Annex B. Methodology

Input-output analysis

This annex provides a brief introduction to input-output analysis, including the formulas used to calculate the indicators reported in this study.

One-country closed economy

A stylized IO-table of a closed economy is depicted in Table B.1. The first $n \times n$ elements of the IO-table record intra- and inter-industry flows of intermediate goods and services, where sales from sector i to j are recorded horizontally and purchases vertically. The $n+1$ column ("Final demand") records sales to final consumers and the $n+1$ row ("Value added") outlays on labour and capital that process raw materials and manufactured inputs into more valuable outputs. The shaded column to the right reports total output (supply) by industry and the shaded column at the bottom total input (use) by industry, which in equilibrium are equal in monetary terms.

Table B.1. Input-output table of a closed economy

Using sector j = 1, 2, …, n

		Intermediate demand			Final demand	Total output
		Sector 1	Sector 2	Sector n		
Supplying sector *i =1, 2 ,… , n*	Sector 1	z_{11}	z_{12}	z_{1n}	f_1	y_1
	Sector 2	z_{21}	z_{22}	z_{2n}	f_2	y_2
	…	…	…	…	…	…
	Sector n	z_{n1}	z_{n2}	z_{nn}	f_n	y_n
	Value Added	w_1	w_2	w_n	*GDP*	
	Total input	y_1	y_2	y_n		

To analyse the interaction between sectors, Leontief (1936) – the intellectual father of IO-analysis – proposed a linear model with fixed input coefficients and constant returns to scale (CRS). The production functions were specified as,

$$(1) \qquad y_j = min\left(\frac{z_{1j}}{a_{1j}}, \frac{z_{2j}}{a_{2j}}, \dots, \frac{z_{nj}}{a_{nj}}; \frac{w_j}{b_j}\right),$$

where y_j denotes the output of sector j, z_{ij} inputs from sector i and w_j inputs of primary production factors. The a_{ij} coefficients in the denominator specify the *minimum input requirements* from sector i to produce one unit of output in sector j. Since there is no substitutability between different types of inputs, firms will employ just the minimum amount of inputs to produce the output demanded by the market,

(2) $z_{ij} = a_{ij} y_j$.

The last term in the production function is the input of primary production factors w_j (value added) which enter with coefficient b_j (which in equilibrium equals $1 - \sum a_{ij}$ under the CRS assumption). This part of the model is not well developed: it is just assumed that there is enough primary factors to supply all sectors of the economy (either because of elastic supply or flexible factor prices). The model is closed by treating final demand as an exogenous "variable".

Under these assumptions, the model boils down to a linear equation system of supply and demand,

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\boldsymbol{y}} + \underbrace{\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}}_{\boldsymbol{f}}.$$

where $\boldsymbol{y}$ denotes the production vector, $\boldsymbol{A}$ the input-output matrix per unit of output and $\boldsymbol{f}$ the final demand vector, and where the product of $\boldsymbol{A}$ and $\boldsymbol{y}$ gives the intermediate demands for inputs. The solution to this equation system (the general equilibrium of the economy) is,

(3) $\boldsymbol{y} = [\boldsymbol{I} - \boldsymbol{A}]^{-1} \boldsymbol{f}$,

where $[\boldsymbol{I} - \boldsymbol{A}]^{-\mathbf{1}}$ is the "Leontief inverse" that computes the total input requirements from each sector to produce the exogenous vector of final demand.[1]

One country open economy model

Let us now introduce exports and imports into the model. Let´s assume that we have data on the total export by sector ($\boldsymbol{x}$) whilst the import vector ($\boldsymbol{m}$) is further divided into intermediate and final goods. The demand from the world market is treated as an exogenous variable just as domestic final demand, whereas the demand for intermediate imported goods and services depends on the domestic production. The open economy model is described by two blocks of linear equations

