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Trends and Cycles
in Labour Productivity
in the Major OECD
Countries

**Giuseppe Nicoletti,
Lucrezia Reichlin**

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TRENDS AND CYCLES IN LABOUR PRODUCTIVITY IN THE MAJOR OECD COUNTRIES

This paper uses a multivariate generalisation of the Beveridge and Nelson methodology to model trends and cycles of business-sector labour productivity in the major OECD countries. The method implies that the trend is the long-term forecast of productivity, given all available information; the cycle is thus interpreted as the total excess growth that one would forecast beyond "normal" rates of productivity (see Evans and Reichlin, 1992). Multivariate trends in productivity were estimated including series that Granger-cause and, possibly, are cointegrated with productivity. The corresponding cycles were compared with those generated by the Hodrick-Prescott filter and with the business-cycle dating of the OECD. The stability and predictive properties of the Beveridge-Nelson and Hodrick-Prescott trends were compared. Finally, the estimated productivity gaps were used as proxies for capacity utilisation in econometric models of price formation in order to assess their empirical content. The sample period considered is 1960 to 1991 and data are quarterly.

* * *

Cette étude utilise une généralisation au cas multivarié de la méthodologie proposée par Beveridge et Nelson pour modéliser les tendances de la productivité du travail dans le secteur des entreprises des principaux pays de l'OCDE. Selon cette méthode, la tendance est la prévision de long terme de la productivité, étant donné l'information disponible à chaque période. Il s'ensuit que le cycle est interprété comme la somme des taux de croissance attendus de la productivité qui excèdent le taux de croissance "normal" (Evans et Reichlin, 1992). Les tendances multivariées de la productivité ont été estimées en incluant dans le modèle des séries qui causent la productivité, au sens de Granger, et qui sont, si possible, co-intégrées avec celle-ci. Les cycles correspondants ont été comparés à ceux estimés selon la méthodologie de Hodrick et Prescott ainsi qu'à la périodisation des cycles proposée par l'OCDE. Une comparaison a été aussi effectuée entre la stabilité et les propriétés de prévision des tendances estimées selon les méthodes de Beveridge et Nelson et de Hodrick et Prescott. Enfin, les écarts entre la productivité du travail et sa tendance estimée ont été inclus comme variables explicatives dans des modèles de formation des prix pour donner une approximation des effets du taux d'utilisation de la capacité de production. Par ce biais, on a essayé d'évaluer la validité empirique des tendances estimées. Les séries utilisées sont trimestrielles et, pour la plupart des pays, la période considérée est celle de 1960 à 1991.

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TRENDS AND CYCLES IN LABOUR PRODUCTIVITY IN THE MAJOR OECD COUNTRIES

Giuseppe Nicoletti and Lucrezia Reichlin¹

Introduction

The decomposition of labour productivity into cyclical and trend components has important implications for macroeconomic analysis. Historical decompositions allow the dating of business cycle peaks and troughs, while "real time" decompositions make it possible to judge the current phase of the cycle, increasing the reliability of economic predictions. More generally, the distinction between transitory and permanent changes in productivity is useful when judging the success of structural reform programmes or assessing the sustainability of current productivity levels. The measurement of trend productivity and output can be used to compute "gaps", which contribute to the understanding of the fiscal policy stance and, when interpreted as deviations from potential, are expected to determine many important macroeconomic variables, such as wage and price inflation, thus providing an important input for the setting of monetary policy.

Attempts to define labour productivity trends for several OECD countries have been mostly based on univariate detrending methods, such as regressions of labour productivity on time or exponential-smoothing procedures. Regressions on time are based on the assumption that trends are deterministic. However, this assumption does not account for the possibility of permanent stochastic shocks to labour productivity, a feature which is at variance with modern theories of economic growth. Exponential smoothing procedures -- such as the popular Hodrick-Prescott filter -- may generate spurious cyclical behaviour by overlooking the statistical properties of the series (Cogley and Nason (1992)). In addition, these procedures are based on a very limited information set including only current and past values of labour productivity and are typically backward-looking at end-of-sample, implying unreliable trend estimates of the most recent period.

In this paper a multivariate version of the methodology by Beveridge and Nelson (1981) is used in order to model business-sector labour productivity trends in the Major OECD countries. The multivariate Beveridge and Nelson (henceforth B-N) methodology has been recently used by Evans (1989), Cochrane (1990), King *et al.* (1991) and Evans and Reichlin (1992) to model the trend of GDP in the United States. In the B-N approach, the trend is defined as the long-term forecast of the level of productivity adjusted for its mean growth. Since the change in trend in each period is the innovation in this forecast, the B-N trend is not a known deterministic function of time but rather a stochastic component of the series that results from the cumulation of past permanent shocks. Evans and Reichlin (1992) showed that the importance of the cyclical component resulting from the B-N methodology depends on the information set used to forecast productivity. Therefore, it is crucial to estimate the trend of productivity within the framework of a multivariate model.

The multivariate B-N model exploits two sources of information ignored by univariate procedures. First, the estimation of trends is based on the simultaneous observation of productivity and other series that partially account for its cyclical movements. Second, it is possible to use information on long-run empirical regularities, such as the stability of the labour share or the average propensity to consume, which link labour productivity to other economic series. In so doing, the multivariate

procedure enlarges the information set on which trend computations are based, making the resulting trend/cycle decomposition more reliable. In the paper a wide range of variables displaying a cyclical and/or long-run relationship with labour productivity was considered and the estimated models included all those variables which had explanatory power.

It is important to note, however, that the B-N definition of the trend as the long-term forecast of the series prevents any structural interpretation of the shocks affecting labour productivity. Therefore, estimated changes in trend cannot be attributed to structural shifts in supply or demand conditions. Similarly, as in other common de-trending procedures, such as linear regressions on time or exponential smoothing, correlations between the estimated trend and cycle in labour productivity cannot shed light on the effects of cyclical developments on long-run productivity growth, an issue which has been recently addressed by Gali and Hammour (1991), Aghion and Saint-Paul (1991) and Caballero and Hammour (1992)².

The paper has two main sections. Section 1 briefly describes the characteristics of the multivariate B-N procedure. The differences between deterministic and stochastic trends are clarified, the definition of the B-N trend as the long-run forecast of the series is made more precise and the advantages, the specification and the estimation of the multivariate model are explained in a non-technical fashion. Technical details can be found in Annex 1. Readers interested in the results alone can skip to Section 2 which reports the results of the empirical investigation. The empirical strategy is the following. The univariate features of a broad range of macroeconomic series are explored and Granger-causality and pair-wise cointegration tests with respect to labour productivity are performed. The series selected according to these criteria are then used to estimate a VAR (possibly including the cointegration restrictions) which is used to compute the multivariate trends and cycles. The estimated cycles are compared with the cycles generated by the Hodrick-Prescott filter and with the periodisation of peaks and troughs of the OECD. The stability properties and predictive performances of the multivariate and Hodrick-Prescott trends are also compared. Finally, the productivity gaps deriving from the multivariate trend estimates are used as proxies for capacity utilisation effects in an econometric model of price formation in order to assess their empirical content. A few concluding remarks close the paper.

1. Trends and cycles

1.1. Deterministic and stochastic trends

It is a well-known empirical regularity that the mean of labour productivity and of many other economic series tends to shift permanently over time while, at the same time, labour productivity is affected by shocks that tend to offset each other in a finite number of periods. These series are said to be non-stationary, since their mean is not constant over time³. Permanent shifts in the series are generally attributed to long-term factors -- such as technical progress -- and their transitory movements to cyclical influences. Accordingly, in each period t , the realisation of a non-stationary series y_t can be decomposed into trend (τ_t) and cycle (c_t):

$$y_t = \tau_t + c_t \quad (1)$$

where c_t is a stationary component⁴. Since the cyclical component is stationary, its dynamics eventually die out over time. Therefore, a shock affecting the cyclical component has no long-run effect on the level of y_t .

Various assumptions can be made about the trend component τ_t . The simplest representation of the trend is a deterministic function of time:

$$\tau_t = f(t) \quad (2)$$

In this case, non-stationarity is entirely captured by the deterministic trend, whereas the cycle represents stationary movements around it. For this reason, the corresponding representation of the series y_t is called trend-stationary (TS). Clearly, if the TS model is correct, in each period the change in trend is also a known function of time and the trend can be predicted perfectly on the basis of past observations⁵. Moreover, while the mean of y_t is trended, its variance is not.

The main characteristic of the TS model is that it does not account for the possibility of permanent *stochastic* shocks to the series, ruling out any non-stationarity in the variance. Yet the notion of variable and persistent shocks corresponds to a widely accepted view of the process of economic growth. At the firm level, labour productivity is expected to be permanently affected by supply shocks, such as technological innovations and changes in human capital. In the aggregate, these changes can be represented as random shocks occurring in each period and having a long-term effect on the level of labour productivity.

In the presence of these shocks, the TS model is clearly inappropriate. Since it defines as cyclical all deviations of growth rates from their mean, this model would wrongly attribute the permanent stochastic shocks to the cycle. This, in turn, has two undesirable implications. First, the cycle resulting from a regression of the series against time, would have spurious cyclical characteristics (Nelson and Kang, 1981). Second, the forecast of the trend, which would be simply given by the extrapolation of a linear regression over time, will diverge from the series as time elapses and its standard error would increase with the forecasting horizon.

In view of these shortcomings, an alternative model of the trend that captures stochastic non-stationarity is used in this paper. The labour productivity series is assumed to be permanently affected by stochastic shocks, whose cumulation over time determines its trend. Such a series is said to be "integrated" because its value at each point in time depends on the cumulated sum of past realisations. Moreover, changes in productivity are assumed to be stationary, ruling out trends in the mean growth rate of the series. A non-stationary series whose first differences are stationary is said to be integrated of the first order, or I(1)⁶. An I(1) series contains a non-stationary stochastic component that is usually called its stochastic trend.

Under certain conditions, the series y_t can be represented as the following autoregressive/integrated/moving-average (ARIMA(p,1,q)) model:

$$\Delta y_t = d + \alpha_1 \Delta y_{t-1} + \dots + \alpha_p \Delta y_{t-p} + u_t + \beta_1 u_{t-1} + \dots + \beta_q u_{t-q} \quad (3)$$

where u_t is a white noise and p and q are the orders of the autoregressive and moving average components, respectively. In order to decompose the series y_t into trend and cycle it is useful to start from the moving-average representation of (3) (see Annex 1):

$$\Delta y_t = \gamma + a(L)u_t \quad (4)$$

where L is the lag operator such that $L^i y_t = y_{t-i}$, and $a(L)$ is the following lag polynomial:

$$a(L) = 1 + a_1L + a_2L^2 + \dots = \sum_{i=0}^{\infty} a_iL^i \quad (5)$$

Given (4), the stochastic trend τ_t and the cyclical component c_t of y_t can be specified in several different ways. Beveridge and Nelson (1981) proposed the following decomposition:

$$\Delta y_t = \gamma + a(1)u_t + \Delta \bar{a}(L)u_t = \Delta \tau_t + \Delta c_t \quad (6)$$

where $\bar{a}(L)$ is defined by $\Delta \bar{a}(L) = a(L) - a(1)$, with $a(1) = \sum_{i=0}^{\infty} a_i$.

In equation (6) the trend is defined as the random-walk component of the series:

$$\tau_t = \gamma + \tau_{t-1} + a(1)u_t \quad (7)$$

and the cyclical component is:

$$c_t = \bar{a}(L)u_t \quad (8)$$

The variance of the stochastic trend is determined by the size of the coefficient $a(1)$, which measures the long-run impact of a shock u_t on the level of y_t . If $a(1) = 0$, the model collapses to the deterministic case, where the shock u_t has only a temporary effect and no stochastic trend is present in the series. However, if $a(1) \neq 0$, the properties of the model are radically different from the deterministic case: i) each shock u_t has a permanent effect on the level of the series; ii) the first difference of the trend is a stochastic process, whose variance is determined by the variance of u_t ; and iii) the series y_t has not only a trend in mean but also a trend in variance⁷.

Summarising, in the stochastic trend model the first difference of labour productivity is a stationary random process around a mean rate of increase (the "drift"). At each moment in time, the series is hit by a shock which affects its trend and, therefore, has a permanent effect on the series itself. While in the TS model the trend increases each period by a fixed amount, a stochastic trend increases each period by some fixed amount on average, but deviates from average by a random amount. If the trend is defined to be the random walk component of the series, as in (6), this amount is unpredictable and the best forecast of the series next year is just its current level plus its mean growth rate⁸.

1.2 Univariate and multivariate Beveridge and Nelson (B-N) trends

In this paper a multivariate methodology for estimating the stochastic trend in labour productivity is used. The methodology is an extension of the univariate approach of Beveridge and Nelson (1981)⁹. Given an ARIMA specification for the series y_t , B-N showed that the stochastic trend defined in equation (7) can be interpreted as the long-term forecast of the level of the series (adjusted for its mean growth):

$$\tau_t = \lim_{k \rightarrow \infty} (E[y_{t+k} | I_t] - dk) \equiv \lim_{k \rightarrow \infty} (E y_{t+k} - dk) \quad (9)$$

where E is the expectation operator and I_t is the information set containing present and past values of y_t .

The forecast in (9) is *the level that y_t will attain after all transitory dynamics work themselves out*. In other words, at each point in time the trend is defined as the sum of the series y_t and all expected future changes in y_t . Intuitively, if y_t is expected to grow in the future, then its current value must be below trend, while if it is expected to decline it must be above trend. Therefore, the trend is defined as the "normal" level of y_t and, at each point in time, the cycle is defined as the difference between the series and this normal level, i.e. it is the negative of the sum of all expected future changes in y_t (in excess of the mean rate of growth d):

$$c_t = y_t - \tau_t = -\sum_{i=1}^{\infty} (E_t \Delta y_{t+i} - d) \quad (10)$$

Since the trend is the long-run forecast of the series given current and past information, in each period the change in trend corresponds to the revision of this forecast implied by the observation of the current shock u_t :

$$\Delta \tau_t = d + \lim_{k \rightarrow \infty} (E_t y_{t+k} - E_{t-1} y_{t+k}) = d + a(1)u_t \quad (11)$$

This revision depends crucially on the size of the non-stationary stochastic component of the series, which is modelled as a random walk -- as in equation (6) above -- and whose size is a function of $a(1)$, the degree of persistence of the shock u_t .

The attractiveness of the B-N approach is that, once the ARIMA model for the series is estimated, the trend can be defined as the optimal long-term forecast generated by the model and the cycle as the gap between the series and this forecast. In this forecasting interpretation, no further assumptions are needed in order to identify the trend, which is unambiguously defined¹⁰.

Given this forecasting interpretation, it is clear that the properties of the estimated labour productivity cycle will depend on the amount of information on which the forecasts are based¹¹. As more (relevant) information becomes available, it is more likely that the fraction of the variance of labour productivity explained by cyclical influences will be larger¹². In particular, there is no reason to limit the information set to current and past values of labour productivity. Since labour productivity moves along the cycle together with other economic series (such as unemployment, for example), useful information on the cyclical properties of labour productivity can potentially be extracted from these series. Similarly, since labour productivity is expected to bear long-run relationships with other series (such as real wages), the constraints implied by the existence of stable long-run productivity ratios should be used as *a priori* information in estimating the long-run forecast. For these reasons, this paper estimates labour productivity trends using the multivariate B-N approach of Evans (1989), Cochrane (1990), King *et al.* (1991) and Evans and Reichlin (1992).

The main difference between the univariate and multivariate approaches is that the trend/cycle decomposition is performed using a vector of series including labour productivity and other stationary (and/or non-stationary) series statistically related to it. Therefore, the long-run forecast of labour productivity -- which is the definition of the trend in the B-N methodology -- is based on information stemming from the current and past values of all the series included in the system. In other words,

while in the univariate model the innovation u_t was defined with respect to an information set containing only the history of y_t , in the multivariate model the relevant innovation for y_t is defined with respect to a larger information set containing the history of all the variables used in the model. In addition, any long-run constraint linking productivity to other series in the system is also used in order to improve the efficiency of the estimates.

The multivariate generalisation of the moving-average representation (4) is:

$$W_t = D + A(L)u_t \quad (12)$$

where W_t is a $p \times 1$ vector of series, which includes first differences of y_t and other (difference) stationary series related to y_t , D is a $p \times 1$ vector of constants, A is a $p \times p$ matrix of lag polynomials and u_t is now a $p \times 1$ vector of white noise shocks. The conditions under which (12) is uniquely identified are discussed in the Annex. Reinterpreting $a(L)$ as the row of the lag polynomial matrix $A(L)$ corresponding to y_t , the moving-average representation (11) leads to the same trend/cycle decomposition as in (5)¹³. Since u_t is now a vector of shocks driving each of the variables in W_t , in each period the change in the stochastic trend of y_t , $\Delta\tau_t = \gamma + a(1)u_t$, depends on a linear combination of the permanent components of all the shocks affecting the system. Similarly, reinterpreting the information set I_t as the set containing the current and past values of all the series in W_t , the definitions of the multivariate B-N trend and cycle are given by equations (8) and (9) above.

