6 Opportunities to learn

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This chapter documents what the Study defined to be a "quadratic equations unit" and the degree to which that unit was implemented in similar ways across countries/economies. Variations in intended length and content are highlighted and, using teaching materials and teacher and student reports, variations in approaches to teaching the focal unit are described. Together, they shed light on what and how different types of opportunities to learn manifested in classrooms.

What students learn in the classroom is shaped by the curriculum. The curriculum defines the objectives, contents and expected outcomes of schooling, and embodies a country's vision and aspirations for its citizens. The curriculum is a window into the knowledge and skills students may need to lead a good life, successfully find their pathway in education and professionally, and become responsible citizens. The curriculum is also one of the most powerful tools in educational policy. For schools and teachers, the curriculum provides a road map and a benchmark guiding professional activities. For students and parents, the curriculum is a promise of what teaching offers them.

There is no single approach to the design and delivery of curriculum. Countries will choose different priorities and they will organise curriculum development differently. For example, the balance between national consistency and local diversity in curriculum varies considerably across countries (OECD, 2018[1]). This diversity makes international comparisons as challenging as they are valuable.

Furthermore, the intended curriculum often differs from that which is implemented. What is defined in mandated syllabi and standards, the *intended* curriculum, interacts with decision making at the school and teacher level to create an *implemented* curriculum in the classroom (Travers and Westbury, 1989[2]). This translates into different "opportunities to learn" (OTL) for students, which are a powerful determinant of their achievement growth (Burstein, 1993[3]; Kuger et al., 2017[4]) and later performance in international assessments (Scheerens, 2017[5]; Schmidt and Maier, 2009[6]).The concept of OTL refers to the subject matter as it is taught and experienced by students.

This chapter looks at opportunities to learn the mathematics unit of quadratic equations. By looking at a micro level, we can better understand how the intended curriculum interacts with teaching and becomes actual learning experiences for students. Exploring these variations at an international, comparative level can be valuable. Not only does it offer countries/economies rich insights into their own systems, but it can help them garner new perspectives that may also stimulate revision and reform.

Key findings

- With the exception of Germany*[1], Kumagaya, Shizuoka and Toda (Japan) (hereafter "K-S-T [Japan]"), and Shanghai (China), teachers typically spent less time teaching the focal unit than expected based on the official curriculum.
- There are large differences between countries in the total teaching time for the unit, reported by teachers, with around 6 to 7.5 hours reported in Colombia, England (UK), Mexico and Shanghai (China), and 10 to 14 hours in Biobío, Metropolitana and Valparaíso (Chile) (hereafter "B-M-V [Chile]"), Germany* and K-S-T (Japan).
- Except for in Shanghai (China), significant variation existed within countries/economies between the median and maximum number of lessons teachers spent on the focal unit, the difference being as high as 16 additional lessons in B-M-V (Chile).
- Teachers encouraged more conceptual reasoning in B-M-V (Chile), Madrid (Spain) and Shanghai (China), and more engagement with graphical representations in England (UK) and Germany*. In K-S-T (Japan) and Shanghai (China) graphical materials were almost non-existent.

- Teachers sequenced content in different ways. For example, teachers introduced quadratic functions early on and faded them off in Germany* and Colombia, whilst in B-M-V (Chile) they were more prominent later in the topic.
- The more cognitively demanding aspects of solving quadratic equations were not common across classrooms. “Completing the square” was relatively popular in Germany*, K-S-T (Japan) and Shanghai (China) but otherwise rarely seen, and “finding roots in quadratic functions” was significantly used only in Colombia, England (UK) and Germany*. In Madrid (Spain) and Mexico, both approaches remained very rare.
- Opportunities to engage in reasoning – e.g. to determine the number and kind of solutions of a given equation – were related to students’ perceptions of high-quality mathematics teaching.

Looking deeper into the foundations of mathematics literacy

Traditionally, mathematics education in lower secondary schools focused on algebraic knowledge and skills; but more recently, the focus of mathematics education has shifted towards more fundamental concepts of mathematical reasoning and literacy. PISA 2018 defines mathematical literacy as “an individual’s capacity to formulate, employ and interpret mathematics in a variety of contexts” (OECD, 2019[7]). Manipulating algebraic expressions and equations is considered a tool for employing mathematics in diverse contexts, rather than just a procedural skill.

The Study examines in great detail students’ opportunities to learn quadratic equations, its focal unit. Quadratic equations is one of the most demanding topics in school algebra (Graf et al., 2018[8]; Kabar, 2018[9]; Kaur, 2014[10]). It consists of procedures (e.g. methods for solving equations) with conceptual understanding (e.g. the concepts of variables and equations) and applications both within mathematics and real-world contexts (see Box 6.1), which can be considered foundations of mathematical literacy (Kieran, 2007[11]; OECD, 2019[7]).

Examining opportunities to learn through the lens of quadratic equations offers a window into what international variation exists in terms of the approaches to teaching algebra and to nurturing mathematically literate students. The opportunities to learn algebra – specifically linear and quadratic equations – were a powerful predictor of students’ performance in PISA 2012 (Kuger et al., 2017[4]) and vary considerably across countries (Kuger, 2016[12]; OECD, 2011[13]).

While previous international studies described variations in curriculum at large, the Study examines the opportunities to learn for the selected focal unit through a variety of data. These include teaching materials from the classroom; a Teacher Log where teachers documented the date, duration and coverage of certain subtopics for each individual lesson within the focal unit; and teacher and student questionnaires. Empirical findings based on these data were mapped against the intended curriculum as detailed in official curricula documents, syllabi, standards and textbooks.

The intended curriculum varies across countries

At the start of the Study, mathematics experts from all participating countries/economies submitted official curriculum materials to describe the intended curriculum for quadratic equations (see *Global Teaching InSights Technical Report* [hereafter the "Technical Report"], Chapter 2). The experts agreed on a joint definition of the focal unit, including a common starting point and a list of subtopics. This definition is presented in Box 6.1. As most countries/economies, with the exception of Germany* and Shanghai (China), split quadratic equations up into small sections of the curriculum, experts had to choose that section which most closely fits to the definition of the focal unit in their national context.

