METHODS AND CODES FOR ESTIMATION OF TOLERANCE IN RELIABLE RADIATION-FREE HIGH-POWER LINAC

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Abstract

Successful development of a reliable and normal hands-on maintainable high-power linac requires minimisation of beam losses along the accelerator. The high sensitivity of the high-power linac focusing channel to random perturbation encourages designers to concentrate their attention on the tolerance estimation problem. The degradation of beam parameters, including transverse size and emittance growth, is caused by channel and beam parameter perturbations. The calculation of channel sensitivity permits to estimate the influence of each perturbing factor on the beam parameters and to determine the main source of perturbation, to find perturbing factors compensation possibilities and to determine the factors required to guarantee beam passage throughout the real channel without losses. Methods for the estimation of tolerance in high-power linac accelerating-focusing channels are considered in the present paper. The realisation of these methods in the LIDOS code package is presented. Monte Carlo simulation results for various types of accelerating channels are discussed.

Introduction

It is generally acknowledged that the main source of particle losses in the extended linac channel is the Coulomb field of space-charge-dominated beam. However, there are the non-Coulomb effects concerned with external field perturbations (external perturbations) and their influence on beam dynamics is comparable with the action of space charge.

It is convenient to split external perturbations into two groups: constructive (regular) perturbations caused by the distinction of a real structure from an ideal one and perturbations caused by random parameter deviations within given tolerances. In the latter case only the probability that the beam size will be no more than a given value can be found. In connection with the strict requirement for channel transparency and also with the complex procedure for channel retuning and readjustment, the confidence level chosen must be sufficiently large.

The calculation of the accelerating/focusing channel is always based on the specific mathematical model involving a description of the external accelerating and focusing forces.

In this model as a rule, the focusing field linear dependence on transverse co-ordinates and accelerating field axis distribution is used. The channel based on such models will be called ideal. Of course, there are no unprojected losses in the ideal channel.

The degradation of output beam parameters, including transverse size and emittance growth, is caused by channel and beam parameter perturbations (not always small). The influence of perturbations upon the beam output parameters is determined by quantity, which has come to be known as the channel sensitivity. The goals of beam dynamics investigations are:

- Knowledge concerning channel sensitivity – each perturbing factor influences output beam parameters.

- The possibilities of compensating for perturbing factors.

- Redundant factor determination guaranteeing beam passage throughout the real channel without losses.

The cardinal problem is to choose the channel with minimum sensitivity in order to minimise the beam losses during the process of accelerating.

Below we consider the main sources of perturbations, perturbation field characteristics and their influence on output beam parameters.

Main perturbations in different channels

RFQ linac

Random errors independently arising in each cell are the most dangerous. In this case there is a possibility of unfavourable realisation with each subsequent cell amplifying the disturbances of all previous ones.

Let us consider possible types of random perturbation, disturbed field characteristics, their influence on initial beam parameters, and the possibility of theoretical and numerical investigations of the different factors.

- *Electrode modulation deviation*. Main effect – changes in accelerating field amplitude, leading to coherent longitudinal oscillations and longitudinal repulsion. Result – longitudinal phase volume increase at the output of accelerator. Weak effect – Changes in focusing field and transverse effective emittance increase produced by these changes.

- *Aperture radius deviation.* Main effect – changes in focusing field gradient, deviation of accelerator axis from ideal line (axis may be presented as polygonal line), quadrupole symmetry violation. Result – transverse beam mismatching, coherent beam oscillations about real accelerator axis and transverse phase volume increase produced. Weak effect – appearance of new random non-linear field harmonics, changes in accelerating field.

Linac with focusing by quadrupole magnet lenses

- *Lens end displacements regarding linac axis.* These perturbations turn up as a result of errors in lens settings as well as a result of errors during lens manufacturing. Main effect – beam coherent oscillations regarding linac axis and effective emittance growth.

- *Deviations of focusing field gradient in lenses.* Main effect – beam gradually mismatching and growth of transverse oscillation amplitude. In this case beam emittance is retained but effective emittance grows.

- *Median plane rotations of quadrupole lens.* Main effect – beam gradually mismatching and growth of tranverse oscillation amplitude. In this case relations between X-oscillations and Y-oscillations turn up, leading to beam emittance growth. The effective emittance also grows.