From an empirical point of view, the computation of multivariate trends involves three steps. First, the multivariate approach requires the estimation of a VAR model in the variables W_t . The infinite order VAR system corresponding to (12) is obtained by inversion of the matrix $A(L)$ of the moving average representation. In empirical applications, a finite-order approximation of the VAR is estimated by OLS¹⁴. Second, the estimated model is used to compute in each period the conditional forecasts $E(\Delta y_{t+i} | I_t)$ ($i = 1, \dots, K$) and the unconditional forecast $E\Delta y_{t+i}$ (i.e. the constant mean rate of growth of the series)¹⁵. Finally, the cyclical component is computed in each period as in equation (9) summing up the conditional forecasts (net of the unconditional forecast) and the trend is computed as the difference between the series and the cycle. If y_t is related in the long-run with other I(1) series, the relevant restrictions are tested by cointegration analysis and included in the estimated VAR. For example, if an I(1) series x_t were cointegrated with y_t , the multivariate trend of y_t would be the long-run forecast of y_t conditional on the ratio between y_t and x_t being equal to the estimated cointegrating coefficient.

2. Modelling labour productivity trends in the Big 7

2.1 The empirical strategy

The empirical investigation concerned quarterly business sector labour productivity in the Major Seven OECD countries. For each country, the labour productivity series and the other series included in the multivariate trend models were drawn from the Analytical Data Base and the Main Economic Indicators (MEI) of the OECD. Variables were selected among a large set of series, namely *per capita* private consumption, business and housing investment, exports, imports, stockbuilding and the unemployment rate, real wages and short- and long-run interest rates¹⁶. When none of these series yielded satisfactory results, the composite leading indicator series of the MEI database was used. The sample usually spanned the 1960-1992 period, although sample sizes varied from country to country according to the availability of data.

The multivariate B-N methodology incorporates information coming from several other economic series in modelling the stochastic trend of labour productivity. This information is obtained from the estimation of a multivariate model that takes into account the statistical properties of the series. Crucial assumptions are that the labour productivity series is I(1) and that the additional series used to model the trend have explanatory power for labour productivity. Technically, this occurs if the series Granger-cause labour productivity. A stronger requirement is that the series are cointegrated with labour productivity, since cointegration implies Granger-causality.

Model specification involved several steps. First, the univariate statistical properties of the series were explored in order to establish their order of integration. Second, Granger-causality tests were performed in order to select the series potentially informative for labour productivity. Third, pairwise cointegration tests between labour productivity and other I(1) series were performed in order to check if long-run restrictions could be included in the multivariate trend model. Cointegration was considered only when it appeared to have economic meaning, i.e. while *per capita* private consumption and real wages can be expected to be cointegrated with labour productivity in a permanent-income/Cobb-Douglas world, no simple theoretical economic model can explain cointegration between labour productivity and the short-rate.

Given the historical estimates of trends and cycles in labour productivity resulting from the country models, criteria had to be chosen in order to assess the empirical content of the multivariate trend/cycle decompositions. Three such criteria were retained in the paper. The first criterion was to compare labour-productivity cycles resulting from the B-N trend estimates to the cycles resulting from univariate Hodrick-Prescott (H-P) filtering and set both cycles against the periodisation of major cyclical peaks and troughs based on OECD leading indicators (OECD, 1987). Since economists generally grasp cyclical developments better and more quickly than changes in long-run trends, it is natural to start assessing the estimated trend by looking at the resulting labour productivity cycle. The OECD periodisation of peaks and troughs is a useful yardstick since it results from a simple methodology, which is standardised across countries. On the other hand, cycles resulting from the H-P trend computations constitute a relevant term of comparison because of the wide diffusion of the H-P filtering technique, both in the OECD and other major research centres. Details on the H-P technique and on its advantages and drawbacks are given in the Annex.

The second criterion was to assess the ability of both B-N and H-P trends to predict and fit persistent changes in the labour productivity profile. Persistent changes in labour productivity growth have been observed in the 1970s (the so-called "productivity slowdown") and, for some countries, in the 80s. *Ex post*, economists tend to interpret some of these changes as inflexions in trend productivity growth. Full-sample B-N and H-P trends were compared to trends obtained over sub-samples truncated at the point of inflexion in order to check to what extent the computed trends are stable and whether use of the de-trending procedure would have made it possible to anticipate lasting changes in labour productivity growth.

The third criterion was to make sure that B-N trend estimates could be used in estimating behavioral equations. One of the reasons for separating out trend and cyclical components of a series is to obtain proxies for demand pressures which are presumed to drive price inflation in the short-run. Therefore, the performance of B-N productivity-gap estimates in econometric models of price formation was checked.

2.2 Empirical results

Preliminary analysis

Table 1 shows the variables selected for the estimation of the B-N labour productivity trends in each country. It also indicates whether the selected variables were found to Granger-cause labour productivity and whether cointegration restrictions were included in the model. Unit-root tests, performed using the methodology of Perron (1988), indicated that all selected series were I(1). Cointegration analysis, performed according to the methodology of Johansen (1991), supported pair-wise cointegration between private consumption, real wages and labour productivity in all countries except Canada¹⁷. Despite the widespread support for pair-wise cointegration between *per capita* consumption, real wages and labour productivity, the only country in which inclusion of a cointegrating relationship in the model was admissible was the United Kingdom. In the other cases, the inclusion of cointegrating restrictions yielded estimated cyclical profiles with implausible characteristics, such as trends or non-zero means¹⁸. Granger-causality tests were performed on both the levels and first differences of the series. Generally, all the series that were found to Granger-cause productivity were included in the multivariate models. The specifications reported in the Table are the most parsimonious and the most easily interpretable in terms of the implied cyclical profiles of productivity. In most countries private consumption, unemployment and the short-rate were the preferred series. In Canada, the real wage also performed well, while in Japan and Italy only the leading indicator series was retained.

Labour productivity developments and trend/cycle estimates

Table 2 shows basic statistics describing historical developments in labour productivity and how these are explained by the estimates of cycle and trend resulting from both the B-N and H-P methodologies. Statistics are provided for the full sample and for the periods preceding and following the productivity slowdown (for simplicity, the turning point is set in 1974 in all countries). The table also shows the ratio of the variance of the cycle to the variance of the changes in trend -- a synthetic measure of the way in which observed changes in labour productivity are attributed to trend or cyclical factors-- and the correlation between the estimated labour productivity cycle and the corresponding changes in trend.

In most countries both the mean and variance of labour productivity growth have changed substantially after the first oil shock. Mean growth has been reduced by a third in Italy and by half in all other countries except the United Kingdom where the decline was smaller. The variance has declined spectacularly in France, by a third in Japan and Italy and by half in Germany and Canada, while the change was less important in the United States and the United Kingdom.

While both the B-N and H-P trend estimates capture the decline in mean productivity growth over time, the decomposition of the series' variance between cycle and trend -- summarised by the variance ratios -- is very different in the two methodologies. In the H-P case, virtually all the variability in the series is attributed to the cycle, since the trend is smooth by construction. By contrast, the B-N procedure -- in which the trend is defined to be the random-walk component of the series -- does not attribute *a priori* weights to the variance decomposition. While in most countries the variance of the B-N cycle is greater than the variance of the trend, this is not necessarily the case, as exemplified by Japan, Canada and Italy.

Table 1. Selected variables for multivariate labour productivity trends

Country:	Variables:	Cointegration (1):	Granger-causality (2):
United States	Private consumption	No	Yes
	Unemployment	--	Yes
	Short-rate	--	Yes
Japan	Composite leading indicator	--	Yes
Germany	Unemployment	--	Yes
	Short-rate	--	Yes
France	Private consumption	No	Yes
	Short-rate	--	Yes
Italy	Composite leading indicator	--	Yes
United Kingdom	Private consumption	Yes	Yes
	Unemployment	--	Yes
Canada	Private consumption	No	Yes
	Real wage	No	Yes
	Unemployment	--	No

1. Column entries indicate whether a cointegrating relationship between the series and labour productivity was included. Tires indicate that cointegration was not applicable.
2. Column entries indicate whether the series Granger-causes productivity.

Table 2. Basic statistics

Sample Estimation period (2)	United States (1)		Japan (1)		Germany (1)		France (1)		Italy (1)		United Kingdom (1)		Canada (1)								
	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre							
	1959Q1 to 1991Q3	1962Q1 to 1990Q4	1962Q1 to 1990Q4	1962Q1 to 1990Q4	1960Q1 to 1990Q4	1963Q1 to 1991Q4	1960Q1 to 1991Q4	1961Q1 to 1990Q4	1960Q1 to 1991Q4	1961Q1 to 1990Q4	1962Q2 to 1990Q4	1966Q1 to 1991Q4	1966Q1 to 1991Q4	1966Q1 to 1991Q4	1966Q1 to 1991Q4						
	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre	Full	Pre							
Labour productivity growth (3)																					
Mean	1.08	1.64	0.56	4.96	7.24	3.16	3.08	4.08	2.16	3.52	4.76	2.64	3.84	5.64	2.24	2.48	3.04	2.00	1.68	2.28	1.36
Variance	1.07	1.25	0.83	2.50	3.44	1.01	3.58	4.88	2.21	4.54	9.92	0.64	2.53	3.52	1.12	2.24	2.50	1.97	1.66	2.62	1.15
Hodrick-Prescott (3)																					
Trend	Mean	1.16	1.92	0.40	3.36	--	3.12	4.28	2.04	2.84	--	--	3.84	5.72	2.24	2.56	3.20	2.04	1.76	2.72	1.24
Variance	0.13	--	--	0.03	--	--	0.16	--	--	0.03	--	--	0.43	--	--	0.06	--	--	0.08	--	--
Cycle	Variance	1.95	--	--	2.50	--	3.28	--	--	1.26	--	--	4.19	--	--	3.17	--	--	2.61	--	--
Beveridge-Neison (3)																					
Trend	Mean	1.12	1.80	0.52	3.40	--	3.04	4.40	1.84	2.76	--	--	3.32	4.92	2.36	2.60	3.24	2.12	1.72	2.36	1.44
Variance	1.31	--	--	1.44	--	--	1.90	--	--	0.54	--	--	3.44	--	--	1.22	--	--	1.68	--	--
Cycle	Variance	1.47	--	--	0.48	--	3.46	--	--	0.83	--	--	3.09	--	--	7.36	--	--	0.86	--	--
Variance ratios (4)																					
H-P	15.25			78.00		20.50		39.50		9.70		49.50		32.60							
B-N	1.12			0.33		1.82		1.53		0.90		6.05		0.51							

1. Full = full estimation period; Pre = pre-1974 period; Post = post-1974 period.
 2. Sample period used for H-P and B-N estimates.
 3. Mean rate of growth and variance of growth rates (annual rates). The mean is in percentage terms, the variance has been multiplied by 1 000.
 4. Variance of cycle over variance of first-differenced trend.

Beveridge-Nelson cycles, Hodrick-Prescott cycles and the OECD periodisation

Charts 1-7 plot the labour productivity cycles deriving from the B-N (continuous line) and H-P (dashed line) procedures¹⁹. These cycles are compared to the major cyclical peaks (P) and troughs (T) identified by the OECD using the MEI composite leading indicators for the period 1960-1985 (OECD, 1987). The last historical peak was identified in each country using the MEI index of capacity utilisation.

Overall, the B-N procedure yields plausible cyclical profiles. Major upturns and downturns as well as the duration of the cycles correspond to the generally-held priors. Moreover, in most instances peaks and troughs match the OECD-MEI periodisation. Where this does not occur, B-N peaks and troughs either lead or lag slightly the MEI periodisation, with no sign of systematic bias. The most unconventional estimate of the cycle is for Japan, where cyclical variability is estimated to be very low as pointed out by the variance ratio in Table 2. Although the B-N and H-P cycles are generally quite similar, in several instances the B-N procedure yields cyclical profiles that correspond better to received wisdom. Concentrating mainly on the most recent period, notice for instance the following examples:

- In Germany, the B-N cycle captures better than the H-P cycle the overheating of the economy at the end of the 1980s and the unsustainability of the associated productivity gains. While in both procedures the last historical trough leads the MEI periodisation, it is less so in the B-N cycle. On the other hand, the B-N procedure puts in the right perspective the minor recession of 1963, which is largely exaggerated by the H-P cycle.
- In France, the B-N procedure describes better than H-P the period of restrictive policies of the government that followed Mitterand's post-electoral demand boost of 1981. In the B-N cyclical characterisation, the French economy oscillated very slightly around potential throughout the second half of the 1980s and even the last historical peak was quite a modest one when compared to the past.
- In the United Kingdom, the B-N procedure captures extremely well the cyclical nature of the British productivity pick-up during the late 1980s, while the picture offered by the H-P cycle is more confusing.

Stability and fit of the Beveridge-Nelson and Hodrick-Prescott trends

Charts 8a-9a plot labour productivity in the United States and Germany against the full-sample H-P trends and H-P trends computed over sub-samples ending a few periods after a major lasting inflexion in productivity growth had occurred. The truncation periods are the second quarter 1982 for the United States -- a time at which a medium-term pick-up in productivity had started unravelling -- and the second quarter 1980 for Germany -- a period in which the "productivity slowdown" had been going on for some time. The behaviour of labour productivity and the full-sample and sub-sample H-P trends is magnified around the inflexion point in Charts 8b-9b. A similar analysis, truncating the samples in identical periods, is shown for the B-N trends in Charts 10a-11a and 10b-11b. The purpose of the truncation exercise is to answer the following question: "What would an economist using the H-P or, alternatively, the B-N methodology have thought about trend productivity growth some time after an inflexion in trend productivity growth was observed?" Clearly, the issues at stake here are two: Which of the H-P and B-N trend productivity estimates is more stable over time? Is any of the two methodologies superior in predicting lasting changes in productivity on the basis of a few observations?

An examination of the Charts suggests that the B-N procedure performs better on both grounds. The sub-sample H-P trend estimates are both at variance with their full-sample counterparts and misleading as to the future medium-term behaviour of productivity. In the United States, the sub-sample H-P trend predicts a continuing slowdown in labour productivity after 1983. In Germany, the sub-sample H-P trend is unable to capture the slowdown which has occurred in trend productivity at the end of the 1970s. On the contrary, the sub-sample B-N trends closely reflect the lasting changes in productivity occurred in the United States and Germany at the date of truncation. As a consequence, the B-N estimates are also more stable over time.

The superiority of the B-N procedure depends on two related factors. First, the B-N trend is defined to be the long-run forecast of productivity and, given the assumed properties of the series and the available information in each period, this forecast is optimal. In particular, the expectation formation mechanism is forward-looking at each point in time, while the H-P procedure becomes completely backward-looking at the end of the sample. Second, since changes in the B-N trend are defined to be the unpredicted part of current productivity growth, series of innovations having the same sign affect permanently trend productivity growth. In this respect, the B-N approach is quite flexible in approximating in-sample a wide variety of underlying growth processes.

Econometric performance of productivity-gap estimates

Table 3 reports estimates of price equations using the productivity gaps resulting from the B-N and H-P methodologies as proxies for demand pressures. GDP inflation (*inf*) was regressed on its own lags, contemporaneous and lagged growth in labour costs (*inflc*), contemporaneous and lagged productivity gaps (*gap*) and an error-correction term (*ecm*) equating the GDP deflator to unit labour costs in the long-run. Coefficient estimates for the gaps are shaded and a set of regression diagnostics is provided, including adjusted R^2 s (*Adj.R2*), standard errors (*SE*), Lagrange Multiplier tests for autocorrelation (*AR*), autoregressive conditional heteroscedasticity tests (*ARCH*), Chow tests for stability of the regression estimates (*Chow*) and Wald tests for the reduction from the standard model (with two lags of each variable) to the preferred equation estimates reported in the table.

Overall, the econometric performance of the productivity gaps resulting from the B-N trend estimates is satisfactory. The B-N gaps are significant and correctly signed in all equations except the price equation for Germany. On the other hand, the H-P gaps are significant and correctly signed in all equations except the price equation for Canada. Comparison between the two productivity gap estimates suggests that the B-N gaps perform better in Canada; equally well in the United Kingdom, Japan and France; slightly worse in the United States and Italy; and worse in Germany. Given that the H-P estimates display a wider variability by construction, it is not surprising that in a number of cases their significance is higher. It is reassuring, however, that the B-N gaps, which result from the estimation of a multivariate model rather than from *ad hoc* procedures, perform well in behavioral equations concerning a wide spectrum of countries.