The position of the focal unit across grade levels and within the school year is not uniform. In England (UK) and Germany*, the focal unit may officially be taught any time across two or three consecutive school years. In Germany*, the intended curriculum varies across school tracks and states, while in England (UK), and sometimes in Germany*, schools have some flexibility in arranging curriculum across grade levels. On the contrary, in K-S-T (Japan) and Shanghai (China), there is very little variation in when the topic of quadratic equations is taught.

Box 6.1. Focal unit: Quadratic equations

Prerequisite (taught before the start of the unit, maybe years before)**:** Binomial formula $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and other ways of handling algebraic expressions.

Start: Some examples of a quadratic equation are introduced with the goal to "solve for x".

Includes: Solving quadratic equations of a general form such as $ax^2 + bx + c = 0$, either through factorising (e.g. Vieta's rule), through completing the square or through the quadratic formula $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$; and sometimes (B-M-V [Chile], Colombia and Germany*) also through the reduced quadratic formula $x = -p/2 \pm \sqrt{p^2/4 - q}$.

End: Application of quadratic equations to everyday problems.

Variations:

- Different kinds of quadratic equations may be transformed into a standard form.
- Different cases depending on values of a, b, c (or with one, two or no real solutions) may be discussed.
- Graphs (of quadratic functions) may be shown and used for approximate solutions ("finding roots").

Note: This definition of the Focal Unit also specifies the subtopics assessed within the Study.
Blue: subtopics covered in the Teacher Log; underlined: subtopics covered by the teaching material codes.

Table 6.1 characterises each country's/economy's intended curriculum for the focal unit in relation to the grade level, student age, number of lessons and use of graphics. It is worth highlighting that:

- The "intended student age" where students would encounter the content of the focal unit is similar in most countries/economies. Quadratic equations are taught around age 14, with the exception of B-M-V (Chile), where students learn this topic at age 16.
- There were different intended lengths for the unit across countries. As an illustration, these might range roughly from about 6 lessons in Madrid (Spain) to 16 in Colombia.
- The inclusion of certain content varied. The curriculum in B-M-V (Chile), Colombia, England (UK) and Germany* integrated quadratic equations with graphical representations (i.e. quadratic functions). In the remaining countries/economies, teachers were expected to teach the focal unit independent of graphical material because quadratic functions would be covered separately in the same grade or even at a different grade level.
- Where the focal unit does include graphical material, higher number of lessons tend to be allocated to quadratic equations. For instance, B-M-V (Chile) and Colombia, who included graphical material in their unit design, envisioned at least 15 lessons while Madrid (Spain), Mexico and Shanghai (China) envisioned 10 lessons at most.

Table 6.1. Characteristics of the intended and the implemented quadratic equations curriculum

	Grade level		Student age		Number of lessons		Unit includes graphical material (quadratic functions)	
	Intended	Implemented	Intended	Implemented	Intended	Implemented	Intended	Implemented
B-M-V (Chile)	11	11	16	16.5	15	9 (max. 25)	Yes	Yes
Colombia	9	9	14	15.1	16	6 (max. 18)	Yes	Yes
England (UK)	8 - 10	8	14-15	14.8	6-12	7 (max. 15)	Yes	Yes
Germany*	8 - 10	9	14 - 15	14.9	11	13 (max. 19)	Yes	Yes
K-S-T (Japan)	9	9	14	14.8	13	12 (max. 18)	No	No
Madrid (Spain)	9	8	14	13.7	6	(Range 1 - 14)	No	Yes
Mexico	9	9	14	14.7	8 - 10	7 (max. 14)	No	Yes
Shanghai (China)	8	8	14	13.6	10	10 (max. 12)	No	No

Notes: **Intended values** were submitted by national mathematics education experts based on their official curricula and standards (if existing), approved teaching materials or range of exemplary school curricula as of June 2016 (see Technical Report, Chapter 2).
Implemented values are calculated from the Study data.
For implemented values, grade level indicates median classroom grade level, student age indicates median classroom mean student age, number of lessons indicates median number of lessons or full hours reported in the Teacher Log with maximum in brackets; "Yes" indicates that OTL for functions accounts for 10% to 30% of teacher-reported content coverage, while "No" indicates less than 2% accounted for by functions.
In Colombia, England (UK) and Germany*, the number of full hours is given instead of the number of lessons, because the length of lessons may vary between classrooms.
According to curricula on the national level in England (UK), the intended number of hours would be 6 – 12, plus some lessons on quadratic functions if integrated with algebra.
In Colombia, the focal unit includes graphical material (quadratic functions) for a sub-population of students which comprises the majority of students in grade 11 in this country.
In Madrid (Spain), the Teacher Log only contained information about more than one lesson for 35 out of the 85 participating classrooms. Therefore, Teacher Log data from Madrid (Spain) will not be interpreted in the present chapter.
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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The actual time spent was lower than that expected in the intended curriculum

Across countries/economies, the number of lessons or overall teaching time reported by teachers falls short of that established by their respective intended curriculum (Table 6.1). Only in Germany*, K-S-T (Japan) and Shanghai (China) did the teaching time of the implemented curriculum broadly align with that which was intended.

The total number of hours spent on the focal unit in some countries/economies is about half of that in other countries/economies (see Annex 6.A, Table 6.A.1). In Colombia, England (UK), Mexico and Shanghai (China) the total teaching time reported is around 6 to 7.5 hours. In contrast, teachers in B-M-V (Chile), Germany* and K-S-T (Japan) report spending about 10 to 14 hours on the focal unit.

There are also notable differences in the implemented curriculum within countries/economies. The difference between the typical number (median) of implemented lessons and the maximum number is very large in all countries/economies, with the exception of Shanghai (China). In B-M-V (Chile), for example, the teacher who invested the most teaching time reported 16 additional lessons compared to the 9 lessons of a typical teacher.

The differences in length and time spent on the focal unit might be explained by multiple factors. These include the level of fragmentation of the curriculum within a school year, the level of teacher autonomy in implementing the curriculum, local conditions such as school retreats or unforeseen circumstances. These variations suggest that students' opportunities to learn can be very different within countries.