(4a) $\boldsymbol{y} = \boldsymbol{A^D y} + \boldsymbol{f^D} + \boldsymbol{x}$,

(4b) $\boldsymbol{m} = \boldsymbol{A^M y} + \boldsymbol{f^M}$,

where the first block is the supply-equals-demand conditions for domestic goods (superscript ***D***) and the second block supply-equals-demand conditions for imported goods (superscript ***M***). The solution to this block-recursive equation system is:

(5a) $\boldsymbol{y} = [\boldsymbol{I} - \boldsymbol{A^D}]^{-\mathbf{1}} (\boldsymbol{f^D} + \boldsymbol{x})$,

(5b) $\boldsymbol{m} = \boldsymbol{A^M} [\boldsymbol{I} - \boldsymbol{A^D}]^{-\mathbf{1}} (\boldsymbol{f^D} + \boldsymbol{x}) + \boldsymbol{f^M}$.

Note that the open economy version of the Leontief model establishes a direct link between exports and imports flowing from the dual assumptions of fixed input coefficients and no substitutability between domestic and imported inputs.

The marginal impact of an external increase in exports ($\boldsymbol{dx}$) on the domestic production ($\boldsymbol{dy}$) and on the derived imports of intermediate products ($\boldsymbol{dm}$) is calculating by differentiating (5a) and (5b) with respect to $\boldsymbol{x}$:

$$(6a) \quad \boldsymbol{dy} = [\boldsymbol{I} - \boldsymbol{A^D}]^{-1}\boldsymbol{dx},$$

$$(6b) \quad \boldsymbol{dm} = \boldsymbol{A^M}[\boldsymbol{I} - \boldsymbol{A^D}]^{-1}\boldsymbol{dx}.$$

How much domestic value added does the export vector $\boldsymbol{x}$ contains? The *direct* export of value-added of the exporting sector equals: $\boldsymbol{v} \cdot \boldsymbol{X}$, where $\boldsymbol{v}$ is the vector of value-added coefficient and $\cdot$ the element-by-element operator (the Hadamard product). Firms may also export value-added *indirectly* by supplying inputs to other domestic firms that export. The indirect export of this value-added equals $\boldsymbol{v} \cdot \boldsymbol{A^D X}$. The forward supply chain may also go through two stages of domestic processing before export. The value added export of these sub-parts and sub-components is $\boldsymbol{v} \cdot (\boldsymbol{A^D})^2 \boldsymbol{X}$. Continuing the forward iteration to account for evermore indirect supply linkages to domestic firms that export, we get a geometric matrix series that converges to the following vector that tell us how much domestic value added each export product contains:

$$(7a) \quad \vec{\boldsymbol{v}}^{\boldsymbol{x}} = \boldsymbol{v} \cdot \{[\boldsymbol{I} - \boldsymbol{A^D}]^{-1}\boldsymbol{x}\}.$$

The arrow in $\vec{\boldsymbol{v}}^{\boldsymbol{x}}$ indicates that we have calculated the value added content of the vector by following the supply chains forward from each sector to the final point of exports. If we sum all elements of (7) we get the total value added content of the export vector of Australia, and if we sum all services sectors the value added exports of the services sector. We can also calculate how much of the value added that is exported by different sectors of the economy by substituting $\boldsymbol{diag}(\boldsymbol{x})$ for $\boldsymbol{x}$ in (7a),

$$(7b) \quad \vec{\boldsymbol{V}}^{\boldsymbol{x}} = \boldsymbol{v} \cdot \{[\boldsymbol{I} - \boldsymbol{A^D}]^{-1}\boldsymbol{diag}(\boldsymbol{x})\},$$

where $\boldsymbol{diag}(\boldsymbol{x})$ is a diagonal matrix of the export vector (exports on the diagonal elements and zeros on the off-diagonal elements), which in turn can be aggregated into the main sectors of the economy as in the main text (A, B, C, D-S).