3. Conclusions

The multivariate version of the methodology by Beveridge and Nelson (1981) was used to disentangle trend from cycle in business sector labour productivity in the Major Seven OECD countries. The methodology, which is timely and fully data-based, defines the trend as the long-run forecast of the series given currently available information. The resulting estimates were assessed using various empirical criteria. The analysis suggests that the multivariate B-N methodology provides trends that capture slow movements in labour productivity, plausible productivity cycles and productivity gaps that perform well in econometric models of price formation. The B-N trend estimates were shown to

Table 3. Econometric performance of productivity gap estimates
Dependent variable: GDP inflation

Country Period Trend	United States 1960:2 to 1991:3		Japan 1971:4 to 1990:4		Germany 1961:4 to 1990:4		France 1971:4 to 1991:4		Italy 1964:4 to 1991:4		United Kingdom 1962:4 to 1990:4		Canada 1967:2 to 1991:4	
	B-N	H-P	B-N	H-P	B-N	H-P	B-N	H-P	B-N	H-P	B-N	H-P	B-N	H-P
Explanatory variables (1):														
inf(-1)	0.21	0.1	0.26	-0.24	-0.21	-0.24	0.25	0.21	0.33	0.44	0.16	0.15	0.54	0.51
	2.7	1.3	2.6	-2.4	-2.4	-2.8	3	2.5	3.8	5.3			7	5
inf(-2)	0.23	0.18	0.29	-0.09	-0.09	-0.18	-0.19	-0.16			0.16	0.15		0.19
	2.9	2.4	3	1	1	-2.1	-2.2	-1.8			2	1.8		1.8
infic	0.26	0.51	0.2	0.34	0.4	0.4	0.59	0.63	0.24	0.27	0.41	0.39	0.42	0.37
	5.6	7.7	4.3	11.1	12.9	12.9	9.6	10.3	5.5	5.9	7.1	6.3	9.4	7.5
infic(-1)			0.14	0.11	0.11	0.16			-0.1	-0.15	0.17	0.21	-0.15	-0.08
			3	2.8	2.8	4			-2	-2.6	2.7	3	-2.7	-1.5
infic(-2)	0.07	0.07		0.09	0.09	0.09			0.08	0.07	0.08	0.07		-0.1
	1.4	1.5		2.2	2.2	2.4			1.4	1.8	1.4	1.8		-1.8
dgap		0.32					0.32	0.32	0.29	0.32	0.2	0.16		0.2
		5.5					2.6	3.7	4.2	4.7	2.6	2		2.9
dgap(-1)							0.09	0.12	0.21	0.08	0.09	0.17		
							0.8	1.5	2.8	1	1.2	2.1		
gap(-2)			0.2						0.27	0.23			0.24	
			3.3						3.4	4			3.7	
d2gap	0.09		0.42											
	3.4		3.1											
ecm(-1)	-0.06	-0.05		-0.02	-0.02	-0.02	-0.06	-0.05	-0.15	-0.14	-0.04	-0.04	-0.07	-0.04
	-3	-2.3		-1.9	-1.9	-2.8	-3.9	-3.4	-6.9	-5.4	-2	-1.9	-2.9	-1.7
constant	0.002	0.002	0.002	0.009	0.008	0.008	0.03	0.03	0.03	0.02	0.005	0.005	0.005	0.003
	2.9	1.8	1.6	6.5	7.3	7.3	4.3	3.8	6.6	5.8	2.6	2.7	3.6	2.3
Regression statistics (2):														
Adjusted R ²	0.75	0.78	0.76	0.54	0.64	0.64	0.79	0.8	0.79	0.8	0.71	0.71	0.77	0.74
SE%	0.33	0.31	0.62	0.51	0.44	0.44	0.45	0.44	0.71	0.68	0.8	0.8	0.43	0.46
AR(4)	3.2	3	3.7	9.3	2.7	2.7	2.3	3.5	2.6	3.7	3.6	3.5	3.7	2.9
ARCH(4)	6.4	2.1	12.2	17.6	2	3.6	6.4	4.3	7.4	5.2	2.6	6.7	2.9	2.3
Chow	6.3	9.9	9.1	9.4	20.7	18.5	20.2	19.1	13.5	8.9	7.7	12.2	17.6	15.5
Wald	3.1	2.6	9.3	2.2	0.1	0.1	1.2	2.4	0	0.3	0.5	3	1.5	1.7

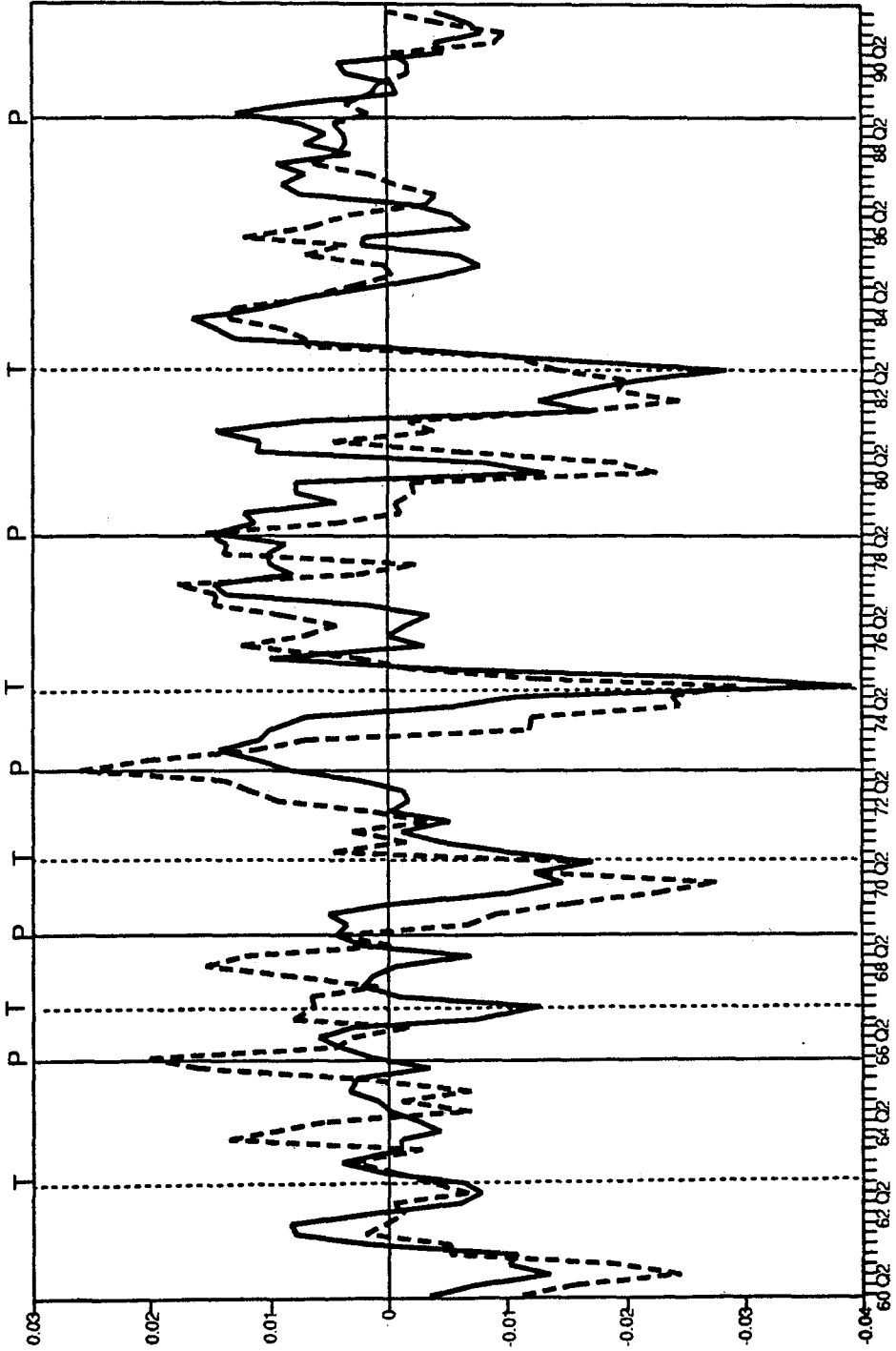
1. Variable definitions: inf = GDP inflation; infic = unit labour cost growth; gap = productivity gap; ecm = error-correction term (price level over unit labour costs). All variables in logs; prefixes d and d2 indicate first and second differences. T-statistics are in italics.

2. AR(4) is the LM test for autocorrelation up to the fourth order; ARCH(4) is the autoregressive conditional heteroscedasticity test up to the fourth order; Chow is the F-steps-ahead test for stability of the regression estimates (truncation period is 1985:4); Wald is the Wald test for the model reduction. All tests are chi-square-distributed. Degrees of freedom are in parentheses for AR and ARCH, they are equal to the prediction periods for Chow and to the number of restrictions for Wald.

be superior on several grounds to the trend estimates resulting from the Hodrick-Prescott filter, a popular exponential-smoothing procedure which relies on the *ad hoc* specification of a smoothing parameter. In particular, the B-N trends were found to be better suited for registering lasting changes in labour productivity growth, such as the "productivity slowdown" of the Eighties.

Chart 1

UNITED STATES
Estimates of labour productivity cycle



--- H-P cycle — Mult. cycle (R,C,U)

Chart 2
JAPAN
Estimates of labour productivity cycle

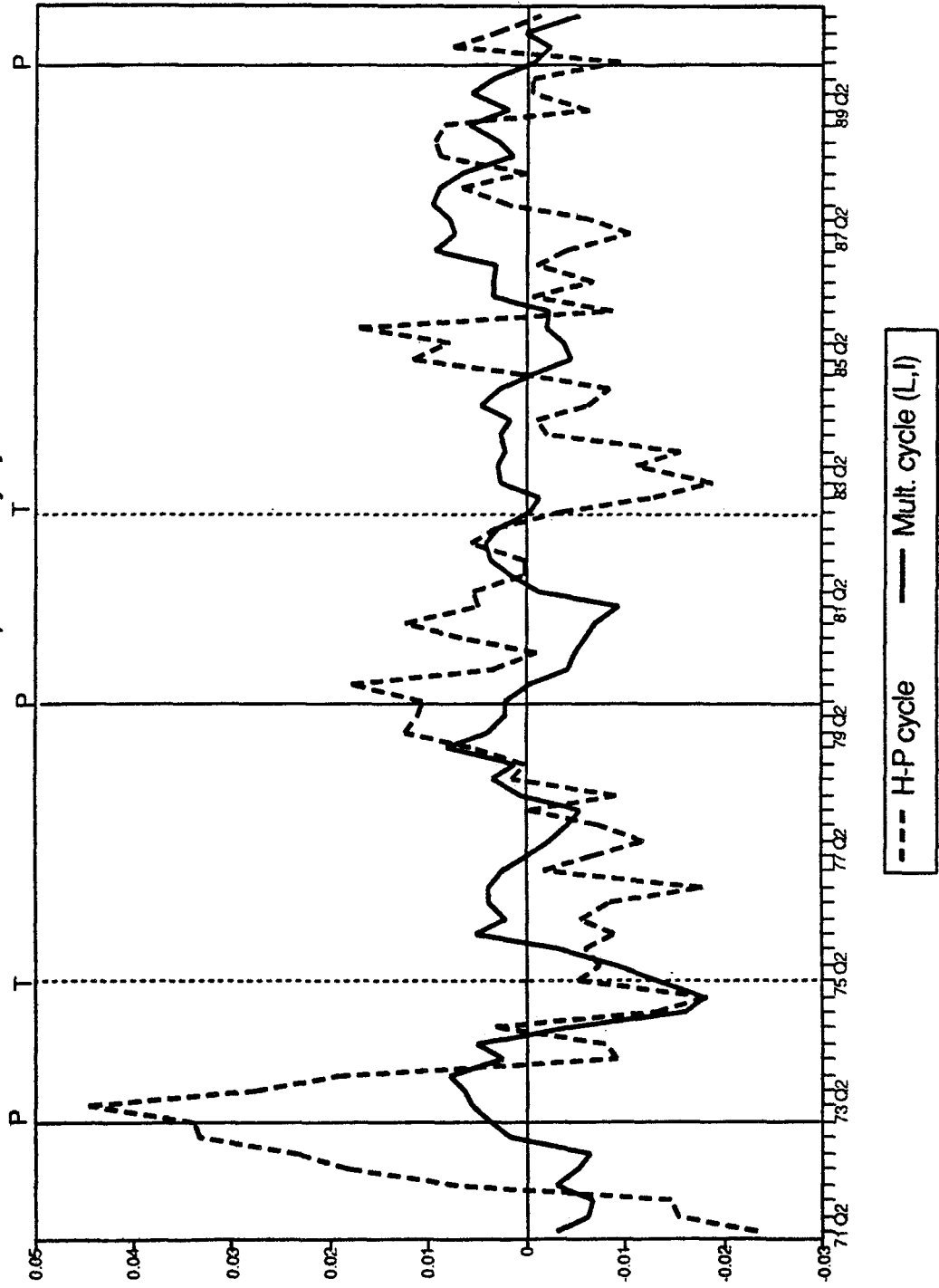
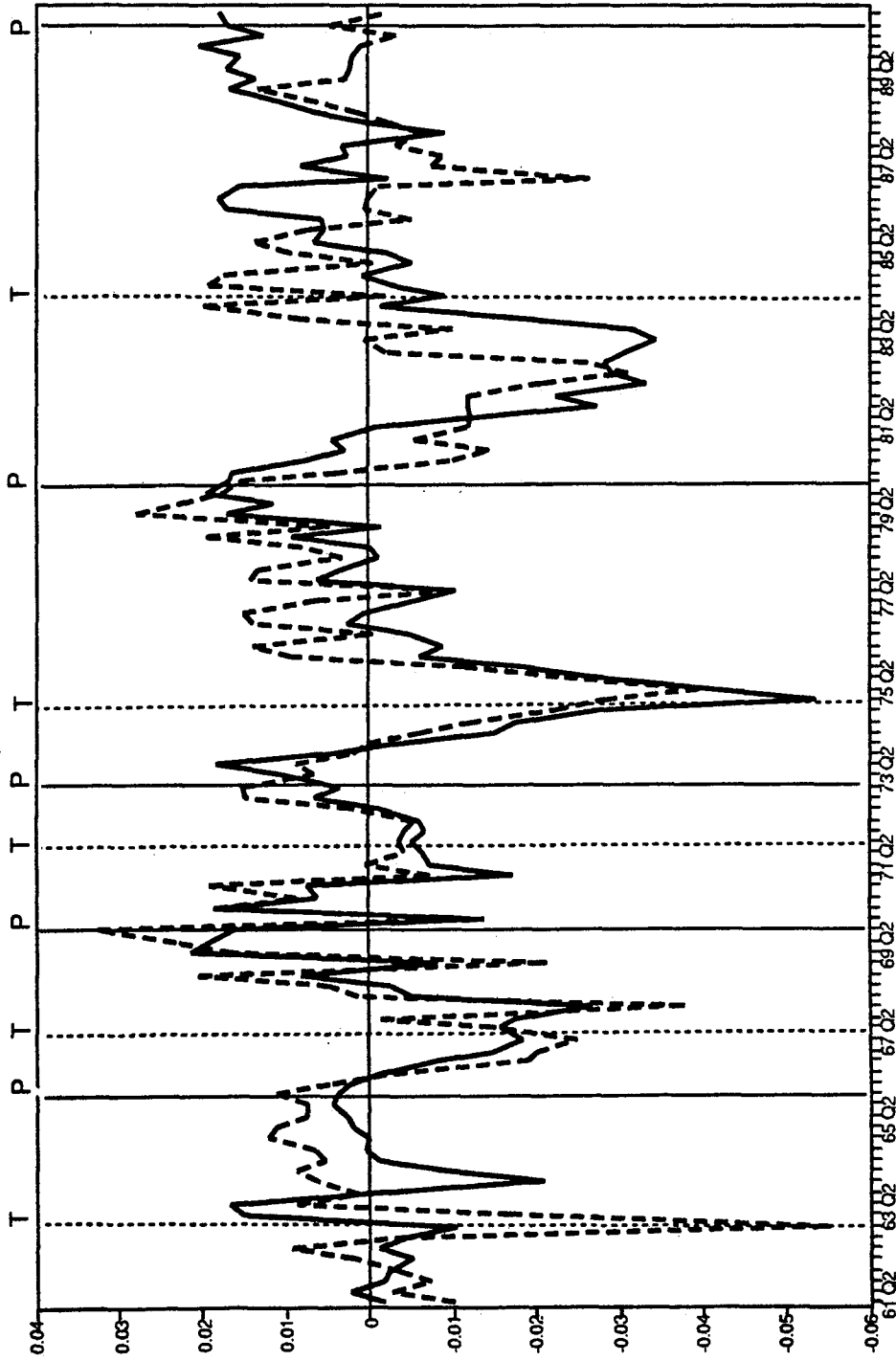