Previous studies have shown that variations in opportunities to learn can have considerable implications in terms of equity, particularly as they are important drivers of student outcomes (Kuger, 2016[12]; OECD, 2011[13]; Patall, Cooper and Allen, 2010[14]). Schmidt et al. (2015) have argued, drawing upon PISA evidence of inequality in OTL and its relationship to student performance, that unequal opportunity to learn mathematics is "one of the key factors driving inequality in schools" (Schmidt et al., 2015[15]).

Students learn different algebraic procedures

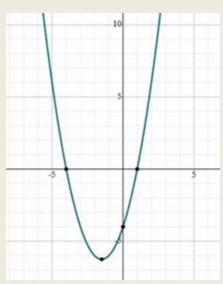
Teachers can prepare students to solve quadratic equations using at least four different approaches: factorising, using the quadratic formula, completing the square and using graphical material. Each of these methods requires different mathematical operations, as detailed in Box 6.2.

Box 6.2. Approaches for solving quadratic equations

As an illustration, the quadratic equation $x^2 + 3x - 4 = 0$ can be solved through the following four approaches:

- Factorising: $x^2 + 3x - 4 = (x + 4)(x - 1)$. The product equals zero if and only if one factor equals zero, which is the case if either $x = -4 \ or \ x = 1$.
- Using the quadratic formula: The solutions of $ax^2 + bx + c = 0$ are $x_{1,2} = (-b \pm \sqrt{b^2 - 4ac}\,) / 2a$. Substituting $a = 1, b = 3, c = -4$, after some transformations, we get $x_1 = 1$ and $x_2 = -4$.
- Completing the square: $x^2 + 3x - 4 = 0$ is equivalent to $x^2 + 3x = 4$. Adding $(3/2)^2$ to both sides of the equation, we get a squared term on the left, and the equation is equivalent to $(x + 3/2)^2 = 4 + 9/4$. Therefore, $x_1 + 3/2 = \sqrt{4 + 9/4}$ and $x_2 + 3/2 = -\sqrt{4 + 9/4}$. Since $\sqrt{4 + 9/4} = \sqrt{25/4} = 5/2$, applying some algebraic transformations, we get $x_1 = 1$ and $x_2 = -4$.
- The fourth approach applies graphical material:

  The given equation corresponds to the quadratic function $y = f(x) = x^2 + 3x - 4$.

  Inspecting the graph of this function, the roots, i.e. the values of x for which $f(x) = 0$, can be identified as -4 and $+1$. These roots are the solutions of the corresponding quadratic equation.

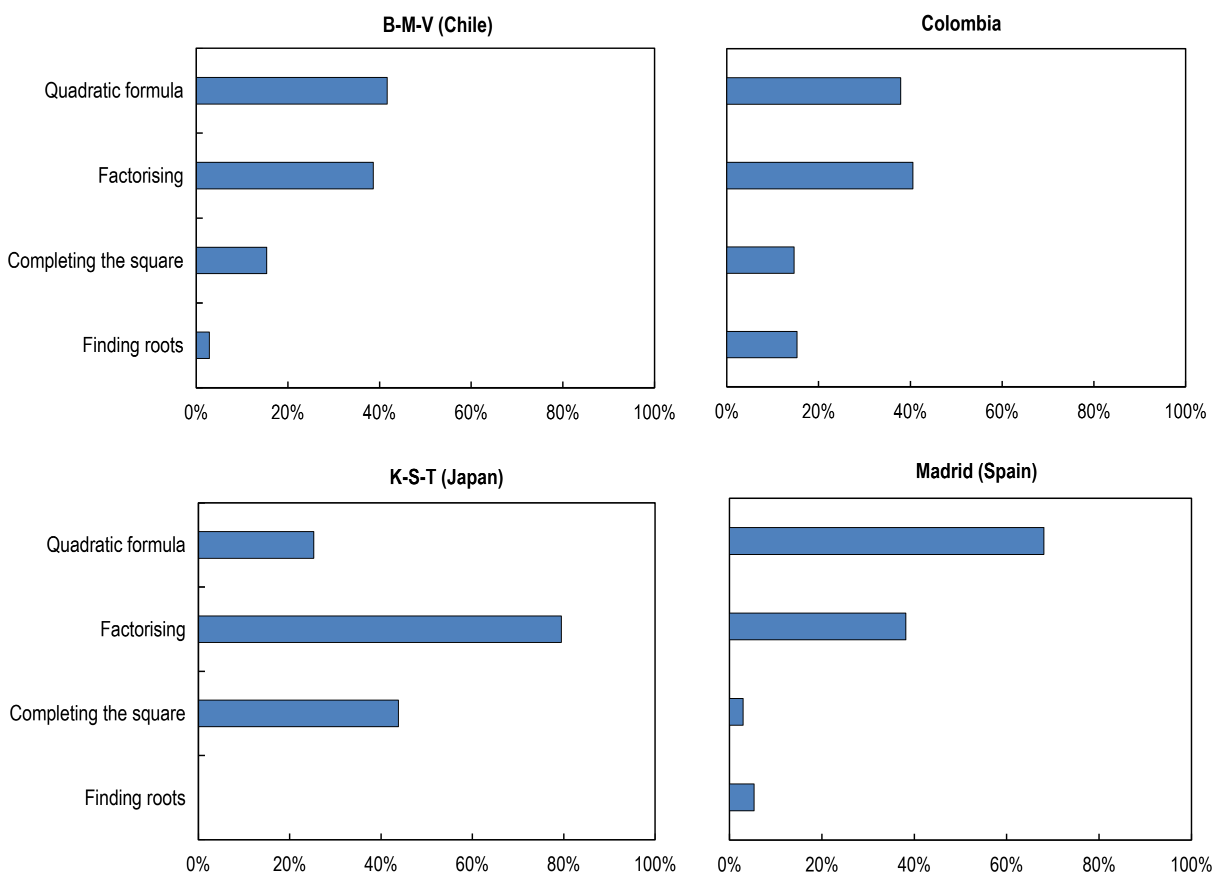
Teachers' choices of what algebraic procedures to teach and how to sequence the progression of learning has implications for students' opportunities to learn. Graf et al. (2018) identify four levels of skill in solving quadratic equations: i) finding solutions by trial and error ("inspection") with no understanding of any specific method; ii) familiarisation with one rationale to solve quadratic equations, most often the factorising method; iii) procedural competence in factoring rational coefficients, in using the quadratic formula and in finding roots graphically; and iv) full understanding of the correspondence between quadratic equations and functions, and the most demanding approach, completing the square.