- *Distinction of vane shape from hyperbole shape.* Linear focusing fields are possible only in the case when vanes have hyperbole form. In practice focusing fields contain non-linear components. There are non-linear components connected with the vane real shape. These components are the same for all lenses and they will be designated as constructive non-linearity. Other non-linear components can be designated as random non-linearity and they are connected with errors during vane manufacturing. Main effect – a coupling between X- and Y-oscillations leading to phase trajectory deformations and effective emittance growth. If the ratio of transverse oscillation period length to focusing period length is near to integer then resonance events can take place. In the non-resonant case maximal amplitude of transverse oscillations does not increase with growth of focusing period number. Random non-linearity also leads to effective emittance growth along with growth of focusing period number.

Linac with focusing by solenoid lenses

- *Lens end displacements regarding linac axis*. Both effects and methods of study are the same as for quadrupole lenses.

- *Deviations of solenoid field induction.* Effects and methods of study are the same as for deviations of focusing field gradient in quadrupole lenses.

Channel sensitivity definition

The mathematical foundation and main treatments are given in the Refs. [1-5], as are the tolerance estimations for various types of focusing and accelerating channels. Below, we investigate the parameters determining the sensitivity of the long channel to external perturbations.

Let us consider as an example the motion of particles in the focusing channel with random errors in focusing field gradients:

$$x'' + \tilde{G}(z)\, x = 0$$

Let us consider $\tilde{G}(z)$ as $\tilde{G}(z) = G(z)\,(1 + \alpha(z))$, where $G(z)$ is non-perturbed field gradient and $\alpha(z)$ is a function for perturbation description.

As shown in the above-cited works, it is necessary to transform the variables x, dx/dz to phase variables r,η. The variable r^2 is quadric in $x,dx/dz$, describing the ellipses matched with the ideal periodical channel. The variable η is an addition due to external perturbations. As a result, the main equation takes the form:

$$\begin{cases} r' = r\,\alpha\, G\,\rho^2 \sin^2(\mu z + \eta + \psi) \\ \eta' = \alpha\, G\,\rho^2 \cos^2(\mu z + \eta + \psi) \end{cases}$$

where μ is phase advance, and ρ is the beam envelope with an emittance of unity in the ideal channel.

The right part of the second equation does not depend on r, so it can be solved separately. If $\eta(z)$ is the solution of the second equation, then the solution of the first equation has the following form:

$$\frac{r(z)}{r_0} = \theta(z) = \exp\left(\int_0^z \alpha G\rho^2 \sin(\mu t + \eta + \psi)\cos((\mu t + \eta + \psi)dt) = \exp(F(z))\right)$$

It can be seen from the obtained relation that distribution of the random value $\theta(z)$ is determined by distribution of random value $F(z)$.

If the focusing channel contains N periods, then $F(z)$ can be presented as the sum of the statistical independent components adequate to corresponding periods L_k:

$$F(N) = \sum_{k=1}^{N} F_k = \sum_{k=1}^{N} \int_{L_k} \alpha G\rho^2 \sin(\mu t + \eta + \psi)\cos(\mu t + \eta + \psi)dt$$

Using an approximate solution of the second equation:

$$\eta(N) = \varphi_0 + \int_0^N \alpha G\rho^2 \cos^2(\mu t + \varphi_0 + \psi)dt = \varphi_0 + \sum_{k=1}^{N} \eta_k = \varphi_0 + \sum_{k=1}^{N} \int_{L_k} \alpha G\rho^2 \cos^2(\mu t + \varphi_0 + \psi)dt$$

we obtain with an accuracy of α^3:

$$F_k = \int_{L_k} \alpha G\rho^2 \left(\tfrac{1}{2}\sin^2(\mu t + \varphi_0 + \psi) - \cos^2(\mu t + \varphi_0 + \psi) \sum_{i=1}^{k} \eta_k \right) dt$$

Suppose $\alpha(z)$ is piecewise constant function that is equal to the relative error of the field gradient inside any element of the channel. In this case, the integral is calculated only along the part of the period containing the perturbation. If σ^2 is the mean square value of the field gradient tolerance, then using statistical independence of errors for different focusing periods and averaged over oscillations we obtain relations for the mean value and dispersion of $F(N)$:

$$M[F(N)] = D[F(N)] = \frac{\sigma^2}{8}\sum_{k=1}^{N}\left(\left|\overline{G\rho^2}\right|\right)_k^2 = \frac{\sigma^2}{8}\sum_{k=1}^{N} S_k$$