Chart 3

GERMANY

Estimates of labour productivity cycle

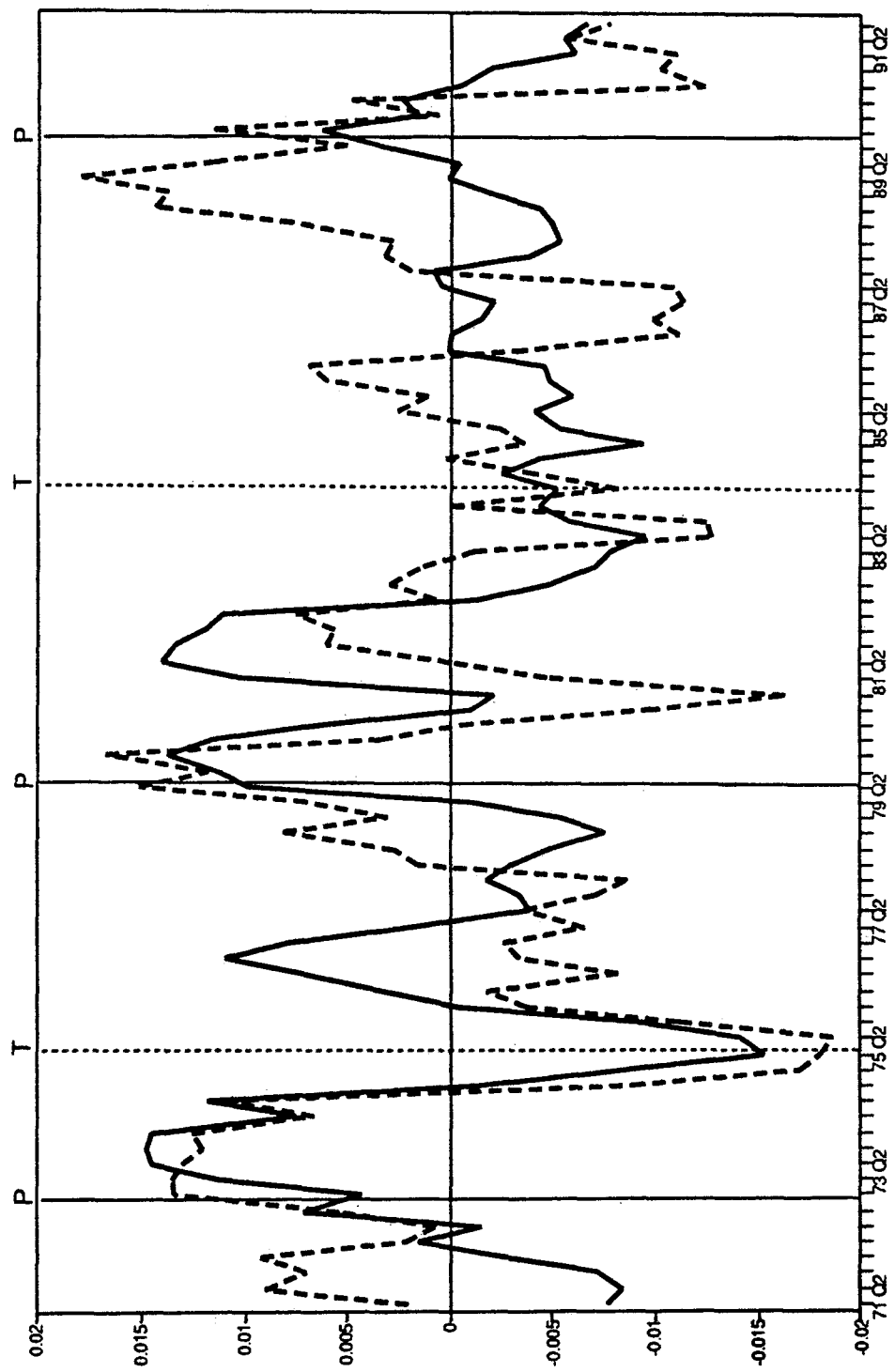


--- H-P cycle — Mult. cycle (U,R)

Chart 4

FRANCE

Estimates of labour productivity cycle



--- H-P cycle — Mult. cycle (C,R)

Chart 5

ITALY

Estimates of labour productivity cycle

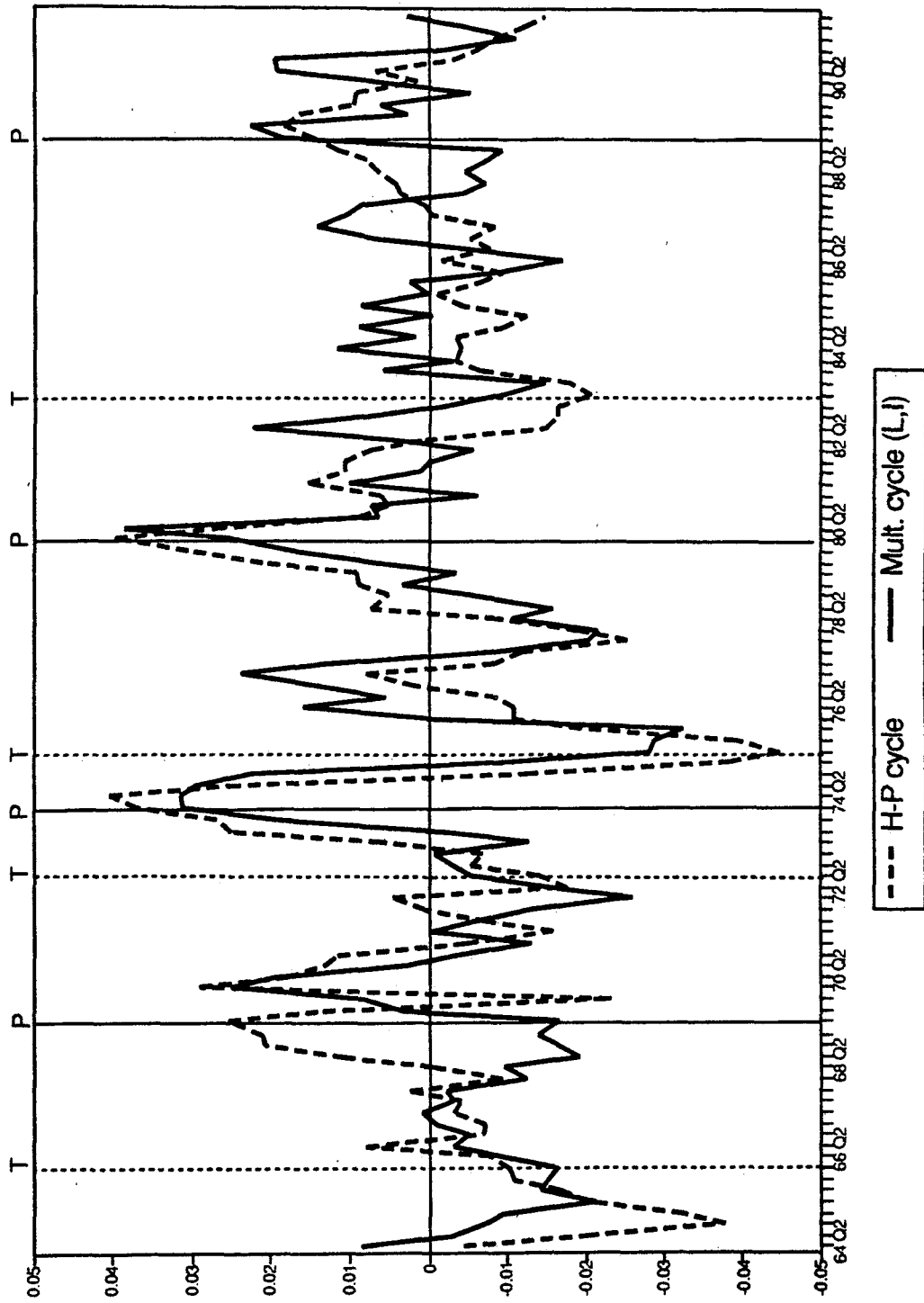


Chart 6

UNITED KINGDOM
Estimates of labour productivity cycle

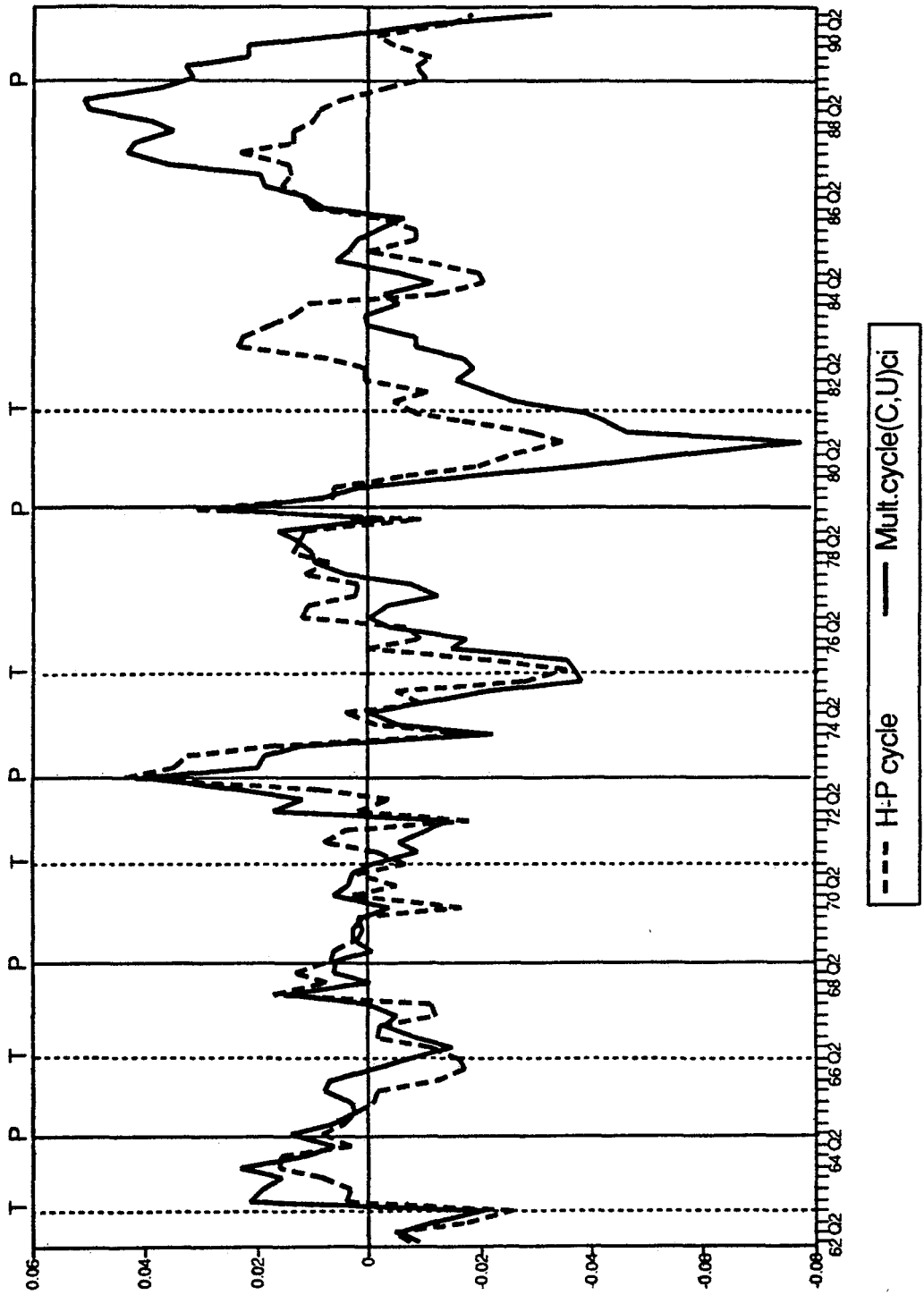


Chart 7
CANADA
 Estimates of labour productivity cycle

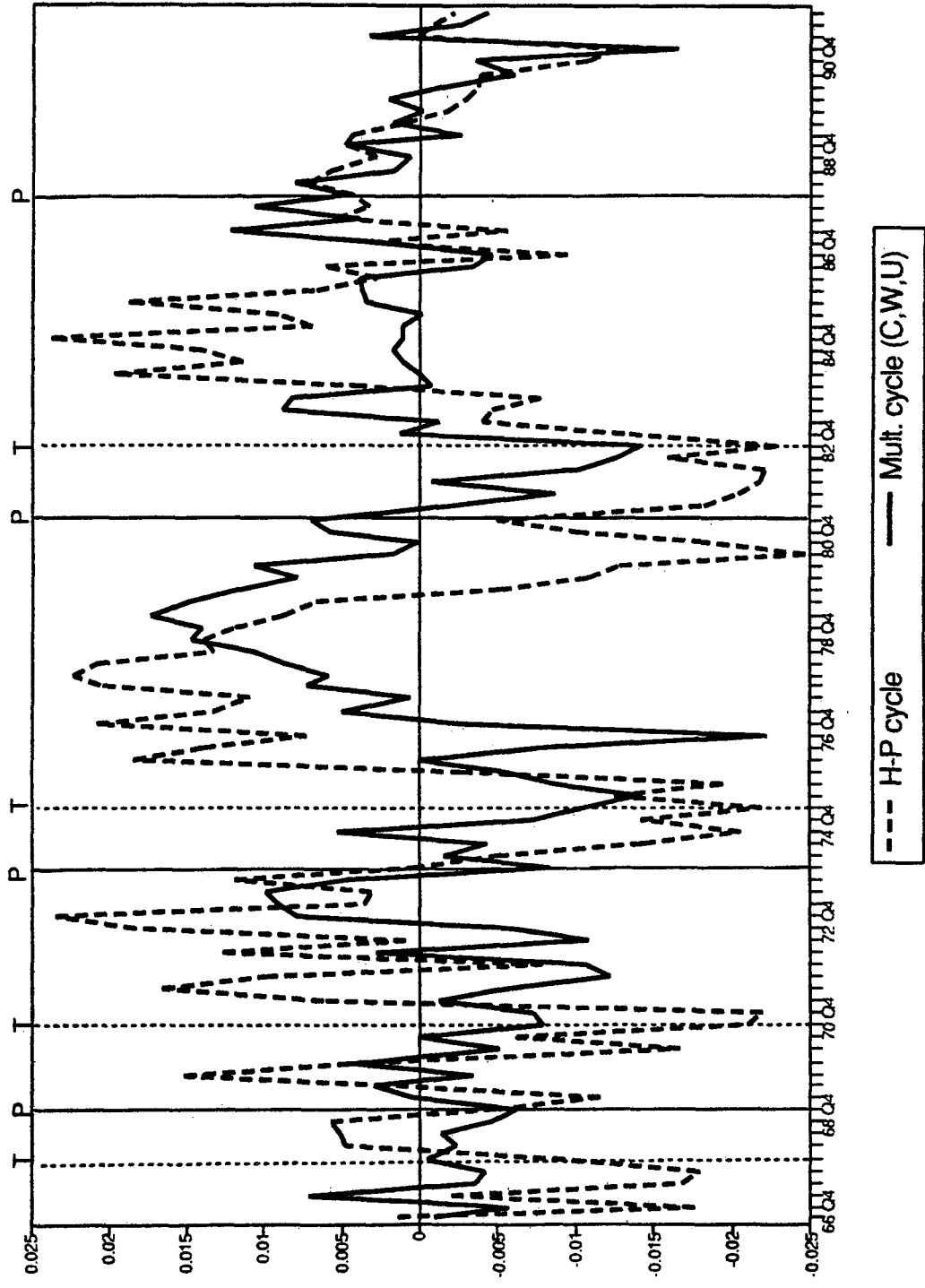


Chart 8a
UNITED STATES
 Hodrick-Prescott vs multivariate trends
 Robustness and fit

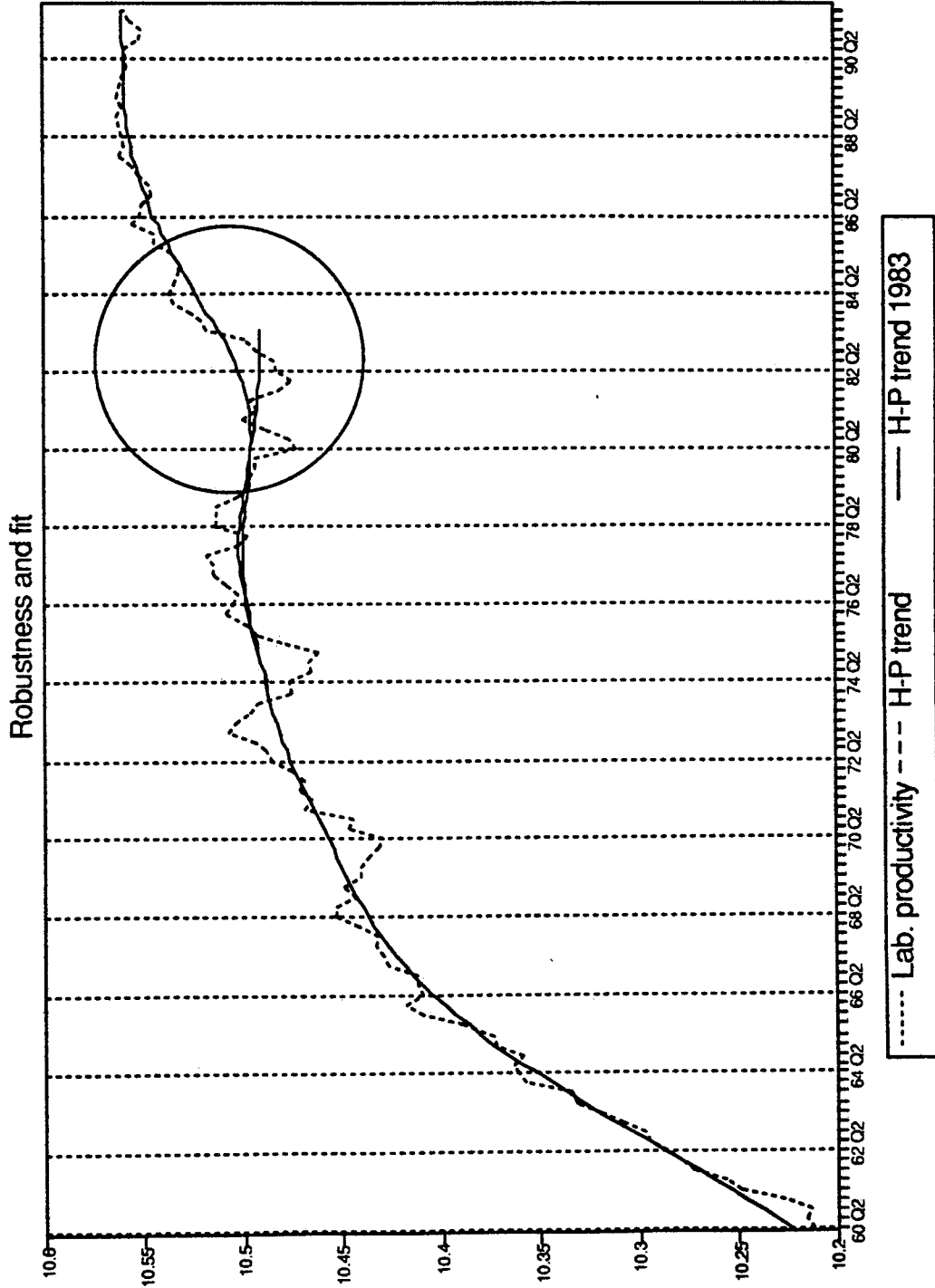


Chart 8b
UNITED STATES
 Hodrick-Prescott vs multivariate trends
 Robustness and fit