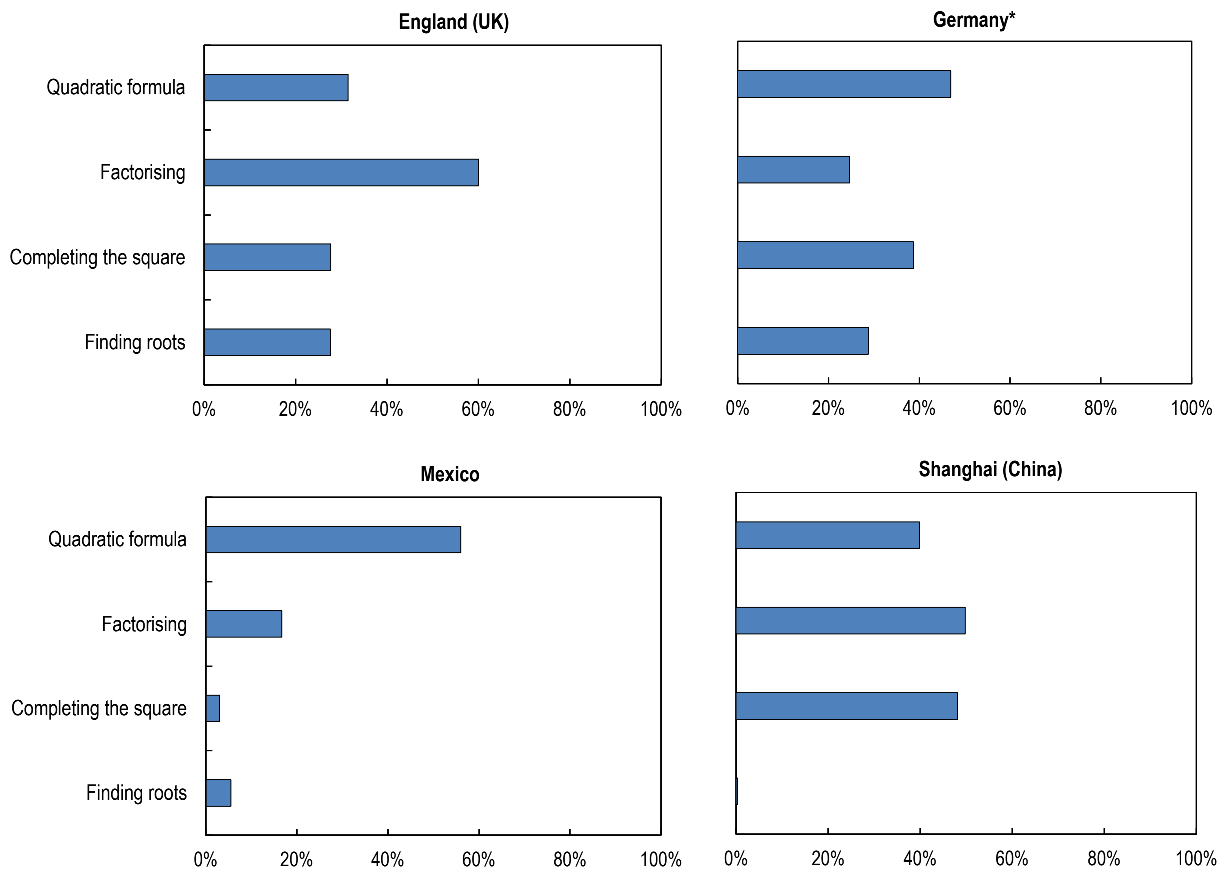
Across countries/economies, the teaching materials submitted by teachers covered minimally demanding approaches, as identified by Graf et al. (2018), more often than highly demanding approaches. However, there are important differences between countries/economies (Figure 6.1). "Factorising" appeared to be the most popular approach in England (UK) and K-S-T (Japan), while "using the quadratic formula" was clearly preferred in Madrid (Spain), Mexico and Germany*.

The more demanding procedures of "completing the square" and "finding roots in quadratic functions" are less common across countries/economies. In Madrid (Spain) and Mexico, teaching materials almost never addressed these approaches. “Completing the square” was relatively popular in Germany*, K-S-T (Japan) and Shanghai (China). “Finding roots in quadratic functions” was significantly used in Colombia, England (UK) and Germany*. These findings from teaching materials are also aligned with teachers’ reports on the methods taught to students, i.e. both are correlated at the classroom level (see Annex 6.A, Table 6.A.2).

Figure 6.1. Methods for solving quadratic equations as present in teaching materials

Proportion of teaching material sets that included the respective method





Notes: Proportion of lessons in which the teaching materials included any reference to the method, averaged across classrooms and raters.
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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Most notably, students did not learn quadratic equations through graphical material in K-S-T (Japan) and Shanghai (China), and rather seldom in B-M-V (Chile), Madrid (Spain) and Mexico. These differences are related to differences in the intended curriculum (Table 6.1) and suggest specificities in the traditions and cultures of mathematical education in each country/economy. This challenges the long-held assumption in some "Western" mathematics traditions that equations and functions need to be taught and learnt together (Leung et al., 2014[16]).

Teachers used different ways of fostering deeper understanding: conceptual reasoning, graphical representations and real-world applications

Teachers can provide students with richer opportunities to learn, not just by teaching them procedures for solving quadratic equations, but also by helping them to develop a deeper mathematical understanding. For example, students can gain conceptual understanding by comparing different types of quadratic equations and reasoning whether there are one, two or no real solutions. Students can also deepen their understanding through graphical representations, analysing the correspondence between quadratic equations and quadratic functions. They can also learn quadratic equations through real world applications, for example by calculating the area of geometrical spaces representing paths, enclosures etc., or the distance a car has travelled after a certain period of acceleration.