As shown in Ref. [4], if σ^2 is the mean square value of the focusing field gradient relative error then the probability that effective emittance growth would be no more than x is determined by the function $P(x) = 1 - e^{-(\ln x)^2/\Delta^2}$, where $\Delta^2 = \frac{\sigma^2}{4}\sum_{k=1}^{N} S_k$. If σ^2 is the error of axis transverse displacement then the probability that the centre of the output beam displacement would be no more than x is determined by the function $P(x) = 1 - e^{-x^2/\Delta^2}$, where $\Delta^2 = \sigma^2\sum_{k=1}^{N} S_k$. We can give the definition of S_k as the sensitivity of a period numbered k. Then total sensitivity of the channel will be the sum of period sensitivities $S = \sum_{k=1}^{N} S_k$.

The regions of stability diagrams for RFQ and some types of quadrupole channels are shown in Figures 1-4. In these regions the isolines of transverse oscillation frequency μ and sensitivity $S = \left|\overline{Q\rho^2}\right|^2$ are plotted. We use traditional co-ordinates: focusing parameter $B = \frac{eU}{m_0 c^2}\kappa\left(\frac{\lambda}{a}\right)^2$ and de-focusing parameter $A = \frac{\pi^2 eU\sigma}{4W_s}\left|\sin\varphi_s\right|$ for the RFQ diagram. For the quadrupole channels we use focusing parameter $Q = \frac{eGL^2}{m_0 c\beta\gamma}$, where G is the focusing field gradient, L is the length of the focusing period, and de-focusing parameter $A = \frac{\pi e E_m L^2}{m_0 c^2 (\beta\gamma)^3 \lambda}\left|\sin\varphi_s\right|$, where E_m is the accelerating wave amplitude and λ is the RF field wave length. In Figures 1-4 μ-isolines correspond to the values (from bottom to top) $\mu = 0$, $\mu = \pi/3$, $\mu = \pi/2$, $\mu = 2\pi/3$, $\mu = \pi$.

Figure 1. Sensitivity *S* of RFQ channel

1 – 40, 2 – 50, 3 – 60, 4 – 70, 5 – 80 (from left to right)

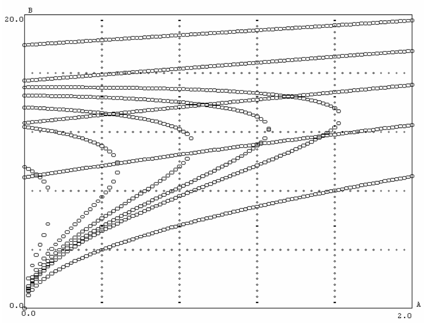


Figure 2. Sensitivity *S* of FODO channel

1 – 30, 2 – 35, 3 – 40, 4 – 45, 5 – 60 (from left to right; Gap/Period = 0.5)

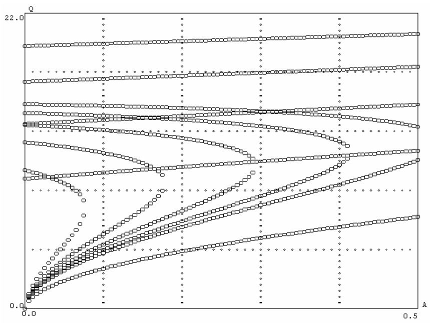


Figure 3. Sensitivity *S* of FDO channel

1 – 70, 2 – 85, 3 – 100, 4 – 115, 5 – 130 (from left to right; Gap/Period = 0.8)

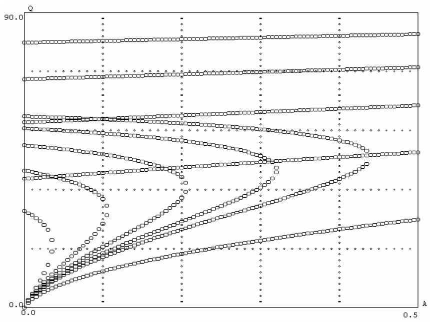
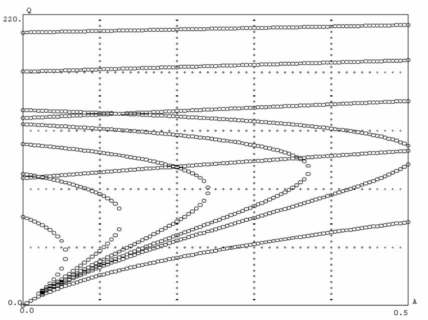


Figure 4. Sensitivity *S* of FDO channel

1 – 100, 2 – 130, 3 – 160, 4 – 190, 5 – 220 (from left to right; Gap/Period = 0.9)



As is seen from these diagrams, even within the boundaries of the operating region of stability diagram the channel sensitivity can be varied two times by value. So for choice and calculation parameters of the long linac channel the reported results must be taken into account.