The domestic input requirements by sector (the embodied domestic value added) is calculated by following the supply chains upstream instead of downstream as in the previous example,

$$(8a) \quad \overleftarrow{\boldsymbol{V}}^{\boldsymbol{D}} = [\boldsymbol{I} - \boldsymbol{A^D}']^{-1} \cdot \boldsymbol{v},$$

where $\boldsymbol{A^D}'$ is the transpose of the $\boldsymbol{A^D}$. The imported value added by each sector is calculated as

$$(8b) \quad \overleftarrow{\boldsymbol{V}}^{\boldsymbol{M}} = [\boldsymbol{I} - \boldsymbol{A^D}']^{-1}\boldsymbol{A^M}',$$

where $\boldsymbol{A^M}'$ is the transpose of $\boldsymbol{A^M}$.

Extending the Leontief model into an inter-country input-output model (ICIO) is straightforward. The starting point is the realisation that the world as a whole is a closed economy and hence can be modelled in the same way as a closed single country model. We formulate the ICIO-model in block matrix notation in order to distinguish between domestic and international transactions.

The OECD-WTO TiVA database used for this exercise is organized in three matrices,

$$(9) \quad \boldsymbol{y} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}}_{mn\times 1}, \quad \boldsymbol{A} = \underbrace{\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}}_{mn\times mn}, \quad \boldsymbol{F} = \underbrace{\begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mm} \end{bmatrix}}_{mn\times m},$$

where $\boldsymbol{y}$ is the global production vector ($\boldsymbol{y_j}$ is the output vector of country j = {1, 2, …, m}), $\boldsymbol{A}$ is the intermediate consumption matrix with domestic IO-links on the diagonal blocs ($\boldsymbol{A_{jj}}$) and international IO-links on the off-diagonal blocks ($\boldsymbol{A_{jk}}$), and where $\boldsymbol{F}$ is the final demand matrix by destination markets. In general equilibrium supply must equal demand in all sectors and countries, taking into account the intermediate consumption used in all production activities:

$$(10) \quad \boldsymbol{y} = \boldsymbol{Ay} + \sum_j \boldsymbol{F_j} = \underbrace{\boldsymbol{A}[\boldsymbol{I}-\boldsymbol{A}]^{-1}(\sum_j \boldsymbol{F_j})}_{intermediate} + \underbrace{(\sum_j \boldsymbol{F_j})}_{final} = [\boldsymbol{I}-\boldsymbol{A}]^{-1}(\sum_j \boldsymbol{F_j}),$$

where *j={1, 2, …, m}* is a country index and $\sum_j \boldsymbol{F}$ the global demand vector (formed by summing the final demand vector over all countries). The export matrix divided between intermediate exports and exports of final products is given by

$$(11) \quad \boldsymbol{x} = \boldsymbol{A^x y} + (\sum_j \boldsymbol{F_j^x}) = \underbrace{\boldsymbol{A^x}[\boldsymbol{I}-\boldsymbol{A}]^{-1}(\sum_j \boldsymbol{F_j})}_{intermediate} + \underbrace{(\sum_j \boldsymbol{F_j^x})}_{final},$$

where $\boldsymbol{A^x}$ is the export part of the $\boldsymbol{A}$-matrix (zeros on the block diagonal) and $\boldsymbol{F^X}$ the export part of the $\boldsymbol{F}$-matrix (zeros on the block diagonal). As in the single country model, the domestic value added content (ignoring returning value added after a production step abroad) is found by following the domestic value chain forward from every sector to the final point of exports,

$$(12) \quad \vec{\boldsymbol{v}}^{\boldsymbol{x}} = \boldsymbol{v} \cdot \{[\boldsymbol{I}-\boldsymbol{A^D}]^{-1}\boldsymbol{x}\},$$

where $\boldsymbol{A^D}$ consists of the domestic blocks of the $\boldsymbol{A}$-matrix with the zeros on the off-diagonal blocks.