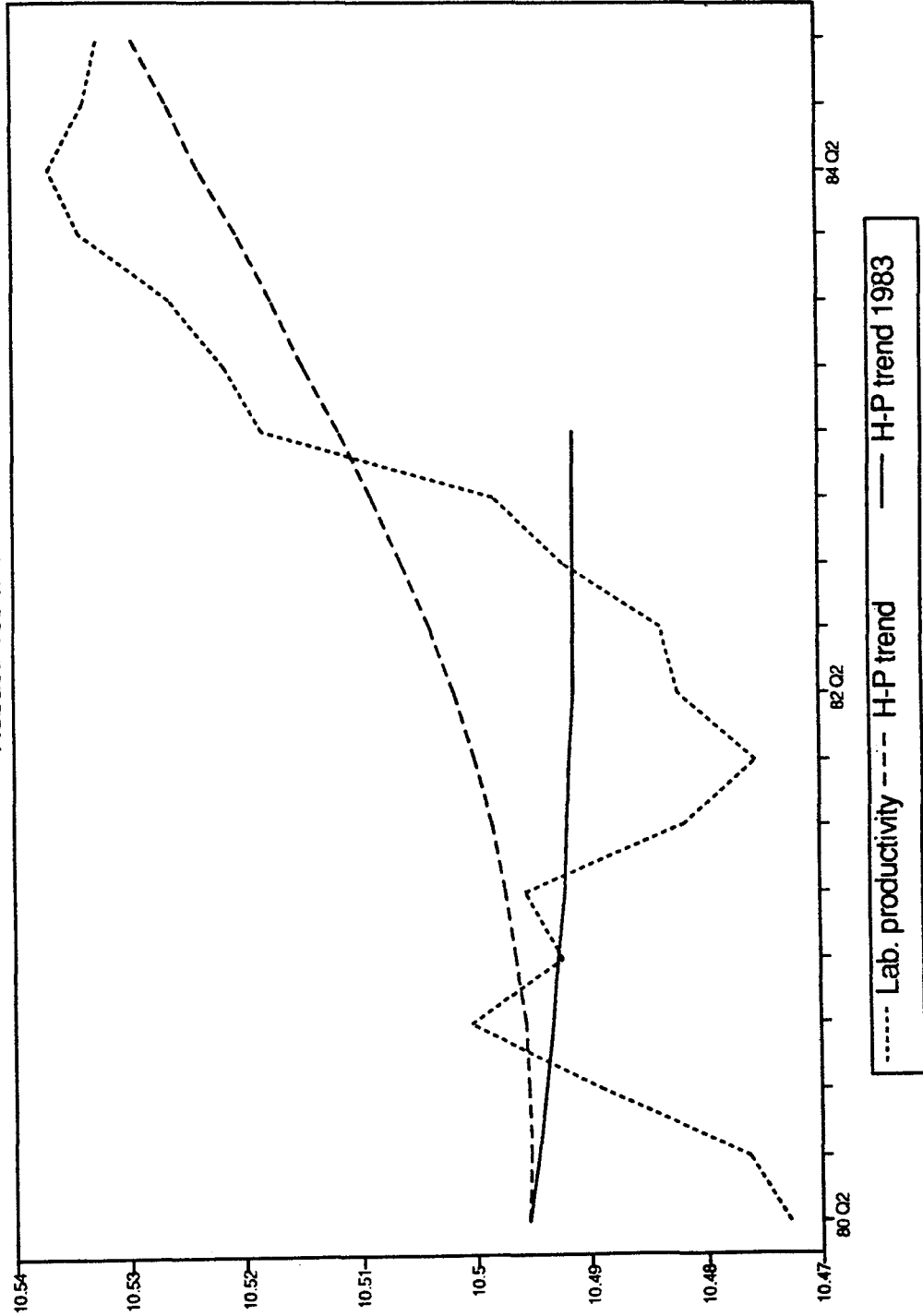


Chart 9a

GERMANY
Hodrick-Prescott vs multivariate trends

Robustness and fit

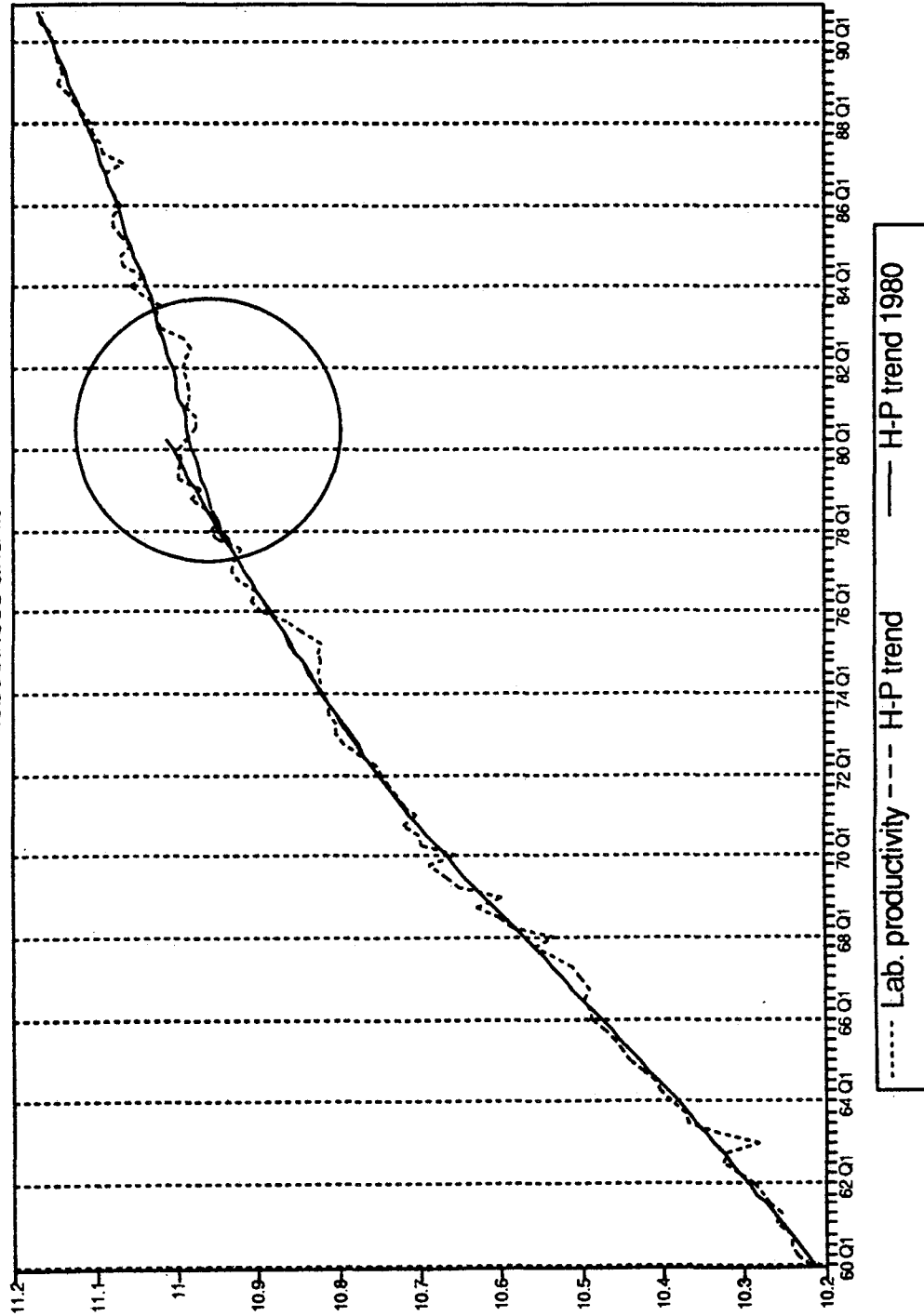


Chart 9b
GERMANY
 Hodrick-Prescott vs multivariate trends
 Robustness and fit

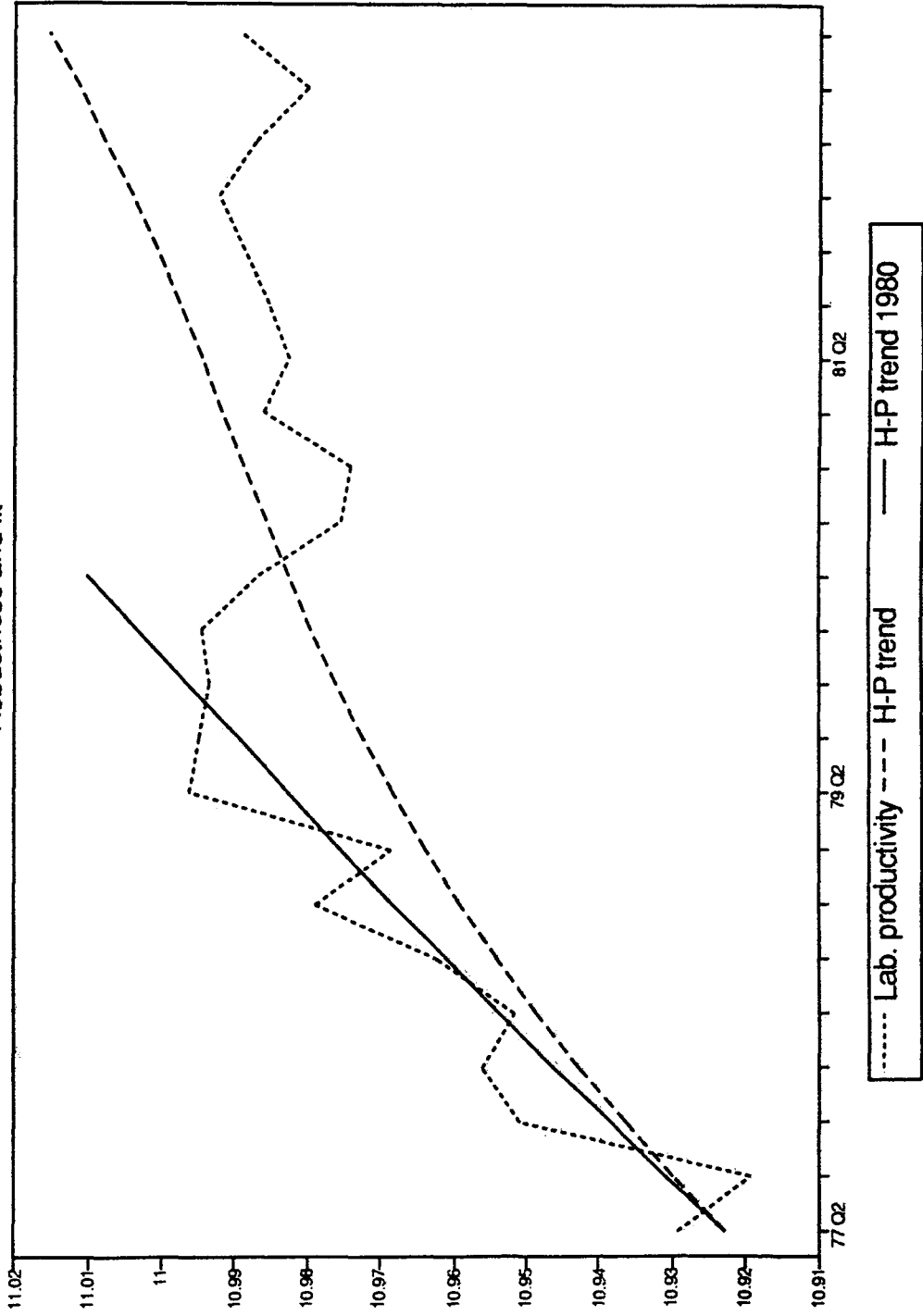


Chart 10a

UNITED STATES
Hodrick-Prescott vs multivariate trends

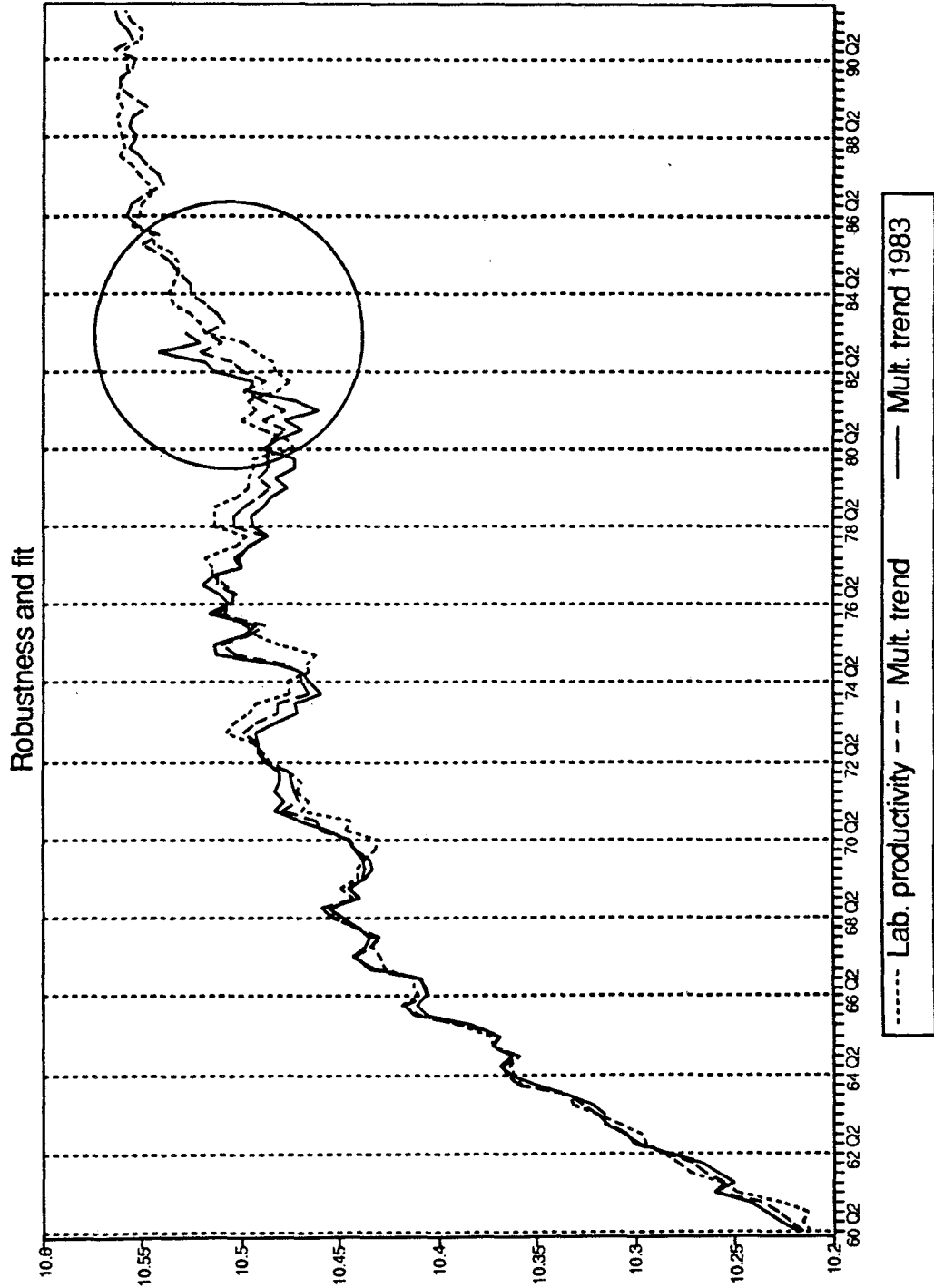


Chart 10b
UNITED STATES
 Hodrick-Prescott vs multivariate trends
 Robustness and fit