Numerical estimations of perturbation influence

Extensive studies of different types of perturbations are being carried out in support of the analytical estimations cited above. Numerical results are in reasonably good agreement with analytical ones. This allows establishing the analytical estimations as good for beam transverse size and emittance growths under perturbations.

Below, we consider several focusing channels as examples and compare their sensitivity as regards random perturbation action.

RFQ channel

Let us consider as an example an RFQ channel operating at 352 MHz and accelerating protons from 0.1 to 5 MeV. The channel length is 8 m, vane voltage was changed from 100 to 123 KV, and bore radius changes along the channel from 4.1 to 3.6 mm (matching section not included). Statistical simulation of beam dynamics was performed in order to estimate variations of output beam parameters due to possible deviations of accelerating and focusing field inside cells. The change of the average radius position for each cell was considered as a perturbation factor. It was assumed that the average radius inside the cell varies linearly. The random value distributed uniformly inside [-δr, δr] was added to the top value of cell average radius. Random values for different cells were chosen independently. Beam simulations were performed for δr =10 μm and δr =25 μm. Beam energy losses, phase width, momentum spread, transverse size, beam centre displacement, beam RMS and total emittances were calculated as integral characteristics. The values were integrated over all particles within one RF period.

Calculations were performed by LIDOS.RFQ.Designer. In order to reduce computation time they were carried out in ideal RF fields. An "ideal" result (δr = 0) was also obtained in ideal RF fields. The number of random realisations was 50 for each type of error.

In general, it can be stated that δr = 10 μm is acceptable whereas δr = 25μm is dangerous, as a probability exists that beam size growth will be large and will be lead to additional beam losses. Thus with δr = 25 μm summary power dissipated on channel walls reaches 2.5 KW with a probability of 0.8.

Figure 5. Lost particle total power

In ideal channel lost particle total power equals 0.134 kW

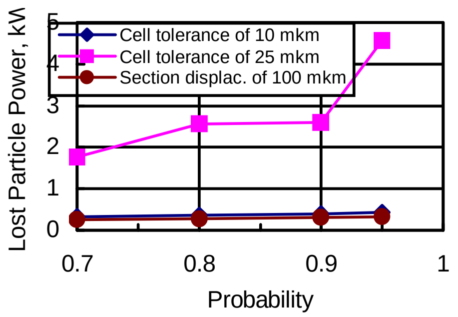
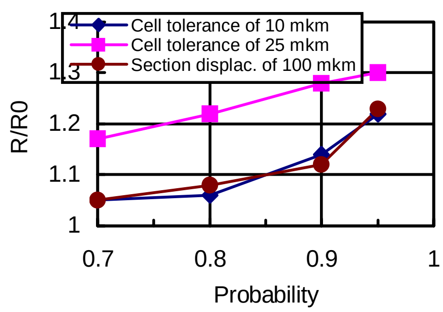


Figure 6. Beam transverse sizes R/R_0

Ideal value equals 1.00



Focusing channel with magnet quadrupole lenses

Let us consider an FDO channel with 300 focusing periods. Suppose that the lens takes 0.1 of the period. For $s = 0.05$ mm we obtain a confidence level for a beam oscillation amplitude, which is no more than 3 mm, of only 0.3.

If we want the same limitation of 3 mm to take place with a probability 0.9, then the necessary RMS value of primary error must be 0.018 mm.

If in the above FDO channel the RMS value of the magnetic field tolerance is equal to 0.5%, the probability of beam transverse size increasing by a factor of no more than 1.2 is equal to 0.2. This probability is equal to 0.9 when the same tolerance RMS value is 0.17%.

Taking into account the lens rotation influence for the considered case we determine that with a probability of 0.9 the beam size increasing factor is no more than 1.3 when the RMS value for the lens rotation angle is equal to $\sqrt{\overline{c^2}} = 0.2$.