By differentiating (9) and (10) with respect to the final demand vectors we can calculate the induced changes in the production and exports of Australia when the demand conditions changes abroad, where particular interest is paid in the main text to demand shifts in Asia and Oceania.

$$(13) \quad \boldsymbol{dy} = \underbrace{\boldsymbol{A}[\boldsymbol{I}-\boldsymbol{A}]^{-1}\sum_j \boldsymbol{dF_j}}_{intermediate} + \underbrace{\sum_j \boldsymbol{dF_j}}_{final},$$

$$(14) \quad \boldsymbol{dx} = \underbrace{\boldsymbol{A^x dy}}_{intermediate} + \underbrace{(\sum_j \boldsymbol{dF_j^x})}_{final},$$

$$(15) \quad \boldsymbol{d}\vec{\boldsymbol{v}}^{\boldsymbol{x}} = \boldsymbol{v} \cdot \{[\boldsymbol{I}-\boldsymbol{A^D}]^{-1}\boldsymbol{dx}\}.$$

Measuring and decomposing productivity at the industry and national level

This annex provides an introduction to the growth accounting methodology in the EU KLEMS manual (Timmer et al, 2007), which grew out of a research project on the productivity slowdown in the European Union funded by the European Commission in 2003-2008. Many other countries have now adopted this manual in their productivity research, including Australia. KLEMS stands for the five categories of inputs that are measured – Capital (K), Labour (L), Energy (E), Materials (M) and Services (S) – alongside the outputs of each sector in order to assess the residual growth in productivity, i.e. growth in output above and beyond the growth in inputs. The KLEMS manual is based on research of Jorgensen and Griliches (1967), Jorgenson, Gollop and Fraumeni (1987), OECD (2001), Jorgenson, Ho and Stiroh (2005) amongst others. A full account of the KLEMS methodology with applications to the European Union and the United States can be found in Timmer et al. (2010) and Jorgenson et al (2005).

Growth accounting at the industry level

Let $j = 1, 2, \ldots, N_j$ index industries in the KLEMS database. The volume index of output of industry j in year t is denoted Y_{jt}. The inputs of production are divided into five volume indices calculated from the national accounts and other statistical sources:

- K_{jt} is a volume index of capital services constructed from 11 classes of capital assets;
- L_{jt} is a volume index of labour services constructed from 18 classes of labour inputs distinguished by gender, educational achievement and age group.
- E_{jt}, M_{jt}, S_{jt} are volume indices of Energy (E_{jt}), Materials (M_{jt}) and Services (S_{jt}) constructed from the use-table of the input-output accounts.

The maximum volume of output that can be produced with the available inputs is called the "production possibility frontier" and is captured by a function f_{jt}

$$(1) \qquad Y_{jt} = f_{jt}\left(K_{jt}, L_{jt}, E_{jt}, M_{jt}, S_{jt}, A_{jt}^{Y}\right),$$

where A_{jt}^{Y} is a productivity index. Under the assumption of constant returns to scale and competitive markets for outputs (Y_{jt}), intermediate inputs (E_{jt}, M_{jt}, S_{jt}) and primary production factors (K_{jt}, L_{jt}), the value of output is equal to the value of the inputs,

$$(2) \qquad P_{jt}^{Y}Y_{jt} = P_{jt}^{K}K_{jt} + P_{jt}^{L}L_{jt} + P_{jt}^{E}E_{jt} + P_{jt}^{M}M_{jt} + P_{jt}^{S}S_{jt},$$

where $P_{jt}^{Y}, P_{jt}^{K}, P_{jt}^{L}, P_{jt}^{E}, P_{jt}^{M}, P_{jt}^{S}$ denotes the corresponding price index to each volume index. Using a translog parameterization of f_{jt} we can define productivity growth (A_{jt}^{Y}) as a Törnqvist (1936) index,