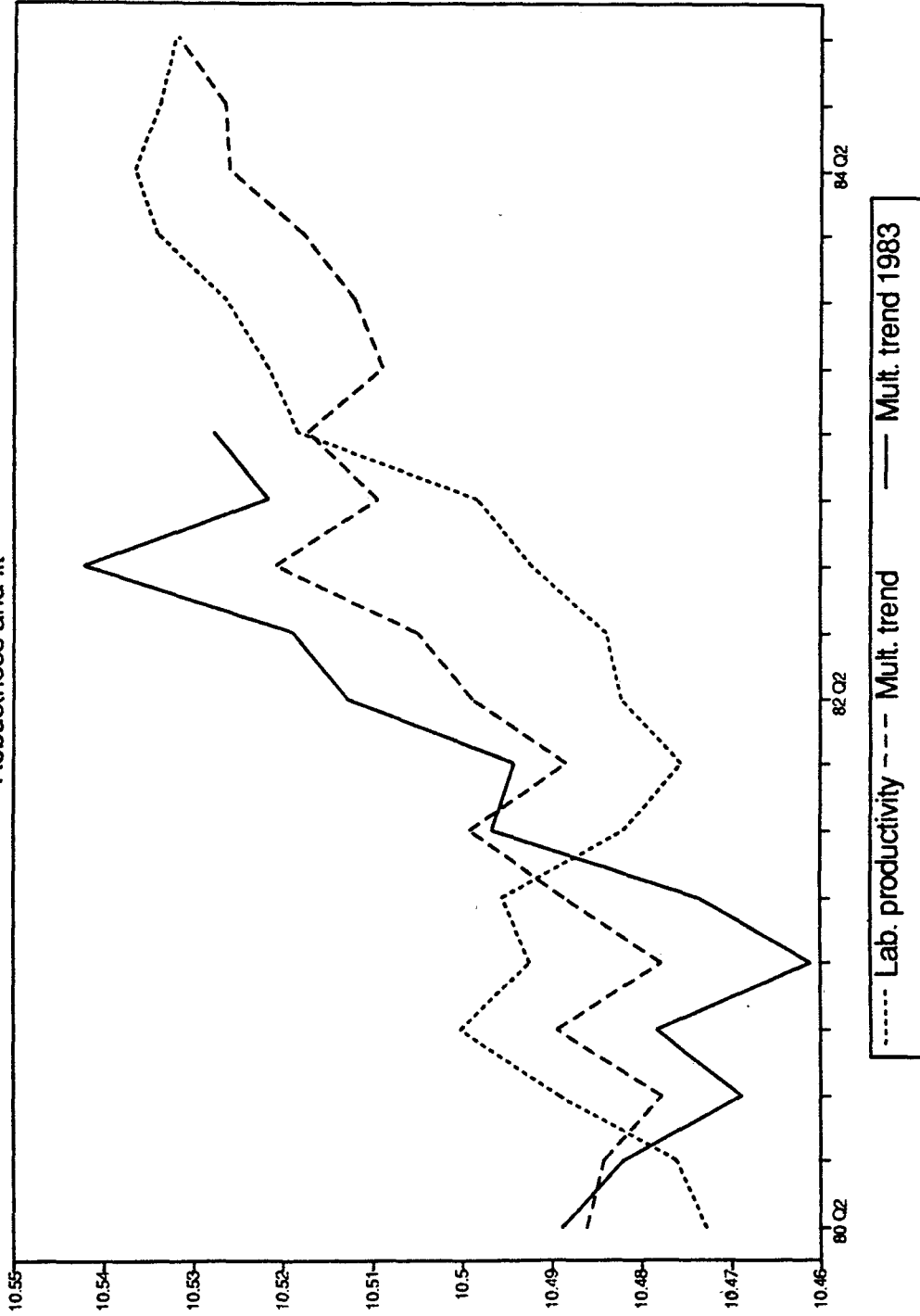


Chart 11a
GERMANY
 Hodrick-Prescott vs multivariate trends

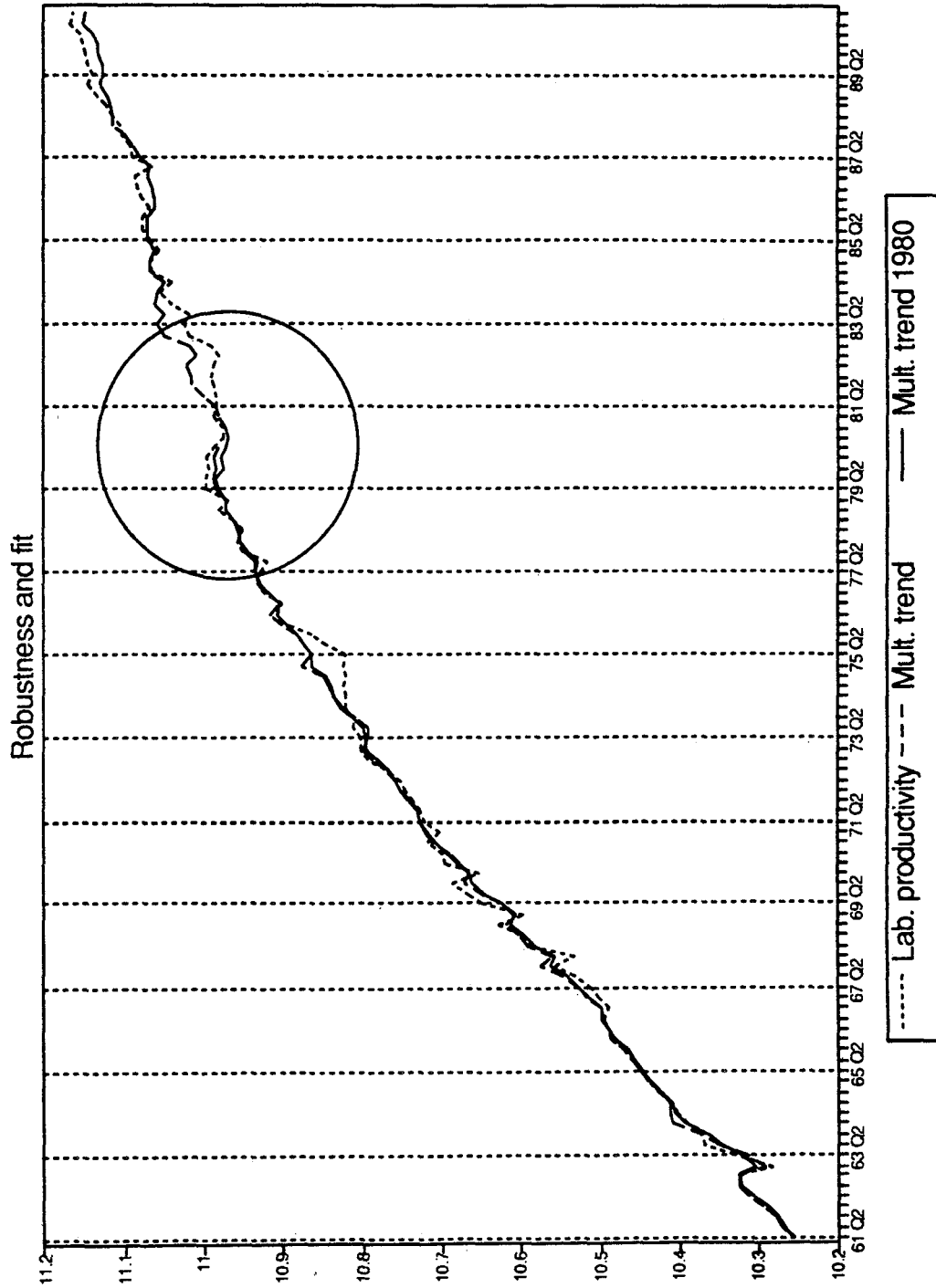
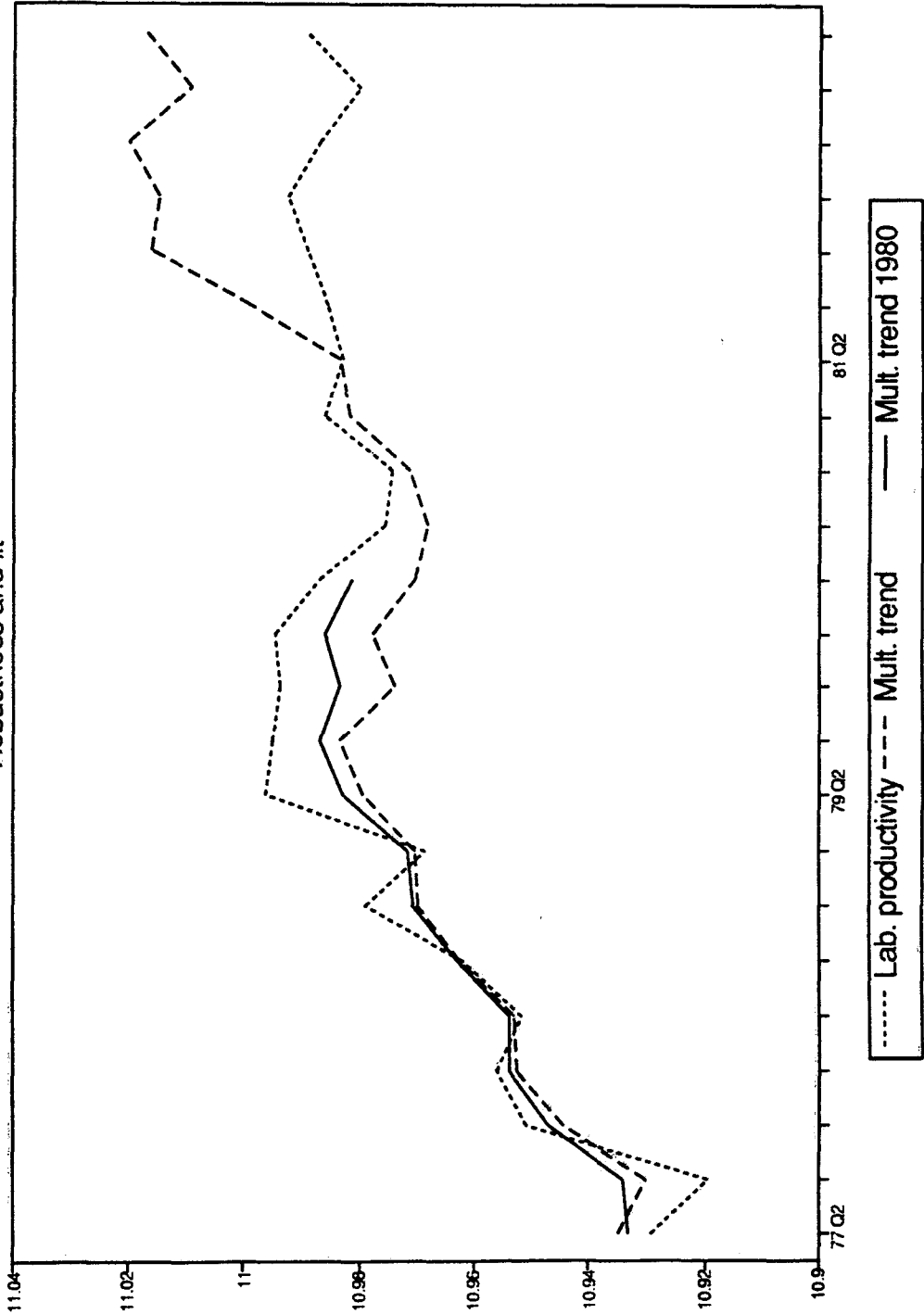


Chart 11b
GERMANY
 Hodrick-Prescott vs multivariate trends
 Robustness and fit



Notes

1. The authors are from Country Studies II Division, Economics Department, OECD and Observatoire français des conjonctures économiques, Paris, respectively. They would like to thank Steve Englander for the many useful comments given during the preparation of this study.
2. In addition, Evans and Reichlin (1993) showed that the multivariate B-N procedure *implies* a negative correlation between the estimated cycle and trend, when the series is positively autocorrelated over time.
3. A series is said to be (weakly) stationary when its mean, variance and autocovariances are independent of time.
4. It is assumed that c_t is generated by an autoregressive/moving-average (ARMA) process.
5. In the linear case the change in trend is a constant. Equation (2) encompasses non-linear deterministic functions, such as polynomials in time. However, the choice of the function is quite arbitrary, unless a growth model suggesting a specific functional form for the trend is assumed. Rappoport and Reichlin (1989) and Perron (1989) have considered segmented trends, which correspond to the notion of a change in the rate of growth "once-in-a-while" at known dates.
6. A series y_t is $I(1)$ if $y_t = d + y_{t-1} + u_t$, where u_t is stationary. A $I(1)$ series has an autoregressive unit root, referring to the coefficient of y_{t-1} above. A series which is stationary in levels is called $I(0)$.
7. The quantity $a(1)$ has been used in the literature to indicate the degree of persistence of shocks (see, for example, Campbell and Mankiw (1987) and Cochrane (1988)).
8. A decomposition of the series where the trend is a random walk and the cycle is stationary is only one of many possible alternatives. Stochastic trends with richer dynamics than the random walk can be envisaged (see, for example, Harvey (1985), Quah (1992) and Lippi and Reichlin (1991, 1993)). In those cases, although the non-stationary component would have the features of persistence and variability discussed in the text, it would also contain a predictable component. For a discussion of the multiplicity of trend/cycle decompositions for $I(1)$ processes, see Quah (1992) and Lippi and Reichlin (1991).
9. See Annex 1 for a detailed derivation of the univariate Beveridge and Nelson decomposition.
10. This is not the case of "structural" interpretations of the trend as in Blanchard and Quah (1989) and King *et al.* (1991). In that approach the *permanent* component is

what results from the dynamic impact of *permanent* shocks; therefore, in order to define it, identification assumptions are needed in order to disentangle permanent *versus* transitory innovations.

11. The relationship between B-N trends and the dimension of the information set was analysed by Evans and Reichlin (1992).
12. Formally, the lower bound of the cycle/trend variance ratio will be higher (see Evans and Reichlin, 1992).
13. See Annex 1 for a detailed illustration of the multivariate Beveridge and Nelson methodology.
14. The finite order of the VAR is chosen in order to obtain white noise OLS residuals.
15. In computing labour productivity trends, four hundred forecasts were computed in each period ($K = 400$).
16. *Per capita* units were computed using business-sector employment data.
17. The detailed results of the unit-root and cointegration tests are available upon request. The failure to reject non-stationarity of the unemployment rate is a perhaps surprising but common finding (see, for example, Barro, 1988 and Bean, 1992).
18. In many cases, the estimated error-correction coefficients attached to the cointegrating relationships had the wrong sign, implying feed-forward rather than feedback mechanisms.
19. Charts showing the B-N trend estimates for the Big Seven OECD countries are shown in Annex 2.

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Annex 1

A. The univariate B-N decomposition

It is assumed that Δy_t admits the Wold representation (5),

$$\Delta y_t = \gamma + a(L)u_t \quad (5)$$

on which the following standard assumptions are imposed:

- i) $a(L)$ has poles outside the unit circle;
- ii) $a(L)$ has roots on or out the unit circle;
- iii) $a(0) = 1$; and
- iv) u_t is a white noise.

Assumption i) implies stationarity of Δy_t . Assumption ii) implies that u_t is an innovation process, i.e. the residual from a linear projection of y_t on its past history:

$$u_t = y_t - \text{Proj}(y_t | y_{t-1}, y_{t-2}, \dots) = y_t - E(y_t | H_t)$$

A representation fulfilling assumptions i), ii) and iv) is called *fundamental* for y_t . Assumption iii) is a normalisation that guarantees that the representation is unique. The presence of a stochastic trend implies $a(1) \neq 0$.

Model (5) implies:

$$\begin{aligned} y_{t+k} &= y_{t-1} + \Delta y_{t+1} + \dots + \Delta y_{t+k} \\ &= y_{t-1} + (k+1)\gamma + a(L)u_t + \dots + a(L)u_{t+k} \end{aligned}$$

Consider now the effect of a shock u_t on y_{t+k} ($u_{t+i} = 0$, for $i \neq 0$):

$$y_{t+k} = y_{t-1} + (k+1)\gamma + (1 + a_1 + \dots + a_k)u_t$$

As k tends to infinity, the long-run effect of u_t on y_{t+k} is given by:

$$\lim_{k \rightarrow \infty} \frac{\partial y_{t+k}}{\partial u_t} = a(1)u_t$$

where $a(1) = \sum_{i=0}^{\infty} a_i$.