Solenoid lenses focusing

If we consider the same period as in the previous examples, where lens length takes 0.1 of period length, then for a quadrupole and solenoid channel sensitivity comparison we obtain the approximate relations $Q_{quad} » 4Q_{sol}$ and $r^2_{quad} » 1.7r^2_{sol}$. From these relations we determine that with the same RMS displacement of the lens ends we obtain approximately the same statistics as with beam centre displacement. In order to receive with the same probability uniform growth of effective emittance, we must have an RMS value of $s_{DH/H}$ 2.5 times bigger then $s_{DB/B}$. All of these elements, along with a missing of rotation error for the solenoid lens, render the solenoid channel much less sensitive to perturbations than the quadrupole one.

The relations and examples cited above show that in an extended focusing channel, external perturbations are of considerable importance for the understanding of effective emittance growth. Ignoring external perturbations can lead to intolerable losses.

Compensation of external perturbations

Compensation of focusing channel axis displacements

The basic sources of axis displacements are inexact channel adjustment and lens magnetic axis displacements. For the partial correction of the first source the adjustment method with maximum correlation between lens displacement must be used. In this case the linac axis may be presented as a smooth curve which is superposed by a random component. A smooth curve does not cause coherent amplitude growth, and the total perturbation effect will decrease.

The magnetic axe displacement cannot be compensated. In order to suppress coherent oscillation amplitude, sections for beam transverse position correction are inserted. In an ideal case, at the output of the correction section the beam runs axially. Correction methods and algorithms were considered in Ref. [5].

Compensation of magnetic field gradient deviations

This perturbation effect may be depressed by establishing a correlation between deviations of adjacent lens magnetic fields. It can be done if there are electromagnetic quadrupoles. The correlation takes place when the lens groups are energised from one power supply. The number of independent error sources reduces and the perturbation effect depresses.

Compensation of quadrupole lens rotations

In this case, the correlation cannot be introduced and this is the reason why requirements concerning the lens rotation tolerance must be strict. In order to eliminate these perturbations, the quadrupole lenses should be replaced by solenoidal ones.

Monte Carlo simulation module for LIDOS.RFQ.Designer code

To choose RFQ channel optimal parameters, random perturbation influences have to be taken into account and an estimation of tolerances must be given. The goal of the LIDOS.RFQ.Designer package [6-11] development is to help the user to solve this problem.

The new version of the package contains the module "Statistics". Random realisations of a channel taking into account deviations of vanes from their ideal positions are generated. From cell to cell the random deviations are statistically independent. To compute the vane surface position we base our calculations on the following parameters: *r* is the distance from axis to cell beginning, *m* is the vane modulation and *L* is the cell length. With perturbations the parameter *r* is changed over a random value Δ. For a cell numbered *k* we use $\tilde{r}_k = r_k + \Delta_k$. In turn the deviations of any vane inside the cell are also statistically independent.

If the channel is divided into sections the other source of perturbation is the independent deviation of section vane ends.

The code simulates beam dynamics for every version of the channel. To decrease the time taken to calculate the statistics, the visualisation of the current version can be switched off. Because it takes the calculation of many random versions to obtain a sufficient statistic (as a rule no less than 50), this procedure is very time consuming, so it is possible to stop calculations and to continue at any time with the previous version.

The "Advisor" module offers supplementary visual information for tolerance estimation. Above, we define parameter *S* as sensitivity of period to perturbations. In this case the relation determining this parameter is as follows:

$$S = \left(B \int_0^1 \rho(\tau) \left|\cos 2\pi\tau\right| d\tau \right)^2$$

where *B* is a focusing parameter, ρ is the envelope of matched beam with the emittance of unity in the ideal channel.

The visual information presents the plot of sensitivity versus number of cells as well as statistic estimation concerning effective emittance growth and beam centre transverse displacement. The error integrals are calculated in accordance with the relations given above: if σ^2 is the mean square value of the focusing field gradient relative error then the probability that effective emittance growth would be no more than x is determined by the function $P(x) = 1 - e^{-(\ln x)^2/\Delta^2}$, where $\Delta^2 = \frac{\sigma^2}{4}\sum_{k=1}^{N} S_k$. If σ^2 is the error of axis transverse displacement then the probability that the centre of output beam displacement would be no more than x is determined by the function $P(x) = 1 - e^{-x^2/\Delta^2}$, where $\Delta^2 = \sigma^2\sum_{k=1}^{N} S_k$. For both cases S_k is the sensitivity of cell numbered k and N is the total number of cells.

Examples of output pictures from the "Statistics" module are shown in Figures 7-10. An RFQ channel with a frequency of 175 MHz and accelerating protons with energies ranging from 0.05 MeV to 2 MeV have been used as an example. The statistics were calculated for two values of tolerance: 10 μm (Figures 7 and 9) and 25 μm (Figures 8 and 10).