$$(3) \qquad \Delta ln\, A_{jt}^{Y} \equiv \Delta ln\, Y_{jt} - \sum_{Z} \bar{v}_{jt}^{ZY}\, \Delta ln\, Z_{jt}\ , \qquad Z \in \{K, L, E, M, S\},$$

where $\Delta ln\, Y_{jt} \equiv \ln Y_{jt} - ln\, Y_{jt-1}$ is the logarithmic volume growth of output from t-1 to t, $\Delta ln\, Z_{jt} \equiv \ln Z_{jt} - ln\, Z_{jt-1}$ the logarithmic volume growth of input Z_{jt}, and $\bar{v}_{jt}^{ZY} = 0.5\left(\frac{P_{jt}^{Z}Z_{jt}}{P_{jt}^{Y}Y_{jt}} + \frac{P_{jt-1}^{Z}Z_{jt-1}}{P_{jt-1}^{Y}Y_{jt-1}}\right)$ the average cost share of input Z_{jt} in the output of Y_{jt}. The assumption of constant returns to scale implies that that the input shares of $K_{jt},\ L_{jt}, E_{jt}\ M_{jt}$ and S_{jt} sum to one $\left(\sum_{Z} \bar{v}_{jt}^{ZY} = 1\right)$. Under this assumption we can use the observed cost shares of the measured inputs in the estimation of the residual productivity growth, i.e. the volume growth of output above and beyond the volume growth of inputs. The residual term is referred to as multi-factor productivity (MFP) in the EU KLEMS manual, also known as total factor productivity. MFP is usually interpreted as "disembodied" technological change that allows more output to be produced with given inputs. It is "disembodied" in the sense that it cannot be linked to any particular input. Technological change may also be "embodied" in, for instance, new generations of computers with faster processors and larger memories to store data. The latter should be captured in the measurement of capital services, but measurement errors cannot be ruled out in practice. See further Chapter 3.6 in Timmer et al (2010).

Rearranging (3) yields the standard growth accounting formula that decomposes the volume growth of output into the contributions of different categories of inputs and the residual MFP growth:

$$(4) \quad \underbrace{\Delta ln\, Y_{jt}}_{\substack{volume\ growth \\ of\ output}} = \underbrace{\bar{v}_{jt}^{KY}\, \Delta ln\, K_{jt}}_{\substack{contribution\ of \\ of\ capital\ services}} + \underbrace{\bar{v}_{jt}^{LY}\, \Delta ln\, L_{jt}}_{\substack{contibution\ of \\ of\ labor\ services}} + \underbrace{\bar{v}_{jt}^{XY}\, \Delta ln\, X_{jt}}_{\substack{contribution\ of \\ of\ int.\ inputs}} + \underbrace{\Delta ln\, A_{jt}^{Y}}_{\substack{contribution\ of \\ MFP\ growth}},$$

where the volume index of intermediate inputs can be decomposed into energy, materials and services inputs: $\bar{v}_{jt}^{XY}\, \Delta ln\, X_{jt} = \bar{v}_{jt}^{EY}\, \Delta ln\, E_{jt} + \bar{v}_{jt}^{MY}\, \Delta ln\, M_{jt} + \bar{v}_{jt}^{SY}\, \Delta ln\, S_{jt}$.

From a policy perspective it may be equally relevant to decompose the growth of *value added* as the growth of output since the value added determines the compensation to the primary factors. Indeed, assuming that wages and capital returns are equal to the marginal productivity in monetary terms, the compensation to the primary factors sum to the value added of the industry: $V_{jt} = P_{jt}^{K} K_{jt} + P_{jt}^{L} L_{jt}$. The decomposition formula for the volume growth of value added takes the following form,

