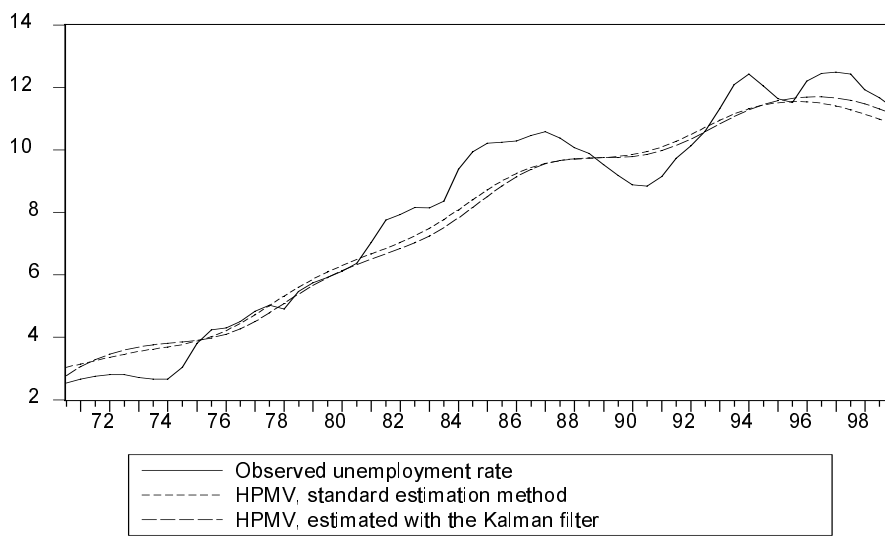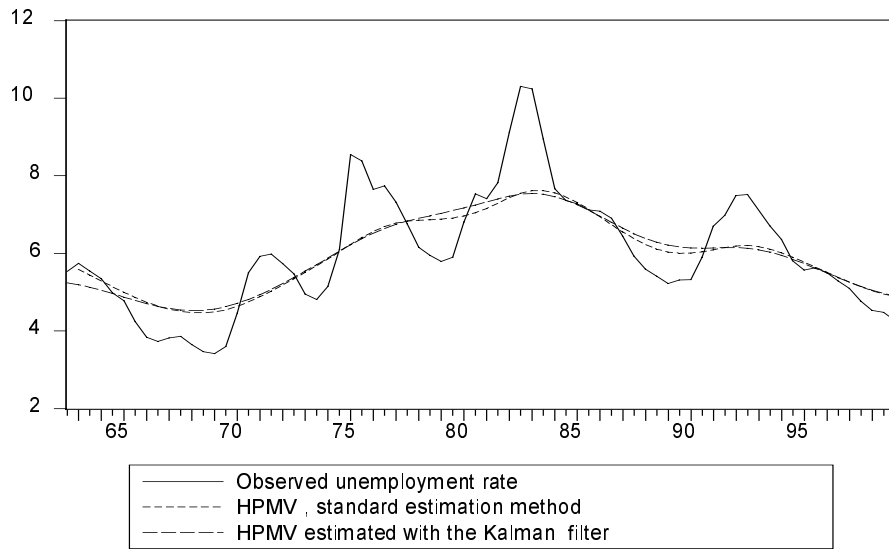Figure A. **HPMV estimates for France and the United States under the two approaches**

Observed unemployment rate
HPMV , standard estimation method
HPMV estimated with the Kalman filter

Observed unemployment rate
HPMV, standard estimation method
HPMV, estimated with the Kalman filter

Figure B. **Estimation under the HPMV and the Kalman filter for the United States**