Defining the trend component τ_t of y_t as the long-term forecast of the level of the series adjusted for its mean growth,

$$\tau_t = \lim_{k \rightarrow \infty} (E[y_{t+k} | I_t] - \gamma k)$$

and taking conditional expectations of y_{t+k} the following expression for the change in trend in period t is obtained (see Beveridge and Nelson, 1981):

$$\Delta \tau_t = \gamma + \lim_{k \rightarrow \infty} (E_p y_{t+k} - E_{t-1} y_{t+k}) = \gamma + a(1)u_t$$

The trend-cycle decomposition deriving from this notion of the trend is:

$$\Delta y_t = \gamma + a(1)u_t + \Delta \bar{a}(L)u_t \quad (\text{BN})$$

where the trend in first differences is:

$$\Delta \tau_t = \gamma + a(1)u_t$$

and the cycle in first differences is:

$$\Delta c_t = \Delta \bar{a}(L)u_t$$

with

$$\bar{a}(L) = \sum_{i=0}^{\infty} \bar{a}_i L^i, \quad \bar{a}_i = - \sum_{j=i+1}^{\infty} a_j$$

Notice that in the decomposition (BN) the trend and the cycle are driven by the same shock u_t .

In the decomposition (BN), the trend is the random walk component of the series, whose variance is $a(1)^2 \sigma_u^2$. The cyclical component is then defined as $c_t = y_t - \tau_t$ and can be interpreted by noting that:

$$c_t = y_t - \lim_{k \rightarrow \infty} (E_p y_{t+k} - \gamma k) = \lim_{k \rightarrow \infty} \left(\gamma k - \sum_{i=1}^k E_t \Delta y_{t+i} \right) = - \sum_{i=1}^{\infty} (E_t \Delta y_{t+i} - \gamma)$$

The cycle is the negative of the sum of expected future output growth rates, given information at t , in excess of the unconditional mean rate of growth γ .

B. The multivariate B-N decomposition

Consider a $k \times 1$ stationary vector W_t , whose first k_1 elements, $\Delta X1_t$, are first differences of $I(1)$ variables, possibly cointegrated, and the remaining $k - k_1$ elements are levels of $I(0)$ variables, $X2_t$. The first element of W_t is Δy_t .

The vector process W_t admits the MA representation (12),

$$W_t = D + A(L)u_t \quad (12)$$

It is assumed that $A(L) = \sum_{j=0}^{\infty} A_j L^j$ is a rational function of L and the following assumptions are made:

- i) The poles of $A(L)$ are outside the unit circle
- ii) The roots of $\det A(L)$ are on or outside the unit circle
- iii) $A_0 = I$
- iv) u_t is now a k -dimensional white noise with $E u_t = 0$ and $E u_t u_t' = \Omega$, Ω positive definite.

Assumptions i), ii) and iv) are the multivariate generalisation of the analogous assumptions made in the univariate context and carry the same implications. The presence of a stochastic trend implies $A(1) \neq 0$.

Partitioning W_t , Γ and $A(L)$ conformably,

$$\begin{aligned} W_t &= [\Delta X1_t \quad X2_t]' \\ A(L) &= [A1(L) \quad A2(L)]' \\ \Gamma &= [\Gamma1 \quad \Gamma2]' \end{aligned}$$

the multivariate generalisation of the Beveridge and Nelson decomposition into a stationary and a non-stationary component can be written as follows:

$$\begin{pmatrix} \Delta X1_t \\ \Delta X2_t \end{pmatrix} = \begin{pmatrix} \Gamma1 \\ 0 \end{pmatrix} + \begin{pmatrix} A1(1) \\ 0 \end{pmatrix} u_t + \begin{pmatrix} (1-L)\bar{A}1(L) \\ (1-L)A2(L) \end{pmatrix} u_t \quad (\text{MBN})$$

where

$$\bar{A}1(L) = -\sum_{j=0}^{\infty} \bar{A}1_j L^j \quad \text{and} \quad \bar{A}1_j = -\sum_{m>j}^{\infty} A1_m$$

If, in the representation (MBN), the vector $X1$ has r cointegrating relationships ($0 \leq r < k1$), then the rank of $A1(1)$ is $k1 - r$. The sum of the first two terms on the right-hand side of (MBN) is interpreted as the first difference in the trend component while the third term is interpreted as the first difference in the cyclical component.

Estimation of $A(L)$ is performed using an approximation to the autoregressive/error-correction representation of (12). Two cases can be distinguished: a) the variables in $X1$ are not cointegrated, then $A(L)$ can be inverted and the system including the variables $\Delta X1$ and $X2$ can be specified and estimated as a standard VAR; b) some components of $X1$ are cointegrated, then estimation requires the specification of a cointegrated (error-correction) VAR, extended in order to include $I(0)$ variables.

Case a)

The rank of $AI(I)$ is k_1 . In this case u_t can be recovered by running a VAR on W_t . The latter is a finite approximation of the following autoregressive representation (the constant term Γ was dropped for simplicity):

$$\frac{A_{ad}(L)}{\det A(L)} W_t \equiv B(L)W_t = u_t$$

where $A_{ad}(L)$ is the adjoint matrix.

Case b) (Evans and Reichlin, 1992)

The determinant of $A(L)$ has roots on the unit circle, so that the rank of $AI(I)$ is $k_1 - r$ ($0 < r < k_1$) and there is at least one cointegration relation. In this case the determinant can be factorised as follows (see Graybill, 1983):

$$\det A(L) = (1-L)\mu(L).$$

where $\mu(L)$ is a lag polynomial. In this case, the adjoint matrix can be written as follows:

$$A_{ad}(L) = [N(L) \quad (1-L)C(L)]$$

Therefore, the autoregressive representation can be expressed in the following form:

$$[N(L) \quad (1-L)C(L)] \begin{pmatrix} \Delta X1_t \\ X2_t \end{pmatrix} = (1-L)\mu(L)u_t$$

or, equivalently,

$$N(L)X1_t + C(L)X2_t = \mu(L)u_t$$

Using standard arguments, the corresponding error-correction representation is:

$$F(L)\Delta X1_t + C(L)X2_t = -\delta z_{t-1} + \mu(L)u_t \quad (CI)$$

where

$$z_{t-1} = \theta X1_{t-1}, \quad N(1) = \delta\theta \quad \text{and} \quad F(L) = \frac{N(L) - N(1)L}{1-L}$$

and δ is $k_1 \times r$, and θ is $r \times k_1$. In this case, estimation is performed on the basis of a cointegrated VAR which is a finite approximation to model (CI).

C. The Hodrick and Prescott filter

The lag structure for the HP filter is determined so as to trade-off goodness of fit against degree of smoothness. In particular, it is obtained by solving the following minimisation problem:

$$\min \sum_{t=1}^N c_t^2 + \lambda \sum_{t=3}^N [(\tau_t - \tau_{t-1}) - (\tau_{t-1} - \tau_{t-2})]^2$$

In the minimand, the first term measures the fit of τ_t to y_t , while the second measures the smoothness of τ_t . The trade-off is given by the parameter λ . For $\lambda = \infty$ a linear trend is obtained and the importance of the cycle is maximised. For $\lambda = 0$, $\tau_t = y_t$ and the cycle is zero.

The solution of the minimisation problem yields:

$$\tau_t = \sum_{k=-\infty}^{\infty} s_k y_{t+k} = S(L)y_t$$

which is a linear time-invariant filter.

In the empirical application a standard value of $\lambda = 1\ 600$ is used.