$$(5) \quad \underbrace{\Delta ln\, V_{jt}}_{\substack{volume\ growth \\ of\ value\ added}} = \underbrace{\bar{v}_{jt}^{KV}\, \Delta ln\, K_{jt}}_{\substack{contibution\ of \\ of\ capital\ services}} + \underbrace{\bar{v}_{jt}^{LV} \Delta ln\, L_{jt}}_{\substack{contribution\ of \\ of\ labor\ services}} + \underbrace{\Delta ln\, A_{jt}^{V}}_{\substack{contibution\ of \\ of\ MFP\ growth}}$$

where $\bar{v}_{jt}^{KV} = \bar{v}_{jt}^{KY} / \bar{v}_{jt}^{VY}$ and $\bar{v}_{jt}^{LV} = \bar{v}_{jt}^{LY} / \bar{v}_{jt}^{VY}$ are the average share of capital and labour in value added, and $\bar{v}_{jt}^{VY} = 0.5 \left(\frac{V_{jt}}{P_{jt}^{Y} Y_{jt}} + \frac{V_{jt-1}}{P_{jt-1}^{Y} Y_{jt-1}} \right)$ the average share of value added in gross output, and $\Delta ln\, A_{jt}^{V} = \frac{1}{\bar{v}_{jt}^{V}} \Delta ln\, A_{jt}^{Y}$ the MFP growth in value added terms that is proportional to the MFP growth in gross output.

Aggregation

Industry accounts can be aggregated into broader sectors of the economy such as primary products, manufacturing, services and the total economy. The KLEMS manual use the direct aggregation approach of Jorgenson, Gollop and Fraumeni (1987),

$$(6) \quad \Delta ln\, Y_{t} = \sum_{j \in \Omega} \bar{w}_{jt}^{Y}\, \Delta\, ln\, Y_{jt},$$

where $\bar{w}_{jt}^{Y} = 0.5 \left(\frac{P_{jt}^{Y} Y_{jt}}{\sum_{j \in \Omega} P_{jt}^{Y} Y_{jt}} + \frac{P_{jt}^{Y} Y_{jt-1}}{\sum_{j \in \Omega} P_{jt}^{Y} Y_{jt-1}} \right)$ is the average two-period share of industry j in the aggregate revenue of all industries in set Ω, and where $\Delta ln\, Y_{jt}$ are substituted from equation (4). Ω can include any combination of industries that are of interest to aggregate for analytical or presentational reasons, for example all services sectors. Similarly, the aggregation of value added is done with the formula

(7) $$\Delta ln\, V_t = \sum_{j \in \Omega} \bar{w}_{jt}^{V}\, \Delta\, ln\, V_{jt},$$

where $\bar{w}_{jt}^{V} = 0.5\left(\frac{V_{jt}}{\sum_{j\in\Omega} V_{jt}} + \frac{V_{jt-1}}{\sum_{j\in\Omega} V_{jt-1}}\right)$ is the average two-period share of industry j in the aggregate value added of all industries in set Ω, and where $\Delta ln\, V_{jt}$ are substituted from equation (5). If Ω include all sectors in the economy, equation (7) decomposes the growth in the gross domestic product (GDP).

Labour inputs

The KLEMS manual defines 18 classes of labour inputs distinguished by gender (male/female), educational achievement (middle school or below, high school, college or above) and age group as a proxy for work experience (15-29, 30-49, and 50+).

Table B.2. Classification of labour inputs

Gender	Education	Age group
Male	Middle school or below (low)	15-29
Female	High school (medium)	30-49
	College or above (high)	50+

The volume index of labour services is constructed by weighing 18 classes of labour inputs formed from the combinations of gender, educational achievements and age group

(8) $$\Delta ln\, L_{jt} = \sum_{l} \bar{v}_{jt}^{l}\, \Delta ln\, H_{jt}^{l},$$

where H_{jt}^{l} is the number of hours worked by labour class l and $\bar{v}_{jt}^{l} = 0.5\left(\frac{P_{jt}^{l} H_{jt}^{l}}{P_{jt}^{L} L_{jt}} + \frac{P_{jt-1}^{l} H_{jt-1}^{l}}{P_{jt-1}^{L} L_{jt-1}}\right)$ the average share of l in the labour compensation of industry j in period t and t-1. The volume index of labour service can be decomposed into total hours worked and changes in the composition of the labour inputs,