Across countries/economies, there was a strong emphasis on algebraic procedures in teaching quadratic equations according to the teaching materials submitted by teachers (Figure 6.2). This finding is not surprising, since quadratic equations require procedures such as transforming algebraic terms (e.g. transforming $5x + 4x$ into $9x$) and solving for x.

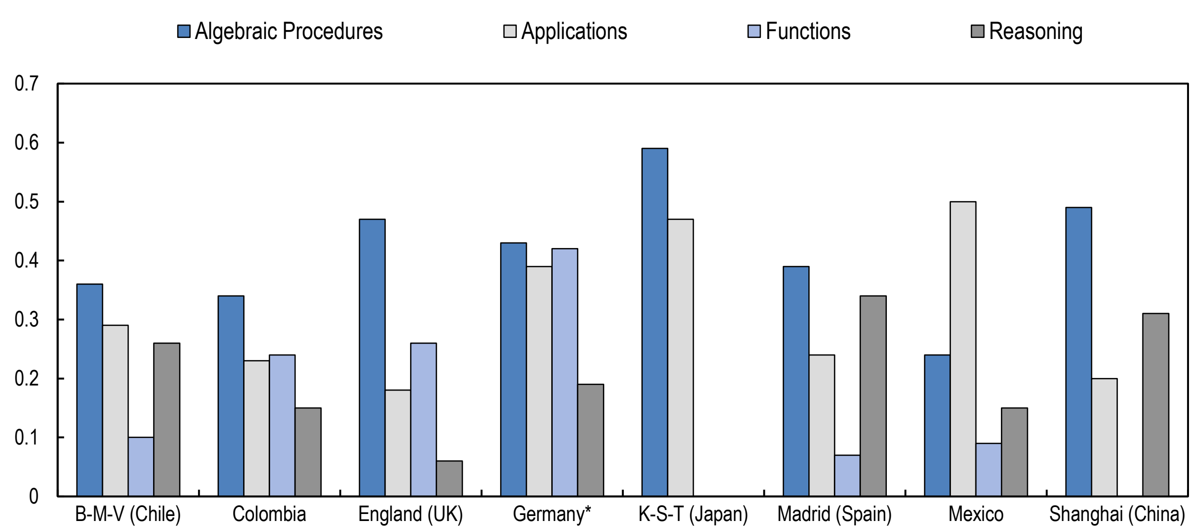
In addition, real-world applications account for a considerable share of learning opportunities in K-S-T (Japan), Germany* and Mexico. Real-world examples in Japanese classrooms are often set in geometrical contexts (with or without diagrams), in line with the traditionally strong focus on geometry (Klieme and Bos, 2000[17]). In Mexico, as documented by the Study's curriculum mapping (see Technical Report), examples typically referred to problems in mechanics.

Across countries/economies there are important differences regarding whether teachers tended to deepen student understanding through conceptual reasoning (comparing types of equations and reasoning about the number of solutions) or through graphical representations (connecting equations with graphs of quadratic functions). For example, teaching materials in B-M-V (Chile), Madrid (Spain) and Shanghai (China) more often provided opportunities to reason about types of equations and solutions, while quadratic functions were used relatively often in England (UK) and Germany*.

By and large, these findings based on the teaching materials submitted by teachers are similar to the information reported by teachers and students (see Annex 6.A, Table 6.A.3). One notable difference is that students reported some experience with quadratic functions and reasoning tasks in K-S-T (Japan). When comparing classrooms, findings from different measures are quite well aligned (see Annex 6.A, Table 6.A.4).

Figure 6.2. Teaching materials including opportunities to learn algebraic procedures, real-world applications, quadratic functions and reasoning

Proportion of lessons in which the teaching materials included any subtopic related to the respective types of OTL, averaged across classrooms, raters and subtopics



Notes: The subtopics are identified in Box 6.1. "Algebraic Procedures" includes the subtopics algebraic expressions, factorising, completing the square and quadratic formula. "Functions" includes the subtopics "graph of quadratic functions" and "finding roots". "Reasoning about types of equations and solutions" includes the subtopics "different cases" and "one, two or no solutions".
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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How teaching of quadratic equations unfolds across the unit varied

One of the biggest challenges in teaching is how to sequence and structure the learning goals of the curriculum unit. The learning progression map developed by (Graf et al., 2018[8]) suggests starting with inspecting some easy kind of quadratic equations, introducing factorisation and/or the quadratic formula later, and using quadratic functions towards the end of the focal unit. Applications should show up increasingly across the unit, as more and more complex modelling problems can be solved. The OTL data from the Teacher Log can be used to test this model. We would expect to find changing patterns of focal subtopics in various stages of the teaching and learning process.

Figure 6.3 provides a sketch of such a process analysis. The Teacher Log from each classroom was divided into three phases: phase 1 included the first 33% of the lessons documented, phase 2 covered the next 33% of lessons and phase 3 covered the rest of the unit. For each phase, an index for the strength of subtopic coverage across all classrooms in the country/economy was calculated. Thus, the lines in Figure 6.3 illustrate the "prototypical development" within each country/economy, rather than individual classrooms. This analysis has not been implemented for Madrid (Spain) because not enough lessons were reported in their Teacher Logs.