Annex 2

This Annex consists of Charts 2.1 to 2.7

Chart 2.1
UNITED STATES
Multivariate trend estimates

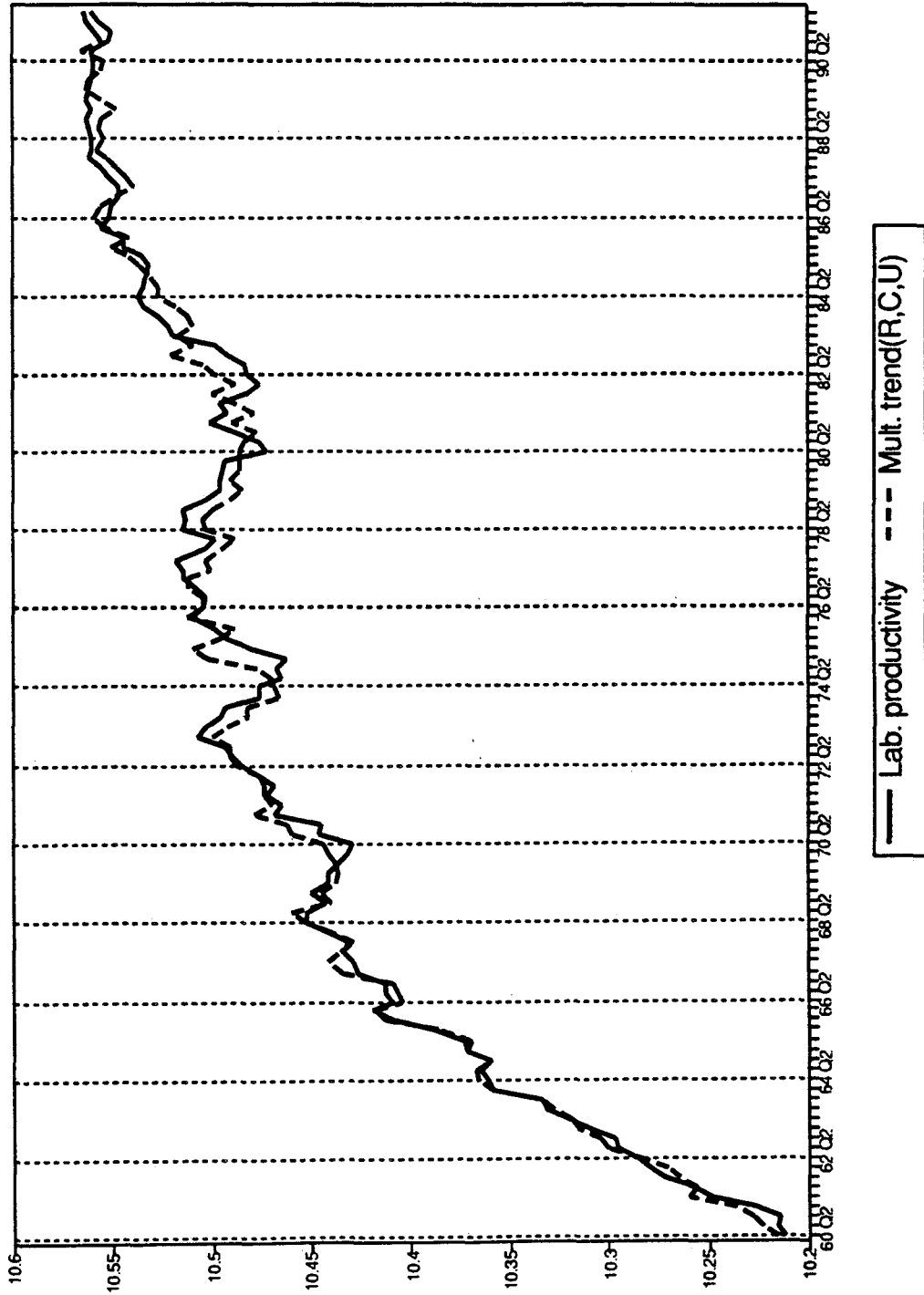


Chart 2.2

JAPAN
Multivariate trend estimates

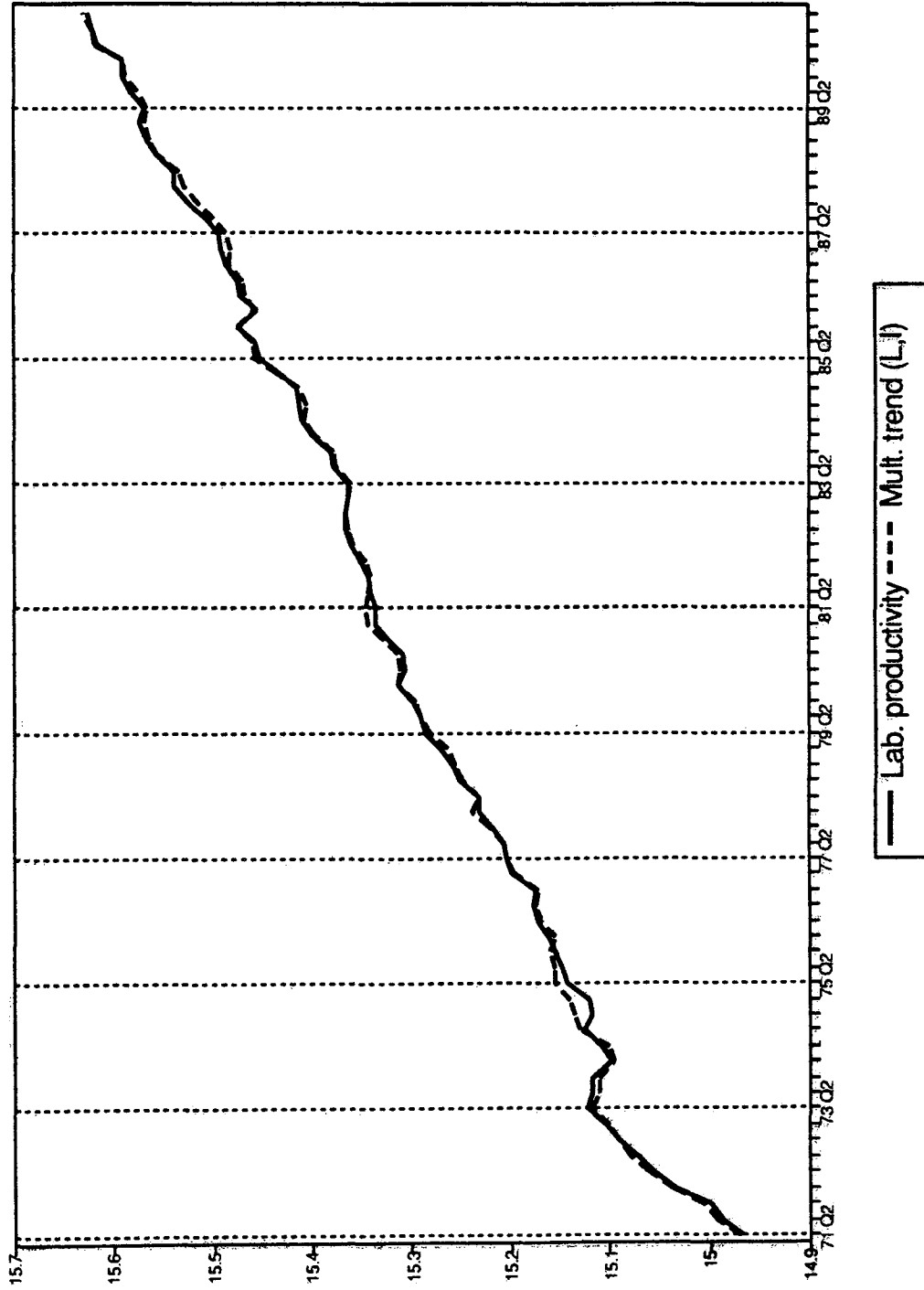


Chart 2.3
GERMANY
Multivariate trend estimates

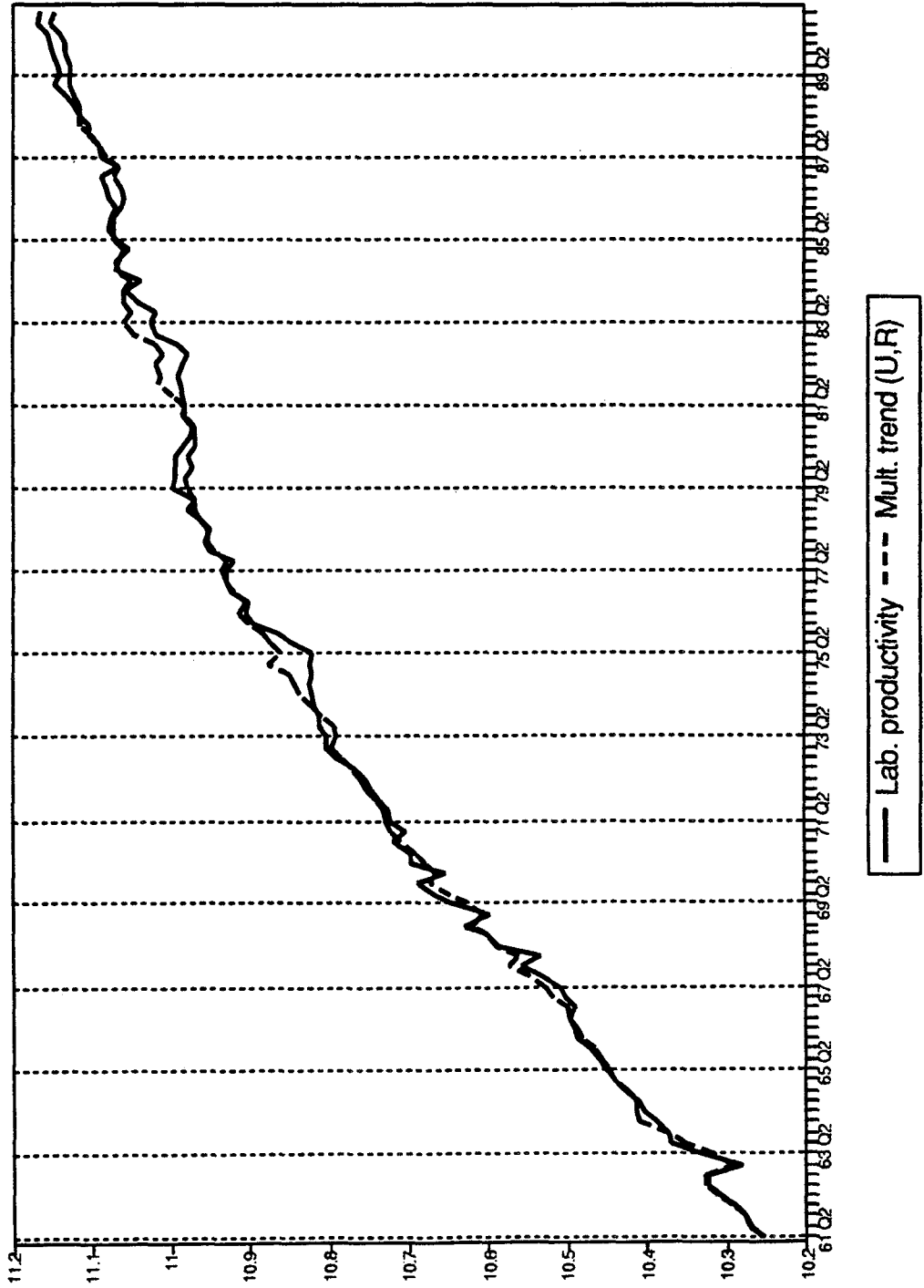


Chart 2.4

FRANCE
Multivariate trend estimates

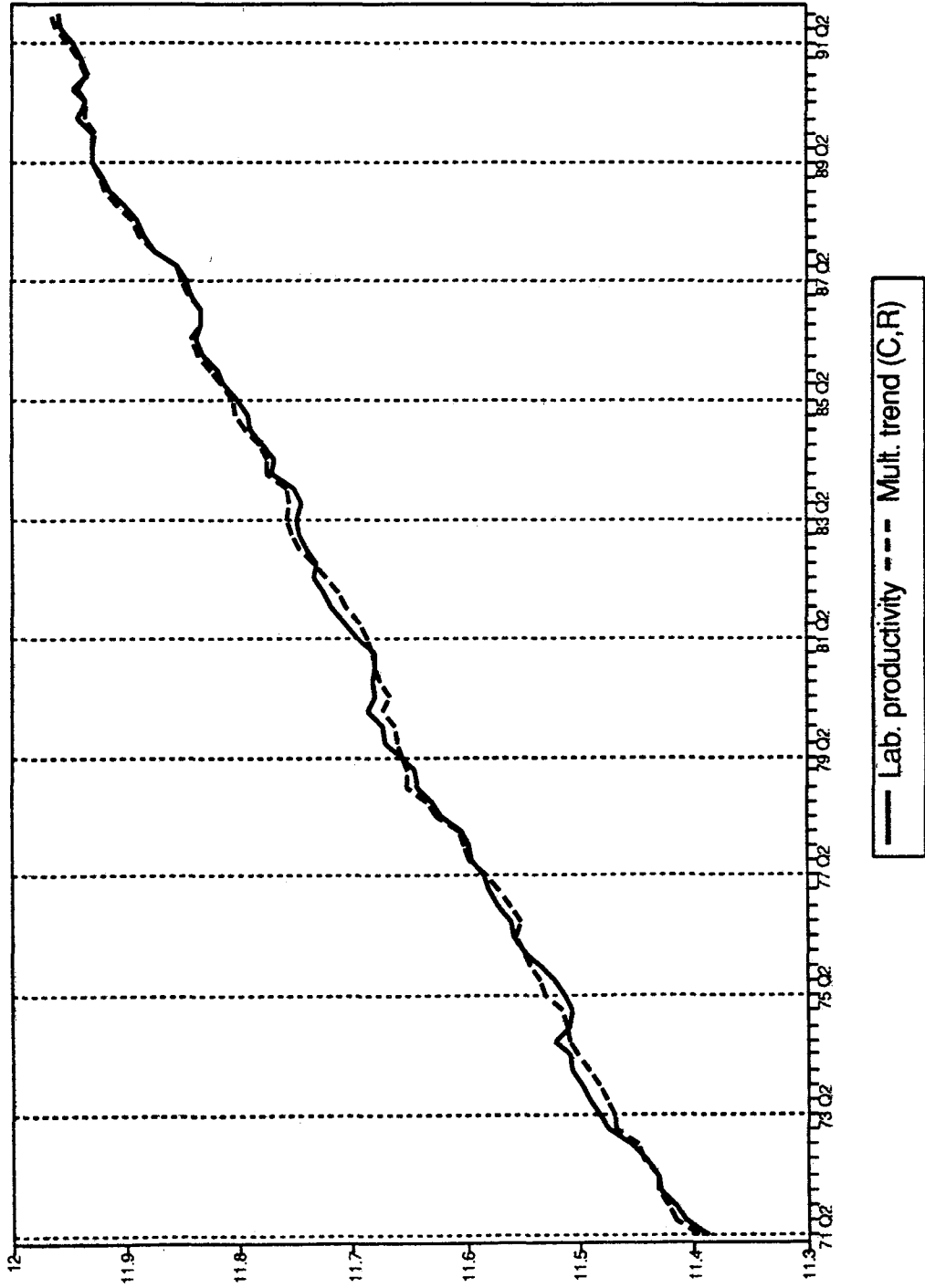


Chart 2.5

ITALY
Multivariate trend estimates

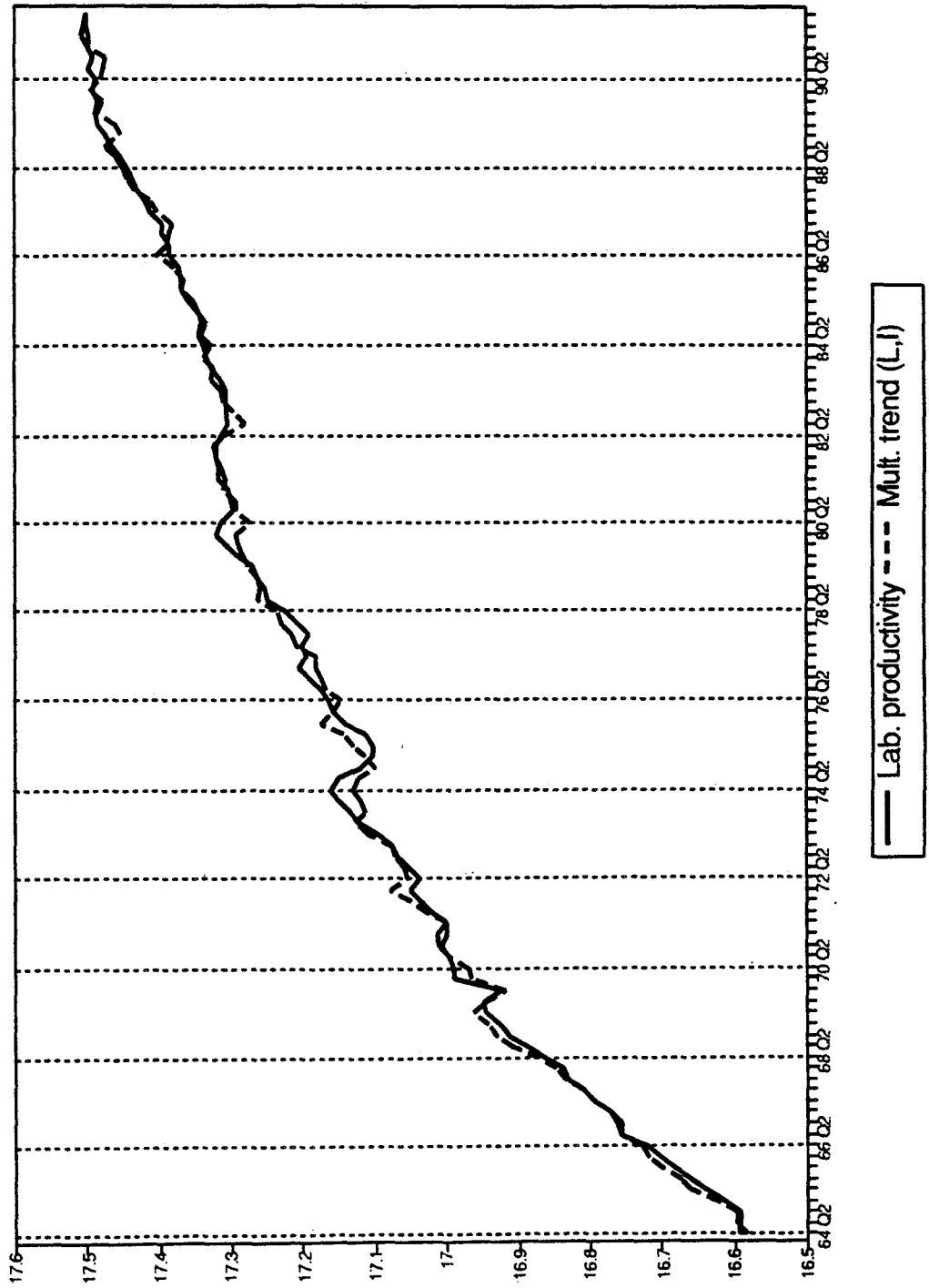


Chart 2.6
UNITED KINGDOM
Multivariate trend estimates

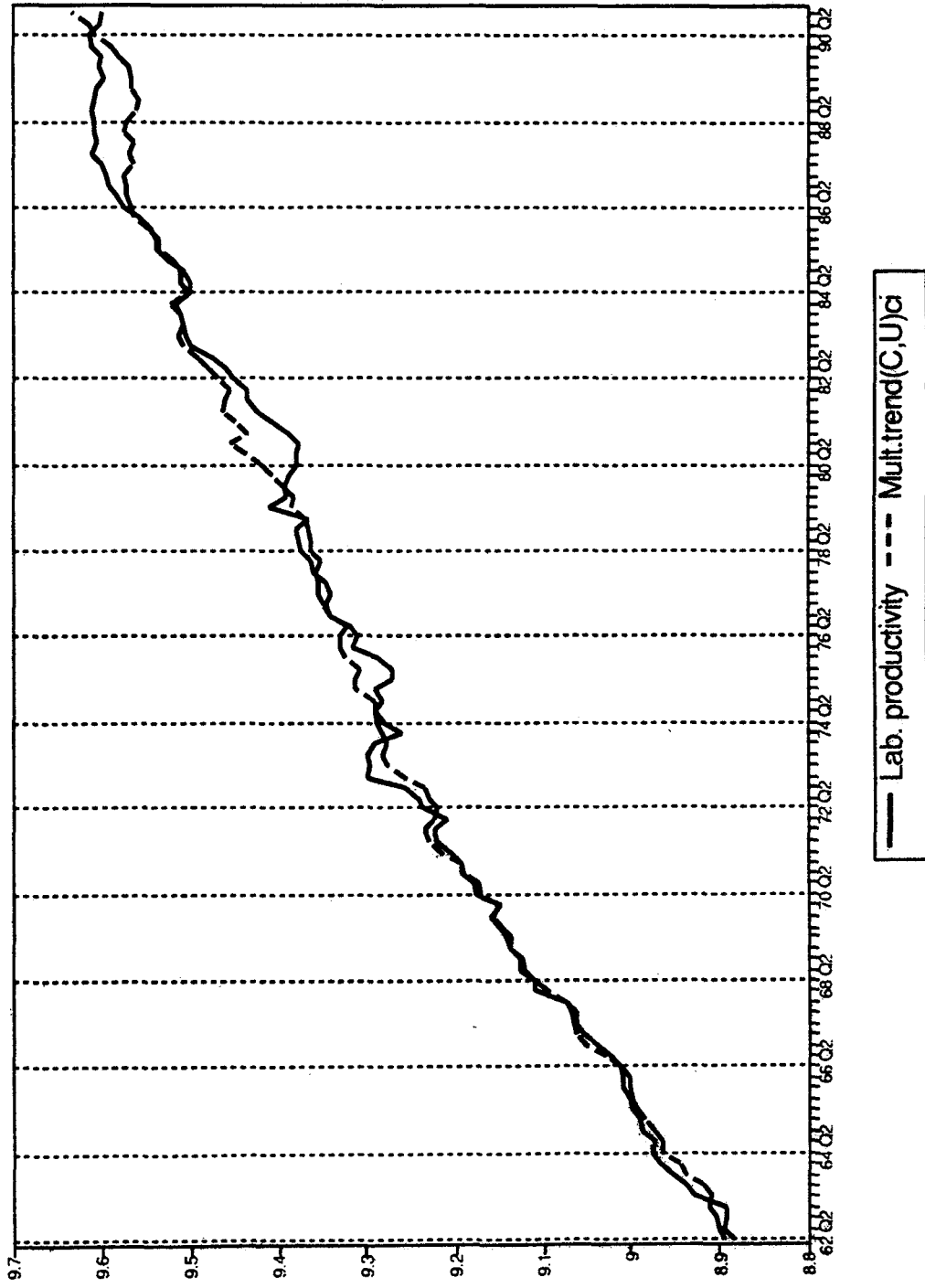
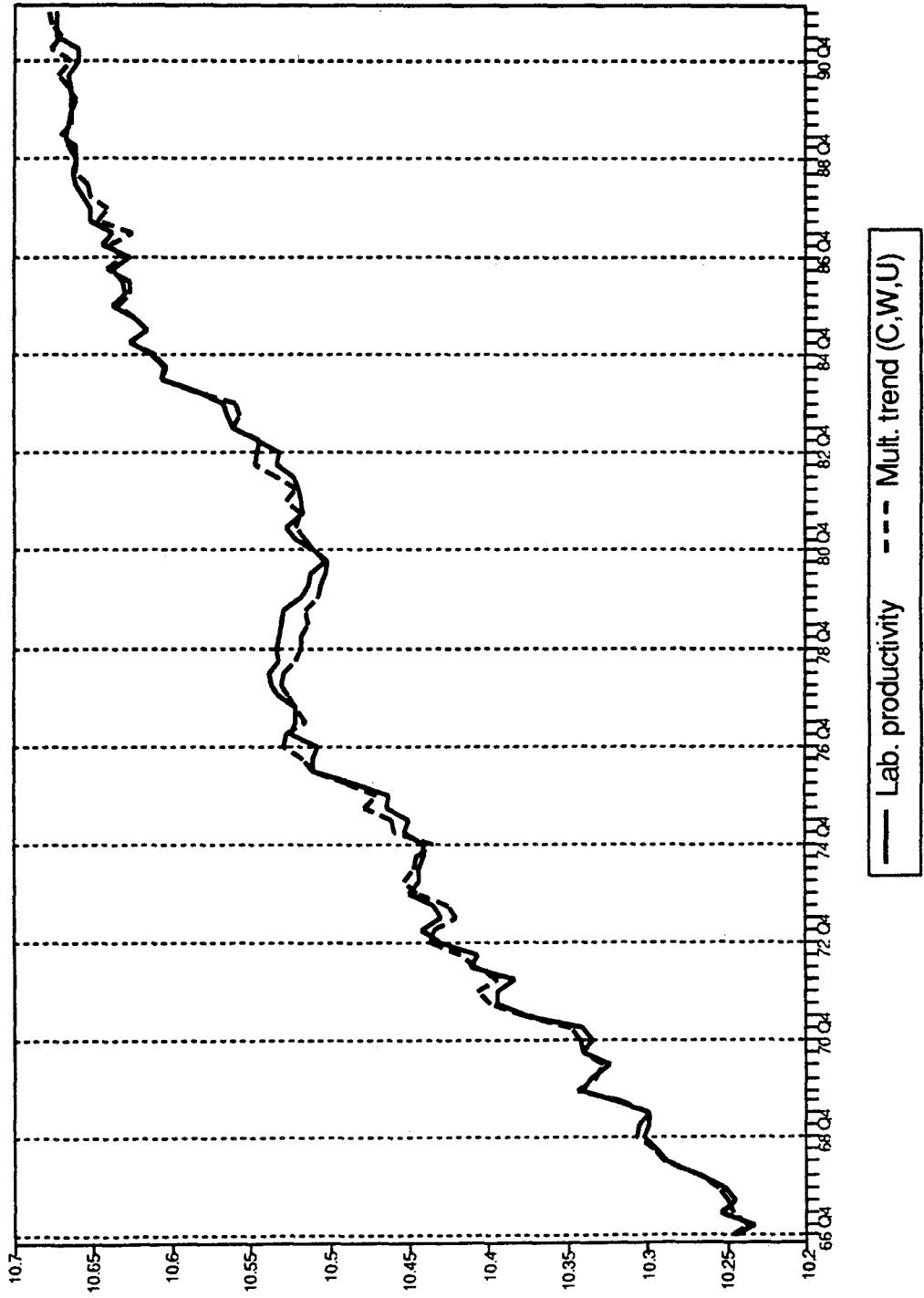


Chart 2.7

CANADA

Multivariate trend estimates



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