(9) $$\Delta ln\, L_{jt} = \sum_{l} \bar{v}_{jt}^{l}\, \Delta ln\left(\frac{H_{jt}^{l}}{H_{jt}}\right) + \Delta ln\, H_{jt} = \Delta ln\, LC_{jt} + \Delta ln\, H_{jt},$$

where $H_{jt} = \sum_{l} H_{jt}^{l}$ is the total numbers of hours worked by all labour classes, and where $\Delta ln\, LC_{jt}$ stands for changes in labour composition. Another useful decomposition is between the three educational classes listed in Table B.1. For example, the low educational group (middle school or below) is constructed by weighing together the hours worked by (low, male, 15-29), (low, male, 30-49), (low, male, 50+), (low, female, 15-29), (low, female, 30-49) and (low, female, 50+), where the weights are given by the two-period average share of each sub-group in the labour compensation of the low educated group:

(10) $$\Delta ln\, L_{jt}^{e} = \sum_{l\in e} \bar{v}_{jt}^{l}\, \Delta ln\, H_{jt}^{l}, \qquad e = \{low, medium, high\}.$$

Capital inputs

The KIP database defines 11 classes of capital assets listed in Table B.3, of which three are classified as Information and Communication Technology (ICT). The estimation of capital assets by industries is done with a modified perpetual inventory method.

The volume index of capital services is constructed by weighing the 11 classes of capital inputs

$$(11) \quad \Delta ln\, K_{jt} = \sum_{k} \bar{v}_{jt}^{k}\, \Delta ln\, K_{jt}^{k},$$

where $\bar{v}_{jt}^{k} = 0.5\left(\frac{P_{jt}^{k} K_{jt}^{k}}{P_{jt}^{K} K_{jt}} + \frac{P_{jt-1}^{k} K_{jt-1}^{k}}{P_{jt-1}^{K} K_{jt-1}}\right)$ is the average share of asset k in the capital compensation of industry j in period t and t-1. Capital services are in turn decomposed into ICT and non-ICT assets,

$$(12) \quad \Delta ln\, K_{jt}^{Z} = \sum_{k \in Z} \bar{v}_{jt}^{k}\, \Delta ln\, K_{jt}^{k}, \qquad Z = \{ICT, NICT\}.$$

Table B.3. Classification of capital inputs

Number	Asset	ICT
1	Residential structures	
2	Non-residential structures	
3	Infrastructure	
4	Transport equipment	
5	Computing equipment	x
6	Communications equipment	x
7	Other machinery and equipment	
8	Products of agriculture and forestry	
9	Other products	
10	Software	x
11	Other intangibles	

Sources of labour productivity growth

A common measure of labour productivity is value added per hour. If we combine (5) and (9) and subtract both sides with $\Delta ln\, H_{jt}$ we get a growth accounting formula for labour productivity:

$$(13) \quad \underbrace{\Delta ln\, V_{jt} - \Delta ln\, H_{jt}}_{\substack{change\ in \\ labor\ productivity}} = \underbrace{\bar{v}_{jt}^{KV}\left[\Delta ln\, K_{jt} - \Delta ln\, H_{jt}\right]}_{\substack{contribution\ of \\ capital\ deepening}} + \underbrace{\bar{v}_{jt}^{LV}\, \Delta ln\, LC_{jt}}_{\substack{contribution\ of \\ changes\ in\ the \\ labor\ composition}} + \underbrace{\Delta ln\, A_{jt}^{V}}_{\substack{contribution\ of \\ MFP\ growth}}.$$

Following Inklaar, Timmer and van Ark (2008), the capital deepening is divided into information and communication technologies (ICT) and other (NICT) assets.