Figure 8. Statistics of *RMS-Emittance XX'*

Mean – 0.28, *sigma – 0.01*, *ideal – 0.28*

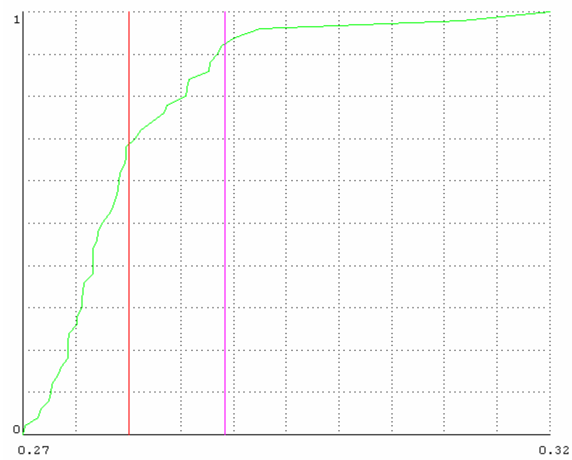


Figure 9. Statistics of *RMS-Emittance XX'*

Mean – 0.34, *sigma – 0.05*, *ideal – 0.28*

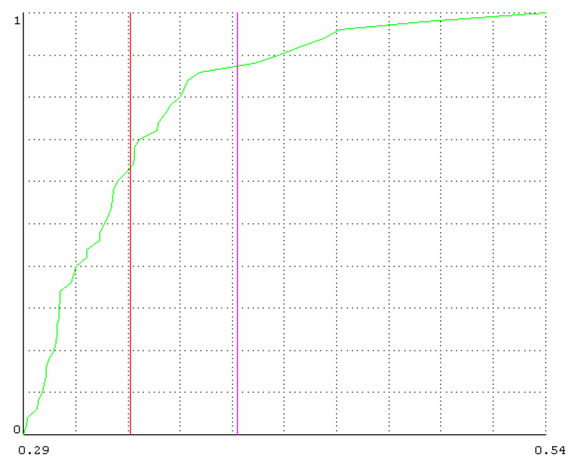


Figure 10. Statistics of *Total Emittance XX'*

Mean – 2.63, *sigma – 0.06*, *ideal – 2.60*

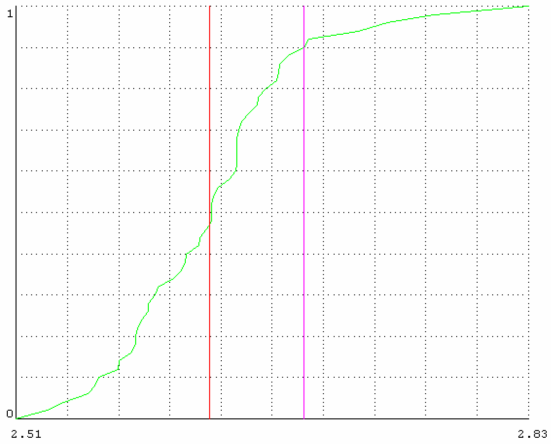
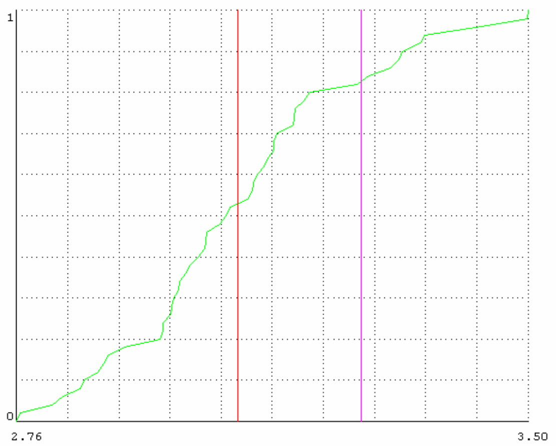


Figure 11. Statistics of *Total Emittance XX'*

Mean – 3.08, *sigma – 0.18*, *ideal – 2.60*



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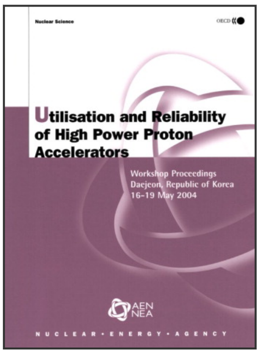

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