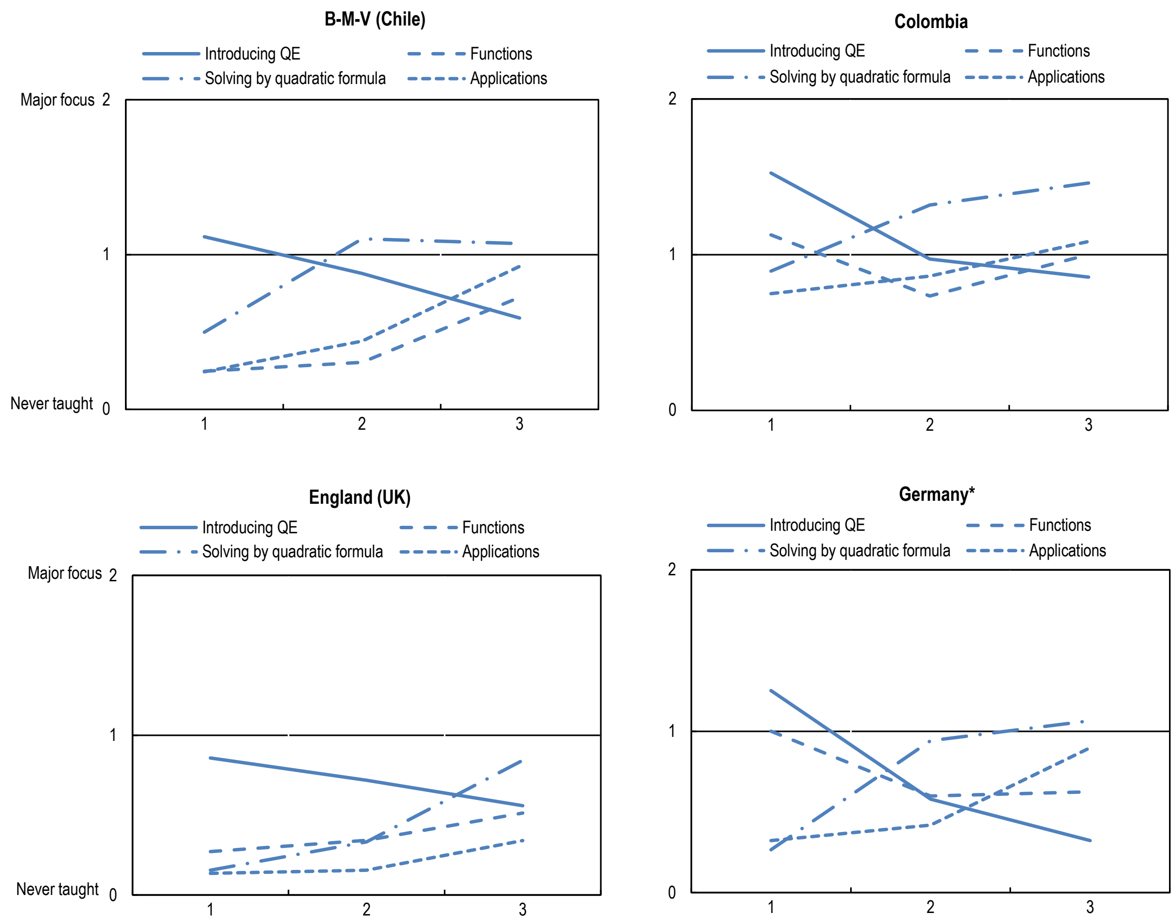
Figure 6.3 includes four subtopics reported by the teachers: introducing some form of quadratic equations, solving equations by the quadratic formula, real-world applications and quadratic functions. For each phase and each subtopic, the coverage of this subtopic, averaged across classrooms and individual lessons, is shown on the vertical (y) axis. This number varies between 0 and 2. A value of 0 means the subtopic was never taught within that phase. A value of 2 means the subtopic was a major focus of instruction in all lessons within that phase. A value of 1 could result from a mix of 0s and 2s, but it can also mean that the subtopic was covered as a minor focus (score 1) across all lessons in the respective phase.

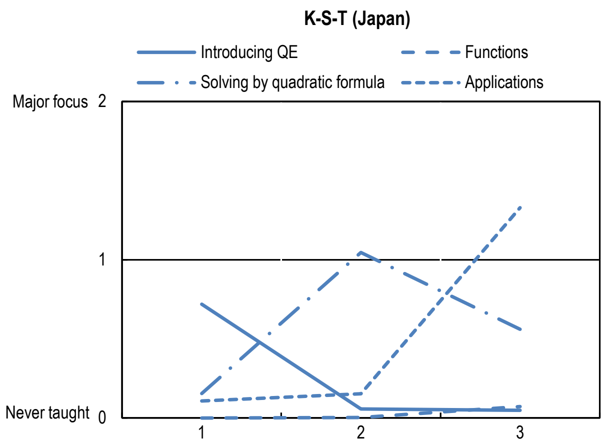
Across the seven countries/economies, the unit typically started with the introduction of different forms of quadratic equations, which was faded out in phase 2 when the quadratic formula was introduced. In phase 3, applications became more and more important, as envisioned by Graf, Fife, Howell and Marquez (2018[8]). This pattern is most clear-cut in K-S-T (Japan) and much less obvious in Mexico.

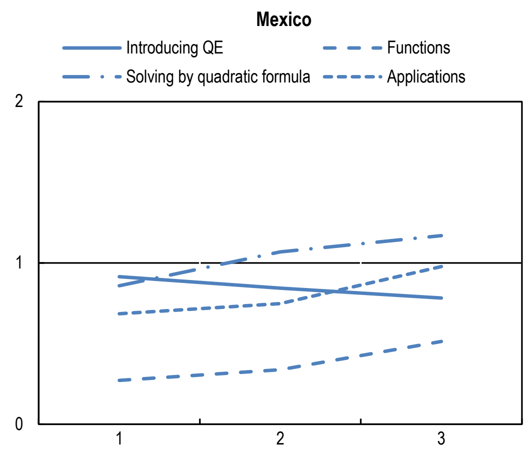
Figure 6.3 again illustrates differences regarding the use of graphical materials: quadratic functions were important elements of the focal unit in B-M-V (Chile), Colombia, Germany*, and to a lesser degree in England (UK) and Mexico, while they were almost non-existent in K-S-T (Japan) and Shanghai (China). Splitting the focal unit up into three phases helps understand differences between B-M-V (Chile), Colombia and Germany* as well: German and Colombian teachers typically introduced quadratic functions earlier, probably to motivate and support the understanding of equations, and faded them off later. Chilean teachers increased the use of quadratic functions from phase 1 until phase 3, probably to support the understanding of applications at the end of the unit.