$$(14) \quad \Delta ln\, K_{jt} = \bar{v}_{jt}^{ICT}\, \Delta ln\, K_{jt}^{ICT} + \bar{v}_{jt}^{NICT}\, \Delta ln\, K_{jt}^{NICT},$$

where $\bar{v}_{jt}^{ICT}$ and $\bar{v}_{jt}^{NICT}$ are the average shares of ICT and non-ICT assets in the capital compensation of industry *j* in period *t* and *t-1*. Substituting (14) into (13), yields the following growth accounting formula for labour productivity

$$(15)\quad \underbrace{\Delta ln\left(\frac{V_{jt}}{H_{jt}}\right)}_{\substack{change\ in\\ labor\\ productivity}} = \underbrace{\bar{v}_{jt}^{KICTV}\Delta ln\left(\frac{K_{jt}^{ICT}}{H_{jt}}\right)}_{\substack{contribution\ of\\ capital\ deepening\\ of\ ICT\ assets}} + \underbrace{\bar{v}_{jt}^{KNICTV}\Delta ln\left(\frac{K_{jt}^{NICT}}{H_{jt}}\right)}_{\substack{contribution\ of\\ capital\ deepening\\ of\ non-ICT\ assets}} + \underbrace{\bar{v}_{jt}^{LV}\Delta ln\, LC_{jt}}_{\substack{contribution\ of\\ changes\ in\ the\\ labor\ composition}} + \underbrace{\Delta ln\, A_{jt}^{V}}_{\substack{contribution\ of\\ MFP\ growth}},$$

where $\bar{v}_{jt}^{KICTV} = \bar{v}_{jt}^{KV}\bar{v}_{jt}^{ICT}$ and $\bar{v}_{jt}^{KVNICT} = \bar{v}_{jt}^{KV}\bar{v}_{jt}^{NICT}$ are the value added share of ICT and non-ICT assets.

Notes

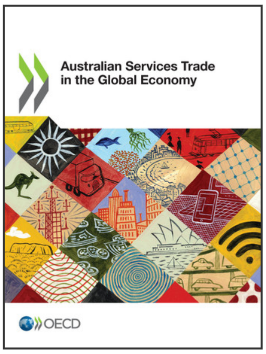
1 . As shown by Miller and Blair (2009, p. 33), provided that $a_{ij} \geq 0$ for all *i* and *j* and $\sum_{i=1}^{n} a_{ij} < 1$ for all j, the Leontief inverse is the solution to an infinite geometric series of $\boldsymbol{A}$,

$$[\boldsymbol{I} - \boldsymbol{A}]^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots ,$$

which is the analogue to a geometric series in standard algebra: $[1 - a]^{-1} = 1 + a + a^2 + a^3 + \cdots$ for $|a| < 1$. The reason why increasingly higher powers of $\boldsymbol{A}$ enter the market clearing condition,

$$\boldsymbol{y} = [\boldsymbol{I} - \boldsymbol{A}]^{-1}\boldsymbol{f} = \boldsymbol{f} + \boldsymbol{Af} + \boldsymbol{A}^2\boldsymbol{f} + \boldsymbol{A}^3\boldsymbol{f} + \cdots = \underbrace{\boldsymbol{A}[\boldsymbol{I} - \boldsymbol{A}]^{-1}\boldsymbol{f}}_{\substack{intermediate\\ consumption}} + \underbrace{\boldsymbol{f}}_{\substack{final\\ consumption}}$$

is that the suppliers of inputs use inputs themselves, which in turn are produced with yet other inputs all the way back to the initial production stage. In equilibrium, the production of each industry must satisfy both the final demand $\boldsymbol{f}$ and the intermediate needs of all sectors in the economy $\boldsymbol{A}[\boldsymbol{I} - \boldsymbol{A}]^{-1}\boldsymbol{f}$.



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