Observed unemployment rate
HP
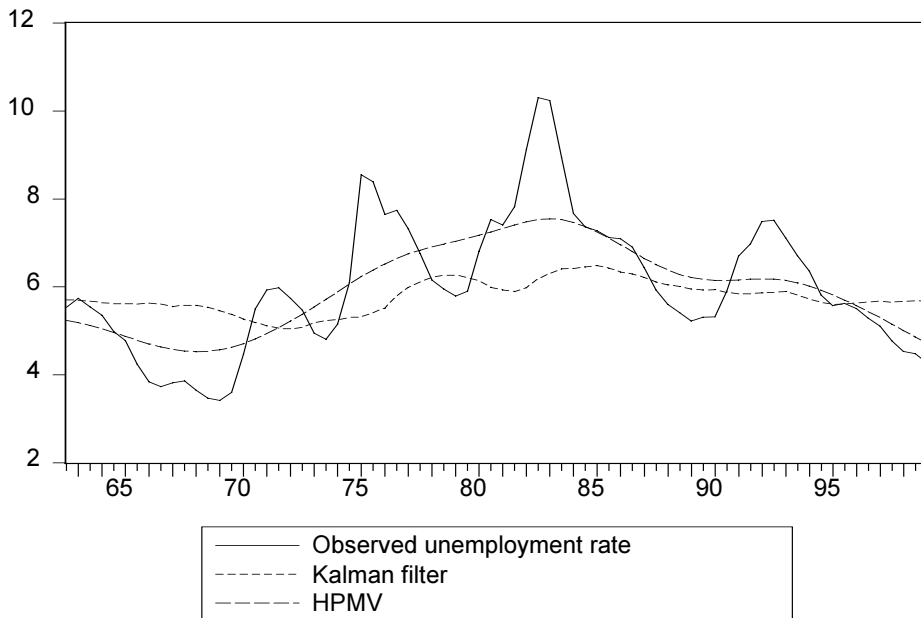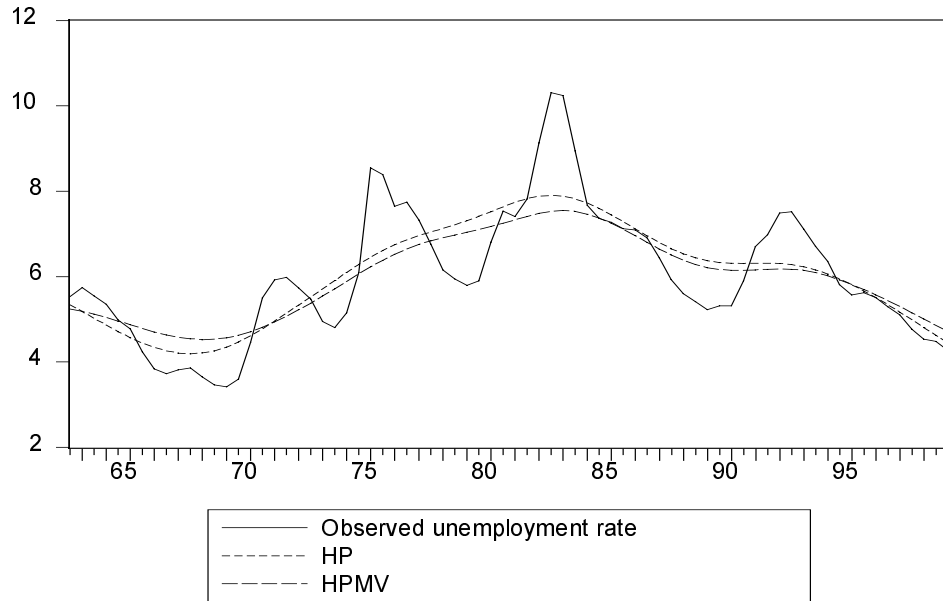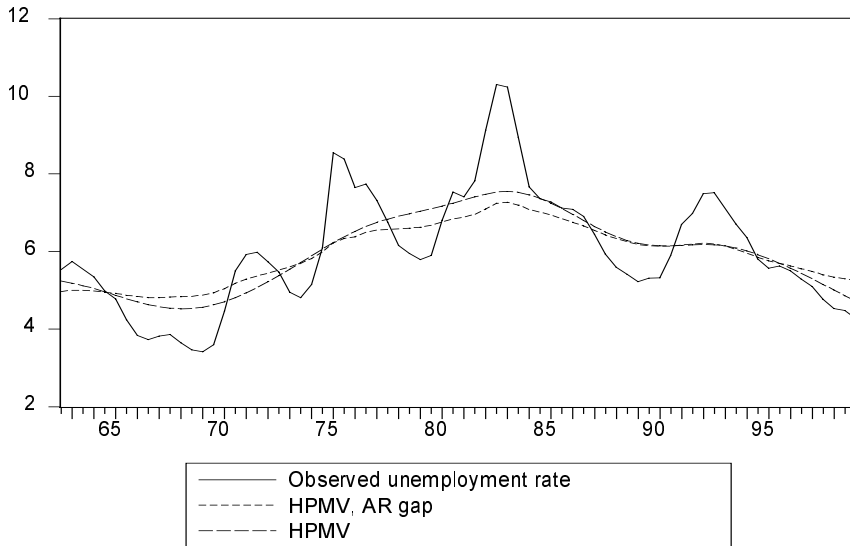HPMV

Observed unemployment rate
Kalman filter
HPMV

Figure C. **Estimation under the HPMV and the Kalman filter for France**

Figure D. Comparison of the HPMV and HPMV with auto regressive gap estimate for the United States and France

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