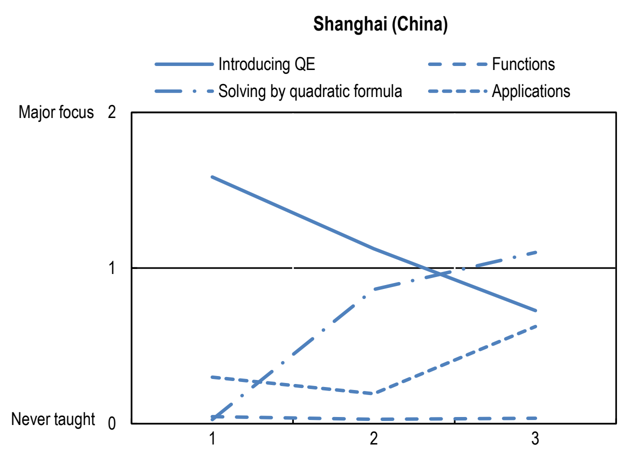
Figure 6.3. Sequencing for four subtopics as reported by teachers

OTL for four subtopics as reported by teachers in three consecutive phases of the unit on quadratic equations









Note: *Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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How the content taught relates to students' perceptions of teaching quality

Is what students are taught in the classroom related to their perceptions of teaching quality? Annex 6.A, Table 6.A.5 shows correlations at the classroom level between opportunities to learn (classified as in Figure 6.2: algebraic procedures, real life applications, quadratic functions, reasoning) and teaching quality, both reported by students. The following significant relationships exist:

- The quality of teacher explanations perceived by students – a measure of the quality of subject matter - was related to all opportunities to learn indices except “applications”: The more mathematical content students had seen, the more they agreed that their teacher explained *why* certain procedures worked. This suggests that, at least from the student’s perception, the quality of subject matter taught is related to content coverage.
- The opportunities indicator for “reasoning” showed an exceptionally strong relationship with “explaining procedures”, the correlation being significant in seven out of eight countries/economies. Also, opportunities for reasoning related to students’ self-reported “cognitive engagement” in five, and to “student participation in discourse” in four countries/economies. Thus, teachers who introduce reasoning tasks – reflecting on the type and number of solutions, over and above solving

quadratic equations procedurally – may succeed in challenging student thinking. At the same time, this opportunity to learn indicator also related to student-reported measures of social-emotional support and classroom management. Reasoning tasks seem to be a good indicator of high-quality mathematics teaching across domains.

- The opportunities indicator for "algebraic procedures" relates to measures of classroom management and social-emotional support: the more opportunities to learn algebraic procedures reported by students, the lower the level of disruptions, and the higher the level of teacher support indicated by them. This suggests that classroom climate might be better when procedures are taught, it is easier to teach procedures when classrooms are well managed and supported, and/or students tend to perceive an extensive coverage of algebraic procedures as well-managed and supportive teaching.

Curriculum reforms typically have the goal of raising opportunities to learn, particularly with regard to developing conceptual understanding in students. In turn, professional development targets the spreading of rich teaching practices that can be implemented with quality. Combining curriculum reforms with professional development seems to be a promising strategy for improving classroom teaching and learning. By unpacking opportunities to learn in different countries/economies, the Study can support the development of a theory of action for this strategy.

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Annex 6.A. Chapter tables

Annex Table 6.A.1. Total teaching time across the focal unit and duration of the unit as reported by teachers

	Total teaching time across the unit (in hours)		Mean duration of the focal unit (in days)[1]
	Mean	Standard deviation	
B-M-V (Chile)	13.88	6.79	39.36
Colombia	5.56	2.69	15.31
England (UK)	7.55	2.22	16.33
Germany*	13.07	3.24	46.16
K-S-T (Japan)	9.99	2.86	69.02
Madrid (Spain)[2]	2.19	1.89	3.34
Mexico	6.47	2.95	19.23
Shanghai (China)	6.09	1.28	14.55

Notes: 1. Number of days between start date and end date reported in the Teacher Log, including days without any mathematics lessons.
2. The fact that more than 50% of teachers in Madrid (Spain) filled out just one line in the Teacher log warrants severe concerns against OTL data from this sample. Therefore, data from Madrid (Spain) will not be interpreted in the present chapter.
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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Annex Table 6.A.2. Alignment between Teacher Log (vertical) and teaching material scores (horizontal) for content coverage

Correlations on teacher level, n = 622

	Completing the square	Factorising	Quadratic formula	Finding roots in functions
Completing the square	0.509***	0.140***	-0.145***	-0.032
Factorising	0.182***	0.221***	-0.180***	0.013
Quadratic formula	0.085*	-0.094**	0.174**	-0.100**
Graphical representation	-0.012	-0.159***	-0.009	0.431***

Source: OECD, Global Teaching InSights Database.

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Annex Table 6.A.3. OTL for algebraic procedures, real life applications, quadratic functions and reasoning about types of equations and solutions – measured in Teacher Logs and Student Questionnaires

	Algebraic procedures (Log)	Applications (Log)	Functions (Log)	Reasoning (Log)	Algebraic procedures (Students)	Applications (Students)	Functions (Students)	Reasoning (Students)
B-M-V (Chile)	7.44	3.02	3.50	3.92	0.93	0.47	0.79	0.91
Colombia	3.82	2.38	2.87	3.01	0.84	0.55	0.82	0.82
England (UK)	6.35	0.86	2.34	2.10	0.82	0.23	0.67	0.72
Germany*	8.86	3.53	5.64	3.84	0.94	0.53	0.89	0.70
K-S-T (Japan)	8.08	3.61	0.26	0.91	0.94	0.30	0.58	0.68
Madrid (Spain)	2.28	0.82	1.56	1.25	0.86	0.34	0.40	0.84
Mexico	5.70	3.08	2.02	3.16	0.86	0.72	0.84	0.88
Shanghai (China)	6.71	1.81	0.22	4.36	0.96	0.31	0.22	0.96

Notes: For a definition of the measures, see Technical Report, Chapter 17h (Teacher Logs) and Chapter 17c (Student Questionnaires).
Measures based on student reports are divided by the respective number of included items to allow for comparability across types of OTL.
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

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Annex Table 6.A.4. Alignment between scores on teaching materials, Teacher Log and Student Post-Questionnaires on four types of OTL

Correlations on teacher level, n = 586

	Algebraic procedures (Materials)	Algebraic procedures (Log)	Algebraic procedures (Students)	Applications (Materials)	Applications (Log)	Applications (Students)	Functions (Materials)	Functions (Log)	Functions (Students)	Reasoning (Materials)	Reasoning (Log)	Reasoning (Students)
Algebraic procedures (Materials)	1											
Algebraic procedures (Log)	0.128**	1										
Algebraic procedures (Students)	0.212**	0.292**	1									
Applications (Materials)	-0.053	0.064	0.095*	1								
Applications (Log)	-0.054	0.457**	0.193**	0.352**	1							
Applications (Students)	-0.401**	-0.076	-0.067	0.181**	0.228**	1						
Functions (Materials)	-0.146**	-0.009	-0.181**	0.059	0.019	0.161**	1					
Functions (Log)	-0.206**	0.243**	-0.062	0.087*	0.351**	0.258**	0.549**	1				
Functions (Students)	-0.284**	0.062	-0.157**	0.138**	0.200**	0.602**	0.424**	0.464**	1			
Reasoning (Materials)	0.121**	-0.115**	0.121**	-0.029	-0.066	0.019	-0.021	0.067	-0.178**	1		
Reasoning (Log)	-0.099*	0.384**	0.141**	-0.021	0.306**	0.100*	0.053	0.373**	0.014	0.314**	1	
Reasoning (Students)	-0.153**	-0.082*	0.376**	-0.027	-0.011	0.241**	-0.203**	-0.03	-0.076	0.391**	0.322**	1

Notes: For a definition of the measures, see Technical Report, Chapter 17h (Teacher Logs) and Chapter 17c (Student Questionnaires).
There is convergent validity for all four types of OTL across measures (dark blue).
Algebraic Procedures correlate with Reasoning, Applications correlate with Functions (light blue).
All measures from the Teacher Log are correlated, because they are defined as the weighted number of lessons covering the respective kind of content, thus confounded by the number of lessons (grey).
Most other correlations are either negative or low, showing discriminant validity.
* Correlation is significant at the 0.05 level (2-tailed). ** Correlation is significant at the 0.01 level (2-tailed).
Source: OECD, Global Teaching InSights Database.

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Annex Table 6.A.5. Relationship between OTL and teaching quality as perceived by students

Correlations on the classroom level

Correlated measures		Country							
OTL	Teaching Quality	B-M-V (Chile)	Colombia	England (UK)	Germany*	K-S-T (Japan)	Madrid (Spain)	Mexico	Shanghai (China)
Algebraic Procedures	Disruptions	0.371**	0.300**	0.262*	0.171	0.198	0.110	0.411**	0.272*
	Teacher Support for Learning	0.206*	0.244*	0.227*	0.069	0.271*	0.090	0.362**	0.212
	Teacher-Student-Relationship	0.191	0.183	0.256*	-0.081	0.264*	0.058	0.407**	0.200
	Explaining Procedures	0.254*	0.279*	0.271*	-0.078	0.293**	0.110	0.503**	0.276*
	Student Participation in Discourse	-0.068	0.161	0.180	-0.068	0.172	0.040	0.219*	0.156
	Self-reported Cognitive Engagement	0.166	0.135	0.235*	0.125	0.422**	0.149	0.232*	0.391**
Quadratic Functions	Disruptions	0.164	0.143	0.022	0.051	0.011	0.025	0.323**	-0.067
	Teacher Support for Learning	0.020	0.266*	-0.064	0.112	0.079	-0.017	0.273**	0.046
	Teacher-Student-Relationship	0.023	0.189	-0.009	0.103	0.017	-0.019	0.289**	0.139
	Explaining Procedures	0.258*	0.411**	0.045	0.099	0.352**	0.075	0.426**	0.221*
	Student Participation in Discourse	0.059	0.131	0.042	0.149	-0.001	0.082	0.241*	0.148
	Self-reported Cognitive Engagement	0.047	0.107	0.152	0.076	0.153	-0.009	0.287**	0.192
Reasoning	Disruptions	0.376**	0.199	0.386**	-0.053	0.054	0.052	0.422**	0.279**
	Teacher Support for Learning	0.266**	0.249*	0.280**	0.110	0.158	0.051	0.273**	0.264*
	Teacher-Student-Relationship	0.274**	0.225*	0.308**	0.000	0.248*	0.064	0.254**	0.200
	Explaining Procedures	0.354**	0.379**	0.356**	0.162	0.427**	0.261*	0.374**	0.264*
	Student Participation in Discourse	0.102	0.267*	0.212	0.041	0.220*	0.146	0.210*	0.237*
	Self-reported Cognitive Engagement	0.047	0.312**	0.346**	0.213	0.477**	0.164	0.260**	0.379**
Applications	Disruptions	-0.097	0.007	-0.118	-0.213	-0.221*	-0.042	0.059	-0.127
	Teacher Support for Learning	-0.072	0.098	-0.046	0.166	-0.070	-0.298*	0.118	-0.008
	Teacher-Student-Relationship	-0.059	0.145	-0.023	0.108	-0.169	-0.171	0.090	0.062
	Explaining Procedures	0.101	0.290**	-0.021	0.162	0.194	-0.096	0.204*	0.204

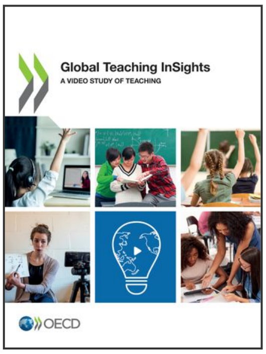
Correlated measures		Country							
OTL	Teaching Quality	B-M-V (Chile)	Colombia	England (UK)	Germany*	K-S-T (Japan)	Madrid (Spain)	Mexico	Shanghai (China)
	Student Participation in Discourse	0.155	0.318**	-0.001	0.196	-0.138	0.038	0.072	0.156
	Self-reported Cognitive Engagement	0.023	0.082	-0.042	0.084	-0.074	0.085	0.107	0.190

Notes: * Correlation is significant at the 0.05 level (2-tailed). ** Correlation is significant at the 0.01 level (2-tailed).
Rows highlighted in light grey: Weak relationship (Correlations are significant in 4 countries/economies).
Rows highlighted in dark grey: Strong relationship (Correlations are significant in the majority of countries/economies).
*Germany refers to a convenience sample of volunteer schools.
Source: OECD, Global Teaching InSights Database.

StatLink https://doi.org/10.1787/888934188101

Notes

[1] Germany* refers to a convenience sample of volunteer schools.



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